

1 On the experimental generation of focusing wave groups on following and adverse sheared 2 currents in a wave-current flume

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7 Abstract

8 Focused waves are often used in physical and numerical studies as a representative condition for
9 extreme waves or as a means to generate very steep and breaking waves at a prescribed location in
10 space and time. They have also been combined with depth-varying currents in investigations of
11 incipient wave breaking, wave breaking induced energy dissipation, and wave-current induced loads
12 on marine structures. A focused wave is created when all the components in a transient wave group
13 come into phase. In the past, linear wave theory and iterative methodologies coupled with the linear
14 Doppler-shifted dispersion relationship have been suggested to account for the presence of a current
15 and achieve the required phase and amplitude focusing. In the majority of cases linear or constant
16 steepness spectra are used, which compared to the measured or theoretical spectra like JONSWAP
17 (Joint North Sea Wave Project), Gaussian and Pierson-Moskowitz (PM) can be termed as unrealistic.
18 The effectiveness of these methodologies also decreases as the nonlinearity increases and therefore in
19 most studies either weakly nonlinear conditions are employed or the focus location is determined
20 empirically. Here, an iterative methodology is suggested which can focus waves of any height at a
21 predetermined temporal and spatial location even for wave groups propagating on a strong following
22 or adverse current. An experimental apparatus developed to generate relatively stable sheared velocity
23 profiles is also described. The depth varying profile of the resulting currents diverges from that of
24 classical wind driven currents and comes closer to profiles measured in field sites important for the
25 deployment of, for instance, tidal and wind energy converters. The methodology is successfully
26 applied to wave groups travelling on still water, following and adverse currents, and the results
27 presented refer to linear, weakly nonlinear and strongly nonlinear focused waves generated for a range
28 of realistic target spectra. The capability to generate wave groups with the same amplitude spectrum

29 at a fixed location for a variety of flow conditions - still water, following and adverse sheared currents
30 – is also illustrated.

31 **Introduction**

32 The constructive interference at a certain point in space and time of numerous wave components of
33 varying frequency and amplitude results in the generation of a large focused wave. When simulating
34 extreme hydrodynamic conditions in a laboratory facility, such a wave possesses comparative
35 advantages. It is significantly higher and steeper than any other wave within the propagating wave
36 group, it occurs at a predefined point in space and time, and it represents an event with a large return
37 period which may take a long time to reproduce within a random wave sequence. Hence the
38 deterministic nature of focused waves makes them suitable candidates for design waves in
39 experimental and analytical investigations of wave loading on marine structures (Tromans et al.,
40 1991).

41 Oceanic field measurements (Taylor and Williams, 2002; Christou and Ewans, 2014) have confirmed
42 previous theoretical considerations (Lindgren, 1970) linking the occurrence of the largest waves to
43 propagating waves groups. On many occasions wave groups will co-exist with currents. In fact, the
44 interaction of wave groups with currents is among the physical mechanisms proposed to explain the
45 formation of rogue waves (Bretherton and Garrett, 1968; Peregrine 1976). For relatively shallow
46 water and relatively strong tidal flows there will be a considerable difference to the flow resistance
47 from the seabed upwards, leading to a sheared current. The presence of surface shear has been
48 associated with variations in the steepness and shape of wave crests, and incipient breaking (Banner
49 and Song, 2002). The combination of very steep and potentially overturning waves with sheared
50 currents entails the local formation of very fast flow regions and thus the potential exposure of marine
51 structures to unusually high forces.

52 Despite its importance, however, experimental investigations on the interaction of waves and
53 especially focused wave groups with sheared currents are scarce. Challenges associated with the
54 generation under laboratory conditions of sheared currents, strongly nonlinear focused waves, and
55 both combined seem to be the main reason for this shortage in measurements. The simultaneous
56 generation of waves and depth-varying currents requires the minimum of interaction between the

57 flow-shaping apparatus and the generated/propagating waves. Usually PVC plates, layers of polyether
58 foam, and honeycomb blocks are used to condition and straighten the current. The required depth-
59 varying velocity profile is provided through the combination of solid and perforated PVC plates of
60 varying heights extending from the bed up to about 10cm below the free surface (Swan et al., 2001;
61 Yao and Wu, 2005). Therefore, the selection of testing conditions is restricted to waves for which the
62 underlying wave kinematics are not drastically disturbed by the flow conditioner/straightener. Wave
63 reflection is an additional problem, mainly for waves generated on adverse currents.

64 For the focusing of experimental unidirectional wave groups three main approaches can be identified.
65 For the simplest approach, linear wave theory is used to calculate the phases of the wave components
66 at the inlet required to produce a wave group focusing at a preselected time and location in the flume
67 (Rapp and Melville, 1990). Empirical methods have also been proposed, where the numerical input
68 for the wavemaker is corrected through an iterative process using surface elevation measurements at
69 the focal point. The Fourier transformation of the elevation time history is used together with the
70 target spectrum to calculate the corrected phases for the new input and the scheme is repeated until all
71 wave components come into phase at focus (Chaplin, 1996). The latter approach was extended to
72 include amplitude modification, and more recently a self-correcting method employing a potential
73 flow solver to replace the physical re-production of the wave groups required for the iterations was
74 suggested (Schmittner et al., 2009; Fernandez et al., 2014). In a different method, the wave board
75 signal required for the generation of a focused wave was computed by backward integration of the
76 Zakharov equation (Shemer et al., 2007). For focusing wave groups in the presence of currents, the
77 empirical method of Chaplin (1996) has been modified to include a linear dispersion relation
78 accounting for the presence of a current with a constant shear (Yao and Wu, 2005).

79 Although effective for small amplitude waves, the efficiency of these methods reduces as the
80 nonlinearity of the wave group increases. As a result, for increasing focused wave amplitudes, the
81 focal point is shifted in both space and time and the quality of focusing reduces considerably.

82 Experimental results with focusing wave groups show that both the location and time of the focused
83 event are dependent upon the nonlinearity of the wave group. Compared to linear prediction, a
84 downshift of up to 1.6m for the focal point and a time-shift of up to 0.6sec for the focusing time were

85 reported (Baldock et al., 1996). It is also noted that, with some exceptions, previous methods have
86 been used with unrealistic target spectra such as top-hat or constant slope; in this work spectra with
87 the same shape as one of the measured/theoretical spectra available in the existing literature (for
88 example JONSWAP (JS), Pierson-Moskowitz (PM) and Gaussian (GS)) are referred to as realistic.
89 The present work combines and adds to previous knowledge and proposes a new methodology and
90 experimental apparatus which increase control over the generation of unidirectional focusing wave
91 groups on following and adverse sheared currents in a wave-current flume. Previously, this approach
92 has been used successfully to generate focused waves on still water and on a following current using a
93 Gaussian target spectrum with peak frequency of 0.6Hz (Stagonas et al., 2014). In the current article,
94 the methodology and the experimental apparatus are described in detail: the cases considered increase
95 substantially to include adverse currents, four different targets including spectra with high frequency
96 tails, and two different peak frequencies per spectrum resulting in wave components travelling on
97 shallow, intermediate and deep water conditions. The proposed methodology is described first before
98 presenting the flow-shaping apparatus that allows the creation of sheared currents with a significantly
99 reduced effect on wave generation. This is followed by an example application and results.

100 **Wave focusing methodology**

101 For wave generation in flumes using a desired target spectrum, linear or 2nd order wave theory is
102 usually combined with the appropriate transfer functions to calculate the required displacement of the
103 wavemaker. However, for strongly nonlinear waves the interaction between the wave components of a
104 propagating wave group reshapes the amplitude spectrum in a way which is not predicted by either
105 linear or 2nd order wave theory, e.g. (Baldock et al., 1996). Complexities increase further for wave
106 groups on currents as, in addition to wave-wave interactions, wave-current interactions also occur and
107 affect the evolution of both the amplitude and phase spectrum (Dingemans, 1997). Empirical
108 methodologies use surface elevation measurements to produce a corrected input signal for the
109 wavemaker and through trial and error overcome the limited capacity of existing wave-current
110 theories to accurately predict the spectral evolution of wave groups on sheared currents (Chaplin,
111 1996). Along these lines, linear wave theory has also been combined with a 2nd order wave-current
112 dispersion equation to generate focused waves on sheared currents (Yao and Wu 2005). As with

113 every other empirical approach, however, corrections were calculated using the fully nonlinear
114 surface elevation signal, which resulted in limited success for strongly nonlinear conditions.
115 In this paper an iterative procedure is described for generating focused wave groups with a target
116 spectrum over currents. The use of a linearised input signal instead of a fully nonlinear wave record
117 distinguishes the proposed methodology from previous attempts. A linearized input signal is the
118 natural choice since the full spectrum of a nonlinear wave group is uniquely defined by its linear part,
119 and since it can be accurately reproduced by any wavemaker employing linear wave theory.
120 Additional key features of the methodology include the use of realistic target spectra and the
121 possibility of using different wave probes for phase and amplitude iterations. For example, for wave
122 groups generated over still water, following and adverse currents, the linearized amplitude spectra can
123 be corrected to match the target spectrum at a point near the wavemaker, while the phase spectra are
124 corrected to zero at a location far away from the wavemaker; hereafter, we refer to the former location
125 as the Amplitude Matching Point (AMP) and to the latter as the Focusing Point (FP).
126 The reasons for separating AMP (Amplitude Matching Point) and FP (Focusing Point) are twofold.
127 Nonlinear wave-wave and wave-current interactions modify the wave group as it travels along the
128 flume. These modifications are usually manifested as energy transfers from lower to higher
129 frequencies and thus attempting to match an amplitude spectrum to the target far away from the
130 wavemaker may entail the generation of a wave group with a non-physical spectrum, especially for
131 high frequency wave components. Therefore, control over the amplitudes and phases of these
132 components is reduced. In addition, increased dissipation of high frequency components along the
133 flume will result in generation of excessively steep high frequency waves leading to premature
134 breaking.
135 At the same time, selecting the AMP to be near the wavemaker and the FP further away provides the
136 opportunity to generate wave groups on variable flow conditions –still water, following/adverse
137 currents with different characteristics - with initially the same linearized amplitude spectrum. Practical
138 experience has shown that setting the AMP at a small distance from the inflow/outflow is beneficial,
139 as it allows for the wave to interact with the flow and develop naturally as it propagates towards the

140 FP. This way, stability and convergence of iterations improves and natural features of the wave
 141 group's interaction with following and adverse currents develop.
 142 The proposed methodology consists of 4 main steps. For the 1st iteration the target spectrum is used as
 143 the initial input to the control system. Here, spectra with and without equilibrium tails such as
 144 Gaussian, JONSWAP and PM are used as targets, but the use of arbitrary spectra is also theoretically
 145 possible. Then, the following steps apply to all subsequent iterations. Firstly for each amplitude
 146 spectrum four wave groups are generated with constant phase shifts of $\Delta\Phi = 0$ (crest focused wave), π
 147 (trough focused wave), $\pi/2$ and $3\pi/2$ (positive and negative slope focused waves). The surface
 148 elevation for each group is measured at the AMP and the FP and the phase-shifted signals are
 149 spectrally decomposed as described later in this section to obtain the linearised signal. The amplitudes
 150 and phases of the linearised spectrum are compared with the target spectrum and a corrected input
 151 spectrum is calculated from:

152

$$a(f_i)_{in}^n = a(f_i)_{in}^{n-1} \times a(f_i)_{tgt} / a(f_i)_{out}^{n-1} \quad \text{Eq. 1}$$

$$\phi(f_i)_{in}^n = \phi(f_i)_{in}^{n-1} - (\phi(f_i)_{tgt} - \phi(f_i)_{out}^{n-1}) \quad \text{Eq. 2}$$

153 where $a(f_i)_{in}^n$ and $\phi(f_i)_{in}^n$ are the input amplitude and phase of the i^{th} frequency of the linearised
 154 spectrum for the n^{th} iteration; $a(f_i)_{in}^{n-1}$ and $\phi(f_i)_{in}^{n-1}$ are the input amplitude and phase of the i^{th}
 155 frequency of the linearised spectrum for the $n^{\text{th}}-1$ iteration; $a(f_i)_{tgt}$ and $\phi(f_i)_{tgt}$ are the target
 156 amplitude and phase for the i^{th} frequency, and $a(f_i)_{out}^{n-1}$ and $\phi(f_i)_{out}^{n-1}$ are the output/measured amplitude
 157 and phase of the i^{th} frequency of the linearised spectrum for the $n^{\text{th}}-1$ iteration. Iterations continue until
 158 the spectral components of the linearised signal come into phase at the FP and their amplitudes at the
 159 AMP match those of the target spectrum to the desired accuracy. For the experiments with waves on
 160 sheared currents considered in the present article, measured amplitudes and phases converged to $\pm 3\%$
 161 of the target within 2 to 3 iterations. As illustrated in the Application Example section below, the
 162 number of iterations required depends on the nonlinearity of the wave, while the accuracy,
 163

164 convergence and overall reliability of iterative focusing techniques is discussed in detail in (Buldakov
165 et al. 2017).

