General Galilei Covariant Gaussian Maps

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We characterize general non-Markovian Gaussian maps which are covariant under Galilean transformations. In particular, we consider translational and Galilean covariant maps and show that they reduce to the known Holevo result in the Markovian limit. We apply the results to discuss measures of macroscopicity based on classicalization maps, specifically addressing dissipation, Galilean covariance and non-Markovianity. We further suggest a possible generalization of the macroscopicity measure defined by Nimmrichter and Hornberger [Phys. Rev. Lett. **110**, 16 (2013)].

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Introduction.—Symmetries have always played a central role in modern physics, especially after their mathematical formulation with the advent of group theory: they underlie the simplicity of nature and manifest the beauty of physical laws. They also serve as a guideline principle for deciding the form of the dynamics [1,2]. Here we are interested in the role of space-time symmetries in nonrelativistic quantum mechanics.

The covariance of the Schrödinger equation, and of the corresponding Liouville–von Neumann equation, under the action of the Galilean group, has been extensively discussed [3–5]. On the other hand, the investigation of Galilean covariance within the context of open quantum systems is still an area of active research [6]. The exact quantum dynamics of a system interacting with the surrounding environment can be very complicated: in general, heavy approximations and heuristical arguments are needed in order to arrive at an explicit useful expression for the system's effective dynamics. In this case, symmetries can be a guiding principle in constructing the effective dynamics, bypassing at least partially the complexity (or impossibility) of a direct calculation by imposing constraints, which are expected to hold not only at the fundamental level, but also at the effective level [7–15].

Space-time symmetries in open quantum systems have been fully analyzed only in the special, but very important, case of a Markovian, completely positive (CP) and trace preserving (TP) dynamics. This dynamics, discussed in the seminal works of Gorini, Kossakowski, Sudarshan, and independently by Lindblad [16,17], is known as the *quantum dynamical semigroup*: it is generated by the *Lindblad superoperator* and can be written as a first order differential equation, called the *Lindblad master equation*. By imposing the additional request of covariance under the action of the Galilei group, Holevo in a series of works [18–21], completely *characterized* translational and Galilei covariant Lindblad master equations, by giving the explicit form of the Lindblad superoperators [22].

The Holevo characterizations play a major role in the description of several important physical phenomena such as environmental decoherence and relaxation phenomena [7–14]. Furthermore, they are also relevant for the foundations of quantum mechanics, where an intrinsic nonunitary dynamics is postulated to solve the measurement problem [23–25], the black hole information paradox [26], or to combine principles of general relativity with quantum mechanics [27].

Although the assumption of Markovianity is often well justified, recent technological advances have lead to investigating several phenomena exhibiting memory effects [28], e.g., ultrafast chemical reactions [29–34], side band cooling [35], and light harvesting in photosynthesis [36–41]. This is little surprising, as the *time resolution* of experimental apparata has increased severalfold in the last decades. It is therefore now clear that non-Markovian dynamics will acquire a more prominent role in the near future: the theoretical investigations are pressed by practical necessity.

In this Letter we will derive the general structure of non-Markovian Galilei covariant Gaussian maps. More specifically, we will consider the non-Markovian Gaussian map introduced in Ref. [42], and we will impose covariance under Galilean space-time symmetries (translations, boosts and rotations). In this way we will obtain a generalization of the Holevo generators [18–21] to the non-Markovian Gaussian case. Using these results, we will discuss measures of macroscopicity based on classicalization maps. Specifically, we will address the role of non-Markovian and dissipative effects, which limit the validity of the macroscopicity measure proposed in Ref. [43].

General framework of Gaussian maps.—Non-Markovian dynamics are in general difficult to analyze: the system and environment form a complicated many-body problem which, without some additional simplifying assumption, remains intractable. On the other hand, the subclass of (non-Markovian) Gaussian maps, still appropriate for the

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description of a vast spectrum of phenomena [44–56], can be analyzed both analytically [42] and numerically [57,58].

