FROM INTRINSIC TO NON-INTRINSIC GEOMETRY:
A STUDY OF CHILDREN'S UNDERSTANDINGS
IN LOGO-BASED MICROWORLDS

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ABSTRACT

The aim of the present study was to investigate the potential for children to use the turtle methaphor to develop understandings of intrinsic, euclidean and cartesian geometrical ideas. Four aspects of the problem were investigated.

a) the nature of the schema children form when they identify with the turtle in order to change its state on the screen;

b) whether it is possible for them to use the schema to gain insights into certain basic geometrical principles of the cartesian geometrical system;

c) how they might use the schema to form understandings of euclidean geometry developed inductively from specific experiences;

d) the criteria they develop for choosing between intrinsic and euclidean ideas.

Ten 11 to 12 year - old children participated in the research, previously having had 40 to 50 hours of experience with Turtle geometry. The research involved three case - studies of pairs of children engaging in cooperative activities, each case - study within a geometrical Logo microworld. The data included hard copies of everything that was said, typed and written.

Issues a) and b) were investigated by means of the first case - study which involved three pairs of children and a microworld embedding intrinsic and coordinate ideas. A model of the children's intrinsic schema and a model of the coordinate schema which they formed during the study were devised. The analysis shows that the two schemas remained separate in the children's minds with the exception of a limited number of occasions of context specific links between the two.

Issue c) was investigated in the second case - study involving one pair of children and a microworld where the turtle was equipped with distance and turn measuring instruments and a facility to mark positions. The analysis illustrates how a turtle geometric environment of a dynamic mathematical nature was generated by the children, who used their intrinsic schema and predominantly engaged in inductive thinking. The geometrical content available to the children within this environment was extended from intrinsic to both intrinsic and euclidean geometry.

Issue d) was investigated by means of the third case - study involving a pair of children and a microworld where the children could choose among circle procedures embedding intrinsic and/or euclidean notions in order to construct figures of circle compositions. The analysis shows that the children employed their turtle schema in using both kinds of notions and did not seem to perceive qualitative differences between them. Their decisions on which type of notion to use were influenced by certain broader aspects of the mathematical situations generated in the study.

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CHAPTER 1

INTRODUCTION

1.1 BACKGROUND TO THE RESEARCH

In the latter half of this century geometry has had a continually reduced role to play in mathematics curricula, at least in the United Kingdom. A major factor has been that Euclidean Geometry, which was taught as a tight deductive system and was considered an area of high status knowledge, came to be regarded as "inappropriate" for primary and secondary education since children could only master its deductive structure by rote learning. Research into children's geometrical understandings, starting from the work of Piaget, has highlighted on the one hand, the formal (in the Piagetian sense) nature of deductive thinking and on the other, pupils' difficulties in achieving such thinking in the context of geometry (Freudenthal, 1973, van Hiele, 1959). Thus the case for teaching geometry as a ready - made deductive system has inevitably become rather weak.

Research in mathematics education in general has given credence to the case put forward by cognitive psychologists and mathematics educators that the process of learning involves the reorganisation of personal experiences by acting on the environment, rather than the passive intake of quantities of information. The advent of the computer and specifically the dramatic increase of availability of micro - computers in classrooms which has begun in the last decade, has provided researchers with the opportunity to create interactive, dynamic computer environments where pupils can take control of their own learning. An increasing amount of research has recently been stimulated, investigating the educational potential of such environments on the one hand, and the learning processes of pupils engaged in activities within such environments on the other.

A good example of such environments is Turtle geometry, an important part of

the Logo computer language, which invites children to give commands to a turtle (a screen cursor with position and heading) to move or turn. The turtle can leave a visible trace when changing its position, thus enabling the children to form shapes and figures on the screen. Apart from increasing evidence that Logo can be a means to generate rich mathematical and programming environments for children, Papert and Lawler argue that turtle geometry has an especially promising characteristic, i.e. that children make particular sense of driving the turtle on the screen because they can identify with it and therefore relate to experienced bodily motion. On the other hand, Turtle geometry incorporates powerful geometric ideas which, according to Papert, belong to Intrinsic (Differential) geometry.

This notion of Turtle geometry seems to fit with the case for geometry made by Freudenthal, i.e. that it has an important role to play in education, if seen through a different perspective, i.e. if, as educators we exploit the relationship between geometry and the experienced space, since it is a unique opportunity to mathematise reality, but we also keep the option open for deductive geometry (Freudenthal, 1973). Furthermore, von Glasersfeld has argued that "the generation of deductive abilities in both logic and mathematics must be based on the practice of inductive inference" (von Glasersfeld, 1985, p.484). Recent research, however, has shown that children do not necessarily use geometrical ideas when doing Turtle geometry (Hillel et al, 1986, Hillel and Kieran, 1987, Leron, 1983).

In the present study, the nature of the "schema" 12 year - old children build when they identify with the turtle is investigated. Although the study is informed by other uses of the word "schema" (see section 2.1.1), there is an attempt throughout the thesis to form a meaning related to the research findings. Furthermore, the investigation extends to the potential for children's use of this schema to understand powerful ideas which are at the basis of geometrical systems other than the Intrinsic, thus extending the geometrical content available to the children from Intrinsic geometry to Euclidean and Cartesian. It was therefore important for the study to focus on both children's learning

processes and understanding of geometrical content.

The research was carried out in a primary school within the Greek educational system. Primary education in Greece is rather formal, i.e. the predominant assumption in the classroom is that the teacher has an amount of knowledge in his/her head and his/her job is to transmit it to the recipiants, i.e. the children. The pupils participating in the study, however, had had one year's experience with Logo in an informal investigative classroom atmosphere.

The four objectives of the study were:

1) to investigate the nature of the schema children form when they identify with the turtle in order to change its state on the screen;

2) to investigate whether it is possible for them to use the schema to gain insights into certain basic geometrical principles of the Cartesian geometric system;

3) how they might use the schema to form understandings of euclidean geometry developed inductively from specific experiences;

4) to investigate the criteria they develop for choosing between intrinsic and euclidean representations of geometrical ideas.

1.2 THE THEORETICAL FRAMEWORK FOR THE STUDY

The theoretical framework of the study is based on the role of Logo and Turtle geometry within a specific view of mathematics education which emphasises the process of learning as an on - going reorganisation of personal experience, rather than an effort to describe some ontological reality.

The "constructivist" perspective regarding the development of knowledge, which seems to be influencing more and more mathematics educators

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(Kilpatrick, 1987), was first considered in "scientific" terms - rather than a purely philosophical question - by Piaget. There are many definitions or descriptions of Constructivism as a theory of knowledge - Piaget himself has described Constructivism several times, according to emphases within particular contexts. According to Sinclair, what Piaget meant by "interactive" or "dialectical" Constructivism incorporates the idea that "the essential way of knowing the real world is not directly through our senses, but first and foremost through our actions... (i.e.) ...all behaviour by which we bring about a change in the world around us or by which we change our own situation in relation to the world" (Sinclair, 1987, p. 28). Epistemological debate on Constructivism, however, has pushed this main idea to extremes such as "all knowing is active and all knowledge is subjective" (Kilpatrick, 1987, p. 10). The constructivist view involves the following main ideas:

- knowledge is actively constructed by the cognising subject, not passively received;

- coming to know is an adaptation process that organizes one's experiential world; it does not discover an independent, pre - existing world.

Although there seems to be general agreement on the former principle (von Glasersfeld, 1985, Cobb, 1986), the latter has raised considerable debate, the main bulk of which, the author believes is of a philosophical nature, rather than an educational one. Acceptance of the last clause, i.e. the questioning of the existence of an ontological reality by arguing that the results of all cognitive construction are necessarily subjective has been labelled "radical Constructivism" (von Glasersfeld, 1985). There have been attempts to deal with the obvious shortcoming of "denouncing" reality, by proposing a definition of objectivity which, as von Glasersfeld puts it, "does not require access to ontology", i.e. "objectivity arises when concepts, relations and operations that I have found to be viable in the management of my own experience, turn out to be viable also when I attribute them to the models of Others which I construct to manage my interaction with them" (von Glasersfeld, 1985, p.99).

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It could be argued however, that the epistemological debate raised by radical Constructivism has a questionable direct relevance to educational practice and that the extremity of the "radical" viewpoint may have been influenced by a polarisation resulting from cognitive scientists' reactions to behaviourism over the last 20 years (Resnick, 1983). The lack of emphasis on the social nature of learning, for instance, does not take into account the reasons for the widely acknowledged discrepancies between solitary and collaborative learning highlighted by Vygotsky's notion of the "zone of proximal development", which he defined as the distance between the actual developmental level during solitary learning and the potential developmental level, determined in situations involving adult guidance or collaboration with more able peers (Vygotsky, 1978).

Mathematics educators, however, have found it useful to adopt key ideas involved in the constructivist perspective, such as the principle of "learning by doing" (Papert, 1972) rather than learning by receiving information, without ignoring the social nature of learning. "I see constructivism as the best way to consider the process of appropriation by which a student makes mathematics his own knowledge. Rather than a pure and lonely construction, the learning of mathematics is for me the difficult appropriation of a social knowledge" (Vergnaud, 1987a, p.53).

It is particularly useful for the present study to employ a theoretical framework for learning mathematics which, on the one hand, incorporates ideas about pupils' learning processes when they are engaged in activities which foster "learning by doing" (often, for example, in the case of Logo activities), and on the other, examines the mathematical content available to the pupils during such situations of active learning. As Vergnaud put it, "the choice of these situations cannot be made without reference to mathematics as a science, and to the developmental process of mathematical schemes and concepts in students' minds... adaptation does cope with the actual world, and not with a purely imaginary science" (Vergnaud, 1987a, p.p.53 and 46).

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Certain notions concerning these two domains, are useful for the theoretical framework of the present study, during which the research focused equally upon the process by which children develop understandings of geometrical ideas in Logo activities and on the nature of those ideas within the structure of wider geometrical domains.

An important notion regarding the learning process is children's developments of operational invariants in situations where they perform actions. The notion is not a new one, since invariants are a recurrent topic in Piaget's work, such as, for instance, the conservation of volume in situations of transferring some liquid from a narrow to a wide glass, a "norm" which, according to Piaget, develops at around the age of 10. However, Vergnaud has focused on this notion from a mathematical perspective, drawing attention to the invariance of relations and to children's implicit and localised use of "powerful properties" or concepts. Vergnaud maintains that children should be given the opportunity to form "theorems in action" - his term for relational invariants -, since "before being objects, concepts are cognitive tools, and many theorems should be 'theorems in action' before being explicit theorems, especially at the primary and secondary level" (Vergnaud, 1987a). The notion of a concept being used as a tool within a situation before becoming an object was put forward by Douady; "We say that a mathematical concept is a tool when our interest is focused on the use to which it is put in solving problems. By object we mean the cultural object, which has a place in the body of scientific knowledge, at a given time, and which is socially recognised" (Douady, 1985, p.35).

For the present study, however, awareness of the mathematical structure of the situations within which pupils formed theorems in action and used concepts as tools was of equal importance to the awareness of pupils' thinking processes. Consequently, the notion of "conceptual field", put forward by Vergnaud, was an important element for the theoretical framework of the study. According to Vergnaud, a concept can be described as a triplet (S, I, Z), where;

S is a set of situations which make the concept meaningful;

I is a set of invariants that constitute the concept;

Z is a set of symbolic representations used to represent the
concept, its properties and the situations it refers to.

For Vergnaud, however, it is not so useful to examine a concept in isolation, but rather within more than one situations, involving on the one hand different properties of the same concept, and on the other, a variety of concepts. Furthermore, pupils' formation of concepts relies on a meaningful application and adaptation of former conceptions. Vergnaud therefore makes a case against studying "small - sized" objects when the objective is to understand the processes by which pupils master mathematics. As an alternative, he offers the notion of "conceptual field", i.e. "...a set of situations, the mastery of which requires a variety of concepts, procedures and symbolic representations tightly connected with one another" (Vergnaud, 1982, p.36). In the present study, situations were designed within which the pupils could explore and solve problems in Turtle geometry. The conceptual field, or the mathematical structure of these situations was carefully analysed by the researcher before the study took place (chapter 3 and sections 6.1.3, 7.1.4 and 8.1.3).

The situations generated in the study, involved pupils' activities within "turtle environments" which were designed by the researcher to have an underlying specific geometrical structure. The notion of a "microworld" seems the most useful for describing such environments. For Papert, microworlds are "places to get to know one's way around a set of concepts, problem situations, activities; places in which the student and teacher can test out ideas in a subject domain of interest" (Weir, 1987, p.12). A main feature of microworlds is that pupils can start exploring without much prerequisite knowledge about the underlying mathematics. They can therefore experiment and try out personal (sometimes wrong) theories. In microworlds, right or wrong are not the decisive criteria... "the child is learning... as a means to get to a creative and personally defined

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end" (Papert, 1980, p.134). According to Lawler, a microworld embodies the simplest model the expert can imagine, for entering a "rich" mathematical area (Lawler, 1982). Hoyles and Noss described the notion of a microworld as a set of Logo based situations constructed so that the pupil will come up against embedded mathematical ideas in the context of meaningful activity (Hoyles and Noss, 1987b). An elaborated example of a microworld is that of Turtle geometry in Logo, where, as will be discussed in chapter 2, the children find an easy entry point by employing their experience of bodily motion to drive the turtle on the screen, but have available a "conceptual field" incorporating powerful ideas belonging to Intrinsic geometry. For the author, the terms "conceptual field" and "Logo microworld" are related, since it could be argued that the latter is a specific case of the former.

In the present research, a case - study method is adopted, involving a detailed observation of pairs of children working collaboratively with a computer by engaging in activities within microworlds designed by the author to have the characteristics of specific conceptual fields. Although the entry point to all three microworlds is the turtle, the embedded concepts not only belong to Intrinsic geometry, as in the standard Turtle geometry microworld, but also to the Euclidean and Cartesian geometrical systems.

ASIDE: The writer of this thesis refers to himself as "the author" in chapters 1 and 2 and as "the researcher" in the remaining parts of the thesis.

1.3 AN OVERVIEW OF THE CONTENTS OF THE THESIS

Although it may be unrealistic to isolate process from content, the dialectic between the two domains, discussed in Capter 1, provided the researcher with a basis for the structure of Chapter 2, where the literature related to the study is reviewed in two parts, the first involving issues concerning children's mathematical learning processes and the second reviewing such processes within specific geometrical contents. Chapter 3 contains an analysis of the general mathematical principles underlying the microworlds in the present

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study. In chapter 4, the methodology for the present study is discussed and an overview of the research is given. The research involved a preliminary phase which is presented in chapter 5. The main research consisted of three case - studies. The findings for each case - study are presented in each of the following three chapters (chapters 6, 7 and 8) respectively. Due to the detailed design of each case - study, it was seen as clearer for the reader to incorporate a presentation of the design at the beginning of each chapter. For instance, the design of the case study presented in chapter 6, is at the front of the same chapter, and so on. The conclusions are presented in chapter 9.

CHAPTER 2

A REVIEW OF THE LITERATURE

2.1 CHILDREN'S MATHEMATICAL LEARNING PROCESSES

The first two parts of section 2.1 contain a review of the literature on the qualitative development of children's thinking and their processes of symbolising in mathematics. Children engaged in Logo activities often have opportunities to form different representations of the same idea (i.e. Logo "code", graphics, "acting out" the turtle's movements) in environments where they can take substantial control over their learning and discover things for themselves. The author found that a review of the literature on the two issues mentioned above provided him with an informative backgound for interpreting the thinking processes of the children participating in the study while they were engaged in activities with Logo. Sections 2.3 and 2.4 contain a review of research over the last ten years concerning the processes of children's learning in open - ended Logo environments.

2.1.1 Knowledge organisation

What we know about the way children think has been greatly influenced by the work of Jean Piaget. He was revolutionary in his approach to learning since, contrary to common belief at the time, he perceived the child as an active learner, an actor, rather than someone who is passively acted on by the environment. Moreover, he regarded cognitive growth as an essentially qualitative change in the organisation of knowledge in the mind rather than a quantitative gathering of increasing amount of knowledge. The nature of cognitive growth, as a qualitative re - organisation of knowledge, is of primary concern in this section.

An important part of Piaget's theory involves the dynamic process by which the child learns, the essence of which can be described as follows; Piaget's

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central notions for describing learning are "scheme", "assimilation", "accommodation" and "equilibration". Assimilation is the application of an existing cognitive structure or scheme to a new situation, while accommodation involves the reorganisation of a scheme as a result of new experiences. "A scheme, for Piaget, is a dynamic totality that ties together all the ingredients of a functional activity and can both accommodate to new situations and assimilate them" (Vergnaud, 1987b, p.231). Equilibration coordinates the three factors which, according to Piaget, influence cognitive growth, i.e. organic growth, experience with the physical world and experience with the social world. Equilibration involves a process of the reorganisation of schemes through assimilation and accomodation.

Although this part of Piaget's theory contains very important ideas about how children learn - the notion of learning as acting on the environment rather than receiving knowledge seldom having generated criticism - there have been attempts to illuminate further the process by which what Piaget called "equilibration", i.e. how knowledge is organised in the mind, comes about (Lawler, 1985, DiSessa, 1982). A common contention between Lawler and diSessa is that knowledge is essentially fragmented and learning takes place via the acquisition and reorganisation of disparate pieces of knowledge.

Lawler's central contention, based on Minsky's theory of "frames" (Minsky, 1975 and 1977), involves the construction of mind as a process of genesis and interaction of "microviews", i.e. fragmentary views of the world. The terms "microworld" and "microview" are central to Lawler's thesis. Microworld is a fragment of the world perceived by the child as disparate. Microviews "are internal, cognitive structures built through interacting with... microworlds and reflecting that fragmentary process of knowing" (Lawler, 1985, p.193). They are like content - specific frameworks into which problems and real life situations are assimilated. However, unlike Piaget's somewhat similar idea of scheme, Lawler focuses on, and tries to explain the relationships between microviews and how they evolve. For instance, he maintains that microviews are linked in an intricate genetic network. Some of them are descendants of

one or more others, some of them co-exist with no apparent relations but possibly with common "ancestral" microviews. He also puts forward the concept of dominant and sub - dominant microviews. There are not only "microviews of knowledge which dominate problem solving behaviour but also... sub - dominant microviews which do not normally dominate behaviour but which with intervention do so" (Lawler, 1985, p.105).

The notion of differing frameworks of knowledge existing simultaneously in the child's mind was also put forward by Booth as a result of a project involving a small - group teaching experiment to investigate children's errors in elementary algebra (Booth, 1984b) highlighted by the earlier CSMS research (Hart, 1980). Analysis of the data from the CSMS project had yielded that children use "naive intuitive strategies" rather than the "proper" mathematics taught them at school (Booth, 1981). Booth noticed, however, that, after participating in the relatively short teaching experiment of the SESM project, the children improved their performance regarding acceptance of "lack of closure" (Collis, 1974) and formalisation of method. She consequently suggested that the cognitive structures necessary for such assimilations were already available to the children and the reason why they did not use them was the inappropriateness of the framework of reference within which they were working, i.e. an arithmetic framework instead of an algebraic framework (Booth, 1984a). In Lawler's terms, the children could have already developed a microview for arithmetic and a microview for algebra, each related to an apparently different ancestral microview. What could have happened as a result of the teaching experiment, was the employment of the microview for algebra, which until that stage, was sub - dominant in the children's problem - solving behaviour.

The idea of simultaneous existence of pieces of knowledge in the mind and their invoking via a priority system, is central to diSessa's proposition of knowledge organisation. He discussed the role of intuitive epistemology in learning through the domain of physics. DiSessa maintained that physical knowledge is based on intuitions which originate in naive interpretations of

personal interactions with the physical world. He called the components of these intuitions "phenomenological primitives", or "p - prims", i.e. the intuitive equivalents of physical laws (diSessa, 1983). P - prims are organised in the mind so that they are evoked to make sense of situations according to a priority system. Two kinds of priority determine whether a p - prim will be invoked to make sense of a specific situation: "cuing priority", which has to do with how likely it is for a p - prim to be called upon, and "reliability priority" referring to the resistance to abandoning a p - prim once it is invoked. For diSessa, experience can initiate a reorganisation of p - prims, for instance a rearranging of cuing priority and reliability priority, the inclusion of new p - prims or the split of a p - prim into two or more. He consequently suggests that they are likely to be responsible for difficulties with the interpretation of situations since "they are high priority naive phenomena which require drastic reduction of priority or rearrangement of priority structure to allow expert - like understanding" (diSessa, 1983, p.30), an argument which the author believes is consistent with Lawler's idea of dominant and sub - dominant microviews (Lawler, 1985) and with Booth's alternative frameworks of knowledge (Booth, 1984a). For diSessa, the difference between common sense and scientific reasoning "is not so much the character or even the content of knowledge, but rather its organisation. Experts have a vastly deeper and more complex priority system" (diSessa, 1983, p.32).

The issues discussed above refer to the process and the nature of cognitive growth. Although there is, of course, agreement that learning in children comes about through their experience with the world, recent researchers do not seem to accept that mental growth is as independent of the nature of these experiences as Piaget seemed to imply. The developing picture of knowledge as fragmented and context - specific and the idea that small discrete pieces of knowledge co - exist in the mind and are invoked according to a dynamic priority system is, in effect, a recognition of the limitations of Piaget's contention that children mostly learn independently and spontaneously; in the literature discussed above, a common implication is that the priority system by which a fragment of knowledge (or a microview, or a p - prim) is employed

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within a situation is strongly related to the nature of this situation. This criticism is informative in understanding children's learning through activities with Logo, where the environment in which the children cause changes has a specific mathematical structure.

Another aspect of Piaget's theory is the well - known, but largely controversial contention that children's thinking develops in stages, each stage characterised by a specific cognitive structure and reached by the child at a specific age. Criticisms of this theory can be split in two categories:

a) criticism within the stage theory, i.e. the rates of development and the consistency of development across tasks or domains (Flavell, 1977, Keating and Clark, 1980). Piaget has acknowledged that development is not consistent across tasks, and has described the inconsistency as "horizontal decalage".

b) existence of other factors influencing the child's observed thinking such as misunderstandings between child and researcher due to the former's inability to use disembedded language (Donaldson, 1978) and the "appropriateness of the framework of reference" within which the child is working (Booth, 1984a, Demetriou and Efklides, 1981).

Although the present study generally benefits from an awareness of those criticisms, certain aspects related to the stage theory are particularly informative regarding children's geometrical activities with Logo where, for instance, children's thinking often involves relations between geometrical ideas and bodily motion (Papert, 1980). The first aspect is Lawler's reasonable assumption that the basis of mind is to be found in the sensorimotor period and his consequent argument that much of the activity of early age is developing communicative links between subsystems of the sensorimotor system, which according to Lawler are: the somatic, locomotive, visual, manipulative and linguistic subsystems (Lawler, 1985). This could provide an indication of why the "turtle metaphor" makes sense to children,

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since using a schema based on movement and turning would be based on early experiences.

The second aspect involves certain characteristics of concrete and formal thinking and of the transition from the former to the latter. Concrete operational thinking involves, for instance, the ability to perform operations and to understand that each operation is reversible. It also involves inductive reasoning, i.e. arriving at a conclusion based on individual experiences. Formal operational thought involves the ability to think about ideas as well as objects. Deductive thinking is a characteristic of formal thinking, since it requires reasoning from the general to the particular. Part of the controversy of the stage theory concerns the age of transition from concrete to formal thinking; Piaget maintained that the transition started at the age of 11, while Collis' observations show that children do not develop formal thinking till the age of 16 and some of them never achieve formal thinking at all (Collis, 1974). According to Collis and Halford, who applied Piaget's stage theory to children's understanding of mathematics (mainly algebra), transition from the concrete to the formal stage involves the child's acceptance of "lack of closure" (Collis, 1974) and the development from understanding binary relations to understanding tertiary relations (Halford, 1978). The most important aspect of the transition for the present study however, is the development from inductive to deductive thinking i.e. when the objects of reasoning become disembedded from personal experience. As will be discussed in section 2.2, Geometry has widely been taught as a tight deductive system, even from the years of primary education. As a consequence, children have only been able to learn Geometry by rote, since they are not ready to understand the mathematical ideas involved in any depth, as argued by Freudenthal (Freudenthal 1973). Piaget would also agree on this point, since he has argued that "...when adults try to impose mathematical concepts on a child prematurely, his learning is merely verbal; true understanding of them comes only with his mental growth" (Piaget, 1953, in Hughes, 1986, p.16). The research issues of the present study are investigated within the context of children learning Geometry in an inductive way, i.e. by trying out things first and building theories about them

after.

2.1.2 The process of symbolising in the learning of mathematics

The present study focuses on the learning of mathematics as a functional activity, i.e. as an activity which is personally meaningful to the pupil. Although the use of symbolic systems is very important in mathematics, little is known about the processes by which children use symbolic representations for functional purposes. Formal symbolic systems are introduced to children very early, from their primary education, in situations with little meaning for them (Mason, 1980). As Hughes argues, the symbols do not help them to solve problems, they do not appear to have any obvious purpose and thus become associated with artificial activities such as doing sums (Hughes, 1986). Not surprisingly, using mathematical symbols is a general problem area in traditional mathematics education. Vergnaud (1984, p.27) states that "- certain symbolic activities are meaningless to many students - it is a difficult job to transform a situation or a word - problem into a symbolic representation".

For example, as a result of research with pre - school children, Hughes (1986, p.95) states that "there seemed to be a large gap between the children's concrete numerical understanding and their use of formal written symbolism". He offers an alternative interpretation of this finding by arguing that "young children do not see the value of using conventional written symbols" (Hughes, 1986, p.122). Although the issue of children's understanding of symbols in mathematics was not the primary concern in the researchers' interpretations of the CSMS (Hart, et al, 1980) and the SESM (Booth, 1984b) research projects, the results show that many children have difficulties in using algebraic symbols during early secondary education, often relying on the use of arithmetic strategies to solve algebraic problems (Booth, 1981).

