FORECASTING THE DURATION OF SHORT-TERM DEFLATION EPISODES

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Accepted for publication in in the *Journal of Forecasting*

KEYWORDS: inflation forecasting, negative inflation, duration forecast, probabilistic forecasting, simulation, multivariate copula analysis.

JEL Codes: C22, C53, E31

ACKNOWLEDGEMENT

Financial support of the Opus 8 project 2014/15/B/HS4/04263 *Modelling macroeconomic uncertainty*, financed by the National Science Centre in Poland, is gratefully acknowledged. We are grateful to the Referee of this paper for constructive comments and suggestions made on the earlier versions of the paper.

ABSTRACT

The paper proposes a simulation-based approach to multi-step probabilistic forecasting, applied for predicting the probability and duration of negative inflation. The essence of this approach is in counting runs simulated from a multivariate distribution representing the probabilistic forecasts, which enters the negative inflation regime. The marginal distributions of forecasts are estimated using the series of past forecast errors, and the joint distribution is obtained by a multivariate copula approach. This technique is applied for estimating the probability of negative inflation in China and its expected duration, with the marginal distributions computed by fitting weighted skew-normal and two-piece normal distributions to ARMA *ex-post* forecast errors and using the multivariate Student-*t* copula.

1. INTRODUCTION

The paper proposes a simple, albeit computationally intensive, way of analysing the joint distributions of multi-horizon probabilistic forecasts. The focus is on evaluating the out-of-sample duration forecast. The concept is illustrated by estimating the duration of negative inflation in China.

There is already a huge body of literature on forecasting the duration of events. In economics, the most popular approaches are grounded within extreme value theory (e.g. Gilli and Këllezi, 2006) and proportional hazard duration modelling. The latter is a development from survival modelling, and is often used for analysing the duration of unemployment (e.g. Lancaster, 1979; Bover *et al.* 2002). For discussion on other approaches see Men *et al.* (2015).

However, these methods do not seem to be fully appropriate for forecasting the duration of short-term deflation episodes, defined by negative inflation. In this paper, we regard deflation in a purely statistical sense, as a decline in the average (weighted) level of consumers' prices in prices, without addressing its possible relation to aggregate demand (see e.g. Atkeson and Kehoe, 2004; Benhabib and Spigel, 2009; for another approach to estimation of the probability of deflation, based on data from inflation surveys and inflation swap rates, see Fleckenstein *et al*., 2017).

Firstly, historical episodes of negative inflation are infrequent or even non-existent for most countries, which makes estimating from its historical appearance inefficient or actually impossible. Secondly, most of the proportional hazard methods rely implicitly or explicitly on the assumption of normality of the forecast distribution. With inflation, this is evidently not a realistic assumption (for the most recent evidence see Chaudhuri *et al.*, 2016). The extreme value models are free of the assumption of normality, but they usually rely on tight and not easily testable assumptions (see e.g. Kotz and Nadarajah, 2000).

The approach we propose allows the probability and duration of events (negative inflation in our case) to be estimated, even if the events did not occur in the past. It is grounded within multi-horizon probability density forecasting, where the marginal distributions of forecasts made for each horizon are estimated from data, and the dependencies between forecasts for different horizons are described by a multidimensional copula. As an illustration, the methodology has been applied to China.

Section 2 of the paper describes the methodology for computing the marginal and joint distributions and evaluating the probability of negative inflation and its expected duration. Section 3 presents the results for China, which indicates that for the period from April 2014 to March 2015, the maximum probability of negative inflation occurring in a given month was about 20%. Further, the expected duration of negative inflation would be about three months if it occurred in the 5th-period forecast, and 1-2 months if it occurred later. Section 4 gives the results of robustness checks that show the results to be reasonably robust to changes in the marginal distributions and copula specifications. Section 5 concludes.

2. METHODOLOGY

To evaluate the joint multi-period distribution of inflation, we consider the density forecast of inflation for the periods $h=1,2,...,H$ made at time 0 as being described by an *H*-dimensional random variable $X=[X_1,X_2,...,X_H]\subset\Re^H$, where X_h is a random variable that represents inflation at time *h*. Let $Y_h=X_h-\mu_h$, where $\mu_h=E(X_h)$; f_{Y_h} denotes the *pdf* of Y_h. The unconditional probability of inflation being below zero, for the forecast horizon *h* is defined as:

$$\Pr(X_h < 0) = \int_{-\infty}^{-\mu_h} f_{Y_h}(s)ds \,. \tag{1}$$

The density function f_{Y_h} is usually not known. It is often estimated using either past forecast errors or data from surveys of professional forecasters (for a comparison of these two approaches see Clements, 2014) or, in some instances, by calibration (e.g. Wallis, 2004). In virtually all the cases analysed in the literature it has been found that the distribution is not normal, or even symmetric.

If there is no model uncertainty, it is usually assumed that there is one point forecast for the horizon *h*, which is the estimate of μ_h, denoted as $\hat{\mu}_h$. It is, however, more realistic to assume that there is some model uncertainty about μ_h. Specifically, there may be K different, competing, models, each producing its own point forecast $\hat{\mu}_{k,h}$, where $k = 1, 2, ..., K$. We assume that the forecasts from these competing models are not necessarily observed and are normally distributed around $\hat{\mu}_h$ with variance κ_h. Under this assumption, the following algorithm for estimating the probability in (1) is proposed:

1. Assuming a particular family of distributions, estimate the density function f_{Y_h} using data on a sequence of past *h*-step-ahead forecast errors around $\hat{\mu}_h$, or data from surveys of forecasters if they are available and reliable. It might be necessary to choose from some competing families of distributions (see Section 3). Earlier results (see Charemza *et al*., 2015a) show that the density functions best-fitted to inflation forecast errors are usually either the weighted skew-normal (WSN) or two-piece normal (TPN) distributions; see Appendix A for the *pdf*'s of these distributions and their characteristics.

2. To account for model uncertainty, generate K realisations of a random variate $\upsilon_{k,h} \sim N(0, \kappa_h)$, and for each $k = 1, 2, ..., K$, estimate the probability of negative inflation for the forecast horizon *h* as:

$$\Pr(X_{k,h} < 0) = \int_{-\infty}^{-(\hat{\mu}_h + \upsilon_{k,h})} \hat{f}_{Y_h}(s)d(s), \qquad k = 1, 2, ..., K, \tag{2}$$

where $\hat{f}_{Y_h}$ is the estimated density function f_{Y_h}. The estimated probability of negative inflation is the arithmetic average of (2) across *k*, and its standard deviation reflects the effect of model uncertainty on the estimate assuming that densities $\hat{f}_{Y_h}$ are the same for all competing models.

