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Luxury Brand Licensing: Competition and Reference Group Effects

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Marketing research has traditionally focused on *centralized* brand-extension strategies where a brand expands its product offerings by controlling the design, production, marketing and sales of new products 'in-house'. However, luxury brands frequently use 'brand licensing' as a *decentralized* brand-extension strategy under which a brand licenses its brand name to an 'external licensee' that designs, produces and sells the new product. Licensing is a time-efficient and cost-effective brand-extension strategy for luxury brands to reach out to their aspirational, low-end consumers ('followers') who value a brand more when more high-end consumers ('snobs') purchase the brand's primary product (i.e., 'positive popularity effect'). On the other hand, over-licensing might dilute the brand for snobs who value brand exclusivity (i.e., 'negative popularity effect'). We develop a game-theoretic model to study luxury brand licensing in the presence of these two countervailing forces. First, in the monopoly setting (a benchmark), we find that the monopoly brand should license only when the negative popularity effect is not too high, and it should prefer 'royalty licensing' over 'fixed-fee licensing' when the negative popularity effect is intermediate. Second, to explicate our analysis, we study the duopoly setting under fixed-fee contracts. In contrast to the monopoly setting, we find that fixed-fee licensing can 'soften' price competition between brands so that licensing is 'always' profitable for both brands under competition. Interestingly, in equilibrium under fixed-fee contracts, competing brands face a prisoner's dilemma and both brands prefer not to license, even though both would be better off if they could commit to fixed-fee licensing. Finally, we expand our analysis of the duopoly model by incorporating royalty licensing in addition to fixed-fee licensing. We find that, in contrast to fixed-fee licensing, royalty licensing can 'intensify' price competition so that both brands have to lower their prices. Consequently, when the positive popularity effect is sufficiently strong, fixed-fee licensing 'dominates' royalty licensing. We also show that, under competition, luxury brands should adopt royalty licensing contracts only when the licensing market is large, and positive and negative popularity effects are small enough.

Key words: Luxury brand; licensing contract; fixed fee; royalty fee; reference groups; competition

1. Introduction

A brand is a name, term, sign, symbol or design that contributes to the value of a product beyond its functional use (Farquhar 1989). A great example is Louis Vuitton: a luxury brand that has US$ 33.6 billion in brand value (Forbes 2018). Luxury brands usually build their initial brand image/reputation by designing, producing and selling unsurpassed quality products in certain categories for discerning customers. For example, Giorgio Armani offers a high-end designer clothing line, Gucci designs and manufactures handbags from fine leather, and Bang & Olufsen makes uniquely designed electronics.

In many instances, once the market for a luxury brand's primary (specialty) products matures, it faces pressure from investors to grow and capture aspirational consumers. To do so, many luxury brands use their strong brand image as a 'platform' and license their brand names for quickly launching products in new categories for aspirational consumers via licensing. Specifically, many luxury brands license their brand names to firms (licensees) with the expertise to design, produce and sell licensed products. For example, Burberry, Gucci and Hugo Boss license their fragrance and/or cosmetics business to Coty–one of world's largest beauty and fragrance companies (Sandle 2017). In the same vein, Bulgari, Ferragamo, Prada and Versace license their eyewear to Luxottica–the world's largest eyewear company (License Global 2018). In 2017, retail sales of licensed goods reached \$271.6 billion, and the bulk of this sales figure was generated from the sales of licensed goods that bear different luxury brand names (Greene 2009; Licensing Industry Merchandisers' Association 2018). In general, licensed products are significantly more affordable products than the primary products (The Fashion Law 2015; License Global 2018). For example, many consumers cannot afford Gucci handbags, but they can show their aspiration by purchasing licensed products such as Gucci fragrance (Centre for Fashion Enterprise 2012). Therefore, licensing creates an opportunity for luxury brands to build a presence for aspirational consumers in the mass market and to venture into new product categories with greater ease.

While appealing, licensing (if not carefully managed) might come at a price and dilute the image of a luxury brand because, under brand licensing, the system is 'decentralized' in the sense that the luxury brand loses its control of sales operations in the new product category to its licensee. When making their purchasing decisions, consumers of luxury brands' primary products (i.e., snobs) value exclusivity and are conscious about the composition (i.e., type and number) of consumers adopting the brand (Bourdieu 1984; Kapferer and Bastien 2009; Amaldoss and Jain 2008). Due to these social effects, in the event that its licensee develops and sells too many licensed products, the image of the luxury brand can be tarnished (Kort et al. 2006; The Economist 2004). Therefore, brand licensing can hurt the sales of a brand's primary products, as experienced by Gucci, Yves Saint Laurent (YSL) and Burberry when their licensing attempts failed (License Global 2018).[1]

Consequently, there are complex trade-offs when luxury brands need to decide whether or not to license their brand names to licensees. On the one hand, they can use licensing to grow and reach out to their aspirational consumers in the mass market who value the brand popularity (i.e., followers). On the other hand, licensing reduces luxury brands' attractiveness for consumers purchasing their primary products in the niche market who care about brands' exclusivity (i.e., snobs). Surprisingly, to the best of our knowledge,

[1] In the 1980s, Gucci licensed its brand name to different licensees who produced more than 22,000 products such as alcohol, key chains and even toilet paper and distributed them through department stores. This licensing strategy backfired because the Gucci brand was diluted and its image was associated with 'drug stores' (Jackson et al. 2002). Gucci gradually recovered its image by limiting the number of its licenses and by having tighter controls over its licensees.

there is no research on brand licensing in the marketing literature despite its importance (especially, for luxury brands), and it is still not clear whether luxury bands should license their brand names in new product categories or not.

Marketing research has traditionally focused on *centralized* brand-extension strategies where the brand extends by controlling the design, production, marketing and distribution of the new product 'in-house' (e.g., see Wernerfelt 1988; Choi 1998; Cabral 2000; Keller and Lehmann 2006; Amaldoss and Jain 2015, and references therein). However, brand licensing is a *decentralized* brand-extension strategy where, through a licensing contract, the brand licenses its name to an 'external licensee', who designs, produces and sells the new product. For this reason, the strategic interactions between the brand and its licensee, and the issue of licensing contracts remain silent in the marketing literature. Therefore, our paper represents an initial attempt to examine how social effects and competition can affect luxury brand licensing.

In this paper, we develop a game-theoretic model to examine how competition and interactions between snobs and followers (i.e., 'reference group' effects) impact on a luxury brand's licensing strategy. We consider two competing *brands* who produce their primary products in the same category (e.g., handbags) and sell them in the same niche market. At the same time, by using either *fixed-fee* or *royalty* contracts, both brands consider licensing their brand names to two competing *licensees* which have expertise in producing affordable products in a different category (e.g., eyewear) and selling them in the same mass market. Under a fixed-fee contract, the licensee pays the brand a lump-sum fixed fee upfront, and the licensee has the right to produce and sell certain products that carry the brand name for an extended time frame (Centre for Fashion Enterprise 2012; Chevalier and Mazzalovo 2012). Under a royalty licensing contract, the brand charges the licensee a per-unit royalty fee for each unit sold (License Global 2018; Greene 2009; Centre for Fashion Enterprise 2012). On the consumer side, we model consumers' segment-specific desire for uniqueness or *reference group effects* by considering two segments, namely, 'snobs' and 'followers'. Snobs in the niche market value exclusivity and they do not want to be associated with followers (i.e., *negative popularity effect*). However, followers in the mass market have a strong desire to assimilate the same brand adopted by snobs so that they value licensees' products more as more snobs purchase the brand (i.e., *positive popularity effect*). Only snobs can afford brands' expensive primary products; therefore, brands offer their primary products to snobs, while licensees offer their licensed products to followers.

As a benchmark, we study the monopoly case and find that the monopolist brand should not license when the snobs' negative popularity effect is too high. We also show that, when the snobs' negative popularity effect is intermediate (neither too high nor too low), a royalty licensing contract is preferred by the monopolist brand over a fixed-fee licensing contract. This is because the brand can use the royalty fee to influence its licensee's selling price and the sales of the licensed product. In doing so, the brand can manage the impact of the negative popularity effect on the snobs' demand for the primary product. In summary, we complement economics literature on patent licensing (e.g., see Bousquet et al. 1998; Beggs 1992; Poddar and Sinha 2002), and identify social effects (or conspicuous consumption) as another rationale behind royalty contracts that are frequently observed in practice (License Global 2018; Greene 2009; Centre for Fashion Enterprise 2012).

We also examine the duopoly setting. To explicate our analysis, we first consider the case when brands can only use fixed-fee contracts to license. Interestingly, in contrast to the monopoly setting, we find that licensing is always beneficial for both brands since fixed-fee licensing creates an *indirect (strategic) effect* that 'softens' price competition between brands so that both brands can afford to increase their selling price without losing market share. This is in contrast to Amaldoss and Jain (2015) who show that a centralized brand-extension strategy (i.e., 'umbrella branding) 'intensifies' price competition between brands and reduces brands' profits from their primary products. As a result, this result challenges a common belief among luxury brand experts (e.g., Kapferer and Bastien 2009; Kapferer 2015) and implies that, under competition, luxury brands can benefit from a decentralized brand extension via 'brand licensing'. We also characterize brands' equilibrium licensing strategies under fixed-fee contracts. We find that licensing is not always optimal for both brands and, in equilibrium, both brands license only when the negative popularity effect is sufficiently low. When the snobs' negative popularity effect is above a certain threshold, each brand would have earned more if they could both commit to licensing via fixed-fee contracts; however, in the absence of such a commitment, we find that both brands would face a prisoner's dilemma and do not license in equilibrium.

Next, we incorporate royalty licensing contracts into our duopoly model and extend our analysis to the case where brands can use either royalty or fixed-fee contracts when they license. We find that a royalty licensing contract can create a new *royalty effect* that 'intensifies' the competition between brands so that both brands will lower their prices when they both license. This result is driven by the fact that, under a royalty licensing contract, both brands can earn more royalties by increasing the followers' demand for the licensed product. Because of the followers' positive popularity effect, both brands can increase the followers' demand in the mass market by increasing the snobs' demand in the niche market. As both brands compete for higher demand in the niche market, the price competition between them is intensified. As a result, when both brands license, prices of their primary products will be lower under a royalty contract than under a fixed-fee contract. When the followers' positive popularity effect is sufficiently high, the competition between brands in the snob market is very intense and the royalty contract is dominated by the fixed-fee contract.

Finally, we characterize licensing strategies of brands in equilibrium where each brand can use a fixed-fee or royalty contract, and we identify cases where two brands use symmetric or asymmetric licensing strategies. We show that, in cases where positive and/or negative popularity effects are sufficiently high, the royalty contract will never dominate and, whenever a brand chooses to license in equilibrium, it will adopt the fixed-fee contract. We find that, in the event when both brands choose to license, a royalty licensing contract is preferred in equilibrium by at least one brand, only when positive and negative popularity effects are both low and the licensing market is large enough. All aforementioned results have important managerial implications, which we shall discuss in §7.

This paper is organized as follows: In the following section, we review the related literature. In §3, we present our model and assumptions. We study the monopoly setting in §4. In §5, we present our analysis of the duopoly setting with fixed-fee contracts. §6 extends our duopoly analysis in 5 by considering royalty licensing contracts. Finally, we discuss the managerial implications of our results in the context of luxury brand licensing and the future research in §7. Proofs of all results in the paper are presented in Appendix C.2.

2. Literature Review

Our paper is related to three research streams: *patent licensing*, *distribution channel* and *conspicuous consumption*. First, the economics literature on patent licensing dates back to Arrow (1962). Using different game-theoretic frameworks, several economists analyzed different licensing strategies of an inventor (licensor). Kamien and Tauman (1986), Katz and Shapiro (1986), Kamien (1992), and Kamien et al. (1992) show that, when there is perfect information, fixed-fee licensing outperforms royalty licensing for the inventor when the inventor (licensor) is an outsider and does not compete with its licensees. However, royalty licensing dominates when the inventor is an insider and competes with its licensees, and/or when there is demand/cost uncertainty or information asymmetry; see Bousquet et al. (1998), Beggs (1992), Gallini and Wright (1990), and Choi (2001). Unlike the economic literature on patent licensing, we examine the issue of luxury brand licensing by considering reference group effects and are able to capture the impact of licensing on luxury brand dilution.

Second, there is extensive literature in marketing and operations management on the distribution channel (or the supply chain). By studying a bilateral monopoly (i.e., an upstream firm (manufacturer) selling its product through a downstream firm (retailer)), the literature identifies the decentralization as the main cause of channel inefficiency and focuses on the vertical integration (or coordination) between channel members through pricing schemes or formal contracts (e.g., see Jeuland and Shugan 1983; Moorthy 1987; Cachon and Lariviere 2005; Cachon 2003). The only exception is Su and Zhang (2008) who show that a bilateral monopoly channel can benefit from decentralization if consumers are forward-looking and can strategically delay their purchases. Cachon and Kök (2010) study a channel with multiple competing upstream firms (manufacturers) that sell their products through a common downstream firm (retailer) by using a two-part tariff, or wholesale-price and quantity-discount contracts. By allowing upstream firms to compete for the business of the downstream firm, they show that a two-part tariff or quantity discount contract intensifies price competition between upstream firms so that they are better off using wholesale price contracts. Ingene and Parry (2000) consider an upstream firm selling through two competing downstream firms and determine the conditions under which a channel-coordinating wholesale-price strategy is preferred by the upstream firm over a two-part tariff. McGuire and Staelin (1983) study a distribution channel where two competing upstream firms (manufacturers) vertically integrate into the downstream (e.g., retailing) or sell their products through two dedicated downstream firms (retailers). They show that, when the products are highly substitutable, it is not optimal for upstream firms (or their channels) to vertically integrate and control the operations of their downstream firms. This is because, in such cases, decentralization within the channel softens price competition between upstream firms and increases their profits as well as the profit of their respective channels. Moorthy (1988) studies the same channel structure as in McGuire and Staelin (1983) and identifies general conditions under which decentralization within a channel is optimal in equilibrium. Unlike the literature on the distribution channel, our paper considers reference group effects and studies a more general channel structure with two upstream firms (brands) that already compete against each other in an existing market (or a product category) and consider expanding into a new market (or a product category) by licensing their brand names and selling through two dedicated competing downstream firms (licensees).

Third, the literature on conspicuous consumption dates back to Veblen (1899) who postulates that individuals consume conspicuous products to signal their wealth and social status. Becker (1991), and Corneo and Jeanne (1997) show that the demand for a product may increase in its price when consumers are followers (conformists) and value a product more when more people purchase it. Amaldoss and Jain (2005a,b) develop a model of conspicuous consumption and analyze how demand and price of a firm are affected by snobs and followers in the monopoly and duopoly settings. Agrawal et al. (2015) analyze the product design and introduction strategies of a firm selling a conspicuous durable product over multiple periods and find that, with exclusivity-seeking consumers, firms introduce products with high durability at low volume and high price. Arifoğlu et al. (2020) consider snobbish consumers with heterogenous (high and low) valuations. They find that exclusivity-seeking consumer behavior leads to buying frenzies and price markdowns. Unlike these papers, we model reference group effects and analyze luxury brands' licensing strategies. There are also several papers that study the impact of conspicuous consumption on pricing and the product management strategies of firms selling multiple products (e.g., Balachander and Stock 2009; Amaldoss and Jain 2008). Unlike these papers, we analyze the implications of reference group effects on brands' licensing strategies.

In this paper we adopt the modeling framework developed by Amaldoss and Jain (2015) to capture: (1) the snobs' negative popularity effect; and (2) the followers' positive popularity effect. However, our paper is fundamentally different in four aspects.

- First, unlike Amaldoss and Jain (2015) who study the 'product line extension' within the *same* category through *umbrella* or *individual branding*, we focus on the 'product category extension' through *brand licensing* and aim to determine when a brand should license its brand name to extend in a *new* product category.

• Second, unlike Amaldoss and Jain (2015) who examine centralized brand-extension strategies (i.e., umbrella and individual branding), we investigate the issue of *brand licensing* that is a decentralized brand-extension strategy where each brand licenses its brand name to an external licensee to produce and sell a different and more affordable product to followers. This enables us to model strategic interactions between brands and their licensees and to capture the impact of decentralization on brands' brand-extension strategies.

• Third, we examine fixed-fee and royalty licensing contracts arising from a decentralized brand extension, whereas such contracting issues do not exist in a centralized brand extension as examined in Amaldoss and Jain (2015). In doing so, we are able to compare fixed-fee and royalty licensing contracts and determine their impact.

• Fourth, we obtain some new findings. We find that fixed-fee licensing 'softens' price competition between brands, and improves brands' profits from their primary products even without taking into account their licensing revenues. This result is fundamentally different from Amaldoss and Jain (2015) who show that umbrella branding 'intensifies' price competition between brands and reduces brands' profits from their primary products. We also find that royalty licensing 'intensifies' the price competition between brands when the positive popularity effect is high. Consequently, under competition, we find that fixed-fee licensing dominates royalty licensing, especially when the followers' positive popularity effect is strong.

Overall, our paper is the first to examine different brand licensing strategies (i.e., fixed-fee and royalty contracts) operating in a decentralized system in the presence of reference group effects and competition.

3. Model Preliminaries

Consider two competing luxury brands A and B that produce and sell the same category of 'speciality' and 'more expensive' product(s) in a niche/exclusive market (e.g., Fendi and Gucci for leather goods). To grow quickly, each brand considers licensing its brand name to its corresponding (external) licensee (say, licensee a for brand A and licensee b for brand B) who has expertise in designing, producing and selling a different category of 'more affordable' product (e.g., cologne) in the mass market that carries the corresponding brand name. We assume that the unit production cost of the brands' primary product is equal to c, which is higher than the unit production cost of licensed goods (produced by licensees) that we normalize to 0.

Market structure. The 'primary' products of both brands (A and B) are sold in market s comprised of high-end, exclusivity-seeking consumers (i.e., 'snobs') with market size equal to 1. The 'licensed' products produced by the licensees (a and b) are sold in a different market f comprised of low-end, aspirational consumers (i.e., 'followers') with market size β (that can be larger than market s). Followers cannot afford brands' primary products that are very expensive, but they can satisfy their aspirations by purchasing affordable licensed products (Centre for Fashion Enterprise 2012; The Fashion Law 2015; Amaldoss and Jain 2015). For tractability, we assume that snobs will never purchase the licensed products (Amaldoss and Jain

2015).[2] In doing so, we ensure that markets s and f are 'separate', and thereby, isolate the 'competition effect' within each market and the 'popularity effect' across markets that can be described as follows.

Within Market *Competition Effect.* For both snobs and followers, a product's value is influenced by *functional* and *social* effects. Within each market s (or f), we use the Hotelling model to capture heterogeneous preferences for the functionality of the product so that all snobs are uniformly distributed over the line $[0,1]$, where brand A's product is located at 0 and B at 1. Hence, for a snob who is located at θ, his/her functional value for brand A's product is $(v_s - t_s\theta)$ and for brand B's product is $(v_s - t_s(1-\theta))$ so that both firms engage in price competition within market s. Here, v_s is the base valuation of the product associated with each brand and t_s represents the 'fit-cost-loss' coefficient.

Using a similar construct, we assume that licensed product a is located at 0 and b at 1, a follower located at θ values product a at $(v_f - t_f\theta)$ and values b at $(v_f - t_f(1-\theta))$ so that licensees a and b engage in price competition within market f.

Cross Market *Social Effects.* Through licensing, a brand's name is exposed to both snobs and followers in markets s and f, which can bring about reference group effects, namely, 'positive' and 'negative' popularity effects among snobs and followers. Snobs despise the popularity of licensed products sold in market f so that a snob's utility derived from purchasing brand I is decreasing in $D_f^{i(e)}$, where $D_f^{i(e)}$ is his/her expectation about the proportion of followers purchasing the licensed product i in market f. Accounting for its functional value, the net utility that a snob located at θ will derive from purchasing product A is given by:

$$U_s^A(\theta) = (v_s - t_s\theta) - \lambda_s \beta D_f^{a(e)} - p^A, \tag{1}$$

where λ_s denotes the snobs' 'negative popularity effect' of licensing a brand in market f, $\beta D_f^{a(e)}$ is the number of brand A's licensed product that snobs expect to be sold in market f, and p^A is the selling price. The net utility for purchasing brand B can be obtained in the same manner. The negative popularity effect λ_s represents luxury consumers' desire to distinguish themselves from the masses by adopting exclusive brands (Bourdieu 1984; The Economist 2004; Kapferer and Bastien 2009). By using λ_s, we are able to capture the effect of licensing on luxury brand dilution (Kort et al. 2006; Colucci et al. 2008; License Global 2018; Patrick and Monga 2020).

Followers in market f interpret the popularity of a brand in market s as a form of endorsement; hence, a follower's utility derived from purchasing licensed product i is 'increasing' in $D_s^{I(e)}$, where $D_s^{I(e)}$ is his/her expectation about the proportion of snobs purchasing the product of luxury brand I in market s. More formally, the net utility that a follower located at θ will derive from purchasing the licensed product a is given by:

$$U_f^a(\theta) = (v_f - t_f\theta) + \lambda_f D_s^{A(e)} - p^a, \tag{2}$$

[2] It is possible that a very small fraction of snobs may purchase the licensed goods. However, we assume that this fraction is negligible.

where λ_f represents the followers' 'positive popularity effect' associated with the sales of the luxury brand in market s, and p^a is the selling price. We can obtain a similar expression for the net utility associated with licensee b's product. The positive popularity effect λ_f represents masses' appreciation of a luxury brand and captures the desire of aspirational consumers to adopt the same brand as high-end consumers (snobs) (Amaldoss and Jain 2008; Bevolo et al. 2011; License Global 2018).

REMARK 1 (WITHIN MARKET SOCIAL EFFECTS). Consumers' individual-level desire for uniqueness within each market (e.g., snobs and followers might want to be different from everyone else in their own market) exists even when a brand does not license and has second-order impact on a luxury firm's brand licensing strategies compared to reference group effects across markets (segment-specific desire for uniqueness). Therefore, for tractability and clarity, we focus on reference group effects and ignore consumers' individual-level desire for uniqueness.

Profit Functions of a Brand and its Licensee: Each brand I ($I = A, B$) licenses its name through a contract that specifies a transfer payment T^I to be collected from its corresponding licensee i ($i = a, b$). Letting D_s^I ($I = A, B$) be the *actual* proportion of snobs purchasing from luxury brand I, and accounting for the transfer payment T^I associated with a licensing contract, the profits of brand I and its corresponding licensee i can be written as:

$$\Pi^I\left(p^I\right) = (p^I - c)D_s^I + T^I, \tag{3}$$

$$\Pi^i\left(p^i\right) = p^i\beta D_f^i - T^I. \tag{4}$$

Licensing Contracts: In this paper, we study *fixed-fee* and *royalty* contracts that are commonly used in the luxury goods industry (Centre for Fashion Enterprise 2012; Chevalier and Mazzalovo 2012). Under a fixed-fee contract, the licensee i pays a fixed fee k^I (lump-sum payment) to luxury brand I (i.e., $T^I = k^I$) upfront and the licensee obtains the right to produce and sell any amount of the licensed product. Under a royalty contract, the brand I ($I = A, B$) charges its respective external licensee i ($i = a, b$) a license fee $T^I = r^I\beta D_f^i$ that depends on the demand in market f, where the 'royalty fee per unit sold' r^I is determined by brand I.[3]

REMARK 2 (MIXED CONTRACTS). To isolate the effect associated with the fixed fee k^I and royalty fee r^I, we consider fixed-fee and royalty contracts separately instead of jointly as in a mixed contract with the transfer payment $T^I = k^I + r^I\beta D_f^i$. We tease out the impact of fixed lump-sum payment k^I and per-unit royalty fee r^I on luxury brand licensing, and show that they have different implications on brands' prices and profits. Identifying the combined effect of a fixed fee and per-unit royalty fee is beyond the scope of this paper.

[3] In practice, the royalties are based on a percentage of the licensee's overall revenue, and this percentage $\alpha^I \leq 1$ is specified by brand I ($I = A, B$). In this case, the transfer payment between brand I and its licensee i ($i = a, b$) is equal to $T^I = \alpha p^i\beta D_f^i$. In line with the literature (e.g., Kamien and Tauman 1986; Poddar and Sinha 2002), we assume that royalties are collected for each unit sold. However, by letting $\alpha^I = r^I/p^i$ for $I = A, B$ and $i = a, b$, it is easy to check that both models are equivalent in our setting.