There has been substantial psychological research on the issue of symbolism in general as a means by which we communicate internal thought. The main theme highlighted by this research is the important role of imagery, as distinct

from language (Paivio, 1978, Skemp, 1971) in the representation of ideas. Some researchers do not distinguish between imagery and perception maintaining a "picture-in-the-mind" view of imagery (Clements, 1982, Casey, 1976), while others do not see a mental image as identical to a picture but as as means to represent an object (Shepard, 1978, Kaufmann, 1979). An analytical review of this literature is not within the scope of the present study. However, Piaget and Inhelder's view of imagery is informative in that they see imagery as a symbolic system which is part of a developmental process, i.e. it changes with age. They emphasise the symbolic nature of imagery, arguing that it is a dynamic symbolic system which develops in parallel to, and in interaction with, logical - verbal thought (Piaget and Inhelder, 1971). For Piaget, "representation is primarily interiorization of action, effective action and accommodation, and later, possible action and accommodation" (Vergnaud, 1987b, p.230).

The present study is primarily concerned with different ways of representing the same idea, since giving commands to the turtle to move or turn in Logo requires the use of a formal symbolic code to convey some action (the Logo language), computer feedback in the form of a graphical representation of that action, and the possibility for the children to perform this action themselves by identifying with the turtle (Papert, 1980). In this sense, Bruner's enactive, iconic and symbolic levels of representation of an idea seem relevant to the process of giving commands to the Logo turtle. In investigating children's cognitive growth, Bruner distinguished three phases of such growth, each corresponding to a mode of internal representation involving action, imagery and language (Bruner, 1966). The phase of enactive representation involves children's ability to respond to questions only in relation to previous practical experience. The iconic phase involves responses which refer to mental images of physical objects or to an inner sense of pattern or structure. Symbolic representations of ideas involve the use of abstract symbols whose meaning must be articulated or defined. Bruner states that "their appearance in the life of a child is in that order, each depending on the previous one for its development, yet all of them remaining more or less intact through life"

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(Bruner, 1964, p.2). Subsequent criticisms of Bruner's theory question the usefulness of interpreting children's activities in the classroom as a direct correspondence to the three phases of representations (Freudenthal, 1983), and Bruner's treatment of images as relatively concrete and static without making more than a passing reference to the possibility that they may evolve into more abstract and dynamic forms, as argued by Presmeg (Presmeg, 1985).

However, the developmental aspect of Bruner's theory, does not relate directly to the turtle geometric environments of the present study, especially with respect to the age of the children; "Bruner's domain of application is the psychology of the very young child, and in this period the phases can meaningfully be filled out" (Freudenthal, 1983, p.135). However, giving commands to the turtle seems to require an almost simultaneous representation of the same idea in three different forms: acting out the idea by playing turtle, using a symbolic code to type it in and receiving a graphical feedback of the implied action. This facet of multiple representations of an idea or action is very vivid in Logo since it applies for every typed - in command. Mason found Bruner's theory useful in addressing the issue of different modes of representing mathematics by emphasising the importance of using all three representations, enactive, iconic and symbolic, in a given meaningful mathematical situation (Mason, 1980). By adopting a principle which is not in accordance with Bruner's theory, i.e. "that symbolic expression must ultimately become enactive if the idea is to be built upon or become a component in a more complex idea", p.10), Mason stressed the importance of "moving along the spiral in which enactive elements provide an iconic representation of some pattern or relationship, to a symbolic articulation, to enactive elements and so on", (Mason, 1980, p.10). A key aspect of this idea is that the "E-I-S spiral" is "relevant to presenting mathematics at all levels". For instance, as mentioned above, algebraic symbols often mean to pupils nothing more than a means to exercise manipulative techniques, i.e. although they might manipulate the symbols on the surface, they find it difficult to understand the meaning.

In an attempt to capture this phenomenon, Skemp maintained that attention is drawn to a syntactical surface structure of symbols, away from a semantic deep structure (Skemp, 1982). In Mason's words, students "only experience other people's algebra, without being encouraged to use algebra to express their own generality, to manifest their own inner perceptions in written form... there must be some access to symbolising so that if and when trouble develops, students have recourse down the spiral to greater confidence and meaning. The attraction of surface structure over deep structure is of course important in the movement up the symbolising spiral, in which symbols become concrete..." (Mason, 1987, p.76). It is the author's belief that the importance of mathematical environments where "moving down" the E-I-S spiral is always possible seems highly relevant to the turtle geometric environment, when children who have come up with some difficulty can always move down to enactive mode, by playing turtle. Accordingly, they can equally move up the spiral to manipulate graphical and "Logo code" symbols, (for instance debugging procedures in the Logo editor), without having to constantly refer back to meaning. An issue which has seldom been addressed, however, is the nature of the relationship between "playing turtle" representations and their signifiers in the form of symbols or graphics.

Vergnaud discusses Bruner's enactive mode of representation in conjunction with the situation the individual is acting upon (Vergnaud, 1987b). He maintains that pupils develop their knowledge within a variety of situations by initially acting upon them, often mastering local or "noncoherent" properties and calls this process "theorems in action" (Vergnaud, 1982). For Vergnaud, theorems in action are representational even though pupils may not be able to put them into words or symbols. He proposes that the production of representations in the pupil's mind involves three types of interactions between three levels of representational entities, the referent, the signified and the signifier;

"1) The referent - signified interaction in which action, chunks and invariants of different levels, inferences, rules and predictions play the main part;

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2) the signified - signifier interaction in which the natural language and other symbolic systems provide aids for identifying invariants for reasoning, for planning and controlling action;

3) the interaction between different symbolic systems" (Vergnaud, 1987b, p.232).

Vergnaud argues that enactive representations imply interactions between the referent and the signified, where representation is related to action. On the other hand, the iconic and the symbolic levels involve signifiers. Although Bruner perceives of the symbolic level as a higher level of representation than the iconic, Vergnaud disagrees, giving the example that language (the best example of a symbolic mode) develops before drawing and reading pictures (the best example of the iconic mode).

2.1.3 Children's learning processes
in the context of Logo activities.

Over the last ten years there has been a substantial amount of research into children's learning with the use of Logo. Since the present study brings into focus both children's learning processes and the geometrical content of the Logo situations in which they are involved (chapters 1 and 4), the author found it useful to review research concerning the process by which children learn in open - ended Logo environments, independently from geometrical content - oriented studies, which are reviewed in section 2.2. The present focus on process is not only due to the special links between learning programming and learning mathematics (Noss, 1985), but also due to the new insights into the way children learn, offered by new technological methods for collecting detailed data and by the child - in - control "active" nature of Logo environments. However, explicit focus on the process of children's learning did not come about automatically in earlier research studies (Feurzeig and Papert, 1969, Howe et al, 1980 and Howe at al, 1982).

2.1.3a) Process versus content

Feurzeig, Papert et al's pioneering 15 month research (1969) involved 12 7 to 9 year - old children of average mathematical ability. The study focused on the children's difficulties in programming and on the role of programming in helping to form an understanding of selected mathematical concepts. Despite the optimistic and encouraging nature of the researchers' conclusions, it is tempting to say that their concluding remarks implicitly acknowledge the need to perceive and research the process of the learning of mathematics rather than the learned content. The researchers accordingly saw the role of Logo as a "conceptual framework" for teaching mathematics which could provide the student "with an active operational universe for constructing and controlling a mathematical process" (Feurzeig et al, 1971, part 4, p.3).

A "premature" focus on content was the main strand of criticisms of the Edinburgh project which extended over a period of six years in three two - year phases (Howe, O'Shea and Plane, 1980, Howe, Ross et al, 1982). The "formative phase" (1974 - 76) involved the teaching of Logo programming to two bottom stream classes of 11 - 13 year olds for about one hour per week. In the subsequent "summative phase" (1976 - 78) the developed materials and strategies were used to teach specific parts of the mathematics curriculum to a bottom stream, end of primary class. The results based on pre and post tests of this group of children and a control group were inconclusive. The study was criticised for its selection of students, choice of school and choice of tests. A second investigation attempting to answer the criticisms by a different choice of school - type, pupils and location of the study (it took place within the school instead of in a laboratory), yielded overall significant results at the 5% level in favour of the Logo group, mainly attributable to the effects on girls (Howe, Ross et al, 1982). The researchers, however, maintained their approach to programming a computer which was later criticised by Noss; "Howe's emphasis on mathematical content, together with a prescriptive pedagogical strategy, implies a relative de - emphasis on the mathematical process involved in programming and concentrates on the modelling of specific mathematical concepts" (Noss, 1985, p. 62). With the benefit of hindsight, it could be argued that a lack of emphasis on process in the work carried out both by Feurzeig and Papert and in Edinburgh implied a lack of awareness of the process - content dialectic, rather than an explicit research or pedagogical choice.

In reaction to this emphasis on content, three subsequent research projects were carried out in the following decade, i.e. the Brookline Logo project (Papert, Watt, di Sessa, Weir, 1979), the Chiltern Logo project (Noss 1985) and the Logo Maths project (Hoyles and Sutherland, in press). A common central aim in all three was to investigate and analyse the processes by which children learned to program in Logo, and specifically, their programming, mathematical and conceptual characteristics. The present study has been

influenced by these three research projects, due to the relevance of the backgound information on such processes, the nature of the created environments for the children and the methods of research and analysis. A brief account of the three projects is given at this point, followed by a synthesis of the findings concerning the process by which children learn to program in Logo.

A comprehensive account of children's programming activities was given by the Brookline project (Papert et al, 1979) which involved 16 sixth grade children working with Logo during the academic year 1977 - 78. Both "average" and "exceptional" achievement children were selected by means of national achievement scores and their teachers' evaluations. The researchers, proposing a one to one child - machine ratio as the norm for the near future, split the children up into groups of four, each group working accordingly with the four available computers, during 40 to 90 minute sessions and with their teacher who had previously had a year - long training in Logo by the researchers. The study's general "teaching objectives" were for the children to:

1. learn to feel in control of the computer;
2. learn the elements of the Logo computer language;
3. learn the subject matter of Turtle Geometry;
4. understand the relation between force and motion (using a
   "dynaturtle");
5. develop problem - solving skills.

The technology was also used in the collection of data which included dribble files of the children's typing and hard copies of graphics and procedures saved on disk. Systematic notes were kept by the teacher, while the researchers conducted regular observations. Although all but two students (both in the lowest quartile of school perfomance) learned to program according to the researchers' criteria, they attributed more importance to a

somewhat different claim that they made: "All students irrespective of perfomance level were engaged by computer activities in the Logo environment; all underwent significant observed learning and we made significant progress towards developing a methodology of channelling this learning toward a mastery of programming." (Papert et al, 1979, p. 1.15).

ASIDE: The "dribble files" facility enables the saving of everything that the children type on a disk file.

Noss (1985) set out to illuminate the mathematical nature of the activities that children engage in while programming in Logo. The study involved 118 children in total, aged from 8 to 11. The children were distributed amongst five classrooms in five schools, each class spanning the full within - school ability range. The children worked with Logo in pairs or threes in the classroom as part of their "routine" schedule, for approximately 75 minutes a week and for a total period of around 18 months. The schools were chosen for their mixed - ability teaching so that the classrooms participating in the study would have a flexible internal organisation and an informal educational atmosphere. The children's own classroom teachers, having had a short training in technical, programming and related educational issues, were responsible for the Logo work in their clasrooms.

The study's objective was to investigate the potential of programming with Logo as a medium for creating a mathematical environment. The research issue mostly relevant to the present study was the investigation of the nature of the children's Logo programming and the mathematical ideas involved in this process. The research consisted of the following three parts:

1. A preliminary phase, illuminating the children's emerging programming strategies.

2. A programming phase, illuminating the children's mathematical activities

via the "combination" of two frameworks; i) certain key programming ideas, i.e. procedures, iteration, subprocedures, editing - debugging, inputs, recursion and ii) a model of the children's "learning modes", i.e.:

a) "making sense of" a new programming idea, i.e. trying it out, acquiring some control over it,

b) "exploring" a new idea by forming links with their already existing ones and,

c) solving problems, i.e. using a programming idea in a goal directed activity, in order to produce a desired outcome.

The researcher analysed the children's mathematical activities with respect to the three learning modes for each of the key programming ideas.

3. Finally, the researcher conducted a series of case - studies of four pairs of 10 - 11 year old children over a period of approximately six months, in order to further investigate two issues arising from the programming phase:

a) the ways in which children acquired new programming ideas and used them in an exploratory or problem solving manner and,

b) ways of intervening which would be effective for the children's learning.

Among the researcher's conclusions is that the children's learning, alternated between the three learning modes described, rather than being of a "developmental stages" nature. Moreover, it was suggested that ample time for exploratory learning may play a crucial role in children's understanding of key programming ideas. The children's work was characterised by a lack of planning throughout the research period, and by initially mainly goal - directed activites gradually giving ground to exploratory activities, which the

researcher suggests may have been a consequence of a feeling of control over the computer.

The most comprehensive case - study research on the potential of Logo as an aid to pupil's learning and thinking in mathematics (Hoyles and Sutherland, in press), involved eight 11 - 14 year - old children starting from their first year of secondary education and extended over a three - year period. The children worked in pairs on the computer during their normal mathematics lessons and the researchers adopted a dual role of observers and teachers, at least one researcher participating in the research at all times. An explicit analysis of their teaching interventions yielded three such types, i.e. those which left the control of the interaction to the pupil, teacher directed tasks in order to achieve specific learning outcomes and "teaching episodes" to introduce new programming or mathematical ideas. The detail in the data collection was substantially increased in comparison to the Brookline and Chiltern projects, involving:

- video recording of all the Logo work;
- audio recording of all the language interactions;
- hard copy of written procedures;
- occasional graphics dumps;
- pupils' written records and plans;
- participant observers' notes.

The nature of the collected data enabled the researchers, not only to throw more light on the children's understanding and use of mathematical ideas and into their problem - solving strategies, but also to investigate other issues emerging from such an educational environment such as the children's collaborative Logo work and the role of the teacher interventions.

As will be shown below, the Brookline, Chiltern and Logo Maths projects provided substantial insight into the way children learn, when programming in

Logo. They showed us that when given the opportunity, the children engage in doing their own mathematics, which are often different than the mathematics an educator might prescribe, and not infrequently, wrong. They argue for the value of such activities, however, since exploration with mathematical ideas provides the opportunity for children to become mathematicians (Papert, 1972), i.e. engage in the process of doing mathematics in meaningful contexts. It is also quite rightly argued that programming is essentially a mathematical activity and at the time when the studies took place (especially the Chiltern project) there was a need to establish this link. In recent studies, however, it could be argued that attempts to investigate children's learning of a specific content domain within a Logo environment would have benefitted from a more thourough separate analysis of the programming and the mathematics children engage in (e.g. Kieren, 1987).

Three "facets" regarding the strategies children seem to build when programming in Logo seem to have emerged from the research projects mentioned above, seen through each project's individual perspective, and refined as one project benefited from the results of the previous. The facets involve the children's adoption of different cognitive styles of programming, their developing problem - solving strategies and the development of their understanding of key programming ideas such as modularity, procedures and subprocedures. The findings are presented in some detail, since they were particularly informative for the present study, both in setting up a Logo environment for the year - long experience of the participating children prior to the main research (chapters 4 and 5) and in providing the author with insights into the interpretation of children's activities during the main study (chapters 6, 7 and 8).

<u>2.1.3b) Programming styles</u>

The Brookline researchers identified a "top - down" / "bottom - up" dimension in the children's programming styles. They used specific examples of individual children (the Brookline project had a 1 - 1 child - machine ratio) to describe these styles. For instance, Kathy developed a strategy of writing small, simple subprocedures and building them up to complex designs, while Donald would plan a superprocedure first, and then move down to define the embedded subprocedures (Papert et al., 1979). These two examples are of course at the two ends of the "continuum" and the researchers include examples of children's work which incorporated both styles, starting for instance with a vague overall plan, defining subprocedures and accordingly refining the plan in the process.

The sexes of the children at the two ends of the continuum in the Brookline research may not have been a mere coincidence. The deeper investigation carried out by the Logo Maths researchers, revealed indications of a preference of a bottom - up approach to programming by the girls and of a top - down approach by the boys, although the pioneering character of such research does not allow any strong assertions. Accordingly, the researchers maintain that gender seems to be related to other dimensions of programming style revealed by the detail of their research, i.e.

- Working towards well defined goals... working towards loosely
   defined goals;

- "Hard" planning... "soft" negotiation;

- Attention to global characteristics.... attention to local
   characteristics;

- Defining a procedure immediately in the editor... trying out a

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procedure in direct drive before defining in editor;

- Systematic... not systematic.

2.1.3c) Problem - solving strategies

In all three projects, the researchers were keen to identify the strategies which the children adopted in their programming. In the Brookline research, the identified strategies were rather closely tied to programming ideas and covered a span of such ideas which seemed somewhat large, in comparison to the more explicitly child - centred learning achievements described later by the lengthier English studies. For instance, such strategies in the Brookline report were:

1. Acquiring the sense of command;
2. Developing the notion of a procedure as an entity;
3. Separating the process from the product of a procedure;
4. Acquiring flexibility in establishing procedure hierarchies (i.e. procedures and subprocedures);
5. Fitting a procedure into a hierarchy (i.e. top - down versus bottom - up);
6. Developing patterned procedures using REPEAT, recursion and iteration;
7. Using variables in Logo procedures.

In the Chiltern project, however, Noss attempted to include a more general dimension to children's strategies "detaching" them from programming ideas by employing his framework of "early strategies" for the children's initial experiences and "learning modes" for their more advanced programming (described earlier). In what seems a further attempt to perceive aspects of the children's activities as de - contextualised from the Logo language, i.e. essentially as activities of thinking and learning in general, Hoyles and

Sutherland (in press) identified the following three categories of programming activities and used them as a framework for their analysis:

1. Working at syntactical level, where children would type in commands focusing on the output without reflecting on how and why the output was achieved;

2. "Making sense of" where children would explore an idea by trying it out and reflecting on what is happening. Not giving the phrase quite the same meaning as Noss (Noss, 1987), the researchers saw this activity as goal - directed or not goal - directed and suggested that pupils should be encouraged to take time to explore new ideas;

3. Goal directed activities, aiming at an outcome. These were described by two dimensions:

a) loosely defined... well defined, refering to the global structure and the outcome of pupils' goals;

b) real world... abstract, refering to the pupils perception of the "realness" of their goal.

They also examined pupils activities from a different perspective, i.e. that of planning, implementing and debugging, all of which can have either a global focus referring to a mental plan, or a local focus referring to graphics or text output. They argued that the sequence of activities depends on individual programming styles and represented those processes in relation to their interaction with the negotiation of a goal.

2.1.3c) Understanding of procedure, subprocedure and modularity

The descriptive characteristic of the Brookline research and the focus of the researchers on the programming that the children actually learned, resulted in a rather optimistic picture (with the benefit of hindsight) regarding their use of procedures and subprocedures in a structured manner. Subsequent studies, however, revealed difficulties not only in children's understandings of structured programming, but also in their adopting a personalised meaningful use for it. In one of the first reactions to previous findings, Leron pointed out that from his experience with 12 year - olds' programming, most children seemed to write linear unstructured lengthy procedures. Moreover, even when encouraged to structure their programs, they would "fall back" to linear programming when left alone (Leron 1983). He related the children's difficulties to their "lack of a clear concept of the interface between subprocedures" and the importance of the turtle state before and after the execution of a subprocedure. In describing the children's mathematical activities while "making sense of", "exploring" and "problem - solving with" procedures and subprocedures, Noss also depicted and described difficulties such as: reluctance to use procedures involving the turtle going over a line more than once, and perceiving the interface between two subprocedures as an entity (Noss 1985).

Shortly after a ten week case - study involving two pairs of 9 - 10 year olds' learning of procedure and variable (Hillel and Samurcay, 1985a), Hillel reported that even children who have had ample experience with procedures and have been involved in discussions of the merits of using them, surprisingly opted for simple direct - drive programming. Offering as a plausible explanation, the children's perception of programming as "drawing with the turtle" (Hillel and Samurcay, 1985b), he later investigated this issue more deeply, giving evidence of "a strong drawing schema underlying the children's choices of goals, productions and planning strategies, as well as, their criterion for success" (Hillel, 1986, p.435). Mendelsohn has also given

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similar evidence in a study of children's programming from a psychological perspective (Mendelsohn, 1985).

Although the researchers of the Logo Maths project agreed with the explanations offered above, they proposed other factors which seemed to influence the programming manner of the children in their research, related to the children's perception of their goals and on the nature of the goals themselves. For the former, they contended that when the children perceive of a goal as a "real world" one, they are more likely to perceive programming as drawing with the turtle. Accordingly, they are more likely to see the need for structure when their goals are well defined abstract ones. For the latter, the researchers report indications that when children have a high degree of field independence they are more likely to use a structured design when programming in Logo.

In addition to drawing results from their longitudinal transcript data of their study's four pupil pairs, Hoyles and Sutherland also gave the pupils an individual structured task (the four squares task consisiting of a row of four equal sized squares horizontally placed at equal distances between them) at the end of each of the three years of the study in order to investigate the children's perception and use of the modular properties of the task and the development of these aspects over the three year period (Hoyles and Sutherland, in press). After analysing the data from these tasks and further ones administered at the end of the third year, the researchers concluded that:

1. Pupils are more likely to use a modular programming style in tasks where the modules are "disconnected" (e.g. the four squares task) than when the modules are interconnected (e.g. the net of a cube);

2. Pupils are more likely to choose modules which when put together will not involve drawing the same line more than once. For example they tended to

choose a rectangle rather than a square in the window and net of cube task. This second conclusion is in accordance with Noss' findings mentioned above and the Canadian researchers' work on 11 - 12 year olds' perceptual and analytical schemas in solving structured tasks (Kieran, Hillel and Erlwanger, 1986). This research will be analysed below.

3. Pupils are more likely to remove the interfacing commands as separate subprocedures when a task involves modules of varying size but invariant interfaces (as in a row of decreasing equidistant squares).

These conclusions could suggest pedagogical techniques for providing environments where key issues of structured programming are meaningful to children. Moreover, the diversity of the tasks throws further light on the relationship between the content nature of a goal or a task and the children's perception of this nature. The problem, however, of whether children who "personalise" aspects of structured programming in this way preserve the same programming manner in other contexts seems to relate to the fragmented, "domain specific" nature of knowledge (diSessa, 1982, Lawler, 1985) rather than to the specific nature of structured programming.

In an effort to describe the pupils' developing use of structure in their programming, Hoyles and Sutherland identified three hierarchical phases of such development, stating that a pupil will not necessarily perform at his/her highest potential phase since the context and nature of a pupil project determines the performance at any particular time. These phases are:

Procedures as product

1. Writing an error free procedure by direct driving and recording commands...

     1a. no evidence of structure within the commands;
     1b. structure emerging within the commands;

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1c. clear evidence of structure within the commands, i.e. using modular ideas but not translating these into a program structure.

Transitional stage

2. Dividing the written record of a set of commands into sequential parts (non modular) and defining these as subprocedures.

3. Using the written record of direct drive work to perceive modularity in the design and defining these modules as subprocedures.

Procedures as processes

4. Perceiving modularity from the outset and using subprocedures to define the modules.

The above findings (from the Brookline, Chiltern and Logo Maths projects) concerning children's processes of learning to program in Logo, provided insights into three important aspects of these processes; the differences between children in cognitive styles of programming, and the variability of their developments of problem - solving strategies and of their understanding of key programming ideas. However, there are more problems to be solved, indications of which have been identified in the above projects and in particular, in the Logo Maths project. Firstly, the influence of interaction between small groups of children on the learning of each child individually. Secondly, on the role of teacher interventions both in classroom situations with children's everyday teachers and in situations of researcher interventions during participant observation. A preliminary analysis of these issues in the Logo Maths project indicates the need for a lot more work to be done in the area.

It seems reasonable to suggest, however, that the illuminative gains from this process - oriented research, have influenced subsequent smaller - scale research developments, now allowing an increasing degree of focus on the mathematical or the programming content of children's learning, whilst taking the children's thinking processes explicitly into account. A detailed review of such research carried out in Canada and England will now follow, due to the relevance of its research principles to the present study, which required a high level of awareness at both the process and the content level (for a discussion of this issue, see chapters 1 and 4). In the following section, however, the review will concentrate on studies focusing on the content of children's learning in a general sense, allowing for a more detailed view of studies specifically dealing with geometry within Logo programming in a later section which will consequently include geometry - specific analyses of studies covering a wider span of issues (e.g. a study on children's understanding of angle in the Logo Maths project).

2.1.4 The Use of Logo for Children's Learning
of Mathematical Content

A research pioneering a more explicit balance between focusing on the process of children's learning and focusing on the content learned (compared to the process - oriented studies reviewed above), was carried out in Canada in 1984 (Hillel and Samurcay, 1985a, b). The researchers focused on a specific programming content, i.e. procedure with variable. The study involved the observation of two pairs of 9 - 10 year old children working with the computer for one hour a week, for a total of ten weeks, the collected data being of the same detail as in the Logo Maths project. The children had had an initial 12 hour experience prior to the study in a slightly restricted Logo environment (e.g. the turtle was slowed down), the researchers emphasising the use of paper and pencil planning, the using of simple procedures as building blocks and drawing attention to the idea of "turtle state" (Hillel, 1985a). The researchers carried out the study in the light of an analysis of the

"conceptual field" of the content which "attempted to establish relationships" among:

- the class of activities of problems;
- the concepts and techniques associated with such activities;
- the cognitive demands of each activity;

Their consequent analysis of the research data was carried out in terms of the mathematical and programming conceptual difficulties encountered by the children and - due to a rather specific task and intervention strategy - on the researchers' "teaching" interventions. Seen through the light of a further 12 - hour observation of the same children in an "extension" of the study (Hillel, 1985b), the researchers report on two sets of conceptual difficulties; the first set involved those in which the children later showed a "greater fluency": the definition and use of general procedures including parametrizing other than length, thinking about variables and spontaneous definition of general procedures. The second set of conceptual difficulties involved those persisting after the children's extended experience, i.e. operating on procedures, resolving inter - procedural relations and coming to terms with the intrinsic nature of the geometry. In his concluding remarks on this "trilogy" of studies, Hillel contended that turtle - geometric activities involve difficult and subtle notions related to both cognitive aspects (e.g. shifting from a "drawing schema" to a "procedural analysis" of figures) and the geometrical and programming content. He concluded with a statement stressing the importance of the children's cognitive processes, which was meant to challenge criticisms on the limitations of content in Logo and seems in accordance with a considerable volume of more recent research - for instance on older and Logo - experienced children (Hoyles and Noss, 1986, 1987a and 1987c); "I argue that, with a little bit of thought and effort, there is enough content in turtle geometry to keep children busy throughout their elementary schooling." (Hillel, 1985b, p.45).

