# Measurement of $\nu_{\mu}$ charged-current single $\pi^{0}$ production on hydrocarbon in the few-GeV region using MINERvA 

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The semiexclusive channel $\nu_{\mu}+\mathrm{CH} \rightarrow \mu^{-} \pi^{0}+$ nucleon(s) is analyzed using MINERvA exposed to the low-energy NuMI $\nu_{\mu}$ beam with spectral peak at $E_{\nu} \simeq 3 \mathrm{GeV}$. Differential cross sections for muon momentum and production angle, $\pi^{0}$ kinetic energy and production angle, and for squared four-momentum transfer are reported, and the cross section $\sigma\left(E_{\nu}\right)$ is obtained over the range $1.5 \mathrm{GeV} \leq E_{\nu}<20 \mathrm{GeV}$. Results are compared to GENIE and NuWro predictions and to published MINERvA cross sections for charged-current $\pi^{+}\left(\pi^{0}\right)$ production by $\nu_{\mu}\left(\bar{\nu}_{\mu}\right)$ neutrinos. Disagreements between data and simulation are observed at very low and relatively high values for muon angle and for $Q^{2}$ that may reflect shortfalls in modeling of interactions on carbon. For $\pi^{0}$ kinematic distributions, however, the data are consistent with the simulation and provide support for generator treatments of pion intranuclear scattering. Using signalevent subsamples that have reconstructed protons as well as $\pi^{0}$ mesons, the $p \pi^{0}$ invariant mass distribution is obtained, and the decay polar and azimuthal angle distributions in the rest frame of the $p \pi^{0}$ system are measured in the region of $\Delta(1232)^{+}$production, $W<1.4 \mathrm{GeV}$.

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## I. INTRODUCTION

Production of single- $\pi^{0,+}$ mesons by $\nu_{\mu}$ charged-current (CC) inelastic scattering on nuclei in the few- GeV region of neutrino energy, $E_{\nu}$, arises from three types of processes. For $E_{\nu} \leq 3 \mathrm{GeV}, \mathrm{CC}(\pi)$ reactions are mostly instances of
neutrino-nucleon scattering wherein a bound nucleon is struck and caused to transition into a baryon resonance that promptly decays into a pion and a nucleon. Production of the $\Delta(1232) P_{33}$ resonance is prominent and causes $\mathrm{CC}(\pi)$ cross sections to rise rapidly from thresholds. As the incident $E_{\nu}$ is increased, however, contributions from higher-mass $N^{*}$ resonances such as the $P_{11}(1440)$, $D_{13}(1520)$, and $S_{11}(1535)$ states become significant. Increasing $E_{\nu}$ also facilitates the onset of deep inelastic scattering, in which the neutrino interacts with a valence or sea quark within a bound nucleon and the exiting quark hadronizes into one or multiple pions. Single-pion production can also arise from neutrino-nucleon scattering that does not involve a resonance. Such processes are referred to as nonresonant pion production and are often treated as a subsample of deep inelastic scattering (DIS) processes that have final-state hadronic invariant masses $W$ less than 1.7 GeV [1].

Charged-current single-pion production features prominently among neutrino interactions that occur within or around the appearance peaks or the disappearance minima in events-versus- $E_{\nu}$ spectra measured by the long baseline neutrino oscillation experiments. This is especially the case for NOvA and DUNE [2,3] since the $\nu_{\mu}$ fluxes of these experiments have maxima at 2.0 and 2.5 GeV , respectively. Improved knowledge of $\mathrm{CC}(\pi)$ is also of keen interest for the T2K and HyperK long baseline experiments [4,5] whose flux spectra peak below 1.0 GeV , especially with regard to $\Delta(1232)$ and nonresonant pion contributions. In long baseline oscillation experiments, neutrino event energies need to be measured to precisions of e.g. $\leq 100 \mathrm{MeV}$ for DUNE and $\leq 50 \mathrm{MeV}$ for T2K [6] in order to resolve the neutrino-sector $C P$ violating phase and the ordering of the neutrino mass states. Consequently new measurements of $\operatorname{CC}(\pi)$ reactions on hydrocarbon and on other nuclei in the few-GeV region of $E_{\nu}$ are urgently needed. Such measurements enable the testing and refinement of the neutrino event generators, e.g. GENIE [7], NEUT [8], NuWro [9], and GIBUU [10]. The predictions of these generators are a crucial element in physics evaluations of oscillated neutrino event spectra.

Among the $\mathrm{CC}(\pi)$ reactions, the channel

$$
\begin{equation*}
\nu_{\mu}+\mathrm{CH} \rightarrow \mu^{-}+\pi^{0}+X(\text { nucleons }) \tag{1}
\end{equation*}
$$

is of particular interest. In contrast to charged pion production, the events of channel (1) have electromagnetic (EM) showers as the dominant part of the visible hadronic system. As a result, channel (1) provides useful information about the responses of detector systems to both signal and backgrounds for the electron-neutrino oscillation appearance measurements, $\nu_{\mu} \rightarrow \nu_{e}$.

While the cross section for $\operatorname{CC}\left(\pi^{0}\right)$ is smaller than for $\mathrm{CC}\left(\pi^{+}\right)$processes, the single- $\pi^{0}$ channel is devoid of CC single-pion coherent scattering (e.g. $\nu_{\mu}+C \rightarrow \mu^{-} \pi^{+}+C$ );
consequently its final-state distributions provide an unencumbered view of $W^{+}$excitation of the nucleon $[11,12]$. Thus for example a four-momentum transfer, $Q^{2}$, distribution measured for $\mathrm{CC}\left(\pi^{0}\right)$ is directly representative of the channel, whereas a $Q^{2}$ distribution obtained with $\mathrm{CC}\left(\pi^{+}\right)$ events includes an elevated event rate at low $Q^{2}$ arising from coherent $\pi^{+}$production-a very different reaction type that may necessitate a further accounting [13]. An additional simplification is that single- $\pi^{0}$ production is composed of carbon-target scattering almost entirely. According to the reference simulation of this analysis, scatters from hydrogen account for less than $3.3 \%$ of the candidate signal sample. The kinematic spectra of the $\pi^{0}$ mesons of (1) provide useful checks on simulation treatments of pion final-state interactions (FSI) that can take place within the struck carbon nucleus. The $\pi^{0}$ distributions in kinetic energy and production angle probe the suite of elastic, inelastic, absorption, and charge-exchange scattering algorithms that are used in these treatments in ways that complement constraints gleaned from charged pion distributions [14,15].

In the present work, a signal event sample for channel (1) is isolated which, after background subtraction, is predicted by the reference model to have a $50 \%$ contribution from the exclusive channel

$$
\begin{equation*}
\nu_{\mu}+\mathrm{n} \rightarrow \mu^{-}+\pi^{0}+\mathrm{p} \tag{2}
\end{equation*}
$$

where the struck neutron is bound within a carbon nucleus. Thus the phenomenology pertaining to reaction (2) can be helpful when evaluating differential cross sections derived from the signal sample and comparing them to MINERvA's published results for the semi-inclusive channels $\nu_{\mu}-\mathrm{CC}\left(\pi^{+}\right)$and $\bar{\nu}_{\mu}-\mathrm{CC}\left(\pi^{0}\right)$ [13]. In contrast to the $\pi^{+} p$ hadronic systems of $\nu_{\mu}-\mathrm{CC}\left(\pi^{+}\right)$, the total amplitude for reaction (2) receives contributions from the $\mathrm{I}=1 / 2$ isospin amplitude, $A_{1}^{C C}$, as well as from the $\mathrm{I}=3 / 2$ isospin amplitude, $A_{3}^{C C}$,

$$
\begin{equation*}
A\left(\nu n \rightarrow \mu^{-} p \pi^{0}\right)=A^{C C}\left(p \pi^{0}\right)=\frac{2}{3}\left(A_{3}^{C C}-A_{1}^{C C}\right) . \tag{3}
\end{equation*}
$$

The amplitudes associated with neutrino-induced baryon resonances having the same spin and the same orbital angular momentum may interfere. The consequence for $\pi^{0} p$ final states is that many more interference terms are possible for their transitions $\left|A^{C C}\left(p \pi^{0}\right)\right|^{2}$ than occur in $\left|A^{C C}\left(p \pi^{+}\right)\right|^{2}$ [16].

## A. $\operatorname{CC}\left(\pi^{0}\right)$ measurements and phenomenology

Differential cross sections for exclusive-channel $\operatorname{CC}(\pi)$ interactions including reaction (2) were obtained during the 1970s and 1980s era of large bubble chamber experiments and the measurements continue to be of interest at the
present time. Neutrino-induced single-pion production at $E_{\nu} \leq 1.5 \mathrm{GeV}$ was studied using hydrogen and deuterium fills in Argonne National Laboratory's (ANL) 12-ft diameter bubble chamber; cross sections and distributions of pion-nucleon invariant mass and of $Q^{2}$ were reported [17,18]. Similar measurements with higher statistics were subsequently obtained using Brookhaven National Laboratory's (BNL) 7-ft deuterium-filled bubble chamber exposed to a neutrino beam with an average energy of 1.6 GeV [19]. In these two experiments, the $\Delta^{++}(1232)$ was found to dominate the $p \pi^{+}$final state with no other resonance structure observed. For the $p \pi^{0}$ and $n \pi^{+}$final states however, $\Delta^{+}$production was accompanied by a broad $N \pi$-mass distribution extending to 2.5 GeV . Measurements of $\mathrm{CC}(\pi)$ reactions at much higher $E_{\nu}$, for $\bar{\nu}_{\mu}$ as well as for $\nu_{\mu}$ scattering, were reported in the late 1980s by experiments using the SKAT heavy liquid bubble chamber at Serpukhov ( $3<E_{\nu}<30 \mathrm{GeV}$ ) [20], and by the deuterium-filled Big European Bubble Chamber at CERN $\left(\left\langle E_{\nu}\right\rangle=54 \mathrm{GeV}\right)$ [21]. The neutrino-induced $\pi^{0} p$ invariant mass distributions of the latter experiments showed the contributions from higher-mass baryon resonances to be increasing relative to the $\Delta(1232)$ contribution in this final state, and in the $\pi^{+} n$ final state as well.

For $\mathrm{CC}\left(\pi^{0}\right)$ production on hydrocarbon targets, there are only two previous measurements. These were carried out relatively recently using neutrino beams with flux maxima occurring well below 2.0 GeV . The MiniBooNE experiment used its spherical, 12 m diameter Čerenkov detector filled with mineral oil $\left(\mathrm{CH}_{2}\right)$ exposed to neutrinos of 0.5 to 2.0 GeV with peak flux at 0.6 GeV . The $\mathrm{CC}\left(\pi^{0}\right)$ cross section was obtained, and differential cross sections for muon and $\pi^{0}$ kinematics and for $Q^{2}$ were reported [22]. The K2K experiment used neutrino scattering on extruded scintillator (polystyrene) bars to obtain the ratio of $\mathrm{CC}\left(\pi^{0}\right)$ to quasielastic scattering at a mean neutrino energy of 1.3 GeV [23].