Rational Expectations Equilibrium: We note that snobs' or followers' expectations, i.e., $D_f^{i(e)}$ $(i = a, b)$ and $D_s^{I(e)}$ $(I = A, B)$, can be different from the actual consumption, i.e., D_f^i $(i = a, b)$ and D_s^I $(I = A, B)$. However, by using the concept of rational expectations equilibrium (Amaldoss and Jain 2005a, 2015; Su and Zhang 2008; Liu and van Ryzin 2008), the actual proportion is equal to the anticipated proportion in equilibrium so that $D_s^I = D_s^{I(e)}$ for $I = A, B$ and $D_f^i = D_f^{i(e)}$ for $i = a, b$.

4. Monopoly

Consider the case when brand A operates as a monopoly located at 0 in market s. Brand A may license its brand name to licensee a located at 0 in market f. For ease of exposition, we shall restrict our analysis to the case when market s is fully covered (i.e., the resulting $D_s^A = 1$), which happens when the base valuation v_s is sufficiently high. (Our results in the monopoly setting continue to hold even when market s is partially covered. We omit the analysis of this case for brevity.)

We consider the following sequence of events. First, brand A decides whether to license or not, and if it licenses, it determines and offers a (fixed-fee or royalty) contract to licensee a. If licensee a agrees the contract, then brand A sets its price p^A for its luxury goods to be sold in market s and licensee a sets its own price p^a for its licensed product to be sold in market f simultaneously. (If licensee a rejects the contract, then no licensing will occur and brand A operates as a monopoly in market s.) Lastly, snobs in market s decide whether to purchase brand A's product, and followers in market f decide whether to purchase licensee a's licensed product. This sequence of events is modeled as a sequential game, and we use backward induction to characterize the equilibrium of this sequential game.

Next, we consider three cases: (i) brand A does not license; (ii) brand A licenses via fixed-fee contract; and (iii) brand A licenses via royalty contract. Then, comparing brand A's optimal profits in these three cases, we determine its equilibrium licensing strategy in §4.4.

4.1. No licensing (NL)

As a benchmark, suppose brand A does not license to licensee a so that $D_f^a = 0$ and $T^A = 0$. It is optimal for brand A to set its price equal to $p^A = v_s - t_s$ so that market s is covered. Thus, brand A's profit in the case of no licensing is given by:

$$\Pi^A(NL) = v_s - t_s - c. \tag{5}$$

4.2. Fixed-fee licensing contract (F)

We now consider the case when brand A licenses its brand name to its licensee a by charging a fixed-fee $T^A = k^A$. In Lemma 1, we characterize the fixed fee and prices of brand A and its licensee a when brand A uses a fixed-fee contract to license.

LEMMA 1. *When brand A licenses its brand name to licensee a by using a fixed-fee contract, the fixed fee and prices are, respectively, given by:*

$$k^A = \begin{cases} \beta\frac{(v_f+\lambda_f)^2}{4t_f}, & \text{if } \lambda_f < 2t_f - v_f, \\ \beta(v_f - t_f + \lambda_f), & \text{if } \lambda_f \geq 2t_f - v_f, \end{cases}$$

$$p^A = \begin{cases} v_s - t_s - \lambda_s \beta \frac{v_f+\lambda_f}{2t_f}, & \text{if } \lambda_f < 2t_f - v_f, \\ v_s - t_s - \lambda_s \beta, & \text{if } \lambda_f \geq 2t_f - v_f, \end{cases}$$

$$p^a = \begin{cases} \frac{v_f+\lambda_f}{2}, & \text{if } \lambda_f < 2t_f - v_f, \\ v_f + \lambda_f - t_f, & \text{if } \lambda_f \geq 2t_f - v_f. \end{cases}$$

Through fixed lump-sum payment k^A under the fixed-fee contract, brand A extracts the entire surplus of licensee a (i.e., $k^A = p^a\beta$) so that licensee a ends up with zero profit (and yet accepts the licensing contract). Lemma 1 shows that the fixed lump-sum payment is strictly increasing in λ_f. This implies that brand A can command a higher fixed licensing fee k^A as followers appreciate the brand's popularity in market s more (i.e., as λ_f increases). Lemma 1 also reveals that licensing causes the monopoly luxury brand to lower its selling price compared to the case of no licensing, i.e., $p^A < v_s - t_s$. This phenomenon is caused by the snobs' *negative popularity effect* λ_s. When it licenses, brand A suffers from a lower margin for its luxury good, but it recovers this loss from the licensing fee k^A to be collected from licensee a.

Brand A's profit under the fixed-fee contract is given by:

$$\Pi^A(F) = \begin{cases} v_s - t_s - c + \beta \frac{v_f+\lambda_f}{2t_f}\left(\frac{v_f+\lambda_f}{2} - \lambda_s\right), & \text{if } \lambda_f < 2t_f - v_f, \\ \\ v_s - t_s - c + \beta\left(v_f + \lambda_f - \lambda_s - t_f\right), & \text{if } \lambda_f \geq 2t_f - v_f. \end{cases} \tag{6}$$

4.3. Royalty contract

Next, we consider the case when brand A uses a royalty contract and charges r^A to licensee a for each unit sold in market f, i.e., $T^A = r^A\beta D^a_f$. Lemma 2 characterizes the royalty fee and prices when brand A licenses by using royalty contract.

LEMMA 2. *Suppose that market s is fully covered. When brand A licenses its brand name to licensee a by using a royalty contract, the per-unit royalty fee and prices are, respectively, given by:*

$$r^A = \begin{cases} v_f + \lambda_f - 2t_f, & \text{if } \lambda_s \leq v_f + \lambda_f - 4t_f, \\ \\ \frac{v_f+\lambda_f+\lambda_s}{2}, & \text{if } v_f + \lambda_f - 4t_f < \lambda_s < v_f + \lambda_f, \\ \\ v_f + \lambda_f, & \text{if } \lambda_s \geq v_f + \lambda_f, \end{cases}$$

$$p^A = v_s - t_s - \frac{\lambda_s\beta}{2t_f}\left(v_f + \lambda_f - r^A\right) \text{ and } p^a = \frac{v_f + \lambda_f + r^A}{2}.$$

From Lemma 2, we observe that as snobs' 'negative popularity effect' λ_s increases, brand A charges a higher royalty fee r^A. With a higher royalty fee r^A, licensee a's marginal cost increases so that it charges a higher price p^a and sells to fewer followers. Therefore, unlike the fixed-fee contract with which brand A extracts all licensing revenues (i.e., $\Pi^a = 0$), brand A can impact on its licensee's price and sales via the royalty fee r^A. In return, because $r^A < p^a$ when $\lambda_s < v_f + \lambda_f$, brand A gives up on some licensing revenues and licensee a can obtain a positive profit (i.e., $\Pi^a > 0$) under the royalty contract.

Brand A's profit under the royalty contract is given by:

$$\Pi^A(R) = v_s - t_s - c + \frac{\beta}{2t_f}(r^A - \lambda_s)\left(v_f + \lambda_f - r^A\right). \tag{7}$$

4.4. Equilibrium licensing strategy of the monopoly brand

To characterize the equilibrium licensing strategy of brand A, we compare brand A's profit without licensing (i.e., $\Pi^A(NL) = v_s - t_s - c$) and with fixed-fee and royalty contracts (i.e., $\Pi^A(F)$ in (6) and $\Pi^A(R)$ in (7)) and obtain the following result.

PROPOSITION 1. *(i) Brand A does not license, i.e.,* $\Pi^A(NL) \geq \max\{\Pi^A(F), \Pi^A(R)\}$, *if, and only if,* $\lambda_s \geq v_f + \lambda_f$.

(ii) When brand A licenses (i.e., $\lambda_s < v_f + \lambda_f$*), the royalty contract dominates the fixed-fee contract, i.e.,* $\Pi^A(R) > \Pi^A(F)$*, if, and only if: (1)* $\lambda_s > \left(\sqrt{2}-1\right)(v_f + \lambda_f)$ *when* $\lambda_f < 2t_f - v_f$*; or (2)* $\lambda_s > v_f + \lambda_f - 2\left(2-\sqrt{2}\right)t_f$ *when* $\lambda_f \geq 2t_f - v_f$.

Proposition 1(i) implies that the licensing decision of a monopoly is driven by the interplay between snobs' negative popularity effect λ_s and followers' positive popularity effect λ_f. Specifically, when the snobs are sufficiently less sensitive towards the popularity of the brand in market f (i.e., when λ_s is lower than a threshold that increases in followers' positive popularity effect λ_f), brand A can afford to license its name to licensee a because the gain from licensing outweighs the loss caused by the lower profit margin in market s.

Proposition 1(ii) shows that, when licensing, brand A prefers the fixed-fee contract when snobs' negative popularity effect is sufficiently low, while it prefers the royalty contract when the negative popularity effect is neither too high nor too low. While brand A can extract the entire surplus of licensee a under the fixed-fee contract, the royalty contract enables the brand to affect it licensee's marginal cost and sales. When λ_s is intermediate, it is important for brand A to strike the balance between sales of its own luxury goods in market s and the royalties to be collected from licensee a (based on the sales of the licensed product in market f). Consequently, the royalty contract dominates the fixed-fee contract in these cases. This result explains why royalty contracts are commonly observed in highly conspicuous markets (Centre for Fashion Enterprise 2012). It also complements the literature and shows that conspicuous consumption can be another rationale behind royalty contracts in addition to uncertain demand (Bousquet et al. 1998), asymmetric information (Beggs 1992; Gallini and Wright 1990; Choi 2001), and competition (Poddar and Sinha 2002).

5. Duopoly: Fixed-Fee Licensing Contracts

To explicate our analysis and ease our exposition, we examine the case when competing brands can only use fixed-fee contracts if they decide to license. (In §6, we expand our duopoly analysis by incorporating royalty contracts.) Because brand A and brand B are symmetric, it suffices to consider three cases: (i) both brands do not license; (ii) both brands license via fixed-fee contracts; and (iii) only one brand licenses via a fixed-fee contract and the other brand does not license. Below, we study each of these three cases, and then, by comparing brands' optimal profits associated with these three cases, we characterize the equilibrium licensing strategies of both brands under fixed-fee contracts in §5.4.

For tractability, both market s and f are fully covered (i.e., $D_s^A + D_s^B = 1$, and $D_f^a + D_f^b = 1$), and brands prefer being a monopoly in market s over competing each other in market s and/or in market f through their licensees (i.e., v_s and v_f are high enough). The sequence of events is similar to that described in §4.

REMARK 3 (RELAXING FULL-MARKET COVERAGE). We show that all our results continue to hold when we relax our full-market coverage assumption and study the duopoly model with fully covered market s and partially covered market f (see Appendix A). We also study the duopoly model where both market s and f are partially covered. In this case, each brand and its licensee become a 'local monopoly' in their respective markets. Hence, there is no competition in both markets and the duopoly model is a simple extension of the monopoly model with two local monopoly brands and their licensees. We omit the analysis of this case for brevity.

5.1. Both brands do not license $(\mathbf{NL}, \mathbf{NL})$

Suppose both luxury brands do not license (so that $D_f^a = D_f^b = 0$) and compete only in the snob market s. Then (1) reveals that a snob located at θ will obtain a net utility $U_s^A(\theta) = v_s - t_s\theta - p^A$ from purchasing A or $U_s^B(\theta) = v_s - t_s(1-\theta) - p^B$ from purchasing B. Hence, the marginal snob θ_s is indifferent between A and B, where $\theta_s = 1/2 + (p^B - p^A)/2t_s$. Because market s is fully covered, the proportion of snobs purchasing from brand A and B are $D_s^A = \theta_s$ and $D_s^B = 1 - \theta_s$, respectively. Substituting D_s^A, D_s^B, and $T^A = T^B = 0$ into (3), we obtain the profits of brand A and B. Then, by considering the first-order conditions, the optimal prices for the case when both brands do not license are equal to $p^A = p^B = c + t_s$. Consequently, by (3), the brands' profits for the case when both do not license satisfy:

$$\Pi^I(NL, NL) = \frac{t_s}{2} \text{ for } I = A, B. \tag{8}$$

Throughout this paper, we use '(X, Y)' to denote the case when brand A chooses licensing strategy X and brand B chooses licensing strategy Y.

5.2. Both brands license via fixed-fee contracts $(\mathbf{F}, \mathbf{F})$

When brands A and B license their brand names to external licensees a and b by charging fixed fees $T^A = k^A$ and $T^B = k^B$, respectively, we get:

LEMMA 3. *When both brands license their brand names to their respective licensees by using fixed-fee contracts, the fixed fee and prices are, respectively, given by:*

$$k^I = \left(t_f + \frac{\beta\lambda_f\lambda_s}{t_s}\right)\frac{\beta}{2}.$$

$$p^I = c + t_s + \frac{\beta\lambda_f\lambda_s}{t_f}, \text{ and } p^i = t_f + \frac{\beta\lambda_f\lambda_s}{t_s}$$

for $I = A, B$ and $i = a, b$.

Lemma 3 shows that, unlike in the monopoly case (F) where the brand licenses via a fixed-fee contract (see p^A in Lemma 1), the brands' prices when they both use fixed-fee contracts are increasing in the snobs' sensitivity to brand popularity λ_s. When both brands use fixed-fee contracts to license, their licensees adopt symmetric pricing strategies (i.e., $p^a = p^b$ by Lemma 3) and share market f equally (i.e., $D_f^i = 1/2$ for $i = a, b$). Consequently, when snobs compare two brands in terms of their exclusivity in market f, brands are identical and the 'negative popularity effect' of each brand cancels each other out so that the net effect is absent.

In addition, Lemma 3 reveals that, relative to brands' equilibrium prices associated with the no-licensing case (NL, NL) presented in §5.1, the prices of both brands are 'higher' when they both license via fixed-fee contracts (i.e., $p^I > c + t_s$ for $I = A, B$). Notice that the term $\beta\lambda_f\lambda_s/t_f$ in p^I as stated in Lemma 3 captures an 'indirect effect' of licensing that can 'soften competition' in markets s and f (see Cabral and Villas-Boas 2005). To examine how fixed-fee licensing softens competition between brands, let us suppose that brand A increases its price by one unit. Then brand B's market share in market s will increase, and this increase in popularity of brand B will make licensee b's product (that carries brand B's name) more attractive to followers in market f (due to the followers' 'positive' popularity effect). Consequently, licensee a's sales will decrease, but it will increase the snobs' valuation of brand A in market s (due to the snobs' 'negative' popularity effect), which affords brand A to increase its price a little bit without affecting its demand. As competition between brand A and B in market s softens, both brands can afford to charge higher prices with licensing (than the case when no brand licenses). Furthermore, as followers' desire to adopt the brand or snobs' sensitivity to the brand popularity in market f increases, a unit increase in brand A's price has more impact on licensee a's market share or snobs' valuations. Consequently, the competition between brands softens more, and brands' prices increase in λ_s and λ_f.

When both brands use fixed-fee contracts to license, their profits are given by:

$$\Pi^I(F, F) = \frac{t_s}{2} + \frac{\beta\lambda_f\lambda_s}{2t_f} + \left(t_f + \frac{\beta\lambda_f\lambda_s}{t_s}\right)\frac{\beta}{2} \tag{9}$$

for $I = A, B$. Notice by (9) and $\Pi^I(NL, NL) = t_s/2$ that $\Pi^I(F, F) > \Pi^I(NL, NL)$ for $I = A, B$. Thus, Lemma 4 follows.

LEMMA 4. *Relative to the no-licensing case, each brand earns more when they both license by using fixed-fee contracts, i.e.,* $\Pi^I(F,F) > \Pi^I(NL,NL)$ *for* $I = A, B$.

Lemma 4 implies that both brands earn more by licensing their brands to their respective licensees by using fixed-fee contract. This result is in contrast to the monopoly case (F) (see Proposition 1) and appears to be counter-intuitive because licensing has a 'negative popularity effect' on snobs' valuations. However, as discussed above, in duopoly when both brands use fixed-fee licensing, the negative popularity effect of licensing is absent and the competition between brands is softened due to the 'indirect effect' across markets s and f. Consequently, both brands can afford to charge higher prices for their primary product and obtain more profits from market s even without taking into account the licensing revenues.

This result is in contrast to Amaldoss and Jain (2015) who show that, when both brands use umbrella branding strategies and extend themselves by producing the new product in-house, the price competition between brands intensifies and brands' profits from their primary products in market s decrease.[4] Also, against the common opinion among luxury brand experts (e.g., Kapferer and Bastien 2009; Kapferer 2015), our result suggests that luxury brands might benefit from decentralization when extending their brands through licensing. As such, brand licensing can be preferred over umbrella branding.

5.3. Only one brand licenses by using a fixed-fee contract $(\mathbf{F}, \mathbf{NL})$

It remains to consider the case when exactly one brand licenses by using a fixed-fee contract. Because both brands and both licensees are symmetric, it suffices to study the case (F, NL) in which brand A licenses its name to licensee a via a fixed-fee contract $T^A = k^A$ so that licensee a operates as a monopoly in market f. Lemma 5 characterizes the fixed fee and prices in this case.

LEMMA 5. *When brand A licenses its brand name to licensee a by using a fixed-fee contract and brand B does not license:*

(i) if $\lambda_s < 3t_s/\beta$, both brands compete in market s, and the fixed fee and prices are, respectively, given by:

$$k^A = \left(v_f + \lambda_f\left(\frac{1}{2} - \frac{\beta\lambda_s}{6t_s}\right) - t_f\right)\beta,$$

$$p^A = c + t_s - \frac{\beta\lambda_s}{3},\; p^B = c + t_s + \frac{\beta\lambda_s}{3},\; \textit{and } p^a = v_f + \lambda_f\left(\frac{1}{2} - \frac{\beta\lambda_s}{6t_s}\right) - t_f;$$

(ii) if $\lambda_s \geq 3t_s/\beta$, brand B becomes a monopoly in market s, and the fixed fee and prices are, respectively, given by:

$$k^A = (v_f - t_f)\beta,\; p^B = v_s - t_s + \frac{\beta\lambda_s}{3},\; \textit{and } p^a = v_f - t_f.$$

[4] The aforementioned indirect effect also emerges in the case when both brands use umbrella branding strategies and produce their new products in-house, as analyzed by Amaldoss and Jain (2015). However, its impact is completely opposite and, *relative to the case of no licensing*, the price competition between brands is intensified. (For consumer tastes that are uniformly distributed between 0 and 1, brands' prices when both use umbrella branding strategies reduce to $p^I = c + t_s - \beta\lambda_f < c + t_s$ for $I = A, B$, see equation (19) in §5.2 in Amaldoss and Jain (2015).) Therefore, the intuition behind the impact of indirect effect under fixed-fee licensing and umbrella branding is very different.

Lemma 5(i) shows that, when the negative popularity effect is low (i.e., $\lambda_s < 3t_s/\beta$), brand A that licenses charges a lower price than brand B that does not license, i.e., $p^A < p^B$. This is because licensing makes brand A less exclusive for snobs due to the negative popularity effect, and, to stay competitive in market s, it has to charge a lower price than brand B. Lemma 5(ii) shows that, when the negative popularity effect is high (i.e., $\lambda_s \geq 3t_s/\beta$), no snob purchases from brand A once it licenses its brand name to licensee a. Thus, brand B operates as a monopoly in market s and licensee a operates as a monopoly in market f.

Both brands are symmetric so that the profit of each brand in the case when only brand B licenses is given by: $\Pi^B(NL,F) = \Pi^A(F,NL)$ and $\Pi^A(NL,F) = \Pi^B(F,NL)$. Then, the brands' profits when only one brand uses fixed-fee contract are given by:

$$\Pi^A(F,NL) = \Pi^B(NL,F) = 2t_s\left(\left(\frac{1}{2} - \frac{\beta\lambda_s}{6t_s}\right)^+\right)^2 + \left(v_f + \lambda_f\left(\frac{1}{2} - \frac{\beta\lambda_s}{6t_s}\right)^+ - t_f\right)\beta, \tag{10}$$

$$\Pi^B(F,NL) = \Pi^A(NL,F) = \begin{cases} 2t_s\left(\frac{1}{2} + \frac{\beta\lambda_s}{6t_s}\right)^2, \text{ if } \lambda_s < 3t_s/\beta, \\ v_s - t_s - c, \qquad \text{if } \lambda_s \geq 3t_s/\beta, \end{cases} \tag{11}$$

where $(x)^+ = \max(x,0)$.

5.4. Equilibrium licensing strategies of duopoly brands under fixed-fee contracts

By comparing brands' profit functions (presented in the previous sections) under different licensing strategies as in a two-player simultaneous-move game, Proposition 2 characterizes the licensing strategy that each brand will adopt in equilibrium under fixed-fee contracts. In preparation, we define the thresholds $\lambda_{sk}^{(1)}$ and $\lambda_{sk}^{(2)}$ as follows:

$$\lambda_{sk}^{(1)} = \begin{cases} \frac{2t_s + \beta\lambda_f - \sqrt{\beta^2\lambda_f^2 + 4t_s\left(t_s - 2\left(v_f - t_f\right)\beta\right)}}{2\beta/3}, \text{ if } v_f \leq t_f + t_s/2\beta, \\ \infty, \qquad \text{if } v_f > t_f + t_s/2\beta. \end{cases} \tag{12}$$

$$\lambda_{sk}^{(2)} = \frac{3\lambda_f t_s - 2t_f t_s + 3\beta\lambda_f t_f + \sqrt{\left(3\lambda_f t_s - 2t_f t_s + 3\beta\lambda_f t_f\right)^2 + 4\beta t_f^3 t_s}}{2\beta t_f/3}, \tag{13}$$

and we let $\lambda_{sk}^{(3)} = \min(3t_s/\beta, \lambda_{sk}^{(2)})$. Also, we define $\lambda_{sk}^L = \min(\lambda_{sk}^{(1)}, \lambda_{sk}^{(3)})$ and $\lambda_{sk}^H = \max(\lambda_{sk}^{(1)}, \lambda_{sk}^{(3)})$.

PROPOSITION 2. *Under fixed-fee licensing contracts, brands' equilibrium licensing strategies can be characterized as follows:*

I. When the base valuation of the licensed product is low so that $v_f \leq t_f + \frac{t_s}{2\beta}$,

(a) both brands do not license if $\lambda_s \geq \lambda_{sk}^H$;

(b) both brands either license or do not license (i.e., two equilibria exist) if $\lambda_{sk}^{(1)} < \lambda_s < \lambda_{sk}^{(3)}$;

(c) only one brand licenses if $\lambda_{sk}^{(3)} < \lambda_s < \lambda_{sk}^{(1)}$;

(d) both brands license if $\lambda_s \leq \lambda_{sk}^L$.