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In support of this argument, other researchers in Canada have subsequently carried out a multitude of studies, extending overall from 1985 to the present (individually or in cooperation), slightly shifting their analysing focus from the process / programming content dimension to that of process / geometrical content. Although these latter studies are reviewed analytically in a later section, two points seem to be of interest here, i.e. the use of "special" Logo primitives and the further light thrown on children's strategies from a cognitive perspective.

A further restriction of the openness of the Logo environment, from the administering of structured tasks, was the use of special primitives which either slowed down the turning of the turtle or constructed simple, specially designed, geometrical figures. For instance, in research carried out by Hillel, Erlwanger and Kieran (1986), the following "primitives" were given to 11 - 12 year old children, as an introduction to Logo:

TRT, TLT (input), a slowed down version of RT and LT;
MOVE (input), a slowed down PU FD (input) PD;
TEE (input), a state - transparent procedure for the figure T;
VEE (input), a state - transparent V with a fixed interior angle of 60.

The children's activities alternated between own projects and set tasks involving complex combinations of Tee's and Vee's, and the researchers started with content - oriented objectives, i.e. to emphasise and foster notions of angle, translation and rotation of figures and decomposition of complex figures in terms of simpler ones other than line segments (fig. 2.1). Their analysis, however, also revealed process - oriented findings of a rather subtle nature, such as the children's tendency to employ a "perceptual" rather than an "analytical" cognitive schema to solve geometrical tasks. The encouraging outcomes of this research (Kieran, Hillel and Erlwanger, 1986) resulted in further such experimentations with special Logo primitives, either keeping the objective to analyse children's cognitive strategies in a decontextualised way

as a "hidden agenda" (Kieran, Hillel, Gurtner, 1987), or reversing the focus of analysis on psychological aspects of children's understandings (Gurtner, 1987).
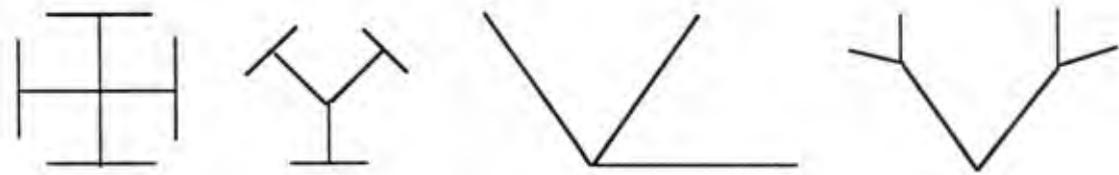
Figure 2.1 Examples of the "Tees and Vees" tasks.

In the former research, highly structured tasks were administered to 12 year old children, composed of explicitly stated inter - relations of simple figures (procedures of which were given as primitives) shown to the children on paper (fig. 2.2). The procedures / building blocks, in this case, were MOVE, TRT, TLR as before, and a state - transparent rectangle procedure with two linear inputs (RECT :A :B). At the beginning of the study, the children's attention would tend to focus on dimensions which appeared directly as inputs to the Logo commands, rather than coordinating other dimensions of the relative placements of the figures. Moreover, their criterion for acccepting a production as a solution was based on visual verification of the output, rather than the structure of a program. However, the researchers report on a development by the end of the year - long study, in all but one of the children, in their awareness that the exactness of the solution lies in the program rather than in the screen output.
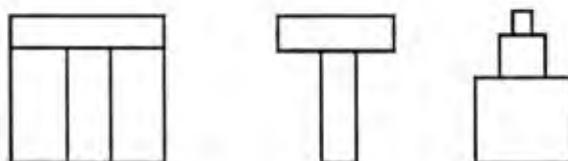
Figure 2.2 Examples from the "Centering" tasks

In a parallel year - long case study, Gurtner's investigation concentrated on children's awarenesses of the reasons or the processes involved in their successful solving of tasks. Gurtner was interested in investigating the gap between success and children's depth in understanding the problem, rightfully arguing that a child might be successful in producing an outcome "without knowing why, sometimes without noticing it" (Gurtner, 1987, p.229). He used the primitives of the previous study, plus the procedure BASELINE (input) which produced a horizontal line segment with the input as its length. He gave the children complex figures with a high degree of precise interdependence between their component parts (fig. 2.3), and several off - computer tasks at later dates during the year, referring to the children's awareness of the tasks' solutions. The analysis revealed that the children have their own spontaneous beliefs about where their programs are faulty, and their own criteria for success, such as a sequential (rather than parallel) dealing of mismatches between a child production and the target figure, and a locality input - specific criterion (a finding similar to the previous study). Gurtner concludes that correct - looking productions do not necessarily mean that a student has solved the expected problem, i.e. that the many ways which lead to a particular target may allow the circumvention of such problems.
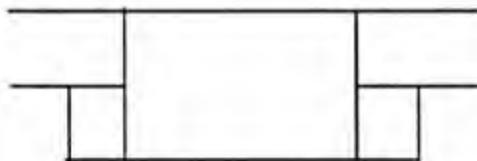
Figure 2.3 The "4 - Tees" figure

The English experience with longitudinal studies of the children's learning processes in Logo programming, led to the developing of further research with a more explicit simultaneous focus on process and content. On the process level, a decontextualised model (UDGS) for learning mathematics was devised from the observations of children's activities with Logo (Hoyles,

1986), and refined in subsequent recent research (Hoyles and Noss, 1987a, b, and 1987c), while used as a framework for designing structured tasks and analysing the children's activities (details of the model are given below). On the content level, on the other hand, the researchers extended the notion of a "microworld" put forward earlier (Papert 1980, Lawler, 1982, Weir, 1987), to incorporate a broader perspective of the learning environment (Hoyles and Noss, 1987b). The two notions of the UDGS model for learning mathematics and a "microworld" are presented through this recent research, since both notions are used in the design and analysis of the present study.

As a pre - amble to an explicit use of the two frameworks, Hoyles and Noss carried out an investigation of the relations between Logo - experienced children's conception of proportion within a Logo environment and their conception of the same notion from school mathematics (Hoyles and Noss, 1986). They devised a set of highly structured tasks on and off the computer, involving the construction of "N" and "Z" shapes, the notion of proportion embedded in the length relationships among the three line segments of the figures. The tasks were given to seven 13 year old children during a day visit to a computer lab in the University. The researchers were familiar with the children's programming background (over 80 hours of Logo programming) since they had started off as participants in the Chiltern project and had been followed up by the researchers during their secondary schooling. The results, yielding a discrepancy between the children's performances in "computer mode" and "pencil and paper mode" (the CSMS ratio test was used, Hart, 1980), led the researchers to re - acknowledge the contrast between a dynamic, investigative response activated by the computer and the limitaton of "paper and pencil mode" to a response in the form of a "fixed answer to a fixed question". Not having knowledge of trigonometry, the children used the computer to try out different lengths and devise their own "theories", discovering for themselves the inadequacies of an additive strategy (Hart, 1980) and exploring a range of multiplicative strategies, given - via the computer - the opportunity to think about the general within the specific. An

important tool for the children was the construction of a procedure with inputs for the lengths, and the attempt to find "general" relationships between the inputs.

In subsequent research, however, instead of the children attempting to "invent" a personal theory when knowledge about the phenomenon was inaccessible to them, they were given a complex tool - in the form of a procedure with inputs - encapsulating specific properties of a geometrical figure (a parallelogram) and encouraged, through structured tasks on and off the computer, to investigate notions embedded within the procedure and notions involved in using the procedure as a whole (Hoyles and Noss, 1987a). For example, a procedure initiating the children's investigational activities was the following:

```
TO SHAPE :SIDE1  :SIDE2
FD :SIDE1 RT 40
FD :SIDE2 RT 140
FD :SIDE1 RT 40
FD :SIDE2 RT 140
END
```

The crucial factor enabling such investigational activity was the ability to change the size of component parts of the figure and therefore explore the essential properties for its construction. Such changes involved the manipulation of either variable or fixed inputs and the consideration of the "effects" of changing one input on the other inputs of the procedure. The analysis of the data was carried out within the explicit process and content frameworks of the UDGS model for learning mathematics (Hoyles, 1986) and the notion of a "microworld" (Hoyles and Noss, 1987b) respectively, the presentation of which seems useful at this point.

According to Hoyles, the UDGS model for learning mathematics reflects a constructivist thesis for mathematics education, attempting to incorporate and foster learning through personally meaningful activity, but also to bring the

mathematics embedded in an activity to "a plane of conscious awareness" (Thom, 1973, Hoyles, 1986), since the children may not be aware of the mathematics they are using, or "may fail to discriminate contextual from mathematical properties" (Hoyles, 1986, p.112). The model involves four dynamically related components, which are:

using: where a concept is used as a tool for functional purposes to achieve particular goals;

discriminating: where the different parts of the structure of a concept used as a tool are progressively made explicit;

generalising: where the range of application of a concept used as a tool is consciously extended from a particular to a more general case;

synthesising: where the range of application of the concept used as a tool is consciously integrated with other contexts of application - that is, where multiple representations of the same knowledge in different symbolic forms derived from different domains, are reformulated into an intergral whole.

The crucial characteristic of the model, by means of which it embodies a constructivist approach to learning mathematics, is the specific role of the activity of using a concept, i.e. that children use a concept first, and then develop understandings of it. It is the relationships between these components, i.e. how transition from one component to the other comes about, and the clarifying of the components themselves, which has been the process - oriented objective of the described research (Hoyles and Noss, 1987a, b and 1987c).

At the content level, the framework of a "microworld" is used, i.e. "Logo - based situations constructed so that the pupil will come up against embedded mathematical ideas in the context of meaningful activity" (Hoyles and Noss,

1987c, p.131). The researchers do not perceive of a "microworld" as only consisting of the technical component, but rather give the term a meaning of the educational situation which can be fostered by the technical component, i.e. by the incorporation of:

a) the pedagogical aspect, playing an important role in provoking "prediction, reflection and evaluation",

b) the pupil component, by taking into account the pupil's perceptions of problems or tasks, built on their previous experiences (Erlwanger, 1973, Hart, 1984, Booth, 1984a), their different working styles, and affective issues, e.g. overcoming the fear of failure (Hoyles and Noss, 1987b), and;

c) the contextual component, i.e. the social setting within which the programming activity takes place.

Hoyles and Noss take the opportunity to state their research intentions of including an explicit focus on mathematical content while maintaining the importance of process, both in curriculum design and in research: "We propose that the next step in research and curriculum development should be to integrate Logo into the curriculum through the construction of mathematical microworlds as set out above. While exploring within Logo microworlds, our hope is that pupils will use mathematical concepts as tools whose functions can be investigated within meaningful projects." (Hoyles and Noss, 1987b, p.591)

It is with this framework in mind that the structured tasks of the above research were perceived by Hoyles and Noss as a component of a parallelogram microworld. A major component of the findings of the two phases of this research (a follow - up study was carried out, taking explicit account of the children's initial and final conceptions, but lacking the backup of longitudinal data), was the developing illumination of how the UDGS model works "in

action", i.e. the understanding of the processes by which the pupils make their way around the model while working in a Logo environment and consequently the role of the environment itself. For instance, the researchers maintain that this role is related to the pupil's ability, through interaction in a Logo microworld, to:

a) synthesise the symbolic desrciptions in terms of programs (or fragments of programs) with the geometric image on paper or on the screen, and;

b) to use the computer as scaffolding for the construction of generalisations (Hoyles and Noss, 1987c).

Moreover, they discuss the role of discrimination within the model; "discrimination involves a synthesis between the geometric and symbolic representations of some part of the concept facilitated by the Logo environment, while generalisation is aided by the scaffolding role of the computer" (Hoyles and Noss, 1987c,p.133). However, although the "synthesising" across contexts activity is considered an integral part of the model, yielding the researchers' contention that it is an important part of learning mathematics, their research does not seem to provide evidence that the designed environments encouraged such activities.

2.1.5 Discussion

Research into the mathematical learning processes of children engaged in Logo programming has provided substantial evidence that Logo can provide a means to generate rich mathematical environments for children to act upon in a personally meaningful way. The specific aspects of these processes in the context of Logo which have been observed and analysed, i.e. their varying programming styles, their problem solving strategies and their understandings of key programming ideas have also provided insights into children's learning in general. However, other aspects related to this process

could be subject to further illumination, such as peer interaction in small -
group learning, the role of teacher interventions and ways of bringing about
change in classrooms so as to inject a more child - centred atmosphere.
Recent contentions concerning the factors involved in children's learning,
imply a growing appreciation of the role of the context - specific and
fragmented nature of knowledge, in contrast to Piaget's holistic view of
qualitative levels of thought. The findings from the process - oriented studies
of children learning with Logo corroborate this view of the nature of
knowledge. Furthermore, recent emphases on the role of symbolising one's
own reality in the learning of mathematics highlight the educational value of
mathematical situations where children can alternate between enactive,
iconic and symbolic representations of ideas. The author suggests that Logo
can be such an environment through Turtle geometry, where children can act
- out turtle movements, and discuss graphical representations of their
symbolising efforts.

However, attempts to study what children learn in Logo environments have
been mainly restricted to specific contents, either from the domain of
programming, or from that of mathematics. A more systematic analysis of
content combined with the design of microworlds, or conceptual fields within
Logo (chapter 1), could provide the basis for research into children's learning
of specific mathematical ideas.

2.2 GEOMETRY AS A CONTEXT WITHIN WHICH CHILDREN DO MATHEMATICS

Before considering children's learning of geometry in a Logo environment, it is useful to take into account other research into the structure of geometrical thinking in relation to developments in the field of geometry as a discipline for education, providing thus a background for interpreting and understanding children's geometrical conceptions during Logo activities. Although there has been some research into children's understanding of geometrical topics, usually carried out from a "psychometric" perspective (Herskowitz and Vinner, 1983, Kramer et al, 1986, Eylon and Razel, 1986, Heink, 1982, Friedlander et al, 1986, Fisher, 1983, Noelting, 1979), it does not seem relevant to the present study, the aim of which is not to suggest that Logo activities will improve performance in traditional geometrical tasks, but rather to investigate the geometry that children do when engaged in open - ended geometrical environments. However, the study is informed by the wide tendency during the second half of this century - at least in the U.K. - to change the role of geometry in education and by the research carried out to establish the nature of geometrical thinking. A review of research into children's geometrical understandings in the context of Logo environments providing children with at least some control of their own learning, is also relevant to the study.

2.2.1 Geometry as a discipline for teaching deduction

The first lesson in geometry on record reflects the Socratic method of instruction of a well prepared teacher posing questions to the pupil designed to lead him to understand the discipline (Socrates to Meno, Freudenthal, 1973). For two millenia, Euclidean Geometry dominated all teaching of geometry, the aim of which was the teaching of deduction (Eves, 1976, Boyer, 1968). From the 17th century, however, other geometrical systems were invented (e.g. Descartes, Riemann, Cauchy, Hilbert) characterised by a "coordinisation" and an "algebrization" of geometry (Freudenthal, 1973). Moreover, after thorough study, logical shortcomings were found in Euclid's

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Elements and new axiomatic systems were proposed by Hilbert and Pasch in the early 20th century.

Although the questioning of geometry as a perfect conceptual system seems to have initially been a catalyst for its diminishing role in educational curricula, research into children's understandings - starting from Piaget in the middle of the century - has been the major factor which "pushed" geometry into the margin of mathematics.

Piaget's research consists of a rather strict application of his theory of stages of cognitive development in conjunction with basic geometrical concepts such as; Conservation and Measurement of Length, Rectangular Co - ordinates, Angles and Curves, Areas and Solids (Piaget et al, 1960). Although Piaget's stage theory has been subject to considerable criticism (section 2.1.1), the main reason for the later tendency of "new mathematics" in the sixties to abandon the teaching of Euclidean geometry was evident in his research; deductive thinking, for Piaget, requires formal thinking which, according to Collis, is aquired as late as the age of 16 and not by all people (Collis, 1974). Euclidean geometry was considered as "inappropriate" for the majority of school children because "it was being taught as a tight deductive system which most children could master only by rote learning" (Kuchemann in Hart, 1981). The teaching of geometry involved the imposing of deductivity on the pupil and thus offering the subject matter as a preorganised structure, rather than allowing the child to experience such organisation. Freudenthal stated that "The deductive structure of traditional geometry has not just been a didactical success. People today believe geometry failed because it was not deductive enough. In my opinion, the reason was rather that this deductivity was not taught as reinvention, as Socrates did, but that it was imposed on the learner... If geometry as a logical system is to be imposed upon the student it would better be abolished." (Freudenthal, 1973, p.402 and 406).

Far from advocating the "abolition" of geometry, however, Freudenthal made an extensive case for its use in education, but in a different role than that of the imposition of ready - made deductivity; "Geometry can only be meaningful if it exploits the relation of geometry to the experienced space. If the educator shirks this duty, he throws away an irretrievable chance. Geometry is one of the best opportunities that exists to learn how to mathematize reality." (p.407)

It is the author's opinion that Freudenthal's perspective of the potential role of geometry in education is, at root, a constructivist position. Initial concrete experiences for the child, using visual, kinetic and kinesthetic cues to experiment and discover properties of geometrical shapes lead to meaningful organising of these properties, an activity which lies at the root of deductive thinking. In this way, the child experiences, for instance, that "defining is more than describing, that it is a means of the deductive organisation of the properties of an object" (Freudenthal, 1973, p.417). Research into children's understandings of geometrical concepts has shown their difficulties in defining or identifying geometrical figures by organising or even using their properties (Pyshkalo 1968, Burger, 1982, APU, 1982, Hart, 1981). For example, the APU findings reveal that 85% of 11 year old children could identify a regular hexagon without obvious distractors present. The percentages however fell dramatically when pentagons were present and / or the hexagon was irregular, ranging from 25% to 43% according to the task (APU, 1982). The author interprets this finding as revealing of the children's defining of the hexagon as "what looks like a hexagon", rather than using the property that it has six vertices, i.e. the children used visual cues rather than adopting an analytical approach. Although the traditional geometrician's view would have been in this case, to teach the definition to the children, the view of geometry proposed by Freudenthal would encourage more experiences of manipulating polygons until they start using the property of the number of vertices for themselves. Then they would be ready, he claims, to understand the definition.

2.2.3 The structure of geometrical thinking

In support of Freudenthal's case for the role of geometry in education, Pierre van Hiele and Dina van Hiele - Geldof completed in 1957 a two - fold research on the structure of geometrical thinking which later influenced both a substantial amount of related research and the structure of curricula internationally. Influenced by Piaget and by their personal experience of high - school geometry teaching, the predominant problem they set out to address was the "futility" of teaching students at a higher level of thought than that which they could attain. A partial outcome of their research was a sequence of levels of geometrical thinking into which people may be classified. In Freudenthal's words about the Van Hieles' research: "As long as the child is not able to reflect on its own activity, the higher level remains inaccessible. The higher level of operation can then, of course, be taught as algorithm though with little lasting consequence." (Freudenthal, 1973, p.130)

P. van Hiele thus formulated a system of thought levels in geometry and D. van Hiele - Geldof concentrated on teaching experiments to raise students' thought levels. Although much attention has been given to the levels of thinking, this is only one of three components of the van Hiele model, the other two being the notion of insight and the phases of learning. Insight is defined by the ability to perfom in a possibly unfamiliar situation, by a competent performance of the acts required by the situation and by the application of an intentional (deliberate and conscious) method to resolve it (van Hiele, 1973 in Hoffer, 1983). The phases of learning proposed by the van Hieles encapsulate a didactical prescription for raising the students' level. Hoffer compares the phases to Polya's principle of consecutive phases (Polya, 1965) and to the learning cycle of Dienes (1963), stating that a common element is the continuity of the generation, refining and extension of ideas by the student (Hoffer, 1983).

The thought levels have mainly been described as they apply to geometry (Hoffer, 1983, Fuys et al, 1985, Usiskin, 1982, Wirszup, 1976). However,

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there have also been attempts to generalise their application to structure courses in other disciplines such as chemistry and economics in the Netherlands. Hoffer proposed a topic - free description of the levels (Hoffer, 1983), which is considered by the author as worthwhile to present in parallel with a geometry - specific description, since it provides an insight into the nature of the levels which are "characterised as differences in objects of thought" (van Hiele, 1959, in Fuys et al, 1985).

level 0, geometry: recognition of shapes by their global appearance;
level 0, topic - free: objects are the base elements of the study;

level 1, geometry: analysis of properties of figures, but no
                         explicit interrelation of figures or properties;
level 1, topic - free: objects are properties that analyse the base elements;

level 2, geometry: interrelation of figures and properties but no
                        organised sequences of statements to justify observations;
level 2, topic - free: objects are statements that relate the properties;

level 3, geometry: deductive reasoning in an axiomatic system;
level 3, topic - free: objects are partial orderings (sequences) of the
                        statements;

level 4, geometry: rigorous study of axiomatic systems;
level 4, topic - free: objects are properties that analyse the partial orderings.

The Soviets used the van Hiele model to analyse their geometry teaching materials for children aged 7 to 15 in 1960, and have carried out research, a major finding of which is that children up to the age of twelve mostly tend to perceive figures as "wholes", and only 10 - 15 % reach level 1, which is needed as a basis for further study of geometry (Pyshkalo, 1968). Extensive research has been carried out more recently in the United States, with similar results with respect to the identification of high school students' thought levels (Chicago project, Usiskin, 1982). Having been introduced to the model by Wirzup in 1974 (Wirzup, 1976) and Freudenthal (1973), educators in the States carried out research in another two aspects of the model, i.e. longitudinal case studies of children (Oregon project, Burger, 1982) to describe their reasoning processes in geometry in terms of the model, and the development of instructional modules to study the effects of such

instruction (Brooklyn project, Fuys te al, 1985).

An attempt to synthesise the findings from the research carried out in the Soviet Union and in the States, which are informative for the present study, reveals the following:

a) when the van Hiele model was used to assess the teaching of geometry in the States, it revealed that children had very little or no experience with geometry at levels 0 and 1 at the elementary period (most geometry textbook material at primary education corresponded to van Hiele level 0) and were subsequently introduced to deductive geometry (levels 2 and 3) in high - school, when their actual background was at level 0 (Geddes, 1982 in Paalz Scally, 1986);

b) results from several studies (Pyshkalo, 1968, Burger 1982, Fyys et al, 1985) agree that a very high proportion (85 - 90% according to Pyshkalo) of sixth grade children (age 12) are at level 0;

c) Fuys argued that the previous results (those of his own study amongst them) correspond to the students' actual level of thinking. However, after analysing rather detailed interview data from 16 sixth graders' experience (4.5 to 6 hours in total for each student) with suitable teaching materials (developed by the Brooklyn project, Fuys et al, 1985), Fuys reported that all but three of the students showed considerable progress in terms of level of thinking, consequently arguing that their potential level was level 1 or even level 2. Similar findings had been reported much earlier by Pyshkalo (Pyshkalo, 1968), while van Hiele had already stressed the importance of instruction, going so far as to state that "progress from one level to the next is more dependent on educational experiences than on age or maturation" (in Fuys, 1984, p.114). Finally, Dreyfus and Thompson (1987) reported a difference between childrens' actual and potential thinking levels.

2.2.4 Logo as an inductive Geometrical experience

There has been recent research in the States, attempting to bring together the van Hiele model of geometrical thinking and children's learning of geometry through Logo programming. The Atlanta - Emory Logo project (1984 - 86, Olive et al, 1986) involved one - semester long Logo courses to classes of around 20 ninth grade minority students. The general research objective was to determine the impact students' interaction with Logo has on their mathematical thinking (Olive and Paalz Scally, 1987). Although the project lends itself to criticism concerning the clarity of the research design, and the development of the courses (Olive, 1985), there have been a number of subsequent analyses of the data - consisting of dribble files and transcripts from clinical interviews - from different perspectives, such as the effects of the Logo experiences on the students' "non - verbal cognitive abilities" (Olive and Lankenau, 1987), the implementation of various models to evaluate students' responses (Olive and Paalz Scally, 1987) and the effects of learning Logo on students' understanding of specific geometric concepts (Paalz Scally, 1987).

In the latter research, there are certain points of interest to the present study. Firstly, the researcher perceived the use of Logo as a means to provide children with experience of geometrical thinking corresponding to level 1, thus explicitly adressing a problem area highlighted by the relevant research on the van Hiele model described above. The interest here lies in the perspective of using Logo as an environment to provide opportunities for inductive thinking as a prerequisite, or an intermediary step towards deductive geometry. Secondly, the interviews concentrated on specific geometrical topics (angles, triangles and quadrilaterals), thus taking explicitly into account some geometrical content area; descriptors of the van Hiele levels were developed applying specifically to each topic. Although no significant differences were found between the experimental and comparison groups of the study - not surprisingly, according to Papert's criticism on "treatment / effect" methodologies in studies involving children's learning with

Logo (Papert, 1985) - Paalz Scally reported indications of individual progress of Logo group children corresponding to the levels.

On the topic of angle, she also reported the only indication from van Hiele / Logo research of the "discrepancies between students' understanding of static angle and their ability to apply that knowledge to tasks that involve turning angle" (Paalz Scally, 1986, p.127). It seems to the author that there are two implicit issues underlying this statement; the need for a more precise awareness of the geometrical content, from both a research and a pedagogical perspective and a deeper understanding of the children's thinking processes. For instance, the process - oriented studies reviewed above have indicated that using an idea in different contexts - such as the static and dynamic view of angle - is by no means an easy task for children, either in Logo environments (e.g. Hoyles and Noss, 1988) or in a more general sense (e.g. DiSessa, 1982, Lawler, 1985). Moreover, recent Logo research (informed by the process - oriented studies) on children's learning of specific geometrical contents has began to reveal that, at least for the concept of angle, children seem to keep dynamic and static definitions in different "mental compartments" (Kieran, 1986a).