Recent theoretical treatments of $\mathrm{CC}\left(\pi^{0}\right)$ can be roughly categorized in terms of eras of endeavor. In the preMiniBooNE era, investigations focused on refining the phenomenology and used the bubble chamber data to obtain cross checks on the formalism [24-28]. With the advent of $\mathrm{CC}\left(\pi^{+, 0}\right)$ measurements reported by MiniBooNE [22,29], the focus shifted to developing explanations for the new data [30-32]. Comparisons between predictions and MiniBooNE results turned up discrepancies in the shapes of distributions of final-state $\pi^{0}$ mesons, highlighting the interplay between pion FSI processes and possible formation time effects [32]. Comparisons with the MiniBooNE data have continued into the present era; however, phenomenological studies have expanded their purview to the higher-energy pion production results reported by MINERvA and to measurements underway by T2K [11,33-35].

## B. CC $\left(\boldsymbol{\pi}^{\mathbf{0}}\right)$ measurement using MINERvA

The fine-grained tracking capability of the MINERvA experiment's central scintillator tracker coupled with the MINOS downstream spectrometer are used to investigate a sample of 6110 events having the topology and kinematics of the signal channel (1). In contrast to previous measurements, this investigation covers the neutrino energy range that is directly relevant to the NOvA and DUNE neutrino oscillation programs. Results are obtained with good statistics on a hydrocarbon target, a medium whose atomic number composition is an excellent match to NOvA's liquid scintillator medium and does not differ greatly from T2K's water medium. This analysis obtains differential cross sections for channel (1) that characterize the kinematics of both the final-state muon and the produced $\pi^{0}$. The results reported here complement and extend MINERvA's previous measurements of $\nu_{\mu}$ and $\bar{\nu}_{\mu}$ CC pion production on hydrocarbon [13-15].

The analysis makes use of MINERvA's fine-grained tracking to undertake measurements that heretofore have never been carried out for neutrino scattering on a hydrocarbon medium. For a subsample of channel-(1) events, a leading final-state proton has been reconstructed in conjunction with the $\pi^{0}$, enabling the final-state hadronic invariant mass to be directly measured.

A further selection on hadronic invariant mass yields a subsample confined to the region of $\Delta(1232)^{+}$production. It is used to examine the decay angular distribution of the $\pi^{0} p$ system in the candidate $\Delta(1232)^{+}$rest frame. In bubble chamber experiments of the 1970s and 1980s, polarization effects were observed in the pure $\mathrm{I}=3 / 2$ channel $\nu_{\mu} p \rightarrow \mu^{-} p \pi^{+}[18,19,21,36]$ and in the mixed isospin channel $\bar{\nu}_{\mu} p \rightarrow \mu^{+} p \pi^{-}$[36]. This work reports the first-ever measurement of the decay polar and azimuthal angles $\theta$ and $\phi$ for the mixed isospin reaction (2). The latter angular distributions are potentially complicated since nearby resonances can interfere strongly with the leading $\Delta^{+}$amplitude [16,37].

## II. OVERVIEW OF DATA AND ANALYSIS

## A. Detector, exposure, and $\nu$ flux

MINERvA uses a fine-grained, plastic scintillator tracking detector $[38,39]$ positioned upstream of the magnetized MINOS near detector [40] to record interactions of neutrinos from the high-intensity NuMI beam at Fermilab [41]. In the present analysis the spectrometer's central, scintillator tracking region serves as the target, with the surrounding electromagnetic and hadronic calorimeters providing containment. The fiducial volume has a hexagonal cross section of 2.0 m minimal diameter, extends longitudinally for 2.4 m , and has a mass of 5400 kg . It consists of 106 planes, each of $\sim 2 \mathrm{~cm}$ thickness, composed of polystyrene scintillator strips oriented transversely to the detector's horizontal axis. The planes are configured into
modules. There are two planes per module, with an air gap of 2.5 mm between each module. The horizontal axis is inclined $3.3^{\circ}$ relative to the beam axis. Three scintillatorplane orientations, at $0^{\circ}$ and $\pm 60^{\circ}$ relative to the detector vertical axis, provide $\mathrm{X}, \mathrm{U}$, and V "views" of interactions in the scintillator medium. The planes alternate between UX and VX pairs, enabling three-dimensional reconstruction of vertices, charged tracks, and electromagnetic showers of neutrino events. The downstream electromagnetic calorimeter (ECAL) is identical to the central tracking region but includes a 0.2 cm ( 0.35 radiation lengths) lead sheet in front of every two planes of scintillator. The readout electronics have a timing resolution of 3.0 ns for hits of minimumionizing particles [42], allowing the efficient separation of multiple interactions within a single $10 \mu$ s beam spill.

The MINOS near detector is located 2 m downstream of MINERvA and serves as the muon spectrometer for MINERvA's central tracker. A muon that exits the downstream surface of MINERvA is tracked by the magnetized, steel-plus-scintillator planes of MINOS, and the muon's momentum and charge are measured. Trajectories of individual muons traversing the two detectors are matched together by correlating the positions, angles, and timings of track segments in each detector.

The data were taken between March 2010 and April 2012 with the NuMI beam line operating in a mode that produces a wide-band neutrino flux whose spectrum peaks at 3.0 GeV , extends from 1 GeV to above 20 GeV , and has $\nu_{\mu}$ content at $93 \%$ purity. The event sample analyzed here is obtained with an integrated exposure of $3.04 \times 10^{20}$ protons on target (POT).

The $\nu_{\mu}$ flux is calculated using a detailed simulation of the NuMI beam line based on GEANT4 [43,44] (version 9.2.p03, physics list FTFP_BERT). The flux simulation is constrained using proton-carbon yield measurements [45-47] together with more recent thin-target data on hadron yields [48]. A further constraint on the flux estimation is derived using the $\nu+e^{-}$scattering rate observed by MINERvA [49].

## B. Neutrino interaction modeling

The reference Monte Carlo (MC) simulation used by this analysis is based upon version 2.8.4 of the GENIE neutrino event generator [7]. The GENIE strategies that underwrite the generation of CC neutrino-nucleus interactions in the simulation are the same as used in previous MINERvA studies of $\mathrm{CC}(\pi)$ and are described in MINERvA publications [13,14]. Recent neutrino measurements and phenomenology developments motivate certain augmentations to the GENIE version; these are implemented using event reweighting and by adding a simulated sample of quasielasticlike $2 p 2 h$ events. All of the augmentations (described below) have been used in recent MINERvA works [50,51].

In brief, the target nucleus is modeled as a relativistic Fermi gas with addition of a high-momentum tail required
to account for short-range correlations [52]. Neutrinoinduced pion production arises from interaction with a single nucleon and proceeds either by baryon-resonance excitation or by nonresonant DIS processes. Baryonresonance pion production is simulated using the ReinSehgal model [53] with incorporation of modern baryon resonance properties [54]. Decays of baryon resonances are generated isotropically in their rest frames, but an exception is made for the $\Delta^{++}$. In MINERvA's GENIE-based simulation, $\Delta^{++}$decays are generated with an angular isotropy at $50 \%$ of the strength predicted by Rein-Sehgal [14]. Nonresonant pion production is modeled according to the formalism of Bodek-Yang [55] with parameters adjusted to reproduce electron and neutrino scattering measurements over the invariant hadronic mass range $W<1.7 \mathrm{GeV}$ [1,56,57]. No allowance is made for interferences among baryon-resonance amplitudes.

The simulation includes a treatment of FSI of pions and nucleons that are created and propagate within the struck nucleus. Accounting for pion FSI is especially important to this analysis because of the large pion-nucleon cross sections that occur in the vicinity of $\Delta$-resonance excitation. In GENIE, an effective model for FSI is used in lieu of a full intranuclear cascade; each pion is allowed to have at most one rescattering interaction while traversing the nucleus [58]. This approximation works well for a light nucleus such as carbon and it makes the simulation amenable to event reweighting. It has been shown by MINERvA studies of $\nu_{\mu}-\mathrm{CC}\left(\pi^{+}\right)$and $\bar{\nu}_{\mu}-\mathrm{CC}\left(\pi^{0}\right)$ that GENIE FSI improves the agreement between simulation and data for pion kinematic distributions [13-15].

Events are added to the GENIE-based simulation to include quasielasticlike $2 p 2 h$ interactions whose generation is based on the Valencia model [59,60], but with the interaction rate raised to match the data rate observed in MINERvA inclusive CC scattering data [61]. For quasielastic scattering, kinematic distortions attributed to long-range nucleon-nucleon correlations are included in accordance with the random phase approximation (RPA) calculation of Ref. [62]. For CC single-pion production, the GENIE prediction for the nonresonant pion contribution has been reduced by $53 \%$ for all pion charge states, as this has been shown to yield a better agreement with the deuterium bubble chamber data $[56,57]$.

## C. Detector response, calibrations, and event isolation

The ionization response of the MINERvA spectrometer to muons, electrons, and hadrons is modeled using GEANT4 [43,44] (9.4.p02, QGSP_BERT). The scale for visible energy is established by requiring agreement between data and simulation for the reconstructed energy deposited by through-going muons that are momentum analyzed using MINOS. The scale for muon dE/dx energy loss in the detector is known to within $2 \%$. Reconstruction of the energy of hadronic showers makes use of
calorimetric corrections. Initially these were estimated from simulation according to procedures detailed in Ref. [38]. Subsequently the corrections were refined and validated using measurements obtained with a scaled-downed replicate of the detector operated in a low-energy particle test beam [39]. The test beam data, in conjunction with in situ measurements, underwrite the determinations of tracking efficiencies and energy responses to protons, charged pions, and electrons, and of the value assigned to the Birks' constant of the scintillator.

The central scintillator tracker has a radiation length of 40 cm , which corresponds to 25 planes for photons traveling normal to the planes. The energies of photon and electron-induced showers are reconstructed by calorimetry using calibration constants determined from the simulation [15,49,63]. For EM showers with visible energies above 700 MeV , the conversion factor from visible energy, $E_{\text {visible }}^{\gamma}$, to true energy, $E_{\text {true }}^{\gamma}$, is constant to good approximation. The EM showers of this analysis however come from $\pi^{0} \rightarrow \gamma \gamma$ decay and have energies in the range of 50 MeV to 1 GeV . Over the lower half of this range, Compton scattering competes with pair production in the total photon-carbon cross section, and so a conversion factor ( $\left.E_{\text {true }}^{\gamma} / E_{\text {visible }}^{\gamma}\right)$ is used that increases linearly, from 1.33 to 1.49, as $E_{\text {visible }}^{\gamma}$ is decreased from 700 to 0 MeV [64].