II. When the base valuation of the licensed product is high so that $v_f > t_f + \frac{t_s}{2\beta}$*,*

(a) only one brand licenses if $\lambda_s > \lambda_{sk}^{(3)}$*;*

(b) both brands license if $\lambda_s \leq \lambda_{sk}^{(3)}$*.*

Figure 1 Brands' equilibrium licensing strategies under fixed-fee contracts

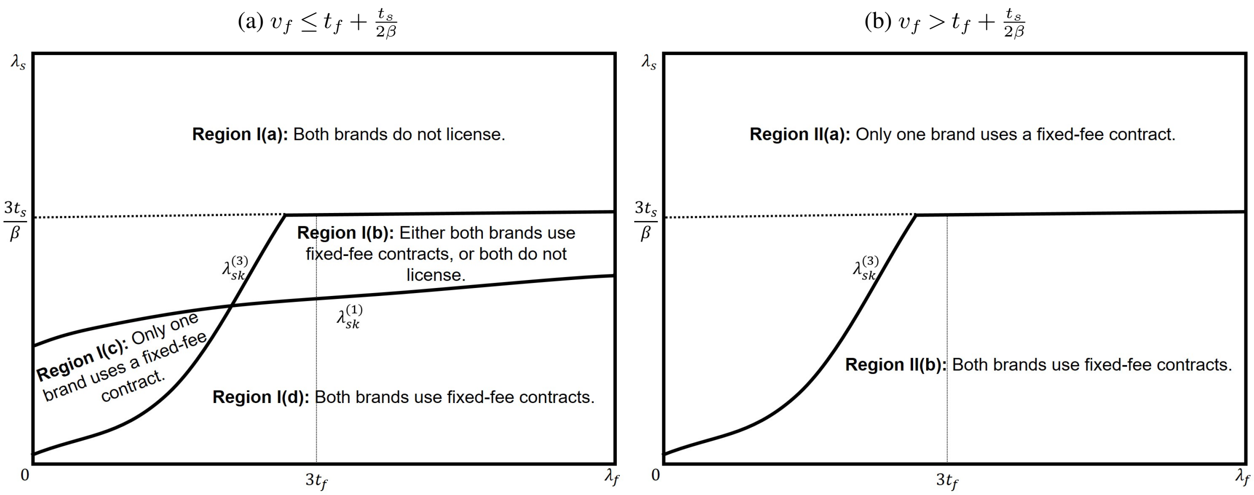


Figure 1a illustrates Proposition 2(I) for the case when $v_f \leq t_f + \frac{t_s}{2\beta}$. Specifically, consistent with Proposition 1 for the monopoly case, Proposition 2(Ia) shows that both brands do not license for sufficiently high λ_s values ($\lambda_s \geq \lambda_{sk}^H$, i.e., region I(a)). However, recall from Lemma 4 that, due to the 'indirect effect' of fixed-fee licensing that can soften competition, fixed-fee licensing is more profitable for the brands as the snobs' negative popularity effect λ_s or the followers' positive popularity effect λ_f increases. Even so, it is interesting to observe that no brand should license when λ_s lies within region I(a). To understand why, recall from Lemma 4 that each brand would be better off if both brands could 'commit' to licensing via fixed-fee contracts. However, in the absence of such a commitment, brands face a prisoner's dilemma and do not license. To better understand the intuition behind this result, consider a scenario where both brands license. In region I(a), the negative popularity effect λ_s is significantly high so that a brand is better off by making its brand more exclusive to please the snobs. In this case, at least one brand will want to deviate and not to license (e.g., $\Pi^B(F, NL) > \Pi^B(F, F)$). Moreover, if one of the brands deviates and does not license, the profit of the other brand significantly decreases due to the negative popularity effect (e.g., $\Pi^A(F, NL) < \Pi^A(NL, NL)$). Hence, both brands end up not licensing in equilibrium in region I(a), facing a prisoner's dilemma.

From Figure 1a, we observe that, given any $\lambda_s < 3t_s/\beta$, when followers' desire to adopt λ_f is sufficiently high (regions I(b) and I(d)), cases where both brands use fixed-fee contracts also become an equilibrium. In region I(b), licensing revenues are significant (due to high λ_f), but the direct negative impact is also significant (due to high λ_s). The latter dominates the former and licensing decreases a brand's profits when the other brand does not license (i.e., $\Pi^A(NL, NL) > \Pi^A(F, NL)$) whereas, since the indirect effect softens competition in both markets and improves profits when both brands license, the former dominates the latter and a brand benefits from licensing when the other brand also licenses (i.e., $\Pi^B(F, F) > \Pi^B(F, NL)$). Thus, there are two equilibria in region I(b). On the other hand, in region I(d), licensing always benefits a brand independent from whether the other brand licenses or not (i.e., $\Pi^A(F, F) > \Pi^A(NL, F)$ and $\Pi^A(F, NL) > \Pi^A(NL, NL)$), and thus, both brands license. This is because licensing softens competition in both markets (due to indirect effect) and/or provides significant additional revenues.

Figure 1a also shows that, for a given λ_s in region I(c), only one brand licenses if followers' sensitivity to brand popularity is sufficiently low (region I(c)). A brand benefits from licensing in region I(c) only when the other brand does not license (i.e., $\Pi^A(F, F) < \Pi^A(F, NL)$ and $\Pi^A(F, NL) > \Pi^A(NL, NL)$). In such cases, licensing revenues are not high enough (due to low λ_f) for both brands to license and licensing decreases their exclusivity. Instead, only one brand licenses and obtains lower profits from market s, yet its total profits increase as it receives all licensing revenues. However, when licensing revenues are sufficiently high (e.g., high β so that the size of market f is sufficiently large), it is never the case that only one brand licenses in equilibrium (i.e., $\lambda_{sk}^{(3)} = 3t_s/\beta$) and region I(c) in Figure 1a disappears.

Figure 1b illustrates Proposition 2(II), which corresponds to the case when $v_f > t_f + \frac{t_s}{2\beta}$. When followers' base valuation is sufficiently high (i.e., $v_f > t_f + \frac{t_s}{2\beta}$), (12) states that $\lambda_{sk}^{(1)} = \infty$. Hence, it is always beneficial for a brand to use a fixed-fee contract when the other brand does not license, i.e., $\Pi^A(F, NL) > \Pi^A(NL, NL)$. Therefore, as shown in Figure 1b, for $v_f > t_f + \frac{t_s}{2\beta}$, unlike in the monopoly case, licensing is always optimal and at least one brand uses a fixed-fee contract in equilibrium. Specifically, both brands license by using fixed-fee contracts when the snobs' negative popularity effect λ_s is low (i.e., $\lambda_s < \lambda_{sk}^{(3)}$), and only one brand licenses via a fixed-fee contract otherwise.

6. Duopoly: Incorporating Royalty Contracts

We now expand our duopoly analysis in §5 by adding royalty contracts into the consideration set so that, when brands consider licensing, they can choose between fixed-fee and royalty contracts. To characterize the brands' equilibrium licensing strategies in this case, we need to compare their profit functions under six different licensing strategies as in a simultaneous-move game with two symmetric players (i.e., brand A and B) and three strategies (i.e., NL, F and R). Due to symmetry, it is sufficient for us to consider three extra cases in addition to those presented in §5: (i) both brands use royalty contracts to license; (ii) one brand (brand A) uses a royalty contract to license and the other brand (brand B) does not license; and (iii) one brand (brand A) uses a royalty contract while the other brand (brand B) uses a fixed-fee contract. Next, by using the same approach as in §5, we analyze each of these three cases while we omit some details for brevity and instead present the key results.

6.1. Both brands license by using royalty contracts $(\mathbf{R},\mathbf{R})$

First, consider the case where brands A and B license their brand names to licensees a and b by charging (per-unit) royalty fees r^A and r^B, respectively. To ensure that equilibrium royalty fees and prices exist in this case,[5] we assume that the followers' positive popularity effect λ_f is sufficiently high so that:

$$\lambda_f \geq \sqrt{\frac{t_f t_s}{2\beta}}. \tag{14}$$

Lemma 6 characterizes equilibrium royalty fees and prices when both brands license via royalty conracts.

LEMMA 6. *When both brands license by using royalty contracts, the royalty fee r^I satisfies:*

$$r^I = \begin{cases} 0, & \textit{if } \lambda_f \geq 3t_f \textit{ and } \lambda_s < \lambda_{sr}^{(1)}, \\ t_f \frac{2\lambda_s\left(2\beta\lambda_f + t_s\right) + 3t_s\left(3t_f - \lambda_f\right)}{3t_f t_s + 2\beta\lambda_f^2}, & \textit{if otherwise,} \end{cases}$$

where $\lambda_{sr}^{(1)}$ is given by (B.5) in Appendix B. Also, the optimal prices of brand I ($I = A, B$) and its licensee i ($i = a, b$) satisfy:

$$p^I = c + t_s + \frac{\beta\lambda_f\lambda_s}{t_f} - \frac{\beta\lambda_f r^I}{t_f} \textit{ and } p^i = r^I + t_f + \frac{\beta\lambda_f\lambda_s}{t_s}.$$

Relative to brand I's equilibrium prices in case both brands use fixed-fee contracts to license (see Lemma 3), Lemma 6 reveals that, under royalty contracts, brands will charge lower prices in market s, while their licensees charge higher prices in market f. To understand why, observe first that the 'indirect effect' (i.e., $\beta\lambda_f\lambda_s/t_f$) that softens the competition in market s continues to persist under royalty contracts. However, under the royalty contracts, the royalties to be collected by each brand depend on the demand of the respective licensed product in market f. At the same time, due to the followers' positive popularity effect λ_f, one can boost the demand for the licensed product in market f by increasing the demand of the brand in market s. For these reasons, each brand has an incentive to lower its price p^I to increase its demand in market s (which causes the licensed product's demand in market f to increase). This price-lowering strategy is caused by the royalties (that depend on the sales of the licensed product in market f), and we shall refer to this effect (i.e., $r^I\beta\lambda_f/t_f$) as the 'royalty effect' that 'intensifies price competition' between brands in market s so that each brand charges a lower price under royalty contracts. To make up for the lower profit margin according to the term $\beta\lambda_f r^I/t_f$ in market s, each brand can leverage its indirect control to push the licensee to increase its price according to an extra term r^I, which is determined by brand I.[6]

[5] We make this assumption for ease of exposition, and the condition in (14) is sufficient but not necessary. There might be cases where that condition is violated but the equilibrium still exists. The condition in (14) is equivalent to assuming that the size of market f is sufficiently large (high β). Therefore, our assumption is in line with practice as the size of licensing market (market f) is much larger compared to the size of the market for a brand's own luxury goods (market s).

[6] In the event that λ_f is high while λ_s is low (i.e., $\lambda_f \geq 3t_f$ and $\lambda_s < \lambda_{sr}^{(1)}$), the royalty effect is much stronger than the indirect effect. Consequently, each brand sets $r^I = 0$ to eliminate the royalty effect and licenses for free.

When both brands use royalty contracts to license, brands' profits satisfy:

$$\Pi^I(R,R) = \frac{t_s}{2} + \frac{\beta\lambda_f\lambda_s}{2t_f} + \frac{\beta}{2}r^I\left(1 - \frac{\lambda_f}{t_f}\right) \tag{15}$$

for $I = A, B$. By comparing brands' profits as stated in (15) against (8) (as in the no-licensing case (NL, NL)) and against (9) (as in the case (F, F) under fixed-fee contracts), we obtain Lemma 7 that involves different threshold values for λ_s (namely, $\lambda_{sr}^{(2)}$ and $\lambda_{sr}^{(3)}$ that are given, respectively, by (B.6) and (B.7) in Appendix B) and threshold values for λ_f (namely, $\lambda_{fr}^L = \min(\lambda_{fr}^{(1)}, \lambda_{fr}^{(2)})$ and $\lambda_{fr}^H = \max(\lambda_{fr}^{(1)}, \lambda_{fr}^{(2)})$, where $\lambda_{fr}^{(1)}, \lambda_{fr}^{(2)} < t_f$ are given, respectively, by (B.9) and (B.10) in Appendix B).

LEMMA 7. *(i) Relative to the case when both brands license via royalty contracts, as in case (R, R), each brand I ($I = A, B$) is better off when both brands do not license (i.e., $\Pi^I(NL, NL) > \Pi^I(R, R)$ for $I = A, B$) if, and only if, $\lambda_s < \lambda_{sr}^{(2)}$ and $\lambda_f \in (t_f, 3t_f)$.*

(ii) Relative to the case that both brands license via royalty contracts, as in case (R, R), each brand I ($I = A, B$) earns more profits in the case that both brands license via fixed-fee contracts (i.e., $\Pi^I(F, F) > \Pi^I(R, R)$ for $I = A, B$) if, and only if: (1) $\lambda_s > \lambda_{sr}^{(3)}$ when $\lambda_f \in \left(\lambda_{fr}^L, \lambda_{fr}^H\right)$ and $\lambda_{fr}^{(1)} \leq \lambda_{fr}^{(2)}$; or (2) $\lambda_s < \lambda_{sr}^{(3)}$ when $\lambda_f \in \left(\lambda_{fr}^L, \lambda_{fr}^H\right)$ and $\lambda_{fr}^{(1)} > \lambda_{fr}^{(2)}$; or (3) $\lambda_f \geq \lambda_{fr}^H$.

Lemma 7(i) asserts that, instead of licensing via royalty contracts, as in the case (R, R), both brands are better off by not licensing when $\lambda_f \in (t_f, 3t_f)$ and $\lambda_s < \lambda_{sr}^{(2)}$ so that the royalty effect ($r^I\beta\lambda_f/t_f$) dominates the indirect effect of licensing ($\beta\lambda_f\lambda_s/t_f$). Hence, licensing via royalty contracts is not beneficial. Clearly, both brands can eliminate the 'royalty effect' by licensing for free ($r^A = r^B = 0$) so that they can benefit from the 'indirect effect' of licensing. In fact, because $\lambda_f > t_f$, (15) reveals that each brand would be better off in equilibrium if both could 'commit' to licensing its name for free by setting $r^A = r^B = 0$. Because such a commitment is absent, brands face a prisoner's dilemma and both charge a positive royalty fee (i.e., $r^A = r^B > 0$ by Lemma 6) and end up with significantly lower profits when they both use royalty contracts.

Lemma 7(ii) shows that, independent from the negative popularity effect λ_s, when followers' brand appreciation is sufficiently high (i.e., $\lambda_f \geq \lambda_{fr}^H$), the fixed-fee contracts perform better than the royalty contracts in cases where both brands license. This implies that, under competition, when the positive popularity effect is sufficiently high, luxury brands always benefit from using fixed-fee contracts. This result is due to the fact that, when the followers' appreciation for the brand λ_f is sufficiently high, royalty effect ($\beta\lambda_f r^I/t_f$) is very high and the price competition between brands is intensified significantly under royalty contracts so that both brands have to lower their prices.

6.2. Only one brand licenses by using a royalty contract $(\mathbf{R}, \mathbf{NL})$

Next, consider the case when only one brand (brand A) licenses via a royalty contract, while the other brand (brand B) does not license. Hence, brand A charges the royalty fee r^A for each unit that licensee a sells (as a monopoly) in market f. Before we present our analysis, let us make two observations. First, because licensee a operates as a monopoly in market f that is fully covered, it is always optimal for brand A to set the royalty fee $r^A = p^a$ to extract licensee a's entire profit. Second, because the market f is fully covered by licensee a's product, it is optimal for licensee a to set its price p^a so as to cover market f and sell to follower located $\theta = 1$. By noting that these two observations are the same as in §5.3, we can conclude that, when only one brand licenses and the other does not, fixed-fee and royalty contracts are equivalent. Thus, the equilibrium prices when only one brand uses a royalty contract are identical to those in Lemma 5 of §5.3, and the brands' profits when only one firm licenses via a royalty contract are given by:

$$\Pi^I(R, NL) = \Pi^I(F, NL), \tag{16}$$

$$\Pi^I(NL, R) = \Pi^I(NL, F) \tag{17}$$

for $I = A, B$, where $\Pi^I(F, NL)$ and $\Pi^I(NL, F)$ are given, respectively, by (10) and (11).

6.3. One brand uses a royalty contract and the other brand uses a fixed-fee contract $(\mathbf{R}, \mathbf{F})$

Now, consider the case where both brands license and use different contracts. Without loss of generality, suppose that brand A uses a royalty contract and charges a per-unit royalty fee r^A to its licensee a while brand B uses a fixed-fee contract and charges a fixed lump-sum payment k^B to its licensee b. To characterize the royalty fee r^A and ensure that brands and their licensees compete in market s and f, respectively, we assume that $\lambda_f \geq t_f$ and $\beta t_f > t_s$ (i.e., followers have sufficient level of aspiration for brand popularity in market s, and the size of market f (or licensing market) is large enough) in this case.[7]

Lemma 8 characterizes brand A's royalty fee r^A, brand B's fixed lump-sum payment k^B, and the prices in market s and f.

LEMMA 8. *When brand A licenses by using a royalty contract and brand B licenses by using a fixed-fee contract, the royalty fee of brand A and the fixed lump-sum payment of the brand B are given, respectively, by:*

$$r^A = \begin{cases} 0, & \text{if } \lambda_f \geq 3t_f \text{ and } \lambda_s < \lambda_{sr}^{(1)}, \\ \frac{t_f\left(9t_f t_s + 4\beta\lambda_f\lambda_s\right)\left(t_f t_s + \beta\lambda_f\lambda_s\right)\left(2\left(t_s + 2\beta\lambda_f\right)\lambda_s - 3t_s\left(\lambda_f - 3t_f\right)\right)}{8\beta^3\lambda_f^4\lambda_s^2 + 18\beta\lambda_f^2 t_f^2 t_s^2 + 2\beta\lambda_s^2 t_f t_s\left(8\beta\lambda_f^2 - t_f t_s\right) + 66\beta\lambda_f\lambda_s t_f^2 t_s^2 + 28\beta^2\lambda_f^3\lambda_s t_f t_s + 54 t_f^3 t_s^3}, & \text{if otherwise,} \end{cases}$$

$$k^B = \beta\left(t_f + \frac{\beta\lambda_f\lambda_s}{t_s} + \frac{-2\beta\lambda_f^2 + 4\beta\lambda_f\lambda_s + 6t_f t_s}{8\beta\lambda_f\lambda_s + 18t_f t_s} r^A\right)\left(\frac{1}{2} + \frac{t_s\left(-\beta\lambda_f^2 + 2\beta\lambda_s\lambda_f + 3t_f t_s\right)}{2\left(9t_f t_s + 4\beta\lambda_f\lambda_s\right)\left(t_f t_s + \beta\lambda_f\lambda_s\right)} r^A\right),$$

where $\lambda_{sr}^{(1)}$ is given by (B.5) in Appendix B. Also, the optimal prices of brands and their licensees satisfy:

$$p^A = c + t_s + \frac{\beta\lambda_f\left(2\lambda_s - r^A\right)}{2t_f} + \beta t_s \frac{2\lambda_s - 3\lambda_f}{8\beta\lambda_f\lambda_s + 18t_f t_s} r^A,$$

[7] This assumption (i.e., $\lambda_f \geq t_f$ and $\beta t_f > t_s$) is sufficient but not necessary for our analysis in this case. It requires followers' positive popularity effect λ_f and the size of licensing market β to be sufficiently high. In addition, it implies the condition in (14) so that our analysis in §6.1 is still valid under this assumption.

$$
\begin{aligned}
p^B &= c+t_s+\frac{\beta\lambda_f\left(2\lambda_s-r^A\right)}{2t_f}-\beta t_s\frac{2\lambda_s-3\lambda_f}{8\beta\lambda_f\lambda_s+18t_ft_s}r^A,\\
p^a &= t_f+\frac{\beta\lambda_f\lambda_s}{t_s}+\frac{2\beta\lambda_f^2+4\beta\lambda_f\lambda_s+12t_ft_s}{8\beta\lambda_f\lambda_s+18t_ft_s}r^A,\\
p^b &= t_f+\frac{\beta\lambda_f\lambda_s}{t_s}+\frac{-2\beta\lambda_f^2+4\beta\lambda_f\lambda_s+6t_ft_s}{8\beta\lambda_f\lambda_s+18t_ft_s}r^A.
\end{aligned}
$$

We observe from Lemma 8 that brand A (that uses a royalty contract) can affect the prices of brand B (that uses a fixed-fee contract) and both licensees by choosing its royalty fee r^A. Licensing via a royalty contract gives brand A extra leverage over brand B and enables it to determine the market shares of brands in market s and licensees in market f. Consequently, brand A always sells to more snobs (i.e., obtains more than half of the snob market) and its licensee (licensee a) attracts more (less) followers when the positive popularity effect is sufficiently higher (lower) than the negative popularity effect.

Brands' profits in this case are given by:

$$
\begin{aligned}
\Pi^A(R,F)=\Pi^B(F,R)=&\left(t_s+\frac{\beta\lambda_f\left(2\lambda_s-r\right)}{2t_f}+\frac{\beta t_s\left(2\lambda_s-3\lambda_f\right)r}{18t_ft_s+8\beta\lambda_f\lambda_s}\right)\\
&\times\left(\frac{1}{2}+\frac{\beta\left(2\beta\lambda_s\lambda_f^2+3t_ft_s\lambda_f+\lambda_st_ft_s\right)r}{2\left(9t_ft_s+4\beta\lambda_f\lambda_s\right)\left(t_ft_s+\beta\lambda_f\lambda_s\right)}\right)+\beta r\left(\frac{1}{2}-\frac{t_s\left(-\beta\lambda_f^2+2\beta\lambda_s\lambda_f+3t_ft_s\right)r}{2\left(9t_ft_s+4\beta\lambda_f\lambda_s\right)\left(t_ft_s+\beta\lambda_f\lambda_s\right)}\right),
\end{aligned}
\tag{18}
$$

$$
\begin{aligned}
\Pi^B(R,F)=\Pi^A(F,R)=&\frac{\left(\left(t_ft_s+\beta\lambda_f\lambda_s\right)\left(9t_ft_s+4\beta\lambda_f\lambda_s\right)-\beta r\left(2\beta\lambda_s\lambda_f^2+3t_ft_s\lambda_f+\lambda_st_ft_s\right)\right)^2}{2t_f\left(9t_ft_s+4\beta\lambda_f\lambda_s\right)^2\left(t_ft_s+\beta\lambda_f\lambda_s\right)}\\
&+\frac{\beta\left(\left(t_st_f+\beta\lambda_f\lambda_s\right)\left(9t_ft_s+4\beta\lambda_f\lambda_s\right)+rt_s\left(-\beta\lambda_f^2+2\beta\lambda_f\lambda_s+3t_ft_s\right)\right)^2}{2t_s\left(9t_ft_s+4\beta\lambda_f\lambda_s\right)^2\left(t_s+\beta\lambda_f\lambda_s\right)},
\end{aligned}
\tag{19}
$$

where $r=r^A$ in Lemma 8, and first equalities in (18) and (19) follow from symmetry of the brands.

6.4. Equilibrium licensing strategies of duopoly brands

We finally characterize brands' equilibrium licensing strategies when they can use either fixed-fee or royalty contracts to license. Characterizing equilibrium for all λ_s and λ_f values in this case is analytically intractable as it requires comparing each brand's profits associated with six different cases (presented in this section and in §5). Proposition 3 characterizes equilibrium when the negative popularity effect is sufficiently high (i.e., $\lambda_s\geq 3t_s/\beta$), and when the negative popularity effect is low and positive popularity effect is high (i.e., $\lambda_s<\lambda_{sr}^{(1)}$ and $\lambda_f\geq 3t_f$).

PROPOSITION 3. *Brands' equilibrium licensing strategies can be characterized as follows:*

(i) If $\lambda_s<\lambda_{sr}^{(1)}$ and $\lambda_f\geq 3t_f$, both brands license by using fixed-fee contracts.

(ii) If $\lambda_s\geq\frac{3t_s}{\beta}$, (a) both brands do not license when $v_f\leq t_f+\frac{t_s}{2\beta}$; and (b) only one brand licenses by using either a royalty or fixed-fee contract when $v_f>t_f+\frac{t_s}{2\beta}$.

Proposition 3 resembles Proposition 2, and the equilibrium licensing strategies when brands can use fixed-fee or royalty contracts have similar characteristics as under fixed-fee contracts when the negative popularity effect is sufficiently high, and when the negative popularity effect is low and positive popularity effect is high (i.e., as illustrated in Figure 1 for $\lambda_s \geq 3t_s/\beta$, and $\lambda_s < \lambda_{sr}^{(1)}$ and $\lambda_f \geq 3t_f$). Hence, Proposition 3 can be interpreted in the same manner as Proposition 2.

Proposition 3(i) shows that, when the negative popularity effect is low and the positive popularity effect is high enough (i.e., $\lambda_s < \lambda_{sr}^{(1)}$ and $\lambda_f \geq 3t_f$), both brands license and prefer fixed-fee contracts, even though either of them could use a royalty contract. The intuition behind this result is as follows. Since the negative popularity effect is low (i.e., $\lambda_s < \lambda_{sr}^{(1)}$), each brand is better off licensing (via a fixed-fee or royalty contract) independent from the strategy of the other brand. Therefore, both brands license. Moreover, since the positive popularity effect is high (i.e., $\lambda_f \geq 3t_f$), both royalty and indirect effects are high if a brand uses a royalty contract so that each brand is better off using a fixed-fee contract, no matter what contract the other brand uses to license. Hence, both brands prefer licensing via fixed-fee contracts for $\lambda_s < \lambda_{sr}^{(1)}$ and $\lambda_f \geq 3t_f$.