The above discussion concerning the limitations of the research involving the van Hiele model and learning geometry with Logo, was restricted to specific research projects. However, there seems to be ground for consideration of certain more general research issues; firstly, whether enough is known about the relationship between the geometrical thinking described by the van Hiele model and children's thinking processes in Logo environments; secondly, whether there has been sufficient analysis of how children's programming strategies relate to the model as a descriptor of children's geometrical thinking; finally, on how the model might explain limitations of children's understandings of geometrical notions within different geometrical systems. These ideas are now discussed.

Firstly, although the van Hiele model has been used - prior to the Logo

studies - as a method for identifying children's level of geometrical thinking, it was considered most useful for understanding more about such thinking and consequently improving the teaching of geometry (Fuys, 1984, Hoffer, 1983). Logo, however, has provided the opportunity to create educational environments of an intentionally different nature than those of conventional schooling. The understanding of children's thinking processes in such environments has been the object of a substantial amount of research. The extent to which the relationship between these processes and the geometrical thinking portrayed by the van Hiele model is understood seems open to question. Consequently, using the model to devise instructional programs for Logo courses based on the model seems somewhat problematic, at least with respect to the time when the Atlanta - Emory Logo project took place. Recent research has been carried out addressing the issue of the use of the model in its de - contextualised form, to understand growth in children's thinking while doing Logo, sensitive to the "ego - syntonic" nature of the "turtle geometric" content (Olson, Kieren and Ludwig, 1987).

Research using the model to determine the effects of Logo programming on children's mathematical thinking falls under a wider category of psychometric research on the effects of Logo "treatments", criticised by Papert (Papert 1985) for its irrelevance to "constructivist" environments, and for its lack of measuring sensitivity. Finally, research with the model as an explicit educational priority and the use of Logo as a means to address problems highlighted by the model (Paalz Scally, 1987) seems to have clearer objectives which, however, are not within the direct interests of the present study.

A second consideration concerning such research is how the programming element of the Logo activities relates to van Hiele levels of thinking and to geometrical content. A distinction between programming and geometry has been taken into account in a recent attempt by Kieren (Kieren, 1987) to explain growth in children's thinking while doing Logo. The analysis is based

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on a study carried out by Ludwig (1986), involving a unit of instruction of motion geometry for Logo experienced 12 year olds, using turtle geometry and adopting the intents of the van Hiele phases of learning. The children were closely monitored and detailed video - taped data was collected. Analysis revealed a progress in the level of children's thinking from level 0 (involving turtle movements used as a "drawing tool") to level 1 thinking, where figures were seen as a group of commands. However, distinguishing between the programming and the geometry aspect of turtle geometric activities involves some contradiction with the application of the van Hiele model on turtle geometry as a whole, leaving questions such as the necessity of using procedures to achieve level 1 thinking, unasked.

Thirdly, a distinction between different geometrical systems has not been explicitly addressed, an issue which applies in general to research on the learning of geometry with the use of Logo. Such content analysis could be useful in interpreting "unexpected" difficulties in children's understanding of geometrical concepts such as static and dynamic angle (Paalz Scally, 1986) and in extending Logo courses such as the one designed by Ludwig (Olson et al, 1987) to contents beyond motion geometry. The issue of the relationship between Turtle geometry and different geometrical systems has been addressed from a mathematical perspective by Papert 1980, Abelson and diSessa, 1981 and Harvey 1985 and is reviewed in the following section.

2.2.5 The geometrical nature of Turtle Geometry

In an analysis of the question 'what mathematics does one learn when one learns Turtle geometry' Papert stated that the turtle is a reconstruction of the qualitative core of a particular mathematical structure, differential geometry; "the turtle program is an intuitive analog of the differential equation, a concept one finds in almost every example of traditional applied mathematics. Differential calculus derives much of its power from an ability to describe growth by what is happening at the growing tip." (Papert, 1980, p. 66)

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To illustrate this, Papert used the example of the construction of a circle with the turtle, by giving it instructions to move forward one step and turn right one degree repeatedly. He described this process as referring "only to the difference between where the turtle is now and where it shall momentarily be. This is what makes the instructions differential. There is no reference in this to any distant part of space outside the path itself. The turtle sees the circle as it goes along, from within, as it were, and is blind to anything far away from it." (Papert, 1980, p. 67). It is in this sense that Papert, and later Abelson and diSessa (1981) and Harvey (1985) characterised Turtle geometry as "Intrinsic".

To illustrate the point further, contrasts have been made between constructions of specific geometrical figures (e.g. square, rectangle, circle) in different geometrical systems, namely the Differential, Euclidean and Cartesian. In the example of the circle for instance, Papert states; "For Euclid, the defining characteristic of a circle is the constant distance between points on the circle and a point, the centre, that is not itself part of the circle... in Descarte's geometry, in this respect more like Euclid's than that of the turtle, points are situated by their distance from something outside of them, that is to say the perpendicular coordinate axes." (Papert, 1980, p.67).

In their analyses of Turtle geometry, Papert, Abelson and diSessa, and Harvey perceive the intrinsic nature of the geometry as the main factor which makes it different from the Euclidean and Cartesian systems. However, emphasising the difference between Cartesian and Turtle geometry, Abelson and diSessa and Harvey also give a global / local dimension as a disrciminating characteristic; "Each instruction... takes into account the turtle's position within the screen as a whole. The "point of view" from which we draw the picture is that of an observer standing above the plane looking down on all of it... By contrast, the turtle geometry metaphor adopts the point of view of the turtle itself; each line is drawn without regard to where the turtle is in global terms." (Harvey, 1985, p.126).

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Moreover, Abelson and diSessa uses an intrinsic / extrinsic dichotomy to characterise properties of geometric figures, stating that "an intrinsic property is one which depends only on the figure in question, not on the figure's relation to a frame of reference." (Abelson and diSessa, 1981, p.13).

In the author's opinion, the latter two dichotomies of local / global descriptions and intrinsic / extrinsic properties of figures seem useful in differentiating Cartesian and Turtle geometry but not so clear regarding Euclidean geometry, even if the dichotomies are considered as a continuum, with Euclidean geometry somewhere in between the two "extremes" (Abelson and diSessa state that Turtle geometry is more intrinsic and more local than Cartesian). For instance, the centre of a circle is an intrinsic property to the circle according to Abelson and diSessa but not an intrinsic characteristic of the circle according to Papert.

Such analysis, however, is not within the scope of the present study which will adopt Papert's perspective of discrimination between geometries, i.e. that the word "intrinsic" characterises the method of constructing a geometrical figure by the absence of reference to any point outside the trace of the figure. If such a reference is involved, the figure is constructed in a non - intrinsic method. A detailed discussion of how these ideas reflect on the microworlds designed for the present study is incorporated in chapter 3.

2.2.6 Children's learning with geometry
in the context of Logo activities

Although there has been a certain amount of recent research focusing on both the geometrical content learned by children doing Turtle geometry and on the process of their learning (Hillel, 1985a, Kieran, 1986b, Lawler, 1985, Hoyles and Sutherland in press, Hillel, Erlwanger and Kieran, 1986), there has been no explicit account taken of the geometrical conceptual field available to the children in a broad sense, i.e. concerning the relation between the geometrical nature of Turtle geometry and other geometrical

systems such as the ones mentioned above. Consequently there is very little known about the potential of Logo to provide the opportunity for children to engage in "constructivist" learning within a wider span of geometrical systems. However, a review of this research is relevant to the present study since it provides a background for understanding children's thinking processes in relation to specific geometrical concepts.

Having carried out substantial work into the children's learning of programming (Hillel, 1984 and 1985a and b), the Canadian researchers began to focus more on the geometrical content of Turtle geometry (Kieran, 1986a and b) and on the thinking schemas formed by the children in relation to this content (Hillel, 1986, Kieran et al, 1986). Hillel carried out a study based on his longitudinal observations of eight to twelve year olds' Logo programming, with the aim of investigating the links between children's thinking processes and the contents of both programming and geometry.

On the process level, the researcher provided evidence that the children's perception of doing Turtle geometry was largely dependent on the "drawing with the turtle" metaphor, i.e. that they developed a "drawing schema", setting themselves "concrete" goals, choosing inputs to commands based on perceptual cues rather than on inherent mathematical relations. Hillel argues that the "drawing schema" is compatible with a "naive programming mode" (Kieren, 1985), but not so with a "planned programming mode". He presents as evidence, children's perceptual organisation of figures with "tight" inherent geometrical or programming relations. Hillel also suggests that the "drawing schema favours a particularly static conception of procedures" since the children not only have difficulties in changing their mental representations of procedures between "procedure as product" and "procedure as process", but also find it difficult to understand geometrical relationships required for appropriate turtle state changes in the interfaces between procedures. However, he agrees with Hoyles and Sutherland (in press), that when children are engaged in solving structured tasks, they are more ready to consider a "planned programming mode" when the tasks are of an "abstract"

nature.

Concerning the geometrical content, Hillel perceived turtle geometry as a "particular type of geometry" the nature of which is independent from the use of computers, but that its embedding within a computer language "not only provides for a very different way of 'doing mathematics' but also brings into play some interesting links between programming concepts and geometric ones" (Hillel, 1986, p.433).

Kieran investigated children's developing understandings of a specific geometrical concept, the angle, through their experiences with Logo (Kieran, 1985, 1986 a and b). There has been a substantial amount of research revealing children's difficulties in understanding the concept of angle in general. Their misconceptions seem to relate to misinterpretations of features of the graphical representation of angle (e.g. the length of the line segments) or features of the plane (e.g. the area between the line segments) as the defining features of the angle (Hershkowits and Vinner, 1984; Hart, 1981; APU, 1982; Noss, 1987). Other recent research (Papert et al., 1979, Noss, 1985, Hoyles and Sutherland in press, Hoyles and Noss, 1987a, Hillel, 1984, Zack, 1986, Paalz Scally, 1986) has indicated that children find difficulties in understanding the concept of angle in Logo and in relating angles to turtle turns, while Noss, corroborating the findings of Papert, suggested that knowledge about length is more deeply rooted than knowledge about angles (Noss, 1987).

Kieran's research consisted of two studies with a difference in the age of the children and their prior experience with formal geometry (Kieran, 1986a). The first study involved nineteen 10 to 11 year old children working with turtle geometry once a week for a year (1984 - 85) on projects of their own choosing. They had had no formal geometry teaching. The researcher carried out three interviews with each child individually, in September, January and June of that year and a follow - up interview in March 1986 (the children continued to work with Logo during the 85 - 86 period). Kieran reported that

prior to the study, the children had a static concept of an angle, and that their experience with Logo led them to form a rotational representation of angle which, however, remained distinct from the former. At the end of the year 84 - 85, Kieran reported:

i) many children were still confused whether an input to a turn command was a rotation or the constructed angle;

ii) it was easier for the children to draw a figure corresponding to a command rather than give the command which was needed for a given figure;

iii) most children retained their static conception of angle;

iv) most children classified angle size in terms of the legth of the arms;

The second study, carried out during 1985 - 86 involved six sixth - formers (12 year olds), working with Logo once a week in the university computer lab under close observation (dribble files were kept and notes on children's and researchers' spoken language were taken). Little evidence is shown in this study of the children's development of the notion of dynamic angle, but some evidence is given of the children beginning to integrate static and dynamic angle notions, in contrast to the fourth graders who "seemed to keep static angles and their measurement in one mental compartment, and dynamic turns and their input in another" (Kieran, 1986a, p.104).

It is interesting that although the initial aims of the research seemed rather insensitive to the dichotomy of a static / dynamic conception of angle (Kieran, 1985), analysis of the data highlighted the importance of the issue for the children's learning of the concept, leading the researcher to further analysis of this issue in particular (Kieran, 1986b). Minsky's hypothesis that children develop an intuitive "turn - schema", ordering rotations according to the amount of turn (Minsky, 1975), was contrasted to evidence from the study (prior to experience with Logo) showing influence of the global size of paper

representations of a rotation on children's conception of angle. The researcher concluded that experience with Logo helped the children refine their "turn - schema" by focusing more on the "sharpness of the point", with respect to exterior turns, but did not seem to have effect on refining the schema with respect to interior turns. Furthermore, although the sixth - graders were able to respond to questions about interior and exterior turns with similar facility, they had major difficulties in understanding the supplementarity relation between interior and exterior turns.

The children's difficulty of understanding the concept of angle, but also the prevailing role of the concept in turtle geometric environments, is related to the fact that the issue was analysed in the Logo Maths project even though it was not an initial objective of the research. Supported by background data collected by administering structured tasks to the case study and extended network pupils and holding structured interviews with the former, the researchers analysed in detail and highlighted the work of Janet - a member of a collaborative working pair - throughout the three years of the study (Hoyles and Sutherland, in press). Through the experience of Janet, who carried her uncertainties with amount of turn and turtle orientation thoughout the three years, the researchers conclude that "pupils may not perceive inputs to RT and LT as rotations in circumstances where the nature of these inputs is determined by the context... in such situations children compute inputs to RT and LT, add and subtract them or compose them at the level of action but do not necessarily synthesise their resulting input to a total amount of turn; put another way children might be adding and subtracting numbers or adding and subtracting actions but not angles!" (Hoyles and Sutherland, in press). In effect, the researchers' process - oriented perspective seems to reveal the importance of the additional problem that pupils (even up to the age of 14) will not necessarily use the mathematics that is there - especially when the projects are chosen by them and are real world representations.

The same issue of the children's limited use of the geometry, was by no means ignored by the researchers in Canada who collaboratively analysed

six 11 - 12 year olds' uses of geometrical concepts in trying to solve structured tasks. The children were introduced to the VEE, TEE, MOVE, TRT and TLT special logo primitives described in section 2.1.4, which produced "vee" and "tee" shaped figures and moved and turned the turtle in a slowed - down fashion. They were introduced to the normal Logo commands at about half way through the year, and were given throughout the year time to work on their own projects. The researchers' analysis was based on the children's work on structured tasks involving complex figures composed of tee and vee shapes linked by "precise" geometrical relations (e.g. fig. 2.1) and concentrated on the children's perceptions of these relations and of the geometrical structure within the VEE and TEE procedures (Hillel et al, 1986, Hillel and Kieran, 1987).

The main outcome from these analyses was the researchers' synthesis of the children's solution schemas across tasks into the following two general schemas:

i) the perceptual schema, where inputs are chosen on the basis of perceptual cues, i.e. without using geometrical relationships or properties and without relating the turtle's state to the previous state; rationale for the choice of inputs was often expressed as "it looks like...";

ii) the analytical schema, where inputs are chosen on the basis of geometrical relationships inherent in the given information and/or "from mathematical knowledge". An example given for the rationale for choosing inputs was; "Since this vee is 60 degrees, then...".

The researchers concluded that most children tend to use the perceptual schema spontaneously, not perceiving the need for a different approach unless the task at hand is unsuccessfully solved (according to their view). In such cases, 11 - 12 year olds are more ready to shift to an analytical schema than younger children (Hillel, 1986, Kieran, 1986a). Moreover, they concluded that the children's perception of the precision of a task influences

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the strategies they use to solve it, a finding which is supported by the Logo Maths project researchers. Finally, emphasis is given on the potential value of such analysis for interpreting children's work and designing Logo activities.

2.2.7 A cognitive persepctive of children's thinking with the turtle

The conclusions from the previous research may be seen through a different perspective in relation to the problem of why children find it so difficult to synthesise the geometrical understandings they develop in the context of Turtle geometry with paper and pencil representations of these understandings. Little research has been carried out concerning the nature of the process by which children identify with the turtle to drive it on the screen, and in particular, the nature of "body - syntonic" learning (Papert, 1980). Papert argues that identifying with the turtle enables children "to bring their knowledge about their bodies and how they move into the work of learning formal geometry" (p.56), stating that the turtle metaphor enables children to make sense of an idea. However, the question of why identifying with the turtle helps children make sense of ideas does not seem to be addressed in detail. Furthermore, subsequent research has shown that children often identify with the turtle without using the available mathematics, but their personal, naive strategies (Leron, 1983, Hillel at al, 1986).

There is equally very little research on children's understandings of the differences and the relations involved in constructing geometrical figures by means of Turtle geometry (and its intrinsic nature) and by methods requiring non - intrinsic points of reference such as, for instance, the centre of a circle or the origin of the coordinate plane.

The research which comes closest to addressing the issues of the nature of learning by identifying with the turtle and children's perception of Turtle

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geometry in relation to other geometries was carried out by Lawler, whose deep investigation of the cognitive development of a six year old child (Miriam) working in Logo environments has been an influence on the present study,for reasons outlined below (Lawler, 1985).

A specific part of Lawler's research into Miriam's cognitive organisation, was to investigate the child's thinking processes when attempting to make sense of a new knowledge domain which could not be linked to her "turtle geometry microview", i.e. the fragment of knowledge she chose to employ. The particular relevance of this part of Lawler's research with the present study was his choice of Turtle geometry and Coordinate geometry as two essentially different content domains, "essentially different in that they connect naturally to different kinesthetic subsystems" (Lawler 1985, p.151). He contended that the "turtle navigation" microview has its roots in the child's very early experiences with movement; he called these experiences a "personal geometry" microview which, "developing from a coordination of the somatic and locomotive sensorimotor subsystems, has those subsystems as its ancestors" (Lawler, 1985, p.163).

On the other hand, Lawler set out to show that the "coordinate microview" has its roots elsewhere, i.e. in the visual subsystem. To do that, he gave Miriam a computer game which involved the use of the coordinate description of the plane in order to change the direction and the position of an entity similar to the turtle. Miriam's main confusions related to her perception of the coordinate values as having an operation - operand structure, for instance she interpreted signs as signifying operations and numerical values as quantities of a change of state. However, Miriam formed a different microview, disconnected from movement or arithmetical knowledge, but related to "visually based knowledges", as a result of her experience with a different computer environment which simply required the naming of locations on the screen.

Two aspects of Lawler's conclusions are important for the present study.

Firstly, that the understanding of Turtle geometry and Coordinate geometry depends on disparate fragments of knowledge which resepectively have their roots in the child's very early locomotion and visual experiences. Secondly, that forming connections between such microviews of different descent is by no means a trivial task. To (successfully) achieve such a connection in Miriam's mind, for instance, Lawler provided her with an experience (involving eye - hand coordination) which was able to function mediatively between descendents of the locomotive and visual subsystems. His respective concluding statement seems provokingly strong: "the connections between late - development cognitive structures can only occur through the mediation of cognitive descendents of the coordinating schemata of the sensorimotor period." (Lawler, 1985, p.184)

In the author's opinion, Lawler's in - depth probing of a child's mind offers limited but very precise evidence of a very close correspondence between the content differences separating two distinct geometrical systems and the respective fragments of knowledge the child applied to understand them. It is also important for the present study that Lawler regards these fragments as descendents of deeply rooted intuitive ideas, deriving from early experiences with motion for Turtle geometry and with vision for Coordinate geometry. Freudenthal has also supported the idea that geometrical knowledge has deep intuitive roots; "Many geometrical objects and concepts have been formed early, most of them at the primary school age and some of them even earlier, though they do not yet bear verbal labels, or at least those labels that we have learned to attach to them in our geometry lessons." (Freudenthal, 1983, p.226).

An example of a misconception, possibly due to 12 year old Gary's separate mental compartments for Turtle geometry and Plane geometry, can be depicted from a case study in the Brookline report (Papert et al, 1979); Gary used a circle procedure (RCIRCLE - input) which caused the turtle to trace along the approximation of a curve resulting from repeated "small" moves and turns. The input, however, was the radius of the circle, i.e. required

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taking into account the "non - intrinsic" centre. Although Gary moved the turtle 80 steps to the left, he then typed RCIRCLE 45, instead of RCIRCLE 40 which would give the diameter of 80 (Papert et al, part 3, p. 7.4).

2.3 CONCLUSION

The reviewed literature has highlighted how the role of geometry in mathematics education has been de - emphasised due to a mismatch between the taught content of deductive Euclidean geometry and the ideas which the children were able to master. Freudenthal's case for reconsidering geometry as a field allowing inductive as well as deductive learning, is supported by substantial evidence,from a wide span of educational systems, that children in primary and secondary education have very little experience in relating geometrical ideas to some meaningful "concrete" reality. Although the work of the van Hieles on children's geometrical thinking has played an important role in this research, it was decided that the van Hiele model could not be of direct use in interpreting children's behaviors in the present study, mainly because there is little known about the relationship between the geometrical thinking described by the model and children's thinking processes in Logo environments. This was felt to be a shortcoming of recent studies using the model in Logo - related research. However, the context - specific descriptions of inductive and deductive geometrical thinking provided by research involving the model has been informative for the present study.

In Turtle geometric open - ended environments children have the opportunity to engage in inductive thinking, since they can explore and develop understandings from experiences within personally meaningful situations. Research into children's thinking processes within such situations has revealed that they do not always use the geometrical ideas embedded in Turtle geometry, often employing perceptual cues, perceiving the turtle as an extension of their hand while drawing. Papert and Lawler suggest that the ideas the children do use in identifying with the turtle to drive it on the screen are based on deeply rooted intuitions concerning bodily motion and that this

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is why Turtle geometry makes sense to children. Recent studies, however, have indicated that children do not relate their understandings of geometrical notions in Logo, to the same notions in "static" environments. Very little is known about the potential for children to use the intuitive "turtle metaphor" to develop understandings within a wider span of geometrical notions than those belonging to Intrinsic geometry. This issue is investigated in the present study.

CHAPTER 3

A CONCEPTUAL FRAMEWORK
FOR THE MICROWORLDS OF THE STUDY

Logo is a programming language developed by Papert and Feurzeig in the late sixties and derived from LISP, a powerful list processing language. It has been characterised as procedural, extensible, interactive, recursive and functional (Noss, 1982, Leron and Zazkis, 1986, Klotz, 1986, Hoyles and Sutherland, in press). Detailed reviews and analyses on its origins and its programming nature can be found in Noss, 1985, Harvey, 1985. An important part of Logo is turtle graphics, where commands to move or turn are given to a screen cursor with position and heading. It is possible for the turtle to leave a trace of its movements, and thus pupils can drive the turtle to make figures on the screen.

The present chapter contains an analysis of the general mathematical principles underlying the Logo microworlds in the present study.

3.1 TURTLE GEOMETRY AND THE INTRINSIC,
EUCLIDEAN AND CARTESIAN GEOMETRICAL SYSTEMS

The present study addresses the problem of whether it is possible for children to use the turtle metaphor in developing understandings of Intrinsic, Euclidean and Cartesian geometrical ideas. Turtle geometry as it is implemented in the standard Logo language, however, has been developed and characterised as Intrinsic geometry (Papert, 1980, Loethe, 1985). It was therefore necessary to design Logo microworlds based on the turtle metaphor which would invite the use of ideas from all three geometrical systems.

As discussed in the review of the literature (2.2.5), there has not been much concern for a rigorous investigation of the nature of Intrinsic geometry with respect to its distinction from Euclidean and Cartesian geometry. Attempts to

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delineate the geometrical identity of Turtle geometry have had differing foci on what it is that matters in the distinctions between Turtle, Euclidean and Cartesian geometry (Papert, 1980, Abelson and diSessa, 1981, Harvey, 1985). In order to discriminate between the three geometries, Papert used the "nucleus" of each geometrical system, its mathematical entity (Papert, 1980). In Turtle geometry the entity, which is the turtle, has a state i.e. a position and a heading. This is not so in Euclidean and Cartesian geometry where the entity is the point on the plane. In Cartesian geometry, however, the point has a location determined by an absolute description of plane locations. This is not the case for Euclidean geometry, where the point does have a location relative to other points or line segments, but not to an absolute locating system. This is the discriminating factor between the entities of Euclidean and Cartesian geometry.

To illustrate how these distinctions offer a way of discriminating among methods of constructing a geometrical figure, Papert used the example of a circle. The Cartesian method involves the location of the points of a circle via an equation relating each point to the perpendicular axes of the coordinate system. A circle is therefore described as; $(x - a)^2 + (y - b)^2 = R^2$, where x and y are the distances of a point of the circle with respect to the axes, a and b are parameters determining the displacement of the circle's centre from the origin and R is the radius. The defining characteristic of a circle in the Euclidean method is the constant distance between the points of the circle and the circle's centre, itself not a part of the circle. Turtle geometry, however, relies upon the notion of constant curvature, i.e. a constant turning for a given forward motion.

In the construction of figures, there are two main factors which discriminate Turtle geometry in general;

1) The turtle's state is uniquely determined by its immediately previous state. This is what makes Turtle geometry differential.

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2) There is no reference to any distant part of space outside the turtle's path. This is what makes Turtle geometry intrinsic.

The differential property was emphasised by Papert in order to highlight Turtle geometry's links to both children's intuitions and a powerful geometrical system which, for instance, provides the base for Newton's physical laws; "Turtle geometry has links both to the experience of a child and to the most powerful achievements in physics." (Papert, 1980, p.67). In the related literature (e.g. Loethe, 1985), and for the purposes of this study, however, Turtle geometry is characterised as Intrinsic geometry, in incorporating both the above "differential" and "intrinsic" principles.

Although the application of both principles makes the turtle the entity of a powerful geometrical system, it is rather restrictive with respect to the turtle's "awareness" of the geometrical plane. In the example of the circle, for instance, the only way for the turtle to go to a given point on the circle is to retrace along the curvature until it reaches the point. The intrinsic turtle can be characterised as "blind" or, more accurately "short sighted"; it has no "awareness" of the plane further than its adjacent state(s). Consequently, for instance, the centre of the circle is non - existant for the turtle.

