## NHDS: THE NEW HAMPSHIRE DISPERSION RELATION SOLVER

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In collisionless astrophysical plasmas, waves and instabilities are well modeled by the linearized Vlasov–Maxwell equations, which have non-trivial solutions only when the complex frequency  $\omega$  solves the hot-plasma dispersion relation. *NHDS* (New Hampshire Dispersion relation Solver) is a numerical tool written in Fortran 90 and first introduced by Verscharen et al. (2013) to solve this dispersion relation under the assumption that the plasma background distribution is a gyrotropic drifting bi-Maxwellian for each species j,

$$f_{0j}(v_{\perp}, v_{\parallel}) = \frac{n_j}{\pi^{3/2} w_{\perp j}^2 w_{\parallel}} \exp\left(-\frac{v_{\perp}^2}{w_{\perp j}^2} - \frac{\left(v_{\parallel} - U_j\right)^2}{w_{\parallel j}^2}\right),\tag{1}$$

in a cylindrical coordinate system aligned with the direction of the background magnetic field  $B_0$ , where  $n_j$  is the density,  $w_{\perp}$  ( $w_{\parallel}$ ) is the perpendicular (parallel) thermal speed with respect to  $B_0$ , and  $U_j$  is the field-aligned drift speed. All floating-point quantities use double precision.

The NHDS code closely follows the formulation of the hot-plasma dispersion relation laid out by Stix (1992). It uses a Newton-secant method to identify those frequencies at which there are non-trivial solutions to the wave equation,

$$\begin{pmatrix} \epsilon_{xx} - \frac{k_z^2 c^2}{\omega^2} & \epsilon_{xy} & \epsilon_{xz} + \frac{k_\perp k_z c^2}{\omega^2} \\ \epsilon_{yx} & \epsilon_{yy} - \frac{k^2 c^2}{\omega^2} & \epsilon_{yz} \\ \epsilon_{zx} + \frac{k_\perp k_z c^2}{\omega^2} & \epsilon_{zy} & \epsilon_{zz} - \frac{k_\perp^2 c^2}{\omega^2} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = 0,$$
(2)

based on an initial guess for  $\omega$ , where  $\epsilon$  is the dielectric tensor, E is the vector of the electric-field Fourier amplitudes, c is the speed of light, and  $\mathbf{k} = (k_{\perp}, 0, k_z)$  is the wavevector. The initial guess defines the plasma mode that the code follows in  $\mathbf{k}$ . The Newton-secant method converges if the absolute value of the determinant of the matrix in Equation (2) is less than a user-defined value. All frequencies are given in units of the proton gyro-frequency  $\Omega_p$  and all length scales in units of the proton inertial length  $d_p$ .

For each of the up to ten plasma components j, the user defines the temperature anisotropy  $T_{\perp j}/T_{\parallel j}$  with respect to  $\boldsymbol{B}_0$ , the value of  $\beta_{\parallel j} \equiv 8\pi n_j k_{\rm B} T_{\parallel j}/B_0^2$ , the relative charge  $q_j/q_{\rm p}$ , the relative mass  $m_j/m_{\rm p}$ , the relative density  $n_j/n_{\rm p}$ , and the normalized drift velocity  $U_j/v_{\rm A}$ , where  $k_{\rm B}$  is the Boltzmann constant,  $v_{\rm A}$  is the proton Alfvén speed, and  $T_{\parallel j}$  is the temperature parallel to  $\boldsymbol{B}_0$ . Furthermore, the ratio  $v_{\rm A}/c$  and the angle of propagation  $\theta$  are user-defined parameters.