To calculate the expected duration of negative inflation, it is necessary to consider the joint distribution of $Y = [Y_1, Y_2, ..., Y_H] \subset \Re^H$. Let f_Y be the joint *pdf* of Y. As the marginal distributions are usually not normal and the dimension *H*, the maximum forecast horizon, is relatively large, direct inference on such a *pdf* is usually not feasible. It is, however, possible to exploit the Sklar (1959) theorem, where F_Y, the multivariate cumulative distribution function, *cdf*, of Y, is given as:

$$F_Y(y_1, ..., y_H) = C\left(F_{Y_1}(y_1), ..., F_{Y_H}(y_H)\right), \tag{3}$$

where $C:[0,1]^H \to [0,1]$ is the *H*-dimensional copula function. Under the assumptions of stationarity and ergodicity, the parameters of the copula function can be estimated from data on past forecast errors. The *pdf* of *Y* can be then expressed by the marginal distributions and the copula function as

$$f_Y(y_1, ..., y_H) = c(u_1, ..., u_H) \times \prod_{h=1}^{H} f_{Y_h}(F_h^{-1}(u_h)) , \tag{4}$$

where $c(u_1,...u_H) = \partial^H C(u_1,...u_H)/\partial u_1...\partial u_H$ and $y_h = F_h^{-1}(u_h)$, $h = 1,...,H$.

In theory, the expected duration of negative inflation and other characteristics of the *H*-dimensional distribution of *Y* can be obtained directly from (4). In practice, however, it might not be feasible to compute the *H*-dimensional integration of (4), as the marginal density functions f_{Y_h} are usually non-normal and might even belong to different families of distributions. A relatively simple, albeit computationally intensive, alternative is to simulate samples of the random variable $U = [u_1,...,u_H]$ on $[0,1]^H$ with a given copula function C as in (3), and then recover the corresponding sample of $y = [y_1,...,y_H]$ as $y_h = F_h^{-1}(u_h)$. This yields the realisations of the *Y*s (and hence the *X*s, as $X = Y + \mu$, $\mu = [\mu_1, \mu_2,..., \mu_H]'$) while maintaining the dependence structure. To evaluate the expected duration of negative inflation within the forecast span that starts at period d ($d = 1,...,H$), it is convenient to consider the following discrete univariate random variable:

$$Z_d = \sum_{i=0}^{H-d} \prod_{j=d}^{d+i} I_j , \tag{5}$$

where $I_h = \mathrm{I}_{X_h<0}$ and $\mathrm{I}_{X_h<0}$ is the indicator function, which is equal to 1 if $X_h < 0$ and to 0 otherwise. Clearly, $Z_d \in \{0,1,...,H-d+1\}$ and its expected value is the duration of negative inflation between periods *d* and *H*. For $d > 1$, the expected length of an episode of deflation that starts at time *d*, meaning that inflation at time $d-1$ was non-negative, can be computed as the expected value of the random variable $\tilde{Z}_d$ defined as:

$$\tilde{Z}_d = (1 - I_{d-1}) \times Z_d . \tag{6}$$

In (6), $\tilde{Z}_d$ represents the expected duration of negative inflation in the ‘constrained’ scenario, that is when negative inflation cannot go beyond the maximum forecast horizon, which leads to $\tilde{Z}_d \le H - d + 1$. The actual duration of negative inflation might in fact be greater than $\tilde{Z}_d$, if it goes beyond the maximum forecast horizon. In consequence it might be useful to compute additionally the joint probability of negative inflation appearing in periods *d* and *H*. If this probability is negligible, it means that $\tilde{Z}_d$ approximates well the duration of negative inflation unconstrained by the maximum forecast horizon.

The computational algorithm is:

1. For each forecast horizon, use data on forecast errors or information from surveys to select the type of marginal density function and estimate its parameters, which are the parameters of the marginal distributions of $Y_1, Y_2,..., Y_H$.

2. Select the type of H-dimensional copula function in (3), estimate its parameters and then simulate $NRepl$ realisations of $U=[u_1,...,u_H]$ from this copula ($NRepl$ is the number of replications).
3. By applying the inverse transformation $y_h = F_h^{-1}(u_h)$ where $h=1,2,...,H$, generate an $NRepl \times H$ matrix of realisations of Y and then one of X by adding $\hat{\mu}=[\hat{\mu}_1, \hat{\mu}_2, ..., \hat{\mu}_H]'$ which represents a vector of point forecasts.
4. Using this $NRepl \times H$ matrix, compute the corresponding matrix of realisations of I_h. Next, for each row, count the runs of consecutive negative inflation episodes, meaning cases where the consecutive simulated realisations are negative, and use the arithmetic mean of these runs as the estimates of the expected duration of negative inflation, as defined by (5) and (6).

In the algorithm above it is important to choose the appropriate multi-dimensional copula function. One of the most commonly used approaches is to apply D-vine copulas (see Kurowicka and Joe, 2010; for applications in forecasting see Smith, 2015). For two reasons, we have, however, decided to use a simpler approach of applying the elliptical Student-*t* copula (see e.g. Demarta and McNeil, 2005). Firstly, the heavy computational burden of the technique proposed is accounted for as the elliptical Student-*t* copula is easier to simulate than a D-vine copula. Secondly, as negative inflation is such an infrequent event, its probability is relatively low and imposing any asymmetry in the dependence (which is typical for most of the D-vine copulas) might distort the estimated duration of negative inflation in a somewhat arbitrary way. In the absence of any a priori information about the type of dependence, it seems rational to assume strong, albeit symmetric, tail dependence, as given by the Student-*t* copula.

In the technique described above there is no dependency of the higher moments of distribution of inflation on time and the observed inflation. If the forecast errors used for estimation are collected over a long period, where inflation goes through different phases, this lack of dependency might not be the case. During high inflation uncertainty might also be high, and it might be low during low inflation (see e.g. Ball, 1992). Consequently, a modification might be needed if the sample includes high and low inflation intervals. Such modification may consist of estimating the parameters of the marginal and joint distributions separately for different regimes. This, however, requires long data series.

3. EMPIRICAL RESULTS FOR CHINA

China's long-term economic strategy of export-stimulating growth led to the Chinese currency being seen as undervalued for relatively long periods of time (see e.g. Rodrick, 2010). One side effect of this policy was weak domestic demand. Although the policy of undervaluation effectively ended in 2014, domestic demand remained weak in 2015 due to low commodity prices and the high real costs of borrowing. It is reduced further by deflationary expectations, which delay spending by consumers.