For each $10 \mu$ s NuMI beam spill, the visible energy in the scintillator as a function of time is divided into "time slices" of tens to hundreds of nanoseconds; each time slice encompasses a single event in the detector. A charged particle initiated by a neutrino interaction traverses the scintillator strips of the central tracker, and its trajectory is recorded as individual energy deposits (hits) having a specific charge content and time of occurrence. The hits are grouped in time, and neighboring hits in each scintillator plane are gathered into objects called clusters. Clusters that have more than 1 MeV of energy are matched among the three views to create a track. The position resolution per plane is 2.7 mm and the track angular resolution is better than 10 mrad [38] in each view.

## III. EVENT RECONSTRUCTION AND SELECTIONS

## A. Muons, protons, and vertex energy

A track that starts in the central tracker exits via the detector's downstream surface, and matches with a negatively charged track that enters the front of MINOS near detector, is taken to be the $\mu^{-}$track of a CC event. Selected events are required to have a muon track that is MINOS matched. This requirement eliminates events having muons with production angle $\theta_{\mu}>25$ degrees $(\approx 26 \%$ of CC interactions in the tracker). The momentum resolution for muons reconstructed using both the MINERvA and MINOS detectors (the rms width of the residual fractional error) is $6.1 \%$. A small inefficiency occurs with muon track
reconstruction due to event pileup, which requires a correction of $-4.4 \%(-1.1 \%)$ to the simulated efficiency for muons of momenta less than (greater than) $3 \mathrm{GeV} / \mathrm{c}$ [13].

Having found and reconstructed a muon track, the reconstruction algorithm searches for additional tracks that share the primary vertex of the muon track. If additional shorter track(s) are found, the vertex position is refitted. Kinked tracks, which are usually the result of secondary interactions, are then reconstructed by searching for additional tracks starting at the end points of tracks previously found. The event primary vertex is required to occur within the central 112 planes of the scintillator tracking region and must be at least 25 cm from any edge of the planes. The fiducial volume contains a target mass of 5.27 metric tons with $3.17 \times 10^{30}$ nucleons.

Events having no reconstructed tracks emanating from the primary vertex other than the muon are retained for further analysis. When one or more extra tracks are found, selections for protons are applied. The ionization $\mathrm{dE} / \mathrm{dx}$ profile of each track is compared to simulated profiles for protons and charged pions based on the Bethe-Bloch formula, and a likelihood ratio is calculated. An event is retained if all prompt nonmuon tracks pass a cut on the proton consistency score [65]; the momenta of such tracks are then assigned according to a track range for the proton hypothesis. To further ensure that final states having $\pi^{+}$ mesons are eliminated, regions around the primary vertex, around each track end point, and around beginning and end points of candidate EM showers that are remote from the primary vertex are examined for instances of Michel electrons. These are low-energy ( $\leq 100 \mathrm{MeV}$ ) EM showers that arise in the decay sequence $\pi^{+} \rightarrow \mu^{+} \rightarrow e^{+}$and appear in time slices that are later, by 0.5 to $16 \mu \mathrm{~s}$, than the candidate-event time. Events associated with a Michel tag are removed.

Protons of kinetic energy $T_{p} \simeq 100 \mathrm{MeV}$ are at the threshold for reconstruction as tracks in the detector. A candidate for channel (1) can have one or more final-state protons that are too short to be tracked, but still gives visible ionization around the primary vertex. This "vertex energy" appears as hits that are not used in reconstruction of the muon or prompt proton tracks. The unused hits that fall within a sphere of 9 cm radius centered on the primary vertex are gathered and their net energy estimated using a calibration established via study of inclusive hadron production.

The distributions of vertex energy for channel (1) candidates after all analysis selections, including the $\pi^{0}$ selections described in the remainder of this section, are shown in Fig. 1 together with predictions from the GENIEbased MC. The background consists of events that pass the signal selection criteria but have final states that, upon emergence from their parent nuclei, are not examples of reaction (1).

Vertex energy consists of ionizations of low-momentum protons (or charged pions) from the primary vertex that


FIG. 1. Vertex energy in data and simulation, of events after all selections. Statistical (systematic) errors are shown for the data (for the MC prediction). The simulation is broken out as signal (clear histograms) and background (shaded histograms). The distribution shapes depend on whether the muon is accompanied by zero reconstructed protons (a), or by one or more reconstructed protons (b).
cannot be reconstructed as tracks. When protons of an event are reconstructed as tracks, their energy is excluded from vertex energy. Thus the energy reach of the vertex-energy distribution for events without reconstructed protons [Fig. 1(a)] is noticeably reduced for the subsample that possesses reconstructed protons [Fig. 1(b)]. However for each of the two subsamples that together comprise the analysis signal sample, the vertex energy distribution is well described by the simulation.

## B. Selections for events with two photons

The candidate sample is subjected to a prefilter prior to $\pi^{0}$ reconstruction. An event is removed if the upstream nuclear targets section has registered time-coincident energy that exceeds 20 MeV . Such activity can be due to interactions originating in a nuclear target or in the earth berm upstream of the detector. Events are removed if the unused visible energy (hits not associated with muons, primary protons, or vertex energy) in the detector (central tracker plus the EM calorimeter, ECAL, and hadronic calorimeter, HCAL, sections) is less than $50 \mathrm{MeV}-$ effectively too little to yield a reconstructable $\pi^{0}$. Events are also removed if the unused visible energy in the detector exceeds 2500 MeV since a single, produced $\pi^{0}$ is highly unlikely to have that much energy. The signal loss incurred by these two cuts is less than $1 \%$.

Events selected in this stage are subjected to $\pi^{0}$ pattern recognition and reconstruction. The goal of pattern recognition is to identify events that have two and only two gamma showers that belong to the decays of singly produced neutral pions, $\pi^{0} \rightarrow \gamma \gamma$. For each event, the unused hit clusters are gathered that are coincident to within 25 ns with the muon track, have ionization that exceeds low-activity cross-talk, and lie within the scintillator or the ECAL regions. Hit clusters found in the X view that are close in polar angle with respect to the vertex (the best-fit origin of the muon track plus any proton tracks), but
can be separated in radial distance from the vertex, are grouped into photon-conversion candidates. Then, for each candidate, clusters in the U and V views that are consistent among the three views are added. Photon candidates must have clusters in at least two views in order to enable their directions to be reconstructed in three dimensions. Additional steps are taken when the above procedures yield a single-photon or three-photon configuration. Searches are repeated using tighter polar angle criteria, and using the U view and V view instead of the X view for the start of search. For three-photon situations, a spurious shower can sometimes be identified on the basis of a straight-line fit to the positions of each cluster plus the event vertex. These extra steps are estimated to have resolved the candidacy of $9.6 \%$ of the final signal sample [64].

The photon reconstruction is then finalized for events that are deemed to have exactly two photon showers. The position, direction, and energy of a photon-conversion shower are determined by the clusters that have been assigned to each of the candidate photons. The photon direction is reconstructed from the cluster energy-weighted slopes in each view. The photon vertex is defined using the closest cluster to the event vertex on the photon direction axis. The photon energy is reconstructed by calorimetry using calibration constants determined by detector response simulations. An event is removed if the conversion distance for the more energetic photon (denoted as $\gamma_{1}$ ) is less than 14 cm ( $\sim 0.28$ conversion lengths) from the primary vertex. This cut is based on optimization of sample purity; it mitigates against nontracked charged particle hits close to the vertex that can fake an EM shower.

Figure 2 shows a data event whose topology satisfies the requirements for retention in the signal sample. Clearly discernible are the final-state muon, the pair of photonconversion showers (one in the scintillator, the other in the ECAL), and a proton that ranges to stopping. The interaction vertex is nearly devoid of extra ionization hits.


FIG. 2. Data event candidate for subsample (2) of the signal channel (1). The neutrino enters from the left and interacts in the scintillator to produce a muon, proton, and two photonconversion showers. The horizontal and vertical axes labels show the module and strip numbers, respectively. The color (online) linear scale shows energy deposited in the strips.

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The final-state muon traverses the scintillator, ECAL, and HCAL regions and exits in the direction of the MINOS near detector. The photon converted in the ECAL, $\gamma_{1}$, is more energetic $(618 \mathrm{MeV})$ than the photon $\gamma_{2}$ in the scintillator $(140 \mathrm{MeV})$. The display is obtained with the Arachne event viewer [66]. The photons and proton of this event are energetic compared to selected-sample norms, but the uncluttered appearance of the event topology is typical of the sample.

## C. The $\pi^{0}$ signal

The photon energies $E_{\gamma 1}, E_{\gamma 2}$ and their opening angle from the vertex, $\theta_{\gamma \gamma}$, are used to calculate the two-photon invariant mass $M_{\gamma \gamma}$,

$$
\begin{equation*}
M_{\gamma \gamma}^{2}=2 E_{\gamma_{1}} E_{\gamma_{1}}\left(1-\cos \theta_{\gamma \gamma}\right) \tag{4}
\end{equation*}
$$

The invariant mass $M_{\gamma \gamma}$ is studied in the data and in the MC simulation. The data distribution is examined after background subtraction, where the background consists of selected events whose out-of-nucleus final-state particle content is not consistent with reaction (1). The background estimation is constrained by fitting the data to four different sideband distributions (see Sec. IV). The peak position for the data distribution was found to be $137.8 \pm 2.6 \mathrm{MeV}$, while the peak position for the MC distribution was $130.3 \pm 1.6 \mathrm{MeV}$. Correction factors were then applied to the absolute scales for electromagnetic energy, separately in the data and the MC, that adjusted the $M_{\gamma \gamma}$ peaks to match the nominal $\pi^{0}$ mass $(134.97 \mathrm{MeV})$. These tunings of EM absolute energy scales have been applied to the distributions of Fig. 3 and to all subsequent figures.

Figure 3 shows the $M_{\gamma \gamma}$ distribution for the data (solid circles), for the signal distribution as predicted by the MC [upper histogram in Fig. 3(a) and histogram of Fig. 3(b)], and for the background distribution predicted by the MC [lower histogram in Fig. 3(a)]. The distributions shown are normalized to the POT of the data exposure, with the normalization of the estimated background [Fig. 3(a), lower histogram] constrained via fitting to four data sidebands as described in Sec. IV. The peak in the data coincides with the $\pi^{0}$ mass. The signal plus background as estimated by the MC follows the shape of the data [Fig. 3(a), upper histogram]. The estimated signal, which is shown in Fig. 3(b) after background subtraction, distributes broadly and skews toward lower invariant mass. The skew reflects the loss of hits from EM showers that travel transversely to the detector's longitudinal axis.