The starting point of our analysis is the most general trace-preserving, completely positive Gaussian map derived in Ref. [42] (we work in interaction picture and adopt Einstein's summation convention):

$$\mathcal{M}_t = \exp_+\left\{\int_0^t d\tau \int_0^t ds D_{jk}(\tau,s) \times (\hat{A}^k_{sL}\hat{A}^j_{\tau R} - \theta_{\tau s}\hat{A}^j_{\tau L}\hat{A}^k_{sL} - \theta_{s\tau}\hat{A}^k_{sR}\hat{A}^j_{\tau R})\right\}, \quad (1)$$

where $\exp_+$ denotes the time-ordered exponential, D_{jk} is a complex valued positive semi-definite matrix, $\hat{A}$ are bounded Hermitian operators, and the subscript L (R) denotes operators acting on the statistical operator ρ from the left (right), e.g., $\hat{A}^k_L\hat{A}^j_R\hat{\rho} = \hat{A}^k\hat{\rho}\hat{A}^j$ with $\hat{A}^k$ Hermitian operators. The correlation matrix $D_{jk}(\tau,s)$ and the operators $\hat{A}^k$ are supposed to encode, phenomenologically, the action of the bath on the system. We note that, by imposing the request of Markovianity

$$D_{jk}(\tau,s) = \delta(\tau - s)\tilde{D}_{jk}(s), \quad (2)$$

where $\tilde{D}_{jk}(s)$ is a complex valued positive semi-definite matrix, the exponent in Eq. (1) takes the well-known Lindblad form.

Since we are interested in space-time symmetries, we now explicitly assume that the Hilbert space $\mathcal{H}_\mathcal{S}$ is $L^2(\mathbb{R}^3)$ (the generalization to the N-particle Hilbert space is straightforward). In this case it is convenient to decompose the operators in Eq. (1) by using the Weyl-Wigner decomposition (in Schrödinger picture) [59]:

$$\hat{\mathbf{A}}_t = \int_{\mathbb{R}^3} d\alpha \int_{\mathbb{R}^3} d\boldsymbol{\beta} \mathcal{A}_t(\boldsymbol{\alpha},\boldsymbol{\beta}) e^{i(\boldsymbol{\alpha}\cdot\hat{\mathbf{x}}+\boldsymbol{\beta}\cdot\hat{\mathbf{p}})}, \quad (3)$$

where $\hat{\mathbf{A}}_t$ may depend explicitly on time, which is encoded in the time-dependency of $\mathcal{A}_t$, and $\hat{\mathbf{x}}$ and $\hat{\mathbf{p}}$ are the standard position and momentum operators. It is then straightforward to show that the map in Eq. (1) becomes (in the interaction picture):

$$\mathcal{M}_t = \exp_+\left\{\int dT \int d\Gamma \mathcal{D}(\boldsymbol{\alpha}_1,\boldsymbol{\beta}_1,\boldsymbol{\alpha}_2,\boldsymbol{\beta}_2,\tau,s)\Theta^{\mu\nu}_{\tau s} \times (e^{i(\boldsymbol{\alpha}_1\cdot\hat{\mathbf{x}}_{s\mu}+\boldsymbol{\beta}_1\cdot\hat{\mathbf{p}}_\mu)}e^{-i(\boldsymbol{\alpha}_2\cdot\hat{\mathbf{x}}_{\tau\nu}+\boldsymbol{\beta}_2\cdot\hat{\mathbf{p}}_\nu)})\right\}, \quad (4)$$

where $dT = d\tau ds$, $d\Gamma = d\alpha_1 d\boldsymbol{\beta}_1 d\alpha_2 d\boldsymbol{\beta}_2$, the integration domains, which we omit to simplify the notation, are $[0,t]\times[0,t]$ and $\otimes_{j=1}^4 \mathbb{R}^3$ for the T and Γ integrals, respectively, $\hat{\mathbf{x}}_s$ is the position operator in the interaction picture at time s, μ and ν denote L or R (left or right operators), $\Theta^{LR}_{\tau s} = \Theta^{RL}_{\tau s} = 1/2$, $\Theta^{LL}_{\tau s} = -\theta_{\tau s}$, $\Theta^{RR}_{\tau s} = -\theta_{s\tau}$ and

$$\mathcal{D}(\boldsymbol{\alpha}_1,\boldsymbol{\beta}_1,\boldsymbol{\alpha}_2,\boldsymbol{\beta}_2,\tau,s) = D_{jk}(\tau,s)\mathcal{A}^j_\tau(\boldsymbol{\alpha}_1,\boldsymbol{\beta}_1)\mathcal{A}^k_s(\boldsymbol{\alpha}_2,\boldsymbol{\beta}_2) \quad (5)$$

is a kernel that satisfies the following symmetry property [60]:

$$\mathcal{D}(\boldsymbol{\alpha}_1,\boldsymbol{\beta}_1,\boldsymbol{\alpha}_2,\boldsymbol{\beta}_2,\tau,s) = \mathcal{D}^*(\boldsymbol{\alpha}_2,\boldsymbol{\beta}_2,\boldsymbol{\alpha}_1,\boldsymbol{\beta}_1,s,\tau). \quad (6)$$

We now impose the relevant Galilei symmetry on the system, constraining the form of the dynamics given by Eq. (4).