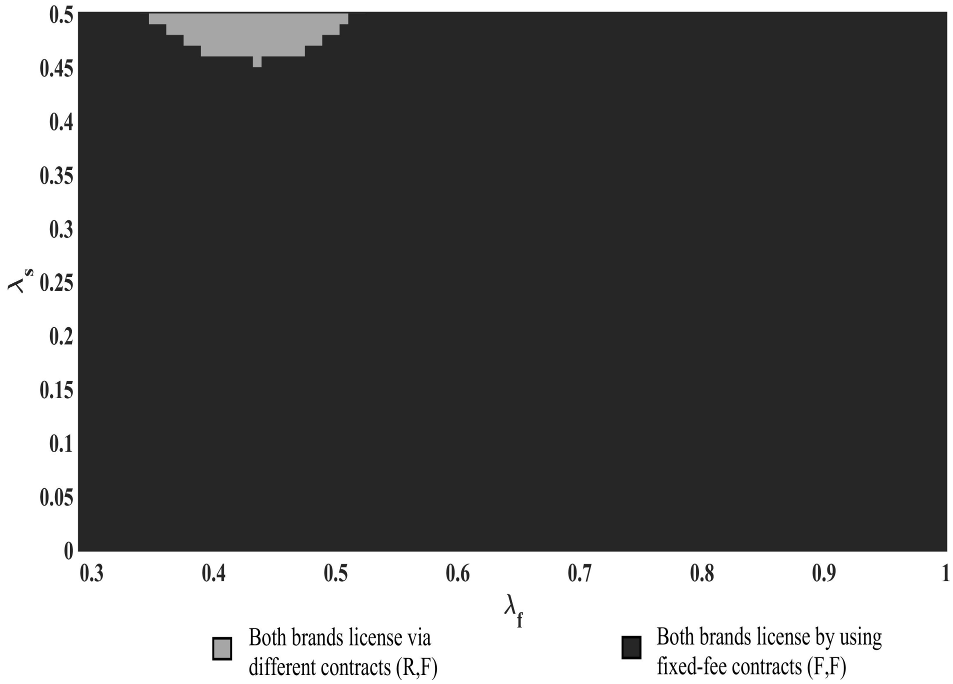
Also, Proposition 3(ii) implies that, as in the monopoly case (see Proposition 1), when followers' base valuation is sufficiently low (i.e., $v_f \leq t_f + \frac{t_s}{2\beta}$) and the snobs' negative popularity effect is very strong (i.e., $\lambda_s \geq 3t_s/\beta$), both brands should not license, even when they can use fixed-fee or royalty contracts, because both brands cannot afford to dilute their brands via licensing. Also, as in §5.4, Proposition 3(ii), coupled with Lemmata 4 and 7, implies that, in some cases, for sufficiently high λ_s values (e.g., $\lambda_s \geq 3t_s/\beta$), both brands would actually be better off if they were able to commit to licensing via fixed-fee or royalty contracts; however, without such a commitment, both they face a prisoner's dilemma under competition and both end up not licensing.

Numerical examples: To obtain a more complete picture about the brands' equilibrium licensing strategies beyond the range of positive and negative effects that are considered in Proposition 3, we conduct an extensive numerical study.[8] We observe from all numerical examples for sufficiently low negative popularity effect (i.e., $\lambda_s < 3t_s/\beta$) that both brands license via fixed-fee contracts in the equilibrium for all values of positive popularity effect when the licensing market is small enough (i.e., $\lambda_f < t_f$ for low enough β), or for high values of the positive popularity effect when the licensing market is large (i.e., $\lambda_f \geq t_f$ for large β). (We omit these numerical examples for brevity.) This has two important implications for low values of negative popularity effect (i.e., $\lambda_s < 3t_s/\beta$): (1) brands never use royalty licensing and always prefer fixed-fee licensing if the licensing market is small; and (2) Proposition 3(ii) is valid and the same intuition applies for all $\lambda_f \geq t_f$.

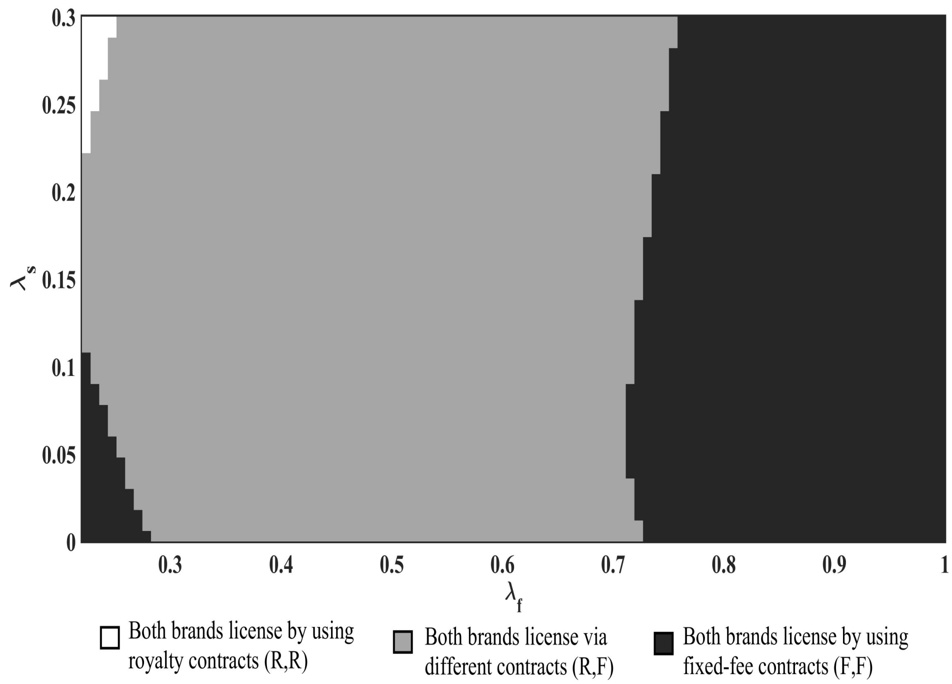
[8] We choose v_s and v_f sufficiently large in all numerical examples so that markets s and f are covered. In addition, to conduct a more extensive numerical study, we relax the condition $\lambda_f \geq t_f$ and $\beta t_f > t_s$ (which ensures that brands and their licenses compete in their respective markets in case (R, F)) and make sure that the condition in (14) is satisfied so that the royalty fee(s) in

Figure 2 **Brands' equilibrium licensing strategies under fixed-fee and royalty contracts for $\lambda_\mathbf{f} < \mathbf{t_f}$ and $\lambda_\mathbf{s} < \mathbf{3t_s}/\beta$ when $\mathbf{t_s} = \mathbf{t_f} = \mathbf{1}$ and $\mathbf{c} = \mathbf{0.5}\$$**

(a) $\beta = 6$



(b) $\beta = 10$



Note: For numerical examples considered in Figures 2a and 2b, we choose v_s and v_f large enough so that so that markets s and f are covered; moreover, only one brand licenses by using either a fixed-fee or royalty contract, as in cases (F, NL) and (R, NL), for $\lambda_s \geq 3t_s/\beta$ while both brands license by using fixed-fee contracts, as in the case (F, F), for $\lambda_s < 3t_s/\beta$ and $\lambda_f \geq t_f$.

Figure 2 illustrates brands' equilibrium licensing strategies for low values of negative and positive popularity effects (i.e., $\lambda_f < t_f$ and $\lambda_s < 3t_s/\beta$) when the licensing market is large enough (i.e., $\beta \in \{6, 10\}$), and $t_s = t_f = 1$ and $c = 0.5\$$. In the figure, snobs' and followers' base valuations (v_s and v_f) are set very high so that markets s and f are fully covered. Figure 2 shows that, when the negative popularity effect is low, royalty licensing is preferred (by at least one brand) only when the positive popularity effect is low and the licensing market is large enough, and it is used in equilibrium in more cases as the licensing market becomes larger. This suggests that, under competition, royalty licensing is used by the brands to impact on their licensees' marginal costs and sales in large markets when the negative and positive popularity effects are low.

Further, from Proposition 3, Figures 2a and 2b, we observe that both brands prefer royalty licensing only when the licensing market (i.e., β) is sufficiently large, the negative popularity effect is neither too high nor too low, and the positive popularity effect is sufficiently low. This is because, in such cases, royalty contracts enable brands to increase marginal costs of their licensees and prevent them to sell too much in market f; moreover, they soften price competition between brands so that they can charge higher prices in market s (i.e., a high indirect strategic effect and a low royalty effect). Lastly, together with Proposition 3, Figure 2 indicates that any combination of no licensing (NL), and fixed-fee and royalty licensing (F and R) is possible so that each of the six cases analysed in §§5 and 6 can be observed in equilibrium.

7. Discussion and Concluding Remarks

Over the last 30 years, many luxury brands have licensed their brand names to licensees so that they can extend their product offerings in new product categories in a cost-effective and time-efficient manner (License Global 2018). While licensing can enable a luxury brand to capture additional revenues from aspirational consumers (or followers), it can lead the brand to lose the direct control over the sales of the licensed products to the licensee. Consequently, licensing can backfire and make the brand less attractive for the exclusivity-seeking consumers (or snobs) who purchase brands' own primary products in the niche market as it was evident when several luxury brands such as Gucci, YSL and Burberry failed when they attempted to license in the 1980s and 1990s (License Global 2018).

To examine these two countervailing forces associated with licensing, we have developed a game-theoretic model to investigate how reference group effects and competition affect luxury brands' licensing strategies. Our analysis provides some useful insights on luxury brand licensing.

• *How do fixed-fee and royalty licensing affect the price of a brand's primary product in the niche market?* Due to snobs' desire for uniqueness, it is intuitive to expect a decrease in the price of a brand's primary product when it licenses. While we have confirmed this intuition in the monopoly setting, we have

cases (R, R) and (R, F) exist. We numerically characterize the royalty fee and prices in the case (R, F) taking into account that brands and licensees do not necessarily compete in their respective markets when we relax the condition $\lambda_f \geq t_f$ and $\beta t_f > t_s$. The condition in (14) is also sufficient to ensure the existence of a royalty fee in the case (R, F).

shown that it is not true in most cases in the duopoly setting. Specifically, in cases where both brands license via fixed-fee contracts, an indirect effect emerges and 'softens price competition' between brands. Therefore, fixed-fee licensing increases prices of brands' primary products in the duopoly setting. In cases where both brands use royalty contracts, in addition to the indirect effect that softens price competition between brands, a royalty effect arises and 'intensifies price competition' between brands. The royalty effect dominates the indirect effect and hence royalty licensing decreases brands' prices only when followers' desire to adopt the same brand as snobs is sufficiently high. This is because, when followers have a strong desire to emulate snobs, both brands compete for the snobs' demand by lowering their prices so that they can attract more followers to purchase their licensed products and increase their overall royalties. These results indicate that the impact of licensing on brands' prices and profits depend critically on contracts being used, and luxury brands should be careful when they determine their licensing contracts.

• *Does licensing always decrease a brand's profit obtained from its own primary product?* Since licensing decreases brand exclusivity for snobs, one could argue that that a brand will obtain a lower profit from snobs if it licenses. Indeed, we have shown that this is true in the monopoly setting. In the duopoly setting, however, we have found that, when both brands use fixed-fee contracts, licensing always increases a brand's profit from its primary product and is always beneficial for both brands. The intuition of this result is primarily driven by the indirect effect of fixed-fee licensing that softens competition between brands. It is interesting to observe that, despite their high-end consumers' desire for exclusivity, competing luxury brands license their brand names, e.g., Chanel and Dior license their brand names to Luxottica and Safilo in eyewear (Luxottica 2020; Safilo 2020). We uncover a plausible reason behind this practice of luxury brands and show that brand licensing can soften price competition between luxury brands and improve profits from their high-end consumers.

• *Is it beneficial for a brand to use royalty licensing to influence its licensee's price and sales of the licensed product when snobs' desire for uniqueness is high?* Our analysis has revealed that, in the monopoly setting, a royalty licensing contract enables a brand to counteract the negative popularity effect more effectively than a fixed-fee licensing contract (under which a brand cannot affect its licensee's sales). Therefore, for high values of snobs' desire for uniqueness, a monopoly brand always benefits from licensing via a royalty contract. On the other hand, under competition in the duopoly setting, we have found that, when the followers' desire to adopt the same brand as snobs is strong, snobs' desire for uniqueness has no impact and both brands are always better off licensing by using fixed-fee contracts, instead of using royalty contracts.

• *Why do some brands never license?* We have found that, in both monopoly and duopoly settings, a brand should not license when snobs' desire for uniqueness is above a certain threshold. In the monopoly setting, the primary motivation for a brand to choose not to license is to avoid the negative impact of licensing on its profits from snobs. In the duopoly setting, however, a brand may not license also due to the lack of a commitment mechanism. In particular, we have shown that, in the duopoly setting, when snobs' desire for uniqueness is very high, each brand would have earned more if they could both commit to licensing via fixed-fee contracts. However, in the absence of such a commitment, both brands face a prisoner's dilemma and do not license in equilibrium. This provides an alternative explanation for why luxury brands like Louis Vuitton and Hermes never license their brand names (Chevalier and Mazzalovo 2012; License Global 2018).

Limitations and Future research. When we developed our model, we have made simplification assumptions for tractability and to obtain clean insights. Consequently, our model has limitations and there are several avenues for future research. First, we have assumed that the snob and follower markets are completely separate so that snobs purchase only luxury brands' primary products while followers purchase only licensed products. Future research can study alternative settings where snobs and/or followers may purchase both luxury brands' own products and licensed products. Second, we have only considered brand licensing though fixed-fee and royalty contracts in order to isolate the effect of fixed fee and per-unit royalty fee. However, brands can also use mixed licensing contracts (i.e., a combination of fixed-fee and royalty contracts) or umbrella branding strategy (i.e., producing in-house) to extend in a new product category. Future work can study mixed licensing contracts and/or umbrella branding strategy. Lastly, we have assumed that consumers in one market are only sensitive towards brand popularity across the other market. However, our model can be generalized by considering cases where consumers are sensitive towards brand popularity within their own market.

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Appendix

A. Duopoly Model with Partially-Covered Follower Market

To complement our analysis in the monopoly setting in §4,[A.1] we extend our duopoly model presented in §5 to cases where follower market (market f) is partially covered. As in §5, we continue to assume that brands can only use fixed-

[A.1] In our monopoly model, we assume that market s is fully covered and consider both fully- and partially-covered market f. For the monopoly results with *fully-covered* market s and *partially-covered* market f, see our results in §4 for restricted values of followers' base valuation (i.e., $v_f < 2t_f - \lambda_f$).

fee contracts if they decide to license. Also, to ensure market f is partially covered and there is competition between brands when only one of them licenses, we restrict our analysis to cases where the following conditions are satisfied:

$$v_f \le t_f, \lambda_f \le 2(t_f - v_f), \text{ and } \lambda_s \le 6t_s/\beta. \tag{A.1}$$

Conditions in (A.1) are sufficient but not necessary for our analysis and the results in this appendix may still be valid in cases where these conditions are violated.

We follow a similar procedure as in §5. First, we analyze three cases: (i) both brands do not license (NL, NL); (ii) both brands license via fixed-fee contract (F, F); and (iii) only one brand licenses by using fixed-fee contract (F, NL). Then, by comparing brands' profits in these cases, we characterize brands' equilibrium licensing strategies. The case (NL, NL) where both brands do not license and only compete in fully-covered market s is already analyzed in §5.1. It only remains to analyze (F, F) and (F, NL).

A.1. Both brands license by using fixed-fee contracts $(\mathbf{F}, \mathbf{F})$

Consider the case where both brands license via fixed-fee contracts. By using (1), rational expectations and the assumption that market s is fully covered (i.e., $D_s^A + D_s^B = 1$), the marginal snob θ_s who is indifferent between purchasing from brand A and B is given by: $U_s^A(\theta_s) = U_s^B(\theta_s)$. In addition, by (2) and market f being partially covered (i.e., $D_f^a + D_f^b < 1$), the utility of a marginal follower θ_f^a who is indifferent between buying licensee a's product and not buying any product is given by $U_f^a\left(\theta_f^a\right) = 0$; and the utility of a marginal follower θ_f^b who is indifferent between purchasing from licensee b and not purchasing any product satisfy $U_f^b\left(\theta_f^b\right) = 0$. Then, solving $U_s^A(\theta_s) = U_s^B(\theta_s)$ for θ_s, $U_f^a\left(\theta_f^a\right) = 0$ for θ_f^a, and $U_f^b\left(\theta_f^b\right) = 0$ for θ_f^b; we obtain:

$$\begin{aligned}
\theta_s &= \frac{1}{2} - \frac{(p_A - p_B)\, t_f + \beta\lambda_s (p_b - p_a)}{2\,(t_f t_s + \beta\lambda_f\lambda_s)}, \\
\theta_f^a &= \frac{2v_f + \lambda_f - p_a}{2t_f} - \frac{\lambda_f t_f (p_A - p_B) + t_f t_s p_a + \beta\lambda_f\lambda_s p_b}{2t_f\,(t_f t_s + \beta\lambda_f\lambda_s)}, \\
\theta_f^b &= 1 - \frac{2v_f + \lambda_f - p_b}{2t_f} + \frac{\lambda_f t_f (p_B - p_A) + \beta\lambda_f\lambda_s p_a + t_f t_s p_b}{2t_f\,(t_f t_s + \beta\lambda_f\lambda_s)}.
\end{aligned}$$

Using $T^I = k^I$ for $I = A, B$, $D_s^A = \theta_s$, $D_s^B = 1 - \theta_s$, $D_f^a = \theta_f^a$ and $D_f^b = 1 - \theta_f^b$ along with (3)-(4), we obtain profits of brands A and B and licensees a and b. By considering the first-order conditions simultaneously, we characterize the optimal prices for brand I ($I = A, B$) and its licensee i ($i = a, b$) as follows:

$$p^I = c + t_s + \frac{\beta\lambda_f\lambda_s}{t_f} \text{ and } p^i = \frac{\lambda_f + 2v_f}{4}\left(1 + \frac{\beta\lambda_f\lambda_s}{4t_f t_s + 3\beta\lambda_f\lambda_s}\right).$$

Note that brands' prices above are exactly the same as those when both brands use fixed-fee contracts and market s and f are fully covered (see Lemma 3 in §5.2). Also, by comparing brands' prices above with those when both brands do not license as in the case (NL, NL) (i.e., $c + t_s$, see §5.1), we observe that brands charge higher prices when they both license via fixed-fee contracts. Therefore, as in our base model when both markets are fully covered, an indirect strategic effect emerges under fixed-fee licensing and softens price competition between brands.

Substituting brands' and licensees' prices in above expressions for marginal snobs and followers, we obtain:

$$\theta_s = \frac{1}{2}, \tag{A.2}$$

$$\theta_f^a = 1 - \theta_f^b = \frac{(\lambda_f + 2v_f)(2t_f t_s + \beta\lambda_f\lambda_s)}{8t_s t_f^2 + 6\beta\lambda_f\lambda_s t_f}. \tag{A.3}$$

By (A.2), brands equally share market s. In addition, Lemma A.1 shows that each licensee has equal share and obtains less than half of market f (i.e., $D_f^a = D_f^b = \theta_f^a < 1/2$). Proofs of all results in this appendix are presented in Appendix A.4.

LEMMA A.1. *When both brands license via fixed-fee contract as in the case (F,F), each licensee covers less than half of market f, i.e., $D_f^a = D_f^b = \theta_f^a \in (0,1/2)$.*

Further, since it is optimal for each brand I to choose its fixed lump-sum payment to extract all revenues from its licensee i, the fixed lump-sum payment is equal to $k^I = p^i \beta D_f^i$. By substituting p^i and $D_f^i = \theta_f^i$ for $i = a,b$, the fixed lump-sum payment of brand I ($I = A,B$) is given by:

$$k^I = \frac{\beta\left(2t_f t_s + \beta\lambda_f\lambda_s\right)\left(t_f t_s + \beta\lambda_f\lambda_s\right)\left(\lambda_f + 2v_f\right)^2}{2\left(4t_f t_s + 3\beta\lambda_f\lambda_s\right)^2 t_f}.$$

Lastly, in this case, brands' profit are given by:

$$\Pi^I(F,F) = \frac{t_s}{2} + \frac{\beta\lambda_f\lambda_s}{2t_f} + \frac{\beta\left(2t_f t_s + \beta\lambda_f\lambda_s\right)\left(t_f t_s + \beta\lambda_f\lambda_s\right)\left(\lambda_f + 2v_f\right)^2}{2\left(4t_f t_s + 3\beta\lambda_f\lambda_s\right)^2 t_f} > \frac{t_s}{2} = \Pi^I\left(NL,NL\right). \tag{A.4}$$

for $I = A,B$, where the last inequality follows from (8). Thus, akin to Lemma 4, we have Lemma A.2 when market f is partially covered, and the same intuition applies.

LEMMA A.2. *Suppose markets s is fully covered and conditions in* (A.1) *are satisfied so that market f is partially covered. Then, relative to the no-licensing case (NL,NL), each brand earns more when they both license by using fixed-fee contract, i.e., $\Pi^I\left(F,F\right) > \Pi^I\left(NL,NL\right)$ for $I = A,B$.*

A.2. Only one brand licenses by using a fixed-fee contract $(\mathbf{F},\mathbf{NL})$

Now, let us consider the case where exactly one brand licenses by using fixed-fee contract (i.e., brand A licenses to licensee a, and brand B does not license). Through a similar procedure as in §A.1 and using the assumption that market s is fully covered and market f is partially covered (i.e., $D_s^A + D_s^B = 1$ and $D_f^a < 1$), we obtain prices of brands and licensee a as:

$$\begin{aligned}
p^A &= c + t_s + \frac{\beta\lambda_s\left(\beta\lambda_s\lambda_f^2 + 5t_f t_s\lambda_f - 2t_f v_f t_s\right)}{2t_f\left(6t_f t_s + \beta\lambda_f\lambda_s\right)},\\
p^B &= c + t_s + \frac{\beta\lambda_s\left(\beta\lambda_s\lambda_f^2 + 7t_f t_s\lambda_f + 2t_f v_f t_s\right)}{2t_f\left(6t_f t_s + \beta\lambda_f\lambda_s\right)},\\
p^a &= \frac{v_f}{2} + \frac{\lambda_f\left(3t_f t_s + \beta\lambda_s(\lambda_f + v_f)\right)}{12t_f t_s + 2\beta\lambda_f\lambda_s}.
\end{aligned}$$

Observe that the brand that licenses (brand A) charges a lower price than the brand that does not license (brand B) in market s since it is less exclusive in the followers' market. Using these prices, we characterize θ_s, the marginal snob who is indifferent between purchasing from brand A and B, and θ_f, the marginal follower who is indifferent between purchasing from licensee a and not purchasing, as:

$$\theta_s = \frac{1}{2} - \frac{\beta\lambda_s t_f t_s\left(\lambda_f + 2v_f\right)}{2\left(2t_f t_s + \beta\lambda_f\lambda_s\right)\left(6t_f t_s + \beta\lambda_f\lambda_s\right)}, \tag{A.5}$$

$$\theta_f = \frac{t_s\left(\lambda_f + 2v_f\right)\left(3t_f t_s + \beta\lambda_f\lambda_s\right)}{\left(2t_f t_s + \beta\lambda_f\lambda_s\right)\left(6t_f t_s + \beta\lambda_f\lambda_s\right)}. \tag{A.6}$$

Next, Lemma A.3 shows that, under conditions in (A.1), there is always competition between brands in market s, and market f is partially covered.

LEMMA A.3. *Suppose conditions in* (A.1) *are satisfied. Then, when exactly one brand licenses via fixed-fee contract as in the case* (F, NL)*, both brands compete in market* s *and brand* A *gets less than half of market* s *(i.e.,* $D_s^A = \theta_s \in (0, 1/2)$*), and licensee* a *does not cover all of market* f *(i.e.,* $D_f^a = \theta_f \in (0,1)$*).*

Lemma A.3 also shows that the brand that licenses (brand A) gets a lower share of market s (i.e., $D_s^A = \theta_s < 1/2$) since it is less exclusive due to licensing.