## D. Estimation of $E_{\nu}, Q^{\mathbf{2}}$, and $W$

The neutrino energy is calculated as the sum over the energies of all reconstructed final-state particles, plus the vertex energy (Sec. III A), plus additional extra energy within the event time slice that is associated with the event and is not included in the particle reconstructions,

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FIG. 3. The $M_{\gamma \gamma}$ distribution from data [solid circles in (a)] and as predicted by the MC (histograms, both plots). In (a), the MC prediction for signal plus background is seen to be in good agreement with the data. In (b), the signal distribution as estimated by the MC is replotted. It distributes broadly about the $\pi^{0}$ mass and exhibits a skew towards lower mass values (b).

$$
\begin{equation*}
E_{\nu}=E_{\mu}+E_{\pi^{0}}+\sum T_{p}+E_{\mathrm{vtx}}+E_{\mathrm{extra}} \tag{5}
\end{equation*}
$$

Here, $E_{\pi^{0}}$ is assembled from the gamma vector momenta and the $\pi^{0}$ rest mass (see Sec. VIII), $T_{p}$ is the kinetic energy of a reconstructed proton, $E_{\mathrm{vtx}}$ is the vertex energy, and $E_{\text {extra }}$ is the extra energy that is left over after reconstruction and is not included in $E_{\mathrm{vtx}}$.

Extra energy is the sum of energies represented by all unused hit clusters that are time coincident with the muon track. The hits include clusters rejected during $\pi^{0}$ reconstruction, clusters lying close to the muon but not used by the tracking algorithm, and all unused clusters that lie within a radius of 30 cm about the primary vertex. Also included in $E_{\text {extra }}$ are ionizations remote from the primary vertex that are induced by scatters of slow neutrons released by breakup of the struck nucleus. Figure 4 shows the energy $E_{\text {extra }}$ that is included in the neutrino estimation, for events without [Fig. 4(a)] and with [Fig. 4(b)] reconstructed protons. The distributions have similar shapes and are adequately described by the GENIE-based simulation.

The sum of final-state energies as in Eq. (5) is used to reconstruct the neutrino energy, $E_{\nu}$. Distributions of


FIG. 4. Distributions of event energy $E_{\text {extra }}$ that is included in the neutrino energy estimation-see the text. The distributions are for events without a reconstructed proton (a), and for events with at least one reconstructed proton (b).
reconstructed $E_{\nu}$ minus true $E_{\nu}$ as evaluated by the reference MC are found to be symmetric and have means consistent with 0 [64]. This $E_{\nu}$ estimator is used throughout the analysis for both the data and simulation. Figure 5 shows the distributions of calculated $E_{\nu}$ for events without a reconstructed proton [Fig. 5(a)] and for events that have at least one reconstructed proton [Fig. 5(b)]. The errors shown with the data points are purely statistical. The simulation predictions are in rough agreement with the data for both distributions. The neutrino energy distribution for the full signal sample has an average neutrino energy of 5.25 GeV (4.4 GeV for events with $E_{\nu}$ below 10 GeV ). The rms width of the MC (reconstructed true) distribution for $E_{\nu}$ is 0.42 GeV .

The four-momentum-transfer squared, $Q^{2}$, and the hadronic invariant mass, $W$, are calculated as follows:

$$
\begin{equation*}
Q^{2}=-\left(k-k^{\prime}\right)^{2}=2 E_{\nu}\left(E_{\mu}-\left|\vec{p}_{\mu}\right| \cos \theta_{\mu}\right)-m_{\mu}^{2} \tag{6}
\end{equation*}
$$

and

$$
\begin{equation*}
W^{2}=(p+q)^{2}=M_{N}^{2}+2 M_{N}\left(E_{\nu}-E_{\mu}\right)-Q^{2} \tag{7}
\end{equation*}
$$

where $k, k^{\prime}$, and $p$ are the four-momenta of the incident neutrino, the outgoing muon, and the struck nucleon,


FIG. 5. Neutrino energy distributions for selected events that do not have, or do have, reconstructed proton(s) in the final state [plots (a) and (b), respectively].
respectively, while $q=k-k^{\prime}$ is the four-momentum transfer and $M_{N}$ is the nucleon mass. For estimation of $W$ as in Eq. (7) there is an underlying assumption that a single struck nucleon is initially at rest. It is useful at this point to distinguish between $W_{\exp }$ and the "true W" of the simulation. The hadronic mass, $W_{\text {exp }}$, is calculated-in both data and simulation-using observable quantities. The variable $W_{\text {exp }}$ can be estimated for every signal event using reconstructed lepton kinematic variables and calculating with Eqs. (6) and (7). This can be done whether or not a final-state proton was reconstructed. The rms widths of MC (reconstructed, true) distributions for the variables $Q^{2}$ and $W_{\text {exp }}$ are $0.02 \mathrm{GeV}^{2}$ and 0.09 GeV , respectively, and both distributions are sharply peaked at 0 [64].

## E. Final selections: the signal sample

Selections that finalize the signal sample of the analysis are now imposed. As previously stated, the muon polar angle with respect to the beam axis is required to be $<25^{\circ}$. Additionally the reconstructed neutrino energy is limited to $1.5 \mathrm{GeV}<E_{\nu}<20 \mathrm{GeV}$. The lower bound ensures good acceptance for muons to be matched in MINOS, and the upper bound removes events that are unlikely to be CC single- $\pi^{0}$ production. An upper bound of 1.8 GeV is placed on $W_{\exp }$ in order to obtain a sample that is enriched in baryon-resonance events. Finally, it is required that $M_{\gamma \gamma}$, calculated according to (4), lies in the range $60 \mathrm{MeV}<M_{\gamma \gamma}<200 \mathrm{MeV}$. The signal region around the nominal $\pi^{0}$ mass is shown by the pair of vertical delimiters in Fig. 3.

The final selected signal sample contains 6110 data events. The purity of the signal sample is calculated using events that have a vertex inside the fiducial volume and a muon track reconstructed using the MINOS near detector. The sample purity is $51 \%$ implying that 3115 of the selected data events are actual occurrences of Eq. (1). The reconstruction efficiency for signal events is $8.4 \%$. The requirement that the muon of an otherwise valid signal event gives a MINOS-matched track accounts for an efficiency loss of nearly $51 \%$.

Background reactions remaining in the selected signal sample are classified into three categories having sizable statistics, plus an "other" category consisting of an odd lot of strange particle production, CC antineutrino events, and neutral current events. The largest background category consists of events that have zero $\pi^{0}$ s but one or more charged mesons emerging from the target nucleus (with $\pi^{+}$ being most probable); this "charged meson(s)" category gives $57 \%$ of the background. Of the produced charged mesons in this category approximately $37 \%$ subsequently interact in the detector and initiate $\pi^{0} \mathrm{~s}$. Other mesons of this category scatter and/or travel transversely in the detector, giving hit clusters that mimic gamma conversions. Backgrounds also arise from events that have at least one $\pi^{0}$
that emerges from the struck nucleus, accompanied by additional pions or kaons. This " $\pi^{0}+$ meson $(\mathrm{s})$ " category accounts for $20 \%$ of background events. A third background arises with events that have zero mesons, but have proton and neutron-induced ionizations that give a fake $\pi^{0}$ in reconstruction. This "zero meson" category accounts for $19 \%$ of backgrounds; the remaining $4 \%$ of background is the other contribution.

## IV. BACKGROUND RATES FROM FITTING TO SIDEBANDS

The overall normalizations for the background categories are determined by fitting the MC simulation to multiple different background-rich samples whose events have topological and kinematic resemblances to the selected signal events. The data events that populate these "sideband samples" do not appear in the selected signal sample as they do not satisfy one or more of the event selections. The individual sidebands have different mixtures of the background categories so that a combined fit has sensitivity to the normalizations of all categories.

The analysis uses four separate sideband samples to achieve good constraints on the normalizations of the three main background categories [64]. The first two sidebands consist of events whose $M_{\gamma \gamma}$ values fall below or above the $\pi^{0}$ invariant mass selection indicated by the vertical bars displayed in Figs. 3 and 6. Figure 6 shows that for the low-side $M_{\gamma \gamma}$ sideband, the MC simulation-before fittinghas the zero-meson and charged-meson(s) background categories contributing at comparable rates, with the $\pi^{0}+\operatorname{meson}(s)$ category giving a much smaller contribution. For the high-side $M_{\gamma \gamma}$ sideband, the zero-meson and chargedmeson(s) are still the leading categories; however the $\pi^{0}+\operatorname{meson}(\mathrm{s})$ category has a larger presence. The $M_{\gamma \gamma}$ low-side and high-side sidebands contain 1424 and 2309 data events, respectively.

A third sideband contains events tagged as having Michel EM showers from end point $\pi^{+}$decays. This Michel sideband (1803 events) has abundant charged-meson(s) background but also contains a sizable $\pi^{0}+$ meson(s) contribution. There are very few zero-meson events in the Michel sideband. In the fourth sideband, the muon is accompanied by a second reconstructed track that has a low likelihood score for the proton hypothesis (3933 events). This low-proton-score sideband is also predicted to be mostly composed of charged meson(s) plus $\pi^{0}+$ meson(s) backgrounds but with their apportionment differing somewhat from that in the Michel-tag sideband. The estimated compositions of the latter sidebands after background tuning by the fit-to-sidebands described below are displayed as component histograms in Figs. 7(a) and 7(b).

The fit to the sidebands is carried out as a single chisquared fit of the simulation prediction to the data, where the data consist of binned values of $\gamma \gamma$ invariant masses


FIG. 6. Data versus MC $\gamma \gamma$ invariant mass distributions before (a) and after (b) revision of the background normalizations to match the values obtained by the overall fit to the four data sideband samples. The low-side and high-side $M_{\gamma \gamma}$ sidebands, denoted by arrows in the plot, are fitted together with the Micheltag and low-proton-score sidebands, while the data of the signal region between 60 and 200 MeV are excluded. The MC agreement with the data in the signal region improves dramatically as the result of constraining the backgrounds in the sideband regions.
taken over all four sidebands. The fit procedure varies the normalizations of the charged-meson(s), zero-meson, and $\pi^{0}+\operatorname{meson}(\mathrm{s})$ categories to minimize the total $\chi^{2}$ in all sidebands at once. The MC estimates of other background and of residual signal content of the sidebands are held fixed. The $\chi^{2}$ uses 50 bins for each of the Michel-tag and low-proton-score sidebands, 30 bins for the high-side $M_{\gamma \gamma}$ sideband, and 6 bins for the low-side $M_{\gamma \gamma}$ sideband. The total of 136 bins, each with good data statistics, yields robust constraints on the background normalizations. The fit improves the $\chi^{2}$ in each of the four sidebands. The $\chi^{2}$ over all sidebands changes from 956.3 to 216.8 for 133 degrees of freedom.

