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A Two-sided Incentive Program for Coordinating the Influenza Vaccine Supply Chain

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Problem definition: The US influenza (flu) vaccine supply chain is decentralized and experiences frequent supply and demand mismatches caused by two key factors: (1) the vaccine production process (yield) is highly uncertain; and (2) individuals are self-interested and do not completely take into account positive and negative externalities that they impose on others. To improve matching of supply and demand, we counteract these factors by developing an ex-ante budget-neutral incentive program. ***Methodology:*** We model the flu vaccine supply chain as a decentralized system consisting of self-interested individuals on the demand side, and a profit-maximizing manufacturer with uncertain yield on the supply side. We use backward induction to characterize the subgame-perfect equilibrium of the sequential game that models the interactions between individuals and the manufacturer. ***Results:*** We develop a two-sided incentive program that proposes 'vaccination incentives' to be given to individuals on the demand side, and 'a menu of transfer payments' between the social planner and manufacturer on the supply side. When the realized vaccine supply is high (or low), our incentive program provides positive (negative) vaccination incentives for individuals to stimulate (or curb) the demand and eliminate positive (or negative) externalities by making vaccination more affordable (or costly). When social benefits from vaccination are significantly high, our incentive program uses a menu of transfer payments to penalize (or subsidize) the manufacturer for low (or high) yield realizations so that it produces the socially-optimal quantity. We show that our incentive program can attain the social optimum, maintain an ex-ante balanced budget (i.e., budget-neutral in expectation), and distribute the maximum social welfare between individuals and the manufacturer arbitrarily. ***Academic/practical relevance:*** We establish the sources of inefficiency in the flu vaccine supply chain. To eliminate the inefficiency, we develop a two-sided incentive program that policymakers can implement to finance vaccines under an ex-ante balanced budget. ***Managerial implications:*** Vaccination incentives to individuals can ensure their access to the vaccine, but they are not enough to entice the manufacturer to ensure vaccine availability. A menu of contracts contingent on realized yield provides necessary incentives to the manufacturer and assures the availability.

Key words: Influenza vaccine, contract, vaccination incentive, self-interested behavior, externality, random yield

1. Introduction

Vaccination is among the most cost-effective medical interventions against the influenza (flu) epidemic (Pauly 2005). However, the US flu vaccine supply chain is highly decentralized and is known to be inefficient due to frequent supply and demand mismatches (Government Accountability Office 2005; Institute of Medicine 2004). These mismatches are caused by two factors arising from the demand and supply sides.

On the demand side, individuals make their vaccination decisions in a self-interested[1] manner and do not account for *all* the 'externalities' (i.e., indirect benefits/costs) that they impose on others' well-being (Galvani et al. 2007; Shim et al. 2012). On the supply side, the manufacturer produces the flu vaccine through a long and uncertain production process (Gerdil 2003; Deo and Corbett 2009).

Various researchers (e.g., Brito et al. 1991) identified 'positive externalities' as the only source of inefficiency on the demand side by showing that not enough people are seeking vaccination because they are self-interested, so they do not consider all indirect benefits from their vaccination (e.g., protecting others due to herd immunity). To eliminate positive externalities and improve the demand-side efficiency, previous research proposed mechanisms/interventions to *increase the demand* (Mamani et al. 2012; Adida et al. 2013). It has also been shown that, in the presence of positive externalities, the manufacturer's lack of incentives causes inefficiency on the supply side and leads to a lower production quantity than that in the social optimum (Mamani et al. 2012; Adida et al. 2013). To elicit the manufacturer to produce more and improve supply-side efficiency, different interventions (e.g., whole-unit discount/cost-sharing contracts) are proposed in the literature (Chick et al. 2008; Adida et al. 2013).

In contrast to the suggestions proposed in the extant literature, the Centers for Disease Control and Prevention (CDC) does not always seek to increase the demand. Specifically, when the supply is limited, it aims to *curb the demand* by recommending that individuals in low-risk groups do not seek vaccination and that they leave the vaccine to high-risk individuals (Meltzer et al. 1999; Galvani et al. 2007).[2] In line with this CDC practice, Arifoğlu et al. (2012) find that mechanisms that focus only to increase the demand, as suggested in the literature, can make the flu vaccine supply chain even more inefficient. This is because, when the supply is limited, the equilibrium demand is higher than the socially-optimal demand and the source of inefficiency on the demand side is 'negative externalities' (e.g., each individual seeking vaccination reduces vaccine availability and increases infection risk for all other individuals). Arifoğlu et al. (2012) also show that, in the presence of negative externalities, the manufacturer can have higher incentives and thus over-produce, especially when the social benefits from vaccination are low. Therefore, mechanisms that intervene only on the supply side to entice the manufacturer to produce more can actually make the flu vaccine supply chain become more inefficient.

The above observations reveal that prior research tends to focus only on developing mechanisms to increase demand or supply in the presence of positive externalities, and there is a need to develop a mechanism

[1] When making their vaccination decisions, some individuals might have altruistic motivations; however, self-interest is more influential and accounts for 75% of vaccination decisions (Galvani et al. 2007; Shim et al. 2012; Hahne 2013).

[2] Flu vaccine cannot be efficiently allocated in practice due to problems in its production and distribution. Priority groups are not mandatory and only a recommendation by CDC. CDC data shows that individuals with high infection cost are not necessarily more stubborn in their search for the flu vaccine. For example, consistently in the past 20 flu seasons, there are always some low-risk individuals (10-40%) who got vaccinated even though a significant number of high-risk individuals (40-80%) did not receive the vaccine (Fiore et al. 2010; CDC 2017, 2018).

that can boost/curb demand and increase/decrease the production under appropriate conditions. At the same time, it is not clear if there exists an incentive mechanism that can coordinate both supply and demand sides of the decentralized flu vaccine supply chain in the presence of positive and negative externalities. Further, even if such coordinating mechanisms exist, it is unclear if any of them are budget-neutral so that no external funding is required. We examine these open research questions in this paper.

In this paper, we consider a decentralized flu vaccine supply chain over a single vaccination season that consists of self-interested individuals and a profit-maximizing manufacturer with uncertain production yield. First, the manufacturer determines its planned production quantity (i.e., chicken eggs to be fertilized); however, due to the inherent yield uncertainty, only a random fraction of this quantity (i.e., vaccine doses) is brought to the market. Then, upon observing the realized vaccine supply, each self-interested individual decides whether to seek vaccination or not by comparing the infection cost (e.g., sickness, healthcare costs, lost wages, etc.) and the vaccination cost (e.g., side effects, vaccine price, administration costs, etc.).[3] When demand exceeds supply, we consider random (or proportional) rationing and assume that the available vaccine is allocated to individuals with equal probability.

By comparing the decentralized supply chain and the social optimum (or the first best), we find that, when the realized vaccine supply is abundant, fewer individuals than the socially-optimal demand will seek vaccination due to *positive externalities*. This behavior will increase the infection risk and decrease the social welfare. On the other hand, when the vaccine supply is limited, more individuals than the socially-optimal demand will seek vaccination due to *negative externalities*. As the limited vaccine supply is rationed, some individuals with higher infection costs may not receive the vaccine and the social welfare decreases. Lastly, on the supply side, depending on infection and vaccination costs, the manufacturer may over- or under-produce, even when there is no demand-side inefficiency. Based on these three findings, we develop a two-sided incentive program to coordinate the supply and demand sides of the flu vaccine supply chain.

First, to counteract the inefficiency on the demand side, we offer a 'vaccination incentive' to individuals that can be negative or positive depending on the realized vaccine supply. This incentive is negative and makes vaccination more costly when the realized vaccine supply is low. By doing so, it aims to curb the demand and eliminate the negative externalities that cause demand-side inefficiency when vaccine supply is limited. In doing so, it can help to ensure that individuals with high infection costs can get vaccinated. Such negative vaccination incentives (especially, for low-risk individuals) are observed in practice. For example, general practitioners (GPs) or healthcare providers are either not subsidized at all or subsidized only for high-risk individuals to increase effective vaccination costs and thereby effectively create negative vaccination incentives, either for all individuals or for only low-risk individuals (Institute of Medicine 2004;

[3] We allow the flu vaccine to be imperfect so that vaccinated and unvaccinated individuals can be infected with different probabilities that depend on vaccination coverage. Because vaccination reduces the infection risk, the probability of infection is lower for vaccinated individuals.

Spencer and Kennedy 2007; Sinha et al. 2018; NHS 2020). On the contrary, when the realized vaccine supply is high, the vaccination incentive that we propose is positive and makes vaccination less costly. This will boost the demand and eliminate the positive externalities which cause demand-side inefficiency for high realizations of the vaccine production. Such positive vaccination incentives are used in practice by making vaccination more accessible (e.g., allowing pharmacies to administer the vaccine) (Sinha et al. 2018); subsidizing GPs for all vaccine administrations (Spencer and Kennedy 2007; Nugent and Knaul 2006); and covering vaccination costs for everyone through vouchers, and public/private insurance (Institute of Medicine 2004; Sinha et al. 2018).

Second, to counteract supply-side inefficiency and align the manufacturer's incentives with the production incentives in the social optimum, we propose a contract that is offered by the social planner to the manufacturer ex ante before the uncertain yield is realized. This contract takes the form of a menu of transfer payments that are contingent on the realized vaccine supply. Specifically, under the contract, when social benefits are significantly high (e.g., high infection costs and low vaccination costs), the manufacturer is penalized for low realized vaccine supply and is subsidized for high realized vaccine supply. By penalizing or rewarding the manufacturer ex post, our contract elicits the manufacturer to choose the socially-optimal production quantity ex ante. Similar payment schemes that depend on the outcome of an uncertain event are commonly used in healthcare, e.g., Payment-by-Result (PbR) and Risk Sharing (RS) contracts used in Italy to pay pharmaceutical firms, and Hospital Readmissions Reduction Program (HRRP) used to reimburse hospitals in the USA (So and Tang 2000; Keohane and Petrie 2017; Arifoğlu et al. 2020).

Through our analysis, we show that the two-sided incentive program we propose can be ex-ante budget-neutral in the sense that the net cost of the program is zero in expectation. This budget-neutral two-sided incentive program also provides the flexibility to the social planner to arbitrarily distribute the welfare between individuals and the manufacturer.

Finally, through an extensive numerical study, we examine the impact of vaccine efficacy on the demand- and supply-side inefficiency and evaluate the performance of our two-sided incentive program. We find that the manufacturer can be better off with imperfect vaccines and thus may have some financial incentives to not develop a more effective vaccine. Interestingly, we also find that a more effective vaccine can make the vaccine supply chain more inefficient. In addition, the numerical study reveals that our incentive program can reduce the social costs by more than 70%. It also indicates that 'one-sided' mechanisms examined in the literature (that focus on demand (or supply) side by ignoring uncertain yield (or individuals' self-interest)) can increase social costs, and our two-sided incentive program outperforms them, especially when the yield uncertainty is high and/or infection costs among the population are low.

This paper is organized as follows. We review the related literature and list our contributions in the following section. In §3, we present the model and assumptions. We analyze the social planner's problem in a

centralized system in §4. Then, in §5, we study the decentralized supply chain with self-interested individuals and the manufacturer. In §6, we develop a two-sided incentive program to coordinate the decentralized supply chain. We present our numerical study in §7 and conclude the paper in §8. All proofs are presented in Appendix B in the online supplement.

2. Related Literature and Contributions

Previous research has studied the role of consumption externalities on individuals' vaccination decisions. The epidemiology literature examines the interaction between individuals' self-interested vaccination decision and the disease progression (Bauch and Earn 2004; Reluga et al. 2006; Galvani et al. 2007). The health economics literature studies the role of externality arising from vaccination and finds that the demand for vaccines is lower than the socially-optimal demand (Brito et al. 1991; Geoffard and Philipson 1997; Philipson 2000). Brito et al. (1991) propose a tax/subsidy payment, which subsidizes vaccinated individuals and taxes unvaccinated individuals, to induce the demand, and Geoffard and Philipson (1997) analyze the role of subsidies in eradicating infectious diseases. We introduce the uncertain yield to the health economics and epidemiology literatures and show that a transfer payment that is contingent on the realized yield is necessary on the supply side to align the manufacturer's incentives, and vaccination incentives given to individuals should depend on the supply-side factors such as the wholesale price and uncertain yield.

In the context of flu vaccine supply chain, several studies examine the coordination issues between firms (e.g., manufacturer, healthcare provider, etc.) in the presence of uncertain yield (Cho and Zhao 2018). Assuming that the government has full control of the demand, Chick et al. (2008) propose a whole-unit discount/cost-sharing contract under which the social planner shares the manufacturer's production cost and induces the production quantity. Chick et al. (2017) extend Chick et al. (2008) by assuming that the manufacturer has a second opportunity and can exert a more costly production effort to fulfill the demand which cannot be satisfied due to the uncertain yield. They show that the manufacturer commands information rent due to information asymmetry and they propose a menu of contracts to attain the social optimum. Dai et al. (2016) study on-time delivery of the flu vaccine and study the coordination between the manufacturer and healthcare provider. They develop contracts that ensure on-time delivery of the vaccine. In our study, we incorporate a new and important element that involves individuals' self-interested behavior, and we find that the coordinating mechanism must entail a vaccination incentive for individuals and a transfer payment for the manufacturer that is contingent on the realized vaccine supply.

Yamin and Gavious (2013) focus only on self-interested individuals and develop a game-theoretical epidemiology model to find the optimal vaccination incentive (subsidy). Mamani et al. (2012) study the coordination issues in vaccine supply chains with self-interested individuals and multiple manufacturers, and they propose subsidy payments to align individuals' incentives with the social optimum. Unlike Mamani et al. (2012) and Yamin and Gavious (2013), we consider the uncertain yield in this paper and show that positive vaccination incentives (subsidy payments) alone cannot eliminate the inefficiency.

By considering both self-interested individuals and uncertain yield, Adida et al. (2013) are the first to develop a two-sided mechanism that proposes a subsidy payment to vaccinated individuals on the demand side and a cost-sharing contract on the supply side. Our paper complements Adida et al. (2013) but is fundamentally different in the following aspects:

• First, in contrast to assuming that all individuals make their vaccination decisions by ignoring the uncertain yield and preorder the vaccine *before* the vaccine supply is realized as considered in Adida et al. (2013),[4] we assume that individuals make their vaccination decisions *after* observing the realized vaccine supply so that yield uncertainty affects the demand. This modeling choice enables us to examine the impact of uncertain yield (supply side) on individuals' rational behavior (demand side) and capture a phenomenon that occurs in practice, i.e., individuals rush to get the vaccine when the realized vaccine supply turns out to be low (Cho and Tang 2013).

• Second, unlike Adida et al. (2013) who assume *efficient* allocation, we consider the inefficiency in the allocation of the flu vaccine (see footnote 2) and assume that, in the event of supply shortages, available vaccine is randomly allocated to individuals who seek vaccination. This modeling choice enables us to show how negative externalities create inefficiency on the demand side. To counteract negative externalities, we develop a vaccination incentive that lowers individuals' utility from vaccination and curbs excess demand when the realized vaccine supply is limited.[5]

• Third, unlike the incentive mechanism in Adida et al. (2013), our two-sided incentive program proposes a menu of contracts between the social planner and the manufacturer that depends on the realized yield to coordinate the supply side and it offers two extra values: (1) it is ex-ante budget neutral and balances out the budget in the long run; and (2) it provides the social planner with flexibility to arbitrarily distribute the welfare between individuals and the manufacturer.

Arifoğlu et al. (2012) identify sources of inefficiencies on demand and supply sides by considering a setting similar to our paper. They show that, when individuals take uncertain yield into account, in addition to positive externalities, negative externalities arise and also lead to inefficiency on the demand side. They assume that vaccination provides perfect protection against infection and do not propose a specific mechanism to coordinate the flu vaccine supply chain. We generalize the main results in Arifoğlu et al. (2012) to the case with imperfect vaccines and find that: (i) the manufacturer might have some financial incentives to

[4] Healthcare providers (not individuals) preorder from the manufacturer by anticipating the rational behavior of individuals (CDC 2005; Rall 2009; American Academy of Pediatrics 2019). Individuals decide whether to get vaccinated or not during the vaccination season after observing the realized vaccine supply.