The calculation of  $\epsilon_{ik}$  entails the evaluation of the modified Bessel function  $I_m(\lambda_j)$  of the first kind and the plasma dispersion function  $Z(\zeta)$ , where  $\lambda_j \equiv k_{\perp}^2 w_{\perp j}^2 / 2\Omega_j^2$ , and  $\zeta$  is a dimensionless complex number. For the evaluation of  $I_m$ , NHDS applies the recursion method supplied by the Numath Library (Clenshaw 1962). It determines the maximum order  $m_{\text{max}}$  of  $I_m$  as the smaller of either a user-defined limit or as the number for which  $I_{m_{\text{max}}}(\lambda_j)$  is less than a user-defined value. NHDS evaluates  $Z(\zeta)$  following Poppe & Wijers (1990) by computing the complex error function  $w(\zeta) = Z(\zeta)/i\sqrt{\pi}$  through one of the following methods, depending on the value of  $|\zeta|$ : a power series, the Laplace continued fraction method, or a truncated Taylor expansion. This combined method is faster than alternative approaches and calculates  $w(\zeta)$  to an accuracy of 14 significant digits for almost all  $\zeta$ .

*NHDS* determines the polarization of the wave solutions as the ratios  $E_y/E_x$  and  $E_z/E_x$  from Equation (2), which translate to ratios of the magnetic-field amplitudes through Faraday's law. In addition, as described by Verscharen & Chandran (2013) and Verscharen et al. (2016), *NHDS* calculates the relative wave energy  $W_k$  and the Fourier amplitudes of the fluctuations in density, bulk velocity, and pressure. The code also calculates the contribution  $\gamma_j$  to the total growth/damping rate Im( $\omega$ ) from each species j as described by Quataert (1998).

For a given wave solution, *NHDS* can determine the value of the self-consistent fluctuating distribution function on a user-defined Cartesian grid in velocity space as described by Verscharen et al. (2016). *NHDS* saves the fluctuating distribution function in *HDF5* files and creates an *XDMF* file for visualization with programs like *ParaView*. This calculation entails the calculation of the Bessel function  $J_m(k_{\perp}v_{\perp}/\Omega_j)$  of order m, which *NHDS* performs through a polynomial Chebyshev approximation. The maximum order  $m_{\text{max}}$  for  $J_m$  is determined in the same way as  $m_{\text{max}}$  for  $I_m$  in the calculation of  $\epsilon_{ik}$ , except that  $m_{\text{max}}$  for  $J_m$  is evaluated for each  $v_{\perp}$ .

Figure 1 shows the dispersion relations of Alfvén/ion-cyclotron (A/IC) and fast-magnetosonic/whistler (FM/W) waves in parallel and perpendicular propagation as well as some of their polarization properties determined with *NHDS*.

The code is publicly available for download (Verscharen & Chandran 2018, Codebase: https://github.com/danielver02/NHDS).

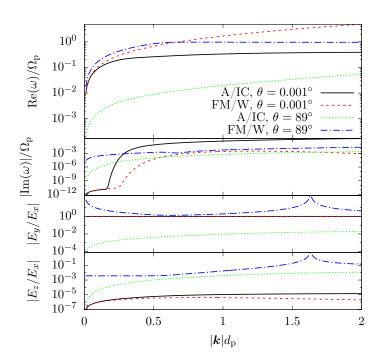


Figure 1. Dispersion relations for the A/IC and FM/W waves in parallel ( $\theta = 0.001^{\circ}$ ) and perpendicular ( $\theta = 89^{\circ}$ ) propagation. The panels show from the top to the bottom: the normalized real part of the frequency, the normalized damping rate, the ratio  $|E_y/E_x|$ , and the ratio  $|E_z/E_x|$  as functions of  $|\mathbf{k}|$ .

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## REFERENCES

Clenshaw, C. W. 1962, Mathematical Tables: Chebyshev	Verscharen, D. & Chandran, B. D. G. 2013, ApJ, 764, 88
Series for Mathematical Functions, Vol. 5,	Verscharen, D., Bourouaine, S., Chandran, B. D. G., et al.
H. M. Stationery Office	2013, ApJ, 773, 8
Poppe, G. P. M. & Wijers, C. M. J. 1990, ACM	<ul> <li>Verscharen, D., Chandran, B. D. G., Klein, K. G., et al. 2016, ApJ, 831, 128</li> <li>Verscharen, D. &amp; Chandran, B. D. G. 2018, NHDS, v1.0, Zenodo, doi:10.5281/zenodo.1227265</li> </ul>
Trans. Math. Softw., 16, 38	
Quataert, E. 1998, ApJ, 500, 978	
Stix, T. H. 1992, Waves in plasmas, American Institute of	
Physics	