166 Although somewhat laborious, the methodology ensures that through the iterations a ‘self-calibration’
167 of the wavemaker is performed for any flow condition in the flume. As an example, and for a
168 wavemaker controlled by directly specifying the time history of its displacement, one would expect
169 that a different calibration is required for still water, following and adverse currents. Nevertheless,
170 through the iterative correction of the input signal a re-calibration of the wavemaker when transferring
171 from, for example, still water to following flow conditions is no longer required. When the iterations
172 are converged the same target output is achieved for all flow conditions at the AMP and FP, without
173 the need to compute new transfer functions.

174 Spectral decomposition, also known as separation of harmonics, is a powerful technique for isolating
175 harmonic components corresponding to Stokes expansion orders. For example, two wave profile time-
176 histories with a constant phase shift of π corresponding to peak and trough focused wave groups can
177 be used to separate even and odd harmonics in the measured surface elevation. In this separation,
178 second order sub- and super-harmonics co-exist in the same record (even harmonics), and the same is
179 true for linear, 3rd and higher order terms (odd harmonics) (Borthwick et al., 2006; Orszaghova et al.,
180 2014). Hence, the signal decomposed in such a way is difficult to use for the calculation of the
181 corrected input as the linearised part is contaminated by higher order nonlinear terms. More recently,
182 combinations of more than 2 experimental records have been used confirming the possibility of a
183 more effective separation of components either in the time or in the frequency domain (Hann et al.
184 2014; Fitzgerald et al. 2014).

185 In the present study the following 4-wave decomposition is used to separate first and higher-order
186 wave components:

$$\begin{aligned}
 S_0 &= \frac{s_0 + s_1 + s_2 + s_3}{4} \\
 S_1 &= \frac{s_0 - is_1 - s_2 + is_3}{4} \\
 S_2 &= \frac{s_0 - s_1 + s_2 - s_3}{4} \\
 S_3 &= \frac{s_0 + is_1 - s_2 - is_3}{4}
 \end{aligned}
 \tag{Eq. 3}$$

187

188 where s_n are complex spectra of fully nonlinear surface elevation signals with phase shifts $\pi n/2$,
 189 $n=0,1,2,3$; S_0 is the complex spectrum of the 2nd order difference components; and $S_{1,2,3}$ are complex
 190 spectra of nonlinear super-harmonics for 1st (linear), 2nd and 3rd orders.

191 An example of the 4-wave spectral decomposition application to wave groups generated with a
 192 wideband Gaussian target spectrum is illustrated in Figure 1. In particular, the time histories of
 193 surface elevation for $\Delta\Phi = 0, \pi, \pi/2$ and $3\pi/2$ at the AMP and the FP are presented in Figures 1 (a) and
 194 (c). These records are used to decompose the spectrum into its linearised (S_1) and nonlinear ($S_{0,2,3}$)
 195 parts shown in Figures 1 (b) and (d). It is noteworthy that for a 4-wave decomposition the 1st order
 196 (linearized) part includes 5th and higher order terms (see Eq. 3), which however have insignificant
 197 amplitudes.

198 An example of the inverse Fourier reconstructed elevation time histories of the 1st (linearized) order,
 199 2nd order sum and difference and 3rd and higher order waves for wave groups propagating on still
 200 water (solid line), adverse (dashed line) and following current (dotted line) is illustrated in Figure 2.
 201 Spurious and reflected waves are clearly distinguished from the free and bound waves of the focused
 202 crest, and can thus be excluded by selecting an appropriate analysis/observation window for the
 203 iterations. Previously, the contamination of the measured signal with unwanted waves was a
 204 significant challenge for any empirical methodology. Here, the ability to exclude them leads to better
 205 control over the wave group and improves convergence.

206 The effects of wave-current interaction on the evolution of wave groups are evident in Figure 2. In
 207 Figure 2 (a), and for the time instant between ± 3 sec, the highest linearized focused event is seen for
 208 the group propagating on an adverse current (dashed line). Since the linearised amplitude spectra for
 209 all cases illustrated are matched at the AMP with high accuracy, the differences observed can be

210 attributed to the effect of the current on the waves as they propagate from the AMP to the FP. The
211 higher energy content of the linearized part agrees well with the reduced amplitudes observed for the
212 second-order sum and third and higher order components of the same group, as shown in Figures 2 (b)
213 and (d).

214 The long wave trough is also seen to be deeper/shallower for the tests on following/adverse current
215 (dotted/dashed lines), while the amplitude and width of the long wave crest is significantly
216 reduced/increased. Since, however, for these experiments – employing linear wave generation –
217 second-order difference components are inevitably contaminated with spurious long wave
218 components, it is not possible to comment on their connection to the evolution of the linearized part; it
219 is also noted that spurious second-order sum components travel with a celerity smaller than that of the
220 group and they are thus separated at focus, see for instance prior to -3sec in Figure 2 (b). Local
221 nonlinear effects responsible for the formation of bound higher-order components are different for
222 different current conditions and result in discrepancies between higher-order components close to $t =$
223 0sec. Similarly, spurious free and reflected components are also different for different current
224 conditions explaining differences at times after and before the main event at focus for second-order
225 difference components. Nevertheless, a detailed investigation of these aspects of wave-current
226 interaction falls outside the scope of this paper.

227 **Experimental apparatus for generating sheared currents in a wave-current flume**

228 The wave-current flume at UCL is 20m long, 1.2m wide and 1m deep. It is equipped with two
229 Edinburgh Design Limited force-feedback ‘piston-type’ wavemakers, installed at each end of the
230 flume. One wavemaker is used to generate waves, while the other acts to absorb the incoming wave
231 energy. Three impellers placed in parallel recirculation pipes are used to drive the flow, which enters
232 and exits the tank vertically. The flow is discharged in the working section of the flume at a distance
233 of 1m in front of each wavemaker, and through a 0.4m deep settling tank fitted with turning vanes and
234 a honeycomb, Figure 3.

235 At the inflow point of ‘up-welling’ type facilities such as the UCL wave-current flume, the free
236 surface is significantly perturbed and leads to increasingly unsteady conditions as the flow speed
237 increases. This in turn has an undesirable effect on wave generation and propagation. At the outflow,

238 a vortex is formed with a size and intensity which increases with the flow speed and results in further
239 unsteadiness of the free surface upstream. For ‘up-welling’ type facilities a delay in flow development
240 due to the formation of a strong recirculation pattern at the inlet has been previously reported,
241 (Robinson et al. 2015). The flow was shown to acquire the uniform velocity desired for their study
242 about 40m downstream from the inlet. Turbulence intensity increased from about 12% to its
243 maximum 20% within the first 10m, while an additional 10m (20m from inlet) were required for its
244 values to reduce below 15%. Following (Giles et al. 2011), representative real sea conditions can be
245 achieved in experimental facilities only if turbulence intensity in the working section is less than 10-
246 15%. Flow development and test repeatability is also negatively affected by high turbulence levels,
247 and the challenges increase further if sheared currents are to be produced. Nevertheless, sheared
248 currents have more practical applications than uniform or logarithmic currents which are most
249 commonly considered in wave-current experiments.

250 The Pentland Firth is a typical example where strong wave-current interaction phenomena occur and
251 affect the deployment of tidal turbine arrays. If the typical conditions in the Pentland Firth are scaled
252 to the size of the UCL flume, then for a water depth of 0.5m the current will have an equivalent
253 velocity of 0.2m/s near the surface. These calculations are based on ADCP field measurements, which
254 also confirm that the shear in these flows differs from the typical wind generated shear, with a
255 collinear reduction of the velocity with depth (Chatzidou and Karunaratna, 2014). In contrast,
256 profiles for wind generated sheared flows are closer to those used in previous studies, where the
257 current speed is high very close to the free surface and reduces to zero less than half way through the
258 water depth (Swan et al., 2001; Yao and Wu, 2005).

259 Wire mesh structures have been successfully used to generate controlled and reproducible regular
260 waves over uniform currents (Kemp and Simons, 1982; Kemp and Simons, 1983). This concept is
261 developed further in the present study to accommodate the requirement to generate sheared currents.
262 The flow conditioning/shaping apparatus shown in Figure 4 consists of 8cm diameter tubes made of
263 flexible galvanised wire mesh with 5cm holes. A 0.5m long, 1.2m wide and 0.88m deep box section
264 formed of vertically and horizontally placed tubes is positioned on top of the inlet/outlet at each end
265 of the flume. An additional filter layer is installed between the pre-existing honeycomb and the box

266 section to further condition the flow. The filter is made of a 5cm thick, 50Pores Per Inch (PPI)
267 polyester foam enclosed between two anodized aluminum meshes with 1cm holes. The additional
268 flow resistance required to generate a sheared profile is introduced by a triangular 0.88m x 0.88m
269 section made of the same wire mesh tubes and attached to the downstream side of the box section; for
270 simplicity we refer to the triangular section as the flow profiler and the rest of the structure as the flow
271 conditioner.

272 The performance and the required shape of the flow conditioner and the profiler were determined
273 through a series of trial and error tests. A Nortek-AS Vectrino+ ADV was used to acquire flow
274 measurements at 4m (AMP), 6.9m, 8.7m (FP) and 13m from the wavemaker, with the first and the
275 last positions located 1m from the end of the profilers at each end of the flume. To characterize the
276 flow, velocity profiles were measured through the vertical, starting about 2cm from the bed and
277 extending up to about 4cm below the free surface. At each point the mean velocity was calculated
278 from 3min long records acquired at the instrument's maximum sampling frequency of 25Hz, a
279 measurement volume of 9.2mm and acoustic pulse length of 2.4mm; for locations with higher
280 turbulence, such as very close to the inlet, the size of the measurement volume and the length of the
281 acoustic pulse were reduced. Seeding was provided with rutile titanium dioxide pigment and only data
282 with correlation values higher than 85% were considered. Mean velocity values were calculated for at
283 least 4000 samples.