Covariance.—Let us consider a locally compact Lie group G and a unitary representation $\hat{\mathbf{U}}_g$, with $g \in G$, on the Hilbert space of the system. Following [61,62] a quantum dynamical map is said to be G covariant if it commutes with the linear transformation $\mathcal{U}_g[\cdot] = \hat{\mathbf{U}}_g \cdot \hat{\mathbf{U}}_g$:

$$\mathcal{M}_t = \mathcal{U}_g^{-1}\circ\mathcal{M}_t\circ\mathcal{U}_g. \quad (7)$$

With reference to the single particle Hilbert space $\mathcal{H}_\mathcal{S}$ ($L^2(\mathbb{R}^3)$) we assume that the Hamiltonian is covariant under the relevant symmetry of the Galilei group $\mathcal{G}$ [63]: specifically, we consider the centrally extended unitary representation ($\hat{\mathbf{U}}_g$) of the Galilei group ($\mathcal{G}$) on $\mathcal{H}_\mathcal{S}$. The generators of infinitesimal translations, boosts, and rotations are (in the interaction picture)

$$\hat{\mathbf{p}} = \hat{\mathbf{p}}, \quad (8)$$

$$\hat{\mathbf{J}} = \hat{\mathbf{x}}\times\hat{\mathbf{p}}, \quad (9)$$

$$\hat{\mathbf{K}} = m\hat{\mathbf{x}}, \quad (10)$$

respectively, where m is the mass of the particle. Exploiting Eq. (4), and the fact that we are considering a unitary representation, it is straightforward to show that Eq. (7) is satisfied if and only if the following condition is satisfied:

$$\int dT \int d\Gamma\mathcal{D}(\boldsymbol{\alpha}_1,\boldsymbol{\beta}_1,\boldsymbol{\alpha}_2,\boldsymbol{\beta}_2,\tau,s)\Theta^{\mu\nu}_{\tau s} \times (e^{i(\boldsymbol{\alpha}_1\mathcal{U}_g[\hat{\mathbf{x}}_{s\mu}]+\boldsymbol{\beta}_1\mathcal{U}_g[\hat{\mathbf{p}}_\mu])}e^{-i(\boldsymbol{\alpha}_2\mathcal{U}_g[\hat{\mathbf{x}}_{\tau\nu}]+\boldsymbol{\beta}_2\mathcal{U}_g[\hat{\mathbf{p}}_\nu])} - e^{i(\boldsymbol{\alpha}_1\hat{\mathbf{x}}_{s\mu}+\boldsymbol{\beta}_1\hat{\mathbf{p}}_\mu)}e^{-i(\boldsymbol{\alpha}_2\hat{\mathbf{x}}_{\tau\nu}+\boldsymbol{\beta}_2\hat{\mathbf{p}}_\nu)}) = 0. \quad (11)$$

This equation constrains the structure of the dynamical map under the Galilean symmetry $g \in \mathcal{G}$. In particular, we will now see how the request of translation (boost) covariance characterizes the structure of the dynamical map.

Translational covariance.—Restricting to the subgroup of translations $\mathcal{T} \subset \mathcal{G}$ we have that

$$\mathcal{U}_{\boldsymbol{a}}[\hat{\mathbf{x}}_t] = \hat{\mathbf{x}}_t + \boldsymbol{a}, \quad (12)$$

$$\mathcal{U}_{\boldsymbol{a}}[\hat{\mathbf{p}}] = \hat{\mathbf{p}}, \quad (13)$$

where $\hat{\mathbf{x}}_t = \hat{\mathbf{x}} + (\hat{\mathbf{p}}/m)t$ is the position operator in the interaction picture at time t, $\boldsymbol{a}$ is a translation vector, and $\mathcal{U}_{\boldsymbol{a}}$ denotes the corresponding linear transformation [see Eq. (7)]. Using Eqs. (12), (13) we obtain from Eq. (11)

$$\int dT \int d\Gamma\mathcal{D}(\boldsymbol{\alpha}_1,\boldsymbol{\beta}_1,\boldsymbol{\alpha}_2,\boldsymbol{\beta}_2,\tau,s)\Theta^{\mu\nu}_{\tau s} \times e^{i(\boldsymbol{\alpha}_1\cdot\hat{\mathbf{x}}_{\mu s}+\boldsymbol{\beta}_1\cdot\hat{\mathbf{p}}_\mu)}e^{-i(\boldsymbol{\alpha}_2\cdot\hat{\mathbf{x}}_{\nu\tau}+\boldsymbol{\beta}_2\cdot\hat{\mathbf{p}}_\nu)}(1 - e^{i(\boldsymbol{\alpha}_1-\boldsymbol{\alpha}_2)\cdot\boldsymbol{a}}) = 0. \quad (14)$$