Figure 6 shows the $\gamma \gamma$ invariant mass distribution for the data compared to the GENIE MC prediction before and after the sideband fit. The sideband fit uses the low-side and high-side $M_{\gamma \gamma}$ regions (designated via arrows), but not the signal region between 60 and 200 MeV .


FIG. 7. Comparisons of $M_{\gamma \gamma}$ distributions of data (solid points) to predictions of the reference MC (histograms), for the Micheltag sideband [Fig. 7(a)] and for the low-proton-score sideband [Fig. 7(b)]. The MC predictions are brought into agreement with the data by adjustments of component background levels as determined by the simultaneous fit to all four sidebands of the analysis.

The $\chi^{2}$ for the entire distribution, with 49 degrees of freedom, changes from 656.6 [Fig. 6(a)] to 88.2 [Fig. 6(b)]. Figures 7(a) and 7(b) show that the sideband fit also brings the MC background model into agreement with the data distributions of the Michel-tag and low-proton-score sidebands. The normalization scale factors obtained by the fit are $0.97 \pm 0.03$ for the charged-meson(s) background category, $0.72 \pm 0.07$ for the $\pi^{0}+$ meson(s) category, and $0.42 \pm 0.04$ for the zero-meson category. The fit tends to reduce the zero-meson category in all four sideband samples [64]. This trend may indicate that the generation and/or subsequent visible scatters of neutrons is overpredicted in the simulation for these quasielasticlike zero-meson events [67].

## V. DETERMINATION OF CROSS SECTIONS

Calculation of the flux-integrated differential cross section per nucleon for kinematic variable $X$ (such as $p_{\mu}, \theta_{\mu}$, and $Q^{2}$, in bins of $i$, proceeds as follows [13-15]:

$$
\begin{equation*}
\left(\frac{d \sigma}{d X}\right)_{i}=\frac{1}{T_{n} \Phi} \frac{1}{\Delta X_{i}} \frac{\sum_{j} U_{i j}\left(N_{j}^{\mathrm{data}}-N_{j}^{\mathrm{bkg}}\right)}{\epsilon_{i}} \tag{8}
\end{equation*}
$$

where $T_{n}$ is the number of nucleons in the fiducial volume, $\Phi$ is the integrated flux, $\Delta X_{i}$ is the bin width, and $\epsilon_{i}$ is the selection efficiency and acceptance. The unfolding function, $U_{i j}$, calculates the contribution to true bin $i$ from reconstructed bin $j$, where the $j$ th bin contains $N_{j}^{\text {data }}$ data candidates and $N_{j}^{\mathrm{bkg}}$ of estimated background events. Calculation of $\sigma\left(E_{\nu}\right)_{i}$, the cross section per bin $i$ of neutrino energy, is carried out using an expression that can be obtained from Eq. (8) by dropping $\Delta X_{i}$ and changing $\Phi$ to $\Phi_{i}$, the neutrino flux for the $i$ th bin of $E_{\nu}$.

The factor $\left(N_{j}^{\text {data }}-N_{j}^{\mathrm{bkg}}\right)$ in Eq. (8) denotes the binned contents of the background-subtracted data distribution, obtained by subtracting the MC background prediction after sideband tuning from the data. The backgroundsubtracted data are subjected to an iterative unfolding procedure [68]. The procedure takes detector resolution smearing into account and corrects reconstructed values to true values according to mappings, $U_{i j}$, determined by the reference simulation. For most of the kinematic variables measured in this work, the unfolding matrices are close to diagonal and the effects of unfolding are minor. The largest spread about the matrix diagonal is exhibited by pion kinetic energy; the matrices for $Q^{2}$ and $E_{\nu}$ also show some population in neighboring off-diagonal bins. For all variables, the biases versus the number of unfolding iterations were studied. The biases, after the four iterations that were used, were always found to be much smaller than other systematic uncertainties.

The bin-by-bin efficiency $\epsilon_{i}$ is estimated using the simulation. For muon momentum, for example, the efficiency rises from $3 \%$ below $2 \mathrm{GeV} / \mathrm{c}$ to $10 \%$ at $3.5 \mathrm{GeV} / \mathrm{c}$ and then remains roughly constant, reflecting the limited tracking acceptance $\left(\theta_{\mu}<25^{\circ}\right)$ for lower-momentum muons in the MINOS near detector. As previously stated, the overall reconstruction efficiency for signal events is $8.4 \%$.

The analysis uses current determinations of the integrated and differential neutrino fluxes over the $E_{\nu}$ range 1.5 to 20 GeV for the NuMI low-energy beam mode [48]. The neutrino flux in bins of $E_{\nu}$ is given in Supplemental Material [69]. The value for the integrated flux $\Phi$ is $2.55 \times 10^{-8} \nu_{\mu} / \mathrm{cm}^{2} /$ POT. Also required is the number of target nucleons inside the fiducial volume: $T_{n}=3.17 \times$ $10^{30}$ nucleons.

## VI. SYSTEMATIC UNCERTAINTIES

The cross-section measurements use the reference simulation to estimate selection efficiencies, detector acceptance and resolutions, distribution shapes of backgrounds, and the neutrino flux. All of these estimations introduce systematic uncertainties. While there are many individual sources of uncertainty, each can be associated with one of five general categories. Detector response uncertainties arising with particle energy scales, particle tracking, and
detector composition comprise the first category. Categories two, three, and four are, respectively, uncertainties with simulation modeling of neutrino interactions, uncertainties residing with the GENIE model for FSI involving produced hadrons, and neutrino flux uncertainties. A fifth category is reserved for other uncertainties which include uncertainties with background fitting and subtraction, uncertainty inherent to the unfolding procedure, and uncertainty arising from inclusion of $2 p 2 h$ events into the simulation.

Most sources of uncertainty for the present work were encountered by previous MINERvA studies of $\mathrm{CC}(\pi)$ interactions and their treatment is described in publications [13-15]. The systematic uncertainty from the neutrino flux is described in detail in Refs. [48,70]. The systematic uncertainties for many quantities are evaluated by shifting the relevant parameter in the simulation within its $\pm 1 \sigma$ band and producing a new simulated event sample. Cross sections are then remeasured using an ensemble of such alternate-reality samples, and a covariance matrix is formed from the results. The procedure is repeated for each systematic source; details are given in [14]. On cross-section plots to follow, the errors shown represent the square roots of covariance diagonal entries. The full correlation matrices are given in Supplemental Material [69].

Systematic uncertainty compositions that are fairly typical of all cross-section determinations of this work are shown in Fig. 8, for muon momentum [Fig. 8(a)] and for pion kinetic energy [Fig. 8(b)]. In the lowest momentum or energy bin of either distribution, the detector response category gives the largest fractional error; this is the result of reduced, variable acceptance for low-momentum muons to intercept MINOS [Fig. 8(a)], and increased uncertainty with tracking of low-energy EM showers [Fig. 8(b)]. At higher momentum or energy bins the detector response uncertainty diminishes and stays with a range of $3 \%$ to $7 \%$. For most bins of either distribution, finite data statistics gives a larger uncertainty than does any single systematic category. The principal interaction cross-section model (GENIE) contributes fractional uncertainties in the range of $4 \%$ to $10 \%$ for both variables, reflecting sizable uncertainties that arise with the modeling of neutrinonucleon pion production. Uncertainties arising from the FSI model are associated with parameters in the GENIE framework, the largest of which is a $50 \%$ uncertainty associated with pion charge-exchange cross sections [71]. The uncertainty propagated to a cross-section measurement from the FSI-parameter uncertainties varies from $2 \%$ to $7 \%$.

Uncertainties assigned to the neutrino flux are subject to constraints provided by the background normalization procedure, enabling this analysis to have flux-related uncertainties that are somewhat smaller than for most other MINERvA measurements. The neutrino flux uncertainties in Fig. 8 are roughly constant, hovering at or slightly below $5 \%$ across all bins of either distribution. Tables of measured


FIG. 8. Systematic error composition of the total fractional uncertainties for the differential cross sections of muon momentum (a) and of $\pi^{0}$ kinetic energy (b). Component histograms show the statistical errors (dotted line) and the contributions from the five systematic-error categories (see the text). Detector response (blue solid-line histograms) gives the leading uncertainty in the lowest bins of either distribution, while statistical uncertainty is leading in higher bins.
cross-section values and of the systematic uncertainty composition for each bin of each measurement are provided in Supplemental Material to this paper [69].

## VII. MUON KINEMATICS OF $\mathrm{CC}\left(\boldsymbol{\pi}^{\boldsymbol{0}}\right)$

## A. Muon momentum

Figure 9 shows the differential cross section for muon momentum, $d \sigma / d p_{\mu}$. For this distribution and others to follow (see Sec. III A), the cross section is flux integrated over the range $1.5 \mathrm{GeV} \leq E_{\nu}<20 \mathrm{GeV}$ and the muon production angle is restricted to $\leq 25^{\circ}$. The general shape of $d \sigma / d p_{\mu}$ is strongly influenced by the $\nu$ flux spectrum; the cross section peaks between 2.0 and 2.5 GeV and falls off rapidly as $p_{\mu}$ increases from $3.0 \mathrm{GeV} / \mathrm{c}$ to beyond $6.0 \mathrm{GeV} / \mathrm{c}$. The GENIE-based simulation (solid-line curve) is in agreement with the data in all bins to within $1 \sigma$ of the statistical plus systematic error on the data distribution. The muon differential cross section can be
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FIG. 9. The flux-integrated muon-momentum differential cross section, $d \sigma / d p_{\mu}$, for muons with $\theta_{\mu} \leq 25^{\circ}$. Data are shown as solid circles; the inner (outer) error bars correspond to statistical (total) uncertainties. The GENIE-based simulation matches the data distribution in shape and absolute rate to within $1 \sigma$ in all bins.
compared to similar measurements for $\nu_{\mu}-\mathrm{CC}\left(\pi^{+}\right)$and $\bar{\nu}_{\mu}-\mathrm{CC}\left(\pi^{0}\right)$ obtained with MINERvA-see Fig. 7 of Ref. [13]. The spectral peaks are observed to nearly coincide for all three data sets, even though the absolute cross sections are rather different. Differences in crosssection magnitudes are to be expected, since the three pion production channels differ in their isospin compositions and in the role played by interferences between vector current and axial vector current contributions, the latter being constructive in the $\nu_{\mu}$ channels and destructive in the $\bar{\nu}_{\mu}$ channel.