284 For the accurate characterization of the flow the use of higher sampling rates and records with more
285 samples is typically recommended, see for example (Rusello et al., 2006; Chanson et al., 2007). To
286 assess the effect of shorter records on mean velocity and maximum turbulent intensity calculations,
287 9min long records were also acquired for the same flow conditions. Calculations were then conducted
288 using the full record but also 3min long segments and results were compared with those for the
289 original data set. Differences in the mean velocity and maximum turbulent intensity were found never
290 to exceed 0.8% and 1.5%, respectively. A similar analysis but using 30min long records was
291 performed to ensure that the flow measurements reported are not contaminated by transient effects
292 associated with the start and operation of the flume and/or large scale turbulent structures. Indeed, for
293 measurements conducted at least 10min after the initiation of the flow the error in the mean velocity

294 was less than 5%. Nonetheless, for all experiments a minimum delay time of 20min was kept between
295 the initiation of the flow and data acquisition.

296 Velocity profiles at the 4 measuring locations along the flume and the 3 locations across the flume are
297 presented in Figure 5. Sheared currents with surface velocities of $\pm 0.2\text{m/s}$ and $\pm 0.4\text{m/s}$ are seen to
298 remain relatively stable in the working section; in Figure 5 positive and negative velocities correspond
299 to currents propagating in opposition to and following with the waves. In principle, for the profiles
300 acquired along the flume, Figure 5 (a) and (b), higher and lower mean velocities are consistently
301 reported at approximately 5cm below the water surface and about 2cm above the glass floor. More
302 importantly, the relatively good overlap between the velocity profiles indicates that downstream
303 changes in the current are not significant and thus quasi-steady flow conditions are confirmed. A
304 similarly good agreement is reported for the profiles taken across the flume at the focus point for the
305 waves, Figures 5 (c) and (d). Nevertheless, at the AMP (4m/1m from the wavemaker/profiler) and for
306 following currents only, the mean velocities are seen to reduce between about 0.35m and 0.45m from
307 the bed before increasing again to reach maximum at the surface, while the velocity profile for the
308 next measuring station (6.9m from the wavemaker) is seen to acquire a shape similar to the profile
309 observed at FP.

310 Previously, experimental (ADV) flow measurements and CFD simulations for an ‘up-welling’ flume
311 showed a reduction in the mean flow velocity 15cm above the bed, and maximum turbulence intensity
312 of (approx.) 12% for profiles acquired approximately 0.8m from the inlet, (Robinson et al., 2015).

313 For the flow measurements presented in this paper, maximum turbulence intensity reduces from about
314 15% at AMP to about 7% at FP for the fastest following current, and from about 13% (AMP) to 5%
315 (FP) for the fastest adverse current; flow measurements conducted at mid depth (e.g. for Figure 5, -
316 $0.2\text{m} < \text{Distance from the bed} < -0.3\text{m}$) gave the highest turbulence intensity observations at all
317 measuring stations. Reduced turbulence levels for adverse currents are attributed to a greater distance
318 between the flow entry location and the impellers, see also Figure 3.

319 **Application example**

320 Examples of the proposed methodology for the generation of focused waves on still water and over
321 the fastest following and adverse currents with $U_s = 0.4\text{m/s}$ are presented first. A wideband Gaussian

322 spectrum with peak frequency $Fr_p = 0.6\text{Hz}$ (GW06) is selected as the target spectrum, and the AMP
323 and the FP are set at 4m and 8.7m from the wavemaker, respectively. In preliminary tests (not
324 presented here) 8.7m was found to be the furthest from the wavemaker location for which the focused
325 wave was not contaminated by spurious long wave reflections. Therefore, in the figure showing the
326 surface elevation time histories, the focus location is set at 0m with the wavemaker at -8.7m. Waves
327 propagate in the negative direction towards the focus time at 0sec. 7 wave probes were used to
328 measure the wave surface elevation with a sampling frequency of 100Hz, Figure 3.

329 For a fixed water depth of 0.5m the wave components of the target spectrum propagate in the shallow
330 to intermediate depth regime; $k_p d = 0.968$, where k_p is the peak frequency wave number. The
331 wavemaker used operates with a discrete input spectrum with $\Delta Fr = 1/128\text{Hz}$, for a selected return
332 period of 128sec. For clarity, the wave groups are categorised based on the linear sum of the target
333 amplitude components at focus (A). As such, wave groups with $A = 0.025\text{m}$ are referred to as linear,
334 and groups with $A = 0.05\text{m}$ and $A = 0.07\text{m}$ as weakly and strongly nonlinear, respectively. Wave
335 groups with constant phase shifts of $\Delta\Phi = \pi n/2$, with $n = 0,1,2,3$, are created and the 1st order /
336 linearised part of the spectrum is isolated using the decomposition technique described in Section 2.1.
337 Focused waves with A up to 0.07m are produced with an excellent agreement between the target and
338 the measured (linearised) amplitude and phase spectrum at the AMP and the FP as shown by the solid
339 lines in Figure 6 (a), (b) and (c). However, during the experiments it was observed that the generation
340 of wave groups was not possible for waves steeper than $H/L > 0.04$ at the AMP; H/L is calculated for
341 the largest wave in the group using a zero down-crossing method to define the wave period and the
342 dispersion equation to calculate L. Attempts to increase further the wave steepness resulted in waves
343 breaking before the AMP, either in the vicinity of the wavemaker or at the flow conditioner, Figure 7.
344 In tests conducted prior to the installation of the flow-shaping apparatus limiting non-breaking
345 conditions were observed for a focused wave with a wide Gaussian spectrum (GW06) and $A = 0.09\text{m}$.
346 Measured at 4m from the wavemaker (AMP), the steepest wave in this wave group had $H/L = 0.05$.
347 For the tests with adverse currents, focused waves with $A = 0.025, 0.05, 0.07$ and 0.09m were
348 successfully generated, Figure 6 (a). In contrast, for experiments with following currents and still
349 water, the highest focused waves generated had $A = 0.05\text{m}$ and $A = 0.07\text{m}$, respectively. In fact, even

350 for wave groups with moderate amplitude, iterations did not fully converge for frequencies higher
351 than about 0.93Hz, dotted line in Figure 6 (b), and an undershoot was observed for the phases of these
352 components at focus, Figure 6 (c). Nevertheless, the latter was not seen to have a noticeable effect on
353 the behavior of the main event at focus most probably due to the small amplitude of these wave
354 components. Examples of surface elevation records at AMP for $A = 0.05\text{m}$ are given in Figure 6 (d).
355 Attempts to produce focused waves with larger amplitudes either in experiments with following
356 current or with still water resulted in waves breaking near the wavemaker.

357 **Results**

358 The applicability of the proposed methodology was investigated for a range of target spectra and for
359 cases where wave frequency blockage due to adverse currents is predicted (Mei, 1983). Experimental
360 conditions summarised in Table 1 include spectra with $\text{Fr}_p = 0.6\text{Hz}$ and $\text{Fr}_p = 0.9\text{Hz}$, following and
361 adverse currents with $U_s = 0.2\text{m/s}$ and $U_s = 0.4\text{m/s}$. Shapes of target spectra can be seen in Figures 8
362 and 9, and bandwidths for the narrow (GN) and wide (GW) band Gaussian spectra were selected to
363 represent the bandwidth of the JONSWAP (JS) and PM spectra, respectively.

364 Small wave amplitudes usually required 2 iterations, while very good focusing was achieved for the
365 largest waves on all flow conditions within 2 or 3 further iterations. Figure 8 illustrates a range of
366 examples for waves generated on following (dotted lines) and adverse (dashed lines) currents. For
367 clarity, cases with waves on still water are omitted from the figure. In particular, Figure 8 (a) shows
368 the fully nonlinear surface elevation at FP for waves generated on following and adverse current with
369 $U_s = 0.2\text{m/s}$, using a PM target spectrum with $\text{Fr}_p = 0.6\text{Hz}$. For both tests, the linearized amplitude
370 spectra match the target well, and a good quality focus is achieved. Encouraging results are also
371 presented for waves with a narrowband Gaussian spectrum (GN) and $\text{Fr}_p = 0.6\text{Hz}$ on currents with U_s
372 $= 0.4\text{m/s}$ (Figure 8 (b)) for waves with a wideband Gaussian spectrum (GW) and $\text{Fr}_p = 0.9\text{Hz}$ on
373 currents with $U_s = 0.2\text{m/s}$ (Figure 8 (c)) and for waves with a JONSWAP spectrum (JS) and Fr_p
374 $= 0.6\text{Hz}$ on currents with $U_s = 0.2\text{m/s}$ (Figure 8 (d)).

375 For waves approaching breaking, increasing the amplitude of the input signal in smaller increments
376 allowed the effective detection of wave breaking. The latter is better illustrated in Figure 9. For a
377 JS09 target spectrum and still water, up to limiting non-breaking waves ($A = 0.07\text{m}$) were

378 successfully produced; however, breaking was observed for the largest waves on following and
379 adverse currents with $U_s = 0.2\text{m/s}$ (Figure 9 (a)). Increasing the input signal for following currents and
380 moderate wave steepness ($A = 0.05\text{m}$) by 40% ($A = 0.07\text{m}$) led to waves breaking before the AMP,
381 resulting in a diverged amplitude spectrum (Figure 9 (c)). It is, however, noteworthy that despite this
382 and the appearance of irregularities on the following wave train, the focus quality of the main
383 crest/event is not significantly affected (Figure 9 (b)). Results for $U_s = 0.4\text{m/s}$ demonstrated a similar
384 trend but for small and moderate wave amplitudes. Considering the slower adverse current, attempts
385 to increase the target amplitude from $A = 0.05\text{m}$ to 0.06m led to amplitude (Figure 9 (d)) and phase
386 (Figure 9 (e)) convergence but breaking was observed between the FP and AMP.

387 Finally, for experiments looking at wave groups with $Fr_p = 0.9\text{Hz}$ on the strongest ($U_s = 0.4\text{m/s}$)
388 adverse currents, wave blocking of high frequency components at the AMP is reported. Blocking of
389 the higher frequency parts of the wave spectrum is a well-documented aspect of wave-structure
390 interaction and for waves on a depth uniform current it occurs at points where the wave group velocity
391 is equal to that of the current, e.g. (Chawla and Kirby, 2002). Figure 10 (a) and (b) show the
392 amplitude spectra of the linearized part at the AMP for wave groups produced with a wideband
393 Gaussian and a JONSWAP target spectrum, while the phase spectrum at focus is presented in Figure
394 10 (c) and (d). For both cases, blocking is seen to occur for wave components with frequencies greater
395 than about 1Hz.

396 **Conclusions**

397 A methodology to focus steep wave groups on currents has been developed and applied in a
398 specifically designed experimental apparatus for creating sheared currents in a wave-current flume.
399 For the ‘up-welling’ type flume available to this study the use of the suggested flow
400 conditioning/profiling arrangement resulted in relatively stable, collinear sheared velocity profiles for
401 currents of different velocities flowing following or opposing the direction of wave propagation, and
402 with turbulence intensity levels acceptable for studies representative of real sea conditions. The
403 conditions reproduced here resemble those reported for the Pentland Firth, UK (Chatzidou and
404 Karunarathna, 2014).