Figure 1 ABOUT HERE

Figure 1 shows the dynamics of annual CPI inflation in China measured monthly from January 2005 to April 2015 (data are from the official website of the National Bureau of Statistics of China, and are also available from the website of e.g. the Federal Reserve

Bank of St Louis https://research.stlouisfed.org/fred2/series/CPALTT01CNM659N#). Although the period of recorded negative CPI inflation was relatively brief, from February to October 2009, inflation nevertheless remained low from February 2012 until the end of the period analysed in April 2015. This created the danger that negative inflation could return as some prices might fall even as the total inflation index remains positive, creating expectations of a further fall and depressing domestic demand even further. This is one reason why it is important to assess the probability of negative inflation in times of low positive inflation.

Table 1 reports the non-stationarity test statistics for the inflation series depicted in Figure 1. These are the Elliot, Rothenberg and Stock test *(ERS*), the Ng and Perron feasible point optimal test (*MPT*), the Ng and Perron generalised least squares test (MZ_{α}), efficient modified Phillips-Perron tests (*MSB* and MZ_t), and the generalised least squares augmented Dickey-Fuller test (ADF). All these tests are computed in the presence and absence of structural breaks; see Ng and Perron (2001) and Carrion-i-Silvestre *et al.* (2009). The results show that 15 out of the 24 tests reject the null hypothesis of a unit root.

Table 1 ABOUT HERE

We have also computed the Robinson (1994) test for fractional integration for all combinations of the fractional levels of integration equal to 0.1, .., 0.2 …, 0.9 and first-order autocorrelations equal to 0, 0.1, …, 0.9 .[1] For all cases the null hypothesis of fractional integration has been strongly rejected. Consequently, we have decided to treat annual inflation in China as a stationary series. However, as this conclusion is not very strong, as the test results are sometimes contradictory, we have conducted additional forecast comparison analysis for the ARMA and ARIMA models, described in Section 4.

Data on ARMA forecast errors resulting from pseudo out of sample forecasting are used to estimate the distribution of Y_h and then X_h. It is assumed that the mean of the competing forecasts is approximated by the ARMA forecast. This seems to be a reasonable assumption, as the forecasts from univariate ARMA models often outperform forecasts from more complex models for sample sizes smaller than 500 (see Constantini and Kunst, 2011; Herwatrz, 2013, Mitchell *et al.* 2014). The pseudo out of sample forecasting was conducted as follows. The univariate ARMA model of inflation was first estimated using the sub-sample from January 2005 to October 2008 and was used for predicting the inflation rate up to 12 steps ahead. Then the entire sample rolls forward by one observation and the model is re-estimated using data from February 2005 to November 2008; predictions are then made for the next 12 months, and so the process continues. Lag lengths have been obtained in the estimation of the ARMA model by minimising the Akaike information criteria in each sub-sample. Finally, the *ex-post* forecast errors are computed as the differences between the realisations and the forecasts.

For the entire time span from January 2005 to April 2015 the highest inflation was equal to 8.7%. In the period covering all forecasts, that is from November 2008 until April 2015, the maximum inflation was 6.5%. As this does not cover instances of high inflation,

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[1] Programmed in Gauss using the specification by Gil-Alana (2005).

usually defined as at least 7% (see e.g. Charemza et al. 2015b), we can assume that, except for the brief deflation episode, the Chinese economy was not in a high inflation regime.

Next, different types of distribution were fitted to the forecast errors. Table 2 gives the measures of goodness of fit (twice-squared Hellinger distance, *HD*, and the Pearson χ^2 measure) obtained by the simulated minimum distance method for the normal, alpha-stable, weighted skew-normal (WSN) and two-piece normal (TPN) distributions. The simulated minimum distance estimation method is described in Charemza *et al*. (2012). In this method, the empirical data grouped in a histogram are approximated by simulated data and the choice of parameters for the best-fitting distribution is made by minimising the twice squared Hellinger distance criterion. The technique is similar to that of Dominicy and Veredas (2013).

Table 2 ABOUT HERE

Table 2 shows that for all forecast horizons, the WSN and TPN distributions fit the data markedly better than the normal and alpha-stable distributions and the TPN outperforms the WSN in 10 cases out of 12. For the subsequent computations presented here, the following main settings have been applied:[2]

a) Marginal distributions have been decided by the best Hellinger fit, so the TPN was used for all forecast horizons except 2 and 4. Selection based on the best χ^2 fit would be the same in all cases.

b) The 12-dimensional elliptical copula applied here is the Student-*t* copula with 4 degrees of freedom. The choice of the copula is somewhat arbitrary. However, using the Student-*t* copula with 4 degrees of freedom gives reasonably strong tail dependence and is partially justified by the results of the robustness check (see Section 4).

c) Simulation of the multidimensional Student-*t* copula requires knowledge of the scatter (dispersion) matrix (see Embrechts *et al.*, 2003). This matrix has been estimated by the method of moments, meaning it is recovered from the Kendall *tau* correlation coefficients computed for all pairs of forecast errors for different horizons (see Demarta and McNeil, 2005), using the empirical data on the ARMA forecast errors for the forecast horizons from 1 to 12. The values of the scatter matrix are in fact quite close to the ordinary Pearson coefficients (see Figures 2a and 2b for more information).

d) Under the assumption of model uncertainty and competing forecasts introduced in Section 2, the number of competing forecasts is set at 1,000. It is assumed that the model uncertainty increases with the increase in the forecast horizon, which is the standard deviation of model uncertainty with κ_h increasing linearly from 10% to 21% relative to the corresponding point forecast.

e) Numerical integration in (2), with f_{Y_h} given by either the WSN or the TPN distribution, turned out to be awkward and imprecise, as both the density functions have a rather tangled analytical form (see Appendix A); this usually causes numerical

[2] Computations were made using Aptech GAUSS on the ALICE High Performance Computer of the University of Leicester.

problems in integration. Consequently, integrals for the marginal probabilities have been computed by simulation, with a simple rejection algorithm with 100,000 replications applied for each f_{Y_h}. In this case the simulation error affects the accuracy of the estimation. The *n out of n* bootstrap is performed to evaluate the magnitude of this error, where re-draws are made n times with replacement from the simulated realisations of Y_h, where n stands for the number of replications and (2) is computed for each re-draw; see e.g. Halt and Martin (1988), and Cheung and Lee (2005).

f) The practical problem in the simulation of the realisations of the 12-dimensional random variable using (3) is in computing the inverse transformation for the WSN and TPN distributions. Unlike for some other distributions, the analytical forms of their inverse transformations are not known. For this reason, a sequential search algorithm has been applied (see Devroye, 1986) to find an approximation. This was the most computer-intensive part of the computations and was set at 100,000. Although it resulted in quite poor approximation for low probabilities of negative inflation below 5%, such low probabilities did not have important practical relevance and turned out not to have a marked effect on the accuracy of the evaluation of the expected duration of negative inflation.