[5] We also developed an *alternative two-sided mechanism* based on the assumptions in Adida et al. (2013): (1) individuals ignore uncertain yield and order the vaccine in advance before observing the realized vaccine supply; and (2) in the event of limited supply, an individual with high infection cost will always receive the vaccine before any individuals with lower infection costs, i.e., efficient allocation. (The analysis of this alternative mechanism is not presented for brevity.) We found that incentivizing individuals through vaccination subsidies only, as suggested by Adida et al. (2013), can adversely impact on the negative externalities and the alternative two-sided mechanism proposed by Adida et al. (2013) can exacerbate the demand-side inefficiency.

develop a less effective vaccine; and (ii) higher vaccine efficacy can increase total supply-chain inefficiency. More importantly, while the coordination issue is not considered in Arifoğlu et al. (2012), we develop a two-sided incentive program that can coordinate the flu vaccine supply chain by eliminating demand- and supply-side inefficiency.

In summary, our paper contributes to the extant literature by: (i) capturing the interaction between individuals' self-interested behavior and manufacturer's uncertain production yield in practice; and (ii) developing a new two-sided incentive program that coordinates a decentralized flu vaccine supply chain in the presence of positive/negative externalities (demand side) and uncertain yield (supply side). Our incentive program is ex-ante budget-neutral and provides the social planner with the flexibility to arbitrarily distribute welfare between individuals and the manufacturer.

3. Model Fundamentals

We now describe our model and notation (see Table 1). We consider a flu vaccine supply chain comprising a welfare-maximizing social planner, a profit-maximizing flu vaccine manufacturer, and a continuum of individuals who make self-interested vaccination decisions. Flu vaccine supply chain operations are known to be challenging because of the inherent supply uncertainties (Cho 2010) and the inherent demand complexities due to the 'externalities' caused by limited vaccine supply and flu transmission among individuals (Arifoğlu et al. 2012).

3.1. Endogenous Supply of Flu Vaccine under Uncertainty: $\mathbf{Q_r}$

The social planner (e.g., government) negotiates with the manufacturer in advance regarding the wholesale price w that the manufacturer receives for each unit of vaccine dose sold. Given the pre-negotiated wholesale price w and the processing cost of each planned production unit c, the manufacturer needs to decide on the 'planned input production quantity' Q (measured in terms of the number of chicken eggs fertilized). However, due to the inherent uncertainty in the production process of the flu vaccine (Chick et al. 2008; Deo and Corbett 2009; Jansen and Ozaltın 2017),[6] the 'actual output production quantity' is $Q_r = Y \cdot Q$ (measured in terms of the number of vaccine doses), where the uncertain yield $Y \in [0, \infty)$ is a random variable that follows a distribution $F(\cdot)$ with mean μ and standard deviation σ.[7] To ensure that the manufacturer has the incentive to produce the vaccine, we assume that $w\mu > c$.

[6] The majority (over 97%) of the flu vaccine in the USA is produced by injecting virus strains into chicken eggs through a very complex process (Palese 2006; Government Accountability Office 2008). Growth characteristics of virus strains inside chicken eggs are highly variable, which makes the uncertain yield an important challenge in the flu vaccine production.

[7] More than one vaccine dose can be obtained from a chicken egg so that the proportional yield Y can be greater than 1 (Hartgroves 2009; Palese 2006; Chick et al. 2008; Deo and Corbett 2009; Arifoğlu et al. 2012). Also, our analysis continues to hold when the proportional yield has a finite upper bound (i.e., $Y \in [0, \overline{Y}]$ where $\overline{Y} < \infty$).

Table 1 Notation in the Analytical Model

		Exogenous Variables
c	:	Unit production cost per planned production quantity
p	:	All additional vaccination costs beyond wholesale price w that a vaccinated individual incurs (e.g., vaccine administration, side effects, etc.)
V_H	:	Utility of a healthy individual
w	:	Manufacturer's wholesale price
x	:	Infection cost of an individual
$\bar{x}$	:	Maximum infection cost within the population
Y	:	Random proportional yield with realization y
μ	:	Mean of the random proportional yield Y
σ	:	Standard deviation of the random proportional yield Y
		Endogenous Variables
K	:	A constant parameter of the transfer payment T under the incentive program
Q	:	Planned input production quantity (measured in number of chicken eggs fertilized)
r	:	Ex-post vaccination incentive under the incentive program
T	:	Ex-post transfer payment to the manufacturer under the incentive program
t	:	Ex-ante tax collected from all individuals under the incentive program
		Probability Distributions
F	:	Distribution of proportional yield Y with support $[0,\infty)$, mean μ and standard deviation σ
G	:	Distribution of infection cost x across the population with support $[0,\bar{x}]$ ($\overline{G}=1-G$ and G is uniform distribution, i.e., $G(x)=x/\bar{x}$)
		Definitions
B	:=	Marginal social benefit from vaccinating an individual with a certain infection cost
h	:=	Ex-post fraction of vaccinated people
h_{zr}	:=	Zero-risk vaccination fraction above which the average number of people infected by an infected individual is less than 1
$h^{(i)}$	:=	Ex-post fraction of vaccinated people in the social optimum (in decentralized supply chain and under the incentive program, respectively) if $i=c$ ($i=d$ and $i=I$, respectively)
Q_r	:=	Realized production quantity (measured in number of vaccine doses), which is a random proportion of the planned production quantity, i.e., $Q_r=Y\cdot Q$
$Q^{(i)}$	:=	Optimal planned input production quantity in the social optimum (in decentralized supply chain and under the incentive program, respectively) if $i=c$ ($i=d$ and $i=I$, respectively)
$S^{(I)}$	:=	Ex-post total cost borne by the social planner due to the incentive program I
U_i	:=	Net utility of an unvaccinated (vaccinated) individual if $i=u$ ($i=v$)
u	:=	The probability of infection for an unvaccinated individual
v	:=	The probability of infection for a vaccinated individual
W	:=	Ex-post social welfare
$W^{(i)}$	:=	Ex-ante expected social welfare in the social optimum (in decentralized supply chain and under the incentive program, respectively) if $i=c$ ($i=d$ and $i=I$, respectively)
$x^{(i)}$	:=	Threshold infection cost (such that only individuals with infection costs higher than that threshold seek vaccination) in the social optimum (in decentralized supply chain and under the incentive program, respectively) if $i=c$ ($i=d$ and $i=I$, respectively)
ϕ	:=	Ex-post vaccine allocation probability or fill rate
π	:=	Manufacturer's expected profit
$\pi^{(i)}$	:=	Manufacturer's expected profit in the social optimum (in decentralized supply chain and under the incentive program, respectively) if $i=c$ ($i=d$ and $i=I$, respectively)

3.2. Sequence of Events

The sequence of events is as follows. First, before the vaccination season, the manufacturer determines its input production quantity Q. Second, at the beginning of the vaccination season, the actual production quantity $Q_r = yQ$ is realized and commonly observed,[8] where y is realized yield. Third, upon observing Q_r, each individual decides whether or not to seek vaccination by considering the underlying trade-offs that we shall explain next. (All unused vaccine will be discarded with zero value due to antigenic drift.)

3.3. Endogenous Demand of Flu Vaccine: $\overline{\mathbf{G}}(\hat{\mathbf{x}})$

There is a continuum of self-interested individuals in a population with a size normalized to 1. To capture heterogeneity across different individuals (Pauly 2005), we assume that each individual enjoys a utility V_H when 'healthy', but incurs an infection cost x when 'infected'.[9] For tractability, we assume that infection costs among individuals are uniformly distributed between 0 and $\bar{x}$, i.e., $x \sim U[0, \bar{x}]$.[10] For ease of exposition, we shall use $G(\cdot)$ to denote the distribution of $x \sim U[0, \bar{x}]$ so that $G(x) = x/\bar{x}$. We assume that an individual's infection cost x is private information but its distribution $G(\cdot)$ is common knowledge.[11]

In addition, we let p denote all additional costs on top of the wholesale price w that an individual incurs due to vaccination (e.g., vaccine administration, side effects, etc.), that is, $w + p$ is the total cost of receiving the vaccine. For tractability, we assume that p is the same for all individuals and is common knowledge. To ease our exposition and to focus on the development of our two-sided incentive program for coordinating a decentralized flu vaccine supply chain, we assume that available vaccine is allocated to individuals seeking vaccination with equal probability (i.e., random rationing). Since random rationing is a stylized representation of vaccine allocation in practice, we also consider the vaccine allocation based on priority groups (e.g., elderly, infants, etc.) and show that all our key results continue to hold. See §6.4 for our discussion on priority groups.

[8] Supply projections for a flu season are calculated based on manufacturer reports and publicized well in advance via CDC's website and the media (Mangan 2016; Searing 2018; CDC 2019). These projections are in line with weekly lot distribution reports announced by the CDC and US Food and Drug Administration (FDA) throughout the vaccination season (FDA 2018; CDC 2019). Thus, in line with our assumption, information about the realized production quantity Q_r is readily available and observed by individuals before the vaccination season starts.

[9] An infected individual may end up in one of four different cases (i.e., no medical care sought, outpatient visit, hospitalization, and death) with some probability. Hence, x represents the expectation of all (direct and indirect) costs that an infected individual incurs in these cases (Meltzer et al. 1999).

[10] This assumption is sufficient to guarantee the uniqueness of the solutions in the decentralized supply chain and social optimum. We numerically tested our results for several combinations of parameter values from practice (see Table 2 in §7) and observed from all numerical examples that our results hold true even when x has exponential distribution.

[11] The distribution of infection costs $G(\cdot)$ may depend on the severity of the flu epidemic or the virulence of flu virus strains. Our assumption that $G(\cdot)$ is common-knowledge implicitly assumes that the virulence of flu virus strains is known when the production starts (i.e., six to nine months before the flu season). This is consistent with research showing that the severity of the flu epidemic can be predicted accurately before the flu season starts by tracking the global circulation of virus strains and/or changes in the structure of hemagglutinin protein in the flu virus (Russell et al. 2008; Wolf et al. 2010).

3.3.1. Infection probabilities $\mathbf{u}(.)$ and $\mathbf{v}(.)$. To capture the fact that the flu virus may be transmitted among individuals through different means, including direct contact, we use a Susceptible-Infected-Recovered (SIR) epidemic model to obtain the infection probabilities that depend on the 'vaccinated fraction of the population' $h \in [0,1]$ (Bauch and Earn 2004; Mamani et al. 2012). (We shall define $h(.)$ more formally later.) Also, because the flu virus mutates and changes its structure frequently (antigenic drift), vaccination does not stop individuals from getting infected (Anderson and May 1991; Cho 2010; Mamani et al. 2012). Hence, for any given vaccinated fraction of population $h \in [0,1]$, there is a probability $u(h)$ for an unvaccinated individual to get infected (by the end of the flu season) and a probability $v(h)$ for a vaccinated individual to get infected, where $u(h) \geq v(h)$. We assume that infection probabilities $u(h)$ and $v(h)$ are common knowledge and continuous in h.

We let u' and v', respectively, be the first derivatives of u and v with respect to their arguments, and define the 'zero-risk vaccination fraction' h_{zr} as:

$$h_{zr} = \begin{cases} \inf\{h \in [0,1] | (1-h)u(h) + hv(h) = 0\}, & \text{if } v(1) = 0, \\ 1, & \text{if } v(1) > 0. \end{cases} \tag{1}$$

The zero-risk vaccination fraction h_{zr} is also known in epidemiology as the 'critical vaccination fraction' that reduces the average number of individuals infected by an infected person below 1 so that there is no infection risk for (vaccinated or unvaccinated) individuals, i.e., by (1), $u(h) = v(h) = 0$ for $h \geq h_{zr}$ (Hill and Longini 2003). For tractability, we shall assume that the infection probabilities $u(h)$ and $v(h)$ satisfy the technical conditions in Assumption 1.

ASSUMPTION 1. *(i) The individual with the largest infection cost $\bar{x}$ will seek vaccination when nobody is vaccinated, i.e., $\bar{x} \cdot u(0) \geq w + p + \bar{x} \cdot v(0)$.*

(ii) Infection probabilities $u(h)$ and $v(h)$ satisfy: $u'(h) < 0$ and $v'(h) \leq 0$ for $h \leq h_{zr}$.

(iii) Each additional vaccination leads to a smaller decrease in expected infection risk across the population, i.e., $(1-h)u(h) + hv(h)$ is convex in h for $h \leq h_{zr}$.

(iv) The gap between the expected infection risk across the population and the infection risk of a vaccinated individual is decreasing as more people are vaccinated, i.e., $(1-h)(u(h) - v(h))$ decreasing in h for $h \leq h_{zr}$.

(v) Each additional vaccination leads to a smaller marginal decrease in the total number of infections that vaccination causes relative to the average, i.e., $h[((1-h)u(h) + hv(h)) - v(h)]$ is concave in h for $h \leq h_{zr}$.

Conditions in Assumption 1 are quite general, and in §7, we shall present specific functional forms for $u(h)$ and $v(h)$ (e.g., considered by Mamani et al. 2012) that satisfy them. Condition (i) guarantees that some individuals will seek vaccination in the decentralized supply chain. Conditions (ii) and (iv) (or conditions

(iii) and (v)) guarantee the uniqueness of the solution in the decentralized supply chain (or social optimum). By conditions (ii) and (iii), the expected infection risk is convex and strictly decreasing for $h < h_{zr}$.[12]

3.3.2. Vaccinated fraction $\mathbf{h}(.)$, and vaccine allocation probability $\phi(.)$. Given the realized vaccine supply Q_r and any threshold $\hat{x}$ such that only individuals with infection cost $x > \hat{x}$ seek vaccination so that the 'vaccine demand' is $\overline{G}(\hat{x}) \equiv 1 - G(\hat{x}) = 1 - \frac{\hat{x}}{\bar{x}}$ (such a threshold always exists, see §5.1), the ex-post vaccinated fraction or 'sales' $h(\hat{x}, Q_r)$ is:

$$h(\hat{x}, Q_r) = \min\left\{\overline{G}(\hat{x}), Q_r\right\}. \tag{2}$$

Also, since we assume random rationing, the (ex-post) vaccine allocation probability or 'fill rate' $\phi(\hat{x}, Q_r)$ can be expressed as follows:

$$\phi(\hat{x}, Q_r) = \frac{h(\hat{x}, Q_r)}{\overline{G}(\hat{x})} = \min\left\{1, \frac{Q_r}{\overline{G}(\hat{x})}\right\}. \tag{3}$$

3.4. Ex-post Social Welfare $\mathbf{W}(\hat{\mathbf{x}}, \mathbf{Q_r})$

For any given realized vaccine supply Q_r and the threshold $\hat{x}$, the ex-post social welfare $W(\hat{x}, Q_r)$ (i.e., total utilities of all individuals (or total individual surplus) plus ex-post revenue from vaccine sales) satisfies:

$$\begin{aligned} W(\hat{x}, Q_r) &= V_H - ph - \int_0^{\hat{x}} [u(h) \cdot x] d(G(x)) - \int_{\hat{x}}^{\bar{x}} [\phi \cdot v(h) \cdot x + (1-\phi) \cdot u(h) \cdot x] d(G(x)) \\ &= V_H - ph - \frac{\bar{x}}{2} G^2(\hat{x})\, u(h) - \frac{\bar{x}}{2}\left(1 - G^2(\hat{x})\right)[\phi \cdot v(h) + (1-\phi) \cdot u(h)], \end{aligned} \tag{4}$$

where $h(.)$ and $\phi(.)$ are functions of $(\hat{x}, Q_r)$ as given in (2) and (3), respectively. In the first line of (4), the first term represents the total utility when all individuals are healthy; the second term represents total net vaccination costs to society (because the actual sales of vaccine is equal to h); the third term is total utility loss of infected individuals who do not seek vaccination (i.e., those individuals with infection cost $x \leq \hat{x}$); and the last term is total utility loss of infected individuals who seek vaccination (by accounting for the vaccine availability probability ϕ). Because $G(\hat{x}) = \hat{x}/\bar{x}$, we obtain the second line of (4).

4. Socially-Optimal Decisions in a Centralized System

We first determine a benchmark solution that is based on the socially-optimal decisions in a centralized system, where the social planner aims to maximize the social welfare by setting manufacturer's planned production quantity to $Q^{(c)}$ and 'dictating' that all individuals should follow a particular threshold policy[13]

[12] Using the SIR epidemic model, Chick et al. (2008) show that this is a valid assumption for a wide range of parameter values of flu epidemic and its vaccine. We observe assumptions that are similar to conditions (iv) and (v) in the literature (e.g., see Mamani et al. 2012).

[13] In the social optimum, the society incurs higher infection costs in cases when individuals with higher infection cost are infected. Therefore, the social planner never vaccinates an individual with a lower infection cost while another individual with a higher infection cost is not vaccinated. This indicates that there is a threshold infection cost $x^{(c)}(Q_r)$ such that only individuals with $x > x^{(c)}(Q_r)$ will seek vaccination.