The GENIE-based prediction in Fig. 9 is less sensitive to pion FSI than are the corresponding predictions for $\nu_{\mu}-\mathrm{CC}\left(\pi^{+}\right)$and $\bar{\nu}_{\mu}-\mathrm{CC}\left(\pi^{0}\right)$. This is because a full account ing of FSI alterations of pionic charge and multiplicity predicts approximately equal feed-in versus feed-out rates for channel (1). The GENIE prediction in Fig. 9 is shown with FSI included; the FSI model has contributed a downwards shift in the predicted distribution of $\sim 4 \%$ in the vicinity of the peak.

In Fig. 9 and in many subsequent figures, the prediction of the NuWro event generator [9] is shown with FSI effects included, providing comparison with the GENIE-based reference simulation as well as an independent prediction for the data. In NuWro the baryon resonance region extends to $W<1.6 \mathrm{GeV} ; \Delta(1232)$ production is calculated using the Adler model [72,73], with nonresonant pion production added incoherently as a fraction of DIS, where DIS is formulated according to the Bodek-Yang model [55]. For its FSI treatment, NuWro uses the Salcedo-Oset model [74] in a hadronic cascade formalism that includes nuclear-medium corrections. Figure 9 shows that for $d \sigma / d p_{\mu}$, NuWro predicts a cross section that is lower through the peak than either the data or the GENIE-based prediction; however it comes into agreement with the data and with GENIE at higher $p_{\mu}$ values. Similar trends with NuWro versus GENIE predictions occur

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FIG. 10. Composition of $d \sigma / d p_{\mu}$ in terms of three neutrinointeraction processes, as predicted by the simulation. Resonance production yielding $\Delta(1232)^{+} \rightarrow \pi^{0} p$ is dominant (uppermost gray-shade histogram), followed by single-pion nonresonant production (bottom histogram), and production of $N^{*}$ states of masses above the $\Delta^{+}$.
for other differential cross sections of this analysis, reflecting differences in cross-section strengths assigned to $\Delta$ and nonresonant DIS production.

Events of the signal channel (1) originate in one of three general processes: (i) pion production via the $\Delta$ (1232) resonance, (ii) pion production via other baryon resonances, and (iii) nonresonant pion production. (As previously noted, coherent single-pion production is absent.) Figure 10 shows the relative strengths of these processes as predicted by the GENIE-based simulation. Here, $\Delta^{+}$production is predicted to account for $\approx 52 \%$ of the rate (uppermost histogram in Fig. 10); production and decay of higher-mass $N^{*}$ resonances gives an additional $\approx 29 \%$, with nonresonant single-pion production accounting for the remaining $19 \%$ of the total rate. The relative contributions among the three processes remain nearly constant over the muon-momentum range of this $d \sigma / d p_{\mu}$ measurement.

## B. Muon production angle

Figure 11 shows the differential cross section as a function of polar angle, $\theta_{\mu}$, with respect to the neutrino beam. The $\theta_{\mu}$ distribution peaks near $9^{\circ}$ and then decreases gradually at larger angles. The cross section is obtained from $0^{\circ}$ to $25^{\circ}$, at which point the diminishing acceptance for muons to reach MINOS precludes further measurement. In contrast to the good match between data and the reference MC observed for $d \sigma / d p_{\mu}$, significant differences are found with the generator predictions $d \sigma / d \theta_{\mu}$. The GENIE-based prediction overshoots the data below $5^{\circ}$, while it consistently falls below the data for angles exceeding $12^{\circ}$. Milder forms of these shape disagreements can be discerned in $d \sigma / d \theta_{\mu^{+}}$for $\bar{\nu}_{\mu}-\mathrm{CC}\left(\pi^{0}\right)$ (data lie below the GENIE prediction at very forward angles) and in $d \sigma / d \theta_{\mu^{-}}$ for $\nu_{\mu}-\mathrm{CC}\left(\pi^{+}\right)$(data fall-off at large angles is more gradual than predicted), see Fig. 6 of Ref. [13]. Since the muon


FIG. 11. Differential cross section for muon production angle, $d \sigma / d \theta_{\mu}$. The GENIE prediction (sum of histograms) exceeds the data for $\left(\theta_{\mu}<5^{\circ}\right)$ and falls below the data for $\left(\theta_{\mu}>12^{\circ}\right)$. The relative contributions from $\Delta^{+}$production, higher-mass $N^{*}$ production, and nonresonant $\pi^{0}$ production show little variation over the measured range of $\theta_{\mu}$.
production angle is measured with a rms resolution 2 mrad , the discrepancy suggests a shortfall with the neutrinocarbon scattering model. These disagreements are related to data-MC disagreements observed at very low and at high $Q^{2}$, as discussed in Sec. IX.

## VIII. PION KINEMATICS OF $\mathrm{CC}\left(\boldsymbol{\pi}^{0}\right)$

Using the three-momenta of the two EM showers in each event, the neutral pion momentum and kinetic energy are calculated as $\vec{p}_{\pi^{0}}=\vec{p}_{\gamma_{1}}+\vec{p}_{\gamma_{2}}$ and $T_{\pi^{0}}=E_{\pi^{0}}-m_{\pi^{0}}$, where $E_{\pi^{0}}=\sqrt{\left|\vec{p}_{\pi^{0}}\right|^{2}+m_{\pi^{0}}^{2}}$. The differential cross section for pion kinetic energy, $d \sigma / d T_{\pi^{0}}$, is shown in Fig. 12. The


FIG. 12. Pion kinetic energy differential cross section. Predictions are shown for GENIE and NuWro; GENIE predictions are displayed without and with pion FSI modeling (dotted-line and solid-line distributions, respectively). The data exhibit a slightly stronger enhancement at $T_{\pi^{0}} \sim 100 \mathrm{MeV}$ than is predicted by the simulation. The enhancement is attributed to intranuclear charge-exchange reactions $\pi^{+} \rightarrow \pi^{0}$ fed by $\pi^{+}$ production from $\nu_{\mu}-\mathrm{CC}\left(\pi^{+}\right)$interactions [11].


FIG. 13. Composition of $d \sigma / d T_{\pi^{0}}$ in terms of underlying pion FSI scattering processes invoked by the GENIE model. The FSI processes are displayed as component histograms lying above the no-scattering case (bottom histogram). The data and MC are shown area normalized to each other.

NuWro prediction lies everywhere below the data, while GENIE generally reproduces the data normalization and shape. The data exhibit a rate enhancement in the region $T_{\pi^{0}}<100 \mathrm{MeV}$, a trend predicted by the pion FSI model used by GENIE [58]. This effect was predicted by U. Mosel using a GiBUU simulation of MINERvA data [11]. The effect is attributed to event feed-in to channel (1) resulting from intranuclear charge-exchange involving $\pi^{+} \rightarrow \pi^{0}$; the latter processes are fueled by the large $\nu_{\mu}-\mathrm{CC}\left(\pi^{+}\right)$cross section. It is also pointed out that the $T_{\pi^{0}}$ region around 240 MeV incurs depletion from pion absorption.

Figure 13 shows the relative contributions of component pion scattering processes to $d \sigma / d T_{\pi^{0}}$ as invoked by the GENIE FSI model. The data and MC distributions are shown area normalized to each other in order to elicit shape differences. According to the GENIE model, $\pi^{0}$ inelastic scattering is the most probable FSI; however charge exchange $\pi^{+} \rightarrow \pi^{0}$ contributes very significantly in the


FIG. 14. Differential cross section for pion angle relative to the neutrino beam direction. The GENIE-based simulation reproduces the data distribution over the full angular range, with inclusion of pion FSI giving a small improvement.
lowest bins of kinetic energy. Figure 13 suggests that modest tuning of the pion FSI cross section may improve the data-MC agreement.

The differential cross section for pion production angle with respect to the beam direction is shown in Fig. 14. The cross section shows most $\pi^{0}$ s to be produced in the Lab forward hemisphere with angles around $35^{\circ}$ being most probable. The GENIE-based simulation is in good agreement with the data over the entire angular range while NuWro falls below the data in the region of the peak. The peak location and shape for $d \sigma / d \theta_{\pi^{0}}$ are similar to those obtained previously for $\nu_{\mu}-\mathrm{CC}\left(\pi^{+}\right)$and $\bar{\nu}_{\mu}-\mathrm{CC}\left(\pi^{0}\right)$ [13]; this outcome was anticipated by a GiBUU simulation of MINERvA 's low-energy exposure [11].

## IX. $\sigma\left(E_{\nu}\right), d \sigma / d Q^{2}, \mathrm{AND} d \sigma / d W_{\exp }$

Figure 15 shows the channel (1) cross section as a function of neutrino energy, $\sigma\left(E_{\nu}\right)$, for events with hadronic invariant mass restricted to $W_{\text {exp }}<1.8 \mathrm{GeV}$. The cross section rises from threshold and its value for $E_{\nu} \geq 6 \mathrm{GeV}$ is described on average by the reference simulation, with NuWro predicting a cross section of similar shape and slightly lower magnitude. According to GENIE, $\Delta$ production is the dominant process for $E_{\nu}<3 \mathrm{GeV}$. Above 4 GeV , the contributions from $\Delta$ production, $N^{*}$ production, and nonresonant pion production become nearly independent of $E_{\nu}$ with relative proportions that are roughly $35: 15: 40$ [64]. The cross section of Fig. 15 can be readily compared to the pion production cross sections reported in Ref. [13]. The measured cross sections become nearly independent of neutrino energy around $E_{\nu}=7 \mathrm{GeV}$. At that point the relative strengths for $\nu_{\mu}-\mathrm{CC}\left(\pi^{0}\right): \nu_{\mu}-\mathrm{CC}\left(\pi^{+}\right)$: $\bar{\nu}_{\mu}-\mathrm{CC}\left(\pi^{0}\right)$ in units of $10^{-40} \mathrm{~cm}^{2}$ per nucleon, according to the measured cross sections, approximately follow the ratios 22:80:19.


FIG. 15. The cross section as a function of neutrino energy for the signal channel (1) whose definition includes $W_{\text {exp }}<1.8 \mathrm{GeV}$. Data (solid circles) are shown with inner (outer) error bars corresponding to statistical (total) uncertainties.