Since this relation must hold $\forall\boldsymbol{a}$, it follows that Eq. (14) is satisfied if and only if the following equality holds:

$$\mathcal{D}(\boldsymbol{\alpha}_1,\boldsymbol{\alpha}_2,\boldsymbol{\beta}_1,\boldsymbol{\beta}_2,\tau,s) = \delta^{(3)}(\boldsymbol{\alpha}_1 - \boldsymbol{\alpha}_2)\mathcal{D}_T(\boldsymbol{\alpha}_1,\boldsymbol{\alpha}_2,\boldsymbol{\beta}_1,\boldsymbol{\beta}_2,\tau,s), \quad (15)$$

where $\mathcal{D}_T$ is a complex valued function, which we rewrite as

$$\mathcal{D}_T(\boldsymbol{\alpha}_1,\boldsymbol{\beta}_1,\boldsymbol{\alpha}_2,\boldsymbol{\beta}_2,\tau,s) = D_{jk}(\tau,s)\tilde{\mathcal{A}}_\tau^{j*}(\boldsymbol{\alpha}_1,\boldsymbol{\beta}_1)\tilde{\mathcal{A}}_s^{k}(\boldsymbol{\alpha}_2,\boldsymbol{\beta}_2). \tag{16}$$

We then insert Eq. (15) into Eq. (4), use Eq. (16), integrate over $\boldsymbol{\alpha}_2$ and relabel $\boldsymbol{\alpha}_1$ as $\boldsymbol{\alpha}$ to obtain

$$\begin{aligned}\mathcal{M}_t = \exp_+\Bigg\{&\int_0^t d\tau\int_0^t ds\int_{\mathbb{R}^3} d\alpha D_{jk}(\tau,s)\\ &\times([F_{sL}^k(\hat{\mathbf{p}},\boldsymbol{\alpha})e^{i\alpha\cdot\hat{\mathbf{x}}_{sL}}][F_{\tau R}^{j\dagger}(\hat{\mathbf{p}},\boldsymbol{\alpha})e^{-i\alpha\cdot\hat{\mathbf{x}}_{\tau R}}]\\ &-\theta_{\tau s}[F_{\tau L}^{j\dagger}(\hat{\mathbf{p}},\boldsymbol{\alpha})e^{-i\alpha\cdot\hat{\mathbf{x}}_{\tau L}}][e^{i\alpha\cdot\hat{\mathbf{x}}_{sL}}F_{sL}^k(\hat{\mathbf{p}},\boldsymbol{\alpha})]\\ &-\theta_{s\tau}[F_{sR}^k(\hat{\mathbf{p}},\boldsymbol{\alpha})e^{i\alpha\cdot\hat{\mathbf{x}}_{sR}}][e^{-i\alpha\cdot\hat{\mathbf{x}}_{\tau R}}F_{\tau R}^{k\dagger}(\hat{\mathbf{p}},\boldsymbol{\alpha})])\Bigg\},\end{aligned} \tag{17}$$

where

$$F_{\tau\mu}^k(\hat{\mathbf{p}},\boldsymbol{\alpha}) = \int d\boldsymbol{\beta}\tilde{\mathcal{A}}_\tau^k(\boldsymbol{\alpha},\boldsymbol{\beta})e^{i\boldsymbol{\beta}\cdot\hat{\mathbf{p}}_\mu} \tag{18}$$

is a completely general operator valued function of the operator $\hat{\mathbf{p}}$. Equation (17) fully characterizes translation covariant CP Gaussian maps.