In the microworlds designed for the present study, it was essential for the mathematical entity to be the turtle with its state of position and heading, since the primary concern was to create situations inviting the use of the turtle metaphor. Consequently, the geometry in these environments was generated in a dynamic way, that is by changing the state of the turtle. It follows therefore, that the present study has a differing perspective than that of Papert in distinguishing among the three geometries; for Papert, the important discriminating factor is the mathematical entity of each system. For the present study, however, it is the method by which the turtle is controlled, i.e. the ideas employed by the pupil in order to change the turtle's state. If these ideas conform to both the differential and intrinsic principles stated above, then the method of changing the state, for the purposes of this study, will be

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characterised as intrinsic.

The microworlds in the study, however, were designed to provide the possibility of changing the turtle's state without conforming to the "strict" intrinsic principles; for instance, the opportunity is provided for referring directly to any point of the turtle's path rather than just the adjacent to the turtle's present state. In such cases, the method for changing the turtle's state is characterised as "non - intrinsic".

In order to illustrate how these ideas are embedded in the Logo language, an account is given at this point, of the state - changing Logo instructions used to design the microworlds of the study. The commands were presented to the children as "primitives", i.e. as the basic tools available for constructing figures on the screen. In each microworld, a subset of these commands was employed, according to the nature of the designed mathematical environment and the research objectives. The specific combination of commands used for each microworld and the designed situations within a microworld are presented in chapters 6, 7 and 8 respectively. However, at this point, it was seen as useful to outline in brief, the general mathematical principles involved in the methods of changing the turtle's state via the Logo commands used in the study. In accordance with those principles, the commands can be split into the following five groups:

Group A

FD (number), BK (number),
      - where "number" is a numerical quantity in unit lengths of turtle
       position change;

RT (number), LT (number),
      - where "number" is a numerical quantity in degrees.

The commands in Group A represent an action - move or turn - and the quantity of that action in the respective metric system - turtle steps or degrees. In the study they are often referred to as the "action - quantity" or "action -

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quantification" commands.

POST (name);
DISTANCE (name);
DIRECTION (name);
        - where "name" is a user - defined name of a location.

The POST command provides the ability to give a name to a position of the turtle. The name can only be given when the turtle is in that position.

The DISTANCE command provides the distance between the turtle's current and some previous position. This previous position can only be specified by a name having been given to it by the POST command.

The DIRECTION command provides the angle between the turtle's current heading and the heading required to face some previous position. The angle is measured from the current heading clockwise. The previous position can again only be specified by a name having been given to it by the POST command.

The POST, DISTANCE and DIRECTION commands have been adapted from Loethe (1985). Loethe's purpose in designing the commands was to define a microworld "of clear and vivid concepts which describe extrinsic geometry..." (Loethe, 1985, p.123). "The main point by the use of these new primitives is that we avoid coordinates by posting important points in advance. This makes these points to elements of the geometric setting of a figure rather than to elements of an absolute coordinate system of the plane; these posts are local to the related procedure and posting in this way guarantees that the figure can be drawn in every position only dependent on the initial state of the turtle." (Loethe, 1986, p.8). In the present study, the POST command was changed so that its use results in a graphical representation of the respective location and of the name of the location given by the pupil (see chapters 6 and 7).

The POST, DISTANCE, DIRECTION commands do not cause a state change in themselves. However, they can be used for referring to previously labelled points (via POST) in the plane which are distant from the turtle's current position.

Group C

SETX (x) - where "x" is a number representing a location on the x axis, in unit lengths of the coordinate axes;
SETY (y) - where "y" is a number representing a location on the y axis in unit lengths of the coordinate axes;
SETPOS (x y) - where "x" and "y" represent the coordinate values of a location on the plane.

The SETX, SETY, and SETPOS commands change the position of the turtle. They are not action - quantity commands since the input here represents a location on the plane rather than the quantity of a turtle action. In standard Logo the turtle metaphor becomes mathematically meaningless, since the commands cause a displacement of the turtle to the named location irrespective of the turtle's heading which remains unaltered. This is in accordance with the mathematical entity in Cartesian geometry, the point, which does not have a heading. In standard Logo, however, the graphical representation of the entity remains the turtle even with the use of the Group C commands. In the present study the commands have been changed - the SETX, SETY and SETPOS commands work only if the turtle is facing towards the target location (chapter 6). In this way, the turtle metaphor is essential for changing the state even when the Cartesian coordinates are used.

Group D

SETH (number) - where "number" represents the heading in degrees in relation to an absolute heading system in which headings are measured clockwise starting from zero, i.e. the turtle facing upwards.

The SETH command is not an action - quantity command, since changing the heading comes about via the description of an absolute direction rather than the quantity of a turtle turn. However, it does not directly require the use of the coordinate system either.

Group E

SETH TOWARDS (x y) - where "x" and "y" are the coordinates of a plane location.

It is obvious that SETH TOWARDS is not an action - quantity command. Heading change is caused via a description of a location, rather than the description of a direction as in the SETH command. There is, therefore, a direct reference to the coordinate system.

The SET commands, i.e. SETPOS, SETX, SETY, SETH, SETH TOWARDS are refered to in the study as the "coordinate" (or the "Cartesian") commands.

The command descriptions given above refer to the general mathematical principles underlying the use of the commands to change the state of the turtle. What follows is a discussion of the relationship between the specific geometrical ideas which can be employed in using the commands to change the turtle's state and the Intrinsic, Euclidean and Cartesian geometrical systems. A convenient way to present this discussion is via an example involving a hypothetical situation which requires a specific change in the turtle's heading and position.

3.1.1 An example

From its starting position, the turtle has drawn a square and has set a marker labelled "A" on the top right hand vertex, as a result of the following instructions: FD 30 RT 90 FD 30 RT 90 POST "A FD 30 RT 90 FD 30 RT 90

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(fig. 3.1). The geometrical ideas embedded in the construction belong to Intrinsic geometry, since the differential and intrinsic principles are met; the quantities of the action - quantity commands (FD and RT) depend entirely on the action and not on any distant part of the plane outside the path of the turtle.
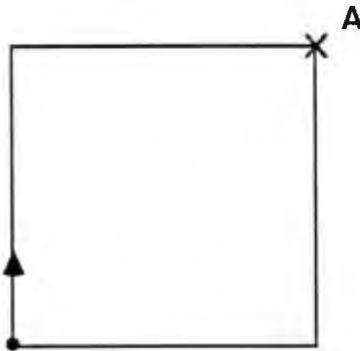
Figure 3.1 An example

The objective in this situation is now to take the turtle directly from its present position to point "A", thus "constructing" the diagonal of the square. It is obvious that what is required is a heading change for the turtle to face point "A" and a position change for the turtle to get there. It is however impossible to retain the differential and intrinsic principles to achieve this since, for instance, reference to point "A" involves a breach of both principles.

The different ways of effecting this heading and position change with the use of the described commands is discussed with respect to the geometrical ideas which can be employed in each occasion.

Changing the heading

a) RT 45

This method employs the action - quantity paradigm. However, the quantity of the action does not depend on the action itself, but on information about parts of the plane outside the turtle's path such as: the internal angle of the square

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and the dissecting property of the diagonal. In other words, knowledge about some of the square's properties involving the plane are a prerequisite for determining the quantity which the turtle has to turn.

b) PR DIRECTION "A (computer feedback is: 45)
   RT 45

This method also employs the action - quantity paradigm. However, knowledge about properties of the figure in the plane is not a prerequisite: the quantity is determined by a "measurement", as if the turtle was equipped with a protractor. Nevertheless, the method is non - intrinsic, since the "measurement" involves reference to a point away from the adjacent positions of the turtle.

c) PR DIRECTION 30 30 (computer feedback is: 45)
   RT 45

This method involves the same ideas as the previous one. The difference is the method of referring to the target point. In both cases referring to the point involves naming it. In the previous case, however, the name is given by the user and the point is necessarily relative to the figure in question. In this case there is an underlying absolute method of naming any location on the plane via the coordinate system. Point 30 30 happens to be related to the square, but naming a point does not necessarily require some relationship between the point and the figure.

d) SETH 45

This is not an action - quantity method of heading change. The numerical input of 45 is a description of a direction on an absolute heading system. There is no reference to point "A". However, knowledge of plane properties of the figure is a prerequisite for determining the turtle's target direction.

e) SETH TOWARDS 30 30

This, also, is not an action - quantity method, since heading change is brought about by a description of a turtle direction. The target direction, however, is described in relation to an absolute location rather than an absolute direction system. This implies reference to a point away from the immediate "vicinity" of the turtle which does not necessarily need to be related to the figure (here, of course, it is). No knowledge about the figure's plane properties is needed.

Changing the position

a) FD 42.46

This method employs the action - quantity paradigm. As in the case of RT 45, the quantity of the action does not depend on the action itself, but on information about parts of the plane outside the turtle's path, such as those required for the pythagoras theorem with the help of which the distance between the two diagonal points can be determined. In other words, knowledge about some of the square's properties involving the plane are a prerequisite for determining the quantity which the turtle has to move.

b) PR DISTANCE "A (computer feedback is 42.26)
   FD 42.26

As in the case of PR DIRECTION "A, the action - quantity paradigm is employed. However, knowledge about properties of the figure in the plane is not a prerequisite: the quantity is determined by a "measurement", this time resembling the turtle equipped with a ruler. Similarly, the method is non - intrinsic, since the "measurement" involves reference to a point away from the adjacent positions of the turtle.

c) PR DISTANCE 30 30 (computer feedback is 42.26)

FD 42.26

In this case, the same ideas apply as in using the PR DIRECTION 30 30 "measurement" followed by the action - quantity command.

d) SETPOS 30 30

This is not an action - quantity method, since position change is brought about by a description of a location. The target location is described in relation to the absolute coordinate system. This implies reference to a point away from the immediate "vicinity" of the turtle which does not necessarily need to be related to the figure (here, of course, it is). No knowledge about the figure's plane properties is needed.

In the present study, the terms intrinsic, euclidean and cartesian geometry, idea or notion (spelt with lower - case initials to distinguish the meaning from the classical terms) are used to convey a specific meaning in accordance with the geometrical ideas analysed above in situations of changing the turtle's state.

a) When the ideas used in a specific state change comply with the intrinsic and differential principles stated above, then they are characterised as intrinsic.

b) When the ideas used in a state change involve a reference to a point in the plane which;

i) is related to the figure under construction,

ii) does not coincide with a position adjacent to the turtle's present position and;

iii) cannot be located on the plane via an absolute plane description method,

then they are characterised as euclidean.

c) When they involve a reference to a point in the plane, which can be located with the use of Cartesian coordinates, then they are characterised as cartesian.

However, this characterisation does not restrict itself to situations of state change. Its application is extended to the method of constructing a geometrical figure. In the previous example, for instance, the notions involved in the method by which the turtle constructed the square by repeatedly moving forward and turning, were intrinsic. The author's awareness that the terms Intrinsic, Euclidean and Cartesian geometry have been much used in various situations and disciplines (e.g. Mathematics, Mathematics Education) led to this need to specify the meaning attributed to them in the present study.

The above analysis involved a particular subset of the Logo language, the state - changing commands, due to their specific relevance to the geometrical principles used in this study. However, all the Logo commands and programming ideas with which the participating children were familiar,were available to them during the research sessions (see chapter 5). Among these, for instance, are the following: REPEAT, procedures, editing procedures, saving and loading files, variables, PU, PD, PE, PRINT. The above analysis can be extended to these commands with respect to their use in generating state changes and constructions of figures. For instance, the following procedure with a variable length input can be written for a square:

```
TO SQUARE :SIDE
REPEAT 4 [FD :SIDE RT 90]
END
```

According to the above, the procedure, the REPEAT command and the

variable input have been used for an intrinsic construction of a square. In the following examples, however, the procedures DIAG1 and DIAG2 involve intrinsic square constructions and euclidean methods to "draw" their diagonals:

```
TO DIAG1                    TO DIAG2
REPEAT 4 [FD 30 RT 90]      REPEAT 2 [FD 30 RT 90]
RT 45 FD 42.26              POST "A
END                         REPEAT 2 [FD 30 RT 90]
                            RT DIRECTION :A
                             FD DISTANCE :A
                            END
```

Finally, the procedure COSQUARE employs a cartesian method to construct a square:

```
TO COSQ
SETPOS 0 30  SETH TOWARDS 30 30
SETPOS 30 30  SETH TOWARDS 30 0
SETPOS 30 0  SETH TOWARDS 0 0
SETPOS 0 0  SETH TOWARDS O 30
END
```

3.2 OVERVIEW OF THE MICROWORLDS IN THE PRESENT STUDY

This chapter consisted of an analysis of the general geometrical principles underlying the Logo commands used in the study. Each of the three microworlds developed for the study incorporates a specific subset of the state - change commands. In summary, the microworlds involved the following:

1) the "Turtle in the Coordinate Plane" (or "T.C.P.") microworld, where the turtle was provided with means of referring to the coordinate system, by either retaining the action - quantity method of changing its state or achieving such changes by location descriptions (chapter 6),

2) the "POST, DISTANCE, DIRECTION" (or "P.D.D.") microworld, where the turtle was given the means to refer to any of its previous positions (chapter 7), and;

3) the "Circle" microworld, where the turtle was provided by four circle - constructing procedures which involved differing intrinsic and euclidean ideas (chapter 8).

The geometrical conceptual field of each microworld is analysed in the respective chapters, as a specific application of the general principles outlined in this section. The particular set of state - changing primitive commands in each microworld is analysed with respect to the method(s) it provides to change the turtle's state, and with respect to the turtle's awareness of the plane when it is equipped with the respective set of primitives. For the purposes of the present study, activities within each microworld were designed for the participating children. The activities for each microworld are analysed in the respective chapters (chapters 6, 7 and 8). The relationship between the research objectives and the microworlds is presented in the following chapter (4.2.2).

CHAPTER 4

AN OVERVIEW OF THE RESEARCH

4.1. A METHODOLOGICAL PERSPECTIVE

The distinction between qualitative and quantitative research has been an increasingly established means of characterising research methodology, in the past decade at least. Recently, however, it has been considered worthwhile to bring this characterisation under scrutiny. Goetz and Lecompte, for instance, maintain that, on the one hand, it is rather naive to classify methodologies according to discrete categories and that continua between two extremes is more realistic (Goetz and Lecompte, 1984). On the other hand, they regard as a more useful means of conceptualising researchers' assumptions of reality and how to explain it, the framing of these assumptions into four dimensions rather than one;

- The inductive / deductive dimension, referring to the place of theory; the inductive perspective implies that the theory is built from the data, while the deductive, that data is found to match some pre - conceived theory.

- The generative / verificative, referring to the position of the evidence within a research and the generalisability of the findings to other populations. Generative research is often inductive and verificative is often deductive.

- The constructive / enumerative, referring to assumptions concerning the analysis of the data. A constructive approach involves the development of the method of analysis during the course of observation and description. Enumeration implies a pre - conceived method involving counting or enumeration.

- Finally, the subjective / objective dimension involving the explanation of reality according to the researchers' own experiences or according to how the

experiences of others match the experiences of the researcher.

Goetz and Lecompte conceptualised each of these dimensions as a continuum, with qualitative research being at the inductive, generative, constructive and subjective end, and the converse for quantitative research.

Since the present study is concerned with the generation and development of mathematical schemas in children's minds in situations within specific geometrical contexts, and since, prior to the study, very little was known about these schemas, the author decided that a qualitative approach would be an effective research strategy. The study involved a year - long preliminary phase during which the objectives of the main research were developed and refined (section 4.2.1, chapter 5). The methodological process of progressive clarification and redefining of the problem areas so as to systematically reduce the breadth of inquiry in order to enable more concentrated attention to the emerging issues has been labelled "progressive focusing" (Parlett and Hamilton, 1977, Atkinson, 1979).

In the present research, however, it was not considered as useful to follow some prescriptive research methodology, but rather, informed by the general methodological issues and assumptions, to allow the research problem itself to determine the method. Parlett and Hamilton have expressed an a corresponding viewpoint more than ten years ago; "Illuminative evaluation is not a standard methodological package, but a general research strategy. It aims to be both adaptable and eclectic. The choice of research tactics follows not from research doctrine, but from decisions in each case as to the best available techniques: the problem defines the methods used, not vice versa." (Parlett and Hamilton, 1977, p.13).

On the one hand, it has been argued that qualitative research involves the researcher adopting a stance of the naive observer (Atkinson, 1979) who initially avoids sharpening his/her problems into specific research hypotheses "until considerable exploratory investigation has occured" (Atkinson, 1979,

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p.53). On the other, it is ultimately necessary for the researcher to be knowledgeable about what he/she is observing. For instance, Vergnaud has argued that in research situations involving children engaged in mathematical thinking and learning, "one cannot observe well what one is not prepared to observe. This presupposes that the contents, and the situations through which these contents are conveyed, are clearly analysed beforehand so that one may be prepared to 'see' the meaning of events and behaviours observed." (Vergnaud, 1982, p.41)

The researcher spent the preliminary year becoming progressively aware of the nature of the learning environment aimed to be generated in the main research and what children's behaviours might mean. However, during the main research sessions, although the researcher used his experience from the preliminary phase, he also explicitly attempted to take into account unpredicted or surprising behaviours.

The main phase of the research involved case studies of pairs of children working cooperatively with a computer in situations which on the one hand were carefully designed by the researcher, but on the other, were open - ended in order to allow for the children to feel in control of their learning by participating in the directing of the work. Case - study work, as a proponent of qualitative research, has been widely practiced in open - ended Logo situations. Apart from Logo being a means for children to do mathematics, it has been widely appreciated as a research tool, due to the opportunities it provides for recording children's activities; Weir, for instance, states that Logo has the potential to "act as a window into the mind of the learner" (Weir, 1987, p.1).

However, the importance of carefully planned "situations", or microworlds - in the broader sense of Hoyles and Noss (Hoyles and Noss, 1987b) - has recently enjoyed increasing appreciation, as is clearly implied by Weir. "Case descriptions are interesting as texts behind which to probe for the 'why' and the 'how' of phenomena and are a prerequisite for carefully controlled large

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group studies. In particular, they help to decide what the appropriate controls might be. However, it is fallacious to assume that students' free activity alone can tell us why particular behaviours do or do not occur. Direct interventionist steps, for example, setting particular tasks designed to probe particular possibilities, are crucial. In due course these probes become incorporated into the learning situation itself, so that the boundary between research and teaching becomes blurred." (Weir, 1987, p.3).

4.1.1 The initial research problem

As discussed in the review of the literature, there has recently been increasing evidence of Logo being used as a means to generate vivid open - ended educational environments within which children have the opportunity to take control of their learning.

Moreover, there has been little - but very rich - evidence (Papert, 1980, Lawler, 1985) that the Logo - turtle metaphor invites children to form an experience - linked schema in identifying with the turtle and thus make sense of mathematical situations arising from driving the turtle on the screen. However, very little is known about whether the geometrical content of turtle metaphor environments can be extended further than that of Intrinsic geometry, so that children might use the schema in wider geometrical contexts.

The problem which the study addressed in a year - long preliminary phase, played an important role in the development of the main research issues. The main question asked was: is intrinsic geometry the only geometrical system within which children may develop geometrical understandings with the use of the turtle metaphor? Furthermore, are there any indications that children use notions from Intrinsic, Euclidean and Cartesian geometry in order to construct geometrical figures with the turtle in "standard Logo" environments?

The researcher, therefore, began the preliminary research with the general

objectives of investigating;

- the problem of extending the geometrical content which children can use for mathematical learning and thinking while programming with turtle geometry and,

- the nature of the mathematical schemes they use and form while engaged in turtle geometric environments.

4.1.2 The overall research setting

The research was carried out in Psychico College, a primary school in Athens. There are two types of schools in Greece, state schools being the large majority, and private schools. Some private schools, one of which is Psychico College, are considered as above average in "status". In this school there are around 700 children distributed among the six years of primary education which, in Greece, begins from the age of five and a half to six years. There are around 30 children in each classroom, i.e. 120 in each year.

Research in mathematics education in Greece has barely got off the ground as yet. Consequently, although the researcher was informed by reviewing the relevant literature on mathematics education based on evidence from educational systems other than the Greek, he could not take for granted that the reviewed issues applied to Psychico College automatically. However, the researcher being brought up through this system, and his role as participant observer in a Logo "club" formed during the preliminary phase (section 4.2.1 and chapter 5) in Psychico College, gave him an insight into the relevance of the literature for the school (see also chapter 5).

In general, the Greek educational system is highly centralised compared to that of the U.K. and characterised by a prescriptive curriculum. The general principle by which education is practiced rather conveys a view of the teacher as the transmitter of the knowledge embedded in a content - defined syllabus,

and the children as the receivers of that knowledge, a view which was supported by the researcher's experience during the preliminary phase of the research.

The children officially start Geometry at the fourth year of primary. Although their Geometry books have been recently improved as regards the "friendliness" of the presentation (e.g. the narrative is more personal, addresses the reader - pupil in a more informal way than before) and there is now a little measurement or experimentation before some definition or theorem, the overall spirit remains akin to the one described above. The content predominantly consists of formal angles, circle and the relationship between diameter and perimeter taught in the fourth year, triangles and some of their properties (e.g. sum of internal angles) taught in the fourth and revisited in the sixth year. Moreover, there are some sections on area and volume in the fifth and sixth years. Discussions with the children's teachers, however, revealed that the teaching of geometry was de - emphasised, mainly for practical reasons, i.e. being at the end of the mathematics "book - syllabus", the contents of which were unrealistically disproportionate to the available time in the school year, resulted in most of it being left out. During the research period, the researcher was aware of whether and what the children were doing in school geometry. However, it is only mentionned in the study in cases of evidence that it influenced their work with the Logo turtle.

Although at present there is no central provision or near - future prospect for the use of computers in Greek primary education, some private primary schools have recently been equipped with computers, the use of which, however, has been limited. As a result of Psychico College being equipped with ten microcomputers at the beginning of the year 1985/86 (details in section 4.2.3), the researcher made a personal agreement with the school's director, himself an educationalist not unaware of issues concerning Logo and primary education and interested in "exploiting" the available technology. The researcher would have access to the school's technology and children (after negotiation) for his research. In exchange, he would help with the

"spreading" of an informal "Logo culture" throughout the school.

It has been the practice that private schools often offer some extracurricular activities for the children, such as the teaching of foreign languages or the running of "clubs" involving activities such as drama, chess, sport, craftwork, etc. Taking the opportunity of this "free activities" hour, the researcher formed and organised a "Logo club" consisting of twenty 5th - year (10 - 11 year old) children, representative (not in the clinical sense) of the span of abilities in the school. The children worked in an informal environment with a Logo experienced (elsewhere) school teacher for 75 minutes each week, and provided the basis for the preliminary phase of the study. The Logo club continued to function in the following year with the same children and a different teacher in similar environments. The children for the case studies of the main research were chosen from the Logo club, during the first two terms of the following year, i.e. 1986/87.

The latter part of the agreement between the researcher and the school's director led, in the following year 1986/87, to the development of a full Logo program involving one informal Logo session per week for all the children in the last four years of the school and all their teachers, each one with their own class. The Logo program involved investigation - oriented sessions with a structure similar to that developing in the Logo club (a summary of the progress of the program is given in appendix B). The only specific relation of this program to the present research, however, is the acknowledgement of the nature and the extent of the Logo experience the main research children were having at the time when the research was in progress; a particular child participating in a case - study of the main research would also have two investigation - oriented Logo sessions per week, one as a member of the club in its second year and one as a member of their normal class in the Logo program session (see chapter 5). Details of the setting of the main case - study research are given in section 4.2.7.

4.2 AN OUTLINE OF THE RESEARCH

The three main case - studies of the research are used to investigate different aspects of the same problem, i.e. whether children can use their turtle metaphor "schema" to develop understandings of intrinsic, euclidean and cartesian geometry. However, each study stands rather on its own, i.e. no child participated in more than one study, and each study involved an individual microworld and activity development, research design, and pilotting. Furthermore, each study involved a considerable degree of detail in its design, since the study relied on careful planning of the research environments. It was therefore seen as clearer for the reader, to present the details of the design of each study at the beginning of the relevant chapter (chapters 6, 7 and 8), allowing at this point for only a brief general outline of the research.

In the following first two sections (4.2.1 and 4.2.2), an outline of the preliminary phase of the research, the emerging research issues and how they relate to the three case - studies of the subsequent main research is given. In sections 4.2.3. and 4.2.4., an account is given of how the case - study microworlds and the activities within them were developed and pilotted. The development of the method for collecting data and the technology used in this process are then presented, followed by an account of the participating children and the criteria for their selection. In section 4.2.7, the case - study research setting is described, concluding in an account of the researcher's activities during a typical 24 hours of the main research phase. Finally, in section 4.2.8, the phases of the analysis of the data and the resulting method of the presentation of the results are described. In chapter 5, a more detailed outline of the preliminary phase of the research is given.

4.2.1 The preliminary phase

The preliminary phase of the research involved the formation of a Logo club in the Greek school during the 1985/86 school year. Twenty 5th - year (10 to

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11 year old) children participated in the club. In cooperation with the school's director, ten girls and ten boys, representative of the school's span of "abilities" were chosen to join the club among a comparatively large number of volunteers.

The aim of the club was;

- to provide the children with an experience of an informal explorative educational environment, which was not automatically assumed by the existing educational system,

- to allow the children to develop considerable experience with Logo programming.

- to address the general research problems described in section 4.1.1, in order to focus on, and refine, the objectives of the study.

The teacher responsible for the club (labelled teacher "F") had already had some prior experience in using Logo with infant children. The Logo sessions took place for 75 minutes per week throughout the year, the children working collaboratively in groups of two or three, in an informal, investigation - oriented, atmosphere. By the beginning of the main research which took place during the first two terms of the following school year (1986/87), the children participating in the three main case - studies, who were all chosen from the Logo club, had had 40 to 55 hours of experience with Logo programming. This was a result of their participation in the Logo club and Logo program activities (see section 4.1.2).