$x^{(c)}(Q_r)$. (Throughout this paper, we use superscript (c) for the social optimum and superscript (d) for the decentralized case.) By considering the sequence of events as described in §3.2, we now determine the socially-optimal production quantity and demand (i.e., $Q^{(c)}$ and $\overline{G}\left(x^{(c)}(Q_r)\right)$) via backward induction.

4.1. Socially-Optimal Demand: $\overline{\mathbf{G}}\left(\mathbf{x^{(c)}(Q_r)}\right)$

In the centralized system, the social planner chooses the threshold $x^{(c)}(Q_r)$ that maximizes the ex-post social welfare given in (4), i.e., $x^{(c)}(Q_r) = \arg\max_{\hat{x}} W(\hat{x}, Q_r)$. To characterize the threshold $x^{(c)}(Q_r)$, we let x_{zr} be the threshold infection cost that corresponds to the zero-risk vaccination fraction h_{zr} given by (1) so that:

$$\overline{G}\left(x_{zr}\right) = h_{zr}. \tag{5}$$

Also, we define $B(\hat{x})$, the marginal social benefit from not vaccinating the individual with infection cost $\hat{x}$, as follows:

$$\begin{aligned} B\left(\hat{x}\right) &\equiv \frac{dW(\hat{x}, Q_r)}{d\hat{x}} \\ &= \frac{p}{\bar{x}} + \frac{1}{2}\left[G^2\left(\hat{x}\right)u'(\overline{G}(\hat{x})) + \left(1 - G^2\left(\hat{x}\right)\right)v'(\overline{G}(\hat{x}))\right] - G\left(\hat{x}\right)\left(u(\overline{G}(\hat{x})) - v(\overline{G}(\hat{x}))\right). \end{aligned} \tag{6}$$

Using the above definitions, Proposition 1 characterizes the threshold $x^{(c)}(Q_r)$ and the socially-optimal demand.

PROPOSITION 1. *For any realized vaccine production quantity $Q_r \geq 0$, there exists a unique socially-optimal threshold policy $x^{(c)}(Q_r) \in [x_{zr}, \bar{x})$ so that its corresponding ex-post vaccination demand (or equivalently, the ex-post vaccinated fraction) satisfies:*

$$\overline{G}\left(x^{(c)}(Q_r)\right) \equiv h^{(c)}(Q_r) = \min\left\{\overline{G}\left(x_1^c\right), Q_r\right\}, \tag{7}$$

where $x_1^c \in [x_{zr}, \bar{x})$

$$x_1^c = \begin{cases} x_{zr}, \text{ if } \lim_{x \to x_{zr}^+} B\left(x\right) \leq 0, \\ \check{x}, \quad \text{if } \lim_{x \to x_{zr}^+} B\left(x\right) > 0, \end{cases} \tag{8}$$

and $\check{x} \in (x_{zr}, \bar{x})$ *satisfies* $B\left(\check{x}\right) = 0$

Proposition 1 reveals that when the realized vaccine supply is abundant (i.e, $Q_r \geq \overline{G}(x_1^c)$), some vaccines will not be used and the social planner will vaccinate just enough individuals such that ex-post social benefits are maximized (i.e., socially-optimal demand is equal to $\overline{G}(x_1^c)$). However, when the realized vaccine supply is limited (i.e, $Q_r < \overline{G}(x_1^c)$), the socially-optimal demand is always equal to the available supply and no vaccine is unused (i.e., $\overline{G}(x^{(c)}(Q_r)) = Q_r$).

4.2. Socially-Optimal Production Quantity: $\mathbf{Q^{(c)}}$

Given the ex-post socially-optimal demand $\overline{G}\left(x^{(c)}(Q_r)\right)$ as stated in Proposition 1, we now solve the social planner's problem in the first stage. The social planner chooses its production quantity so as to maximize the ex-ante expected social welfare. Hence, the social planner solves:

$$W^{(c)} = \max_{Q}\{W^{(c)}(Q) \equiv \mathbb{E}_Y[W(x^{(c)}(YQ), YQ)] - cQ\}, \tag{9}$$

where the (ex-post) social welfare function W and the threshold $x^{(c)}(Q_r)$ are given by (4) and (7), respectively. Proposition 2 characterizes the socially-optimal planned production quantity $Q^{(c)}$.

PROPOSITION 2. *The socially-optimal planned production quantity $Q^{(c)}$ satisfies:*

$$\int_0^{\frac{\overline{G}\left(x_1^c\right)}{Q^{(c)}}} \bar{x}[-B\left(x_2^c\left(yQ^{(c)}\right)\right)] \cdot y dF(y) = c, \tag{10}$$

where $B(\cdot)$ and x_1^c, respectively, satisfy (6) and (8), and, for $Q_r > 0$, $x_2^c(Q_r) \in [0, \bar{x})$ is given by:

$$x_2^c(Q_r) = \overline{G}^{-1}\left(\min\{1, Q_r\}\right). \tag{11}$$

5. Decentralized Supply Chain

We now examine the decentralized supply chain in which individuals make their own vaccination decisions and the manufacturer determines its own planned production quantity. By considering the sequence of events as described in §3.2, we characterize the equilibrium of the decentralized supply chain via backward induction.

5.1. Decentralized Demand: $\overline{\mathbf{G}}\left(\mathbf{x^{(d)}}\left(\mathbf{Q_r}\right)\right)$

To begin, we first determine an individual's vaccination decision after observing the realized vaccine production quantity Q_r. An individual with infection cost x will obtain an expected utility equal to $U_u(x) = V_H - u(h) \cdot x$ if he chooses not to seek vaccination. However, if he chooses to seek vaccination, his expected utility is equal to $U_v(x) = \phi \cdot [V_H - (w+p) - v(h) \cdot x] + (1-\phi) \cdot (V_H - u(h) \cdot x)$. Hence, an individual with infection cost x will seek vaccination if and only if $U_v(x) > U_u(x)$ (or equivalently, when $x(u(h) - v(h)) > w + p$). This condition yields:

LEMMA 1. *In an ex-post equilibrium in the decentralized supply chain, given realized vaccine supply Q_r, there exists a unique threshold $x^{(d)}(Q_r)$ such that an individual with infection cost x will seek vaccination if and only if $x > x^{(d)}(Q_r)$.*

We now characterize $x^{(d)}(Q_r)$, the threshold that determines the ex-post demand in the decentralized supply chain. To do so, we define $x_1^d \in (x_{zr}, \bar{x})$ and $x_2^d(Q_r) \in [0, \bar{x})$, respectively, as follows:

$$x_1^d\left[u\left(\overline{G}\left(x_1^d\right)\right) - v\left(\overline{G}\left(x_1^d\right)\right)\right] = w + p, \tag{12}$$

$$x_2^d(Q_r)\left[u\left(Q_r\right) - v\left(Q_r\right)\right] = w + p, \tag{13}$$

for $Q_r > 0$. We characterize the ex-post equilibrium demand in the decentralized supply chain as in Proposition 3(i), and then we compare it with available supply as in Proposition 3(ii) and with the socially-optimal demand as in Proposition 3(iii). Proposition 3(ii) reveals that, when the supply is limited (i.e., $Q_r < \overline{G}(x_1^d)$), more individuals seek vaccination so that the resulting equilibrium demand exceeds the available supply.

PROPOSITION 3. *(i) For any realized vaccine supply $Q_r > 0$, there exists a unique threshold $x^{(d)}(Q_r)$ in the decentralized supply chain so that the corresponding ex-post demand satisfies:*

$$\overline{G}(x^{(d)}(Q_r)) = \begin{cases} \overline{G}(x_2^d(Q_r)), \text{ if } Q_r \le \overline{G}\left(x_1^d\right), \\ \overline{G}(x_1^d), \qquad \text{if } Q_r > \overline{G}\left(x_1^d\right). \end{cases} \tag{14}$$

(ii) In the decentralized supply chain, the ex-post equilibrium demand is greater than the available supply (i.e., $\overline{G}\left(x^{(d)}(Q_r)\right) > Q_r$) if and only if $0 < Q_r < \overline{G}\left(x_1^d\right)$.

(iii) The ex-post equilibrium demand in the decentralized supply chain is larger than the ex-post socially optimal demand (i.e., $\overline{G}\left(x^{(d)}(Q_r)\right) > \overline{G}\left(x^{(c)}(Q_r)\right)$) if and only if $0 < Q_r < \overline{G}\left(x_1^d\right)$.

Using Propositions 1 and 3, we draw Figure 1 to illustrate the ex-post demand in the social optimum and in equilibrium of the decentralized supply chain. We observe from Figure 1 that, when the realization of production quantity is sufficiently high ($Q_r \ge \overline{G}\left(x_1^d\right)$), the socially-optimal demand is higher than that in the decentralized supply chain. For such values of the realized production quantity, 'positive externalities' cause demand-side inefficiency. Specifically, due to herd immunity, each individual, if vaccinated, creates indirect benefits for the rest of the society by protecting people around them. However, individuals are self-interested and do not consider all (indirect) social benefits (i.e., positive externalities) when deciding whether to seek vaccination or not, so that fewer people seek vaccination in the decentralized supply chain than in the social optimum.

Figure 1 Ex-post Demand in Social Optimum and Decentralized Supply Chain

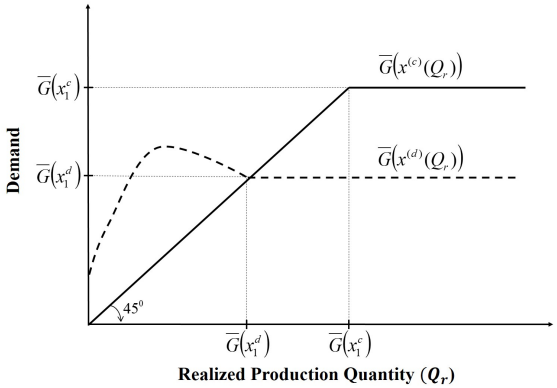


Figure 1 also shows that, when the realized production quantity is sufficiently low ($Q_r < \overline{G}\left(x_1^d\right)$), the equilibrium demand is greater than the socially-optimal demand. In such cases, 'negative externalities' lead

to the demand-side inefficiency. In particular, when the vaccine supply is limited, each individual seeking vaccination reduces the vaccine availability and thereby decreases expected utilities of all other individuals seeking vaccination (e.g., individuals with high infection costs can be rationed and may not get vaccinated). However, when deciding whether to seek vaccination or not, individuals are self-interested and do not take into account all (indirect) social costs (i.e., negative externalities). Consequently, more people seek vaccination in the decentralized supply chain than in the social optimum.

5.2. Decentralized Production Quantity: $\mathbf{Q^{(d)}}$

Anticipating ex-post equilibrium demand $\overline{G}(x^{(d)}(Q_r))$ as characterized in Proposition 3, the manufacturer chooses its production quantity $Q^{(d)}$ to maximize its ex-ante expected profit by solving:

$$\pi^{(d)} = \max_{Q}\{w \cdot \mathbb{E}_Y \left[h\left(x^{(d)}(YQ), YQ\right)\right] - cQ\}, \tag{15}$$

where h is given in (2). Proposition 4 characterizes the manufacturer's production quantity in the decentralized supply chain and compares it with the production quantity in the social optimum.

PROPOSITION 4. *(i) The equilibrium production quantity in the decentralized supply chain $Q^{(d)}$ satisfies:*

$$\int_0^{\frac{\overline{G}\left(x_1^d\right)}{Q^{(d)}}} w \cdot y dF\left(y\right) = c, \tag{16}$$

where x_1^d is given by (12).

(ii) Relative to the socially-optimal production quantity $Q^{(c)}$ given in (10)*, the equilibrium production quantity in the decentralized supply chain is lower, i.e., $Q^{(d)} < Q^{(c)}$.*

Proposition 4(ii) shows that, when vaccines are imperfect (i.e., $v(h) > 0$), the manufacturer in the decentralized supply chain produces less than the socially-optimal production quantity. (This result generalizes the result presented in Arifoğlu et al. (2012) that assume vaccines are perfect (i.e., when $v(h) = 0$).) Hence, in addition to the fact that the demand side is inefficient, this result implies that the supply side is also inefficient. This is mainly due to the manufacturer's self-interest: the manufacturer cares only about its own profit and does not take social benefits from each vaccination into consideration when choosing its production quantity.

6. Coordination: A Two-Sided Incentive Program

To improve the social welfare in the decentralized flu vaccine supply chain, we now propose a two-sided incentive program that is contingent on the manufacturer's ex-post realized production quantity Q_r. Specifically, it offers: (i) a 'transfer payment' $T(Q_r)$ to the manufacturer (on the supply side), where this transfer payment can be a subsidy (or a penalty) when $T(Q_r)$ is positive (or negative); and (ii) a 'vaccination incentive' $r(Q_r)$ to each vaccinated individual (on the demand side), where this incentive can be positive or negative.

To operationalize this two-sided incentive program $I=(r(Q_r),T(Q_r))$, we consider the following sequence of events. Before the production starts, in addition to the wholesale price contract w, the social planner offers a menu of transfer payments $T(Q_r)$ to the manufacturer, which is contingent on the ex-post production quantity Q_r. Given the wholesale price w and the menu of transfer payments $T(Q_r)$, the manufacturer chooses its production quantity $Q^{(I)}$. At the beginning of the vaccination season, the actual production quantity $Q_r=y\cdot Q^{(I)}$ is realized and commonly observed. Then the social planner announces the vaccination incentive $r(Q_r)$ to all individuals. Notice that the vaccination incentive $r(Q_r)$ is determined after the actual production quantity Q_r is realized. Then, given the 'effective vaccination cost' $w+p-r(Q_r)$, each individual decides whether to seek vaccination or not. Finally, the social planner delivers (or collects) the transfer payment $T(Q_r)$ to the manufacturer and vaccination incentive $r(Q_r)$ to vaccinated people.

6.1. Demand ($\overline{\mathbf{G}}\left(\mathbf{x^{(I)}(Q_r)}\right)$) and Production ($\mathbf{Q^{(I)}}$) under the Incentive Program

We begin our analysis by studying each individual's vaccination decision in the second stage, given the realized production quantity Q_r and the vaccination incentive $r(Q_r)$. By noting that the effective cost of vaccination is now $(w+p-r(Q_r))$, we can use the same approach as described in §5.1 to show that an individual with infection cost x will seek vaccination if and only if the revised condition $x(u(h)-v(h))>(w+p-r(Q_r))$ holds, which yields:

LEMMA 2. *In an ex-post equilibrium in the decentralized supply chain under the incentive program $I=(r(Q_r),T(Q_r))$, given realized vaccine supply Q_r, there exists a unique threshold $x^{(I)}(Q_r)$ such that an individual with infection cost x will seek vaccination if and only if $x>x^{(I)}(Q_r)$.*

Anticipating the ex-post demand under the incentive program $\overline{G}\left(x^{(I)}(Q_r)\right)$, the manufacturer determines the production quantity $Q^{(I)}$ ex ante by maximizing its expected profit $\pi^{(I)}(Q)$. Specifically, the manufacturer solves:

$$\pi^{(I)}=\max_{Q}\left\{\pi^{(I)}(Q)\equiv\mathbb{E}_Y\left[w\cdot h(x^{(I)}(YQ),YQ)+T(YQ)\right]-cQ\right\}, \tag{17}$$

where the first and second terms inside the expectation are the ex-post revenue to be obtained from vaccine sales (h is given in (2)) and the transfer payment to be received from the social planner, respectively.

6.2. The Coordinating Incentive Program

We now identify a specific incentive program $I=(r(Q_r),T(Q_r))$ that can coordinate the decentralized flu vaccine supply chain so that manufacturer's and individuals' incentives are aligned with the social optimum, i.e., $Q^{(I)}=Q^{(c)}$ and $x^{(I)}(Q_r)=x^{(c)}(Q_r)$ for all Q_r. Specifically, we consider the incentive program with the following parameters:

$$r(Q_r)=w+p-\bar{x}\left(1-h^{(c)}(Q_r)\right)\left(u\left(h^{(c)}(Q_r)\right)-v\left(h^{(c)}(Q_r)\right)\right), \tag{18}$$

$$T(Q_r)=K-(w+p)h^{(c)}(Q_r)-\frac{\bar{x}}{2}\left[\left(1-h^{(c)}(Q_r)\right)^2\left(u\left(h^{(c)}(Q_r)\right)-v\left(h^{(c)}(Q_r)\right)\right)+v\left(h^{(c)}(Q_r)\right)\right], \tag{19}$$

where K is a constant (that we shall specify later) and $h^{(c)}(Q_r) = \overline{G}(x^{(c)}(Q_r))$ is given by (7). Next, Proposition 5 shows that above incentive program coordinates the decentralized flu vaccine supply chain.