Figures 2a-2b illustrate the dependencies between the distributions of forecast errors for different horizons by showing selected results for two-dimensional cases. In both figures, the upper diagram panels show scatter diagrams of forecast errors made for the horizons 1 and 2, and 1 and 3 (Figure 2a), and 2 and 4, and 6 and 10 (Figure 2b). The types of distributions that have been fitted to the forecast errors are indicated in brackets, with TPN for forecast horizon 1, WSN for forecast horizon 2, etc. Above these panels are the characteristics describing the dependence between these distributions: Kendall τ and Pearson ρ correlation coefficients, and the corresponding elements of the 12×12 scatter matrix of the multivariate Student-*t* copula (the elements [1,2] and [1,3] on Figure 2a and [2,4] and [6,1] on Figure 2b). The bootstrapped *p*-values are shown for Kendall τ in parentheses. The middle panels give the two-dimensional *pdf*s of the Student-*t* copulas with 4 degrees of freedom fitted to the data. For better visualisation, the bottom panels present the contour plots of the corresponding *pdf*s from the middle panels.

Note that the marginal distributions for the forecast horizons 1-3 and 6-10 are both TPN with different parameters; both are WSN for the forecast horizons 2-4, and for 1-2 one of the marginal distributions is TPN and the other is WSN (for the estimates of their parameters see Appendix B). As expected, the figures indicate clear and strong dependence between the forecast errors for the different forecast horizons. They also show that such dependence is markedly different for each pair of forecast horizons. The *pdf*s and their contours indicate skewness of the distributions and, generally, stronger dependence for the upper tail than for the lower tail. In other words, large positive forecast errors where inflation was underestimated tend to be more strongly dependent than the large negative errors which resulted from an overestimation of inflation. It is evident that all the distributions in Figures 2a and 2b are negatively skewed.

Figure 2a ABOUT HERE

Figure 2b ABOUT HERE

Table 3 gives the probabilities of negative inflation with their standard errors, computed with estimated marginal distributions for the period from April 2014 to March 2015. In order to focus on the probabilistic aspect of forecasting and avoid additional errors caused by the imperfectness of a single point forecast for this period, it is assumed that the April 2014-March 2015 point forecasts were perfect, so $\hat{\mu}_h = \mu_h$. In other words, it is assumed that forecasts were perfect for each month and equal to the inflation actually realised. This essentially corresponds to the pseudo out of sample forecast error approach that is applied here. It may be noted that the bootstrapped standard deviations are quite similar to the standard deviation obtained in an ordinary way. They are all relatively small, so the evaluation of particular probabilities can be regarded as reasonably precise.

Table 3 ABOUT HERE

Table 4 presents the basic results for the evaluation of the expected duration of negative inflation for the period from April 2014 to March 2015, as defined by (5) and (6), and under the settings given by (a)-(f) above. Note that according to Table 3, the probabilities of negative inflation for the forecast horizons from 1 to 5 are smaller than 5%. Hence for these forecast horizons, the accuracy of the computation of the inverse probability transformation for the joint distribution based on simulated data is relatively low. In consequence, we present aggregated results for the forecast horizons from 1 to 5.

If negative inflation occurs in horizons 1 to 5, the cumulative probability of which is 3.28%, its expected duration is 2.81 months, with a standard deviation of 1.86, so that it is unlikely that it would last until the end of the forecast period. This is confirmed by the probabilities close to zero of negative inflation appearing in periods 1 to 5 or 12 (see the last two columns of Table 4). If negative inflation occurs over a longer horizon, its expected duration (up to horizon 12) gradually declines from 2.48 months in horizon 6 to 1.35 months for horizon 11. In this case, the corresponding probability of negative inflation appearing in periods $h < 12$ and $h = 12$ is substantial. This indicates that, if the maximum forecast horizon was longer than 12, the expected duration of negative inflation would have been longer than that given in Table 4.

The difference between the ‘probability of negative inflation at horizon *h*’ and ‘starting at forecast horizon *h*’ is such that the former includes possible cases of negative inflation which covers horizon *h*, and the latter is computed under the assumption that at horizon *h*-1, inflation was non-negative. For instance, for h=6, that is for September 2014, the probability that inflation would be negative is 0.22. If negative inflation covers September 2014, its overall expected length is 2-3 months, as the expected value is 2.48. The probability that negative inflation begins in September 2014 is 0.04 and its expected duration is 1.9 months.

Table 4 ABOUT HERE

4. ROBUSTNESS CHECK

We can identify two main potential reasons for lack of robustness of the results given in Section 3. Firstly, there might be model uncertainty, meaning that the type of forecasting model might be incorrect or, even if it is correct, it may be estimated imprecisely. Considering model uncertainty, natural alternatives to the ARMA model applied here are the ARIMA and ARFIMA models. ARIMA models in particular seem to be admissible alternatives to ARMA models, as the results of unit root testing are favourable to the ARMA model but are not fully conclusive (see Table 1). In order to check the robustness in this respect, computations have been repeated for the ARIMA models with the order of integration equals to unity. However, for all forecast horizons the forecast RMSEs for ARIMA turned out to be higher than those for ARMA. The differences, however, are not significant according to the Diebold-Mariano (1995) and the modified Diebold-Mariano (Harvey *et. al*, 1997) tests. As the results given in Section 3 ruled out the possibility of fractional integration for inflation in China, we have decided not to consider ARFIMA models here.

Another approach for evaluating the robustness of the results to model uncertainty is to check whether the uncertainty about the point forecasts causes distortions in the estimated probabilities. In this case, we do not assume anything about the type of forecasting model. Instead, we assume that there might be a large number of competing forecasting models, without any certainty about which one is better. The differences between their forecast can be expressed by a random variable. The standard errors in Table 3 above have been obtained under the assumption that this uncertainty is expressed by a normally distributed random variable, while Table 5 also shows the probabilities of negative inflation and their standard deviations for two other distributions: the uniform and the gamma, where the shape and scale parameters are both equal to unity for the gamma distribution. These account for possible distributional misspecification, as the competing models produce forecasts that are symmetrically distributed with equal probability (the uniform distribution), and highly asymmetric with the mass to the left of the mean (gamma distribution). The distributions have been centred around the true values of inflation and scaled by standard deviation, so that the differences in the negative inflation probabilities and their standard deviations can be attributed solely to the differences in the shape of the distributions. For each case, 1,000 forecasts have been generated and evaluated.