After having discussed with teacher F the pedagogical framework, the setting up, the classroom organisation and the content of the Logo club activities, the researcher payed three extended visits to the school during the preliminary phase, allowing for equivalent time spans between the visits. A record of F's account and the children's perceptions of the activities in each session was

kept during the researcher's absence. In all three occasions of his visiting the club, he carried out preliminary research by administering structured tasks to all the children with the purpose of monitoring their progress in Logo programming and probing the nature of the geometrical ideas they used in attempting to solve the tasks. During the second visit in March 1986, at the beginning of which the children had had 13 hours of Logo sessions, the researcher engaged in participant observation by taking the role of F for 15 hours of Logo sessions in total (some extra sessions were allowed to take place), thus acquiring a personal view of the classroom atmosphere and of the children's work. At the beginning of the following year 1986/87, prior to the main research, the researcher administered a set of structured tasks as part of the process of probing the geometrical ideas used by the children to solve them (appendix A.5 and A.6). The researcher's conclusions from all the structured tasks are presented in chapter 5.

4.2.2 The research issues and their relation
to the case - studies of the research

Analysis of the data from the preliminary phase, which provided indications of the children using the turtle metaphor and specific intrinsic, euclidean and plane description notions in order to construct geometrical figures, allowed the development of the main issues to be addressed by the study.

The four objectives of the study were:

1) to investigate the nature of the schema children form when they identify with the turtle in order to change its state on the screen;

2) to investigate whether it is possible for them to use the schema to gain insights into certain basic geometrical principles of the cartesian geometric system;

3) how they might use the schema to form understandings of euclidean

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geometry developed inductively from specific experiences;

4) to investigate the criteria they develop for choosing between intrinsic and euclidean representations of geometrical ideas.

Issues 1 and 2 were investigated by means of the "T.C.P. microworld" study, presented in chapter 6, and consisting of three individual case - studies, each of which involved the encouraging of the development of a separate learning sequence, which formed a "learning path" from intrinsic to cartesian methods of changing the turtle's state (see chapter 3). A separate pair of children participated in each case - study, by engaging in the activities designed for each path. All three paths led to common activities involving changes of the turtle's state in the "Turtle in the Coordinate Plane" ("T.C.P.") microworld, where the children could choose the method of changing the state of the turtle. In their attempts to control the turtle by using the available "cartesian commands" (chapter 3), the children initially used the turtle schema they had formed prior to the study, thus providing the researcher with an insight into the nature of this schema (issue 1). The children's subsequent uses of cartesian notions in controlling the turtle enabled the researcher to engage in the investigation of issue 2.

Issue 3 was investigated by means of the "P.D.D. microworld" study presented in chapter 7, which consisted of a case - study of a pair of children engaged in activities within the "POST, DISTANCE, DIRECTION" ("P.D.D.") microworld. Initially, the activities were task - oriented and mainly consisted of constructions of irregular and isosceles triangles, a process which involved the employment of euclidean notions and triangle properties. The latter part of the case - study involved open - ended projects where the children had the choice of using the P.D.D. primitives or the procedures resulting from their triangle constructions, thus providing the researcher with further insight into issue 3.

Issue 4 was investigated by means of the "Circle microworld" case - study

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presented in chapter 8, involving a pair of children. In the first part of this study, the children engaged in a learning sequence, the object of which was the development of understandings concerning the functioning of four Logo procedures to construct a circle. The procedures were such that their use invited the employment of differing intrinsic and / or euclidean notions. In the second part of this study, structured tasks were administered to the children, requiring the construction of figures involving circle combinations. The investigation focused on which circle procedure the children chose for each task, and on their reasons for doing so.

Although the studies were designed for the investigation of the issues as described above, subsequent repeated analyses of the data, of a particular study for instance, unexpectedly revealed useful insights into *one of the* research issues from the four stated above, for which the study was not specifically designed. For example, the criteria developed by the children in the "P.D.D. microworld" study, for choosing to use intrinsic or euclidean notions of angle provided further research insights into issue 4. For reasons of clarity, the analysis of the data of each of the main case - studies is given in the chapter presenting the respective case - study. When data from one study is used as supporting evidence for the issue investigated by another, it is mentioned in the discussion section, as for example in section 7.5.

4.2.3 The development of the microworld for each case - study

Informed by the principles concerning the notion of a microworld (discussed in chapters 1 and 2), the researcher initially engaged in the programming involved for the technical component of each of the microworlds. They were all pre - pilotted in order to test their functionality, and the clarity of the error messages (see appendices C, D and E). Changes were also made as a result of the main pilot studies which were carried out in an english school in a research setting similar to that of the main study. The technology used for the pre - pilot and main pilot sessions was different to that used in the study. The former consisted of a B.B.C. (B) microcomputer, and the Logo version

used was B.B.C. LOGOTRON (L.C.S.I.) Logo. The latter consisted of an APPLE IIC microcomputer, and the Logo version used was APPLE LOGO II. The transfer of the programs for the microworlds, although not a trivial task, was completed satisfactorily. Details are given in the sections describing the design for each study.

4.2.4 The development of the activities within each microworld for each case - study

The activities designed for each study involved an interplay between open - ended and task - oriented activities. The latter were designed so as to allow the pupils some choice as to the method or strategy used to achieve the set goal. The research interest was focused on the process by which the children set out to solve the task rather than the actual goal that was set. The activities were pre - pilotted and the resulting refined versions then fully pilotted in the main pilot study. Some activities involved documentation and some were verbally presented to the children. An account of the activities and an analysis of the tasks is given at the beginning of the respective study. The documentation (where applicable) is given in appedix C, D and E.

4.2.5 Development of the method for collecting data.

Informed by the literature on the one hand, but also through the experience of the preliminary phase of the research on the other, the researcher was convinced of the importance of collecting data which would subsequently help in reconstructing what happened during the case - studies in as much detail as possible. The importance of the rich generation of data has recently been appreciated in case - studies involving children's thinking processes in Logo environments (Hoyles, Sutherland and Evans, 1987, Lawler, 1985). Lawler, for instance, argues that; "Inasmuch as the experimenter has an imperfect theory of mind, is insensitive to the importance of specific incidents, or cannot comprehend the mass of observations as it is developing (these conditions are always true), the strategy of choice is to create a corpus of

sufficient richness and permanence that it may be queried as subsequent interpretation proceeds." (Lawler, 1985, p.19).

During the preliminary phase of the research, however, a considerable proportion of the collected data was not directly relevant to the research issues. A primary concern in the design of the main case - studies, therefore, was the generation of situations centreing on the research issues. "Progressive focusing... reduces the problem of data overload, and prevents the accumulation of a mass of unanalysed material". (Parlett and Hamilton, 1977, p.15).

All the data collected during the main research, was produced and processed by the researcher. The finilised data collection for all three case - studies, consisted of the following;

- audio taping of everything that was said during each case study,

- "soft" (i.e. on computer disk) and "hard" (i.e. on paper) copies of verbatim transcriptions translated into English,

- soft and hard copies of everything the children typed,

- hard copies of graphics screen dumps,

- soft and hard copies of all the procedures the children chose to save on disk,

- the researcher's notes, and;

- the children's prompted and unprompted notes on paper.

4.2.6 The children participating in each study
and the criteria for their selection

The data collected during the first year of the Logo club (preliminary phase of the research), was used for the following purposes with respect to this study:

a) to build a profile of the students' general attainments in their class,

b) to form a background knowledge of the development of their intrinsic schema and their more general Logo progress and;

c) to develop criteria for their participation in the study, i.e. willingness and motivation, some evidence of having formed an intrinsic schema, not belonging to an extreme end of the ability range, and being able to work in a sound collaborative spirit with their respective partner.

4.2.7 The setting of the main research sessions

The main research sessions consisted of three participants; the researcher and the respective pair of children. They were held in a small room in the school provided for this purpose. The researcher was aware, on the one hand, of arguments for research settings within children's everyday "normal" activities in their classroom (Noss, 1985, Sutherland, 1988). However, the setting in the present study falls somewhere between two extremes, that of "normal" classroom situations and that of isolated researcher - to - child situations as, for instance, in Lawler's research; in making a case for such methodology, Lawler claimed that "...the Intimate Study represents a sensible approach to studying the process of learning, despite the manifest difficulty of the task and the method's vulnerabilities to criticism. Further, I claim there is no other empirical approach with such promise of telling us anything important about the accommodation of mental structures through experience in the language - capable mind" (Lawler, 1985, p.17).

In the present study, the researcher's objective was to observe children's behaviours in situations prepared with a considerable degree of specificity (see the "task analysis sections in chapters 6,7, and 8). Moreover, it was not within the study's objectives to investigate such situations within classroom dynamics. Furthermore, it was felt that the external stimuli of the classroom setting would enhance fluctuations in the children's attention. Since the research activities were designed to be in a specific order and the experience with one activity could influence children's thinking in another, it was decided that a rather "stable" external environment was essential for the study. The experience of the pilot study, where in some sessions interruptions of various kinds were not infrequent, supported the researcher's decision.

The researcher, however, did attempt to provide a setting which would at least not seem "artificial" to the children. The research sessions were held in a small room which was familiar to the children as their "museum" (all sorts of things collected by the children were on exhibition). The vocabulary used for the children's activities was consistant with their other Logo activities. For example, the Greek word for "concept" was used for a Logo procedure, instead of the verbatim translation of the word "procedure". Moreover, words which were meaningful in Greek were used for the words "saving" (the Greek word for "preserving") or the Logo editor (the Greek word for "writing book"). The "private tutoring" - type environment which resembled the research sessions, was not unfamiliar to the children since they were all having private tuition for one subject or another. For the studies which involved several research sessions with a pair of children (e.g. the P.D.D. microworld study and the Circle microworld study), the pair stayed after school in agreement with the parents. The other studies took place during school hours.

For the researcher, a typical 24 hours during the time of the research would involve a 90 minute research session after school (i.e. at 4.30 pm), followed by an immediate printing of the dribble file so that a hard copy would be available in the next morning. The next day would involve transcribing the previous day's session with the help of the dribble file and the notes, and

making a hard copy of the transcription. Following that, the dribble file would be played back (the technology for this facility was not readily available at the time in England - it was borrowed from John Olive, University of Atlanta) and paused to print suitable screen dumps. The collected data would then be given a code name and filed away. Time was allowed for preparing the next session.

4.2.8 Phases of the analysis

The researcher initially studied the data, attempting to reconstruct in his mind what happened during each case - study. After repeated analyses, certain episodes reflecting children's insights or difficulties related to the research issues, began to emerge from the research situation. The researcher subsequently synthesised such episodes according to the specific research issues which they illuminated. The presentation of the analysis is structured in accordance with the research issues, rather than reflecting the time sequence in which the episodes took place or the sequence in which the tasks were presented to the children. Accordingly, the titles in the sections involving the presentation of the analysis refer directly to the research issues. However, the time sequence and the order in which the activities took place are made clear in the research design sections and are constantly referred to during the presentation of the data. Furthermore, the full set of data collected during one research session is given in appendix F.

CHAPTER 5

THE PRELIMINARY PHASE OF THE STUDY

5.1 INTRODUCTION

The increase in the availability of microcomputers and the Logo language in primary schools could enhance the likelihood that more and more children will have had some experience with Turtle geometry in the coming years, as part of their primary education. The general aim of the study presented in this thesis, was to investigate the potential of children extending their experiences of using the turtle metaphor beyond the conventional Logo environment, to a wider geometrical area within which they would be able to use their turtle schema to do mathematics. It was accordingly decided that the children participating in this study should have had some prior experience with Turtle geometry for the following two reasons:

1) so that they would have acquired the experience which the researcher perceived as essential for their taking part in the designed microworld environments of the main research, i.e. their participation in an informal environment involving investigative activities in the context of Turtle geometry;

2) so that the researcher would have the opportunity to carry out an informal preliminary investigation of the children's activities, enabling him to focus on, and refine the initial general objectives of the study which are given in section 4.1.1.

The opportunity to create such an environment in the primary Psychico College did not leave the researcher without some scepticism; for reasons explained above, an informal child - centred classroom atmosphere would be more of a "forerunner" rather than something which would arise automatically within the Greek educational system (see sections 4.1.2 and 4.2.1). It was therefore important prior to the forming of the Logo club (section 4.2.1), to

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establish a child - centred pedagogical framework for the functioning of the club, through discussions with teacher F, the teacher undertaking the running of the club. A report of the forming and the function of the Logo club is given in section 5.2. The above factors concerning the setting of the Logo club within the Greek educational system warrant a descriptive account of the atmosphere generated during the club's activities, which is given in section 5.3.

The significance of the results of this phase of the research is relatively de - emphasised due to its methodological role within the context of the whole study (section 4.1). Within this context, however, the research objective of the preliminary phase (objective 2) was met by an informal investigation concerning:

a) the children's developing programming strategies during four administrations of the "Four Squares" task, presented in section 5.4, and;

b) indications of their use of intrinsic and non - intrinsic geometrical notions during their attempts to solve a series of geometrical structured tasks, discussed in section 5.5.

5.2 THE FORMING AND THE FUNCTION OF THE "LOGO CLUB"

As discussed above, the forming of the club had a dual purpose; that of providing the children with the necessary experience to participate in the study and that of providing the researcher with the base for preliminary investigation which would enable him to develop and refine the research objectives. The club consisted of 20 5th - year children (10 to 11 year olds), which were picked by the school's director so that the group would be as representative of the school's span of "abilities" as possible and would consist of 10 boys and 10 girls (these constraints were imposed by the researcher).

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The children worked in groups of 2 or 3 throughout the year for 75 minutes a week, split into two sessions during the first term, but, after the researcher's request, merging the sessions to one 75 minute session each week for the rest of the year. The teacher ("F") responsible for the club (teaching E.F.L. in the school) had had some experience of using Logo with young children elsewhere. At the outset of the club activities, the researcher and teacher F established guidelines to encourage an informal, group work classroom atmosphere. The researcher visited the club three times during the year administering structured tasks as a means to monitor the children's progress in Logo programming and carry out preliminary investigations of the nature of the geometrical notions the children were using to solve the tasks. The latter investigation continued as a result of a fourth "batch" of structured tasks given to the children prior to the study, at the beginning of the year 1986/87. Furthermore, as a means of acquiring a personal experience of the classroom atmosphere created during the club sessions and in order to contribute to the encouragement of a child - in - control spirit, the researcher undertook the running of the club during his second visit in March 1986 for 15 hours in total, the children having had 13 hours of Logo sessions prior to that.

The children programmed in "direct - drive" for 5 - 6 hours, used the REPEAT command and procedures for the following 25 hours and variable inputs for 12 hours (hard copies of their procedures saved on disk were kept). The researcher and teacher F attempted to introduce the new programming features in a meaningful way by linking their function to experiences the children had had in recent activities. For example, the researcher introduced the notion of variable during his second visit, as a means to change the size of a house, a project which the majority of the children had chosen to engage in. Apart from the occasions of the introduction of new Logo commands, F and the researcher were keen not to direct the children's activities.

5.3 THE CLASSROOM ATMOSPHERE

At the end of the year, the researcher felt that his skepticism concerning the formation of an informal classroom environment was well - founded. However, the children did show signs of taking control of their projects and of learning the rules of a social game which was new to them, i.e. that it is legitimate and often rewarding to try to solve problems, that cooperation in small groups is accepted and encouraged and that the object is not to solve something set by the teacher, but to engage in something of personal interest. A brief description of this process is presented below, in section 5.3.2 and an outline of the data on which it was based is given in section 5.3.1.

5.3.1 Collection of data

The following data was collected in order to form a picture of the classroom atmosphere:

a) **Data collected on a single occasion:**

- The "Pupil profile" questionnaires, adapted from the Logo Mathematics Project (Hoyles, Sutherland and Evans,1985). These were filled in by F as a result of her impression of each pupil in the club in cooperation with each pupil's Greek teacher (appendix A.1).

- The "Pupil questionnaires", which were also adapted from the same source as the "pupil profiles", translated into Greek and answered in writing by the pupils. The questionnaires concerned the pupils' attitudes to the Logo club and to Mathematics (appendix A.2).

The profiles and the pupil questionnaires were filled in during the researcher's first visit to the club (December, 1985).

- At the end of the school year, the children wrote an essay consisting of their

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opinions on their experiences concerning the club.

b) Data collected at the end of each session (where applicable):

- Throughout the year, each pupil spent the last five minutes of each session answering a brief questionnaire (the "Logo log"), adapted from the Logo Mathematics project and translated into Greek, concerning his/her perceptions of his/her activities during the session (appendix A.3).

- A "Record sheet" was filled in by F at the end of each session regarding her perception of the activities of each group of pupils. It was adapted from the Logo Mathematics project and modified during the researcher's first visit to the club (appendix A.4).

- Hard copies of the children's procedures were kept, when a group would save procedures on disk

- During his undertaking of the responsibility for the sessions, the researcher kept notes (immediately after each session) regarding his impression of the classroom atmosphere

5.3.2 Results

Through his experience as participant observer, the researcher identified indications of the following difficulties the children had with taking control of their learning:

They would expect the teacher to provide the answers to almost any problem. Their initial reactions to a problem would be to raise their hand and wait for the teacher to come and solve it. They seemed to find it hard to realise that a large proportion of the problems they met could be solved by them, if they thought and discussed them. They would treat the outcomes of their activities on the screen as "right" or "wrong", rather than part of some process. There

were difficulties in communicating ideas and suggestions within groups. They would not readily take initiatives to start off new projects, often asking the teacher to provide the ideas. They seemed impatient with problem situations often abandoning a goal at the first difficulty. They would not readily expand or elaborate a project - they were more happy to start on a new one.

However, throughout the year, the children's independence from the teacher seemed to grow, encouraged by teacher F's and the researcher's attitude of not providing ready - made answers, encouraging the children to solve their own problems and responding positively in such instances. Moreover, their written responses from the Logo logs indicated a substantial degree of enjoyment during the sessions and progressively more articulate and precise descriptions of what they had done in a session and their plans for the next session. The latter issue, i.e. the increasing explicitness of their plans, seems to suggest that the children were developing an increasing awareness of a continuity of a project through more than one session, which was not the case from the start.

The record sheets supported the indications that the children were highly motivated throughout the year, although it was difficult to determine the influence of the "novelty" factor and the social "status" of the Logo club children within the school, who were the only ones using the computers at the time. However, the groups did engage in self - set projects, teacher F's impression through the record sheets being that there was an increasingly high degree of involvement and awareness of what was "going on" during a project within each group. The children's essays at the end of the year provided some indications of personal involvement in meaningful situations as is suggested in the following extracts, translated from Greek;

Maria: "When the lessons were finished I felt I had a very good time we had fun, we learned and the most important we are looking forward to the new year to learn more. The computers are not simply machines they need your brain and your patience to work."

Valentini: "...we did not only learn to make shapes with Logo. We knew many more things. By the end of the year we had learned to use our brain more practically and above all, to cooperate... because without realising we discussed, we thought, in a few words we learned to cooperate."

5.3.3 Relevance to the main research

The indications of the children's progressive acceptance of the open - ended classroom dynamics were important for their subsequent participation in the case - studies of the main research, where the activities involved an interplay between researcher - set (but open - ended) tasks and personal projects. The children's relative readiness to take control of their own learning provided the researcher with a rich data base for the investigation of the nature of such learning. It also gave more meaning to the results, due to the children's learning through their experiences with the case - study microworlds by causing self - initiated changes in mathematical situations.

5.4 THE CHILDREN'S PROGRAMMING STRATEGIES

5.4.1 Collection of data

The researcher felt that the data collected by means of the record sheets and the procedures saved on disk, showed only end results of the children's programming, rather than process. It was therefore decided to administer a structured task (the "Four squares" task, adapted from the Logo Mathematics project) to the children on all three occasions of the researcher's visits and to collect more detailed data - by means of the dribble files technique - of all the children's typing in their attempts to solve the task (fig. 5.1). During the first visit the tasks were given to the children in pairs, to solve collaboratively (see appendix A.5). As a result of a preliminary analysis of the data, however, it was decided that information on the children's individual programming would give a considerably clearer picture of their conceptualisations of the task.
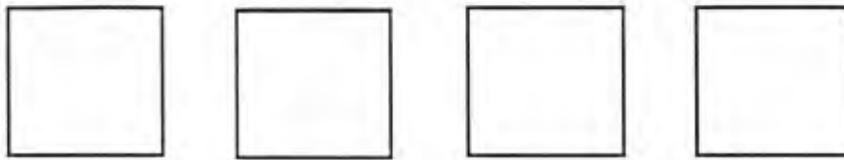
Figure 5.1 The "Four Squares" task

During the second visit, the researcher gave the four squares in a vertical formation, to reduce the element of children using the same strategy because they "remembered" what they had done in a previous occasion. On both second and third visits, the researcher did not mention procedures or other programming techniques, and told the children they could solve the task in any way they liked, when queried on this issue. The four squares were accordingly given (printed on paper) to each child to construct on a computer (appendix A.5). One teaching session was available on all occasions, and a hard copy of the dribble file for each child was kept. During the third administering of the task, programs were employed so that the dribble file would show the contents of the editor and thus enable the researcher to trace the children's "debugging" or developing of their procedures. These programs were from then on used throughout the research.

5.4.2 Relevance of the results to the main research.

The analysis of the data regarding the children's strategies to construct the four squares yielded two main points in relation to the main case - study research. Firstly, that the children's programming was not atypical of that concerning similarly aged children with similar experience, reported in other studies (Noss, 1985, Hillel, 1986). Although replicability was not within the objectives of the study, the researcher's awareness of the children's programming styles and strategies provided him with an insight into their behaviours during the case - study research. A more detailed account of the children's programming strategies can be found in appendix H.

Secondly, it became apparent that for the purposes of interpreting the

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children's activities during the case - studies, a distinction between the programming aspect and the geometrical aspect of their strategies would be useful. Consequently, during the analysis of the data from the main research, the geometrical notions the children used in order to construct figures, and the programming strategies they employed (e.g. using procedures or subprocedures and modularity) were considered separately.

5.5 THE CHILDREN'S USE OF GEOMETRICAL NOTIONS

5.5.1 Collection of data

The researcher administered a series of structured tasks involving interconnected geometrical figures (squares and rectangles) on each of his three visits (see appendix A.6). As in the four squares task, from the second visit onwards the tasks were given to the children individually to construct on a computer, and dribble files of their typing were kept. A fourth administering of the tasks was carried out prior to the main research, at the beginning of the following school year, 1986/87 (fig 5.9).
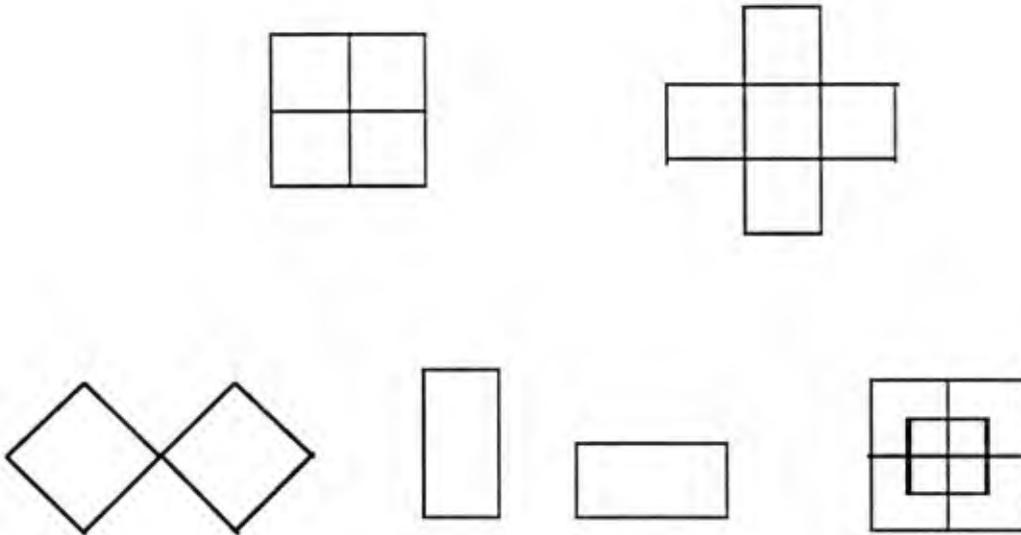
Figure 5.9 The structured tasks administered prior to the main study

5.5.2 Relevance of the results to the main research

An informal analysis of the data indicated that the children's programming strategies were comparable to those employed in the "Four squares" task. However, the children did not perceive modularity in the task figures to the extent exemplified in the "Four squares" task. This finding corroborates the results from the Logo Mathematics project, where Hoyles and Sutherland reported that children tend to see modularity more easily when the modules are disconnected from each other. Furthermore, during the fourth administering of the tasks, the children's programming was not quite up to the level achieved at the end of the previous year, possibly due to the long Summer break.

The analysis of the data, however, supported the argument which emerged from analysing the data from the "Four squares" task, i.e. that a distinction between the children's programming and the geometry they used would be useful in interpreting their strategies during the main research. Moreover, the

analysis of the children's attempts to solve the tasks shown in figure 5.9, contributed to the researcher's decision to use two further distinctions to interpret the main study data.

Firstly, the children's identifying with the turtle did not necessarily mean that they used the geometrical notions embedded in turtle geometry; they often drove the turtle on the screen by "trying out" inputs to turns or moves in a perceptual way, a strategy reported also in other research (Kieran, et al, 1986). Leron described this phenomenon as follows; "...the turtle may be a 'maths speaking creature', but we cannot automatically assume that the children always listen to what it is saying..." (Leron, 1983, p.349). The researcher found it useful to distinguish between the children's "turtle schema", i.e. the schema formed and employed in their identifying with the turtle, and the geometrical notions embedded in changing the turtle's state, which the children used, or ignored.

Furthermore, for the purposes of this study, the geometrical notions the children used during their attempts to construct the figures in the structured tasks, are characterised by the geometrical system in which the notions belong (an analysis of these ideas is presented in chapter 3). In changing the state of the turtle, therefore, a child could either use;

a) perceptual cues,

or a geometrical idea belonging to;

b) intrinsic geometry,

c) euclidean geometry,

d) cartesian geometry.