PROPOSITION 5. *In a decentralized supply chain that operates under the incentive program* $I = (r(Q_r), T(Q_r))$ *in (18) and (19) with a given constant* K*:*

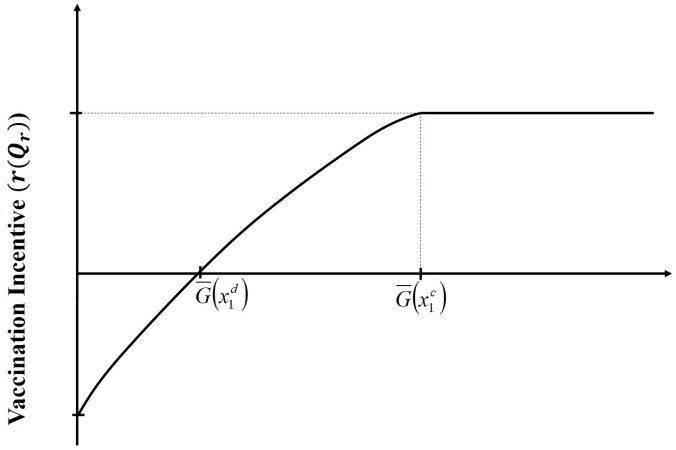
(i) All individuals voluntarily behave in line with the social optimum in an ex-post equilibrium so that $x^{(I)}(Q_r) = x^{(c)}(Q_r)$ *for any realized vaccine supply* Q_r*.*

(ii) The manufacturer chooses the socially-optimal ex-ante production quantity in equilibrium voluntarily so that $Q^{(I)} = Q^{(c)}$*.*

Proposition 5 indicates that we need a vaccination incentive $r(Q_r)$ to coordinate the demand side and a transfer payment $T(Q_r)$ to coordinate the supply side. We next explain how vaccination incentive $r(Q_r)$ in (18) and the transfer payment $T(Q_r)$ in (19) work and coordinate the demand and supply sides, respectively.

6.2.1. Coordinating vaccination incentive: $\mathbf{r(Q_r)}$. To understand how the vaccination incentive works on the demand side, we draw Figure 2 to illustrate $r(Q_r)$ in (18). Figure 2 shows that, when the uncertain yield Y is accounted for, the vaccination incentive $r(Q_r)$ can be positive or negative and take the form of subsidy or penalty. Specifically, when the realized vaccine supply is sufficiently high, i.e., $Q_r > \overline{G}\left(x_1^{(d)}\right)$, the demand in decentralized supply chain is always less than that in social optimum (see Figure 1) due to the positive externalities; therefore, to induce more individuals to seek vaccination, our incentive program provides positive vaccination incentive (i.e., $r(Q_r) > 0$) as illustrated in Figure 2.

Figure 2 Individual vaccination incentive $\mathbf{r(Q_r)}$ in (18)



On the other hand, when the realized vaccine supply is low enough, i.e., $Q_r \leq \overline{G}(x_1^d)$, the demand in decentralized supply chain exceeds the socially-optimal demand due to negative externalities (see Figure 1). In such cases, as shown in Figure 2, to curb the excess demand and ensure that individuals with high infection costs can get vaccinated, our incentive program creates negative vaccination incentive (i.e., $r(Q_r)) < 0$).

REMARK 1 (VACCINATION INCENTIVES IN PRACTICE). We commonly observe vaccination incentives in practice. For example, positive vaccination incentives are used in practice by making vaccination more accessible, subsidizing GPs for all vaccine administrations, and covering vaccination costs for everyone through vouchers, and public/private insurance (Institute of Medicine 2004; Nugent and Knaul 2006; Spencer and Kennedy 2007; Sinha et al. 2018). Similarly, general practitioners (GPs) or healthcare providers are either not subsidized at all or subsidized only for high-risk individuals to increase effective vaccination costs, which effectively creates negative vaccination incentives, either for all individuals or for only low-risk individuals (Institute of Medicine 2004; Spencer and Kennedy 2007; Sinha et al. 2018; NHS 2020). Specifically, the vaccination incentive that we propose can be implemented in practice by using *electronic voucher cards* as recommended by the Institute of Medicine (IOM) (see chapter 7; Institute of Medicine 2004).

6.2.2. Coordinating transfer payment: $\mathbf{T(Q_r)}$. To understand how the transfer payment $T(Q_r)$ works on the supply side, we consider a case where the social planner does not subsidize the manufacturer (i.e., $T(Q_r) = 0$) and only intervenes in the demand side to elicit individuals through $r(Q_r)$ in (18). We call this 'no demand-side inefficiency case' since there is not demand-side inefficiency. In this case, individuals' incentives are aligned with the social optimum, and the ex-post demand is equal to $\overline{G}(x^{(c)}(Q_r))$, which is given by (7), so that the manufacturer's optimal production quantity Q satisfies:

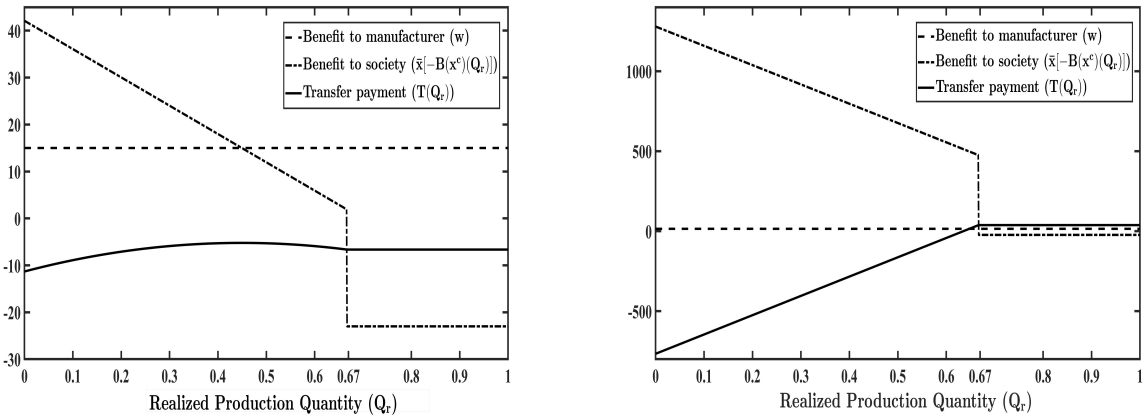
$$\int_0^{\frac{\overline{G}\left(x_1^c\right)}{Q}} w \cdot y dF\left(y\right) = c. \tag{20}$$

By comparing the left-hand side of (10) and that of (20), for any realized production quantity $Q_r < \overline{G}(x_1^c)$, the marginal benefit to the society (as a whole in the centralized system) from producing each dose of vaccine is equal to $\bar{x}[-B(x^{(c)}(Q_r))]$ whereas the marginal benefit to the manufacturer (even when there is no demand-side inefficiency) is equal to w. This 'difference' in marginal benefits to the society and manufacturer causes inefficiency on the supply side. Hence, to entice the manufacturer to choose the socially-optimal production quantity $Q^{(c)}$, the transfer payment to the manufacturer $T(Q_r)$ makes the manufacturer's marginal benefit equal to that of the society and thereby aligns its incentives with the social optimum. To demonstrate this better, by using two problem instances (i.e., $\bar{x} = \$100$ and $\bar{x} = \$2,000$) from our numerical study in §7, we draw Figure 3 to illustrate the coordinating transfer payment (i.e., $T(Q_r)$ in (19)), and ex-post marginal benefit of the manufacturer and society from selling a dose of vaccine (i.e., w and $\bar{x}[-B(x^{(c)}(Q_r))]$, respectively).

Figure 3a shows that, when the infection costs among the population are low (i.e., $\bar{x} = \$100$), the marginal benefit to the society is less than that to the manufacturer (i.e, $w > \bar{x}[-B(x^{(c)}(Q_r))]$) for sufficiently high realized production quantity $Q_r \geq \overline{G}(x_1^c) = 0.67$. Then it can be shown that the expected marginal benefit to the manufacturer in the case of no demand-side inefficiency with $T(Q_r) = 0$ is higher than that to the

Figure 3 **Transfer payment to the manufacturer under the coordinating incentive program (i.e., $\mathbf{T(Q_r)}$ in (19)) with $\mathbf{K = \underline{K}}$ as given in (23), and ex-post marginal benefit from selling a dose of vaccine to the manufacturer (i.e., $\mathbf{w}$) and to the society (i.e., $\bar{\mathbf{x}}[-\mathbf{B}(\mathbf{x^{(c)}}(\mathbf{Q_r}))]$, where $\mathbf{B}(.)$ is given by (6))**

(a) $\bar{x} = \$100$ (b) $\bar{x} = \$2,000$



Note: Two problem instances (i.e., $\bar{x} = \$100$ and $\bar{x} = \$2,000$) in this figure are from our numerical study in §7 with parameter values: $\mu = 1$, $\sigma = 0.4$, $p = \$23$, $w = \$15$, and $c = \$3$; and infection probabilities $u(h)$ and $v(h)$ as, respectively, given in (25) and (26), where $R_0 = 2.5$, $\alpha = 0.9$, and $\eta = 3.3$. By Propositions 2 and 4, the planned production quantities in the decentralized and centralized systems are equal to: $Q^{(d)} = 0.046$ (4.6% of the population) and $Q^{(c)} = 0.76$ (76% of the population) for $\bar{x} = \$100$, respectively; and $Q^{(d)} = 0.8$ and $Q^{(c)} = 1.92$ for $\bar{x} = \$2,000$, respectively.

society. Thus, when $\bar{x} = \$100$, the manufacturer produces more in the case of no demand-side inefficiency than in the social optimum (i.e., Q satisfying (20) is equal to 0.84 and is higher than the socially-optimal production quantity $Q^{(c)} = 0.76$). To discourage the manufacturer from this overproduction, Figure 3a illustrates that, for $\bar{x} = \$100$, our incentive program always penalizes the manufacturer (i.e., $T(Q_r) < 0$) but makes medium yield realizations more attractive by penalizing the manufacturer less (i.e., $T(Q_r)$ is non-monotone). This reduces the manufacturer's production quantity but not more than necessary so that it targets socially-optimal production quantity.

Figure 3b shows that the society's marginal benefit is significantly higher than that of the manufacturer for all $Q_r \leq \overline{G}(x_1^c) = 0.67$ when the infection costs among the population are high (i.e., $\bar{x} = \$2,000$). In this case, when there is no demand-side inefficiency and $T(Q_r) = 0$, it can be shown that the manufacturer chooses a very low production quantity and under-produces (i.e., the optimal Q satisfying (20) is equal to 0.84 and is lower than the socially-optimal production quantity $Q^{(c)} = 1.92$). To encourage the manufacturer to produce more, Figure 3b shows that, for $\bar{x} = \$2,000$, our incentive program penalizes the manufacturer (i.e., $T(Q_r) < 0$) when its realized production quantity Q_r is low (and the penalty decreases as the realized production quantity increases), and it rewards the manufacturer (i.e., $T(Q_r) > 0$) when its realized production quantity is very high. In doing so, the incentive program entices the manufacturer to increase its production quantity Q to the socially-optimal level $Q^{(c)}$.

Although the yield (Y) is uncertain and outside its control, the manufacturer can increase or decrease the realized quantity ($Q_r = Y \cdot Q$) by choosing its planned production quantity Q. By penalizing or rewarding the manufacturer ex post, our transfer payment $T(Q_r)$ elicits the manufacturer to choose the socially-optimal production quantity $Q^{(c)}$ ex ante.

REMARK 2 (TRANSFER PAYMENT IN PRACTICE). The transfer payment that we propose can be implemented through a contract between government and the manufacturer, e.g., CDC vaccine contract (CDC 2020). Similar contracts (or payment schemes) that depend on the outcome of an uncertain event are commonly used in healthcare: For example, to entice manufacturers to develop more effective drugs, Italy uses PbR and RS contracts under which a manufacturer is initially paid for the medication used to treat a person but it is later penalized and required to return all or part of the payment if the patient does not respond to the treatment (Keohane and Petrie 2017). In addition, to entice hospitals to exert readmission reduction efforts in the USA, the Centers for Medicare & Medicaid Services (CMS) implements HRRP under which hospitals are penalized if their actual 30-day excess readmission ratios (ERR) exceed median ERR within their peer group (CMS 2017; Arifoğlu et al. 2020).

6.2.3. Program participation and budget neutrality. While Proposition 5 states that the incentive program $I = (r(Q_r), T(Q_r))$ in (18) and (19) can entice the manufacturer and individuals to behave in a socially-optimal manner, this incentive program may not be *implementable* because it can make the manufacturer and/or overall utilities of all individuals worse off compared to the case with no incentive program, so that either the manufacturer is not willing to participate or no policymaker is in its favor. Further, the incentive program may require external funding. Next, we derive conditions such that our incentive program is implementable (i.e., it weakly improves the manufacturer's profit and expected total utilities of all individuals) and ex-ante budget-neutral (in expectation).

For any K, let us define the total ex-post net cost borne by the social planner (i.e., the amount paid to the manufacturer and those vaccinated individuals) under the two-sided incentive program $I = (r(Q_r), T(Q_r))$ as:

$$\begin{aligned} S^{(I)}(Q_r) &= T(Q_r) + r(Q_r) \cdot h^{(c)}(Q_r), \\ &= K - \frac{\bar{x}}{2}((1 - (h^{(c)}(Q_r))^2)u(h^{(c)}(Q_r)) + (h^{(c)}(Q_r))^2 v(h^{(c)}(Q_r))), \end{aligned} \tag{21}$$

where $h^{(c)}(Q_r) = \overline{G}(x^{(c)}(Q_r))$, which is given by (7). Notice that the total net cost under the incentive program to the social planner $S^{(I)}(Q_r)$ is increasing in K. Also, we define two terms $\overline{K} > \underline{K} > 0$ as follows:

$$\overline{K} = V_H + \pi^{(d)} - W^{(d)}, \tag{22}$$

$$\underline{K} = V_H + \pi^{(d)} - W^{(c)}, \tag{23}$$

where $W^{(c)} = \mathbb{E}_Y\left[W(x^{(c)}(YQ^{(c)}), YQ^{(c)})\right] - cQ^{(c)}$ and $W^{(d)} = \mathbb{E}_Y\left[W(x^{(d)}(YQ^{(d)}), YQ^{(d)})\right] - cQ^{(d)}$ are, respectively, the total expected social welfare in social optimum and decentralized supply chain. In

addition, $\pi^{(d)}$ is the manufacturer's equilibrium profit in the decentralized supply chain as given in (15). Also, we define the term $K_{bn} > 0$ that satisfies:

$$K_{bn} = \frac{\bar{x}}{2}\mathbb{E}_Y\left[(1-(h^{(c)}\left(YQ^{(c)}\right))^2)u(h^{(c)}\left(YQ^{(c)}\right)) + (h^{(c)}\left(YQ^{(c)}\right))^2 v(h^{(c)}\left(YQ^{(c)}\right))\right]. \quad (24)$$

Recall from Proposition 1 that everybody seeking vaccination is vaccinated in the social optimum, i.e., $h^c(Q_r) = \bar{G}(x^{(c)}(Q_r)) \leq Q_r$ and $\phi(x^c(Q_r, Q_r), Q_r) = 1$. Then, from (4), K_{bn} is the expected total utility loss of all (vaccinated and unvaccinated) individuals due to infection in the social optimum.

Using the above definitions, Proposition 6 specifies conditions under which the manufacturer is willing to participate in our incentive program, and our incentive program improves the expected total utilities of all individuals and/or is ex-ante budget-neutral.

PROPOSITION 6. *Suppose that the social planner offers an incentive program $I = (r(Q_r), T(Q_r))$ as stated in (18)-(19), where $T(Q_r)$ depends on the value of K.*

(i) If one selects K that has $K > \underline{K}$, then, relative to the case of no incentive, the manufacturer's expected profit (i.e., $\pi^{(I)}$) is higher under the incentive program $I = (r(Q_r), T(Q_r))$ (i.e., $\pi^{(I)} > \pi^{(d)}$).

(ii) If the term K_{bn} satisfies $K_{bn} < \overline{K}$, then relative to the case of no incentive, the expected total utilities of all individuals are higher under the incentive program $I = (r(Q_r), T(Q_r))$ for any value of K.

(iii) If one selects $K = K_{bn}$, then the incentive program $I = (r(Q_r), T(Q_r))$ is ex-ante budget-neutral (in expectation); i.e, $\mathbb{E}_Y\left[S^{(I)}(YQ^{(c)})\right] = 0$.