Boost covariance.—Restricting to the subgroup of boosts $\mathcal{B}\subset\mathcal{G}$ we have that

$$\mathcal{U}_{\boldsymbol{b}}[\hat{\mathbf{x}}_t] = \hat{\mathbf{x}}_t + t\boldsymbol{b}/m, \tag{19}$$

$$\mathcal{U}_{\boldsymbol{b}}[\hat{\mathbf{p}}] = \hat{\mathbf{p}} + \boldsymbol{b}, \tag{20}$$

where $\boldsymbol{b} = m\boldsymbol{v}$ is a momentum vector (a particle of mass m boosted with velocity $\boldsymbol{v}$) and $\mathcal{U}_{\boldsymbol{b}}$ denotes the corresponding linear transformation [see Eq. (7)]. Imposing boost covariance, and following the analogous steps as for the characterization of translational covariance, we obtain the following equality:

$$\begin{aligned}\mathcal{D}(\boldsymbol{\alpha}_1,\boldsymbol{\alpha}_2,\boldsymbol{\beta}_1,\boldsymbol{\beta}_2,\tau,s) = \delta^{(3)}&\left(\boldsymbol{\beta}_1-\boldsymbol{\beta}_2+\boldsymbol{\alpha}_1\frac{s}{m}-\boldsymbol{\alpha}_2\frac{\tau}{m}\right)\\ &\times\mathcal{D}_B(\boldsymbol{\alpha}_1,\boldsymbol{\alpha}_2,\boldsymbol{\beta}_1,\boldsymbol{\beta}_2,\tau,s),\end{aligned} \tag{21}$$

where $\mathcal{D}_B$ is a complex valued function. Performing the following change of variables: $\boldsymbol{\beta}_1\to\boldsymbol{\beta}_1-\tau\alpha_1/m$ and $\boldsymbol{\beta}_2\to\boldsymbol{\beta}_1-s\alpha_2/m$, using Eq. (21), we can then rewrite Eq. (4) as

$$\begin{aligned}\mathcal{M}_t = \exp_+\Bigg\{&\int_0^t d\tau\int_0^t ds\int_{\mathbb{R}^3} d\boldsymbol{\beta} D_{jk}(\tau,s)\\ &\times([F_{sL}^k(\hat{\mathbf{x}}_s,\boldsymbol{\beta})e^{i\boldsymbol{\beta}\cdot\hat{\mathbf{p}}_L}][F_{\tau R}^{j\dagger}(\hat{\mathbf{x}}_\tau,\boldsymbol{\beta})e^{-i\boldsymbol{\beta}\cdot\hat{\mathbf{p}}_R}]\\ &-\theta_{\tau s}[F_{\tau L}^{j\dagger}(\hat{\mathbf{x}}_\tau,\boldsymbol{\beta})e^{-i\boldsymbol{\beta}\cdot\hat{\mathbf{p}}_L}][e^{i\boldsymbol{\beta}\cdot\hat{\mathbf{p}}_L}F_{sL}^k(\hat{\mathbf{x}}_s,\boldsymbol{\beta})]\\ &-\theta_{s\tau}[F_{sR}^k(\hat{\mathbf{x}}_s,\boldsymbol{\beta})e^{i\boldsymbol{\beta}\cdot\hat{\mathbf{p}}_R}][e^{-i\boldsymbol{\beta}\cdot\hat{\mathbf{p}}_R}F_{\tau R}^{k\dagger}(\hat{\mathbf{x}}_\tau,\boldsymbol{\beta})])\Bigg\},\end{aligned} \tag{22}$$

where

$$F_{\tau\mu}^k(\hat{\mathbf{x}}_\tau,\boldsymbol{\beta}) = \int d\alpha\tilde{\mathcal{A}}_\tau^k(\boldsymbol{\alpha},\boldsymbol{\beta}-\tau\alpha/m)e^{i\alpha\cdot\hat{\mathbf{x}}_{\tau\mu}} \tag{23}$$

is a completely general operator valued function of the operator $\hat{\mathbf{x}}_\tau$. This equation completely characterizes boost covariant CP Gaussian maps.

Translation-boost covariance.—We now require both translation and boost covariance. The dynamical map $\mathcal{M}_t$ must satisfy condition Eq. (15) as well as condition Eq. (21), i.e.,

$$\begin{aligned}\mathcal{D}(\boldsymbol{\alpha}_1,\boldsymbol{\alpha}_2,\boldsymbol{\beta}_1,\boldsymbol{\beta}_2,\tau,s) = \delta^{(3)}&(\boldsymbol{\alpha}_1-\boldsymbol{\alpha}_2)\\ &\times\delta^{(3)}\left(\boldsymbol{\beta}_1-\boldsymbol{\beta}_2+\boldsymbol{\alpha}_1\frac{s}{m}-\boldsymbol{\alpha}_2\frac{\tau}{m}\right)\\ &\times\mathcal{D}_{TB}(\boldsymbol{\alpha}_1,\boldsymbol{\alpha}_2,\boldsymbol{\beta}_1,\boldsymbol{\beta}_2,\tau,s).\end{aligned} \tag{24}$$