Table 5 ABOUT HERE

The results given in Table 5 indicate that the differences in the estimated probabilities are not large for cases where the probability of negative inflation is relatively substantial and exceeds 15%. However, for such probabilities the standard deviations for the uniform and gamma distributions are markedly higher than that for the normal distribution. Although the uniform and highly asymmetric gamma distributions can be regarded as quite extreme cases of misspecification, this result shows that a possible deviation of the distribution of forecast uncertainty from normality may well result in an increase in the standard errors of the estimated probabilities. However, its effect on the probabilities themselves is negligible.

The second potential reason for the lack of robustness might be misspecification of the functional form of the copula model used for the approximation of forecast errors. More precisely, the results might depend on setting types of marginal distributions, type of copula and strength of the copula dependence. It is important to find out how far these settings might affect the outcomes.

Consequently, the following robustness check has been performed. The computations were repeated for three different sets of marginals: (1) decided by the minimum of the Hellinger distance for each forecast horizon, which is TPN for all horizons but 2 and 4, for which it is WSN; (2) using WSN for all forecast horizons; and (3) using TPN for all forecast horizons. Next, three different types of copula were used: two Student-*t* copulas, one with the number of degrees of freedom equal to 4 and one with it at 10, and the normal copula. Finally, three different types of scatter matrix were used: (1) computed from the data on forecast errors for different horizons as a Pearson ρ coefficient; (2) estimated by the method of moments; and (3) with all elements arbitrarily set at 0.9. Combining these settings gives 27 different models for which the computations described in Section 3 were made.

As a concise indicator of robustness, we use the standard deviations across the 27 models of the expected duration of the episodes of negative inflation. These are presented in Table 6, together with the minimum and maximum expected duration for all periods from 2 to 11. Table 6 shows that for most forecast horizons, the results are relatively robust to changes in the specification of the marginal distributions and the copula. The standard deviations of the expected durations are smaller than one except for the forecast horizons 2, 4 and 5.

To find out which model has the biggest impact on robustness, we evaluated the average standard deviation of the duration of negative inflation for all the models, and then for all the models excluding one (see Table 7). The ratio of the latter to the former gives an indication of how much the exclusion of a particular model affects the overall variability of the results.

Table 6 ABOUT HERE

The results in Table 7 indicate robustness in the sense that there was no model specification which could affect the dispersion of the duration of negative inflation by more than 2%. We can, therefore, conclude that the technique proposed here is reasonably robust to changes in the specification of the marginal distributions and the copula.

Table 7 ABOUT HERE

5. CONCLUSIONS

The proposed methodology of inference on the joint distribution of forecast errors is conceptually straightforward. For non-trivial marginal distributions of forecasts the technique is quite demanding in computer power, as precise computation of the inverse transformations might be tedious. Nevertheless, it allows for quite simple analysis of the marginal and joint forecast distributions in the relatively complex cases where the marginal distributions might belong to different families. Our results appear to be relatively robust to model uncertainty, choice of the copula, and the type of marginal distributions. However, the out of sample predictive performance has not been evaluated and should be a topic of further research.

An application presented here is the evaluation of the probabilities of negative inflation and its expected duration for China. The results show that in 2014-2015, the *ex-ante* probability of negative inflation was indeed not negligible, touching 20% for the 12-months-ahead forecast horizon. However, if negative inflation really did happen, it would most probably not last longer than three consecutive months before the end of the forecast horizon. Another possible application of the technique proposed can be for evaluating the length of time for which inflation can be within or outside a specific interval, for instance, the target zone set by monetary authorities.

Apart from the heavy computational burden, the practical disadvantage of the proposed technique is that the expected duration of events can be evaluated only within the forecast span, meaning it cannot exceed the maximum forecast horizon. This can be overcome if the forecasts can be made for relatively long periods, and the interest is in horizons markedly shorter than the maximum. In this case, it is likely that the probabilities of events appearing at the period of interest and also the maximal forecast horizon would be small, so that the evaluated expected length of duration can be regarded as practically unconstrained by the limited forecast span.

Another further development would be in applying a wider definition of deflation, allowing for its relationship with aggregate demand. This would imply that a joint distribution of inflation and growth is used, which allows for conducting a conditional inference, hopefully contributing to the continuing discussion on the relationship between deflation and real sphere.

Appendix A: the WSN and TPN distributions

The random variable Z with WSN (weighted skew-normal) distribution, as defined by Charemza, *et. al* (2015a), has six parameters, $\alpha, \beta, \tau_{low}, \tau_{up}, \rho, \sigma$, and is given by:

$$Z = X + \alpha \cdot Y \cdot I_{Y>\tau_{up}} + \beta \cdot Y \cdot I_{Y<\tau_{low}} \quad ,$$

where: $I_{\{\bullet\}}$ is indicator function of a set $\{\bullet\}$, $\tau_{low} < \tau_{up}$; $\alpha, \beta \in \mathbb{R}$; $\sigma^2 \in \mathbb{R}^+$; and $|\rho| \leq 1$, and

$$(X,Y) \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2 & \rho\sigma^2 \\ \rho\sigma^2 & \sigma^2 \end{bmatrix} \right).$$

For $\sigma = 1$, its probability density function (*pdf*) is given by:

$$f_{\mathrm{WSN}_1}(t) = \frac{1}{\sqrt{A_\alpha}} \varphi\left(\frac{t}{\sqrt{A_\alpha}} \right) \Phi\left(\frac{B_\alpha t - mA_\alpha}{\sqrt{A_\alpha(1-\rho^2)}} \right) + \frac{1}{\sqrt{A_\beta}} \varphi\left(\frac{t}{\sqrt{A_\beta}} \right) \Phi\left(\frac{-B_\beta t + kA_\beta}{\sqrt{A_\beta(1-\rho^2)}} \right)$$
$$+ \varphi(t) \cdot \left[\Phi\left(\frac{m - \rho t}{\sqrt{1-\rho^2}} \right) - \Phi\left(\frac{k - \rho t}{\sqrt{1-\rho^2}} \right) \right] \quad ,$$

where φ and Φ denote respectively the density and cumulative distribution functions of the standard normal distribution, and $A_\tau = A(\tau) = 1 + 2\tau\rho + \tau^2$, $B_\tau = B(\tau) = \tau + \rho$.

The random variable with TPN distribution is defined by its *pdf*:

$$f_{TPN}(t; \sigma_1, \sigma_2, \mu) = \begin{cases} A \exp\left\{ -(t-\mu)^2 / 2\sigma_1^2 \right\} & \text{if } t \leq \mu \\ A \exp\left\{ -(t-\mu)^2 / 2\sigma_2^2 \right\} & \text{if } t > \mu \end{cases} \quad , \quad t \in \mathbb{R} \quad ,$$

where $A = \sqrt{2/\pi} \cdot (\sigma_1 + \sigma_2)^{-1}$. The parameters are $\sigma_1, \sigma_2 \in \mathbb{R}^+$ and $\mu \in \mathbb{R}$.