Using (A.6) and $D_f^a = \theta_f$ and the fact that brand A sets its fixed lump-sum payment to extract all revenues from its licensee (i.e., $k^A = p^a \beta D_f^a$), we obtain the fixed lump-sum payment of brand A as:

$$k^A = \beta \frac{t_s \left(3t_f t_s + \beta\lambda_f\lambda_s\right)^2 \left(\lambda_f + 2v_f\right)^2}{2\left(2t_f t_s + \beta\lambda_f\lambda_s\right)\left(6t_f t_s + \beta\lambda_f\lambda_s\right)^2}.$$

Then substituting optimal prices above, $D_s^A = 1 - D_s^B = \theta_s$, $T^A = k^A$, and $T^B = 0$ into (3), profit of brand I for the case when only one brand licenses via fixed-fee contract is given by:

$$\Pi^A(F,NL) = \Pi^B(NL,F) = \frac{\left(12t_f^2 t_s^2 + \beta^2\lambda_f^2\lambda_s^2 + 7\beta\lambda_f\lambda_s t_f t_s - 2\beta\lambda_s t_f v_f t_s\right)^2}{4t_f\left(2t_f t_s + \beta\lambda_f\lambda_s\right)\left(6t_f t_s + \beta\lambda_f\lambda_s\right)^2} + \beta \frac{t_s\left(3t_f t_s + \beta\lambda_f\lambda_s\right)^2\left(\lambda_f + 2v_f\right)^2}{2\left(2t_f t_s + \beta\lambda_f\lambda_s\right)\left(6t_f t_s + \beta\lambda_f\lambda_s\right)^2}, \quad \text{(A.7)}$$

$$\Pi^B(F,NL) = \Pi^A(NL,F) = \frac{\left(12t_f^2 t_s^2 + \beta^2\lambda_f^2\lambda_s^2 + 9\beta\lambda_f\lambda_s t_f t_s + 2\beta\lambda_s t_f v_f t_s\right)^2}{4t_f\left(6t_f t_s + \beta\lambda_f\lambda_s\right)^2\left(2t_f t_s + \beta\lambda_f\lambda_s\right)}. \quad \text{(A.8)}$$

A.3. Equilibrium under fixed-fee contracts

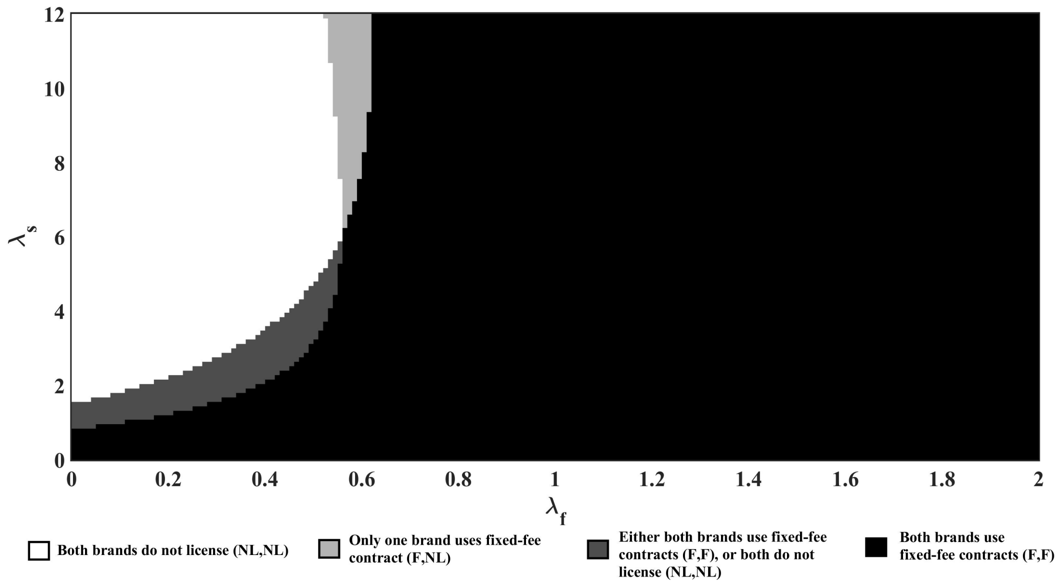
Finally, comparing brands' profits in cases (NL, NL), (F, F) and (F, NL) against each other, we characterize brands' equilibrium licensing strategies. Unfortunately, brands' profits in cases (F, F) and (F, NL) are complex functions of λ_s and λ_f. As a result, a clean analytical characterization of the equilibrium as in our base model for fully-covered market s and f (see Proposition 2 and Figure 1 in §5.4) is not possible when market f is partially covered. Therefore, we use numerical examples.

Figure A.1 illustrates brands' equilibrium licensing strategies under fixed-fee contracts when $v_f = 1,\ t_s = 2,\ t_f = 1,\ c = 0.5,$ and $\beta = 2$. In the figure, v_s is chosen very large so that market s is fully covered and all other parameters are chosen in line with (A.1) so that market f is partially covered.

Figure A.1 shows that, for low values of negative popularity effect (i.e., $\lambda_s \leq 12$), brands have similar equilibrium licensing strategies as in the case where both markets are fully covered (e.g., see Proposition 2 and Figure 1 in §5.4 for $\lambda_s \leq 3t_s/\beta$).[A.2] Specifically, both brands license via fixed-fee contracts if the positive popularity effect is high (i.e., black region); and no brand licenses if the positive popularity effect is low (i.e., white region). In all other cases, depending on the relative values of negative and positive popularity effects, two different equilibrium strategies might emerge: (i) only one brand licenses via fixed-fee contract (i.e., light grey region), and (ii) either both brands license via fixed-fee contracts or none of the brands licenses (i.e., dark grey region). Note by Lemma A.2 that, in white and dark grey regions in Figure A.1, brands face a prisoner's dilemma and none of them licenses, even though both brands would be better off if they could commit to licensing. However, such a commitment mechanism does not exist so that both brands end up not licensing.

[A.2] We do not consider large values of the negative popularity effect (i.e., $\lambda_s > 6t_s/\beta = 12$) in Figure A.1 to ensure that market f is partially covered and both brands compete in market s. In cases where λ_s is large and market f is partially covered (i.e., $\theta_f < 1$), the brand that does not license will still become a monopoly in market s (i.e., $\theta_s = 0$) so that at least one brand will always want to not license. Thus, as in Figure 1, either both brands will not license (i.e., (NL, NL)) or only one brand will license (i.e., (F, NL)) in equilibrium when the negative popularity effect is large and market f is partially covered.

Figure A.1 Brands' equilibrium licensing strategies under fixed-fee contracts in cases where snob market (market s) is fully covered and follower market (market f) is partially covered when $\mathbf{v_f = 1,\ t_s = 2,\ t_f = 1,\ c = 0.5,}$ and $\beta = 2$



Note: In the figure, v_s is very large so that market s is fully covered; and all other parameter values are chosen so as to satisfy sufficient conditions in (A.1) (i.e., $v_f \le t_f$, $\lambda_f \le 2(t_f - v_f)$ and $\lambda_s \le 6t_s/\beta$) so that market f is partially covered.

A.4. Proofs

Proof of Lemma A.1: To prove the lemma, since $D_f^a = D_f^b = \theta_f^a$, where θ_f^a is given by (A.3), it is enough to show that $\theta_f^a \in (0, 1/2)$. From (A.3), $\theta_f^a > 0$. Also, note that

$$4t_s t_f^2 + 3\beta\lambda_f\lambda_s t_f - (\lambda_f + 2v_f)(2t_f t_s + \beta\lambda_f\lambda_s) = \beta\lambda_f\lambda_s((3t_f - 2v_f) - \lambda_f) + 2t_f t_s(2(t_f - v_f) - \lambda_f) > 0$$

where the inequality follows from $v_f \le t_f$and $\lambda_f \le 2(t_f - v_f)$ by (A.1). This, by (A.3), implies that $\theta_f^a < 1/2$. $\square$

Proof of Lemma A.3: Since $D_s^A = 1 - D_s^B = \theta_s$, where θ_s is given by (A.5), it is enough to show that $\theta_s \in (0, 1/2)$ to prove the first part of the lemma. By (A.5), it is obvious that $\theta_s < 1/2$. Then, it remains to show that $\theta_s > 0$. Note that $\lambda_f t_s + 2v_f t_s \le 2t_f t_s$ by $\lambda_f \le 2(t_f - v_f)$ in (A.1) and $\beta\lambda_s t_f \le 6t_f ts$ by $\lambda_s \le 6t_s/\beta$ in (A.1). This implies that

$$\beta\lambda_s t_f t_s(\lambda_f + 2v_f) < (2t_f t_s + \beta\lambda_f\lambda_s)(6t_f t_s + \beta\lambda_f\lambda_s)$$

so that $\theta_s > 0$ by (A.5).

Next, we show that $\theta_f \in (0, 1)$ so that $D_f^a = \theta_f < 1$ and licensee a does not cover market f. From (A.6), $\theta_f > 0$. By $\lambda_f t_s + 2v_f t_s \le 2t_f t_s$ (as proven above), the numerator is less than the denominator in (A.6) and $\theta_f < 1$. $\square$

B. Definitions of Threshold λ_s and λ_f Values in §6

In this appendix, we define the threshold λ_s and λ_f values that we use in §6 of the paper. Let us define functions $g_j(\lambda_f)$ and $h_j(\lambda_f)$ for $j = 1, 2$ as follows:

$$g_1(\lambda_f) = 2\beta\lambda_f^3 - 4\beta\lambda_f^2 t_f + 4\beta\lambda_f t_f^2 + \lambda_f t_f t_s + 2t_f^2 t_s, \tag{B.1}$$

$$g_2(\lambda_f) = 2\beta^2\lambda_f^3 + 4\beta\lambda_f^2 t_s + 2\lambda_f t_s^2 - \beta\lambda_f t_f t_s - 2t_f t_s^2, \tag{B.2}$$

$$h_1(\lambda_f) = 3t_f t_s(3t_f - \lambda_f)(\lambda_f - t_f), \tag{B.3}$$

$$h_2(\lambda_f) = t_s(3t_s - 2\beta t_f)\lambda_f^2 - 12t_f t_s^2\lambda_f + 6t_f^2 t_s^2. \tag{B.4}$$

We define threshold λ_s values in §6 as follows:

$$\lambda_{sr}^{(1)} = \frac{3t_s(\lambda_f - 3t_f)}{2(t_s + 2\beta\lambda_f)}, \tag{B.5}$$

$$\lambda_{sr}^{(2)} = \frac{g_1(\lambda_f)}{h_1(\lambda_f)}, \tag{B.6}$$

$$\lambda_{sr}^{(3)} = \frac{g_2(\lambda_f)}{h_2(\lambda_f)}, \tag{B.7}$$

$$\lambda_{sr}^{(5)} = \frac{3t_s}{2\beta}\left(\frac{\lambda_f}{t_f} + \frac{4\lambda_f\beta(\lambda_f - 3t_f)(\lambda_f - t_f)}{3t_f^2 t_s + 2\beta t_f\lambda_f^2} + \sqrt{\left(\frac{\lambda_f}{t_f} + \frac{4\lambda_f\beta(\lambda_f - 3t_f)(\lambda_f - t_f)}{3t_f^2 t_s + 2\beta t_f\lambda_f^2}\right)^2 + 3\frac{4\beta(\lambda_f - 3t_f)(\lambda_f - t_f)}{3t_f t_s + 2\beta\lambda_f^2}}\right). \tag{B.8}$$

Lastly, we let thresholds $\lambda_{fr}^{(1)}$ and $\lambda_{fr}^{(2)}$ be unique $\lambda_f \in (0, t_f)$ values that, respectively, satisfy:

$$g_2(\lambda_{fr}^{(1)}) = 0, \tag{B.9}$$

$$h_2(\lambda_{fr}^{(2)}) = 0. \tag{B.10}$$

C. Technical Details

C.1. Auxiliary results

In this appendix, we derive four auxiliary results that we will use to prove main results in the paper. First, given the prices of brands and their licensees, the following lemma characterizes marginal snob θ_s and marginal follower θ_f in the rational expectations equilibrium of the duopoly setting when both brands license. We use Lemma C.1 to determine the demand in cases (F,F), (R,R) and (R,F) and to prove Lemmata 3, 6 and 8.

LEMMA C.1. *Suppose that both markets s and f are fully covered. In the rational expectations equilibrium when both brands license as in cases (F,F), (R,R) and (R,F), given brands' and licensees' prices (i.e., p^I for $I = A, B$ and p^i for $i = a, b$), the marginal snob θ_s who is indifferent between purchasing from brand A and B, and the marginal follower θ_f who is indifferent between purchasing from licensee a and b are given, respectively, by:*

$$\theta_s = \frac{1}{2} + \frac{t_f(p^B - p^A) - \beta\lambda_s(p^b - p^a)}{2t_f t_s + 2\beta\lambda_f\lambda_s}, \tag{C.1}$$

$$\theta_f = \frac{1}{2} + \frac{\lambda_f(p^B - p^A) + t_s(p^b - p^a)}{2t_f t_s + 2\beta\lambda_f\lambda_s}. \tag{C.2}$$

Proof of Lemma C.1: By (1), a snob located at θ will obtain a net utility U_s^I from purchasing brand I's product, where:

$$U_s^A(\theta) = v_s - t_s\theta - \lambda_s\beta D_f^{a(e)} - p^A \text{ and } U_s^B(\theta) = v_s - t_s(1-\theta) - \lambda_s\beta D_f^{b(e)} - p^B.$$

Because market f is fully covered ($D_f^{b(e)} = 1 - D_f^{a(e)}$), the marginal snob θ_s who is indifferent between purchasing from brand A and B (i.e., $U_s^A(\theta_s) = U_s^B(\theta_s)$) is given by:

$$\theta_s = \frac{1}{2} + \frac{p^B - p^A + \beta\lambda_s - 2\beta\lambda_s D_f^{a(e)}}{2t_s}.$$

Similarly, because market s is fully covered ($D_s^{B(e)} = 1 - D_s^{A(e)}$), the marginal follower θ_f who is indifferent between purchasing licensee a and b is given by:

$$\theta_f = \frac{1}{2} + \frac{p^b - p^a - \lambda_f + 2\lambda_f D_s^{A(e)}}{2t_f}.$$

By rational expectations, $D_s^{A(e)} = D_s^A = \theta_s$, and $D_f^{a(e)} = D_f^a = \theta_f$. This observation enables us to solve for θ_s and θ_f simultaneously and obtain θ_s and θ_f that are, respectively, given by (C.1) and (C.2). $\square$

Second, by comparing brands' profits in cases (NL, NL), (F, F), and (F, NL), we obtain Lemma C.2, where $\lambda_{sk}^{(1)}$ is given in (12) and $\lambda_{sk}^{(3)} = \min(3t_s/\beta, \lambda_{sk}^{(2)})$ with $\lambda_{sk}^{(2)}$ defined in (13). This lemma specifies the conditions under which it is beneficial for a brand to license via a fixed-fee contract depending on whether the other brand does not license or licenses via fixed-fee contract. We use Lemma C.2 to characterize the equilibrium licensing strategies of brands under fixed-fee contracts in Proposition 2 in the paper.

LEMMA C.2. *Suppose that both markets s and f are fully covered.*

(i) If one brand does not license, it is always beneficial for the other brand to license via the fixed-fee contract (i.e., $\Pi^A(F, NL) > \Pi^A(NL, NL)$, or $\Pi^B(NL, F) > \Pi^B(NL, NL)$) if, and only if, $\lambda_s < \lambda_{sk}^{(1)}$.

(ii) If one brand licenses via a fixed-fee contract, it is always beneficial for the other brand to license via the fixed-fee contract (i.e., $\Pi^A(F, F) > \Pi^A(NL, F)$, or $\Pi^B(F, F) > \Pi^B(F, NL)$) if, and only if, $\lambda_s < \lambda_{sk}^{(3)}$.

Proof of Lemma C.2: We will prove each part of the lemma separately.

Part (i): In this part, to determine cases, we characterize the benefit to a brand from licensing via fixed-fee contract when the other brand does not license. Without loss of generality, assume that brand A licenses while brand B does not license. We let $\Delta(F, NL) = \Pi^A(F, NL) - \Pi^A(NL, NL)$ be the benefit of using fixed-fee contract for brand A when the other brand does not license. By symmetry (i.e., $\Pi^B(NL, F) = \Pi^A(F, NL)$ and $\Pi^A(NL, F) = \Pi^B(F, NL)$), the benefit to brand B from licensing via fixed-fee contract when brand A does not license is also equal to $\Delta(F, NL)$. By (8) and (10) , we have

$$\Delta(F, NL) = 2t_s\left(\left(\frac{1}{2} - \frac{\beta\lambda_s}{6t_s}\right)^+\right)^2 + \left(v_f + \lambda_f\left(\frac{1}{2} - \frac{\beta\lambda_s}{6t_s}\right)^+ - t_f\right)\beta - \frac{t_s}{2}.$$

Consider two cases: (a) $v_f < t_f + t_s/2\beta$, and (b) $v_f \geq t_f + t_s/2\beta$.

Case (ia): In this case, $\lim_{\lambda_s \downarrow 0} \Delta(F, NL) = \frac{1}{2}\beta(\lambda_f + 2(v_f - t_f)) > 0$, $\lim_{\lambda_s \uparrow 3t_s/\beta} \Delta(F, NL) < 0$ (since $v_f < t_f + t_s/2\beta$) and $\Delta(F, NL)$ is strictly decreasing in λ_s. This indicates that there exists a unique $\lambda_{sk}^{(1)} \in (0, 3t_s/\beta)$ such that for $\lambda_s < \lambda_{sk}^{(1)}$, both brands are better off when only one brand licenses via fixed-fee contract. In all other cases ($\lambda_s \geq \lambda_{sk}^{(1)}$), the brand that licenses (brand A) is better off if she does not license. Solving for λ_s such that $\Delta(F, NL) = 0$, we obtain $\lambda_{sk}^{(1)}$ in (12) for $v_f < t_f + t_s/2\beta$.

Case (ib): In this case, $\Delta(F, NL) > 0$ for all λ_s, i.e., $\lambda_s < \lambda_{sk}^{(1)} = \infty$. Thus, both brands are better off when only one of them licenses via fixed-fee contract compare to the case where no brand licenses.

Part (ii): In this part, we characterize the benefit to a brand from licensing via fixed-fee contract when the other brand also licenses via a fixed-fee contract. To that end, we let $\Delta(F,F)$ denote the benefit to a brand from licensing via a fixed-fee contract when the other brand uses a fixed-fee contract. Note that $\Pi^A(F,F)=\Pi^B(F,F)$ by (9) and $\Pi^A(NL,F)=\Pi^B(F,NL)$ by (11). Therefore, the benefit from licensing via fixed-fee contract when the other brand uses a fixed-fee contract is the same for brand A and B. First, consider, $\lambda_s<3t_s/\beta$. In this case, by (9) and (11),

$$\Delta(F,F)=\frac{t_s}{2}+\frac{\beta\lambda_f\lambda_s}{2t_f}+\frac{\beta t_f}{2}+\frac{\beta^2\lambda_f\lambda_s}{2t_s}-2t_s\left(\frac{1}{2}+\frac{\beta\lambda_s}{6t_s}\right)^2.$$

Note that $\lim_{\lambda_s\downarrow 0}\Delta(F,F)>0$, $\lim_{\lambda_s\uparrow\infty}\Delta(F,F)<0$, and $\Delta(F,F)$ is a concave quadratic function of λ_s. This indicates that there exists $\lambda_{sk}^{(2)}\in(0,\infty)$ such that $\Delta(F,F)$ is positive for $\lambda_s\le\lambda_{sk}^{(3)}$ and it is negative for $\lambda_s\in\left(\lambda_{sk}^{(3)},3t_s/\beta\right)$, where $\lambda_{sk}^{(3)}=\min\left(3t_s/\beta,\lambda_{sk}^{(2)}\right)$. Solving for λ_s such that $\Delta(F,F)=0$, we obtain $\lambda_{sk}^{(2)}$ in (13). By (13), $\lambda_{sk}^{(2)}$ is increasing in λ_f, and $\lim_{\lambda_f\downarrow 0}\lambda_{sk}^{(2)}=\frac{3t_s}{\beta}\left(\sqrt{1+\frac{\beta t_f}{t_s}}-1\right)$ and $\lim_{\lambda_f\uparrow\infty}\lambda_{sk}^{(2)}=\infty$.

Second, consider $\lambda_s\ge 3t_s/\beta$. By (9) and (11),

$$\Delta(F,F)=\frac{t_s}{2}+\frac{\beta\lambda_f\lambda_s}{2t_f}+\frac{\beta t_f}{2}+\frac{\beta^2\lambda_f\lambda_s}{2t_s}-(v_s-t_s-c).$$

In this case, by Lemma 5(ii), the brand that does not license (e.g., assume brand B without loss of generality) is a monopoly in market s when only one brand licenses and $\Delta(F,F)<0$ since we assume that a brand prefers being a monopoly in market s when the other brand licenses over competing the other brand in market s and market f when they both license (i.e., v_s is sufficiently large). $\square$

Third, by comparing the profits when only one brand licenses by using a royalty contract, as in the case (R,NL) against the profits in cases (NL,NL)) and (R,R), we get Lemma C.3, [C.1] where $\lambda_{sk}^{(1)}$ is given by (12), and we let $\lambda_{sr}^{(4)}=\min(3t_s/\beta,\lambda_{sr}^{(5)})$ with $\lambda_{sr}^{(5)}$ in (B.8) in Appendix B. Lemma C.3 characterizes conditions under which only a brand licenses via a royalty contract, and it is used to prove Lemma C.4 below and Proposition 3 in the paper.

LEMMA C.3. *Suppose that both markets s and f are fully covered, and the condition in* (14) *is satisfied.*

(i) If one brand does not license, then it is always beneficial for the other brand to license via a royalty contract (i.e., $\Pi^A(R,NL)>\Pi^A(NL,NL)$ and $\Pi^B(NL,R)>\Pi^B(NL,NL)$) if, and only if, $\lambda_s<\lambda_{sk}^{(1)}$.

(ii) If one brand licenses via a royalty contract, then it is always beneficial for the other brand not to license (i.e., $\Pi^A(NL,R)>\Pi^A(R,R)$ and $\Pi^B(R,NL)>\Pi^B(R,R)$): (a) if, and only if, $\lambda_s\ge\lambda_{sr}^{(4)}$ when $\lambda_f\le t_f$; (b) if $\lambda_s\ge 3t_s/\beta$ when $t_f<\lambda_f<3t_f$; and (c) if, and only if, $\lambda_s\ge 3t_s/\beta$ when $\lambda_f\ge 3t_f$.

Proof of Lemma C.3: The first part of Lemma C.3 follows from Lemma C.2(i) by (16) and (17). Next, we will prove the second part of the lemma. To that end, we will analyze the benefit to a brand from licensing via a royalty contract when the other brand uses the royalty contract. Note that $\Pi^A(R,R)=\Pi^B(R,R)$ by (15) and $\Pi^A(NL,R)=\Pi^B(NL,R)$ by (17). Therefore, the benefit from licensing via a royalty contract when the other brand uses a royalty contract is the same for brand A and B, (i.e., $\Pi^A(R,R)-\Pi^A(NL,R)=\Pi^B(R,R)-\Pi^B(NL,R)$). We let $\Delta(R,R)$ denote that benefit. Now, we will consider two cases: (1) $\lambda_s\ge 3t_s/\beta$, and (2) $\lambda_s<3t_s/\beta$.

[C.1] We cannot show Lemma C.3(ii) analytically when $t_f<\lambda_f<3t_f$ and $\lambda_s<3t_s/\beta$. In such cases, several numerical examples confirm that there are similar threshold λ_s values less than $3t_s/\beta$ that characterize cases where a brand is better off not licensing when the other brand uses a royalty contract. For brevity, we do not present these numerical examples in this paper.

Case 1: In this case, we will consider two more sub-cases: (a) $\lambda_f \leq t_f$, and (b) $\lambda_f \geq 3t_f$.