Replacing Eq. (24) in Eq. (4), performing again the following change of variables: $\boldsymbol{\beta}_1\to\boldsymbol{\beta}_1-\tau\alpha_1/m$ and $\boldsymbol{\beta}_2\to\boldsymbol{\beta}_1-s\alpha_2/m$, one obtains

$$\begin{aligned}\mathcal{M}_t = \exp_+\Bigg\{&\int_0^t d\tau\int_0^t ds\int d\alpha\int d\boldsymbol{\beta}\mathcal{F}(\boldsymbol{\alpha},\boldsymbol{\beta},\tau,s)\\ &\times\big(e^{i(\boldsymbol{\alpha}\cdot\hat{\mathbf{x}}_{sL}+\boldsymbol{\beta}\cdot\hat{\mathbf{p}}_L)}e^{-i(\boldsymbol{\alpha}\cdot\hat{\mathbf{x}}_{\tau R}+\boldsymbol{\beta}\cdot\hat{\mathbf{p}}_R)}\\ &-\theta_{\tau s}e^{-i(\boldsymbol{\alpha}\cdot\hat{\mathbf{x}}_{\tau L}+\boldsymbol{\beta}\cdot\hat{\mathbf{p}}_L)}e^{i(\boldsymbol{\alpha}\cdot\hat{\mathbf{x}}_{sL}+\boldsymbol{\beta}\cdot\hat{\mathbf{p}}_L)}\\ &-\theta_{s\tau}e^{-i(\boldsymbol{\alpha}\cdot\hat{\mathbf{x}}_{sR}+\boldsymbol{\beta}\cdot\hat{\mathbf{p}}_R)}e^{i(\boldsymbol{\alpha}\cdot\hat{\mathbf{x}}_{\tau R}+\boldsymbol{\beta}\cdot\hat{\mathbf{p}}_R)}\big)\Bigg\},\end{aligned} \tag{25}$$

where $\mathcal{F}(\boldsymbol{\alpha},\boldsymbol{\beta},\tau,s)$ is a completely general real valued function. In this case the functional dependence of the map on the position and momentum operator is fixed [65]. This equation fully characterizes translation and boost covariant CP Gaussian maps.

Rotation covariance.—For completeness, we also discuss rotation covariance. Restricting to the subgroup of rotations $\mathcal{R}\subset\mathcal{G}$ we have

$$\mathcal{U}_R[\hat{\mathbf{x}}_s] = R\hat{\mathbf{x}}_s, \tag{26}$$

$$\mathcal{U}_R[\hat{\mathbf{p}}] = R\hat{\mathbf{p}}, \tag{27}$$

where R is a generic rotation matrix and $\mathcal{U}_R$ the corresponding linear transformation [see Eq. (7)]. Using the relation $\boldsymbol{a}\cdot(R\boldsymbol{b}) = (R^{-1}\boldsymbol{a})\cdot\boldsymbol{b}$, where $\boldsymbol{a}$, $\boldsymbol{b}$ are generic vectors, and recalling that the integral measure $d\alpha d\boldsymbol{\beta}$ is invariant under rotations, we perform the change of variables $\alpha\to R\alpha, \boldsymbol{\beta}\to R\boldsymbol{\beta}$ in Eq. (11), which gives the condition

$$\mathcal{D}(R\alpha_1,R\boldsymbol{\beta}_1,R\alpha_2,R\boldsymbol{\beta}_2,\tau,s) = \mathcal{D}(\boldsymbol{\alpha}_1,\boldsymbol{\beta}_1,\boldsymbol{\alpha}_2,\boldsymbol{\beta}_2,\tau,s). \tag{28}$$

Equation (4), with the function $\mathcal{D}$ satisfying the symmetry given by Eq. (28), characterizes rotational covariant CP Gaussian maps. This concludes the characterization of CP Gaussian maps covariant under Galilean symmetries.

Markovian limits.—The CP Gaussian covariant maps derived here above reduce to the well-known Markovian CP Gaussian covariant maps in the Markovian limit. In particular, we immediately re-obtain the Holevo structures for the generators of the covariant quantum dynamical semigroup by imposing the request of Markovianity as given by

Eq. (2). Under this assumption it is straightforward to show that Eq. (17) reduces to $\mathcal{M}_t = \exp_+\{\int_0^t ds\mathcal{L}_s\}$, where

$$\mathcal{L}_s = \int d\alpha \tilde{D}_{jk}(s)\bigg(F^k_{sL}(\hat{\mathbf{p}},\boldsymbol{\alpha})e^{i\alpha\cdot\hat{\mathbf{x}}_{sL}}F^{j\dagger}_{sR}(\hat{\mathbf{p}},\boldsymbol{\alpha})e^{-i\alpha\cdot\hat{\mathbf{x}}_{sR}} - \frac{1}{2}F^{j\dagger}_{sL}(\hat{\mathbf{p}},\boldsymbol{\alpha})F^k_{sL}(\hat{\mathbf{p}},\boldsymbol{\alpha}) - \frac{1}{2}F^k_{sR}(\hat{\mathbf{p}},\boldsymbol{\alpha})F^{k\dagger}_{sR}(\hat{\mathbf{p}},\boldsymbol{\alpha})\bigg), \tag{29}$$

is the generator of the translational covariant semigroup.