Proposition 6(i) shows that the social planner should choose the parameter K sufficiently large so that the manufacturer receives enough transfer payment under our incentive program and is willing to participate. In addition, Proposition 6(ii) asserts that the expected total utility loss of all individuals due to infection in the social optimum (i.e., K_{bn}) must be small enough so that the improvement in the expected total utilities of all individuals under our incentive program is sufficiently high. Proposition 6(i) and (ii) together imply that, if it is carefully designed, our incentive program can benefit the manufacturer and the rest of the society, so it is implementable.

Further, Proposition 6(iii) indicates that, if the social planner can use its reserve to implement our incentive program as stated in (18)-(19) with $K = K_{bn}$, then the incentive program will 'balance out' in the long run and achieve the socially-optimal solution at no cost (in expectation). However, for this ex-ante budget-neutral incentive program to be implementable, it should improve the expected total utilities of all individuals without making the manufacturer worse off so that the manufacturer participates (i.e., $\overline{K} > K_{bn} > \underline{K}$ by Proposition 6(i) and (ii)). This indicates that the ex-ante budget-neutral incentive program with $K = K_{bn}$ may not always be implementable (e.g., the manufacturer will not participate when $K_{bn} < \underline{K}$). To overcome this challenge, in §6.3, we shall develop a 'modified' version of our incentive program so that it is always 'ex-ante budget-neutral' (in expectation) and implementable.

6.3. A Modified Budget-Neutral Incentive Program

We now 'modify' our incentive program in §6.2 to make sure that it is self-sufficient and has a balanced budget in the long run (i.e., budget-neutral in expectation). Under the 'modified incentive program', in addition to $(r(Q_r), T(Q_r))$ as stated in (18)-(19), the social planner collects/returns a constant tax/benefit t from/to all individuals (i.e., $I = (t, r(Q_r), T(Q_r))$) before the flu vaccination season or the yield uncertainty is realized. (The constant t is a tax to be collected when it is positive and it is a benefit to be returned when it is negative.) This constant tax/benefit t can be considered as part of annual healthcare-related tax (e.g., social security and medicare taxes in the USA, and national insurance contribution in the UK) which, whether they use healthcare services or not, governments collect from everyone so as to cover healthcare costs for the society (IRS 2020; GOV.UK 2020).

Because t is collected/returned from/to all individuals in advance (independent from whether they are vaccinated or not), it will not change individuals' vaccination decisions. Hence, given the vaccinated fraction h and realized production Q_r, an individual with infection cost x seeks vaccination if and only if $x(u(h) - v(h)) > w + p - r(Q_r)$. Therefore, with $r(Q_r)$ given in (18), individuals will voluntarily behave in line with the social optimum under the modified incentive program. Similarly, given the transfer payment $T(Q_r)$ in (19), the manufacturer will voluntarily produce the socially-optimal production quantity $Q^{(c)}$ under the modified incentive program since its profit is independent of t.

However, note that, when $t > 0$ so that it is a tax to be collected from all individuals, imposing a tax t on all individuals under the modified incentive program will reduce the total expected utilities of all individuals and can make them worse off overall compared to the case with no incentive program. Consequently, the modified incentive program may not be implementable. Proposition 7 shows that, if the tax t and the constant K are carefully chosen, the modified incentive program $I = (t, r(Q_r), T(Q_r))$ in (18) and (19) will always be implementable and ex-ante budget-neutral.

PROPOSITION 7. *Suppose that the social planner offers the modified incentive program* $I = (t, r(Q_r), T(Q_r))$ *that has* $t = K - K_{bn}$*, where* K_{bn} *is given by* (24)*, and* $r(Q_r)$ *and* $T(Q_r)$ *given, respectively, in (18) and (19). Then, the modified incentive program* $I = (t, r(Q_r), T(Q_r))$ *is always 'ex-ante budget-neutral' (in expectation), i.e.,* $t = \mathbb{E}_Y\left[S^{(I)}(YQ^{(c)})\right]$*. In addition:*

(i) Under the modified incentive program, the total expected utility of all individuals and expected profit of the manufacturer is, respectively, equal to $(V_H - K)$ *and* $(K + W^{(c)} - V_H)$*.*

(ii) If one selects K *that has* $K > \underline{K}$*, then, relative to the case of no incentive, the manufacturer's expected profit is higher under the modified incentive program* $I = (t, r(Q_r), T(Q_r))$*.*

(iii) If one selects K *that has* $K < \overline{K}$*, then, relative to the case of no incentive, the expected total utilities of all individuals are higher under the modified incentive program* $I = (t, r(Q_r), T(Q_r))$*.*

The seasonal flu epidemic occurs every year, and, in a given flu season, depending on the realized production quantity Q_r, the modified incentive program as given in Proposition 7 might have a budget deficit (i.e., $t - S^{(I)}(Q_r) < 0$) or a budget surplus (i.e., $t - S^{(I)}(Q_r) > 0$). However, Proposition 7 implies that, if implemented over the long run, the total budget surplus due to the modified incentive program is enough to cover the total budget deficit (i.e., $t = \mathbb{E}_Y\left[S^{(I)}(YQ^{(c)})\right]$). Thus, the modified incentive program is ex-ante budget-neutral and balances out its budget in the long run.

Further, Proposition 7(i) indicates that, under the modified incentive program with $t = K - K_{bn}$, the social planner can arbitrarily distribute ex-ante social welfare among individuals and the manufacturer (e.g., by choosing K arbitrarily). More importantly, Proposition 7(ii) and (iii) reveal that, when the parameter K is set neither very high nor very low (i.e., $\underline{K} < K < \overline{K}$), the modified incentive program is always implementable and improves individuals' total expected utility without making the manufacturer worse off.

6.4. Allocation Based on Priority Groups

Our analysis so far ignores priority groups and assumes that the available vaccine is randomly allocated among all individuals seeking vaccination. However, we can easily extend our model by incorporating the vaccine allocation based on priority groups as in Arifoğlu et al. (2012). To do so, one can divide the population into two priority groups based on the infection cost x: high-risk group with $x \geq z$; and low-risk group with $x < z$, where $z \in [0, \bar{x}]$ is exogenously specified, and it is common knowledge and independent of the realized production Q_r.[14] Individuals in the high-risk group have a higher priority, and the vaccine available at the beginning of the vaccination season is made available to individuals in the high-risk group first and then to individuals in the low-risk group if any vaccine remains. There is random rationing within each priority group so that individuals in the same priority group have an equal chance to obtain the vaccine when the supply is limited (Arifoğlu et al. 2012; CDC 2018).

In the social optimum, the social planner has complete control and can dictate to individuals whether to seek or to not seek vaccination in the social optimum. It is never optimal for the social planner to ask more individiuals to seek vaccination than the available vaccine, as this rations individuals with high infection costs and decreases social welfare. Therefore, priority groups are irrelevant in the social optimum, and the socially-optimal demand and production quantity are, respectively, equal to $\overline{G}(x^{(c)}(Q_r))$ in (7) and Q^c in (10).

In the decentralized supply chain, priority groups affect the availability of flu vaccine for individuals and thus is relevant only in case of limited supply. Therefore, when the realized production is sufficiently high (i.e., $Q_r \geq \overline{G}(x_1^{(d)})$), the ex-post demand in the decentralized supply chain with priority groups is the same as that without priority groups and equal to $\overline{G}(x_1^{(d)})$. When the realized production is low (i.e., $Q_r <$

[14] This corresponds to the Advisory Committee on Immunization Practices (ACIP) practice of announcing the priority groups before the realization of vaccine supply (Institute of Medicine 2004).

$\overline{G}(x_1^{(d)})$), prioritizing individuals with high infection costs reduces the demand compared to the case without the priority groups by discouraging some individuals with low infection costs from seeking vaccination; however, the ex-post demand with priority groups is still more than the available supply Q_r. This implies that both positive and negative externalities still exist and cause inefficiency on the demand side. In particular, priority groups have no impact on the positive externalities when supply is unlimited but decrease the extent of negative externalities when supply is limited. It also implies that, in the decentralized supply chain with priority groups, the ex-post vaccinated fraction (i.e., vaccine sales) is the same and equal to $h^{(d)}(Q_r) = \min\{Q_r, \overline{G}(x_1^d)\}$ so that the manufacturer's profit is the same and its production quantity is equal to $Q^{(d)}$ as given in (16). Consequently, by $Q^{(c)} > Q^{(d)}$ in Proposition 4(ii), the manufacturer's incentives are still not aligned with those of the society, and priority groups have no impact on the supply-side inefficiency.

Lastly, independent of the priority groups, our modified incentive program in §6.3 aligns manufacturer's and individuals' incentives with the social optimum. Therefore, it continues to coordinate the decentralized supply chain even in the presence of priority groups.

7. Numerical Study

We now present an extensive numerical study to better understand the impact of vaccine efficacy (see §7.1) and the value of our modified incentive program, as stated in Proposition 7 (see §7.2). For our numerical study, we use the following infection probabilities $u(h)$ and $v(h)$ associated with a particular form of the SIR[15] model that has been considered by Mamani et al. (2012):

$$u(h) = \begin{cases} \frac{1-\eta(1-\alpha)h}{1-h}\left(1-\alpha h-\frac{1}{R_0}\right), & \text{if } h < h_{zr}, \\ 0, & \text{if } h \geq h_{zr}, \end{cases} \tag{25}$$

$$v(h) = \begin{cases} \eta(1-\alpha)\left(1-\alpha h-\frac{1}{R_0}\right), & \text{if } h < h_{zr}, \\ 0, & \text{if } h \geq h_{zr}, \end{cases} \tag{26}$$

where η is a constant,[16] α is the vaccine efficacy (e.g., vaccine is perfectly effective when $\alpha = 1$), R_0 is the average number of individuals infected by an infected person, and $h_{zr} = (R_0 - 1)/\alpha R_0$ if $R_0 > 1$ (and $h_{zr} = 0$ if $R_0 \leq 1$). We relax condition (i) in Assumption 1 (i.e., there may be no demand for the vaccine) and choose $\alpha > 1 - 1/\eta$ to ensure that conditions (iv) and (v) in Assumption 1 are satisfied.

To make our welfare-maximizing formulation consistent with the cost-minimizing formulations examined in the literature (Galvani et al. 2007; Chick et al. 2008), we normalize V_H to 0 and interpret $-W^{(c)}$ and

[15] The SIR epidemic model and corresponding infection probabilities in Mamani et al. (2012) assume that the initial infected fraction of the population is very small, and hence, the initial fraction of population that is susceptible is almost equal to 1. Taking into account that some fraction of the population can be initially not susceptible to infection (e.g., 40-60%) as in Katriel and Stone (2010), and Yamin and Gavious (2013), we obtain infection probabilities for vaccinated and unvaccinated individuals. Using these infection probabilities, we performed a numerical analysis and showed that all insights in our numerical study in this section are still valid.

[16] The constant η is chosen such that the infection probability in (26) is a good approximation for the infection probability of the vaccinated population. The actual infection probability can be obtained from a SIR epidemic model with imperfect vaccinations. See Mamani et al. (2012) for further details.

$-W^{(d)}$ as the ex-ante total 'social costs' in the social optimum and decentralized supply chain, respectively. As the distribution of random yield Y, we use gamma with mean $\mu = 1$ (Chick et al. 2008) and truncate it at $Y = 4$. We choose the rest of the parameter values from the literature to the extent possible, as summarized in Table 2. We consider a range of values for sensitivity analysis, but, for brevity, we present results only for a subset of parameter combinations in Table 2.

Table 2 Parameter values

Parameters	Value	Source
Coefficient of variation of the yield, $CV \equiv \frac{\sigma}{\mu}$	$\{0.2, 0.4, 0.6, 0.8\}$	Chick et al. (2008)
Highest infection costs, $\bar{x}$ (\$)	$\{100, 500, 2000\}$	Galvani et al. (2007)
Basic reproduction ratio, R_0	$\{1.5, 2, 2.5, 3\}$	Mamani et al. (2012)
The constant, η	3.3	Mamani et al. (2012)
Vaccine efficacy, α	$\{0.8, 0.85, 0.9, 0.95, 1\}$	Mamani et al. (2012)
Production cost per unit egg, c (\$)	$\{1.5, 3\}$	Deo and Corbett (2009); Koh and Paxson (2009)
Ratio of wholesale price to production cost, w/c	$\{2, 5\}$	CDC (2009)
Vaccination costs, p (\$)	23	Meltzer et al. (1999)

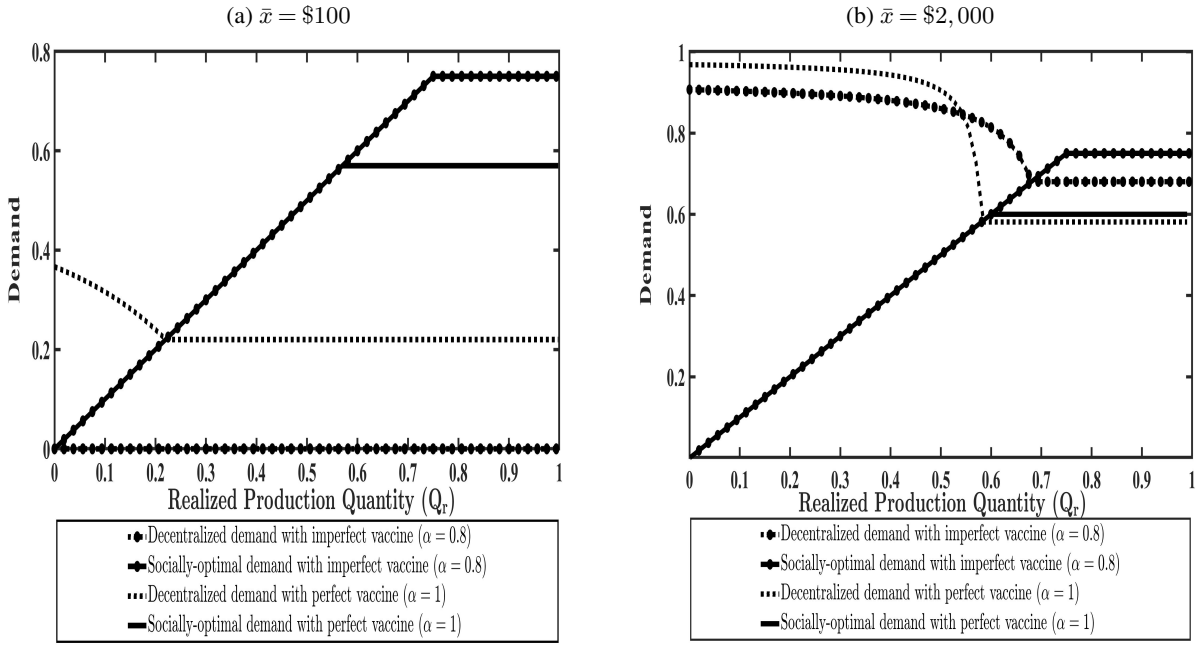
7.1. Impact of Vaccine Efficacy

We first study the impact of vaccine efficacy on ex-post demand and negative/positive externalities in §7.1.1, and then, in §7.1.2, we discuss how the vaccine efficacy affects the supply chain inefficiency.

7.1.1. Ex-post demand. Figure 4 illustrates the ex-post demand in the decentralized supply chain and social optimum for $\bar{x} \in \{100, 2000\}$ and $\alpha \in \{0.8, 1\}$ when the coefficient of variation $CV = 0.4$, $R_0 = 2.5$ and $w = \$15$. We observe from Figure 4 that the impact of vaccine efficacy on the demand depends on the realized yield, and that a more effective vaccine does not necessarily make the demand side more efficient. With a more effective vaccine, the social planner can provide the same level of immunization by vaccinating fewer individuals, and the demand in the social optimum decreases. Moreover, the increase in vaccine efficacy makes the vaccine more valuable by decreasing the infection risk more for vaccinated individuals when the vaccinated fraction is low, whereas it makes the vaccine less valuable by decreasing the infection risk more for unvaccinated individuals when the vaccinated fraction is high. When the infection cost is low ($\bar{x} = \$100$) as in Figure 4a, nobody seeks vaccination due to low direct benefits when the vaccine is imperfect ($\alpha = 0.8$). Therefore, as its efficacy increases (e.g., $\alpha = 1$), the vaccine becomes more valuable and more people demand it, which increases negative externalities (for low Q_r) and decreases positive externalities (for high Q_r).

On the other hand, when infection cost is high ($\bar{x} = \$2,000$) as in Figure 4b, the number of individuals seeking vaccination is very high and the vaccinated fraction is mostly bounded by realized production. Consequently, a more effective vaccine (e.g., $\alpha = 1$) is more valuable so is demanded more when the

Figure 4 Ex-post demand in decentralized supply chain and social optimum



vaccinated fraction is low (low Q_r) whereas it is less valuable so is demanded less when the vaccinated fraction is high (high Q_r). Thus, higher vaccine efficacy increases negative externalities for sufficiently low Q_r and decreases positive externalities for sufficiently high Q_r. Interestingly, when infection cost is high, it decreases negative externalities and increases positive externalities for intermediate Q_r.