Sub-case (1a): In this case (i.e., $\lambda_s < 3t_s/\beta$ and $\lambda_f \leq t_f$), by (15), (17) and (C.8), we have

$$\Delta(R,R) = \frac{\beta\Omega(\lambda_s)}{18t_f t_s\left(3t_f t_s + 2\beta\lambda_f^2\right)},$$

where

$$\Omega(\lambda_s) = -\beta t_f\left(3t_f t_s + 2\beta\lambda_f^2\right)\lambda_s^2 + 27t_f t_s^2(\lambda_f - 3t_f)(\lambda_f - t_f) + 3\lambda_f t_s\left(3t_f t_s + 2\beta\lambda_f^2 + 4\beta(\lambda_f - 3t_f)(\lambda_f - t_f)\right)\lambda_s. \quad \text{(C.3)}$$

Note that $\Omega(\lambda_s)$ is a concave quadratic function of λ_s and its discriminant is nonnegative for $\lambda_f \leq t_f$. Thus $\Omega(\lambda_s) = 0$ has two real roots. Moreover, for $\lambda_f \leq t_f$, its smaller root is always negative and its bigger root $\lambda_{sr}^{(5)}$ is always positive and given by (B.8). Then it follows that $\Omega(\lambda_s) > 0$ if, and only if, $\lambda_s < \lambda_{sr}^{(4)}$, where $\lambda_{sr}^{(4)} = \min\left(3t_s/\beta, \lambda_{sr}^{(5)}\right)$ when $\lambda_f \leq t_f$.

Sub-case (1b): Note by (B.5) that $\lambda_{sr}^{(1)} \in (0, 3t_s/\beta)$ for $\lambda_f \geq 3t_f$ since it is increasing in λ_f and $\lim_{\lambda_f \uparrow \infty} \lambda_{sr}^{(1)} = 3t_s/4\beta$. First consider $\lambda_f \geq 3t_f$ and $\lambda_s < \lambda_{sr}^{(1)}$. In this case, when both brands license, the optimal royalty fee is equal to zero by (C.8) and hence by (15) and (17), we have $\Delta(R,R) = \frac{\beta\lambda_s}{18t_f t_s}\phi(\lambda_s)$, where

$$\phi(\lambda_s) = 9\lambda_f t_s - 6t_f t_s - \beta\lambda_s t_f.$$

Note that $\phi(\lambda_s)$ is decreasing in λ_s and $\lim_{\lambda_s \uparrow 3t_s/\beta} \phi(\lambda_s) > 0$ for $\lambda_f \geq 3t_f$. This implies that $\Delta(R,R) > 0$ for $\lambda_f \geq 3t_f$ and $\lambda_s < \lambda_{sr}^{(1)}$.

Now consider $\lambda_f \geq 3t_f$ and $\lambda_s \geq \lambda_{sr}^{(1)}$. In this case, by (15) and (17), we have

$$\Delta(R,R) = \frac{\beta\Omega(\lambda_s)}{18t_f t_s\left(3t_f t_s + 2\beta\lambda_f^2\right)},$$

where $\Omega(\lambda_s)$ is given by (C.3). For $\lambda_f \geq 3t_f$, the discriminant of quadratic function $\Omega(\lambda_s)$ is nonnegative and hence $\Omega(\lambda_s) = 0$ has two real roots. In addition, the smaller root is always negative and the bigger root $\lambda_{sr}^{(5)}$ is always positive in this case. Moreover, by (B.8), the bigger root $\lambda_{sr}^{(5)}$ is always greater than $3t_s/\beta$. Then it follows by $\Omega(\lambda_s)$ being a concave quadratic function that $\Delta(R,R) > 0$ if, and only if, $\lambda_s < 3t_s/\beta$ when $\lambda_f \geq 3t_f$ and $\lambda_s \geq \lambda_{sr}^{(1)}$. Thus $\Delta(R,R) > 0$ in this case.

Case 2: In this case, by Lemma 5(ii), (16) and (17), the brand that does not license when only one brand uses a royalty contract is a monopoly in market s and $\Delta(R,R) < 0$ since we assume that a brand prefers being a monopoly in market s when the other brand licenses over competing the other brand in market s and market f when they both license (i.e., v_s is significantly large).

To summarize, $\Delta(R,R) \leq 0$: (i) if, and only if, $\lambda_s \geq \lambda_{sr}^{(4)} = \min\left(3t_s/\beta, \lambda_{sr}^{(5)}\right)$ when $\lambda_f \leq t_f$ by Case 1a and Case 2; (ii) if $\lambda_s \geq 3t_s/\beta$ when $t_f < \lambda_f < 3t_f$ by Case 2; and (iii) if, and only if, $\lambda_s \geq 3t_s/\beta$ when $\lambda_f \geq 3t_f$ by Case 1b and Case 2. Hence, the second part of the lemma follows. □

Lastly, by comparing brands' profits in case (R,F) with those in all other cases, we obtain Lemma C.4, where $\lambda_{sr}^{(1)}$ is given by (B.5). Lemma C.4 is used to characterize brands' equilibrium licensing strategies in the duopoly setting in Proposition 3.

LEMMA C.4. *Suppose that both markets s and f are fully covered, and assume that $\lambda_f \geq t_f$ and $\beta t_f > t_s$.*

(i) In the case where both brands license (via fixed-fee or royalty contract), a brand is always better off using a fixed-fee contract (i.e., $\Pi^A(F,R) > \Pi^A(R,R)$ and $\Pi^A(F,F) > \Pi^A(R,F)$) if $\lambda_s < \lambda_{sr}^{(1)}$ and $\lambda_f \geq 3t_f$.

(ii) If both brands license by using different (fixed-fee or royalty) contracts, then it is always beneficial for one of the brands to not license (i.e., $\Pi^A(F,NL) = \Pi^A(R,NL) > \Pi^A(R,F)$ and $\Pi^B(NL,F) = \Pi^B(NL,R) > \Pi^B(R,F)$) if $\lambda_s \geq 3t_s/\beta$.

Proof of Lemma C.4: We prove each part in the lemma separately.

Part (i): When $\lambda_s < \lambda_{sr}^{(1)}$ and $\lambda_f \geq 3t_f$, $r^I = 0$ for $I = A, B$ in case (R,R) by Lemma 6 so that $\Pi^I(R,R) = \frac{t_s}{2} + \frac{\beta\lambda_f\lambda_s}{2t_f}$ for $I = A, B$ by (15); and $r^A = 0$ in case (R,F) by Lemma 8 so that by (18) and (19),

$$\begin{aligned}\Pi^A(R,F) &= \frac{t_s}{2} + \frac{\beta\lambda_f\lambda_s}{2t_f},\\ \Pi^B(R,F) &= \frac{t_s}{2} + \frac{\beta\lambda_f\lambda_s}{2t_f} + \left(t_f + \frac{\beta\lambda_f\lambda_s}{t_s}\right)\frac{\beta}{2}.\end{aligned}$$

By (9), this implies that, when $\lambda_s < \lambda_{sr}^{(1)}$ and $\lambda_f \geq 3t_f$,

$$\Pi^A(F,F) - \Pi^A(R,F) = \Pi^B(R,F) - \Pi^B(R,R) = \frac{\beta t_f}{2} + \frac{\beta^2\lambda_f\lambda_s}{2t_s} > 0$$

so that a brand is always better off using fixed-fee contract when the other brand licenses by using either fixed-fee or royalty contract.

Part (ii): By Lemma 5(ii), (16) and (17), the brand that does not license when only one brand uses fixed-fee or royalty contract is a monopoly in market s. Then, $\Pi^B(R,NL) = \Pi^B(F,NL) > \Pi^B(R,F) = \Pi^A(F,R)$ (or $\Pi^A(NL,R) = \Pi^A(NL,F) > \Pi^A(R,F) = \Pi^B(F,R)$) (i.e., one of the brands is better of not licensing in case (R,F)) by our assumption that a brand prefers being a monopoly in market s when the other brand licenses over competing the other brand in market s and market f when they both license (i.e., v_s is sufficiently high). □

C.2. Proofs of the results in the paper

Proof of Lemma 1: We use backward induction to prove Lemma 1 and first consider the consumers' problem. If licensee a sells its licensed good in market f, then a follower located at θ will purchase if his/her net utility $U_f^a(\theta) = v_f - t_f\theta + \lambda_f D_s^{A(e)} - p^a \geq 0$. As we restrict our analysis for the case when brand A will set its price to ensure that the entire market s is fully covered, followers in the market f anticipate that, i.e., $D_s^{A(e)} = 1$. In this case, by (2), the marginal follower θ_f who is indifferent between purchasing and not purchasing licensee a's product is given by:

$$\theta_f = \frac{v_f + \lambda_f - p^a}{t_f}. \tag{C.4}$$

Similarly, by rational expectations, snobs in market s can anticipate that $D_f^{a(e)} = \theta_f$ so that the demand for licensee a's product is equal to $\beta\theta_f$. Combine this observation with the fact that market s is fully covered, the snob located at $\theta = 1$ will purchase luxury brand A's product, if his/her net utility $U_s^A(1) = v_s - t_s - \lambda_s\beta\theta_f - p^A \geq 0$. This implies that it is optimal for brand A to set its price $p^A = v_s - t_s - \lambda_s\beta\theta_f$.

Next, we consider luxury brand A's and licensee a's problems. Given $T^A = k^A$, $D_f^a = \theta_f$ as stated in (C.4) and $D_s^{A(e)} = 1$, it follows from (4) that licensee a's profit is given by:

$$\Pi^a(p^a) = p^a\beta\frac{v_f + \lambda_f - p^a}{t_f} - k^A.$$

By considering the first-order condition along with the bound on (i.e., $p^a \in [v_f + \lambda_f - t_f, v_f + \lambda_f]$ so that θ_f in (C.4) satisfies $0 \le \theta_f \le 1$), we get:

$$p^a = \begin{cases} \frac{v_f+\lambda_f}{2}, & \text{if } \lambda_f < 2t_f - v_f, \\ v_f + \lambda_f - t_f, & \text{if } \lambda_f \ge 2t_f - v_f. \end{cases}$$

By substituting p^a into (C.4), we can retrieve θ_f and then the corresponding brand A's optimal price $p^A = v_s - t_s - \lambda_s\beta\theta_f$, which can be rewritten as:

$$p^A = \begin{cases} v_s - t_s - \lambda_s\beta\frac{v_f+\lambda_f}{2t_f}, & \text{if } \lambda_f < 2t_f - v_f, \\ v_s - t_s - \lambda_s\beta, & \text{if } \lambda_f \ge 2t_f - v_f. \end{cases}$$

Lastly, under fixed-fee contract, it is optimal for brand A to set the fixed lump-sum payment $k^A = p^a\beta\theta_f$ to extract the entire surplus of its licensee (i.e., $p^a\beta\theta_f - k^A$) so that the licensee a ends up with zero profit (and yet licensee a will accept the licensing contract). Therefore, fixed lump-sum payment k^A is given in Lemma 1. $\square$

Proof of Lemma 2: We characterize the prices and the royalty fee in the monopoly setting under royalty contract by using backward induction. First, we study consumers' problem. Given the licensee's price p^a, the marginal follower θ_f is given by (C.4). By using rational expectations so that $D_f^a = \theta_f$, brand A sets its price equal to $p^A = v_s - t_s - \lambda_s\beta\theta_f$ to ensure that market s is fully covered (i.e., $D_s^A = 1$). By substituting $D_f^a = \theta_f$ and $T^A = r^A\beta D_f^a$ in (4), we obtain licensee a's profit as follows:

$$\Pi^a(p^a) = (p^a - r^A)\beta\frac{v_f + \lambda_f - p^a}{t_f}.$$

Then, using first order condition and imposing bounds on p^a so that θ_f in (C.4) is in between 0 and 1, we determine the optimal price p^a as a function of r^A:

$$p^a = \begin{cases} \frac{v_f+\lambda_f+r^A}{2}, & \text{if } r^A \le v_f + \lambda_f, \\ v_f + \lambda_f, & \text{if } r^A > v_f + \lambda_f. \end{cases}$$

Through substitution, we obtain

$$\theta_f = \begin{cases} 1, & \text{if } r^A \le v_f + \lambda_f - 2t_f, \\ \frac{v_f+\lambda_f-r^A}{2t_f}, & \text{if } v_f + \lambda_f - 2t_f \le r^A \le v_f + \lambda_f, \\ 0, & \text{if } r^A > v_f + \lambda_f. \end{cases}$$

Then, by using $p^A = v_s - t_s - \lambda_s\beta\theta_f$, $D_s^A = 1$ and $T^A = r^A\beta\theta_f$, we use (3) to obtain brand A's profit as a function of its royalty fee r^A:

$$\Pi^A(r^A) = \begin{cases} v_s - t_s + (r^A - \lambda_s)\beta, & \text{if } r^A < v_f + \lambda_f - 2t_f, \\ v_s - t_s + (r^A - \lambda_s)\beta\frac{v_f+\lambda_f-r^A}{2t_f}, & \text{if } v_f + \lambda_f - 2t_f \le r^A \le v_f + \lambda_f, \\ v_s - t_s, & \text{if } r^A > v_f + \lambda_f. \end{cases}$$

Solving for the royalty fee r^A that maximizes brands' profit above, we determine the optimal royalty fee r^A as follows:

$$r^A = \begin{cases} v_f + \lambda_f - 2t_f, & \text{if } \lambda_s \le v_f + \lambda_f - 4t_f, \\ \frac{v_f+\lambda_f+\lambda_s}{2}, & \text{if } v_f + \lambda_f - 4t_f < \lambda_s < v_f + \lambda_f, \\ v_f + \lambda_f, & \text{if } \lambda_s \ge v_f + \lambda_f. \end{cases}$$

Substituting optimal royalty fee above back into p^A and p^a, we obtain the prices in Lemma 2. $\square$

Proof of Proposition 1: First, we prove part (i) of the proposition. Let us define $\check{\lambda}_{sk}^{(1)} \in (0, v_f + \lambda_f)$ as follows:

$$\check{\lambda}_{sk}^{(1)} = \begin{cases} \frac{v_f+\lambda_f}{2}, & if \ \lambda_f < 2t_f - v_f, \\ v_f + \lambda_f - t_f, & if \ \lambda_f \geq 2t_f - v_f. \end{cases} \tag{C.5}$$

Comparing (5) and (6), $\Pi^A(F) \leq \Pi^A(NL)$, if and only if, $\lambda_s \geq \check{\lambda}_{sk}^{(1)} \in (0, v_f + \lambda_f)$. Similarly, comparing (5) and (7), $\Pi^A(R) \leq \Pi^A(NL)$ if, and only if, $\lambda_s \geq v_f + \lambda_f$. Thus, $\max\{\Pi^A(F), \Pi^A(R)\} \leq \Pi^A(NL)$ if, and only if, $\lambda_s \geq v_f + \lambda_f$, and hence, part (i) follows.

Next, we prove part (ii) of the proposition. By part (i), licensing is not optimal under both fixed-fee and royalty contracts for $\lambda_s \geq v_f + \lambda_f$; moreover, licensing is optimal under the fixed-fee contract for $\lambda_s < \check{\lambda}_{sk}^{(1)} \in (0, v_f + \lambda_f)$. This implies that, for $\check{\lambda}_{sk}^{(1)} \leq \lambda_s < v_f + \lambda_f$, the royalty contract dominates the fixed-fee contract.

Now, consider $\lambda_s < \check{\lambda}_{sk}^{(1)}$. From (6) and (7), for $\lambda_f < 2t_f - v_f$, we have $\lambda_s < \check{\lambda}_{sk}^{(1)} = (v_f + \lambda_f)/2$ by (C.5) and

$$\Pi^A(F) - \Pi^A(R) = \frac{\beta}{8t_f}\left(v_f + \lambda_f - \left(1+\sqrt{2}\right)\lambda_s\right)\left(v_f + \lambda_f + \left(\sqrt{2}-1\right)\lambda_s\right).$$

Thus $\Pi^A(F) - \Pi^A(R) > 0$ and the fixed-fee contract dominates for $\lambda_s \leq (v_f + \lambda_f)/\left(1+\sqrt{2}\right)$, and the royalty contract dominates for $\lambda_s \in \left((v_f + \lambda_f)/\left(1+\sqrt{2}\right), (v_f + \lambda_f)/2\right)$ when $\lambda_f < 2t_f - v_f$. Now consider $\lambda_f \geq 2t_f - v_f$. In this case, $\lambda_s < \check{\lambda}_{sk}^{(1)} = v_f + \lambda_f - t_f$ by (C.5). Thus, from (6) and (7), $\Pi^A(F) - \Pi^A(R) > 0$ for $\lambda_s \leq v_f + \lambda_f - 4t_f$, and for $\lambda_s > v_f + \lambda_f - 4t_f$, we have

$$\Pi^A(F) - \Pi^A(R) = \frac{\beta}{8t_f}\left(\lambda_s - v_f - \lambda_f + 2\left(2+\sqrt{2}\right)t_f\right)\left(v_f + \lambda_f - 2\left(2-\sqrt{2}\right)t_f - \lambda_s\right).$$

Note that $\Pi^A(F) - \Pi^A(R) > 0$ for $\lambda_s \in \left(v_f + \lambda_f - 4t_f, v_f + \lambda_f - 2\left(2-\sqrt{2}\right)t_f\right)$ and $\Pi^A(F) - \Pi^A(R) < 0$ for $\lambda_s \in \left(v_f + \lambda_f - 2\left(2-\sqrt{2}\right)t_f, v_f + \lambda_f - t_f\right)$. Then it follows that the fixed-fee contract dominates for $\lambda_s < v_f + \lambda_f - 2\left(2-\sqrt{2}\right)t_f$, and the royalty contract dominates for $\lambda_s \in \left[v_f + \lambda_f - 2\left(2-\sqrt{2}\right)t_f, v_f + \lambda_f - t_f\right)$ when $\lambda_f \geq 2t_f - v_f$. $\square$

Proof of Lemma 3: We use backward induction to prove Lemma 3 and start by studying consumers' problem first. Both brands license in this case and market s and f are fully-covered so that, by Lemma C.1, marginal snob θ_s and follower θ_f are given by (C.1) and (C.2), respectively. Then, using (C.1) and (C.2), $T^I = k^I$ for $I = A, B$, $D_s^A = 1 - D_s^B$, and $D_f^a = 1 - D_f^b$ along with (3)-(4), we obtain profits of brands A and B and licensees a and b as follows:

$$\begin{aligned}
\Pi^A\left(p^A, k^A\right) &= \left(p^A - c\right)\left(\frac{1}{2} + \frac{t_f\left(p^B - p^A\right) - \beta\lambda_s\left(p^b - p^a\right)}{2t_f t_s + 2\beta\lambda_f\lambda_s}\right) + k^A, \\
\Pi^B\left(p^B, k^B\right) &= \left(p^B - c\right)\left(\frac{1}{2} + \frac{t_f\left(p^A - p^B\right) - \beta\lambda_s\left(p^a - p^b\right)}{2t_f t_s + 2\beta\lambda_f\lambda_s}\right) + k^B, \\
\Pi^a\left(p^a\right) &= p^a\beta\left(\frac{1}{2} + \frac{\lambda_f\left(p^B - p^A\right) + t_s\left(p^b - p^a\right)}{2t_f t_s + 2\beta\lambda_f\lambda_s}\right) - k^A, \\
\Pi^b\left(p^b\right) &= p^b\beta\left(\frac{1}{2} + \frac{\lambda_f\left(p^A - p^B\right) + t_s\left(p^a - p^b\right)}{2t_f t_s + 2\beta\lambda_f\lambda_s}\right) - k^B.
\end{aligned}$$

By considering the first-order conditions and solving for prices simultaneously, we characterize the optimal prices for brand I ($I = A, B$) and its licensee i ($i = a, b$) as follows:

$$p^I = c + t_s + \frac{\beta\lambda_f\lambda_s}{t_f} \text{ and } p^i = t_f + \frac{\beta\lambda_f\lambda_s}{t_s}.$$

We now determine brands' optimal fixed licensing fees. By using the fact that licensees share market f equally so that $D_f^i = 1/2$ for $i = a, b$, the profit of licensee i ($i = a, b$), as stated in (4), can be simplified as $\Pi^i = p^i\beta/2 - k^I$ for any given fixed fee k^I. By $p^i = t_f + \beta\lambda_f\lambda_s/t_s$ for $i = a, b$, it is optimal for brand I to set the fixed fee equal to $k^I = p^i\beta/2 = (t_f + \beta\lambda_f\lambda_s/t_s)\,\beta/2$ to extract the entire surplus from its licensee. $\square$

Proof of Lemma 5: To begin, let us consider the snobs in market s. Because licensee a operates as a monopoly that covers the entire market f (i.e., $D_f^a = 1$), a snob located at θ can obtain net utilities $U_s^A(\theta) = v_s - t_s\theta - \lambda_s\beta - p^A$ and $U_s^B(\theta) = v_s - t_s(1-\theta) - p^B$ from purchasing brand A and B, respectively. Then, the marginal snob θ_s who is indifferent between purchasing A versus B is given by:

$$\theta_s = \frac{1}{2} + \frac{p^B - p^A - \lambda_s\beta}{2t_s}. \tag{C.6}$$

By rational expectations and market s being fully covered, followers anticipate that the demands for brand A and B are $D_s^{A(e)} = D_s^A = \theta_s$ and $D_s^{B(e)} = D_s^B = (1-\theta_s)$, respectively. Then, the net utility to be obtained by a follower located at θ who purchases licensee a's product is equal to $U_f^a(\theta) = v_f - t_f\theta + \lambda_f\theta_s - p^a$. To ensure that the entire market f is covered by licensee a's product, it is optimal for licensee a to set its price $p^a = v_f - t_f + \lambda_f\theta_s$ so that the follower located at $\theta = 1$ will purchase its licensed product.

Using (C.6), the fact that $p^a = v_f - t_f + \lambda_f\theta_s$, $D_s^A = 1 - D_s^B = \theta_s$, $D_f^a = 1 - D_f^b = 1$, $T^A = k^A$ and $T^B = 0$ (as brand B does not license), we can use (3) and (4) to express the profit functions for brands A and B and the only licensee a as functions of p^A and p^B.

$$\begin{aligned}
\Pi^A\left(p^A, k^A\right) &= \left(p^A - c\right)\left(\frac{1}{2} + \frac{p^B - p^A - \lambda_s\beta}{2t_s}\right) + k^A,\\
\Pi^B\left(p^B\right) &= \left(p^B - c\right)\left(\frac{1}{2} + \frac{p^A - p^B + \lambda_s\beta}{2t_s}\right),\\
\Pi^a\left(p^a\right) &= \left(v_f - t_f + \lambda_f\left(\frac{1}{2} + \frac{p^B - p^A - \lambda_s\beta}{2t_s}\right)\right)\beta - k^A.
\end{aligned}$$

Also, by considering the first-order conditions associated with the profit functions of brands A and B simultaneously (due to the underlying price competition between both brands in market s) and by considering the bounds associated with θ_s (i.e., $\theta_s \in [0,1]$), we can determine the equilibrium price of both brands and the licensee a as follows.

Case 1: $\lambda_s < 3t_s/\beta$. In this case, the optimal prices are given by:

$$p^A = c + t_s - \frac{\beta\lambda_s}{3},\ p^B = c + t_s + \frac{\beta\lambda_s}{3} \text{ and } p^a = v_f + \lambda_f\left(\frac{1}{2} - \frac{\beta\lambda_s}{6t_s}\right) - t_f.$$

Subsituting above prices in (C.6), we have $\theta_s = 1/2 - \beta\lambda_s/6t_s \in (0, 1/2)$ in this case. Thus, both brands compete in market s and brand B that does not license obtains more than half of market s.

Case 2: $\lambda_s \geq 3t_s/\beta$. In this case, for the prices in case 1, (C.6) reveals that $\theta_s = 1/2 - \beta\lambda_s/6t_s \leq 0$. Hence, as the negative popularity effect is high (i.e., $\lambda_s \geq 3t_s/\beta$), $\theta_s = 0$ so that no snob will purchase brand A once it licenses its brand name to licensee a. Consequently, brand B operates as a monopoly in market s and licensee a operates as a monopoly in market f. While brand A's price p^A is irrelevant, brand B and licensee a will set their prices to ensure their respective markets are fully covered so that:

$$p^B = v_s - t_s \text{ and } p^a = v_f - t_f.$$

Lastly, we determine the optimal fixed lump-sum payment of brand A. In both cases above, licensee a's profit is equal to $p^a\beta - k^A$ since market f is fully covered. Then, it is optimal for brand A to set the fixed fee equal to $k^A = p^a\beta$ to extract the entire surplus from its licensee. This observation together with Case 1 and 2 above implies that the optimal fixed fee k^A satisfies: $k^A = \left(v_f + \lambda_f\left(1/2 - \beta\lambda_s/6t_s\right)^+ - t_f\right)\beta$, where $(x)^+ = \max\{x, 0\}$. □

Proof of Proposition 2: Using Lemma C.2, we characterize the equilibrium strategies of brands with fixed-fee contract. Recall that we define $\lambda_{sk}^L = \min(\lambda_{sk}^{(1)}, \lambda_{sk}^{(3)})$ and $\lambda_{sk}^H = \max(\lambda_{sk}^{(1)}, \lambda_{sk}^{(3)})$. In addition, as in the proof of Lemma C.2, we let $\Delta(F, NL)$ and $\Delta(F, F)$, respectively, be the benefit to a brand from licensing via fixed-fee contract when the other brand does not license and when the other brand licenses via fixed-fee contract. Recall by Lemma C.2 that $\Delta(F, NL) > 0$ if, and only if, $\lambda_s < \lambda_{sk}^{(1)}$, and $\Delta(F, F) > 0$ if, and only if, $\lambda_s < \lambda_{sk}^{(3)}$, which we will use below. We prove each part of the proposition separately.