Analogously, by considering the Markovian limit of the boost and translation covariant map derived in Eq. (25), we obtain the following generator:

$$\mathcal{L}_s = \int d\alpha \int d\boldsymbol{\beta}\tilde{\mathcal{F}}(\boldsymbol{\alpha},\boldsymbol{\beta},s)(e^{i(\boldsymbol{\alpha}\cdot\hat{\mathbf{x}}_{sL}+\boldsymbol{\beta}\cdot\hat{\mathbf{p}}_L)}e^{-i(\boldsymbol{\alpha}\cdot\hat{\mathbf{x}}_{sR}+\boldsymbol{\beta}\cdot\hat{\mathbf{p}}_R)} - 1), \tag{30}$$

where $\tilde{\mathcal{F}}$ is a positive valued function. Equations (29) and (30) correspond to the Holevo results for covariance under translation and boost-translation, respectively [20,21].

Macroscopicity measure.—More and more experiments are nowadays probing quantum mechanics in novel regimes, exploring in particular the boundary between quantum and classical [66–70]. It becomes relevant to define a *measure* that quantifies how far a given experiment pushes this boundary. This is a nontrivial task: what is the measure of *macroscopicity* that correctly accounts for complexity, size, mass, or some other feature of the system being explored?

Beginning with Leggett [71,72] several measures of macroscopicity have been proposed [43,73–78]. Among them, the one given by Nimmrichter and Hornberger in Ref. [78] has become quite popular in the matter-wave interferometry community because of its simplicity and versatility: they define as a *macroscopicity measure* a real number that quantifies how well an experiment tests a *minimal modification of quantum mechanics*. Specifically, they suggest the following measure:

$$\mu = \log(\tau/1s) \tag{31}$$

with τ the biggest excluded *time scale* in which quantum superpositions are suppressed by the minimal modification of quantum mechanics.

They further assume that the *minimal modification of quantum mechanics*, for a single particle with mass m, is described by a Markovian nonunitary TP, *CP*, Galilean covariant (translations, boosts and rotations) and time translation invariant map. This amounts to the nonunitary map generated by Eq. (30), where they choose the following parametrization of the correlation function:

$$\tilde{\mathcal{F}}(\boldsymbol{\alpha},\boldsymbol{\beta},s) = \frac{1}{\tau}g(\boldsymbol{\alpha},\boldsymbol{\beta}), \tag{32}$$

where g is a positive, isotropic phase-space distribution normalized to unity (a Gaussian function with variances σ_α, σ_β) and τ gives the *time scale* in which superpositions are suppressed by the minimal modification (for further details see Refs. [43,78]).

The measure μ defined in Eq. (31) thus relies on the assumptions characterizing the minimal modification. Among these, Markovianity and Galilei covariance are usually taken for granted as they are a building block of the most successful nonrelativistic theories: quantum and classical mechanics. However, technological advances have come to the point of questioning the validity of these two assumptions; on top of this, minimal modifications need not satisfy them *a priori*. We take an example from the literature of collapse models, which can be seen as instances of *minimal modifications* of quantum mechanics in the spirit of Ref. [43]. X-ray measurements [79] pose rather strong bounds on the collapse parameters [66]; however, the strength of the bounds depends critically on whether the collapse model is Markovian or not [80,81]. The reason is that such experiments explore the $\approx 10^{18}$ Hz region of the spectrum, meaning that the time resolution which is probed is $\approx 10^{-18}s$. Any cutoff in the spectrum of the collapse noise smaller than such frequencies weakens significantly the bound. A similar behavior is expected to occur for a macroscopicity measure that correctly includes non-Markovian effects. Markovianity might be verified only under a suitable temporal coarse graining of the underlying dynamics. In general if the time resolution of the experiment is longer than the correlation times associated to the modifications of the theory, then the Markovian assumption is justified, as any non-Markovian dynamics with finite correlation times may be approximated by its Markovian limit [82].