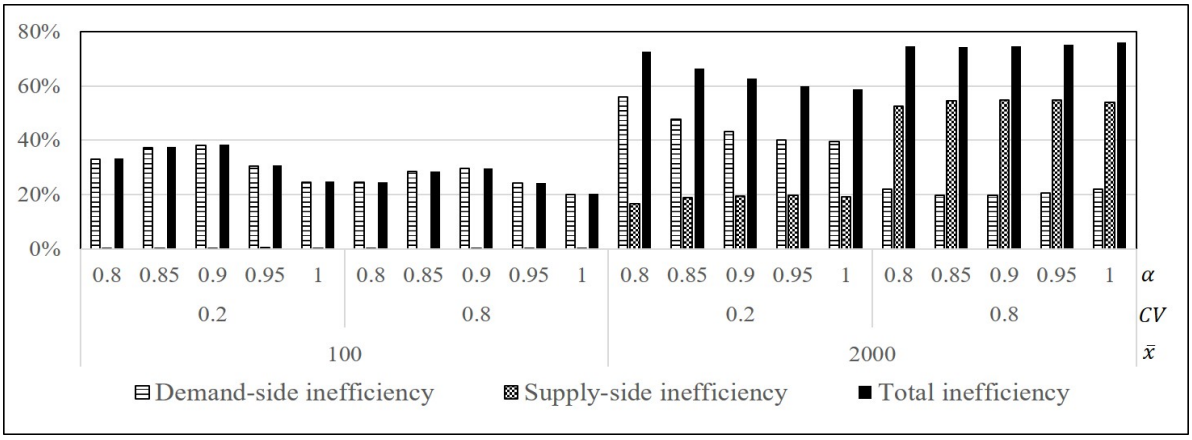
Overall, Figure 4 shows that the net effect of vaccine efficacy on the demand side is not clear ex ante. More importantly, it indicates that the manufacturer can sell more and increase its profit by producing an imperfect vaccine (e.g., vaccinated fraction and hence sales is (weakly) higher for $\alpha = 0.8$ and $\bar{x} = \$2,000$ in Figure 4b). Therefore, the vaccine manufacturer may have some financial motivations to produce a less effective flu vaccine.

7.1.2. Supply-chain inefficiency. Figure 5 depicts demand- and supply-side inefficiency and total supply-chain inefficiency[17] for $\alpha \in \{0.8, 0.85, 0.9, 0.95, 1\}$, $CV \in \{0.2, 0.8\}$ and $\bar{x} \in \{100, 2000\}$ when $R_0 = 2.5$ and $w = \$15$.

We observe from Figure 5 that total supply-chain inefficiency can be significant (e.g., more than 60%) and that the vaccine efficacy impacts on the inefficiency in the supply chain differently, depending on how costly the infection is (i.e., high/low $\bar{x}$) and/or how uncertain the production yield is (i.e., high/low CV). When infection cost is low (i.e., $\bar{x} = \$100$), supply-chain inefficiency is mainly due to individuals' self-interest on the demand side. This is because, for $\bar{x} = \$100$, the manufacturer's and society's expected marginal benefits

[17] We let $W^{(NDI)}$ as the social welfare in *no demand-side inefficiency case* (as described in §6.2.2) and measure demand-side inefficiency by $\left[1 - W^{(NDI)}/W^{(d)}\right] \times 100\%$, supply-side inefficiency by $\left[(W^{(NDI)} - W^{(c)})/W^{(d)}\right] \times 100\%$, and total supply-chain inefficiency by $\left[1 - W^{(c)}/W^{(d)}\right] \times 100\%$.

Figure 5 Impact of vaccine efficacy on supply-chain inefficiency



(i.e., the left-hand side of (10) and (16)) are very close; however, due to low direct vaccination benefits (especially, when the vaccine efficacy is low), demand in the decentralized supply chain is significantly less than that in social optimum (e.g., see Figure 4b for $\alpha = 0.8$). Moreover, recall from §7.1.1 that, for $\bar{x} = \$100$, negative externalities increase while positive externalities decrease in vaccine efficacy. Increase in negative externalities dominates and demand-side inefficiency increases for low enough α whereas decrease in positive externalities dominates and demand-side inefficiency decreases for high enough α. Therefore, as Figure 5 shows, supply-chain inefficiency first increases and then decreases in vaccine efficacy for $\bar{x} = \$100$. This implies that a more effective vaccine does not always increase supply-chain efficiency.

Further, we observe from Figure 5 that total supply chain inefficiency is significant and supply-side inefficiency is higher than demand-side inefficiency when infection cost and yield uncertainty are both high (i.e., $\bar{x} = \$2,000$ and $CV = 0.8$). For $\bar{x} = \$2,000$, production in decentralized supply chain is significantly lower due to misalignment between the manufacturer's and society's incentives. Therefore, under high yield uncertainty (i.e., $CV = 0.8$), it becomes more likely to end up in cases with limited supply, where negative externalities are significant (see Figure 4b). As a result, when $\bar{x} = \$2,000$ and $CV = 0.8$, supply-chain inefficiency is very high and mainly due to the manufacturer's self-interest on the supply side. Figure 5 also shows that, for $\bar{x} = \$2,000$, the vaccine efficacy has a major impact on supply chain inefficiency only when the yield uncertainty is low (i.e., $CV = 0.2$). This implies that, when the infection cost is high, efforts to improve vaccine efficacy will pay off only when the vaccine supply is more reliable.

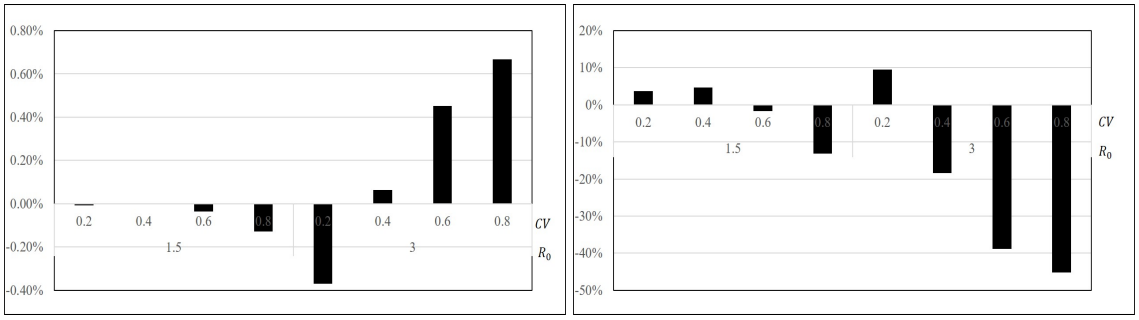
7.2. Value of Coordination

Next, we first analyze the impact of coordinating the demand or supply side on the value of coordination of the other side in §7.2.1, and then, in §7.2.2, we compare our two-sided incentive program against two mechanisms in prior literature.

7.2.1. Interaction between demand- and supply-side coordinations. Figure 6 illustrates the percentage increase[18] in the value of coordination on one side due to the coordination on the other side for $CV \in \{0.2, 0.4, 0.6, 0.8\}$, $R_0 \in \{1.5, 3\}$ and $\bar{x} \in \{100, 2000\}$ when $\alpha = 0.9$ and $w = \$15$. The figure shows that demand- and supply-side coordination can increase or decrease the value of coordination on the other side, and hence can be strategic complements or substitutes.

Figure 6 Increase in value of coordination on demand or supply side due to coordination on the other side

(a) $\bar{x} = \$100$ (b) $\bar{x} = \$2,000$



When the infection cost is low (i.e., $\bar{x} = \$100$), we observe from Figure 5 that the supply-chain inefficiency is mainly due to individuals' self-interest on the demand side (i.e., low supply-side inefficiency); therefore, the interaction between demand- and supply-side coordination is not significant (less than 1%) as Figure 6a shows. On the other hand, when the infection cost is high (i.e., $\bar{x} = \$2,000$), both demand- and supply-side inefficiencies are high so that, as Figure 6b shows, coordinating the demand or supply side has a significant impact on the value of coordination on the other side, especially when the flu is very infectious, and the production yield is highly uncertain (i.e., $R_0 = 3$, and $CV > 0.4$). Interestingly, Figure 6b shows that demand- and supply-side coordinations increase/decrease each other's value and are strategic complements/substitutes when the production yield is less/more uncertain. This implies that coordinating each side of the supply chain has more impact on the overall inefficiency when the production yield is highly uncertain.

7.2.2. Two-sided vs. one-sided coordination. There are two forms of incentive mechanisms in the literature that aim to coordinate only one side of the flu vaccine supply chain. First, the *coordinating mechanism ignoring the supply side* (e.g., the tax/subsidy mechanism of Brito et al. (1991)) aims to coordinate the demand side by ignoring the manufacturer's self-interest and the limited supply due to uncertain yield. This mechanism in the literature corresponds to the case where the social planner implements the

[18] We let $W^{(NDI)}$ and $W^{(NSI)}$, respectively, be the social welfare in no demand-side inefficiency case (as described in §6.2.2) and in the *no supply-side inefficiency case* (where the manufacturer always behaves in line with the social optimum while self-interested individuals make their vaccination decisions). Then, we measure the percentage increase in the value of coordination on one (demand or supply) side due to coordination on the other (supply or demand) side by $\left[(W^{(NDI)} + W^{(NSI)} - W^{(c)} - W^{(d)})/W^{(d)})\right] \times 100\%$.

socially-optimal demand $\overline{G}(x_1^c)$ without accounting for the supply side. Second, the *coordinating mechanism ignoring the demand side* (e.g., the whole-unit discount/cost-sharing contract of Chick et al. (2008)) intends to coordinate the supply side by ignoring individuals' self-interested behavior and assuming that the demand is exogenously controlled by the social planner. This mechanism in the literature corresponds to the case where the social planner implements the socially-optimal quantity $Q^{(c)}$ without accounting for the self-interested behavior of individuals. Figure 7 illustrates the value[19] of our two-sided mechanism and aforementioned one-sided mechanisms in prior literature for $CV \in \{0.2, 0.4, 0.6, 0.8\}$, $R_0 \in \{1.5, 3\}$ and $\bar{x} \in \{100, 2000\}$ when $\alpha = 0.9$ and $w = \$15$.

Figure 7 Reduction in social costs under different coordinating mechanisms

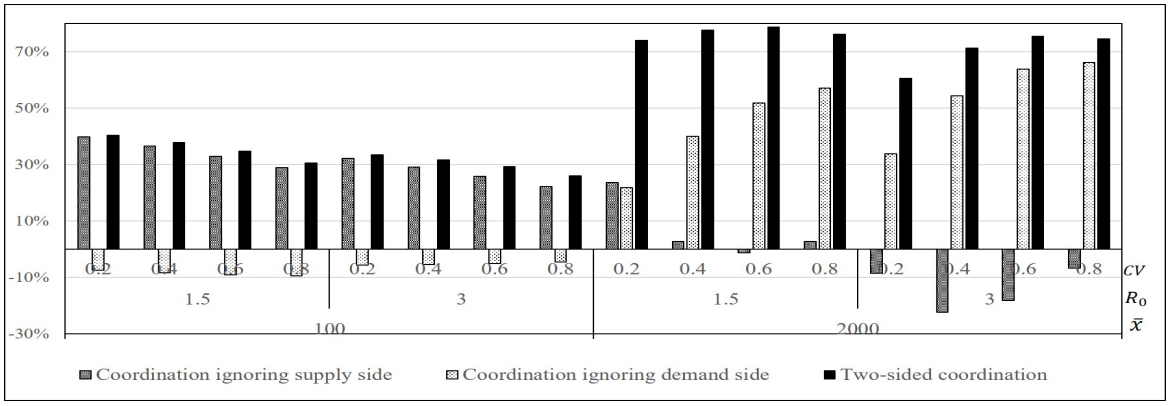


Figure 7 shows that our two-sided incentive program creates significant social value (e.g., it reduces social costs by more than 70%), especially when the infection cost is very high (i.e., $\bar{x} = \$2,000$). When the infection cost is low (i.e., $\overline{x} = \$100$), there is a significant misalignment between individual and social benefits, and few individuals seek vaccination, as explained above. In such cases, accounting for individuals' self-interested behavior is very important. Thus, as is shown in Figure 7 for $\bar{x} = \$100$, the coordination mechanism ignoring the demand side performs poorly and increases social costs because it ignores the self-interested individual behavior, whereas the coordination mechanism ignoring the supply side performs as well as the two-sided coordination because it considers self-interested individual behavior.

On the other hand, when the infection cost is high (i.e., $\bar{x} = \$2,000$), the supply-chain inefficiency is mostly due to the uncertain yield (i.e., high supply-side inefficiency), especially when the yield uncertainty is high (see §7.1.2). Thus, from Figure 7 for $\bar{x} = \$2,000$, we can draw three conclusions. First, the coordination mechanism ignoring the demand side (or self-interested individuals) performs better than the one that ignores the supply side (or manufacturer's self-interest and uncertain yield). Second, when the infectiousness of flu virus is high ($R_0 = 3$), the coordination mechanism ignoring the supply side performs poorly and

[19] We measure the value of our two-sided incentive program and aforementioned two one-sided mechanisms in the literature according to the percentage reduction in the total social costs compared to that in the decentralized supply chain, i.e., $\left[1 - W^{(c)}/W^{(d)}\right] \times 100\%$ and $\left[1 - W^{(o)}/W^{(d)}\right] \times 100\%$, where superscript '$o$' corresponds to a one-sided mechanism.

increases social costs in most cases, whereas the coordination mechanism ignoring the demand side always decreases social costs. Third, vaccine supply becomes more uncertain so that the supply-side inefficiency increases as the CV increases. Hence, when $\bar{x} = \$2,000$, as CV increases, the coordination mechanism ignoring the demand side and focusing on the supply side surpasses the coordination mechanism ignoring the supply side and performs almost as good as the two-sided coordination.

Overall, our two-sided incentive program outperforms both one-sided coordinating mechanisms examined in the literature in all cases, as shown in Figure 7. This finding reveals that, to coordinate a flu vaccine supply chain, it is very important to account for the interaction between the self-interested individual behavior that affects demand, and the manufacturer's self-interest and yield uncertainty that affect supply.

8. Conclusion and Policy Implications

The US vaccine market has experienced frequent supply and demand mismatches due to its highly decentralized structure. This has raised serious concerns among health officials and policymakers since vaccines grant significant personal and social benefits that give them their unique status in the healthcare system. After their study in 2003, the Institute of Medicine (IOM) committee on vaccines concluded that, to finance the vaccines, the USA needs a comprehensive strategy that provides adequate incentives to individuals and vaccine manufacturers in order to achieve the right balance in assuring access to vaccines while also encouraging their availability (Institute of Medicine 2004). In this paper, we have provided one such strategy for flu vaccines.

Specifically, we have developed a two-sided incentive program to coordinate a decentralized flu vaccine supply chain with self-interested individuals and a manufacturer with uncertain production yield. On the demand side, this incentive program creates negative vaccination incentives for individuals to counteract negative externalities and curb the demand if the realized vaccine supply is limited, whereas it creates positive vaccination incentives for individuals to counteract the positive externalities and increase the demand when the realized vaccine supply is abundant. On the supply side, it proposes a menu of contracts between the social planner and the manufacturer which penalizes the manufacturer for low realized vaccine supply and subsidizes it for high realized vaccine supply. Our two-sided incentive program creates value for the manufacturer and all individuals in the sense that the expected the manufacturer's profit and expected total utilities of all individuals are higher than those in the decentralized supply chain.

Moreover, the incentive program that we propose offers additional flexibility for the social planner to distribute welfare among individuals and the manufacturer. More importantly, it is ex-ante budget-neutral (in expectation) and does not require any external funding in the long run. Further, by using parameter values in the prior research, we have conducted an extensive numerical study. We have found that our two-sided incentive program can reduce the social costs significantly (e.g., more than 70%) and that, contrary to their intended purposes, incentive mechanisms that aim to coordinate only one side of the flu vaccine supply

chain (ignoring the other side) can actually increase the social costs instead of decreasing them. We also find that vaccine manufacturers may have some financial motivations to develop imperfect (less effective) vaccines, and that more effective vaccines can increase the supply-chain inefficiency.