Although the children were programming in the "standard" Turtle geometry

using an intrinsic notion
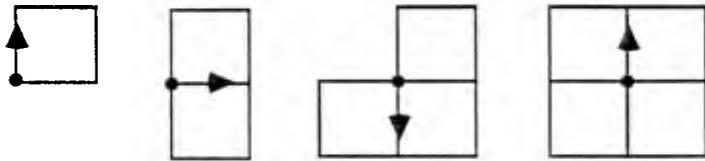
generating squares
by iterating
move and turn
sequences

using a euclidean notion

placing a cross at the
centre of a square

using a cartesian notion
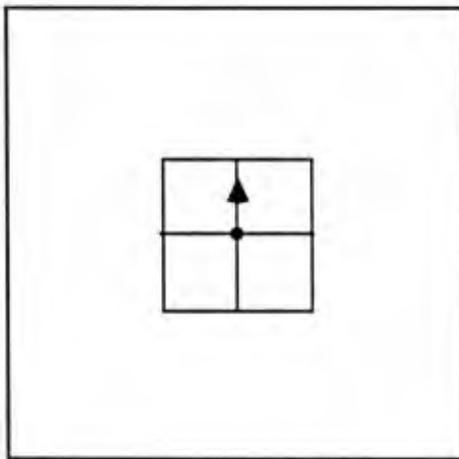
placing a figure at the
centre of the screen

the turtle's heading is signified by an arrow

the turtle's position is signified by a dot

Figure 5.10 Examples of using intrinsic, euclidean and cartesian notions
while constructing a structured task

environment, there were indications of all four cases in the data from the structured tasks. That is, the children would use ideas involving plane relationships between points, line segments or figures (euclidean ideas), or ideas involving some awareness of the plane (cartesian ideas), in the sense discussed in chapter 3. An example of each case is given in figure 5.10.

5.6. CONCLUSION

The classroom environment generated in the preliminary study provided the children with experience of taking control of their own learning. The researcher was thus able to choose children from the club so that encouragement for such learning in the situations generated within the microworlds of the case - studies, would be meaningful for the children. Furthermore, the personal working relationship established during the preliminary year between the researcher and the children, and the researcher's insights into each case - study child's thinking played an important part in the generation of an investigative atmosphere during the main research on the one hand and in the interpretation of the children's behaviours on the other.

The analysis of the data on the children's programming strategies to solve structured tasks enabled the researcher to clarify his ideas on the meaning of children's behaviours during the main research. Aiming to illuminate the nature of the schema the children formed in identifying with the turtle, he decided to initially perceive of the schema independently, without pre - determining what mathematics the children might use when driving the turtle on the screen. It was also decided, to analyse the geometrical aspect of the children's activities as distinct from the programming aspect. The researcher's perspective was that the distinctions of programming strategies from mathematical strategies on the one hand, and the turtle schema from the three geometrical systems on the other, would be valuable tools for interpreting the data of the main research.

The preliminary study was consequently crucial in establishing a general framework for analysing the data and in enabling the refining of the main research issues formulated in section 4.2.2.

CHAPTER 6

FROM INTRINSIC TO CARTESIAN GEOMETRY

6.1 RESEARCH DESIGN

6.1.1 Objectives.

As discussed in chapter 2, Lawler's research illustrates a substantial degree of disparity between Intrinsic and Cartesian geometry for a six year old child, by describing her failure to form a "microview" about the latter, based on her personal / turtle geometry microview (section 2.2.7). Lawler also illustrated the child's reluctance to abandon the use of her intrinsic "thinking schema" and use a different conceptual base (he used the term "ancestral microview") to make sense of cartesian concepts.

As discussed in chapter 4 (section 4.2.2), the general aims of the study presented in this chapter were to investigate issues 1 and 2, i.e.

1) the nature of the schema children form when they identify with the turtle in order to change its state on the screen, and

2) whether it is possible for them to use the schema to form understandings of certain basic geometrical principles of the cartesian geometric system;

The method employed involved the encouraging of the development of three separate learning sequences, one pair of children participating in each sequence. All three sequences were designed to invite the forming of links between intrinsic and cartesian methods of changing the state of the turtle. However, a different conceptual base for describing the plane was embedded in the initial part of each sequence, thus inviting the forming of different links between intrinsic and cartesian geometry. All the sequences (the word "paths" is also used to denote the general progression of the embedded

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notions from intrinsic to cartesian geometry) consisted of three "categories" of activities (fig. 6.1.1). The specific research objective for each category was:

aim of category 1: to illuminate the process by which the children formed understandings of a systematic description of the plane;

aim of category 2: to illuminate the nature of the children's understandings of the absolute coordinate and heading systems, while using a non - intrinsic method to change the turtle state in the coordinate plane;

aim of category 3: to investigate if and how they used their intrinsic schema to relate intrinsic and coordinate notions while choosing a method of changing the turtle state in the coordinate plane.

6.1.2 Overview of the tasks.

The activities designed for the three pairs of children who participated in the study were split into three categories (fig. 6.1.1), in accordance with the above task - specific research objectives. Each pair of children started from a different set of activities in the first category, before progressing to the common activities of the second and third categories. Figure 6.1.1. illustrates what the screen looked like during each activity and the commands available to the children for that specific activity. An analysis of the specific tasks involved in each activity is given as an introduction to the presentation of the findings of the respective activity (sections 6.2.2, 6.3.2, 6.4.2, 6.6.1 and 6.7.1).

6.1.2a) Category 1

The general aim of the first category of activities, was to illuminate the children's formation of three different "conceptualisations" of notions involved in the description of the plane. The activities involved locating positions on the plane in different ways, according to each pathway. The first pathway

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Categories

Path 1: Natassa and Ioanna

Path 2: Maria and Korina

Path 3: Anna and Loukia

1 describing the plane

(A) creating a plane description microview

available commands:

PLACE X Y (number)
DODOTS
(joins points in the order they were placed)
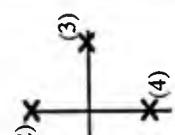
(B) intrinsic use of a chess - type grid

available commands:

PR DISTANCE (name)
PR DIRECTION (name)
FD DISTANCE (name)
RT DIRECTION (name)
FD, BK (quantity)
RT, LT (quantity)
PU, PD
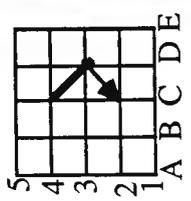
(C) intrinsic construction and use of a grid

available commands:

POST (name)
PR DISTANCE (name)
PR DIRECTION (name)
FD, BK (quantity)
RT, LT (quantity)
PU, PD

2 non - intrinsic turtle control

common activities

(D)

available commands:

SETH (value)
SETX (value)
SETY (value)
WRITE (name)

(80 -60)

available commands:

SETPOS (name)
SETH TOWARDS (name)
WRITE (name)
PU, PD

(E)

(-90 70)
(100 -40)
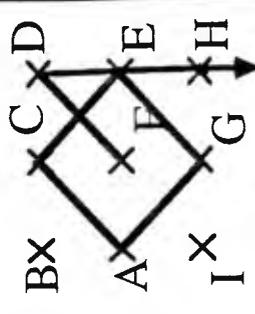
3 locating and measuring in the "turtle in the coordinate plane" microworld
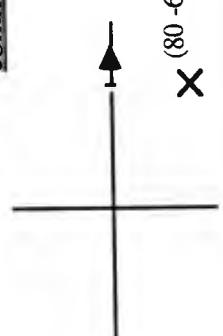
common activities

available commands:

FD (quantity)
RT (quantity)
BK (quantity)
LT (quantity)
PU, PD

PR DISTANCE (name)
PR DIRECTION (name)
FD DISTANCE (name)
RT DIRECTION (name)

SETH (value)
SETX (value)
SETY (value)

SETPOS (name)
SETH TOWARDS (name)
WRITE (name)

(F)

(-20 90)
(-80 -40)
(70 -70)

Figure 6.1.1 A "categories by paths" structure of the study.

(path 1, fig. 6.1.1) involved placing points on the Cartesian plane, in a coordinate non - action environment (no visible turtle). The aim was to investigate the way the children made sense of the method of locating and the numerical naming system. The second pathway involved joining points of a grid with a "chess - type" naming method, by using a turtle equipped with angle and length measuring instruments (i.e. the DIRECTION and DISTANCE commands, see chapter 3), in order to find out the way the children integrated notions of the locating method to their intrinsic schema. The third pathway involved the construction of a simple grid by the children themselves with the use of the POST/DISTANCE/DIRECTION (P.D.D.) microworld (see chapter 3), and subsequently their use of this grid to make shapes by joining its points with the turtle. The aim was to find out what meaning the children gave to constructing and using a description of the plane, and to illuminate how they used their intrinsic schema to make sense of the grid's geometrical properties.

6.1.2b) Category 2

The aim of the second category was to investigate the process by which the children developed an understanding of a non - intrinsic controlling of the turtle which required the use of ideas belonging to coordinate geometry (fig. 6.1.1). The investigation concentrated both on how the children made sense of using the coordinate system to control the turtle, and on the ideas they used to explain the issues involved. The tasks initially involved taking the turtle to specific locations in the coordinate plane using commands which refer directly to the structure of the absolute coordinate and heading systems (i.e. SETH, SETX, SETY). The second part of the tasks involved the use of commands referring directly to the locations themselves (i.e. SETPOS, SETH TOWARDS) for taking the turtle to locations and measuring the distance between them, using the DISTANCE command.

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6.1.2c) Category 3

The third category aimed at illuminating the process by which the children developed an integrated use of intrinsic and coordinate notions in performing actions which required choosing the method of controlling the turtle in the coordinate plane. The task involved driving the turtle in the coordinate plane and making angle and linear measurements between locations (fig. 6.1.1). The available commands provided a choice between employing intrinsic or non - intrinsic notions. The investigation concentrated on two levels of notions: firstly on the interplay between static and dynamic interpretations of lengths and angles, and secondly on the interplay between intrinsic and cartesian notions used to perform actions and collect information (see section 6.1.3).

6.1.3 Analysis of the conceptual field
of the "Turtle in the Coordinate Plane" (T.C.P.) microworld.

This study is based on the develop ment of three different conceptual pathways from intrinsic to coordinate geometry. The embedded geometrical notions in the activities of the learning sequences were initially of a different geometrical nature, thus forming different microworlds. However, all three sequences concluded with activities within the same final microworld, where the turtle can be driven in the coordinate plane, preserving the two - fold heading and position state, but allowing for a choice of intrinsic or non - intrinsic control of the turtle (fig. 6.1.1, category 3). For reasons of clarity, this section deals with analysing the way in which the final microworld (i.e. the T.C.P. microworld) involves notions from both representational systems. The microworlds in the category 1 and 2 activities involve specific sets of notions forming bridges from one geometry to the other. Although these notions become apparent from the analysis in this section, they will be explicitly described, in the "task analysis" sections.

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At this point, however, it is important to analyse the differing nature of; a) the notion of the state of the mathematical entity, b) the method for changing the state of the entity and c) the extent of awareness of the plane, in the two geometries, in order to clarify the ways of controlling the turtle in the T.C.P. microworld.

6.1.3a) Change of state.

As discussed in chapter 3, in Turtle geometry, the state consists of position and heading. Changing the state requires an action (move or turn) followed by a quantification of that action. The action describes the nature of the change which is about to take place (i.e. change of position or change of heading), while the quantification is dependent on the action (e.g. in RT 90, the 90 degrees refer to the turn rather than an absolute system of describing the heading on the plane).

In any coordinate system (e.g. Cartesian or Polar) the state (position or heading) of any point can be described in an absolute way. The description is relative only to the origin of the plane, and is independent of previous states. Therefore, changes in such a plane can be performed by an absolute description of the end state of the change, i.e. the important factor in a state - change moves away from the point itself, and rests on the location descriptions. In the coordinate plane, the mathematical entity (the point) can be fully described by its position. Action ceases to have much meaning, since to change the state, a description of the new location suffices (e.g. in SETPOS 30 40, heading, turn, backwards or forwards are meaningless).

6.1.3b) Awareness of the plane.

The intrinsic nature of Turtle geometry restricts mathematical awareness of the space around the mathematical entity (i.e. the turtle). The turtle's state is uniquely defined by the immediately previous state, i.e. the heart of Turtle

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geometry is the nature of the state itself, changes are generated by the turtle's own actions.

On the contrary, awareness of the plane is what Coordinate geometry is all about. The nature of the mathematical entity has no special meaning since changes happen by describing new locations. The description of the coordinate plane is absolute (does not depend on the entity) and systematic, and is a very important part of the geometry itself.

6.1.3c) Changing the state in the T.C.P. microworld.

In the T.C.P. microworld, the turtle has the ability to make measurements with the DISTANCE and DIRECTION commands. Changing the turtle's state can therefore be achieved by the following sets of commands:

set a) FD, BK, RT, LT, some quantity;

set b) DISTANCE "name",
       DIRECTION "name",
where "name" is the two coordinates of a point;

set c) SETH "value",
       SETX "value",
       SETY "value",
where "value" directly refers to the structure of the absolute coordinate and heading systems;

set d) SETH TOWARDS "name",
       SETPOS "name",
where "name" is the two coordinates of a point.

The commands in set a) are intrinsic, the quantification depending on the

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action only (e.g. FD 30 means "forward 30 steps"). Combining the commands in sets a) and b) preserves the action - quantification characteristic, but the quantification can be determined by the relationship between the present and the desired state of the turtle (e.g. "PR DIRECTION 30 -30" outputs the quantity for a right turn from where the turtle is to the point 30 -30). The state of change does not have to belong to the turtle's path (as in the P.D.D. microworld), there is an absolute system of describing locations. The method of controlling the turtle is substantially different in the c) and d) sets. What causes a change of state is the word SET, which in this context, logically implies the description of the state of change via an absolute method for such a description. Consequently, there is no action element from the current state to the desired state, and therefore no quantification of an action, but a name (a description) of the desired state only (e.g. SETPOS 40 40 changes the position by simply describing the end position of the turtle).

Moreover, in Coordinate geometry, the state of the point can be fully described by its position, which makes the heading redundant (e.g. SETPOS can move the turtle in a direction which is different to that of its heading). The purpose of the T.C.P. microworld is to provide an environment where there is a close interdependency between intrinsic and coordinate notions.

Consequently, a factor which is mathematically superfluous, but "binds" the two geometries together was imposed: in order to change the turtle's position, it has to be facing towards the position of change. This factor imposes a heading and position state for the T.C.P. microworld's entity, the turtle. For example, to take the turtle from a zero heading and a (0 0) position to point (-20 90), there is a need to turn the turtle to face towards point (-20 90) and then change its position, otherwise an error message appears on the screen.

The children in the study were provided with the T.C.P. microworld only as the final activity of three initially differing conceptual pathways, whose research aim was to illuminate on the one hand, how they might make sense of

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different categories of concepts belonging to Coordinate geometry (initially, either by incorporating their intrinsic schema or not) and on the other, how they progressively integrated coordinate and intrinsic notions according to the respective pathway.

6.1.4 Methodology.

An illuminative approach was employed for the case study of each pair of children, under the general principles discussed in chapter 4, with the researcher as participant observer. The learning environments were designed so that the children's activities would on the one hand be relevant to the research issues and on the other would increase the likelihood of revealing their thinking processes. In this sense, designing the T. C. P. microworld, and the conceptual pathways leading to it, had a two - fold aim; that of creating learning environments and that of using those environments as research tools. The role of the researcher was accordingly two - fold, i.e. participating in the pedagogical component of the microworld (Hoyles and Noss 1987b), and carrying out the research.

The pilot study consisted of two phases. The preliminary phase involved trying out the microworld programs, the new primitives, and a rough trial structure for the three pathways. As a result of the preliminary phase, a trial structure of the learning paths was designed and tested out in a detailed main pilot study.

6.1.4a) The main pilot study.

The main pilot study took place in an English school, employing the structure of the main study, i.e. three pairs of children, with each pair of which the whole set of the activities pertaining to the respective pathway was tried out. The research took place in a room with one computer, the researcher and a pair of children at a time. The children were aged 13, and had had one to

three years of Logo experience. The research lasted for three school hours per pair of children (i.e. per pathway). Data was collected by the following means:

a) all the children's typing using "dribble files" (see section 2.1.3);

b) researcher's notes;

c) audio taping and transcriptions.

The following changes were made as a result of the pilot study:

<u>a) Global changes</u>

It was decided that more time was needed for each pair of children in order to allow the respective experiences and concepts to mature. This would enable the choice between commands at the final task to be genuine, rather than due to insufficient experience with a certain set of commands. Moreover, more time would result in a higher degree of experience with the pathways which in turn would allow more clarity from a research point of view.

Imposing a position change relative to the heading (see section 6.1.3) was made clear and stated at the first instance of its use. It was decided that method of data collection was appropriate, the only shortcoming being that a clearer picture of what was going on on the screen was essential. Dribble playback programs were therefore implemented; after the end of a research session, the option to activate the dribble playback program for the dribble file collected during the session, provided the researcher with the opportunity to pause the playback and make a printout of the screen representing the state of affairs at specific moments of the session.

The programming needed for the setting up of the activities was improved

from the points of view of clarity to the user, efficiency and practicality. Transferring the programs from the B.B.C. machine which was used in the pilot study to the APPLE IIC used in the main research was no trivial task (e.g. for the APPLE machine, there is no way of linking the text screen to the graphics screen to make labels). However, Apple Logo II enabled the redefining of primitives which made the microworlds much more consistent with the Logo syntax (e.g. FD outputs "I DONT KNOW HOW TO FD" (appendix C) in appropriate cases such as the activities in category 2, fig. 6.1.1). The implementation of colour was also used, mainly in order to counter screen resolution deficiencies by using a different colour for graphics with different meaning, i.e. labels, plane description lines, turtle path, location representations (x - signs). Certain changes were made in the researcher's intervention strategy in order to clarify and improve the research outcome (e.g. the frequent encouraging of the children to explain what they were doing and why, either to their peer or to the researcher).

b) Local changes:

As a result of the pilot study, certain local and detailed changes were made at task or activity level. For example, in the path 1 initial activities (fig. 6.1.1), the children could place points (i.e. x - signs) on screen locations and join them up with lines with the DODOTS command. The DODOTS command was altered so that the figure did not close automatically unless there was a placing of a point to do so. In the path two initial activities (fig 6.1.1), the grid was named systematically (instead of A, B, C, etc.), in order to incorporate the concept of systematic naming of points on a plane in this pathway. In the coordinate plane representations, the coordinate axes were extended to the edge of the screen, to avoid misconceptions regarding the infinity of the length of the axes. In all the activities where axes appeared, the calibrating markings were changed to lines of two sizes, in order to make the counting clearer and therefore avoid mistakes that were due to screen effects. The introduction to the activities of the second category was changed in order to

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make them more interesting to the children.

6.1.4b) The main study.

The main study involved three pairs of children from the logo club, Natassa and Ioanna, Maria and Korina, Anna and Loukia. The research was carried out during school hours in the research room. Each pair of children participated in three 90 to 120 minute sessions in a total period of no more than a week for each pair. During the research, the children participated normally in their Logo club and school program Logo activities. The machine and the Logo version used for the research was the children's familiar (from their school activities) Apple IIC and Apple Logo II respectively. During the research sessions, they were faced towards the machine, the researcher seated behind them, so that their collaboration was unrestricted unless there was an intervention from the rearcher (they had to turn round to face him).

The research data consisted of;

a) audio taping of everything that was said,

b) soft and hard copies of verbatim transcriptions translated in English,

c) soft and hard copies of everything the children typed,

d) hard copies of graphics screen dumps by playing back the dribble files, and pausing them to print,

e) researcher's notes and children's prompted and unprompted notes on paper.

During the whole of the research, the children had paper and pens in front of them to use if and when they wanted. The researcher's notes consisted

mainly of recording observations that would otherwise slip through the mechanical data collection net, e.g. the children often used their fingers to talk about screen locations, either when responding to research questions or spontaneously. A record of precise moments of spontaneous use of paper and pen was also kept. When the children spoke words in English (e.g. names of commands), the words were transcribed in capital letters. The same convention is used in the presentation of data throughout the study. The dribble playback facility was used for numerous graphics dumps which, together with the hard copies of the children's typings facilitated the transcription of the audio tapes.

6.2 CATEGORY 1 ACTIVITIES, NATASSA AND IOANNA FORMING A SCHEMA FOR THE COORDINATE SYSTEM

6.2.1 The children.

Natassa was characterised by her teacher as a hard worker, but *"irresponsible in her actions and thoughts"*, i.e. that she was far from achieving her potential in school. She was classed as an average student in general and in mathematics, lacking initiative in her classroom activities. During the first Logo club year, her teacher F., believed that *"through working with Logo she is beginning to show far more initiative in every day matters"*. The researcher believes that negotiating with her peers (Loukia and Ioanna) helped her gain confidence in arguing a point in a constructive way. This is supported by her opinion on what she gained from her Logo club experience: *"With this club, I learn to think and to cooperate. I learn to work out solutions more easily and quite quickly."* However, although she seemed to be confident with ideas she understood, she felt rather "unsafe" with experimenting. Her programming attainment in the structured tasks throughout the preliminary year was above average compared to the other children in the club (her strategy in the Four Squares task is represented in figure 5.2, E and F, appendix H)

Ioanna, although well motivated, was characterised by her teachers as a below average student. At the beginning of the year 85 - 86, her teacher said *"she was all over the place"* (i.e. disorganised), and had some problems in communicating with her peers. She was not a dominant personality and in her Logo club group, preferred to discuss with her peers (Loukia and Natassa) and compromise as to what is to be done - she would often be upset when they were not prepared to do so. However, although initially she showed a lack of initiative to pursue a Logo investigation, her perseverence grew substantially during the year. Furthermore, she showed progress in her programming, characterised by a "shift" from group C to group F in figure 5.2, appendix H.

The activities involved placing points on the coordinate plane by naming locations and joining the points up with lines, in the order in which they were placed on the plane. Initialising the program resulted in the coordinate axes appearing on the screen, callibrated in units of ten. There was no turtle and no movement on the screen throughout the activities, nor were there any labels. There were only two commands that the children could use, PLACE and DODOTS, which were given to them on paper. The first word placed a point on the plane, denoted by an x sign. The inputs to PLACE were the two cordinate values and the numerical order of this particular point. DODOTS joined the points up in the order they were placed (fig. 6.2.1).


```
PLACE -80 0 1
PLACE 0 80 2
PLACE 80 0 3
PLACE 0 -80 4
PLACE -80 0 5
DODOTS
```
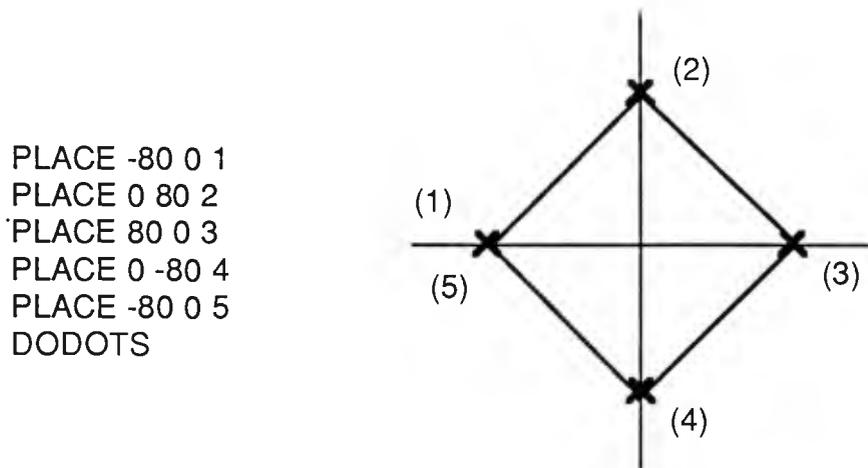
Figure 6.2.1: N. and I. : "Constructing a rhombus"

This is a non - action environment, where naming a location causes its graphical representation (an "x" - sign). Consequently, there is no entity to give operation (action) commands to, number does not mean a quantifier for an operation (Logo or algebraic), and the signs do not denote operations. The sequentiality factor (having no strong relation to coordinates) was imposed partly in order to strengthen the clarity of the research issues, i.e. to avoid the "noise" caused by its absence. The way it was embedded in the task was by the ordering of the named points with numbers, and the slowed

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down visible execution of the DODOTS command. Joining up locations gave on the one hand a meaningful "drawing shapes" element to the activity, and on the other, an insight into the children's mental image of planned figures, i.e. the finished version as a consistent reference or imposing a mental image with intrinsic characteristics by "constructing" the shape in their minds in a step by step manner as they are placing points on the screen.

The beginning of the activities involved explaining the coordinate system and the use of the two new words to the children. The third input to PLACE was explained as denoting sequentiality, the researcher making sure that the children distinguished the meaning of this input from the coordinate values.

After a few "nudges" from the researcher while the children tried out placing points of their choice, they were asked to carry out projects of their own. Their choice of "real - life" (e.g. a bow - tie), abstract (e.g. square, circle) or non - planned projects and their strategies for making sense of and using the coordinate system in meaningful contexts was in focus during the researcher's investigation on the nature of the "thinking schema" they would adopt.

6.2.3 Findings.

6.2.3a) Focusing on coordinate locations versus focusing on coordinate values.

After being introduced to the "mechanics" of the coordinate system (axes and their names, 10 unit calibration, plus and minus regions of axes), and to the locating and naming method in the context of using the PLACE command, the children's strategies for the several trials they made of placing points on the screen, fell into two broad categories:

a) **in focusing on the location first** (e.g. by putting a finger on the screen) **and then finding out the coordinates for that location** (e.g. by

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"counting" while a finger remained on the location) and,

b) **in focusing on the coordinates as numbers or values, and then looking to see where the location was.** Typing numbers in was done either supposedly at random, or with a focus on particular number cases (e.g. comparing 130 0 with 130 -0).

6.2.3b) Imposing a sequence schema on coordinate locations.

The first project was a rhombus, which the children drew on paper beforehand, and then placed the points on the screen in a clockwise manner, starting from the left hand point, i.e. -80 0. Their predominant strategy was to point at a location and then find the coordinates for it. This was a first indication of the children using a "sequentiality" schema, i.e. realising that the order in which the points were placed was directly linked to the final effect. This is illustrated by Ioanna's comment when they had placed the fourth point (0 -80), and were considering using the DODOTS command:

I: "No, it won't close, we should go back to the beginning." (fig. 6.2.1)

The children carried this "sequence" schema on to their next project, the initial plan for which was to make a circle inside the rhombus, but which, after the three first points turned out to be in a vertical formation, changed to a bow - tie. The interest here lies in the planned method for constructing the circle (fig. 6.2.2a), i.e. starting from the left and placing one point after another progressively closer with no particular focus on equal distance from the centre (the children drew the point at the centre as an afterthought), and how **this sequential circular formation was applied in the construction of the bow tie** (fig. 6.2.2 a,b,c).