405 Focused wave groups with the same initial amplitude spectrum near the wavemaker are reproduced
406 for a range of flow conditions, including still water, following and adverse sheared currents with
407 different magnitudes. Target spectra with and without equilibrium tails are used as targets, and phase
408 focusing at the same location is illustrated for all cases including those with strongly nonlinear waves.
409 The well-known problem of spatial and temporal down-shifting of the focus point is thus overcome.
410 Clearly, the need to generate waves with 4 phase shifts for each iteration is a disadvantage, somewhat
411 balanced, however, by the ability to use the corrected input signal to generate the same wave on the
412 same flow conditions in future experimental expeditions involving, for example, structures installed at
413 the focus location.
414 To the best of the authors' knowledge this is the first time that such detailed control of the generation
415 of focusing wave groups on sheared currents has been achieved. Here this work is considered an
416 improvement on existing methodologies for conducting experimental studies into wave-current and
417 wave-current-structure interaction.

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514 **Tables**

515 **Table 1:** Summary of experimental conditions for waves and currents.

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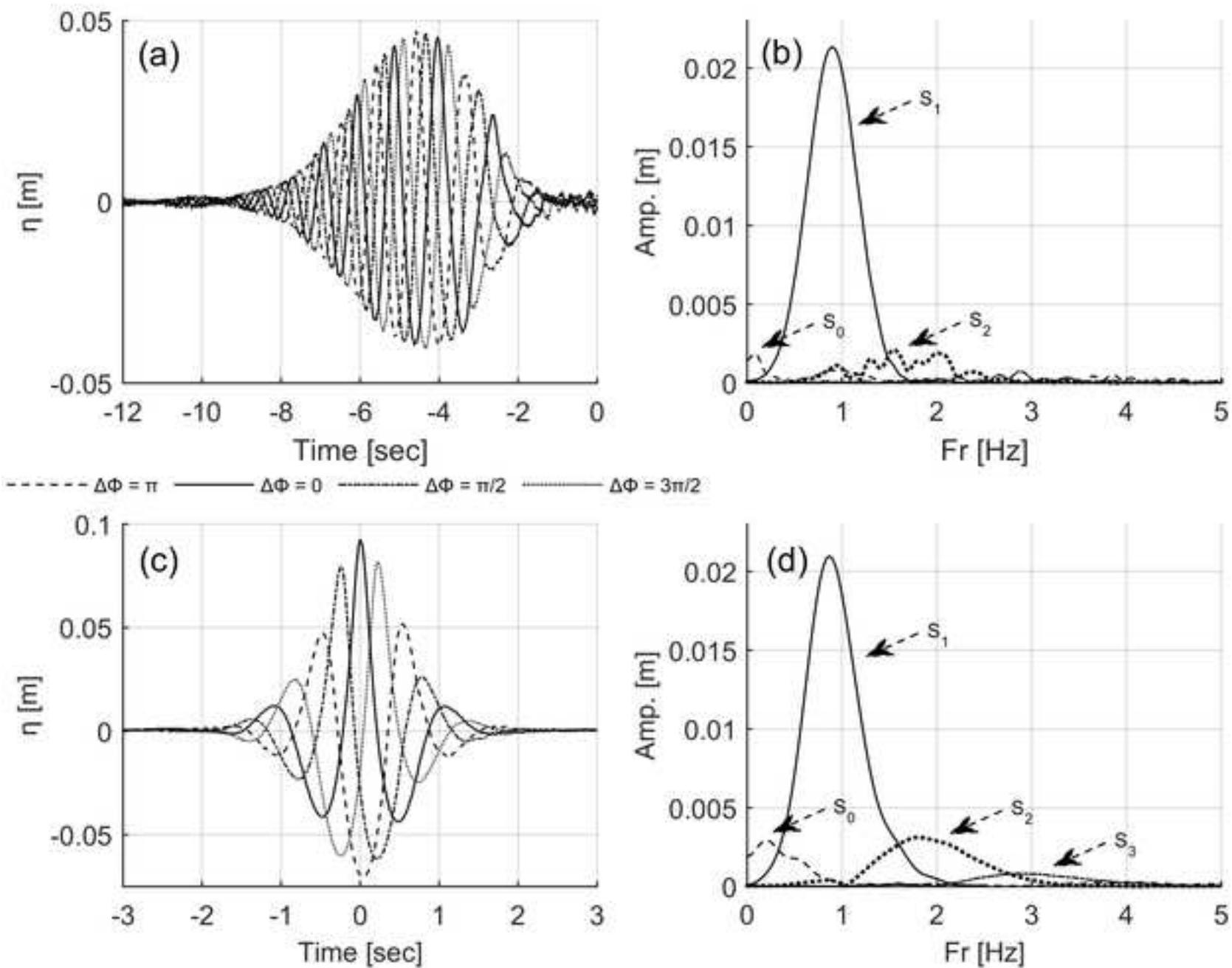
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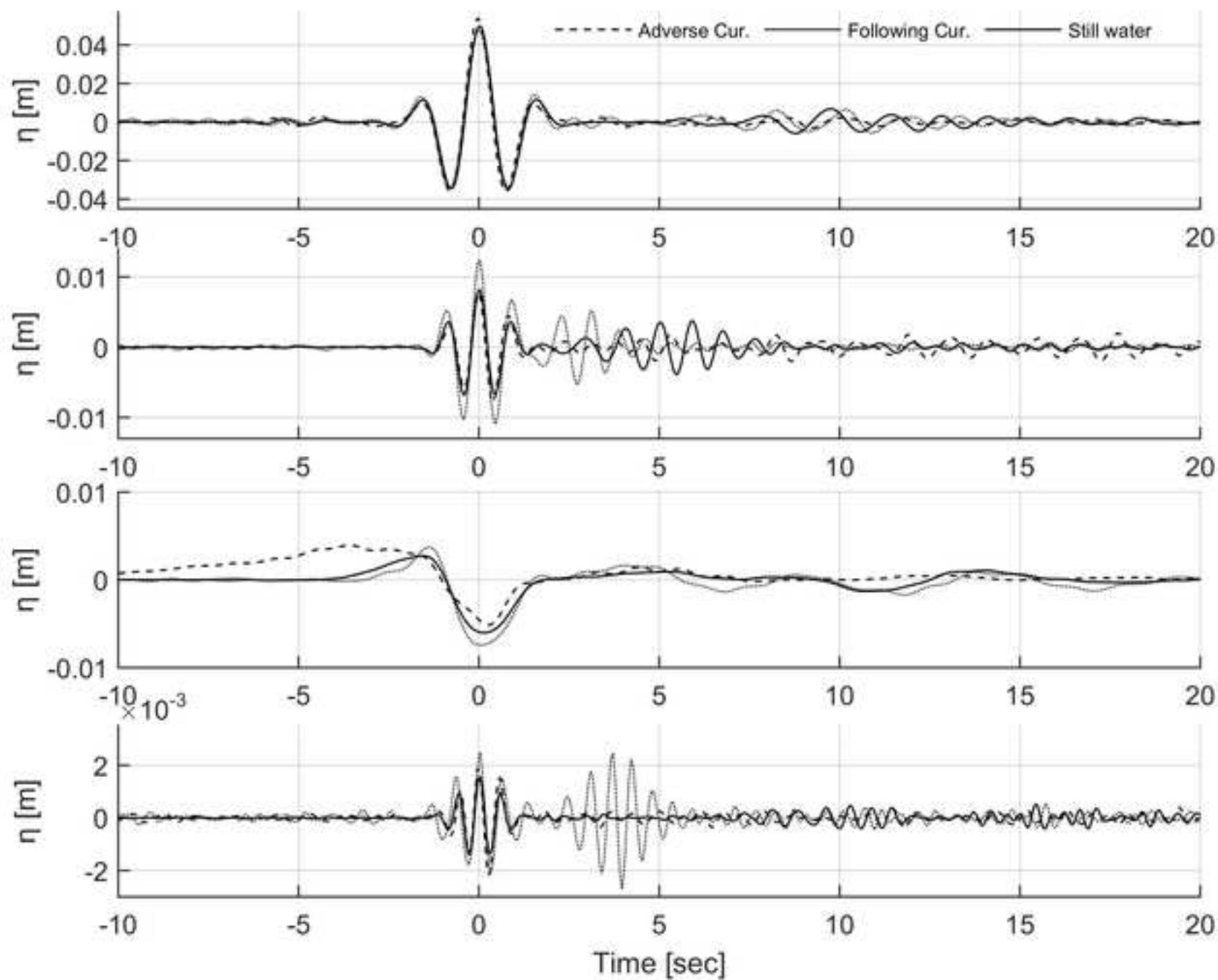
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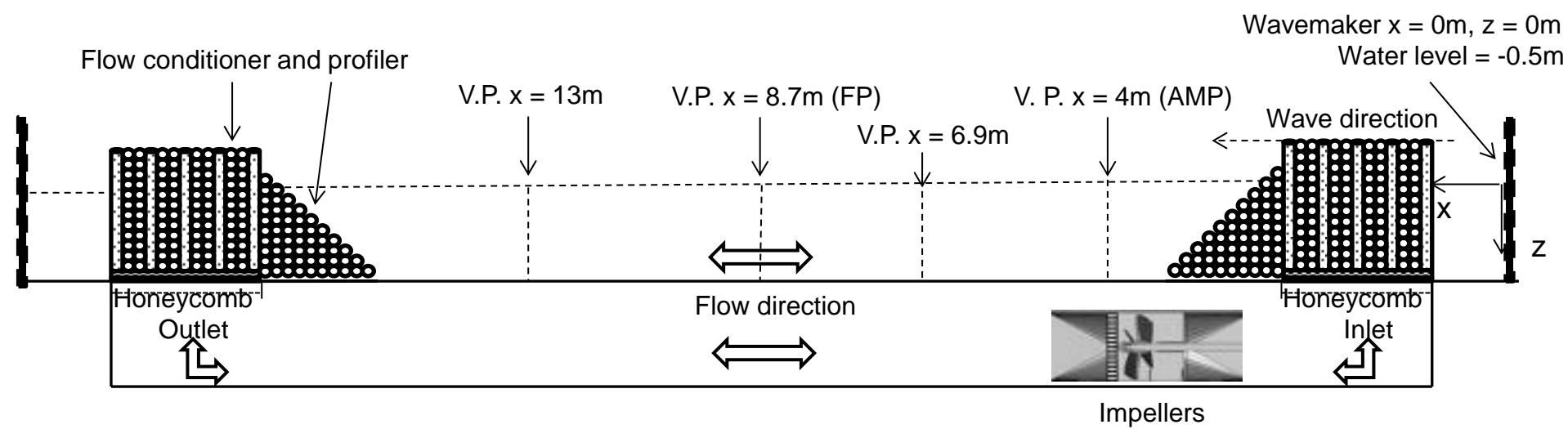
Wave conditions			Flow conditions	
Spectrum	Peak frequency [Hz]	Amplitude [m]	Current	Surface Velocity [m/s]
Wide Gaussian (GW)	0.6Hz	0.025, 0.05, 0.07	Still water, Adverse, Following	0, ±0.2, ±0.4
	0.9Hz	0.025, 0.05, 0.07	Still water, Adverse, Following	0, ±0.2, ±0.4
Narrow Gaussian (GN)	0.6Hz	0.025, 0.05, 0.07	Still water, Adverse, Following	0, ±0.2, ±0.4
	0.9Hz	0.025, 0.05, 0.07	Still water, Adverse, Following	0, ±0.2, ±0.4
JONSWAP P (JS)	0.6Hz	0.025, 0.05, 0.07	Still water, Adverse, Following	0, ±0.2, ±0.4
	0.9Hz	0.025, 0.05, 0.07	Still water, Adverse, Following	0, ±0.2, ±0.4
Pierson-Moskowitz (PM)	0.6Hz	0.025, 0.05, 0.07	Still water, Adverse, Following	0, ±0.2, ±0.4
	0.9Hz	0.025, 0.05, 0.07	Still water, Adverse, Following	0, ±0.2, ±0.4