Our results have important policy implications on the IOM committee's recommendations about financing the vaccines in the USA. First, we confirm that the *vaccine payment system* proposed by the committee that provides vaccination incentives to individuals through government subsidies and vouchers can ensure individuals' access to vaccines and eliminate the demand-side inefficiency. However, for the flu vaccine, where the yield uncertainty plays an important role, these vaccination incentives should be modified taking the realized yield into consideration, specifically because the yield affects the coverage and hence the social benefits from each vaccination. Second, in contrast to the committee's expectations, we show that vaccination incentives given to individuals are not sufficient to entice the manufacturer to produce more and ensure the availability of flu vaccines. Our finding confirms the concerns raised by the National Vaccine Advisory Committee (NVAC) that the vaccine payment system proposed by the IOM committee will not provide enough incentives to the manufacturer (Hinman 2005). To ensure availability of flu vaccines, we develop a contract between the social planner and the manufacturer that depends on the realized yield. Third, we show that financing vaccinations does not necessarily require external funding and can be designed to have a balanced budget in the long run.

There are several directions for future research. We do not consider the healthcare providers and pharmacies that actually vaccinate people. Future work can incorporate all parties involved in the flu vaccine supply chain and develop incentive mechanisms to align their incentives with the social optimum. Also, we assume that all individuals are self-interested and behave rationally. Our work can be extended by considering irrational behavior of individuals. Finally, in our model, vaccination occurs only in a single period, and all vaccines arrive at the beginning of that period. Future work can model the vaccination season as multiple periods, and analyze individuals' decisions regarding when to vaccinate by considering the arrival of vaccines over time.

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Online Supplement:
A Two-sided Incentive Program for Coordinating the Influenza Vaccine Supply Chain

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A. Definitions and Auxiliary Results

In this appendix, we make definitions and derive auxiliary results that help us to prove our main results in the paper. To that end, we define

$$\varphi_1(x) = x\left(u\left(\overline{G}(x)\right) - v\left(\overline{G}(x)\right)\right) - (w+p), \tag{A.1}$$

$$\varphi_2(x, Q_r) = x\left(u(Q_r) - v(Q_r)\right) - (w+p), \tag{A.2}$$

for $Q_r > 0$ and $x \in [0, \bar{x}]$. Certainly, $x_2^c(Q_r)$ exists and is equal to $\bar{x} \cdot \min\{Q_r, 1\}$ since $G(x) = \min\{x/\bar{x}, 1\}$ for all $x \geq 0$.

We use Lemma A.1 to characterize the demand in social optimum and equilibrium of the decentralized supply chain.

LEMMA A.1. *Define $x_{zr} \in (0, \bar{x}]$ as in* (5)*.*

(i) There exists $x_1^d \in (x_{zr}, \bar{x})$ satisfying (12) *(i.e., $\varphi_1(x_1^d) = 0$) such that $\varphi_1(x) < 0$ if $x < x_1^d$, and $\varphi_1(x) > 0$ if $x > x_1^d$.*

(ii) $B(x)$ as given in (6) *is strictly decreasing for $x \geq x_{zr}$; and $B(x) > 0$ if $x < x_1^c$, and $B(x) < 0$ if $x > x_1^c$, where $x_1^c \in [x_{zr}, \bar{x})$ satisfies* (12)*.*

(iii) $x_1^d > x_1^c$, where $x_1^c \in [x_{zr}, \bar{x})$ and $x_1^d \in (x_{zr}, \bar{x})$, respectively, satisfy (7) *and* (12)*.*

Proof of Lemma A.1: We prove each part of the lemma separately.

Part (i): By (A.1), we have $\varphi_1(x) = -(w+p) < 0$ for $x \leq x_{zr}$ by (1), and

$$\frac{d\varphi_1(x)}{dx} = u\left(\overline{G}(x)\right) - v\left(\overline{G}(x)\right) - G(x)\left(u'\left(\overline{G}(x)\right) - v'\left(\overline{G}(x)\right)\right) > 0 \tag{A.3}$$

for $x > x_{zr}$, where equality and inequality in (A.3) follow from G being uniform and Assumption 1(iv), respectively. In addition, $\lim_{x \uparrow \bar{x}} \varphi_1(x) > 0$ by Assumption 1(i) and (A.1). Then, by $\varphi_1(x) = -(w+p) < 0$

for $x \leq x_{zr}$ and $d\varphi_1(x)/dx > 0$ for $x \in (x_{zr}, \bar{x})$, this implies that there exists a unique $x_1^d \in (x_{zr}, \bar{x})$ that satisfies (12) or $\varphi_1(x_1^d) =$ such that $\varphi_1(x) < 0$ if $x < x_1^d$, and $\varphi_1(x) > 0$ if $x > x_1^d$.

Part (ii): From (6), $B(x) = p/\bar{x} > 0$ for $x \leq x_{zr}$ by (1), and $\lim_{x\uparrow\bar{x}} B(x) < 0$ by Assumption 1(i). Moreover, by Assumption 1(iii) and (v), $B(x)$ in (6) is decreasing in x for all $x > x_{zr}$. Note that $B(x)$ may be discontinuous at $x = x_{zr}$ (i.e., $\lim_{x\uparrow x_{zr}^-} B(x) \neq \lim_{x\downarrow x_{zr}^+} B(x)$) depending on the form of infection probabilities. If $\lim_{x\downarrow x_{zr}^+} B(x) \leq 0$, then $B(x) < 0$ for all $x > x_{zr}$ since $B(x)$ is decreasing for $x > x_{zr}$. Thus, when $\lim_{x\downarrow x_{zr}^+} B(x) \leq 0$, for $x_1^c = x_{zr}$, $B(x) > 0$ if $x < x_1^c$, and $B(x) < 0$ if $x > x_1^c$. On the other hand, if $\lim_{x\downarrow x_{zr}^+} B(x) > 0$ there exists an $\check{x} \in (x_{zr}, \bar{x})$ satisfying $B(\check{x}) = 0$ such that $B(x)$ is negative (or positive) for all $x \in (\check{x}, \bar{x})$ (or $x \in (x_{zr}, \check{x})$). This, by $B(x) = p/\bar{x} > 0$ for $x \leq x_{zr}$ and $B(x)$ being decreasing in x for $x > x_{zr}$, implies that, when $\lim_{x\downarrow x_{zr}^+} B(x) > 0$, for $x_1^c = \check{x}$, $B(x) > 0$ if $x < x_1^c$, and $B(x) < 0$ if $x > x_1^c$.

Part (iii): From part (i), $x_1^d \in (x_{zr}, \bar{x})$. Thus, if $x_1^c = x_{zr}$, i.e., $\lim_{x\downarrow x_{zr}^+} B(x) \leq 0$, then $x_1^c < x_1^d$. Next, we show that $x_1^c < x_1^d$ if $x_1^c = \check{x}$, i.e., $\lim_{x\downarrow x_{zr}^+} B(x) > 0$. Note by (6) and (12) that

$$B\left(x_1^d\right) = \frac{1}{2}G^2\left(x_1^d\right)u'\left(\overline{G}\left(x_1^d\right)\right) + \frac{1}{2}\left(1 - G^2\left(x_1^d\right)\right)v'\left(\overline{G}\left(x_1^d\right)\right) - \frac{w}{\bar{x}} < 0$$

by Assumption 1(ii). Moreover, from the part (ii), $B(x)$ is decreasing for all $x \in (x_{zr}, \bar{x}]$ and $B(x_1^c) = 0$ when $x_1^c = \check{x}$. This implies that $x_1^c < x_1^d$. □

B. Proofs

Proof of Proposition 1: We first characterize the threshold infection cost that determines the socially-optimal demand (i.e., $x^{(c)}(Q_r)$). Then, by using this threshold, we show that the socially-optimal demand satisfies (7). By (4), we have

$$W(x, Q_r) = \begin{cases} V_H - pQ_r - \frac{\bar{x}}{2}u(Q_r) + \frac{\bar{x}}{2}(1 + G(x))Q_r(u(Q_r) - v(Q_r)), & \text{if } x < x_2^c(Q_r) \\ V_H - p\overline{G}(x) - \frac{\bar{x}}{2}G^2(x)u\left(\overline{G}(x)\right) - \frac{\bar{x}}{2}(1 - G^2(x))v\left(\overline{G}(x)\right), & \text{if } x_2^c(Q_r) \leq x \leq \bar{x} \end{cases}$$

for $x \in [0, \bar{x}]$ and $Q_r > 0$, where $x_2^c(Q_r)$ is given by (11). Next, we find x that maximizes the ex-post social welfare $W(x, Q_r)$.

By (1), for $x < x_2^c(Q_r)$, the ex-post social welfare $W(x, Q_r)$ is increasing in x if $Q_r < \overline{G}(x_{zr}) = h_{zr}$ and it is constant in x if $Q_r \geq \overline{G}(x_{zr})$. That is, for all $Q_r > 0$, ex-post social welfare is non-decreasing in x for $x < x_2^c(Q_r)$. Therefore, the threshold infection cost in the social optimum $x^{(c)}(Q_r)$ cannot be less than $x_2^c(Q_r)$. Now, we consider $x \geq x_2^c(Q_r)$. Note that, for $x \geq x_2^c(Q_r)$,

$$\begin{aligned} \frac{\partial W(x, Q_r)}{\partial x} &= \frac{1}{\bar{x}}\Big[p - \bar{x}G(x)\left(u\left(\overline{G}(x)\right) - v\left(\overline{G}(x)\right)\right) \\ &\qquad + \frac{\bar{x}}{2}\left[G^2(x)u'\left(\overline{G}(x)\right) + \left(1 - G^2(x)\right)v'\left(\overline{G}(x)\right)\right]\Big] \\ &= B(x) \end{aligned}$$

for $Q_r > 0$ where $B(x)$ is given by 6. By Lemma A.1(ii), $B(x) > 0$ for $x < x_1^c$ and $B(x) < 0$ for $x > x_1^c$, where x_1^c is given by (8). Then, if $x_2^c(Q_r) > x_1^c$, the ex-post social welfare $W(x, Q_r)$ is always decreasing in x so that it is socially optimal to vaccinate all individuals with $x \geq x_2^c(Q_r)$ (i.e., $x^{(c)}(Q_r) = x_2^c(Q_r)$). However, if $x_2^c(Q_r) \leq x_1^c$, the ex-post social welfare attains its maximum at $x = x_1^c$ (i.e., increasing (decreasing) in x for $\geq x_2^c(Q_r) \leq x \leq x_1^c$ ($x > x_1^c$)) so that $x^{(c)}(Q_r) = x_2^c(Q_r)$. Consequently, by using (11), the socially-optimal demand is equal to $\overline{G}\left(x^{(c)}(Q_r)\right) = \min\left\{Q_r, \overline{G}(x_1^c)\right\}$. □

Proof of Proposition 2: By Proposition 1 and using (9), the first and second derivatives of $W^{(c)}(Q)$ are, respectively, given by:

$$\frac{dW^{(c)}(Q)}{dQ} = \int_0^{\frac{\overline{G}\left(x_1^c\right)}{Q}} \bar{x}[-B(x_2^c(yQ)) \cdot y] dF(y) - c,$$

$$\frac{d^2W^{(c)}(Q)}{dQ^2} = \bar{x}B(x_1^c)\frac{\overline{G}^2(x_1^c)}{Q^3} f\left(\frac{\overline{G}(x_1^c)}{Q}\right) + \int_0^{\frac{\overline{G}\left(x_1^c\right)}{Q}} \bar{x}\frac{dB(x_2^c(yQ))}{dx} y dF(y) < 0,$$

for $Q \geq 0$, where $x_2^c(Q_r)$ is given by (11), and $B(x)$ is given by (6). The inequality above follows from $B(x_1^c) \leq 0$ and $B(x)$ being decreasing in $x > x_1^c$ (by Lemma A.1(ii)), and indicates that $W^{(c)}(Q)$ is concave in Q. Note that

$$\lim_{Q \downarrow 0} \frac{dW^{(c)}(Q)}{dQ} > w\mu - c > 0,$$

where the first inequality follows from (6) by Assumption 1(i) and (ii), and the second inequality follows from our assumption that $w\mu > c$. Then by $\lim_{Q\uparrow\infty} dW^{(c)}(Q)/dQ < 0$ and $W^{(c)}(Q)$ being concave, it follows that there exists a unique $Q^{(c)} \in (0, \infty)$ satisfying $dW^{(c)}\left(Q^{(c)}\right)/dQ = 0$, and $Q^{(c)}$ is the unique maximizer of $W^{(c)}(Q)$. □

Proof of Lemma 1: For any given realized production quantity, let $\tilde{h}$ be the vaccinated fraction in the equilibrium. Suppose that an individual with infection cost $\hat{x} \in [0, \bar{x}]$ does not seek vaccination. Since the individual with $\hat{x}$ does not seek vaccination, we have $\hat{x}(u(\tilde{h}) - v(\tilde{h})) < w + p$. For any $x < \hat{x}$, this condition is satisfied and hence individuals with $x < \hat{x}$ do not seek vaccination. Therefore, there exists a thresholds $x^{(d)}(Q_r)$ such that all individuals with $x > x^{(d)}(Q_r)$ seek vaccinations while all individuals with $x < x^{(d)}(Q_r)$ do not. □

Proof of Proposition 3: We prove each part of the proposition separately.

Part (i): First, we characterize the equilbrium demand. To that end, we define $\psi(x, Q_r)$ as the excess of utility to the marginal individual with infection cost x when seeking vaccination over not seeking. It turns out that

$$\psi(x, Q_r) = x(u(h(x, Q_r)) - v(h(x, Q_r))) - (w + p)$$

for all $Q_r > 0$, and $x \in [0, \bar{x}]$, where h satisfies (2). Note by (2) and Assumption 1(iv) that $\psi(x, Q_r)$ is non-decreasing in x so that the threshold $x^{(d)}(Q_r)$ that determines the equilibrium demand is the minimum

x such that $\psi(x, Q_r) > 0$ for any $Q_r > 0$. (In cases such an x does not exist, the threshold is equal to $\bar{x}$ and nobody seeks vaccination.)

Using (11) and (2), $\psi(x, Q_r)$ becomes

$$\psi(x, Q_r) = \begin{cases} \varphi_2(x, Q_r), \text{ if } x < x_2^c(Q_r), \\ \varphi_1(x), \qquad \text{if } x_2^c(Q_r) \leq x \leq \bar{x}, \end{cases} \tag{B.1}$$

for all $Q_r > 0$, where φ_1 and φ_2 are given, respectively, by (A.1) and (A.2). Notice that $\psi(x, Q_r)$ is continuous in x for all $Q_r > 0$ since φ_1 and φ_2 are both continuous in x, and $\varphi_1(x_2^c(Q_r)) = \varphi_2(x_2^c(Q_r), Q_r)$. Now, consider two cases: $x_2^c(Q_r) \leq x_1^d$ and $x_2^c(Q_r) > x_1^d$, where x_1^d and $x_2^c(Q_r)$ are, respectively, given by (12) and (11).

Case 1 ($x_2^c(Q_r) \leq x_1^d$): In this case, by (B.1), $\psi(x, Q_r) = \varphi_2(x, Q_r)$ for $x < x_2^c(Q_r)$. Note by (A.1) and (A.2) that $\lim_{x \downarrow 0} \varphi_2(x, Q_r) < 0$ and $\lim_{x \uparrow x_2^c(Q_r)} \varphi_2(x, Q_r) = \varphi_1(x_2^c(Q_r)) < 0$, where the second inequality follows from $x_2^c(Q_r) < x_1^d$ and $\varphi_1(x) < 0$ for all $x < x_1^d$ (by Lemma A.1(i)). This, by φ_2 being increasing in x by (A.2), implies that $\varphi_2(x, Q_r) < 0$ for $x < x_2^c(Q_r)$. Thus, $\psi(x, Q_r)$ is negative for $x \in [0, x_2^c(Q_r))$. Then, it remains to analyze the excess utility in $\psi(x, Q_r)$ for $x \geq x_2^c(Q_r)$. By (B.1), $\psi(x, Q_r) = \varphi_1(x)$ for $x \in [x_2^c(Q_r), \bar{x}]$. Then, by $x_2^c(Q_r) \leq x_1^d$ in this case and by Lemma A.1(i), it follows that $\psi(x, Q_r) > 0$ if, and only if, $x > x_1^d$ (i.e., $x^{(d)}(Q_r)$ is unique and equal to $x_1^{(d)}$). Therefore, the equilibrium demand is equal to $\overline{G}(x_1^d)$ when $x_2^c(Q_r) \leq x_1^d$ (i.e., $Q_r \geq \overline{G}(x_1^d)$ by (11)).