The assumption of Galilean covariance (translation and boost), even if it seems an innocent assumption, forces the nonunitary dynamics to produce an infinite growth of the system's energy on long time scales [85]. Galilean covariant maps must be then understood only as a good approximation that can be used in experiments that run for sufficiently short times, such that dissipative phenomena are negligible. In experiments with a long running time, the results could be influenced by dissipative phenomena and consequently the assumption of Galilean covariant dynamics is too restrictive. We consider a second example taken from collapse models. A recent experiment succeeded to cool a cloud of cold atoms to temperature less than 50^{+50}_{-30} pK [87]. They measured the spreading of the cloud over time, which would be affected by modification of quantum mechanics. The analysis performed in Ref. [69] shows that the predictions of collapse models depend on whether dissipative effects are taken into account (Fig. (8) of Ref. [69] shows that the bounds on the collapse model drastically change with the thermalization temperature T, which quantifies the dissipation in the model). Again, a similar dependence on dissipation is expected by a macroscopicity measure, which takes dissipative effects into account.

To summarize, although μ is a reasonable choice for the measure of macroscopicity in many instances, novel experiments probing the very short and very long time scales need a different measure of macroscopicity due to Non-Markovian and dissipative effects, respectively. For such cases we propose to use the (translational covariant and non-Markovian) map given in Eq. (17) as the *minimal modification*, with an appropriately chosen correlation function $D(t,s)$ and operators $F_{\tau\mu}(\hat{\mathbf{p}},\boldsymbol{\alpha})$, where for simplicity we consider that the sum over j, k contains only one term. We can still use Eq. (31) to define the measure of macroscopicity, where now $\tau \to \tau(\tau_c, T)$ is the biggest excluded *time scale*, for fixed parameters (τ_c, T), in which quantum superposition is suppressed by the minimal modification. Here τ_c is the correlation time of the correlation function $D(t,s)$ and T is the temperature measuring dissipative effects.

To be more concrete we suggest the exponential correlation function

$$D(t,s) = \frac{1}{2\tau_c} e^{-|t-s|/\tau_c} \tag{33}$$

and the Gaussian operators

$$F_{\mu}(\hat{\mathbf{p}},\boldsymbol{\alpha}) = \sqrt{\frac{1}{\tau}\frac{m^2}{m_0^2}\left(\frac{r_c}{\sqrt{\pi}\hbar}\right)^3} e^{-(r_c^2/2\hbar^2)[(1+k_T)\boldsymbol{\alpha}+2k_T\hat{\mathbf{p}}_\mu]^2}, \tag{34}$$

where $k_T = (\hbar^2/8m_0 r_c^2 k_B T)$, $m_0 = 1$ amu is a reference mass, k_B is Boltzmann's constant, r_c is a free length parameter analogous to the spread σ_α in Eq. (32), and τ gives the *time scale* in which the superpositions of a reference object with mass m are suppressed. In the Markovian ($\tau_c \to 0$) and nondissipative ($T \to \infty$) limit, we reobtain the measure of macroscopicity proposed by Nimmrichter and Hornberger with $\sigma_\beta \to 0$ [see Eqs. (31), (32)].

This new measure depends critically on the values of τ_c and T. To illustrate this, we have studied the classicalization map in the regime of small distances and low momentum transfer in one spatial dimension [88]. Specifically, we have considered a simple ideal experiment capable of resolving the time evolution of the spread of the wave packet of a freely evolving particle. The associated macroscopicity measure is investigated in the non-Markovian and dissipative regimes [89] (cf. Supplemental Material S4 and Fig. S2), showing how it depends on the correlation time τ_c and temperature T.

Summary.—We have analyzed Galilean symmetries in non-Markovian Gaussian *CP* maps. The two main results of this Letter are the characterization of translational and of Galilei (translation-boost) covariant non-Markovian *CP* Gaussian maps given by Eqs. (17) and (25), respectively. These maps are a generalization of the well-known Holevo results, which we reobtain in the Markovian limit. We have also provided the corresponding unravelling given by stochastic Schrödinger equations in a form suitable for nonperturbative numerical analysis [95]. As mentioned in the introduction, these results can find applications in several fields of research [7–14]. We have also analyzed the role that non-Markovian and dissipative effects play in the construction of a macroscopicity measure. We have shown that experiments probing the quantum-to-classical boundary on very short or very long time scales might not be adequately described by the macroscopicity measure in Ref. [43], and a more general definition is needed, as the one we propose, based on Eqs. (17), (33), and (34).

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