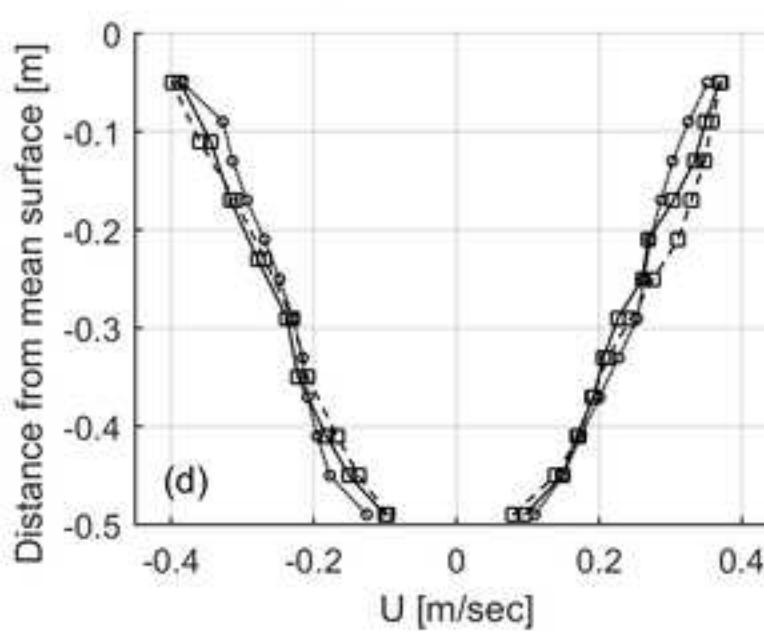
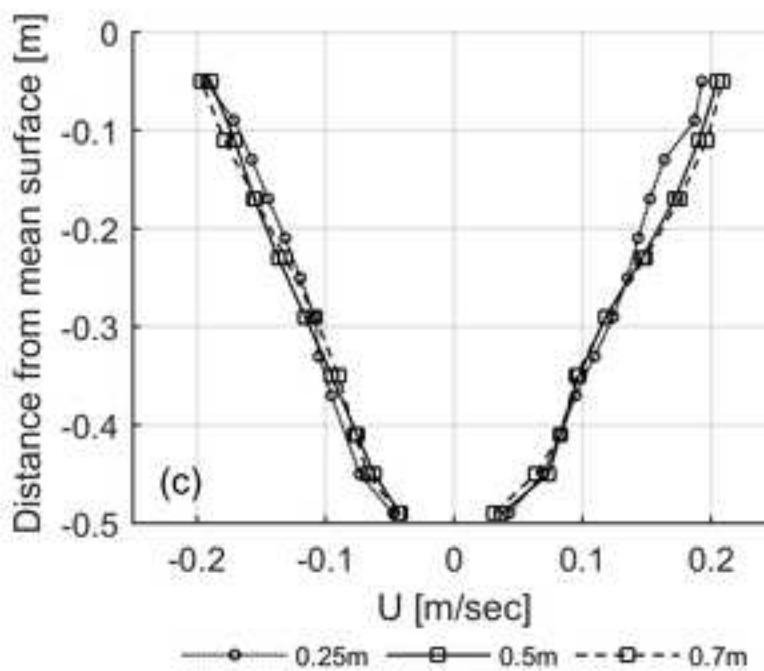
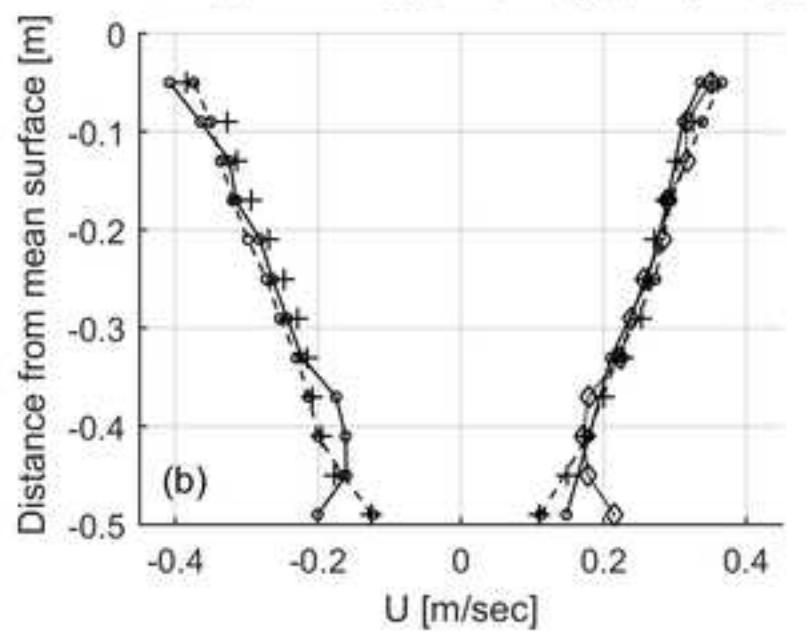
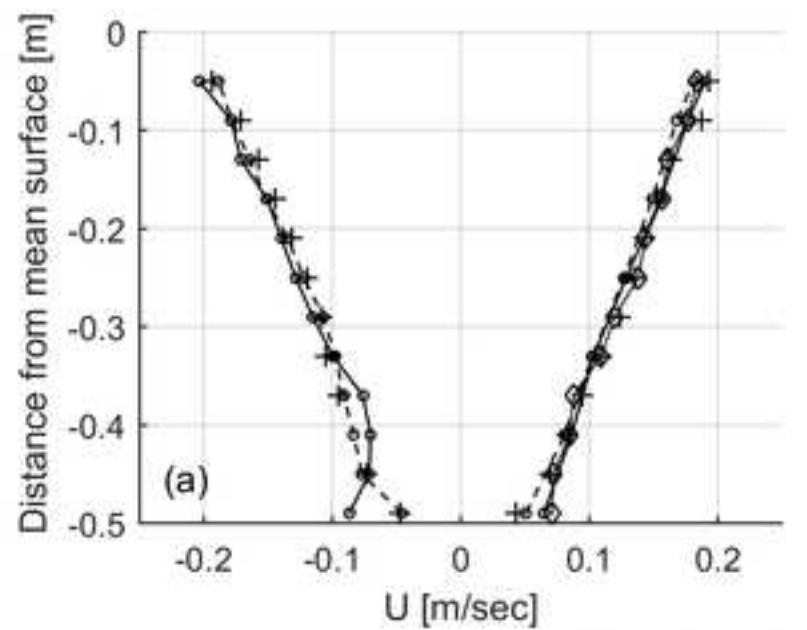
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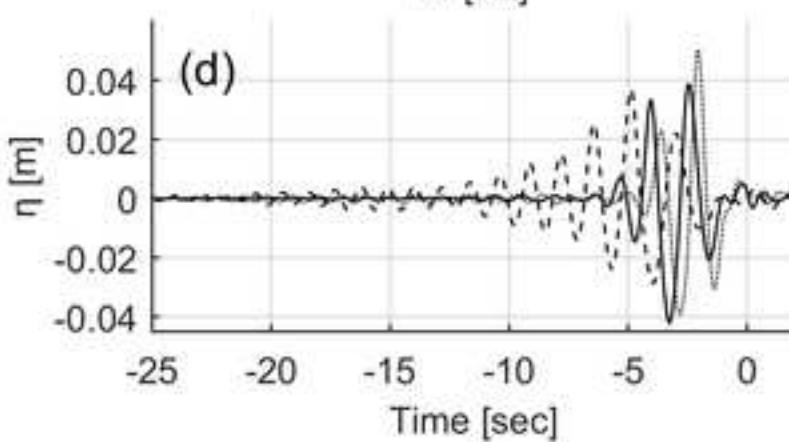
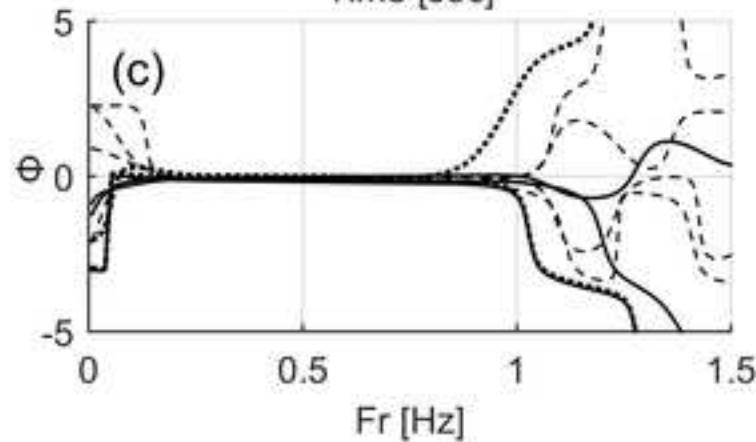
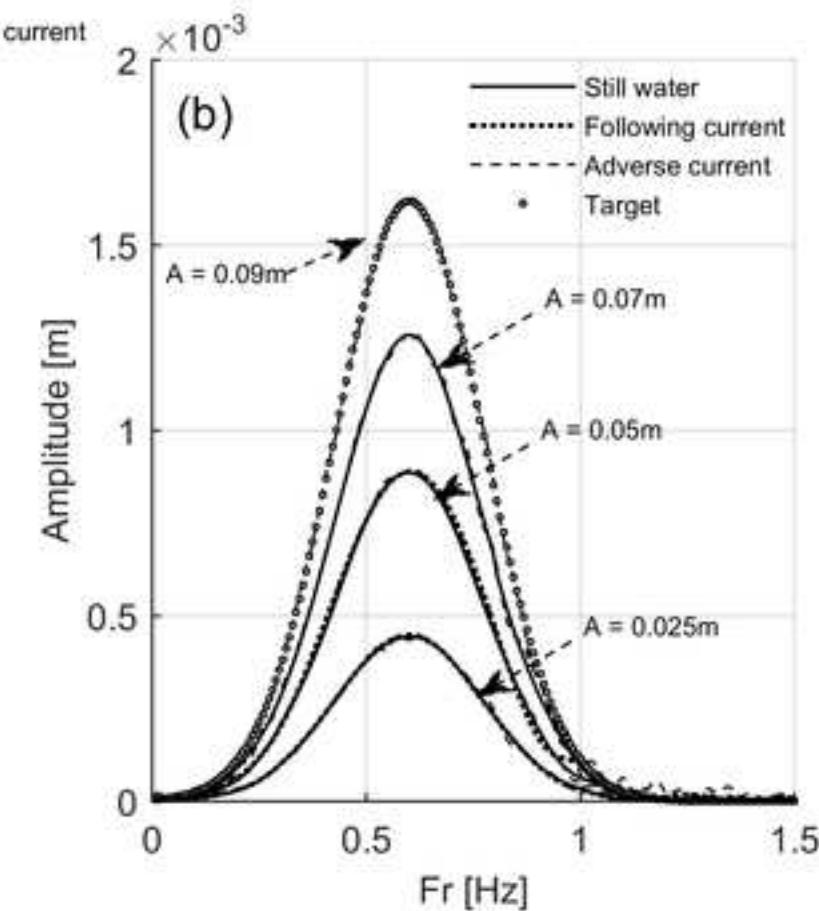
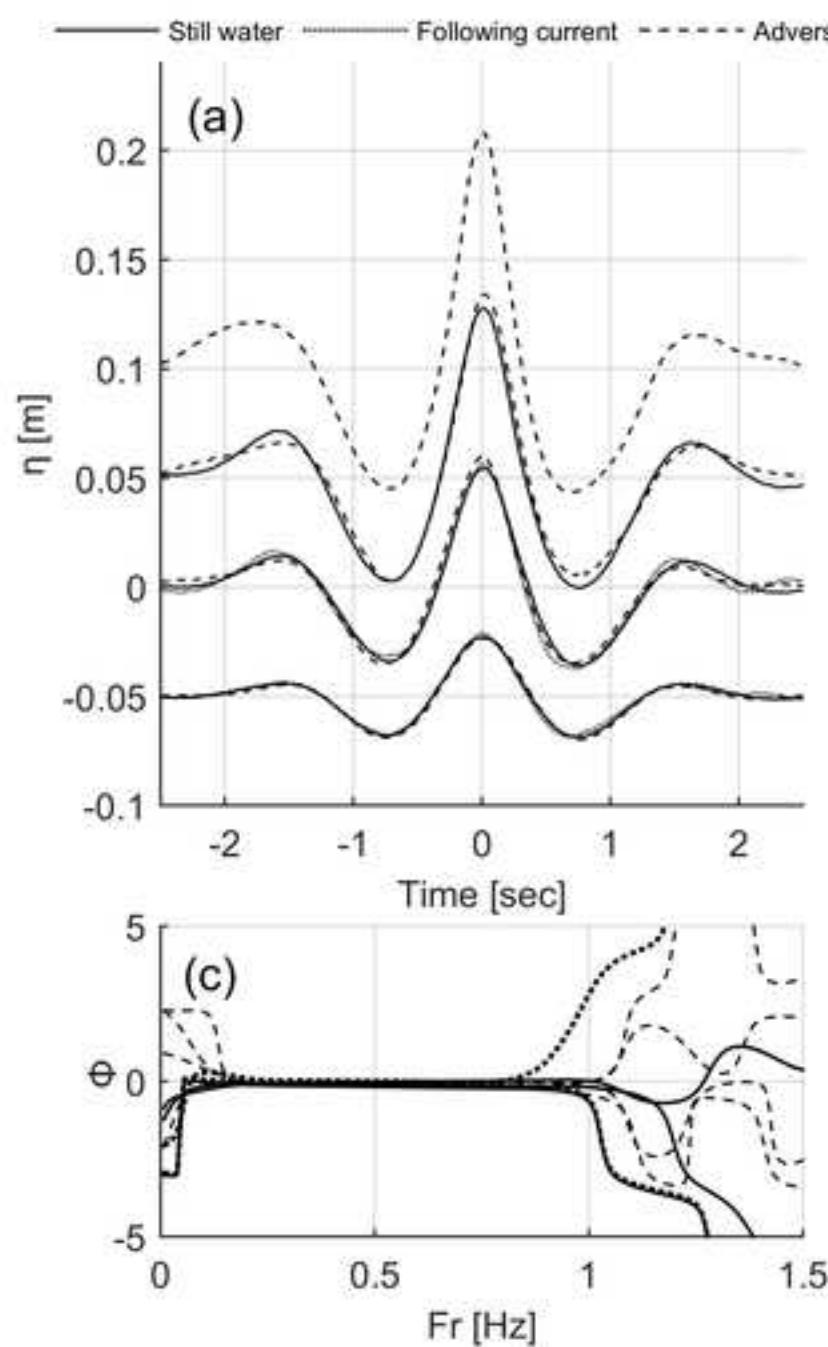
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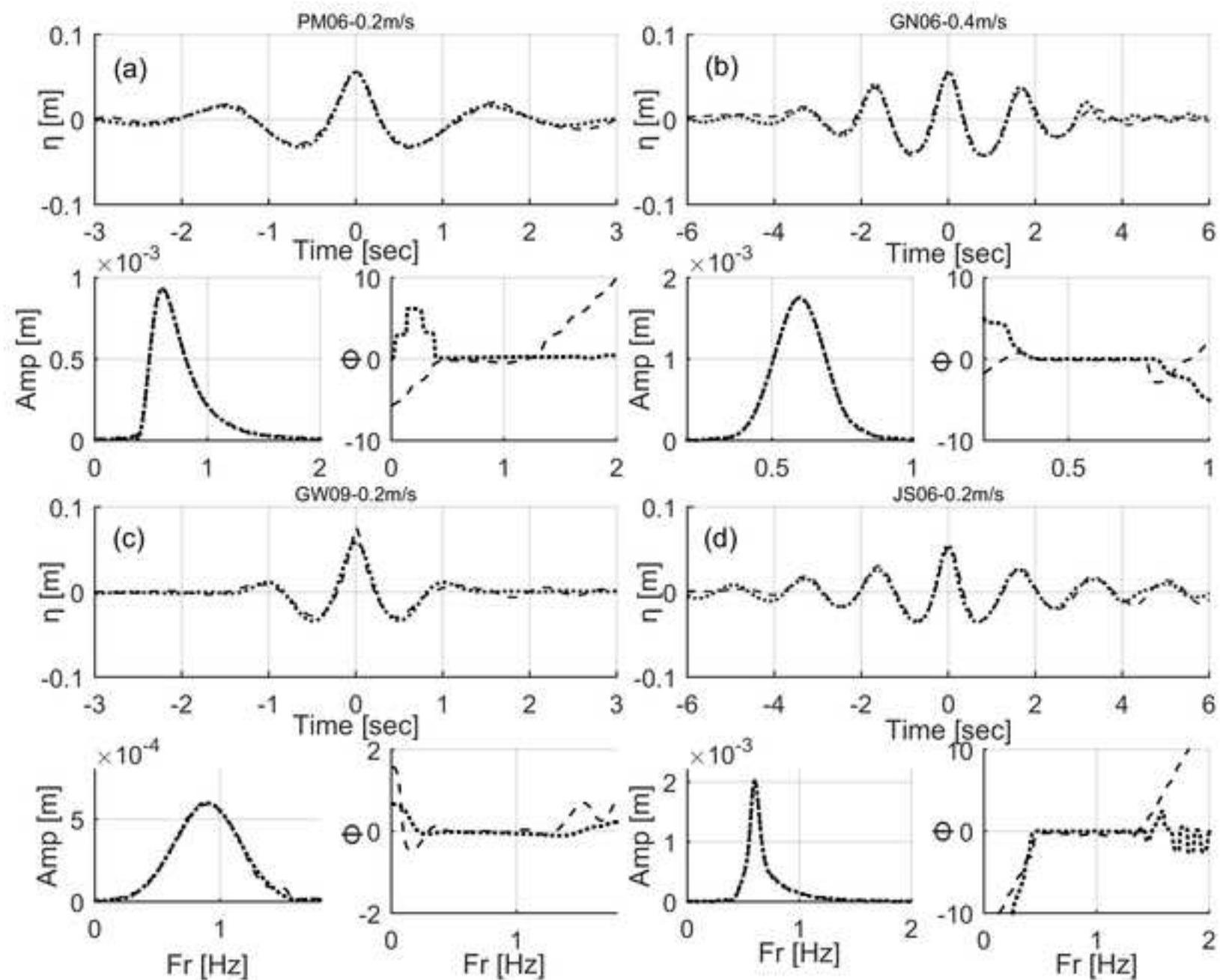