Part I: In this part, $v_f \le t_f + t_s/2\beta$ so that $\lambda_{sk}^{(1)} < \infty$ by (12). Now we consider four cases: (a) $\lambda_s \le \lambda_{sk}^L$, (b) $\lambda_{sk}^{(1)} < \lambda_s < \lambda_{sk}^{(3)}$, (c) $\lambda_{sk}^{(3)} < \lambda_s < \lambda_{sk}^{(1)}$, and (d) $\lambda_s \ge \lambda_{sk}^H$.

Case I(a): In this case, $\Delta(F, NL) > 0$ and $\Delta(F, F) > 0$, that is, it is optimal for a brand to license independent from whether the other brand licenses or not. Thus, both brands license and use fixed-fee contract for $\lambda_s \le \lambda_{sk}^L$.

Case I(b): In this case, $\lambda_{sk}^{(1)} \le \lambda_{sk}^{(3)}$ so that $\lambda_{sk}^L = \lambda_{sk}^{(1)}$ and $\lambda_{sk}^H = \lambda_{sk}^{(3)}$. Then, for $\lambda_s \in (\lambda_{sk}^L, \lambda_{sk}^H)$, $\Delta(F, NL) \le 0$ and $\Delta(F, F) > 0$; therefore, it is optimal for a brand to license when the other brand uses fixed-fee contract and to not license when the other brand does not license. In other words, the best response of a brand is to use the same strategy as the other brand. Thus, there are two Nash equilibria in this case: (i) both brands use fixed-fee contract, and (ii) no brand licenses.

Case I(c): In this case, $\lambda_{sk}^{(1)} > \lambda_{sk}^{(3)}$ so that $\lambda_{sk}^L = \lambda_{sk}^{(3)}$ and $\lambda_{sk}^H = \lambda_{sk}^{(1)}$. Thus, for $\lambda_s \in (\lambda_{sk}^L, \lambda_{sk}^H)$, $\Delta(F, NL) > 0$ and $\Delta(F, F) \le 0$; therefore, it is optimal for a brand to not license when the other brand uses fixed-fee contract and to use fixed-fee contract when the other brand does not license. In other words, the best response of each brand is to use the opposite strategy of the other brand. As a result, only one brand uses fixed-fee contract in this case.

Case I(d): In this case, $\Delta(F, NL) \le 0$ and $\Delta(F, F) \le 0$, which implies that it is optimal for a brand not to license independent from whether the other brand licenses or not. Thus, both brands do not license.

Part II: Lastly, we assume $v_f > t_f + t_s/2\beta$ and prove the second part of the proposition. In this case, $\lambda_{sk}^{(1)} = \infty$ by (12), and $\lambda_{sk}^L = \lambda_{sk}^{(3)}$ and $\lambda_{sk}^H = \lambda_{sk}^{(1)} = \infty$. For $\lambda_s \le \lambda_{sk}^L$, $\Delta(F, NL) > 0$ and $\Delta(F, F) > 0$ by Lemma C.2. Hence each brand always prefers using fixed-fee contract whether the other brand licenses or not, that is, both brands license. However, for $\lambda_s > \lambda_{sk}^L$, $\Delta(F, NL) > 0$ and $\Delta(F, F) \le 0$ by Lemma C.2, that is, the best response of a brand is to use the fixed-fee contract when the other brand does not license and to not license when the other brand licenses. Therefore, for $\lambda_s > \lambda_{sk}^L$, either brand A or brand B licenses and only one brand uses fixed-fee contract. □

Proof of Lemma 6: Since both brands license in this case and market s and f are fully-covered, marginal snob θ_s and follower θ_f are given by (C.1) and (C.2), respectively, by Lemma C.1. In addition, $D_s^A = 1 - D_s^B = \theta_s$ and $D_f^a = D_f^b = \theta_f$ by rational expectations. Using this and substituting $T^A = r^A\beta D_f^a$, and $T^B = r^B\beta D_f^b$ into (3)-(4), we obtain profits of brands and their licensees as follows:

$$\Pi^A\left(p^A, r^A\right) = \left(p^A - c\right)\left(\frac{1}{2} + \frac{t_f\left(p^B - p^A\right) - \beta\lambda_s\left(p^b - p^a\right)}{2t_f t_s + 2\beta\lambda_f\lambda_s}\right) + r^A\beta\left(\frac{1}{2} + \frac{\lambda_f\left(p^B - p^A\right) + t_s\left(p^b - p^a\right)}{2t_f t_s + 2\beta\lambda_f\lambda_s}\right)$$

$$\begin{aligned}
\Pi^B\left(p^B, r^B\right) &= \left(p^B - c\right)\left(\frac{1}{2} + \frac{t_f\left(p^A - p^B\right) - \beta\lambda_s\left(p^a - p^b\right)}{2t_f t_s + 2\beta\lambda_f\lambda_s}\right) + r^B\beta\left(\frac{1}{2} + \frac{\lambda_f\left(p^A - p^B\right) + t_s\left(p^a - p^b\right)}{2t_f t_s + 2\beta\lambda_f\lambda_s}\right)\\
\Pi^a\left(p^a\right) &= \left(p^a - r^A\right)\beta\left(\frac{1}{2} + \frac{\lambda_f\left(p^B - p^A\right) + t_s\left(p^b - p^a\right)}{2t_f t_s + 2\beta\lambda_f\lambda_s}\right)\\
\Pi^b\left(p^b\right) &= \left(p^b - r^B\right)\beta\left(\frac{1}{2} + \frac{\lambda_f\left(p^A - p^B\right) + t_s\left(p^a - p^b\right)}{2t_f t_s + 2\beta\lambda_f\lambda_s}\right).
\end{aligned}$$

Using first-order conditions, we obtain prices as a function of royalty fees as follows:

$$\begin{aligned}
p^A &= c + t_s + \frac{\beta\lambda_f}{t_f}\left(\lambda_s - r^A\right) + \beta\left(r^A - r^B\right)\frac{2\beta\lambda_f^2\lambda_s + 3\lambda_f t_f t_s + \lambda_s t_f t_s}{9t_f^2 t_s + 4\beta\lambda_f\lambda_s t_f}\\
p^B &= c + t_s + \frac{\beta\lambda_f}{t_f}\left(\lambda_s - r^B\right) + \beta\left(r^B - r^A\right)\frac{2\beta\lambda_f^2\lambda_s + 3\lambda_f t_f t_s + \lambda_s t_f t_s}{9t_f^2 t_s + 4\beta\lambda_f\lambda_s t_f}\\
p^a &= t_f + r^A + \frac{\beta\lambda_f\lambda_s}{t_s} + \left(r^B - r^A\right)\frac{3t_f t_s + 2\beta\lambda_f\lambda_s - \beta\lambda_f^2}{9t_f t_s + 4\beta\lambda_f\lambda_s}\\
p^b &= t_f + r^B + \frac{\beta\lambda_f\lambda_s}{t_s} + \left(r^A - r^B\right)\frac{3t_f t_s + 2\beta\lambda_f\lambda_s - \beta\lambda_f^2}{9t_f t_s + 4\beta\lambda_f\lambda_s}.
\end{aligned}$$

Substituting above prices in brand A's profit function, we obtain its profit as follows:

$$\begin{aligned}
\Pi^A\left(r^A\right) = {}& \beta\left(r^A - r^B\right)\frac{2\beta\lambda_f^2\lambda_s + 3\lambda_f t_f t_s + \lambda_s t_f t_s}{9t_f^2 t_s + 4\beta\lambda_f\lambda_s t_f}\left(\frac{1}{2} + \beta\left(r^A - r^B\right)\frac{2\beta\lambda_f^2\lambda_s + 3\lambda_f t_f t_s + \lambda_s t_f t_s}{2\left(t_f t_s + \beta\lambda_f\lambda_s\right)\left(9t_f t_s + 4\beta\lambda_f\lambda_s\right)}\right)\\
& \left(t_s + \frac{\beta\lambda_f}{t_f}\left(\lambda_s - r^A\right)\right)\left(\frac{1}{2} + \beta\left(r^A - r^B\right)\frac{2\beta\lambda_f^2\lambda_s + 3\lambda_f t_f t_s + \lambda_s t_f t_s}{2\left(t_f t_s + \beta\lambda_f\lambda_s\right)\left(9t_f t_s + 4\beta\lambda_f\lambda_s\right)}\right)\\
& + r^A\beta\left(\frac{1}{2} + t_s\left(r^A - r^B\right)\frac{\beta\left(\lambda_f - \lambda_s\right)^2 - 3t_f t_s - \beta\lambda_s^2}{2\left(t_f t_s + \beta\lambda_f\lambda_s\right)\left(9t_f t_s + 4\beta\lambda_f\lambda_s\right)}\right).
\end{aligned}$$

By symmetry, the profit of brand B ($\Pi^B\left(r^B\right)$) is given by $\Pi^A\left(r^A\right)$ where r^A and r^B are replaced by r^B and r^A, respectively. Hence, we focus only on brand A's problem in what follows. Taking the derivative, we have

$$\begin{aligned}
\frac{d\Pi^A\left(r^A\right)}{dr^A} = \frac{\beta}{2}\Bigg(& \frac{-3\lambda_f t_s + 2\lambda_s t_s + 9t_f t_s + 4\beta\lambda_f\lambda_s}{9t_f t_s + 4\beta\lambda_f\lambda_s} + r^B t_s\frac{2\beta\lambda_s^2\left(2\beta\lambda_f^2 - t_f t_s\right) + \beta\lambda_f\lambda_s\left(27t_f t_s + 2\beta\lambda_f^2\right) + 27t_f^2 t_s^2}{\left(t_f t_s + \beta\lambda_f\lambda_s\right)\left(9t_f t_s + 4\beta\lambda_f\lambda_s\right)^2}\\
& -2r^A\frac{4\beta^3\lambda_f^4\lambda_s^2 + 14\beta^2\lambda_f^3\lambda_s t_f t_s + 8\beta^2\lambda_f^2\lambda_s^2 t_f t_s + 9\beta\lambda_f^2 t_f^2 t_s^2 + 33\beta\lambda_f\lambda_s t_f^2 t_s^2 - \beta\lambda_s^2 t_f^2 t_s^2 + 27t_f^3 t_s^3}{t_f\left(t_f t_s + \beta\lambda_f\lambda_s\right)\left(9t_f t_s + 4\beta\lambda_f\lambda_s\right)^2}\Bigg).
\end{aligned}$$

Note that

$$\frac{d^2\Pi^A\left(r^A\right)}{d{r^A}^2} = -\beta\frac{\left(4\beta^2\lambda_f^4 + 8\beta\lambda_f^2 t_f t_s - t_f^2 t_s^2\right)\lambda_s^2 + \beta\lambda_f\lambda_s t_f t_s\left(33t_f t_s + 14\beta\lambda_f^2\right) + 9\beta\lambda_f^2 t_f^2 t_s^2 + 27t_f^3 t_s^3}{t_f\left(t_f t_s + \beta\lambda_f\lambda_s\right)\left(9t_f t_s + 4\beta\lambda_f\lambda_s\right)^2}.$$

Observe that $d^2\Pi^A\left(r^A\right)/d{r^A}^2 < 0$ and $\lim_{r_A\uparrow\infty} d\Pi^A\left(r^A\right)/dr^A < 0$ by our assumption that $\lambda_f \geq \sqrt{t_f t_s/2\beta}$. Also we have

$$\begin{aligned}
\lim_{r^A\downarrow 0}\frac{d\Pi^A\left(r^A\right)}{dr^A} = \frac{\beta}{2}\Bigg(& \frac{2\lambda_s\left(2\beta\lambda_f + t_s\right) + 3t_s\left(3t_f - \lambda_f\right)}{9t_f t_s + 4\beta\lambda_f\lambda_s}\\
& + r^B t_s\frac{2\beta\lambda_s^2\left(2\beta\lambda_f^2 - t_f t_s\right) + \beta\lambda_f\lambda_s\left(27t_f t_s + 2\beta\lambda_f^2\right) + 27t_f^2 t_s^2}{\left(t_f t_s + \beta\lambda_f\lambda_s\right)\left(9t_f t_s + 4\beta\lambda_f\lambda_s\right)^2}\Bigg). \qquad \text{(C.7)}
\end{aligned}$$

Note by (14) (i.e., $\lambda_f \geq \sqrt{t_f t_s/2\beta}$), the second term inside the parenthesis above is always positive. Consider two cases: (i) $\lambda_f \leq 3t_f$, or $\lambda_s \geq \lambda_{sr}^{(1)}$ when $\lambda_f > 3t_f$, and (ii) $\lambda_s < \lambda_{sr}^{(1)}$ when $\lambda_f > 3t_f$, where $\lambda_{sr}^{(1)}$ is given by (B.5).

Case (i): In this case, $\lim_{r_A\downarrow 0} d\Pi^A\left(r^A\right)/dr^A > 0$ by (C.7). This by $\Pi^A\left(r^A\right)$ being concave and $\lim_{r_A\uparrow\infty} d\Pi^A\left(r^A\right)/dr^A < 0$ implies that there exists unique $r^A \in (0,\infty)$ that satisfies first-order condition, i.e.,

$d\Pi^A(r^A)/dr^A = 0$. Similarly, by symmetry, there exists a unique $r^B \in (0,\infty)$ that satisfies $d\Pi^B(r^B)/dr^B = 0$. Solving $d\Pi^I(r^I)/dr^I = 0$ for r^I ($I = A, B$), the optimal royalty fee for brand I ($I = A, B$) is unique and given by:

$$r^I = t_f \frac{2\lambda_s(2\beta\lambda_f + t_s) + 3t_s(3t_f - \lambda_f)}{3t_f t_s + 2\beta\lambda_f^2}.$$

Case (ii): In this case, by (C.7), $\lim_{r_A \downarrow 0} d\Pi^A(r^A)/dr^A \le 0$ if, and only if, $r^B \le \bar{r}$, where $\bar{r}$ is positive in this case and given by:

$$\bar{r} = \frac{(3t_s(\lambda_f - 3t_f) - 2\lambda_s(t_s + 2\beta\lambda_f))(t_f t_s + \beta\lambda_f\lambda_s)(9t_f t_s + 4\beta\lambda_f\lambda_s)}{t_s\left(27t_f^2 t_s^2 + 2\beta^2\lambda_f^3\lambda_s + 4\beta^2\lambda_f^2\lambda_s^2 - 2\beta\lambda_s^2 t_f t_s + 27\beta\lambda_f\lambda_s t_f t_s\right)}.$$

Similarly, by symmetry $\lim_{r_B \downarrow 0} d\Pi^B(r^B)/dr^B \le 0$ if, and only if, $r^A \le \bar{r}$.

First let us characterize all equilibria where a brand sets its royalty fee less than or equal to $\bar{r}$. Without loss of generality, assume that brand B sets its royalty fee $r^B \le \bar{r}$. The best response of brand A in this case is to set $r^A = 0$ since its profit is always decreasing in r^A (by $\lim_{r_A \downarrow 0} d\Pi^A(r^A)/dr^A \le 0$ and $\Pi^A(r^A)$ being concave). When $r^A = 0$, best response of brand B is also to set $r^B = 0$ since $\bar{r} > r^A = 0$ and brand B's profit is always decreasing in r^B. This indicates that $r^I = 0$ ($I = A, B$) is the only equilibrium where a brand sets $r^I < \bar{r}$ in this case.

Next we characterize all equilibria where a brand sets its royalty fee greater than $\bar{r}$. Again assume that brand B sets its royalty fee $r^B > \bar{r}$. Since $\lim_{r_A \downarrow 0} d\Pi^A(r^A)/dr^A > 0$ for $r^B > \bar{r}$, the best response of brand A in this case is to set its royalty fee $r^A > 0$ such that $d\Pi^A(r^A)/dr^A = 0$. First suppose that r^A satisfying $d\Pi^A(r^A)/dr^A = 0$ is less than or equal to $\bar{r}$, i.e., $r^A \le \bar{r}$. By the discussion in above paragraph, the best response of brand B is to set its royalty fee equal to zero, i.e., $r^B = 0$, when $r^A \le \bar{r}$. This is a contradiction to our initial assumption that $r^B > \bar{r}$. Now suppose that r^A satisfying $d\Pi^A(r^A)/dr^A = 0$ is greater than $\bar{r}$, i.e., $r^A > \bar{r}$. In this case, since $\lim_{r_B \downarrow 0} d\Pi^B(r^B)/dr^B > 0$, $\lim_{r_B \uparrow \infty} d\Pi^B(r^B)/dr^B < 0$, and $\Pi^B(r^B)$ is concave, brand B will set its royalty fee r^B such that $d\Pi^B(r^B)/dr^B = 0$. This implies that r^A and r^B must simultaneously satisfy $d\Pi^A(r^A)/dr^A = 0$ and $d\Pi^A(r^B)/dr^B = 0$. Solving for r^A and r^B, we obtain

$$r^A = r^B = t_f \frac{2\lambda_s(2\beta\lambda_f + t_s) + 3t_s(3t_f - \lambda_f)}{3t_f t_s + 2\beta\lambda_f^2} < 0$$

where the inequality follows from $\lambda_s < \lambda_{sr}^{(1)}$ and $\lambda_f > 3t_f$ in this case. However, note that this a contradiction to our assumption that brand B sets its royalty fee $r^B > \bar{r}$. Therefore, in case (ii), $r^I > \bar{r} > 0$ ($I = A, B$) cannot be an equilibrium.

Summarizing above analysis, for $\lambda_f \ge 3t_f$ and $\lambda_s < \lambda_{sr}^{(1)}$, $r^I = 0$; otherwise,

$$r^I = t_f \frac{2\lambda_s(2\beta\lambda_f + t_s) + 3t_s(3t_f - \lambda_f)}{3t_f t_s + 2\beta\lambda_f^2}$$

for $I = A, B$. Then plugging $r^A = r^B$ above in price expressions, prices of brand I ($I = A, B$) and its licensee i ($i = a, b$) are, respectively, equal to $p^I = c + t_s + \beta\lambda_f(\lambda_s - r^I)/t_f$ and $p^i = r^I + t_f + \beta\lambda_f\lambda_s/t_s$. □

Proof of Lemma 7: We will prove each part of the lemma separately.

Part (i): In this part, we will show that both brands are better off when they both do not license compared to the case when they both license by using royalty contract if, and only if, $\lambda_s > \lambda_{sr}^{(2)}$ and for $\lambda_f \in (t_f, 3t_f)$. By Lemma 6, the royalty fee of brand I ($I = A, B$) when both brands license by using royalty contract is given by:

$$r^I = \begin{cases} 0, & \text{if } \lambda_f \ge 3t_f \text{ and } \lambda_s < \lambda_{sr}^{(1)}, \\ t_f \frac{2\lambda_s(2\beta\lambda_f + t_s) + 3t_s(3t_f - \lambda_f)}{3t_f t_s + 2\beta\lambda_f^2}, & \text{if otherwise.} \end{cases} \tag{C.8}$$

By (8), (15) and (C.8), $\Pi^I(R,R) > \Pi^I(NL,NL)$ for $I = A, B$ when $\lambda_f \geq 3t_f$ and $\lambda_s < \lambda_{sr}^{(1)}$, or $\lambda_f \leq t_f$.

Now consider all other cases, i.e., $\lambda_f \geq 3t_f$ and $\lambda_s \geq \lambda_{sr}^{(1)}$, or $\lambda_f \in (t_f, 3t_f)$. In all these cases, by (8), (15) and (C.8),

$$\Pi^I(R,R) - \Pi^I(NL,NL) = \frac{\beta}{2t_f}\left(\lambda_f\lambda_s - (\lambda_f - t_f)\, t_f \frac{2\lambda_s(2\beta\lambda_f + t_s) + 3t_s(3t_f - \lambda_f)}{3t_f t_s + 2\beta\lambda_f^2}\right)$$

for $I = A, B$. After some simplifications, we have

$$\Pi^I(R,R) - \Pi^I(NL,NL) = \frac{\beta}{2t_f\left(2\beta\lambda_f^2 + 3t_f t_s\right)}\left(g_1(\lambda_f)\lambda_s - h_1(\lambda_f)\right), \tag{C.9}$$

for $I = A, B$, where $g_1(\lambda_f)$ and $h_1(\lambda_f)$ are given, respectively, by (B.1) and (B.3). Note that $h_1(\lambda_f) > 0$ for $\lambda_f \in (t_f, 3t_f)$ and $h_1(\lambda_f) \leq 0$ for $\lambda_f \geq 3t_f$. Also note that $g_1(\lambda_f)$ is (strictly) convex for $\lambda_f > t_f$ and $\lim_{\lambda_f \downarrow t_f} dg_1(\lambda_f)/d\lambda_f > 0$ which indicates that $g_1(\lambda_f)$ is increasing for $\lambda_f > t_f$. This by $\lim_{\lambda_f \downarrow t_f} g_1(\lambda_f) > 0$ futher implies that $g_1(\lambda_f) > 0$ for $\lambda_f > t_f$.

Now consider $\lambda_f \in (t_f, 3t_f)$. In this case, $g_1(\lambda_f) > 0$ and $h_1(\lambda_f) > 0$, and by (C.9), both firms are better off from not licensing, i.e., $\Pi^I(NL,NL) > \Pi^I(R,R)$ for $I = A, B$, if $\lambda_s < \lambda_{sr}^{(2)}$, where $\lambda_{sr}^{(2)} = h_1(\lambda_f)/g_1(\lambda_f)$. Finally consider $\lambda_f \geq 3t_f$. In this case, $g_1(\lambda_f) > 0$ and $h_1(\lambda_f) \leq 0$, therefore, $\Pi^I(R,R) > \Pi^I(NL,NL)$ for $I = A, B$ by (C.9). □

Part (ii): In this part, we characterize cases where each brand is better off when both use fixed-fee contracts relative to the case when both use royalty contracts. For $\lambda_f \geq t_f$, it follows from (9) and (15) that: $\Pi^I(F,F) > \frac{t_s}{2} + \frac{\beta\lambda_f\lambda_s}{2t_f} \geq \Pi^I(R,R)$ for $I = A, B$. Thus, each brand is better off when they both use fixed-fee contracts compared to the case when they both use royalty contract when $\lambda_f \geq t_f$.