The two-piece normal, TPN, distribution was originally proposed by John (1982) and was developed further by Kimber (1985). It has often been used for approximating distributions of forecast errors in constructing probabilistic forecasts of inflation and output (see e.g. the seminal paper by Wallis, 2004).

The parameters of the WSN and TPN density functions were estimated by the simulated minimum distance method (*SMDE*, see Charemza *et al.*, 2012). In order to achieve identification of the WSN, it has been assumed that $m = -k = 1$ and $\rho = 0.75$, reducing the number of parameters estimated to three, these being α , β and σ .

Appendix B: estimates of the parameters of marginal distributions

for. hor.	type of distribution	if WSN: $\hat{\alpha}$ / if TPN: $\hat{\sigma}_1^2$	if WSN: $\hat{\beta}$ / if TPN: $\hat{\sigma}_2^2$	if WSN: $\hat{\sigma}$ / if TPN: $\hat{\mu}$
1	TPN	0.6955 (0.0068)	0.0185 (0.0256)	0.4625 (0.0361)
2	WSN	-1.280 (0.2258)	-0.454 (0.0965)	0.6429 (0.0208)
3	TPN	0.9148 (0.4199)	0.5127 (0.2130)	0.2014 (0.3700)
4	WSN	-1.465 (0.2563)	-0.7817 (0.0050)	1.0370 (0.1505)
5	TPN	0.0191 (0.2670)	2.344 (0.0119)	-1.7680 (0.0863)
6	TPN	2.1640 (0.0337)	0.0131 (0.3161)	1.1340 (0.4311)
7	TPN	2.1100 (0.1701)	0.0696 (0.1700)	1.0990 (0.2668)
8	TPN	2.2200 (0.1552)	0.0124 (0.0075)	1.1100 (0.2951)
9	TPN	2.3420 (0.6821)	0.0400 (0.0951)	1.2310 (0.1749)
10	TPN	2.5130 (0.8324)	0.0428 (0.1037)	1.2240 (0.3808)
11	TPN	2.8700 (0.4871)	0.3530 (0.5239)	1.2150 (0.7365)
12	TPN	3.0780 (0.1803)	0.1013 (0.2926)	1.4650 (0.0503)

Legend: standard errors of the estimates are given in brackets.

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Table 1: Unit root tests for inflation

	ERS	*MPT*	MZ_{α}	*MSB*	MZ_{t}	*ADF*
no break	2.10**	2.04**	-12.03**	0.20**	-2.45**	-2.34
1 break	5.70	5.61	-29.40***	0.13***	-3.83***	-3.04**
2 breaks	14.71	13.53	-15.79**	0.18**	-2.78***	-2.40
3 breaks	15.93	14.22	-13.71**	0.19**	-2.61***	-2.39

Legend: (1) *, **, *** indicate the test statistics are significant at the 10%, 5% and 1% significance levels.

Table 2: Goodness of fit measures for inflation forecast errors

f.hor	Normal		alpha-stable		WSN		TPN	
	HD	χ^2	*HD*	χ^2	*HD*	χ^2	*HD*	χ^2
1	14.992	17.522	51.444	76.905	4.370	4.167	**3.105**	**3.185**
2	8.601	9.715	40.933	52.456	**5.792**	**5.691**	6.270	6.166
3	0.964	0.964	26.126	26.530	0.748	0.757	**0.330**	**0.300**
4	1.422	1.463	16.180	16.663	**1.200**	**1.249**	1.238	1.347
5	17.876	21.315	20.384	24.933	15.050	17.490	**3.446**	**10.100**
6	6.945	7.475	6.047	6.132	5.436	5.989	**2.648**	**2.732**
7	5.815	5.741	7.531	7.234	4.365	4.341	**2.242**	**2.271**
8	7.681	9.462	9.068	9.677	6.144	7.259	**1.244**	**1.310**
9	3.304	3.674	1.929	1.965	2.288	2.457	**0.645**	**0.654**
10	7.421	8.469	5.488	5.572	5.730	6.592	**2.726**	**3.058**
11	2.429	2.498	1.395	1.537	1.509	1.523	**0.471**	**0.492**
12	4.414	5.757	8.456	9.157	3.394	4.216	**1.013**	**1.016**

Legend: Boldfaced entries indicate minimum *HD* and χ^2 statistics for the given forecast horizon across the distributions.

Table 3: Estimated probabilities of short episodes of deflation, April 2014-March 2015

Date	*f.hor*	*infl.%*	*prob.defl.*	*st.dev.*	*boot.st.dev.*
Apr 2014	1	2.0	0.0003	0.0000	0.0002
May 2014	2	2.5	0.0000	0.0000	0.0000
June 2014	3	2.0	0.0097	0.0006	0.0011
July 2014	4	1.9	0.0221	0.0010	0.0014
Aug 2014	5	1.8	0.0007	0.0036	0.0002
Sep 2014	6	1.5	0.2225	0.0049	0.0041
Oct 2014	7	1.4	0.2290	0.0059	0.0042
Nov 2014	8	1.7	0.2034	0.0047	0.0041
Dec 2014	9	1.8	0.1941	0.0045	0.0039
Jan 2015	10	2.1	0.1838	0.0032	0.0039
Feb 2015	11	2.3	0.1981	0.0026	0.0039
Mar 2015	12	2.3	0.2154	0.0025	0.0041

Legend: *f.hor* is the forecast horizon; *infl%* is observed headline annual inflation recorded monthly; *prob.defl* is the probability of negative inflation for the month indicated, computed according to (2); *st.dev.* is its standard deviation computed across the simulated competing forecasts; *boot.st.dev.* is standard deviation computed by the *n out of n* bootstrap. The standard deviations are functions of the imposed forecast uncertainty and hence are, to an extent, arbitrary.