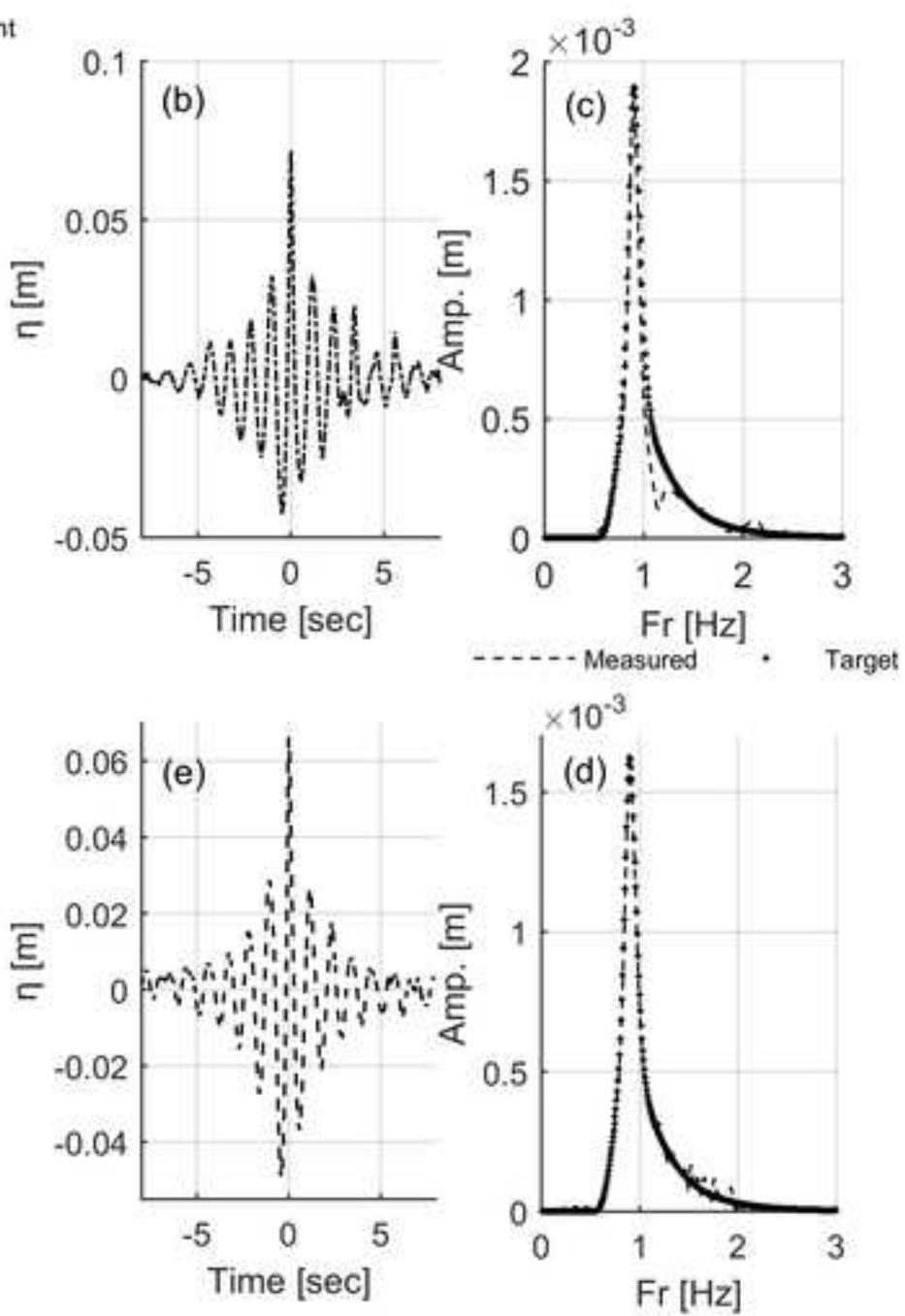
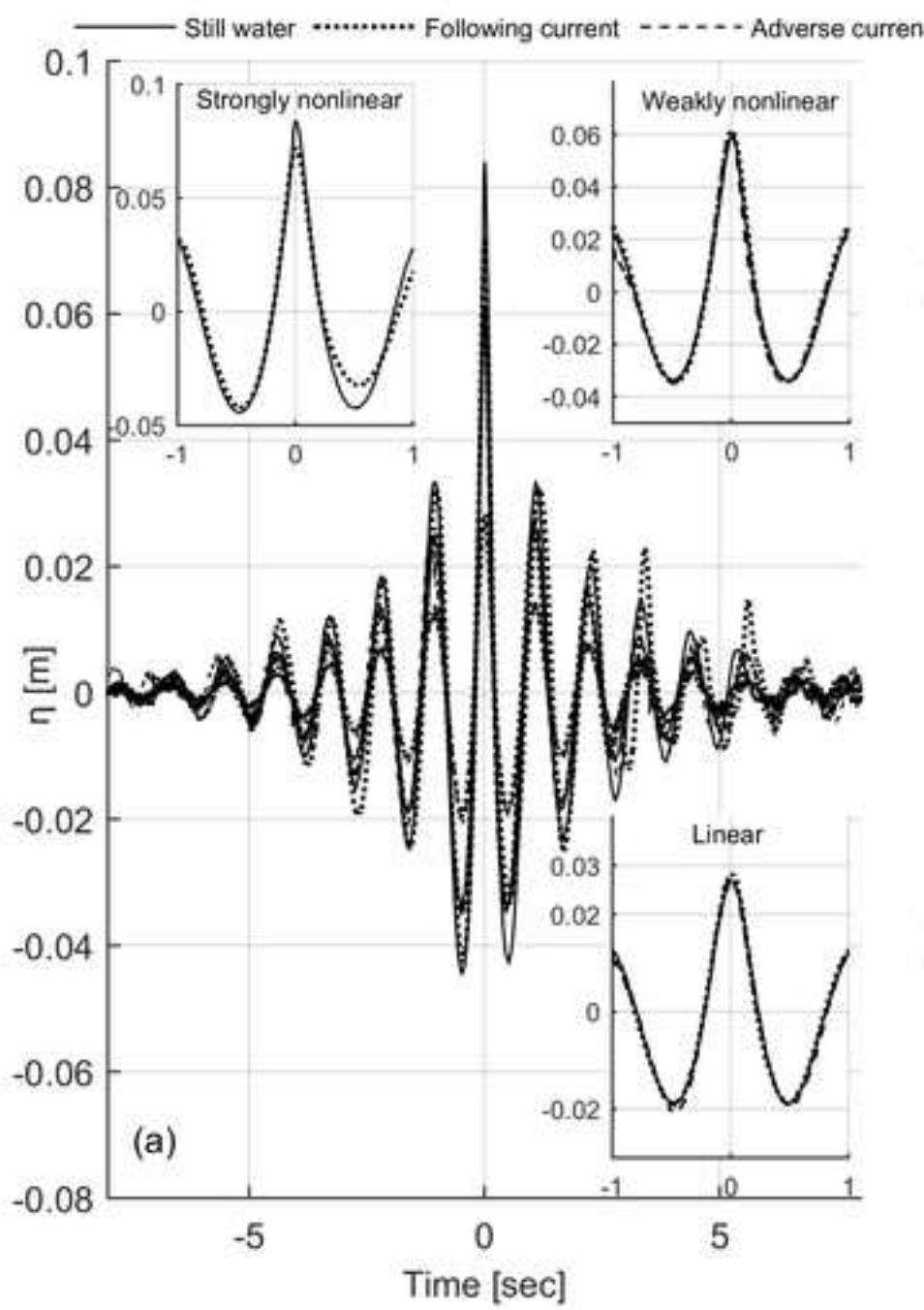