Case 2 ($x_2^c(Q_r) > x_1^d$): In this case, by (B.1), $\psi(x, Q_r) = \varphi_1(x) > 0$ for all $x \geq x_2^c(Q_r)$ by Lemma A.1(i). Thus, it remains to consider $x < x_2^c(Q_r)$ to characterize the threshold $x^{(d)}(Q_r)$. By (B.1), $\psi(x, Q_r) = \varphi_2(x, Q_r)$ for all $x < x_2^c(Q_r)$ so that, by (A.2), $\lim_{x \downarrow 0} \psi(x, Q_r) < 0$ and $\lim_{x \uparrow x_2^c(Q_r)} \psi(x, Q_r) > 0$, where the second inequality follows from the continuity of $\psi(x, Q_r)$ and $\varphi_1(x) > 0$ for all $x > x_1^d$ (by Lemma A.1(i)). This by $\varphi_2(x, Q_r)$ being increasing in x (by (A.2)) implies that there exists $x_2^d(Q_r) \in (0, x_2^c(Q_r))$ such that $\psi(x_2^d(Q_r), Q_r) = \varphi_2(x_2^d(Q_r), Q_r) = 0$, and $\psi(x, Q_r)$ is negative (or positive) for $x < x_2^d(Q_r)$ (or $x > x_2^d(Q_r)$), where $x_2^d(Q_r)$ is given by (13). Therefore, $x^{(d)}(Q_r)$ is unique and equal to $x_2^d(Q_r)$ so that the equilibrium demand is equal to $\overline{G}(x_2^d(Q_r))$ if $x_2^c(Q_r) > x_1^d$ (i.e., $Q_r < \overline{G}(x_1^d)$ by (11)).

Part (ii): From part (i), $x^{(d)}(Q_r)$ is equal to $x_2^d(Q_r) \in (0, x_2^c(Q_r))$ so that, by (11), $\overline{G}\left(x^{(d)}(Q_r)\right) > Q_r$ for all $Q_r < \overline{G}(x_1^d)$, and $x^{(d)}(Q_r)$ is equal to $x_1^d \in (x_2^c(Q_r), \bar{x})$ so that $\overline{G}\left(x^{(d)}(Q_r)\right) < Q_r$ for all $Q_r > \overline{G}(x_1^d)$.

Part (iii): If $Q_r < \overline{G}(x_1^d)$, then $\overline{G}\left(x^{(d)}(Q_r)\right) > Q_r = \overline{G}\left(x^{(c)}(Q_r)\right)$ by part (ii) and Proposition 1 since $x_1^c < x_1^d$ by Lemma A.1(iii). Similarly, if $Q_r \geq \overline{G}(x_1^d)$, then $\overline{G}\left(x^{(d)}(Q_r)\right) = \overline{G}(x_1^d)$ by Proposition 3(i) and $\overline{G}\left(x^{(c)}(Q_r)\right) = \min\left\{\overline{G}(x_1^c), Q_r\right\} \geq \overline{G}(x_1^d)$ since $x_1^c < x_1^d$ by Lemma A.1(iii). Thus, $\overline{G}\left(x^{(d)}(Q_r)\right) > \overline{G}\left(x^{(c)}(Q_r)\right)$ if, and only if, $Q_r < \overline{G}(x_1^d)$. $\square$

Proof of Proposition 4: We prove each part of the proposition separately.

Part (i): By (15), the first and second derivatives of $\pi^{(d)}(Q)$ are, respectively, given by:

$$\frac{d\pi^{(d)}(Q)}{dQ} = w\int_0^{\frac{\overline{G}\left(x_1^d\right)}{Q}} ydF(y) - c,$$
$$\frac{d^2\pi^{(d)}(Q)}{dQ^2} = -w\frac{\overline{G}^2(x_1^d)}{Q^3} f\left(\frac{\overline{G}(x_1^d)}{Q}\right) < 0,$$

for $Q \geq 0$. Note that

$$\lim_{Q\downarrow 0}\frac{d\pi^{(d)}(Q)}{dQ} = w\mu - c > 0, \text{ and } \lim_{Q\uparrow\infty}\frac{d\pi^{(d)}(Q)}{dQ} = -c < 0,$$

where the first inequality follows from our assumption that $w\mu > c$. Then it follows that there exists a unique $Q^{(d)} \in (0,\infty)$ satisfying $d\pi^{(d)}\left(Q^{(d)}\right)/dQ = 0$ and $Q^{(d)}$ is the unique maximizer of $\pi^{(d)}(Q)$.

Part (ii): We prove this part by contradiction. Contrary to our claim, suppose that $Q^{(d)} \geq Q^{(c)}$. By (6), we have

$$c > \int_0^{\frac{\overline{G}\left(x_1^d\right)}{Q^{(c)}}} \bar{x}[-B\left(x_2^c\left(yQ^{(c)}\right)\right)\cdot y]dF(y) > \int_0^{\frac{\overline{G}\left(x_1^d\right)}{Q^{(d)}}} wydF(y).$$

We get the first inequality above by (10), and by $x_1^c < x_1^d$ (by Lemma A.1(iii)) and $B(x) < 0$ for $x > x_1^c$ (by Lemma A.1(ii)). The last inequality follows from our supposition that $Q^{(d)} \geq Q^{(c)}$ and $-\bar{x}B\left(x_2^c\left(\overline{G}\left(x_1^d\right)\right)\right) > w$ (by (12), (6) and Lemma A.1(ii)). However, the final result is a contradiction to Proposition 4(i) and hence $Q^{(d)} < Q^{(c)}$. □

Proof of Lemma 2: For any $Q_r > 0$, let $\tilde{h}^I$ be the vaccinated fraction in the equilibrium under the incentive program $I = (r(Q_r), T(Q_r))$. Assume that the individual with infection cost $\hat{x}$ does not seek vaccination, i.e., $\hat{x}(u(\tilde{h}^I) - v(\tilde{h}^I)) + r(Q_r) < w + p$. Notice that for any $x < \hat{x}$,this condition is satisfied and hence individuals with $x < \hat{x}$ do not seek vaccination. This indicates that under the incentive program I, there is a threshold $x^{(I)}(Q_r)$ such that only individuals with infection cost $x \geq x^{(I)}(Q_r)$ seek vaccination. □

Proof of Proposition 5: We prove each claim in Proposition 5 separately.

Part (i): We define $\psi^I(x, Q_r)$ as the excess of utility under the incentive program to the marginal individual with infection cost x, when seeking vaccination over not seeking. In addition, we let

$$\tau(x, Q_r) = x\left(u(h(x, Q_r) - v(h(x, Q_r))\right) \tag{B.2}$$

for $x \in [0, \bar{x}]$ and $Q_r > 0$, where h is given by (2). Then, by (18) and $h^{(c)}(Q_r) = h(x^c(Q_r), Q_r) = \overline{G}(x^c(Q_r))$, the excess utility under the incentive program can be written as follows:

$$\psi^I(x, Q_r) = \tau(x, Q_r) - \tau\left(x^{(c)}(Q_r), Q_r\right), \tag{B.3}$$

for all $Q_r > 0$ and $x \in [0, \bar{x}]$, where $x^{(c)}(Q_r)$ is given by (7). By using $h(x, Q_r) = \min\{\overline{G}(x), Q_r\}$ from (2), notice that, for any given $Q_r > 0$, $\tau(x, Q_r)$ as given in (B.2) is non-decreasing in x by Assumption

1(ii) and (iv). By (B.3), this implies that $\psi^I(x, Q_r)$ is also non-decreasing in x so that the threshold that determines the demand under the incentive program $x^{(I)}(Q_r)$ for $Q_r > 0$ is the minimum x, if any, such that $\psi^I(x, Q_r) > 0$. Next, we characterize this threshold x for all $Q_r > 0$ (i.e., $x^{(I)}(Q_r)$).

By (B.3), $\psi^I(x, Q_r) = 0$ at $x = x^{(c)}(Q_r)$ which by $\psi^I(x, Q_r)$ being non-decreasing in x implies that $\psi^I(x, Q_r) \leq 0$ for all $x \leq x^{(c)}(Q_r)$. Therefore, the threshold $x^{(I)}(Q_r)$ cannot be less than $x^{(c)}(Q_r)$. Then, it remains to consider $x > x^{(c)}(Q_r)$. By (7) and (11), $x^{(c)}(Q_r) = \max\{x_1^c, x_2^c(Q_r)\} \geq \max\{x_{zr}, x_2^c(Q_r)\}$, where the inequality follows from $x_1^c \geq x_{zr}$ by Lemma A.1(ii). Then it follows from (1) and (2) that, for $x > x^{(c)}(Q_r) \geq \max\{x_{zr}, x_2^c(Q_r)\}$, $h(x, Q_r) < \min\{h_{zr}, Q_r\}$ by (5) and (11). Then, for $x > x_2^{(c)}(Q_r)$, $h(x, Q_r) = \overline{G}(x)$ by (2) so that, by (B.2), $\tau(x, Q_r) = x\left(u(\overline{G}(x)) - v(\overline{G}(x))\right)$ is strictly increasing in x by Assumption 1. By (B.3) and $\psi^I(x, Q_r) \leq 0$ for $x \leq x_2^{(c)}(Q_r)$, this implies that $\psi^I(x, Q_r) > 0$ if, and only if, $x > x_2^{(c)}(Q_r)$. Therefore, $x^{(I)}(Q_r) = x^{(c)}(Q_r)$ for all $Q_r > 0$ so that the demand under the incentive program is always equal to the socially-optimal demand.

Part (ii): Recall from part (i) that $h^{(c)}(Q_r) = h\left(x^{(c)}(Q_r), Q_r\right) = h\left(x^{(I)}(Q_r), Q_r\right)$ for all $Q_r \geq 0$. Then, by (17), it turns out that

$$\pi^{(I)}(Q) = \mathbb{E}_Y\left[wh\left(x^{(c)}(YQ), YQ\right) + T(YQ)\right] - cQ,$$

for all $Q \geq 0$. Using $h\left(x^{(c)}(Q_r), Q_r\right) = \min\left\{\overline{G}(x_1^c), Q_r\right\}$ (by (7)) and (19), we obtain

$$\begin{aligned}
\frac{d\pi^{(I)}(Q)}{dQ} &= -\int_0^{\frac{\overline{G}(x_1^c)}{Q}} \frac{\bar{x}}{2}\left(\left(1-yQ\right)^2 u'(yQ) + \left(1-\left(1-yQ\right)^2\right) v'(yQ)\right) y dF(y) \\
&\quad + \int_0^{\frac{\overline{G}(x_1^c)}{Q}} \left(\bar{x}\left(1-yQ\right)\left(u(yQ) - v(yQ)\right) - p\right) y dF(u) - c \\
&= \int_0^{\frac{\overline{G}(x_1^c)}{Q}} \bar{x}[-B(x_2^c(yQ)) \cdot y] dF(y) - c
\end{aligned}$$

for all $Q \geq 0$, where $B(x)$ satisfies (6) and $\overline{G}(x_2^c(Q_r)) = Q_r$ for $Q_r < \overline{G}(x_1^c)$. By (10), $B(x)$ being decreasing in x for $x > x_{zr}$ and $x_1^c > x_{zr}$ (see Lemma A.1(ii) and (iii)), and $x_2^c(Q_r)$ being decreasing in Q_r (see (11)), this implies that $\pi^{(I)}(Q)$ is maximized at $Q = Q^{(c)}$. □

Proof of Proposition 6: We prove each part of the proposition separately.

Part (i): We first prove the existence of a positive $K \geq \underline{K}$. From (23), we have

$$\begin{aligned}
\underline{K} &= V_H + \pi^{(d)} - W^{(c)} \\
&= w\mathbb{E}_Y\left[h\left(x^{(d)}\left(YQ^{(d)}\right), YQ^{(d)}\right)\right] + p\mathbb{E}_Y\left[h\left(x^{(c)}\left(YQ^{(c)}\right), YQ^{(c)}\right)\right] \\
&\quad + \frac{\bar{x}}{2}\mathbb{E}_Y\left[\left(1-h\left(x^{(c)}\left(YQ^{(c)}\right), YQ^{(c)}\right)\right)^2\left(u\left(h\left(x^{(c)}\left(YQ^{(c)}\right), YQ^{(c)}\right)\right) - v\left(h\left(x^{(c)}\left(YQ^{(c)}\right), YQ^{(c)}\right)\right)\right)\right] \\
&\quad + \frac{\bar{x}}{2}\mathbb{E}_Y\left[v\left(h\left(x^{(c)}\left(YQ^{(c)}\right), YQ^{(c)}\right)\right)\right] + c\left(Q^{(c)} - Q^{(d)}\right) \\
&> 0
\end{aligned}$$

since $Q^{(c)} > Q^{(d)}$ by Proposition 4.

Next, we prove that the manufacturer's profit is higher under the incentive program with $K \geq \underline{K}$. By (17) and Proposition 5, the expected profit of the manufacturer under the incentive program is equal to

$$\pi^{(I)} = \mathbb{E}_Y \left[wh\left(x^{(c)}\left(YQ^{(c)}\right), YQ^{(c)}\right) + T\left(YQ^{(c)}\right)\right] - cQ^{(c)} = W^{(c)} + K - V_H \geq \pi^{(d)}$$

for $K \geq \underline{K}$, where the second equality follows from (9) and (19), and the inequality follows from $K \geq \underline{K} = V_H + \pi^{(d)} - W^{(c)}$. As a result, the manufacturer is not worse off ex ante.

Part (ii): In this part, we show that the expected total utility of all individuals under the incentive program with any K, the expected total utility of all individuals is high compared to the case of no incentive if $K_{bn} < \overline{K}$. The expected total utility of all individuals in the equilibrium of decentralized supply chain is equal to $W^{(d)} - \pi^{(d)}$. On the other hand, for any K, the expected total utility of all individuals under the incentive program with transfer payments (18)-(19) is equal to

$$W^{(c)} + \mathbb{E}_Y \left[\left(r\left(YQ^{(c)}\right) - w\right) h^{(c)}\left(YQ^{(c)}\right)\right] + cQ^{(c)} = V_H - K_{bn}, \tag{B.4}$$

where the equality follows from (9) and (18). Note by (22) that if $K_{bn} < \overline{K}$, the expected total utility of all individuals under the incentive program is greater than that in the decentralized supply chain, i.e., $V_H - K_{bn} \geq W^{(d)} - \pi^{(d)}$.

Part (iii): In this part, we show that the incentive program with $K = K_{bn}$ in (24) is ex-ante budget-neutral. By (21), expected total cost of subsidies to the manufacturer and vaccinated individuals with $K = K_{bn}$ is given by:

$$\mathbb{E}_Y \left[S^{(I)}(YQ^{(c)})\right] = K_{bn} - \mathbb{E}_Y \left[\frac{\bar{x}}{2}((1 - (h^{(c)}\left(YQ^{(c)}\right))^2)u(h^{(c)}\left(YQ^{(c)}\right)) + (h^{(c)}\left(YQ^{(c)}\right))^2 v(h^{(c)}\left(YQ^{(c)}\right)))\right] = 0,$$

where the second equality follows from (24). □

Proof of Proposition 7: By (21), under the incentive program $I = (t, r(Q_r), T(Q_r))$ with $t = K - K_{bn}$, and $(r(Q_r), T(Q_r))$ given in (18) and (19), the expected total cost due to subsidies to the manufacturer and vaccinated individuals is equal to $\mathbb{E}_Y \left[S^{(I)}(YQ^{(c)})\right] = K - K_{bn}$. Note that for $t = K - K_{bn}$, total tax collected ex ante is equal to the expected total cost due to subsidies to the manufacturer and individuals, i.e., $t = \mathbb{E}_Y \left[S^{(I)}(YQ^{(c)})\right]$. Hence, the incentive program with $t = K - K_{bn}$ is ex-ante budget-neutral.

We now prove three parts of the proposition. Note that, under the incentive program with or without the tax t, the transfer payment is the same and given by (19). Then, it follows from part (i) in the proof of Proposition 6 that the manufacturer's profit under the modified incentive program is equal to $\pi^{(I)} = W^{(c)} + K - V_H$ so that part (ii) of Proposition 7 follows from Proposition 6. In addition, the expected total

utility of all individuals under the incentive program $I = (t, r(Q_r), T(Q_r))$ with $t = K - K_{bn}$ and transfer payments (18)-(19) is equal to

$$W^{(c)} - K + K_{bn} + \mathbb{E}_Y \left[\left(r\left(YQ^{(c)}\right) - w \right) h^{(c)}\left(YQ^{(c)}\right) \right] + cQ^{(c)} = V_H - K, \tag{B.5}$$

where the equality follows from (9) and (18). Thus, part (i) of the proposition follows. This, by (22), implies that if $K < \overline{K}$, the expected total utility of all individuals under the incentive program is greater than that in thedecentralized supply chain, i.e., $V_H - K \geq W^{(d)} - \pi^{(d)}$. $\square$