Table 4: Expected duration of the episodes of negative inflation

f.hor (*h*)	*prob.defl.*	*prob.defl. starting*	*exp.length*	*exp.length starting*	*prob.defl at f.hor h and 12*	*cond.prob. defl at f.hor h and 12*
≤5	0.0328	0.0328	2.81 (1.86)	2.82 (1.86)	0.0000	0.0000
6	0.2225	0.0421	2.48 (1.59)	1.90 (1.40)	0.0985	0.4784
7	0.2290	0.0645	2.25 (1.35)	2.06 (1.29)	0.0934	0.4984
8	0.2034	0.0454	1.97 (1.12)	1.75 (1.05)	0.1345	0.5390
9	0.1941	0.0592	1.69 (0.88)	1.56 (0.83)	0.1141	0.4955
10	0.1838	0.0605	1.47 (0.72)	1.38 (0.72)	0.2030	0.4782
11	0.1981	0.1462	1.35 (0.48)	1.10 (0.47)	0.1651	0.5118
12≤	0.2154	0.1429	1.00≤ (0.00)	1.00≤ (0.00)	0.3379	1.000

Legend: *prob.defl* is as in Table 3, with probabilities for forecast horizons 1 to 5 cumulated; *prob.defl.starting* is the probability of negative inflation starting at a given forecast horizon; *exp.length* is the expected duration of negative inflation observed at the given horizon, see (5); *exp.length.starting* is the expected duration of negative inflation within the forecast span which starts at the given horizon, see (6); *prob.defl at h and 12* is the probability of negative inflation at period *h* and 12, and *cond.prob. defl at h and 12* is the probability of negative inflation in periods *h* and 12, if negative inflation happens in period *h*. In columns 4 and 5 figures in brackets denote standard deviation of the random variable defined by (5).

Table 5: Effects of model uncertainty on the estimated probabilities of negative inflation

	normal		Uniform		Gamma	
f.hor	*prob.defl.*	*st.dev*	*prob.defl.*	*st.dev*	*prob.defl.*	*st.dev*
1	0.0003	0.0000	0.0008	0.0000	0.0003	0.0000
2	0.0000	0.0000	0.0000	0.0000	0.0008	0.0000
3	0.0097	0.0006	0.0061	0.0005	0.0096	0.0007
4	0.0221	0.0010	0.0231	0.0049	0.0225	0.0012
5	0.0007	0.0036	0.0832	0.0003	0.0007	0.0000
6	0.2225	0.0049	0.2144	0.1701	0.2222	0.0412
7	0.2290	0.0059	0.1958	0.1913	0.2257	0.0461
8	0.2034	0.0047	0.1783	0.4547	0.2062	0.1161
9	0.1941	0.0045	0.1921	0.3364	0.1948	0.0890
10	0.1838	0.0032	0.2083	0.3031	0.1771	0.9462
11	0.1981	0.0026	0.1942	0.7973	0.1964	0.2009
12	0.2154	0.0025	0.2230	0.9786	0.2133	0.2503

Table 6: Robustness measures of the expected duration of negative inflation

f.hor	st.dev	minimum	maximum
2	1.221	0.000	4.147
3	0.882	0.000	2.389
4	1.210	1.000	5.367
5	2.319	1.238	7.625
6	0.787	1.236	3.275
7	0.669	1.210	2.960
8	0.565	1.202	2.656
9	0.443	1.153	2.348
10	0.329	1.136	1.994
11	0.157	1.105	1.520

Table 7: Contribution of particular models to the robustness check

Model excluded:	Marginal distr. Type	Scatt. mat.	Copula type	St dev ratio
1	All WSN	Pearson	Student-*t*, 4 dof's	0.9833
2	All WSN	Pearson	Student-*t*, 10 dof's	0.9848
3	All WSN	Pearson	normal	0.9865
4	All TPN	0.9	Student-*t*, 4 dof's	0.9873
5	Mixed TPN/WSN	Pearson	Student-*t*, 4 dof's	0.9876
6	Mixed TPN/WSN	MM	normal	0.9878
7	All TPN	0.9	Student-*t*, 10 dof's	0.9882
8	All TPN	0.9	normal	0.9884
9	Mixed TPN/WSN	MM	Student-*t*, 10 dof's	0.9846
10	**Mixed TPN/WSN**	**MM**	**Student-*t*, 4 dof's**	**0.9902**
11	Mixed TPN/WSN	Pearson	Student-*t*, 10 dof's	0.9915
12	All WSN	MM	Student-*t*, 4 dof's	0.9929
13	All TPN	MM	normal	0.9934
14	All TPN	MM	Student-*t*, 10 dof's	0.9934
15	Mixed TPN/WSN	0.9	normal	0.9942
16	All WSN	MM	Student-*t*, 10 dof's	0.9942
17	All TPN	MM	Student-*t*, 4 dof's	0.9945
18	All WSN	MM	normal	0.9947
19	Mixed TPN/WSN	0.9	Student-*t*, 10 dof's	0.9954
20	Mixed TPN/WSN	Pearson	normal	0.9957
21	Mixed TPN/WSN	0.9	Student-*t*, 4 dof's	0.9971
22	All WSN	0.9	Student-*t*, 4 dof's	0.9996
23	All WSN	0.9	normal	1.0000
24	All WSN	0.9	Student-*t*, 10 dof's	1.0000
25	All TPN	Pearson	Student-*t*, 4 dof's	1.0660
26	All TPN	Pearson	Student-*t*, 10 dof's	1.0730
27	All TPN	Pearson	normal	1.0770

Legend: Mixed TPN/WSN denotes the models where the marginal distributions for the forecast horizons 2 and 4 are those of WSN, and the remaining cases are those of TPN. In the column labelled Scatt. mat., Pearson, Pearson correlation coefficients have been used as the estimates of the elements of the scatter matrix; MM means that these elements have been obtained by the methods of moments as in Demarta and McNeil (2005), and 0.9 means that they all have been pre-assigned and are equal to 0.9. Entries for the model discussed in detail in Section 3 are boldfaced.