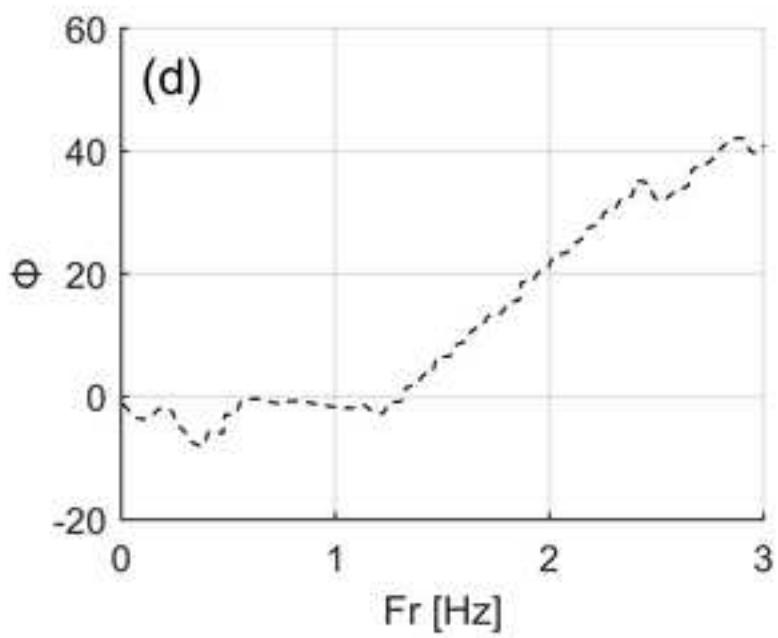
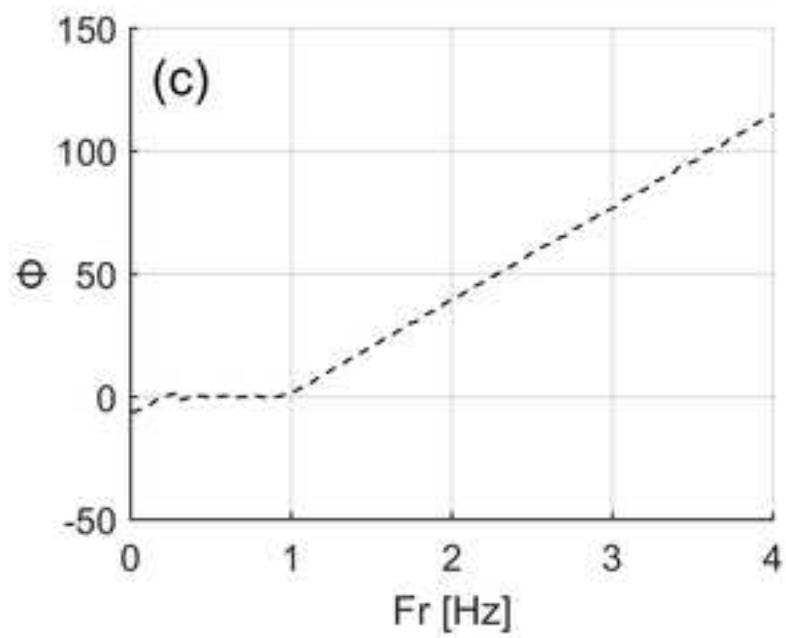
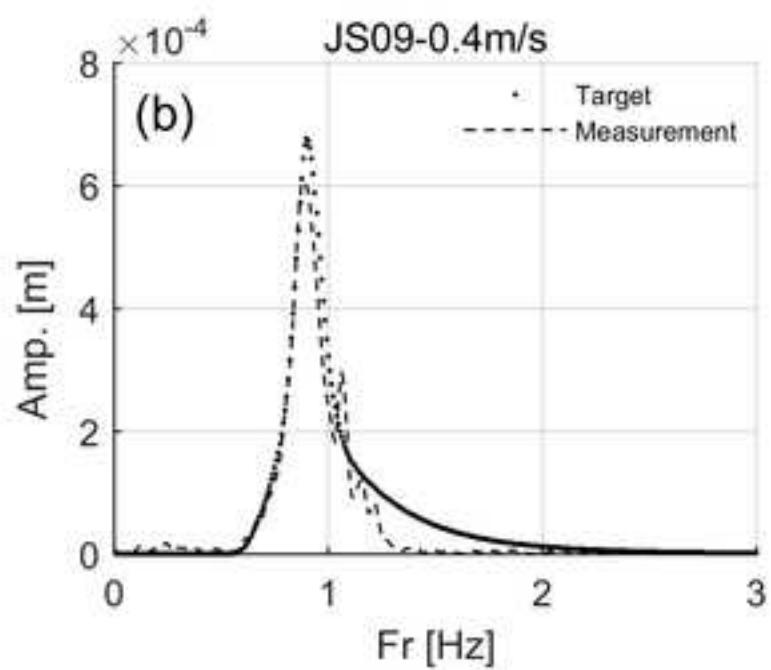
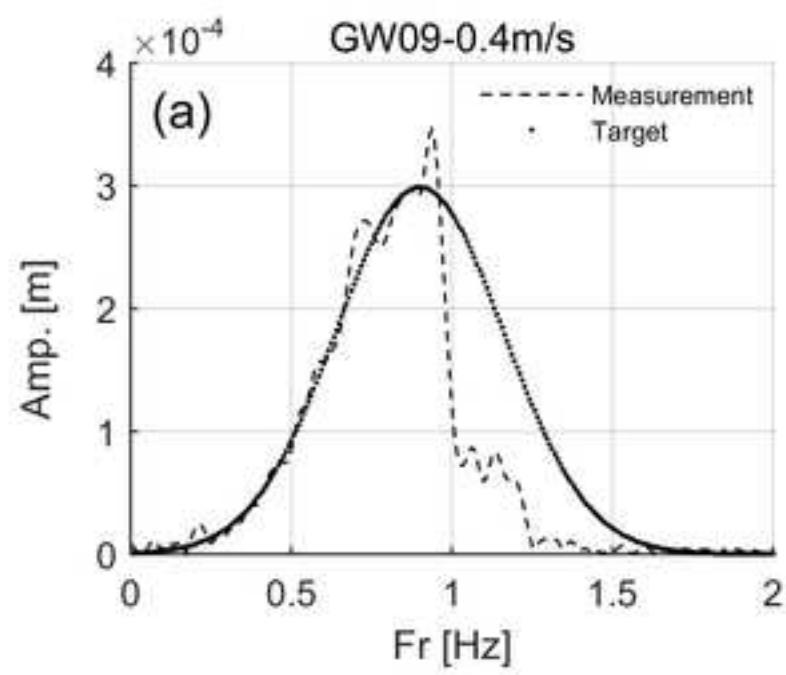