The squared four-momentum transfer from the lepton system, $Q^{2}$, is calculated using Eq. (6), which incorporates both lepton and hadron information [via Eq. (5)]. Figure 16 shows the differential cross section versus $Q^{2}$ determined by this analysis. The data exhibit a rate reduction at $Q^{2}$ below $0.2 \mathrm{GeV}^{2}$ larger than that predicted by the GENIEbased MC. A similar data-MC disagreement was observed at low $Q^{2}$ in the MINERvA $\bar{\nu}_{\mu}-\mathrm{CC}\left(\pi^{0}\right)$ sample [13], and data suppressions at low- $Q^{2}$ for $\Delta$-enriched event samples have been reported by MiniBooNE [22,75] and by MINOS [76]. On the other hand, the reference simulation falls below the data in the region $Q^{2}>0.4 \mathrm{GeV}^{2}$. From Eq. (6) it is clear that these data-MC differences are related to those in Fig. 10 for muons produced at small and large muon polar angles. Note that above $0.2 \mathrm{GeV}^{2}$, NuWro predicts a flatter spectrum than does GENIE and thereby trends more similarly to the data; however the absolute scale for $d \sigma / d Q^{2}$ predicted by NuWro is clearly too low.

For neutrino quasielastic scattering in nuclei such as carbon, it is well known that Pauli blocking produces a turnover of event rate at low $Q^{2}$ [77]. Additionally it is estimated on the basis of RPA and 2 p 2 h that multinucleonnucleon correlations give rise to low- $Q^{2}$ suppression and high- $Q^{2}$ enhancement of rate in the case of CCQE-like scattering [60,78,79]. For $\mu^{+} \Delta(1232)$ channels produced in carbonlike nuclei modeled as a Fermi gas, the effect of Pauli blocking has been calculated in Ref. [80]. Pauli suppression is shown to be confined to $Q^{2}<0.2 \mathrm{GeV}^{2}$ and to $W<1.4 \mathrm{GeV}$. Thus Pauli blocking of $\Delta$ and $N^{*}$ states, which is not included in the reference simulation, plausibly accounts for a modest portion of the low- $Q^{2}$ suppression exhibited by the data in Fig. 16. It is possible that NNcorrelation effects of the kind targeted by RPA calculations may also be present in neutrino-nucleus baryon-resonance production; however this has yet to be demonstrated with a calculation. If the calculation of Ref. [60], which


FIG. 16. Differential cross section, $d \sigma / d Q^{2}$, for channel (1). Data versus GENIE disagreements are evident for $Q^{2}$ near $0.0 \mathrm{GeV}^{2}$ and for $Q^{2}>0.4 \mathrm{GeV}^{2}$. These are related to the data-MC discrepancies observed at small and large muon polar angles.
is for quasielastic scattering (and not baryon-resonance production), is indicative of the strength and $Q^{2}$ dependence of RPA distortion, then RPA in the absence of 2 p 2 h is conceivably capable, in conjunction with Pauli blocking, of generating the data-MC disagreement shown in Fig. 16. Admittedly, the latter scenario is speculative and it may be at odds with a recent theoretical treatment that shows RPA correlations to have reduced effect in calculations based on a realistic nuclear ground state [81]. Another possibility, recognized for many years [80], is that the normalization or functional form of the axial-vector form factors of neutrinonucleus scattering may need modification. Unfortunately there is a dearth of theoretical guidance on how to approach this.

Figure 17 displays the reaction composition of $d \sigma / d Q^{2}$ as predicted by the GENIE-based MC. Baryon-resonance production, especially $\Delta(1232)$ production, is expected to dominate at low $Q^{2}$. However as $Q^{2}$ is increased, nonresonant pion production gradually overtakes the baryonresonance contributions to become the dominant single process for $Q^{2}$ beyond $1.0 \mathrm{GeV}^{2}$.

The relative contributions from $\Delta(1232)$ production, higher-mass $N^{*}$ production, and nonresonant pion production may be expected to change with neutrino energy, with $N^{*}$ and nonresonant contributions becoming more important with increasing $E_{\nu}$. From a different perspective, namely that of weak-interaction hadronic currents, $d \sigma / d Q^{2}$ can be considered as the sum of vector $(V)$ and axial-vector $(A)$ terms, plus the vector times axial vector (VA) interference term which is constructive for the case of CC neutrino scattering. For incident $E_{\nu}$ of 1 to a few GeV , VA interference may exceed the vector contribution; however its presence diminishes very rapidly at higher $E_{\nu}$ [82]. And of course, the phase space available to the final state grows with $E_{\nu}$. The interplay of these effects can give rise to an $E_{\nu}$ dependence for $d \sigma / d Q^{2}$. To elicit such a


FIG. 17. Reaction composition for $d \sigma / d Q^{2}$ for the sample $\nu_{\mu}-\mathrm{CC}\left(\pi^{0}\right)$. Production of $\Delta^{+}(1232)$ (uppermost histogram) dominates the $Q^{2}$ region below $0.4 \mathrm{GeV}^{2}$. Production of higher-mass $N^{*}$ states and of nonresonant $\pi^{0}$, secomes increasingly important at higher $Q^{2}$.
dependence, determinations of $d \sigma / d Q^{2}$ have been carried out for two different energy ranges, namely $1.5<E_{\nu}<$ 4 GeV and $4<E_{\nu}<10 \mathrm{GeV}$. Figure 18 shows $d \sigma / d Q^{2}$ for signal distributions separated into the two energy ranges according to reconstructed $E_{\nu}$. The difference in the rate of falloff with increasing $Q^{2}$ between the two samples is evident.

For the purpose of providing a phenomenological characterization of the data trend, the shapes of $d \sigma / d Q^{2}$ for $0.25<Q^{2}<2.0 \mathrm{GeV}^{2}$ are fitted to an exponential decay function, and the slope parameters, $\Gamma \equiv 1 / Q_{0}^{2}$, are obtained separately for the low- $E_{\nu}$ and high- $E_{\nu}$ ranges. The slope values from the fits are

$$
\begin{align*}
\Gamma_{\text {low } E_{\nu}} & =2.55 \pm 0.26 \mathrm{GeV}^{-2} \quad \text { and } \\
\Gamma_{\text {high } E_{\nu}} & =0.93 \pm 0.21 \mathrm{GeV}^{-2} \tag{9}
\end{align*}
$$

Thus the slope of $d \sigma / d Q^{2}$ flattens as $E_{\nu}$ is raised from a few GeV to multi- GeV values. This trend is roughly reproduced by the GENIE-based reference simulation.

The event distribution of reconstructed hadronic mass, $W_{\text {exp }}$, in the data and as predicted using GENIE or NuWro,


FIG. 18. The $d \sigma / d Q^{2}$ distributions for signal events separated into low (a) and high (b) ranges of $E_{\nu}$. For events with $0.25<Q^{2}<2.0 \mathrm{GeV}^{2}$, the falloff with increasing $Q^{2}$ is approximately exponential and exhibits a steeper slope for the lower- $E_{\nu}$ sample.


FIG. 19. Distribution of $W_{\exp }$, the hadronic mass estimated using Eqs. (6) and (7). The data peak in the vicinity of the $\Delta(1232)^{+}$resonance. The reference simulation roughly matches the data rate, but the predicted shape is shifted towards higher $W_{\text {exp }}$ values relative to the data.
is shown in Fig. 19. The reference simulation and NuWro as well fail to match the data shape in regions below and above the broad spectral peak near the $\Delta(1232)^{+}$resonance. The disagreement is of an unusual kind; the predictions are shifted towards higher $W_{\text {exp }}$ relative to the data. The average displacement is estimated by imposing an overall shift on the prediction and calculating the $\chi^{2}$ with respect to data for each trial. A shift of 20 MeV gives the minimum $\chi^{2}$, improving the $\chi^{2} /$ d.o.f. from 6.9 to 1.8 . As indicated by Eq. (7) from which $W_{\text {exp }}$ is calculated, a portion of this shift may reflect the data-MC disagreement in $Q^{2}$ exhibited by Figs. 16 and 17. According to simulation trials, the $Q^{2}$ offsets can introduce shifts of up to 10 MeV in the distribution of $W_{\text {exp }}$. Equation (7) is founded on the unrealistic assumption that target nucleons are struck while at rest, and this makes the MC predictions sensitive to shortfalls in modeling of interactions on nuclei. The differences in modeling approaches used by the GENIE and NuWro neutrino generators provide some perspective. Alteration of Fermi-motion modeling and inclusion of in-medium modification of the $\Delta(232)$ resonance [83] may generate offsets of $\sim 5 \mathrm{MeV}$. A shift of similar magnitude may also arise from constructive interference between the $\Delta$ and nonresonant pion production amplitudes [16]. The latter mechanism is not treated by either generator.

## X. SUBSAMPLES WITH $\boldsymbol{p} \boldsymbol{\pi}^{0}$ SYSTEMS

To study the production of high-mass $N^{*}$ states and especially of the $\Delta(1232)^{+}$, it is useful to define two subsamples of the analysis signal sample. Events of these subsamples are required to have a leading proton in the final state with kinetic energy $T_{p}>100 \mathrm{MeV}$. For the first subsample, designated hereafter as the $p \pi^{0}$ sample, the signal definition requirement $W_{\exp }<1.8 \mathrm{GeV}$ is left unchanged. For the second subsample, designated as the $\Delta$-enriched


FIG. 20. Cross section versus $M_{p \pi^{0}}$ for the $p \pi^{0}$ sample, requiring $T_{p}>100 \mathrm{MeV}$ with $W_{\text {exp }}<1.8 \mathrm{GeV}$. Curves predicted by the reference simulation show that hadronic FSI tends to broaden and mute baryon-resonance structures. In the $\Delta(1232)^{+}$region however, the data exhibit a resonance shape that is more pronounced than that predicted by either the GENIE or NuWro generators.
subsample the $W_{\text {exp }}$ range is restricted to $W_{\text {exp }}<1.4 \mathrm{GeV}$. The added requirements amount to a new signal definition for each subsample; consequently the procedures used for the signal sample of channel (1), namely background fitting, background subtraction, unfolding, and efficiency correction, have been carried out anew for each subsample. For either subsample, the invariant mass, $M_{p \pi^{0}}$, can be calculated from the reconstructed $\pi^{0}$ and the leading proton. The resolution (rms width) for $M_{p \pi^{0}}$ is 0.10 GeV .

The $p \pi^{0}$ sample prior to background subtraction consists of 3316 events with an estimated signal purity of $54 \%$. Figure 20 shows the differential cross section in $M_{p \pi^{0}}$ obtained with this sample. The GENIE-based simulation is seen to be strongly affected by the FSI model. The presence of multiple baryon-resonance states above 1.4 GeV is clearly discernible in the simulation without FSI (shortdash curve), but the FSI effectively washes away the $N^{*}$ peaks to give a distribution (solid curve) that more nearly describes the data. Both GENIE and NuWro however appear to underestimate the amount of $\Delta^{+}$production that is indicated by the data.