Figure 1: Inflation in China, January 2015-April 2015

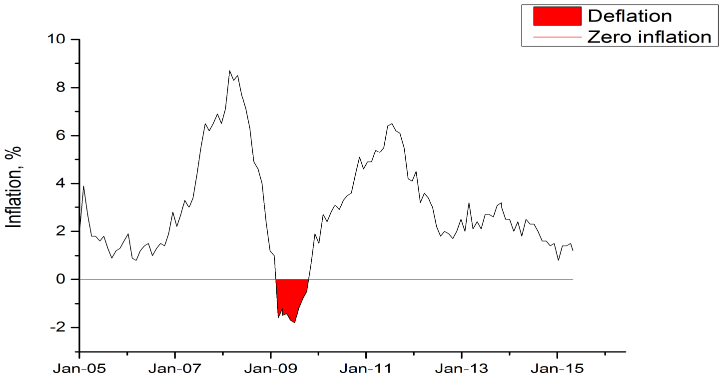


Figure 2a: Characteristics of two-dimensional distributions of forecast errors

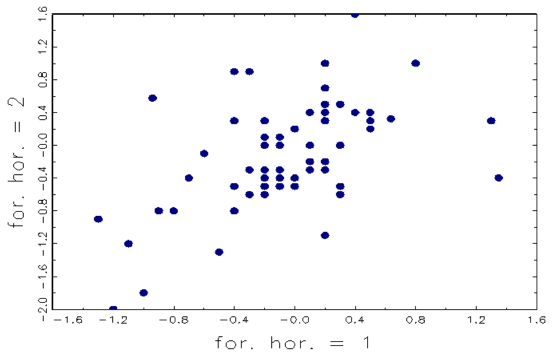
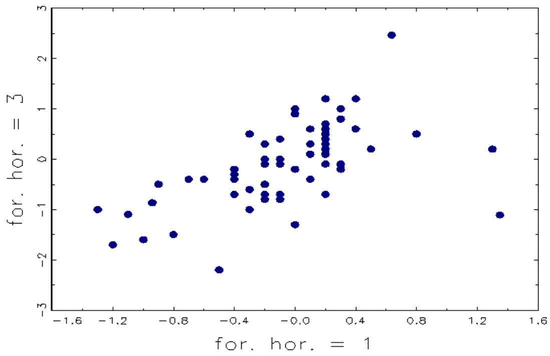
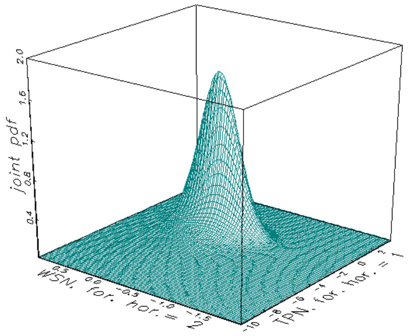
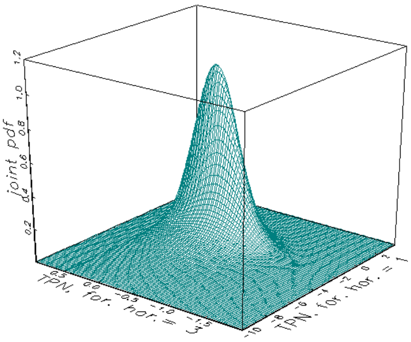
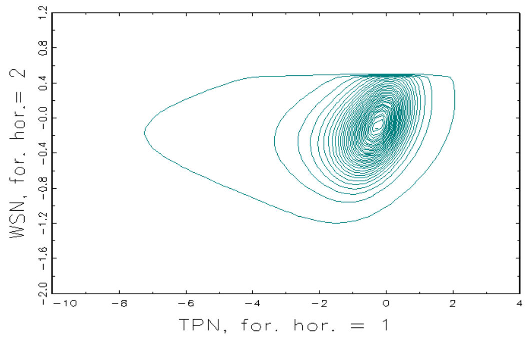
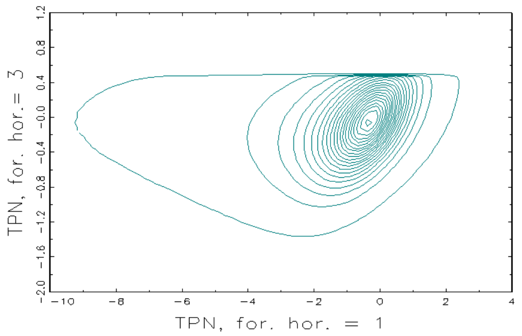
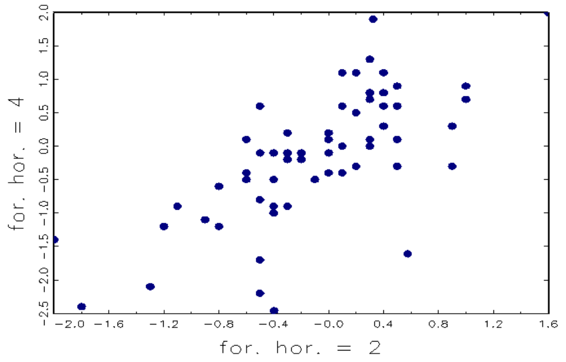
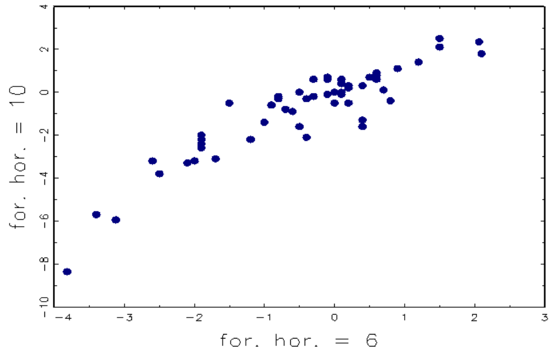
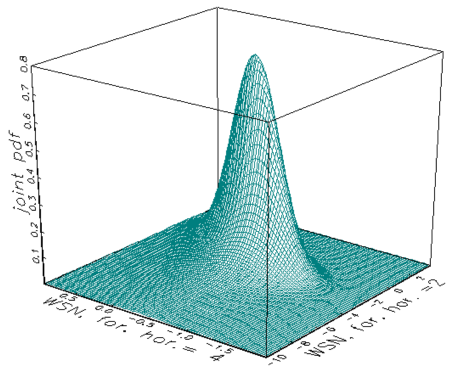
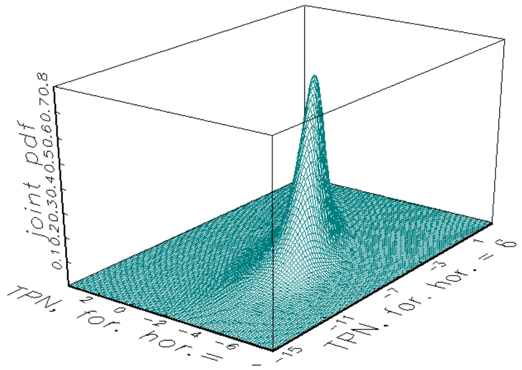
forecast hors 1 and 2 (TPN & WSN)			forecast hors 1 and 3 (TPN & TPN)		
Kendall τ (*p*-value)	Pearson's ρ	Scatter matrix element	Kendall τ (*p*-value)	Pearson's ρ	Scatter matrix element
0.375 (0.000)	0.529	0.555	0.455 (0.000)	0.559	0.656
scatter diagram			scatter diagram		
					
joint *pdf*			joint *pdf*		
					
contours of joint *pdf*			contours of joint *pdf*		
					

Figure 2b: Characteristics of two-dimensional distributions of forecast errors

forecast hors 2 and 4 (WSN & WSN)			forecast hors 6 and 10 (TPN & TPN)		
Kendall τ (*p*-value)	Pearson's ρ	Scatter matrix element	Kendall τ (*p*-value)	Pearson's ρ	Scatter matrix element
0.526 (0.000)	0.700	0.736	0.695 (0.000)	0.864	0.914
scatter diagram			scatter diagram		
					
joint *pdf*			joint *pdf*		
					
contours of joint *pdf*			contours of joint *pdf*