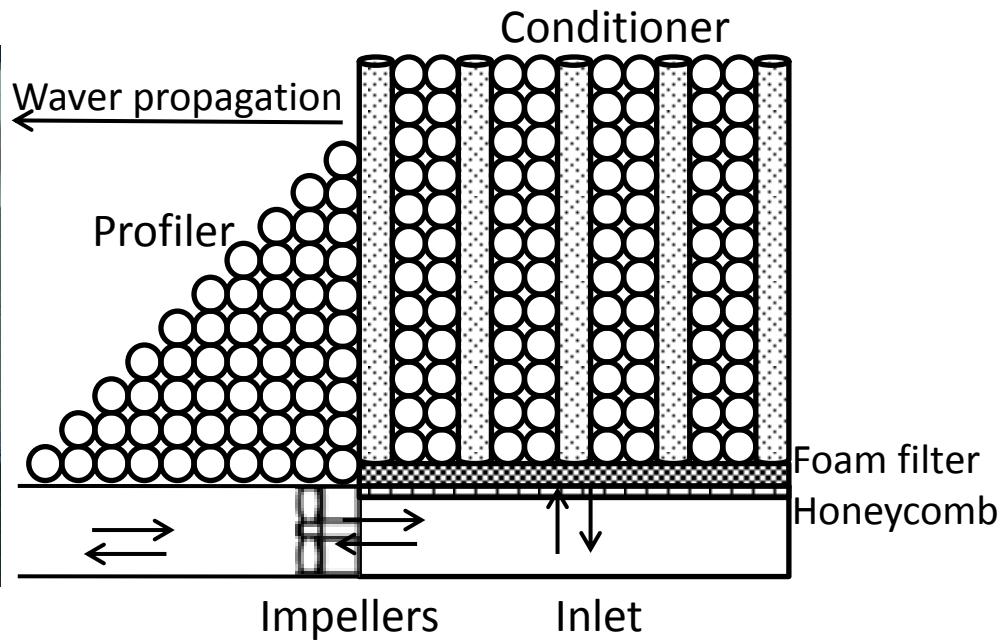
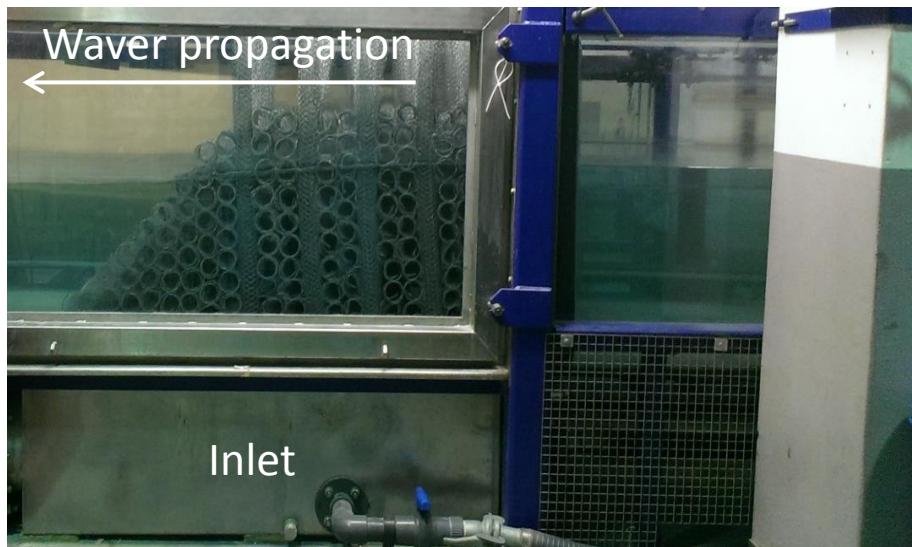


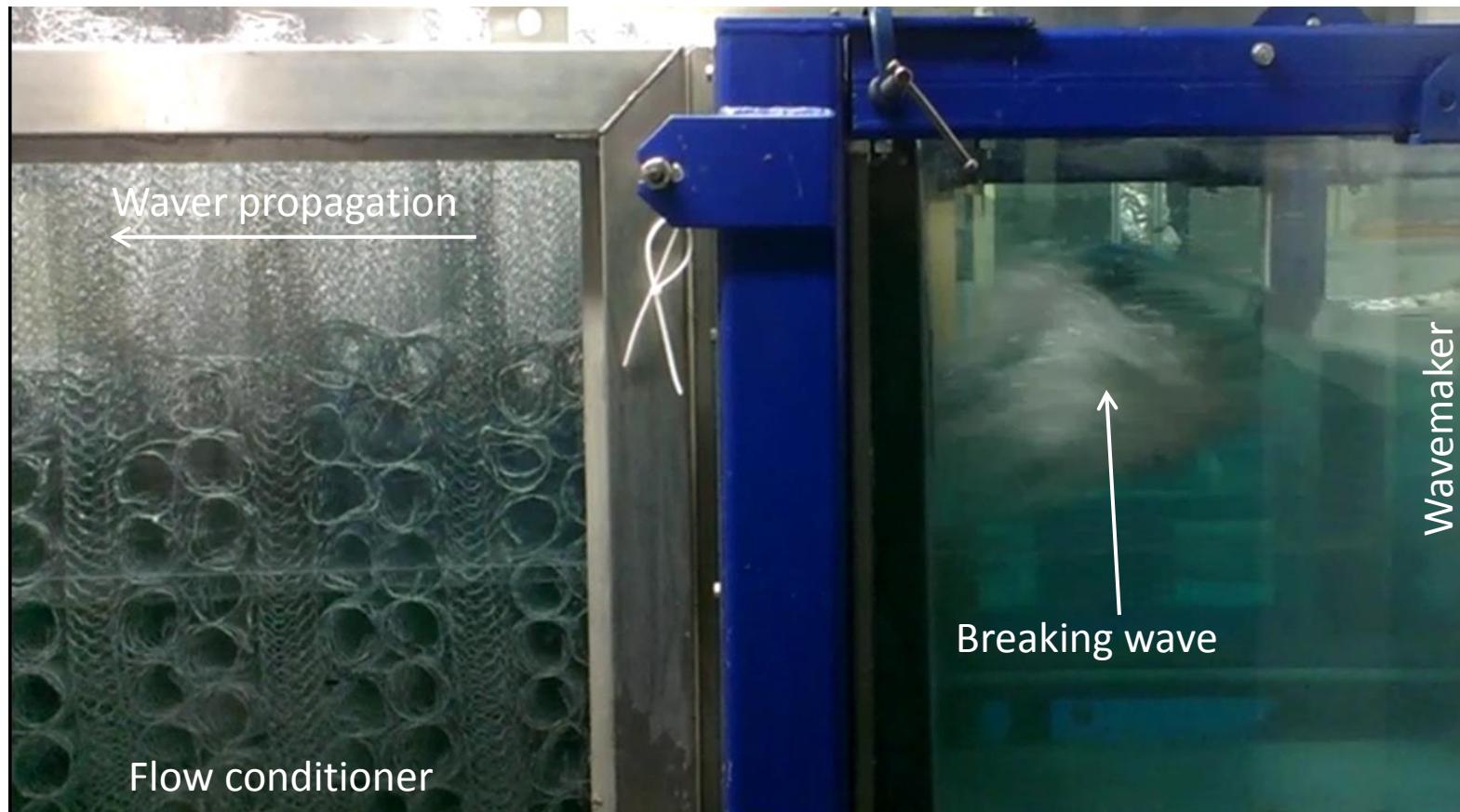












Captions

Figure 1: (a), surface elevation measurements at AMP for wave groups generated on still water with $\Delta\Phi = 0, \pi, \pi/2, 3\pi/2$. (b) and (d), decomposed amplitude spectra at AMP and FP; for $S_{0,1,2,3}$ see Eq.3. (c), surface elevation measurements at FP for $\Delta\Phi = 0, \pi, \pi/2, 3\pi/2$. Target spectrum: wideband Gaussian (GW), $Fr_p = 0.9\text{Hz}$ and $A = 0.07\text{m}$.

Figure 2: Surface elevation time histories reconstructed at FP using Inverse Fourier Transformation. (a) linearised, (b) 2nd order sum, (c) 2nd order difference, and (d) 3rd and higher order part. Target spectrum: wideband Gaussian, $Fr_p = 0.6\text{Hz}$, $A = 0.05\text{m}$ and $U_s = 0.4\text{m}$.

Figure 3: 2D stream-wise slice of the UCL wave-current flume. $(x, z) = (0\text{m}, 0\text{m})$ is set at the intersection of the wavemaker with the water level. Marked with V.P. are the locations for the mean velocity profile measurements. For all tests, wave probes were set at $x = 4, 5.7, 6.9, 7.7, 8.2, 8.45$ and 8.7m .

Figure 4: Photograph (on the left) and schematic (on the right) of the flow shaping arrangement.

Figure 5: (a) and (b), mean velocity profiles measured along the flume. Negative and positive velocity values correspond to cases on following and adverse currents, respectively. For (a) and (c) $U_s = 0.2\text{m/s}$, while for (b) and (d) $U_s = 0.4\text{m/s}$.

Figure 6: (a), time histories of surface elevation measured at focus. For clarity a constant amplitude shift of $\pm 0.05\text{m}$ has been added. Linear ($A = 0.025\text{m}$), weakly ($A = 0.05\text{m}$) and strongly ($A = 0.07\text{m}$) nonlinear waves groups are plotted at $-0.05, 0\text{m}$ and $+0.05\text{m}$, respectively. For adverse currents, waves with $A = 0.09\text{m}$ were also generated and are plotted with a dashed line at 0.1m . (b), linearised amplitude spectra at the AMP. (c), linearised phase spectra at focus, and (d), fully nonlinear surface elevation measurements at the AMP for wave groups with $A = 0.05\text{m}$. Target spectrum: wideband Gaussian (GW), $Fr_p = 0.6\text{Hz}$, $U_s = 0.4\text{m/s}$.

Figure 7: Snap-shop of a wave breaking near the wavemaker (not shown), for GW06 and $A = 0.07\text{m}$ on following current with $U_s = 0.2\text{m/s}$

Figure 8: Surface elevation measurements at focus, linearised amplitude spectra at the AMP and phase spectra at the FP are presented for different test cases. (a), target spectrum: Pierson-Moskowitz (PM), $Fr_p = 0.6\text{Hz}$, $U_s = 0.2\text{m/s}$. (b), experiments with a narrowband Gaussian (GN) target spectrum,

$Fr_p = 0.6\text{Hz}$, $U_s = 0.4\text{m/s}$. (c) target spectrum: wideband Gaussian (GW), $Fr_p = 0.9\text{Hz}$, $U_s = 0.2\text{m/s}$. (d) JONSWAP (JS), $Fr_p = 0.6\text{Hz}$, $U_s = 0.2\text{m/s}$. Still water measurements are omitted for clarity. For all graphs, dotted lines: waves on following current and dashed lines: waves on opposing current.

Figure 9: (a), time histories of surface elevation at focus for $A = 0.025\text{m}$, $A = 0.05\text{m}$ and $A = 0.07\text{m}$. (b) and (c), surface elevation measurements at the FP and linearised spectrum at the AMP for $A = 0.07\text{m}$ on following current, $U_s = 0.2\text{m/s}$. (d) and (e), time histories of surface elevation at focus and linearised spectrum at the AMP for $A = 0.06\text{m}$ on adverse current, and $U_s = 0.2\text{m/s}$. Target spectrum JONSWAP, $Fr_p = 0.9\text{Hz}$.

Figure 10: Linearised amplitude ((a) and (b)) and phase ((c) and (d)) spectra at the AMP and the FP, respectively. All wave groups were generated with $Fr_p = 0.9\text{Hz}$ and $A = 0.025\text{m}$, on adverse current with $U_s = 0.4\text{m/s}$. Target spectrum: (a) and (c), wideband Gaussian and (b) and (d), JONSWAP.