Study of produced $\Delta^{+}$states in isolation from the highermass resonances is carried out using the $\Delta$-enriched subsample. The subsample contains 1522 events and has a $46 \%$ signal purity. Figure 21(a) shows $d \sigma / d M_{p \pi^{0}}$ for the subsample, comparing the data (solid points) to GENIE and NuWro predictions. The GENIE prediction with FSI included (solid curve) falls below the data in the $\Delta$ region $1.15<M_{p \pi}<1.30 \mathrm{GeV}$. However the predicted distribution is broader, which, with respect to total rate, partially compensates for its rate shortfall at the resonance peak. NuWro predicts a smaller cross section at the $\Delta$ peak than does GENIE, but gives a similar cross section at higher invariant mass values.


FIG. 21. Proton-pion invariant mass for the $\Delta$-enriched subsample with $W_{\text {exp }}<1.4 \mathrm{GeV}$ and a reconstructed proton in each event. GENIE with FSI [solid curve in (a)] is broader and falls below the data near the $\Delta^{+}$peak. The $M_{p \pi^{0}}$ distribution is dominated by produced $\Delta^{+}$states [uppermost histogram in (b)]. For both GENIE and NuWro, the predicted shape is shifted towards higher $M_{p \pi}$ relative to the data.

Figure 21(b) shows the subsample in terms of three main interaction categories. According to GENIE, the subsample is composed of $74 \% \Delta^{+}, 10 \%$ higher-mass $N^{*}$ production, and $16 \%$ nonresonant pion production. The $M_{p \pi}$ prediction relative to the data exhibits the same trend towards higher hadronic invariant mass values as observed in Fig. 19.

## A. Polarization of the $\mathrm{p} \boldsymbol{\pi}^{0}$ system

Bubble chamber investigations of neutrino-induced $\Delta^{++}$ production in the few- GeV region of $E_{\nu}$ found the resonant state to be polarized, as evidenced by nonisotropic $\pi^{+}$ angular distributions of $\Delta^{++}$decays measured in the $p \pi^{+}$ rest frame $[18,21,84]$. In general, the decay angular distributions of produced $\Delta$ states are sensitive to production mechanisms and to interferences among different partial wave amplitudes; hence measurements for other $\Delta$ charge states can provide new tests of resonance production models [ $16,36,37]$. In this work the produced $p \pi^{0}$ systems of the $\Delta$-enriched subsample are examined for deviations from isotropic decay that may reflect polarization effects.

The decay angles for $\pi^{0}$ mesons emitted in the $p \pi^{0}$ rest frame are calculated using formalism suited to this measurement $[72,84]$. The coordinate system used in the $p \pi^{0}$ rest frame is established event by event as follows: The four-momenta of the neutrino, muon, pion, and proton are Lorentz boosted into the $p \pi$ rest frame. In that frame, the axes of a right-handed coordinate system are defined with the $z$ axis along the momentum-transfer direction $\left(\vec{p}_{\nu}-\vec{p}_{\mu}\right)$, the $y$ axis along the production plane normal $\left(\vec{p}_{\nu} \times \vec{p}_{\mu}\right)$, and $x$ axis along the direction of the crossproduct $(\hat{y} \times \hat{z})$. The zenith angle $\theta$ is the angle between the pion momentum, $\vec{p}_{\pi}$, and the $z$ axis. The azimuthal angle $\phi$ is the angle between the projection of $\vec{p}_{\pi}$ on the $x-y$ plane and $x$ axis.

The distributions of zenith-angle cosine, $\cos (\theta)$, for the data and as predicted by the GENIE-based simulation and by NuWro are shown in Fig. 22(a). For the predictions by the neutrino generators, the two-body decays of baryon


FIG. 22. Angular distributions in zenith-angle cosine (a) and azimuthal angle $\phi$ (b) for pion emission in the $p \pi^{0}$ rest frame, from the data (solid circles), the GENIE-based simulation (solid lines), and the prediction of NuWro (dashed lines). The generator models assume $\Delta^{+}$resonances to decay isotropically in their rest frames and they roughly describe the data; however mild discrepancies are discernible in the forward hemisphere $[\cos (\theta)>0.0]$ and with respect to left-right (a)symmetry about $\phi=180^{\circ}$.
resonances, including $\Delta^{+} \rightarrow p+\pi^{0}$, are generated isotropically in their rest frames. (An exception is made in MINERvA's GENIE implementation for $\Delta^{++}$decays, where polarization [for $\cos (\theta)$ ] at $50 \%$ of the strength prescribed by the Rein-Sehgal model is used [14].) According to the GENIE, $p \pi^{0}$ systems that do not experience FSI would distribute fairly isotropically. In the presence of FSI however a peak develops in the backwards hemisphere for $\cos (\theta)<-0.5$ as shown by the dotted-line and solid-line curves in Fig. 22(a). A similar trend is predicted by NuWro, and indeed the data show a mild upswing as $\cos (\theta)$ approaches -1.0 . This peaking in the backwards direction is a pion FSI effect. As can be seen in Fig. 12, FSI enhances the low-momentum component of the pion spectrum. When slow pions are boosted to the $p \pi^{0}$ rest frame (the boost direction in the Lab being roughly aligned with the direction of proton momenta), they are projected into the backwards hemisphere where they are antialigned with the protons and with the boost direction which defines the z axis.

In the forward hemisphere the data also show a mild increase in rate at very forward directions; moreover the data lie above the GENIE and NuWro predictions throughout the forward hemisphere. While the absolute event rates in the very forward hemisphere are sensitive to the value set as the proton threshold requirement $\left(T_{p}>100 \mathrm{MeV}\right)$, the tendency for the data to exceed the predictions throughout the forward hemisphere is invariant to changes in this threshold setting. The overall trend in data versus prediction based upon isotropic decay is suggestive of weak polarization excitation of density matrix elements associated with a $Y_{2}^{ \pm 1}(\theta, \phi)$ angular dependence [16].

Figure 22(b) compares the data and simulation prediction for the distribution of azimuthal angle $\phi$. Here, FSI is predicted to give an overall reduction in rate [as is predicted for $\cos (\theta)$ ]; however neither FSI nor details of the subsample selection introduce any angular distortions; the shape of the $\phi$ distribution is predicted to be flatconsistent with isotropy-by both event generators. To zeroth order the data are also isotropic; however there are deviations at the $<2 \sigma$ level that suggest a left-right asymmetry relative to the X-Z plane, with $0^{\circ}<\phi<$ $180^{\circ}$ defining the right side of the plane.

In summary, the data-when compared to simulations based upon isotropic decays for produced $\Delta^{+}$states-show modest deviations from isotropy with respect to both $\cos (\theta)$ and $\phi$. These anisotropies can be quantified using two test statistics, namely a forward/backward, data-versus-MC, ratio of ratios to express asymmetry about the plane at $\cos (\theta)=0.0$, and a left/right, data-versus-MC, ratio of ratios to express asymmetry with respect to $\phi=180^{\circ}$. These ratios are akin to the asymmetry parameters reported by the ANL bubble chamber experiment [18]. Let $N_{F(B)}$ and $N_{L(R)}$ designate the number of events in forward (backward) and left (right) hemispheres, respectively,
and let $R_{F / B}$ and $R_{L / R}$ designate $N_{F} / N_{B}$ and $N_{L} / N_{R}$, respectively; then
$\frac{R_{F / B}^{(\mathrm{data})}}{R_{F / B}^{(\text {GENIE })}}=1.99 \pm 0.41, \quad \frac{R_{L / R}^{(\mathrm{data})}}{R_{L / R}^{(\text {GENIE })}}=1.36 \pm 0.27$.
Here, the prediction of the GENIE-based simulation is taken to represent isotropy, and anisotropy is gauged in terms of a ratio-of-ratios deviation from unity. Thus the data exhibit a $\sim 2 \sigma$ anisotropy that favors pion emission into the forward hemisphere, and a $\sim 1 \sigma$ anisotropy that favors emission into the left hemisphere with $180^{\circ}<\phi<360^{\circ}$. Comparing to the asymmetries observed with $\Delta^{++}$states produced in events of the ANL bubble chamber experiment (with $0.5<E_{\nu}<6 \mathrm{GeV}$ ) [18], the anisotropies in $\cos (\theta)$ and $\phi$ are of comparable magnitudes and of opposite and same sign, respectively.

## XI. CONCLUSIONS

The measurements of this work provide a multifaceted view of semiexclusive $\nu_{\mu}-\mathrm{CC}\left(\pi^{0}\right)$ scattering on carbon. Differential cross sections are presented for muon variables $p_{\mu}$ and $\theta_{\mu}$ and for $T_{\pi}$ and $\theta_{\pi}$ of the final-state $\pi^{0}$. The per event $E_{\nu}$ is estimated from muon kinematics plus a sum over calorimetric measures of final-state hadronic energy, and cross sections are thereby determined as functions of $E_{\nu}, Q^{2}$, and $W_{\text {exp }}$. From the signal sample, events having a proton above the reconstruction threshold $\left(T_{p}>100 \mathrm{MeV}\right)$ are selected. Differential cross sections in proton- $\pi^{0}$ invariant mass are reported for events with $W_{\exp }<1.8 \mathrm{GeV}$, and also for events with the further selection $W_{\text {exp }}<1.4 \mathrm{GeV}$. Using events of the latter subsample, the angular emission of pions in the $p \pi^{0}$ rest frame is examined for evidence of polarization, and differential cross sections for zenith-angle $\cos (\theta)$ and for azimuthal angle $\phi$ are obtained.

The $\nu_{\mu}$ - $\mathrm{CC}\left(\pi^{0}\right)$ cross-section distributions are compared to a modified GENIE simulation used by other recent MINERvA studies and to the predictions of NuWro. They are also compared to previous MINERvA measurements of the single-pion production channels $\nu_{\mu}-\mathrm{CC}\left(\pi^{+}\right)$and $\bar{\nu}_{\mu}-\mathrm{CC}\left(\pi^{0}\right)$ [13-15]. Summary tables for the cross-section measurements that may facilitate data comparisons and phenomenological study are available in Supplemental Material [69].

These measurements promote the development of neutrino interaction models that will, one day, encompass the physics that underwrites CC single-pion production from nuclear targets. Since the $\operatorname{CC}\left(\pi^{0}\right)$ channel occurs at a significant rate in long baseline neutrino detectors, and since the differential cross sections obtained span the working $E_{\nu}$ range used by all accelerator-based $\nu$ oscillation experiments, the results reported here will enable continued improvement in precisions achievable with oscillation measurements.

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