Next, we consider $\lambda_f < t_f$. We will show that there exists $0 < \lambda_{fr}^L < \lambda_{fr}^H < t_f$ such that the fixed-fee contract dominates for $\lambda_f \geq \lambda_{fr}^H$ and is dominated by the royalty contract for $\lambda_f \leq \lambda_{fr}^L$, and for $\lambda_f \in \left(\lambda_{fr}^L, \lambda_{fr}^H\right)$, whether the fixed-fee contract dominates or not depends on the value of λ_s. Using (9), (15) and (C.8), and through some algebra, we have

$$\Pi^I(F,F) - \Pi^I(R,R) = \frac{\beta}{2t_s\left(2\beta\lambda_f^2 + 3t_f t_s\right)}\left(g_2(\lambda_f)\lambda_s - h_2(\lambda_f)\right), \tag{C.10}$$

for $I = A, B$, where $g_2(\lambda_f)$ and $h_2(\lambda_f)$ are given, respectively, by (B.2) and (B.4). Note that $g_2(\lambda_f)$ is (strictly) convex in λ_f, and $\lim_{\lambda_f \downarrow 0} g_2(\lambda_f) < 0$ and $\lim_{\lambda_f \uparrow t_f} g_2(\lambda_f) > 0$. This indicates that there exists a unique $\lambda_{fr}^{(1)} \in (0, t_f)$ such that $g_2\left(\lambda_{fr}^{(1)}\right) = 0$, $g_2(\lambda_f) < 0$ for all $\lambda_f < \lambda_{fr}^{(1)}$ and $g_2(\lambda_f) > 0$ for all $\lambda_{fr}^{(1)} < \lambda_f < t_f$. Similarly $h_2(\lambda_f)$ is either (strictly) convex or concave in λ_f, and $\lim_{\lambda_f \downarrow 0} h_2(\lambda_f) > 0$ and $\lim_{\lambda_f \uparrow t_f} h_2(\lambda_f) < 0$. Thus, there exists a unique $\lambda_{fr}^{(2)} \in (0, t_f)$ such that $h_2\left(\lambda_{fr}^{(2)}\right) = 0$, $h_2(\lambda_f) > 0$ for all $\lambda_f < \lambda_{fr}^{(2)}$ and $h_2(\lambda_f) < 0$ for all $\lambda_{fr}^{(2)} < \lambda_f < t_f$. Define $\lambda_{fr}^L = \min(\lambda_{fr}^{(1)}, \lambda_{fr}^{(2)})$ and $\lambda_{fr}^H = \max(\lambda_{fr}^{(1)}, \lambda_{fr}^{(2)})$, and consider three cases: (a) $\lambda_f \leq \lambda_{fr}^L$, (b) $\lambda_f \in \left(\lambda_{fr}^L, \lambda_{fr}^H\right)$, (c) $\lambda_f \in \left(\lambda_{fr}^H, t_f\right)$.

Case (a): In this case, $g_2(\lambda_f) \leq 0$ and $h_2(\lambda_f) \geq 0$, and by (C.10), the fixed-fee contract is dominated by royalty contract, i.e., $\Pi^I(F,F) \leq \Pi^I(R,R)$ for $I = A, B$.

Case (b): If $\lambda_{fr}^{(1)} \leq \lambda_{fr}^{(2)}$, $\lambda_{fr}^L = \lambda_{fr}^{(1)}$ and $\lambda_{fr}^H = \lambda_{fr}^{(2)}$. For $\lambda_f \in \left(\lambda_{fr}^L, \lambda_{fr}^H\right)$, $g_2(\lambda_f) > 0$ and $h_2(\lambda_f) > 0$, and by (C.10), the fixed-fee contract dominates the royalty contract if, and only if, $\lambda_s > \lambda_{sr}^{(3)}$, where $\lambda_{sr}^{(3)} = h_2(\lambda_f)/g_2(\lambda_f)$. Similarly, if $\lambda_{fr}^{(1)} > \lambda_{fr}^{(2)}$, $g_2(\lambda_f) < 0$ and $h_2(\lambda_f) < 0$ for $\lambda_f \in \left(\lambda_{fr}^L, \lambda_{fr}^H\right)$, and the fixed-fee contract dominates the royalty contract if, and only if, $\lambda_s < \lambda_{sr}^{(3)}$.

Case (c): In this case, $g_2(\lambda_f) \geq 0$ and $h_2(\lambda_f) < 0$, or $g_2(\lambda_f) > 0$ and $h_2(\lambda_f) \leq 0$, and $\Pi^I(F,F) > \Pi^I(R,R)$ for $I = A, B$ by (C.10). Combined with $\Pi^I(F,F) > \Pi^I(R,R)$ for $\lambda_f \geq t_f$ for $I = A, B$, this implies that fixed-fee contract dominates the royalty contract for $\lambda_f \geq \lambda_{fr}^H$. $\square$

Proof of Lemma 8: We prove Lemma 8 in two parts.

Part (i): In the first part, we will prove an auxiliary result that we use to prove the lemma. Let us define

$$\Theta_1(\lambda_f,\lambda_s) = \beta t_f\left(2\beta\lambda_s\lambda_f^2 + 3t_f t_s \lambda_f + \lambda_s t_f t_s\right)\left(2\left(t_s + 2\beta\lambda_f\right)\lambda_s - 3t_s\left(\lambda_f - 3t_f\right)\right), \tag{C.11}$$

$$\Theta_2(\lambda_f,\lambda_s) = t_s t_f\left(-\beta\lambda_f^2 + 2\beta\lambda_s\lambda_f + 3t_f t_s\right)\left(2\left(t_s + 2\beta\lambda_f\right)\lambda_s - 3t_s\left(\lambda_f - 3t_f\right)\right), \tag{C.12}$$

$$\Theta_3(\lambda_f,\lambda_s) = 54t_f^3t_s^3 + 8\beta^3\lambda_f^4\lambda_s^2 + 18\beta\lambda_f^2t_f^2t_s^2 + 2\beta\lambda_s^2t_ft_s\left(8\beta\lambda_f^2 - t_ft_s\right) + 66\beta\lambda_f\lambda_st_f^2t_s^2 + 28\beta^2\lambda_f^3\lambda_st_ft_s \tag{C.13}$$

We will show that

$$0 < \frac{\Theta_1(\lambda_f,\lambda_s)}{\Theta_3(\lambda_f,\lambda_s)} < 1 \text{ and } -1 < \frac{\Theta_2(\lambda_f,\lambda_s)}{\Theta_3(\lambda_f,\lambda_s)} < 1$$

when $\beta t_f > t_s$, $\lambda_f \geq t_f$ and $2\left(t_s + 2\beta\lambda_f\right)\lambda_s - 3t_s\left(\lambda_f - 3t_f\right) > 0$ (i.e., $t_f \leq \lambda_f \leq 3t_f$ and $\lambda_s \geq 0$, or $\lambda_f > 3t_f$ and $\lambda_s \geq \lambda_{sr}^{(1)}$, where $\lambda_{sr}^{(1)}$ is given by (B.5)). First, we show that $0 < \Theta_1(\lambda_f,\lambda_s)/\Theta_3(\lambda_f,\lambda_s) < 1$. Clearly, $\Theta_1(\lambda_f,\lambda_s) > 0$ for $2\left(t_s + 2\beta\lambda_f\right)\lambda_s - 3t_s\left(\lambda_f - 3t_f\right) > 0$ and $\Theta_3(\lambda_f,\lambda_s) > 0$ by $\lambda_f \geq t_f > t_s/\beta$. To prove $\Theta_1(\lambda_f,\lambda_s)/\Theta_3(\lambda_f,\lambda_s) < 1$, It is enough to show that $\Theta_3(\lambda_f,\lambda_s) - \Theta_1(\lambda_f,\lambda_s) > 0$. Note by (C.11) and (C.13), we have

$$\begin{aligned}\Theta_3(\lambda_f,\lambda_s) - \Theta_1(\lambda_f,\lambda_s) = {} & \beta\left(\lambda_f - t_f\right)\left(9t_ft_s + 4\beta\lambda_f\lambda_s\right)\left(2\beta\lambda_f^2\lambda_s + 3\lambda_ft_ft_s + \lambda_st_ft_s\right)\\ & + 4\beta\lambda_s^2t_ft_s\left(2\beta\lambda_f^2 - t_ft_s\right) + 4\beta^2\lambda_f^3\lambda_st_ft_s + 54\beta\lambda_f\lambda_st_f^2t_s^2 + 54t_f^3t_s^3\\ & > 0,\end{aligned}$$

where the inequality follows from $\lambda_f \geq t_f > t_s/\beta$.

Next, we show that $\Theta_2(\lambda_f,\lambda_s)/\Theta_3(\lambda_f,\lambda_s) < 1$. Note by (C.12) that, for $2\left(t_s + 2\beta\lambda_f\right)\lambda_s - 3t_s\left(\lambda_f - 3t_f\right) > 0$, $\Theta_2(\lambda_f,\lambda_s)/\Theta_3(\lambda_f,\lambda_s) < 1$ when $-\beta\lambda_f^2 + 2\beta\lambda_s\lambda_f + 3t_ft_s \leq 0$ (i.e., $\lambda_s \leq \frac{\lambda_f}{2} - \frac{3t_ft_s}{2\beta\lambda_f}$ for $\beta\lambda_f^2 > 3t_ft_s$); however, when $-\beta\lambda_f^2 + 2\beta\lambda_s\lambda_f + 3t_ft_s > 0$ (i.e., $\lambda_s > 0$ for $\beta\lambda_f^2 \leq 3t_ft_s$, and $\lambda_s > \frac{\lambda_f}{2} - \frac{3t_ft_s}{2\beta\lambda_f}$ for $\beta\lambda_f^2 > 3t_ft_s$), $\Theta_2(\lambda_f,\lambda_s)/\Theta_3(\lambda_f,\lambda_s) < 1$ if, and only if, $\Theta_3(\lambda_f,\lambda_s) - \Theta_2(\lambda_f,\lambda_s) > 0$. Note that

$$\begin{aligned}\Theta_3(\lambda_f,\lambda_s) - \Theta_2(\lambda_f,\lambda_s) = {} & 2\beta\left(4\beta^2\lambda_f^4 + 4\beta\lambda_f^2t_ft_s - 2\lambda_ft_ft_s^2 - t_f^2t_s^2\right)\lambda_s^2\\ & + 2t_ft_s\left(16\beta^2\lambda_f^3 + 4\beta\lambda_f^2t_s + 18\beta\lambda_ft_ft_s - 3t_ft_s^2\right)\lambda_s + 3t_ft_s^2\left(-\beta\lambda_f^3 + 9\beta\lambda_f^2t_f + 3\lambda_ft_ft_s + 9t_f^2t_s\right).\end{aligned}$$

Taking partial derivative with respect to λ_s, we have

$$\begin{aligned}\frac{\partial\left(\Theta_3(\lambda_f,\lambda_s) - \Theta_2(\lambda_f,\lambda_s)\right)}{\partial\lambda_s} & = 4\beta\left(4\beta^2\lambda_f^4 + 4\beta\lambda_f^2t_ft_s - 2\lambda_ft_ft_s^2 - t_f^2t_s^2\right)\lambda_s\\ & \quad + 2t_ft_s\left(16\beta^2\lambda_f^3 + 4\beta\lambda_f^2t_s + 18\beta\lambda_ft_ft_s - 3t_ft_s^2\right)\\ & > 4\beta\left(4\beta^2\lambda_f^4 + t_f^2t_s^2\right)\lambda_s + 2t_ft_s\left(16\beta^2\lambda_f^3 + 18\beta\lambda_ft_ft_s + t_ft_s^2\right) > 0\end{aligned}$$

where the inequality follows from $t_s < \beta t_f \leq \beta\lambda_f$. Thus, $\Theta_3(\lambda_f,\lambda_s) - \Theta_2(\lambda_f,\lambda_s)$ is increasing in λ_s. By $\lim_{\lambda_s\downarrow 0}\left(\Theta_3(\lambda_f,\lambda_s) - \Theta_2(\lambda_f,\lambda_s)\right) > 0$ for $\beta\lambda_f^2 \leq 3t_ft_s$, and $\lim_{\lambda_s\downarrow\frac{\lambda_f}{2} - \frac{3t_ft_s}{2\beta\lambda_f}}\left(\Theta_3(\lambda_f,\lambda_s) - \Theta_2(\lambda_f,\lambda_s)\right) > 0$ for $\beta\lambda_f^2 > 3t_ft_s$. This implies that $\Theta_3(\lambda_f,\lambda_s) - \Theta_2(\lambda_f,\lambda_s) > 0$ when $\lambda_f \geq t_f$ and $-\beta\lambda_f^2 + 2\beta\lambda_s\lambda_f + 3t_ft_s > 0$. Consequently, $\Theta_2(\lambda_f,\lambda_s)/\Theta_3(\lambda_f,\lambda_s) < 1$ for all $\lambda_f \geq t_f$ and $\lambda_s \geq 0$ when $\beta t_f > t_s$.

Lastly, we show that $\Theta_2(\lambda_f,\lambda_s)/\Theta_3(\lambda_f,\lambda_s) > -1$ for all $\lambda_f \geq t_f$ and $\lambda_s \geq 0$ when $\beta t_f > t_s$ and $2(t_s+2\beta\lambda_f)\lambda_s - 3t_s(\lambda_f-3t_f) > 0$. By (C.12) and (C.13), we have

$$\begin{aligned}\Theta_2(\lambda_f,\lambda_s)+\Theta_3(\lambda_f,\lambda_s) &= 8\beta^3\lambda_f^4\lambda_s^2+24\beta^2\lambda_f^2\lambda_s^2t_ft_s+3\beta\lambda_f^3t_ft_s^2+2\beta\lambda_f\lambda_s^2t_ft_s^2+9\beta\lambda_f^2t_f^2t_s^2+84\beta\lambda_f\lambda_st_f^2t_s^2\\ &\quad+54t_f^3t_s^3+8\beta\lambda_f^2\lambda_st_ft_s(3\beta\lambda_f-t_s)+2\beta\lambda_s^2t_ft_s^2(\lambda_f-t_f)+3t_f^2t_s^2(-3\lambda_ft_s+2\lambda_st_s+9t_ft_s+4\beta\lambda_f\lambda_s)>0\end{aligned}$$

for $\lambda_f \geq t_f$ and $\beta t_f > t_s$ when $2(t_s+2\beta\lambda_f)\lambda_s - 3t_s(\lambda_f-3t_f) \geq 0$.

Part (ii): Now, we prove Lemma 8 by using part (i). Note that $D_s^A = 1-D_s^B = \theta_s$ and $D_f^a = 1-D_f^b = \theta_f$ by rational expectations. Then, by (C.1) and (C.2), and substituting $T^A = r^A\beta D_f^a$, and $T^B = k^B$ into (3) and (4), we obtain profits of brands and their licensees as follows:

$$\begin{aligned}\Pi^A(p^A,r^A) &= (p^A-c)\frac{-t_f+p^Bt_f+t_ft_s+\beta\lambda_f\lambda_s+\beta\lambda_sp^a-\beta\lambda_sp^b}{2t_ft_s+2\beta\lambda_f\lambda_s}\\ &\quad+r^A\beta\frac{-\lambda_fp^A+\lambda_fp^B-p^at_s+p^bt_s+t_ft_s+\beta\lambda_f\lambda_s}{2t_ft_s+2\beta\lambda_f\lambda_s},\\ \Pi^B(p^B,k^B) &= (p^B-c)\left(1-\frac{-t_f+p^Bt_f+t_ft_s+\beta\lambda_f\lambda_s+\beta\lambda_sp^a-\beta\lambda_sp^b}{2t_ft_s+2\beta\lambda_f\lambda_s}\right)+k^B,\\ \Pi^a(p^a) &= (p^a-r^A)\beta\frac{-\lambda_fp^A+\lambda_fp^B-p^at_s+p^bt_s+t_ft_s+\beta\lambda_f\lambda_s}{2t_ft_s+2\beta\lambda_f\lambda_s},\\ \Pi^b(p^b) &= p^b\beta\left(1-\frac{-\lambda_fp^A+\lambda_fp^B-p^at_s+p^bt_s+t_ft_s+\beta\lambda_f\lambda_s}{2t_ft_s+2\beta\lambda_f\lambda_s}\right)-k^B.\end{aligned}$$

Using first-order conditions, we obtain prices for a given royalty fee r^A as follows:

$$\begin{aligned}p^A &= c+t_s+\frac{\beta\lambda_f(2\lambda_s-r^A)}{2t_f}+\beta t_s\frac{2\lambda_s-3\lambda_f}{18t_ft_s+8\beta\lambda_f\lambda_s}r^A,\\ p^B &= c+t_s+\frac{\beta\lambda_f(2\lambda_s-r^A)}{2t_f}-\beta t_s\frac{2\lambda_s-3\lambda_f}{18t_ft_s+8\beta\lambda_f\lambda_s}r^A,\\ p^a &= t_f+\frac{\beta\lambda_f\lambda_s}{t_s}+\frac{r^A}{2}+\frac{\beta\lambda_f^2+\frac{3}{2}t_ft_s}{9t_ft_s+4\beta\lambda_f\lambda_s}r^A,\\ p^b &= t_f+\frac{\beta\lambda_f\lambda_s}{t_s}+\frac{r^A}{2}-\frac{\beta\lambda_f^2+\frac{3}{2}t_ft_s}{9t_ft_s+4\beta\lambda_f\lambda_s}r^A.\end{aligned}$$

Plugging above prices into brands' profit functions and letting $\Pi^A(r^A) = \Pi^A(p^A,r^A)$ and $\Pi^B(k^B) = \Pi^B(p^B,k^B)$, we obtain:

$$\begin{aligned}\Pi^A(r^A) &= \left(t_s+\frac{\beta\lambda_f(2\lambda_s-r^A)}{2t_f}+\beta t_s\frac{2\lambda_s-3\lambda_f}{18t_ft_s+8\beta\lambda_f\lambda_s}r^A\right)\left(\frac{1}{2}+\frac{1}{2}\beta r^A\frac{2\beta\lambda_s\lambda_f^2+3t_ft_s\lambda_f+\lambda_st_ft_s}{(9t_ft_s+4\beta\lambda_f\lambda_s)(t_ft_s+\beta\lambda_f\lambda_s)}\right)\\ &\quad+\beta r^A\left(\frac{1}{2}-\frac{1}{2}r^A\frac{t_s(-\beta\lambda_f^2+2\beta\lambda_s\lambda_f+3t_ft_s)}{(9t_ft_s+4\beta\lambda_f\lambda_s)(t_ft_s+\beta\lambda_f\lambda_s)}\right),\end{aligned}$$

$$\Pi^B(k^B) = \left(t_s+\frac{\beta\lambda_f(2\lambda_s-r^A)}{2t_f}-\beta t_s\frac{2\lambda_s-3\lambda_f}{18t_ft_s+8\beta\lambda_f\lambda_s}r^A\right)\left(\frac{1}{2}-\frac{1}{2}\beta r^A\frac{2\beta\lambda_s\lambda_f^2+3t_ft_s\lambda_f+\lambda_st_ft_s}{(9t_ft_s+4\beta\lambda_f\lambda_s)(t_ft_s+\beta\lambda_f\lambda_s)}\right)+k^B.$$

Note that given the royalty fee of brand A, it is always optimal for brand B to set the lump-sum payment $k^B = p^b\beta(1-\theta_f)$ so as to extract all licensing profits from its licensee b, i.e.,

$$k^B = \beta\left(t_f+\frac{\beta\lambda_f\lambda_s}{t_s}+\frac{-\beta\lambda_f^2+2\beta\lambda_f\lambda_s+3t_ft_s}{9t_ft_s+4\beta\lambda_f\lambda_s}r^A\right)\left(\frac{1}{2}+\frac{t_s(-\beta\lambda_f^2+2\beta\lambda_s\lambda_f+3t_ft_s)}{2(9t_ft_s+4\beta\lambda_f\lambda_s)(t_ft_s+\beta\lambda_f\lambda_s)}r^A\right).$$

Next, we find brand A's optimal royalty fee. Taking the derivative of brand A's profit function, we get

$$\frac{d\Pi^A\left(r^A\right)}{dr^A} = \frac{\beta\left(2\lambda_s t_s - 3\lambda_f t_s + 9t_f t_s + 4\beta\lambda_f\lambda_s\right)}{2\left(9t_f t_s + 4\beta\lambda_f\lambda_s\right)} + r^A\frac{d^2\Pi^A\left(r^A\right)}{d\left(r^A\right)^2},$$

where

$$\frac{d^2\Pi^A\left(r^A\right)}{d\left(r^A\right)^2} = -\frac{\beta\left(4\beta^3\lambda_f^4\lambda_s^2 - \beta\lambda_s^2 t_f^2 t_s^2 + 8\beta^2\lambda_f^2\lambda_s^2 t_f t_s + 14\beta^2\lambda_f^3\lambda_s t_f t_s + 33\beta\lambda_f\lambda_s t_f^2 t_s^2 + 9\beta\lambda_f^2 t_f^2 t_s^2 + 27t_f^3 t_s^3\right)}{t_f\left(t_f t_s + \beta\lambda_f\lambda_s\right)\left(9t_f t_s + 4\beta\lambda_f\lambda_s\right)^2}.$$

Observe that both $d^2\Pi^A\left(r^A\right)/d\left(r^A\right)^2$ is constant in λ_s and λ_f, $d^2\Pi^A\left(r^A\right)/d\left(r^A\right)^2 < 0$ by $\lambda_f \geq t_f > t_s/\beta$, and that

$$\lim_{r^A\downarrow 0}\frac{d\Pi^A\left(r^A\right)}{dr^A} = \frac{\beta\left(2\lambda_s\left(t_s + 2\beta\lambda_f\right) - 3t_s\left(\lambda_f - 3t_f\right)\right)}{2\left(9t_f t_s + 4\beta\lambda_f\lambda_s\right)}.$$

Next we consider two cases: (1) $\lambda_f > 3t_f$ and $\lambda_s \leq \lambda_{sr}^{(1)} = 3t_s\left(\lambda_f - 3t_f\right)/\left(2\left(t_s + 2\beta\lambda_f\right)\right)$, and (2) $\lambda_f \leq 3t_f$, or $\lambda_f > 3t_f$ and $\lambda_s > \lambda_{sr}^{(1)}$.

Case 1: In this case, $\lim_{r^A\downarrow 0} d\Pi^A\left(r^A\right)/dr^A \leq 0$, which by $d^2\Pi^A\left(r^A\right)/d\left(r^A\right)^2 < 0$ implies that $d\Pi^A\left(r^A\right)/dr^A \leq 0$ for all r^A so that $r^A = 0$. Using above equilibrium prices and substituting $r^A = 0$, we have $p^I = c + t_s + \beta\lambda_f\lambda_s/t_f$ for $I = A, B$ and $p^i = t_f + \beta\lambda_f\lambda_s/t_s$ for $i = a, b$. As a result, $\theta_s = \theta_f = 1/2$.

Case 2: In this case, $\lim_{r^A\downarrow 0} d\Pi^A\left(r^A\right)/dr^A > 0$, which, by $d^2\Pi^A\left(r^A\right)/d\left(r^A\right)^2 < 0$ and $\lim_{r_A\uparrow\infty} d\Pi^A\left(r^A\right)/dr^A < 0$, implies that the optimal royalty fee of brand A $r^A \in (0,\infty)$ satisfies $d\Pi^A\left(r^A\right)/dr^A = 0$ and is given in Lemma 8. Plugging this royalty fee above, we obtain equilibrium prices. Then, substituting these equilibrium prices in (C.1) and (C.2), we get

$$\theta_s = \frac{1}{2} + \frac{\Theta_1\left(\lambda_f,\lambda_s\right)}{2\Theta_3\left(\lambda_f,\lambda_s\right)} \text{ and } \theta_f = \frac{1}{2} - \frac{\Theta_2\left(\lambda_f,\lambda_s\right)}{2\Theta_3\left(\lambda_f,\lambda_s\right)},$$

where $\Theta_1\left(\lambda_f,\lambda_s\right)$, $\Theta_2\left(\lambda_f,\lambda_s\right)$, and $\Theta_3\left(\lambda_f,\lambda_s\right)$ are given, respectively, by (C.11), (C.12) and (C.13). By part (i), for $\beta t_f > t_s$, $\theta_s \in (1/2,1)$ and $\theta_f \in (0,1)$ for $\lambda_f \geq t_f$ and $\lambda_s \geq 0$ in this case. □

Proof of Proposition 3: The first part of Proposition 3 follows from Lemma C.4(i). Now we prove the second part. By case 2 from §5.3, (16) and (17), in cases (F,NL) and (R,NL) when $\lambda_s \geq 3t_s/\beta$, the brand that does not license (brand B) becomes a monopoly in market s and the brand licenses (brand A) gets profit equal to $\beta(v_f - t_f)$. Then, by (8), (16) and (17), it follows that only one brand licenses via either fixed-fee or royalty contract (i.e., $\Pi^B(R,NL) > \Pi^B(R,R)$ and $\Pi^B(F,NL) > \Pi^B(F,F)$, and $\Pi^A(NL,NL) \geq \Pi^A(F,NL) = \Pi^A(R,NL)$) if $\lambda_s \geq 3t_s/\beta$ and $v_f > t_f + t_s/2\beta$; and no brand licenses (i.e., $\Pi^B(R,NL) > \Pi^B(R,R)$ and $\Pi^B(F,NL) > \Pi^B(F,F)$, and $\Pi^A(F,NL) = \Pi^A(R,NL) < \Pi^A(NL,NL)$) if $\lambda_s \geq 3t_s/\beta$ and $v_f \leq t_f + t_s/2\beta$. □