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PLACE -30 -30 1
PLACE -30 0 2
PLACE -30 30 3

(a)   (b)   (c)
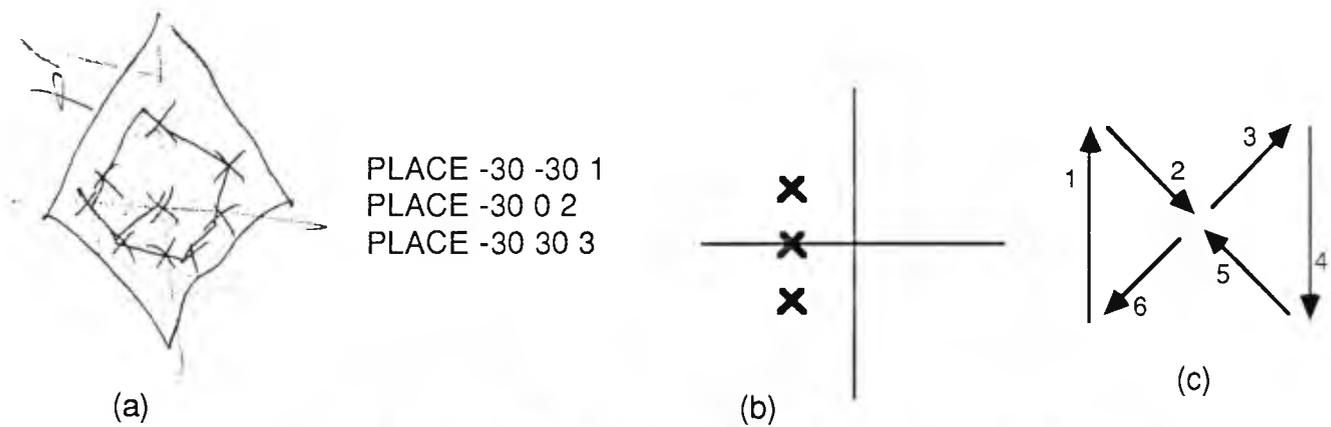
Figure 6.2.2: N. and I. : "From a Circle to a bow-tie"

It is suggested that this formation for the circle resembles the sequentiality characteristic of the "intrinsic" Logo method for constructing circles, with which the children had ample experience in their club and normal class activities. However, the children did apparently attempt to incorporate the notion of equal distances from the centre when they gave equal values for the x coordinate (-30) for the first three points (fig. 6.2.2b). It seems, therefore, that lack of understanding why the first three points were in a vertical straight line formation, was due to a confusion between a fixed distance of a point in the plane from the origin and a fixed value for the partially locating x coordinate. The bow tie construction had the characteristic that there was no need to "bring in" a property or an idea with which the children were not comfortable; it only involved the sequential placing of one point after another in a "circular" fashion (fig. 6.2.2c). It may be therefore, that **this method for constructing the bow tie was an "extension" of their strategy for the circle,** i.e. one which did not involve taking into account a property which the children were not clear about in this context. This is supported by the fact that for their next project, which was a large bow tie in a clear plane, the children did not place a point on the origin, adopting a strategy resembling the figure "8" horizontally (fig. 6.2.3).
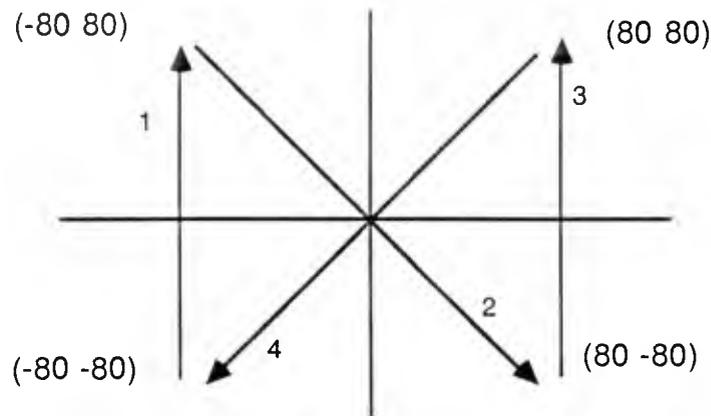
Figure 6.2.3: N. and I. : "The large bow-tie

6.2.3c) Changing from a location, to a "region of the plane" focus.

During their first bow tie, the children adopted the method of concentrating on the location first, by placing a finger on the precise desired point (e.g. -30 30), and then finding the coordinate values for it by keeping one finger on the point and "counting" on one axis at a time. Their concentration on the locating method persisted till the end of this figure. However, in the large bow tie (fig. 6.2.3), there was a shift of focus in the locating method from the first point (-80 80) where the children again counted on the axes, to realising that the values for each point would either be 80 or -80. **They therefore concentrated more on the plus and minus regions of the axes, rather than the values themselves.**

6.2.3d) Developing a method to relate coordinate values to a location.

The children then decided to place some points "at random" i.e. without any pre - specified plan for a shape. The researcher asked the children to show the positions on the screen after the respective numbers had been typed but before the children pressed the RETURN key. The children, used to being asked to provide factual answers, **attempted to incorporate simultaneously, three separate issues:**

a) the order of the axes,

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b) the regions denoted by the signs and;

c) the meaning of the numbers, regarding the location.

This seemed a difficult task for the children. For example, for point 33 99 they pointed at a possible 99 33 point, presumably getting the order of the axes wrong, but the meaning of the numbers and the axis regions right. Also, at -0 -98 they pointed at a point around -50 -98, disregarding the meaning of the number in the first coordinate. **However, towards the end of the session, the children started to pay more attention to the method of reaching the right location,** by counting on one axis and holding their fingers on the spot to count on the other. They seemed to discover for themselves that what was needed was not a speedy answer, but a correct one (the researcher did not change the way of asking the questions), and the means for that was a method with which they were starting to feel more comfortable.

6.2.3e)Employing coordinate values in order to use a geometrical notion

The children "fell upon" the notion of symmetry for the first time, in their project to make the letter A (fig. 6.2.4).
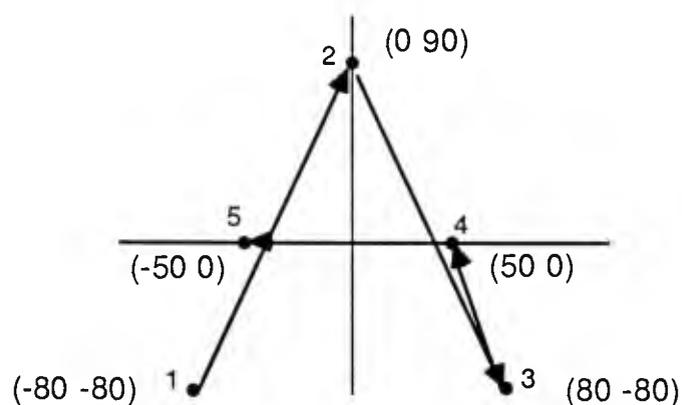
Figure 6.2.4: N. and I. : The letter "A"

They placed the three first points on the screen (-80 -80, 0 90, 80 -80), thinking about the locations first and then putting in the values. Natassa's

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reason for the 90 value in the second point was to make the letter large and for the third she was quite fluent in what seemed to be an implicit use of the notion of symmetry regarding the y - axis. She used the notion again later, to place the final point of the letter A, i.e. to make the line (50 0) (-50 0). In these two instances, Natassa seemed to focus on the location of the symmetrical point first, and then think about the coordinate values. However, it is suggested that after the second time she noticed what happened to the actual values of the coordinates and formed a theory on those values, of how to make symmetrical points regarding the y - axis: by changing the signs in both values. She tried to implement this theory in their next project, a star, having placed the (-60 60) point and wanting to locate its symmetrical (fig. 6.2.5):

N: "Now... PLACE... 60 -60, i.e. the opposite... and you'll write 5 too in the end (fifth placement). *Now... you'll write... why did it go down there? I told it... 60 from here and -60... right. Oh, I should have said 60 60."*

(0 90)

(-60 60)   (60 60)

(-80 -80)   (80 -80)
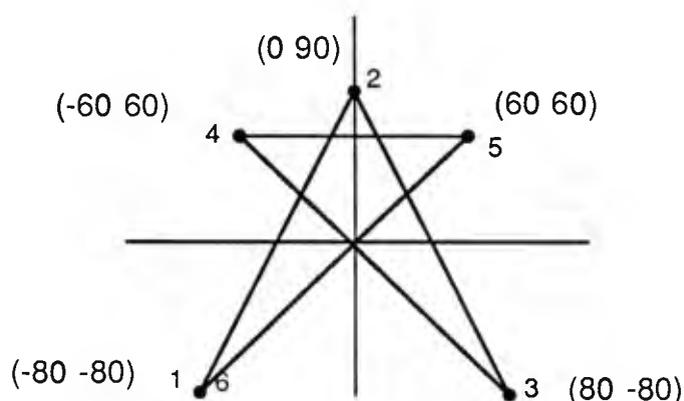
Figure 6.2.5: N. and I. : The "Star"

It is suggested that Natassa's confusion was due to lack of discrimination between the algebraic meaning of the signs ("...opposite...") and their coordinate meaning, i.e. regions of the axes. When she realised her mistake, her focus was back on the location and not on the number values.

The children's first reaction to the problem of finding the location "half way"

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between the second and third point of the letter A (fig. 6.2.4), was typical, taking into account their minimal experience with investigative informal learning (see chapter 5): Ioanna suggested abandoning the whole project and doing something else. Encouragement to continue, brought Natassa to place the following two points: 50 0 and -50 0.

Notice how her mind seemed to work: **firstly, she seemed to analyse the location she was looking for into the two coordinate values.** The 0 value for the y axis indicates that she used her knowledge about coordinates to place the point on the intersection of the x axis and the segment (0 90) (80 -80), which was invisible at that point. **Secondly, once she decided upon the 50 0 values, she implicitly seemed to use** (in a similar strategy as before for the points -80 -80 and 80 -80) **the notion of symmetry regarding the y - axis.** The only value (x coordinate) which she did not have a logical method to determine, she used her perception and made an estimate.

6.3 CATEGORY 1 ACTIVITIES. MARIA AND KORINA INTEGRATING NOTIONS OF A SYSTEMATIC DESCRIPTION OF THE PLANE INTO THE INTRINSIC SCHEMA

6.3.1 The children

Korina's teachers thought she was a very hard worker and above average in all the subjects. However, it seems that she did not particularly like mathematics and found it difficult, her favourite subject being language *"I like Greek because I have more of a vocation for it"*. She was not particularly dominant, and preferred to work on her own unless she felt "intellectually superior" to her partners. During her first Logo club year, the researcher felt that she did not engage very much in her group's projects and did not really attempt to have a go at the keyboard. Both her peers were boys, one of them was keen, but of equivalent everyday classroom "ability". However, the researcher believes that Korina improved in her degree of participation, after

he encouraged the group to cooperate more closely *"we learned to cooperate as a team and say our opinions to the others. Also to take initiatives and not to listen to the others only".* During the preliminary year her programming to solve structured tasks never went beyond direct - drive. However, in the final "batch" of tasks immediately prior to the beginning of the main research, she showed clear evidence of using the geometrical ideas involved in the tasks and being in control of the commands she gave to the turtle.

Maria was characterised a weak student by her teachers. She seemed rather nervous and hesitant in her interactions with her peers, but was not a passive member in her Logo club group. At the beginnng of the year, she said she would prefer to work on her own and in her Logo group she did not get much hands - on experience, her peers being more dominant. However, in the essay she wrote at the end of the year, she described the club's activities very explicitly and summatively showing on the one hand, quite a high degree of engagement, but on the other, that her perception of her role in the computer room was that of a member of the club rather than of her group. Although her programming in solving the tasks did not involve the use of procedures, from early on she showed evidence of being in control of navigating the turtle and of using the geometrical ideas embedded in the tasks. Furthermore, her confidence distinctly increased during the last "batch" of structured tasks.

6.3.2 Task analysis

These activities involved using length and angle measuring instruments to make shapes with the turtle, by joining points on a simple grid with a "chess - type" system for naming locations. Initialising the program draws a 4 x 4 grid of 30 turtle unit squares, the turtle at its centre (fig. 6.3.1). Single numeric (vertical) and letter (horizontal) labels appear on the screen. Naming a location requires combining a horizontal and a vertical label (e.g. E4). The children could use the following commands which were given to them on paper: PR DISTANCE name, PR DIRECTION name, FD DISTANCE name, RT DIRECTION name, FD quantity, RT quantity, PU, PD.
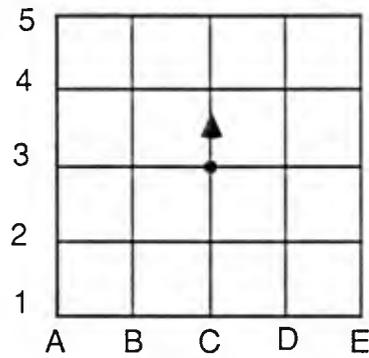
Figure 6.3.1: M. and K. : Initialising the task

These activities involved two new conceptual domains for the children, the first one being the use of the turtle instruments in simple measurement and combined "action - measurement" operations (see chapter 7). The second domain is the method of describing locations. The grid constitutes a systematic description of discrete plane locations with no origin.

In mathematical terms, the turtle does not know of any other part of the plane than the 25 locations. It may travel outside the lines formed by the grid but the only determinant of its path are the locations themselves. The concept of naming points with numbers and signs is also absent. Although numbers are involved in the naming process, they are designed so as not to convey an arithmetical meaning, since they are always joined with a letter (e.g. A1, E4). The environment is action based with the turtle driven in the plane to draw figures.

The action characteristic of this environment is directly linked to intrinsic notions of driving the turtle. The aim of the task is to track the children's reasoning and the nature of the links they make to prior experience: whether they maintain a turtle identification pattern and consider everything else through this perspective, or whether the additional elements of this environment become the important factors in their programming strategies.

6.3.3 Findings

6.3.3a) Developing a locating method.

The realisation that in order to give a name to a point on the screen grid (fig. 6.3.1), one needed to a) combine horizontal and vertical directions and b) make a name out of two labels rather than one, did not seem to be a trivial task for the children. When they were first asked to give a name to a point, they tended to provide a one label answer. The first insight into the naming method came after they had given the name D for point D4, and were asked to give a name for D3:

R: "What's the name of that point? (point D3)"

K: "Three."

M: "It's D because it's vertical..."

R: "What is it, D or 3?"

M: "D3."

R: "Why D3?"

M: "Eh, because it's in between let's say, it's both D and 3."

It is interesting that **the existence of a more sophisticated method seemed to occur to them only when they came up with a problem regarding their existing method of single labels, i.e. that they had the same name for two different points.** The children found the new method satisfactory, having been encouraged to try it out for more points.

6.3.3b) Integrating measurements with the action - quantity schema.

Before introducing the measuring instruments, the researcher employed a technique also used in the P.D.D. microworld study (chapter 7), by asking the children to take the turtle to a specific point labelled on the screen, in order to investigate how far they would pursue an "approximation" tactic based on perceptual cues and to lay the ground for a meaningful introduction to

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measurements. Apart from their lengthy approximating efforts, **it seemed very difficult for the children to realise that there could be some other method for determining distances or turns than the intrinsic action - quantity method.**

The following incident illustrates the strength of their intrinsic, action - quantity schema, in their very first use of the DIRECTION and DISTANCE commands. The researcher had discussed with the children that the turtle has a ruler and a protractor, as a means to reach a point on the grid accurately. He was about to tell them about the syntax of the new commands (DIRECTION first) in the context of taking the turtle from C3 (heading at 0), to D5 (fig. 6.3.1).

Korina did not seem to have yet made a connection between the use of the protractor and the action - quantity schema; she considered the measuring instrument as something which would automatically (without realising how) perform a turtle action:

(R: "So, what do we want to ask her?")
K: "(the degrees)...to get to D5."

However, she soon made an insightful remark, **apparently connecting the protractor with her present strategy for changing the turtle's direction, and also discriminating the two states of the turtle:**

K: "To turn the turtle so that she aims exactly at the place of D5, and, like that, afterwards..."

Notice her careful wording: the action is there *("...turn the turtle..."),* but the determinant of the quantity is an external location *("...at the place of D5..."),* and the reason for using the protractor is accuracy *("...so that she aims exactly...").* Also, although the final aim is to take the turtle to D5, this is only the first step i.e. the change of heading. Changing the position is a different thing *("...and like that, afterwards...").*

Korina's new strategy, however, was challenged by Maria, who's reaction was to go back to an approximating "perceptual" mode to change the position. When the children typed in the DIRECTION command and made exclamatory remarks at the result, Maria's reaction was:

M: "And now let's tell it FD how much?"
K: "FD... fourty...FD..."
M: "If this is 45 this should be..."
K: "But we said we want to arrange with precision how much it should be."
M: "Eh, then we'll say... we want the ruler."

It is suggested that Korina initially went back to her old method, but soon "brought on" her new strategy for state change. It is seen as important that **she seemed to generalise her strategy from direction change and apply it to position change,** and her reason (the phrase "common denominator" could be used) was to achieve the accuracy that the old method could not provide.

After practising at taking the turtle to several grid points, the children then carried out two projects, a square by joining B3, C4, D3, C2 and B3 (fig. 6.3.2), and the letter A by joinning A1, C5, E1, D3, and B3 (fig. 6.3.3). Two issues will be highlighted here. Firstly the development of the children's rationale for deciding which state changing method to use, and secondly their progress from "noticing" certain geometrical properties to using them to determine the quantities of actions.

6.3.3c) Rationale for method of state change.

For the first issue, a description is given of how the children started off using the combined action / measurement commands (i.e. RT DIRECTION :D3, FD DISTANCE :D3). The first incident where they decided to use a geometrical property and break the commands down in order to use their intrinsic schema

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is then illustrated, followed by a discussion of the progress of such decision - taking according to their geometrical knowledge concerning the desired state change.

Having planned the construction of the square by stating the points they would take the turtle to, the children's first decision to use a combined action / measurement command was in the context of changing the turle's heading from 0 to face D3, after having taken the turtle to C4 with a FD 30 action - quantity command (fig. 6.3.2). **It is therefore suggested that they did not simply carry on with the type of command they were using before (combined action / measurement), but seemed to make a conscious decision on its use.** They adopted this strategy for a while, turning the turtle, taking it to D3 and back again to C4 (BK DISTANCE :C4) having forgotten to put the pen down to "draw" the line.

It seems, however, that while the construction of the square was progressing, the children noticed in an implicit way the "regularities", or properties of the figure. This is the first incident where they decided to go back to a simple measurement, and use the outcome repeatedly, in straightforward action - quantity commands:

(turtle at D3, facing towards C2, fig. 6.3.2)

M: "...but now, to know about the others too (she means lengths) *we can ask it how much..."*

*K: "Yes, let's make it appear (*she means the measurement outcome) *so that we don't have to bother* (waste time)... *so PRINT again, PR."* (result on screen)

M: "Ah, 42... write it so that we remember... (she writes it on paper) *and then we'll tell it FD.. em... 42...4...2...5...9."*
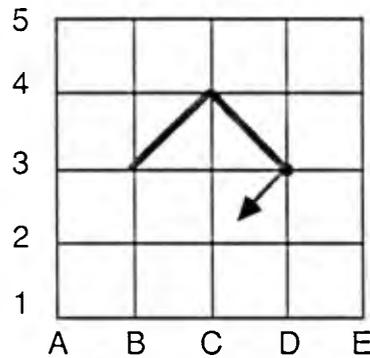
Figure 6.3.2: M. and K. : The "Rhombus"

They then used the same strategy for the turns and finished the square by changing states with the action - quantity method. For this project, perhaps it could be argued that the properties of equal sides and turns of a square was not a great challenge to the children, and therefore they found it quite simple and straightforward to make one measurement and use it three times in their familiar action - quantity method.

However, in their letter A project (fig. 6.3.3), where linear and turn properties were not so obvious, **the children surprisingly made intense efforts to "work out" the quantity in their minds and use it with the move and turn commands.** They developed a strategy of using the instruments for measurements only when they had a reason to find out the result, and in combined action measurement mode when they could not work out a quantity and did not see why the measurement result would be of use to them. The unexpectedness of their strategy lies in the fact that they did not need to work out or think about any quantities. They only had to use the combined action - measurement commands, with which they were quite familiar, and state the names of points according to desired state changes. **It is suggested, therefore, that the main factor in the children's tendency to work out the quantities and use them with the move and turn commands, was the strength of the intrinsic schema in their minds, i.e. that ultimately, this is what made real sense to them, and they would only use another method if it had something more to offer than the old one (e.g. accuracy).**

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ASIDE: The large number of decimal places in the numerical outputs resulting from the use of the DISTANCE and DIRECTION commands could - with the benefit of hindsight - be seen as a methodological shortcomming. However, analysis of the data indicated that this did not interfere with the research issues. Moreover, the researcher asked the children whether it was a distracting factor and received a negative answer in all cases. This applies to the whole of the main study, i.e. chapters 6, 7 and 8.

6.3.3d) From "noticing" to using geometrical properties.

Indicative of the previous argument, is the relative ease with which the children seemed to carry the experience of using the properties of the square, on to their letter A project. Their first thought in their initial aim to take the turtle to point A1, was to work out the distance C3 A1 (fig. 6.3.3) by multiplying the number they had found for the diagonal of a grid square by two.

Another example is their strategy for the turn at the same point. In the square project, the children had explained the 90 degree measurement outcome as a two - times 45 degree turn:

(turtle at C2, facing towards B1, the measurement involved the turn from B1 towards B3, fig. 6.3.2)
M: "Because, as it's like that (towards B1, finger on the point), *45 it will go to the line* (finger rotates 45 degrees) *and another 45 it will go to the side* (finger rotates another 45 degrees)."

For the turn from zero heading to face towards A1, in the A project (fig. 6.3.3), Maria used the same strategy of 45 degree finger rotations:

M: "...I'll tell you now... if I turn there it's 45 (finger rotating on screen) *and 45 90 and 45... 135. Yes that's right."*

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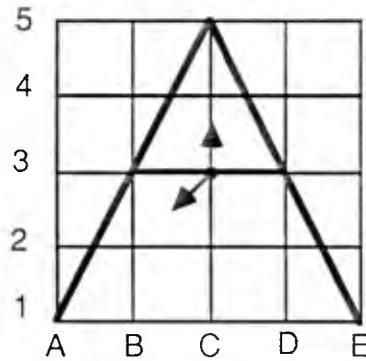
Figure 6.3.3: M. and K. : The letter "A"

In their square project, the children implicitly seemed to notice and use firstly the equal sides and then the equal turns property. At the end of the square, the researcher interviewed them to find out what they thought the property was and came up with an unexpected answer: **the children had not thought of the "regularity" properties as belonging to the square under construction, but as belonging to the squares of the already existing grid.** The argument that could be put forward here is that the grid squares were "concrete" (they existed visually) for the children, since they were on the screen and they had already had experience of using their properties when they were practicing the use of the instruments. On the other hand, the square was under construction, which could mean that at that moment, using its properties would require abstracting its full image.

6.4 CATEGORY 1 ACTIVITIES ANNA AND LOUKIA USING THE INTRINSIC SCHEMA TO CONSTRUCT AND USE A SYSTEMATIC DESCRIPTION OF THE PLANE

6.4.1 The children.

Anna has been characterised by her teachers as "*an average - above average student*". During her everyday school life, she was perceived as intrinsically motivated and keen to find things out for herself "*by experimenting with what knowledge she had at hand*". She has a confident

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personality and shows *"strong leadership qualities"*. In the Logo club, Anna had the experience of trying to cooperate with a very dominant boy who tended to take over the keyboard, and a very unnconfident girl. Anna was actively interested in achieving negotiations with both her peers and discussed the problem with them and with the teacher at hand (researcher or teacher F). Her programming strategies in solving the Four Squares tasks involved the use of procedure and showed progress according to the criteria set in appendix H, illustrated in figure 5.2 by a "shift" from group D to goup E.

Loukia has a very extrovert and sociable personality in her normal class, and according to teacher F, is sometimes dominant in *"a not so positive way"*. She was said to be an average student in all subjects except maths, where her teacher said that *"she often had problems"*. During the first Logo club year, she met (and caused) a lot of cooperation problems in her group, these frequently ending in quarrels and crises. However, during the latter part of the first year and the former of the second, she held a lot of discussions with her peers (Natassa and Ioanna) and between them, they devised methods of much more efficient cooperation. It is the researcher's opinion that Loukia made a lot of progress in a less egocentric attitude both towards her learning *("usually in our lesson we blame the computer or the turtle while it is us who made the mistake")* and towards her peers *("First I discuss it with my team and then we all find the answer")*. Although her programming in the Four Squares tasks was above average (progressed from D to F, fig. 5.2, appendix, H), she programmed in direct - drive for the other structured tasks, and showed a surprising lack in her use of the embedded geometrical properties.

6.4.2 Task analysis.

These activities involved the construction of a simple 2 x 2 square grid with the use of the POST command (see chapter 3), and the joining of grid points to make shapes of the children's own choice, by using the DISTANCE and DIRECTION commands. The children could use the following commands which were given to them on paper: PR DISTANCE name, PR DIRECTION

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name, FD quantity, RT quantity, PU, PD.

Constructing the grid in this method, on the one hand employs intrinsic "action - quantification" notions, and on the other, requires an understanding of the notion of the grid, i.e. its usefulness in location descriptions and its geometrical properties. The grid provides a description of locations which are systematically arranged on the plane but without a systematic (logical) method of naming. The locations are named one by one by a letter label. Using the measuring instruments to join locations requires separate measurement and action - quantification activities.

The task was designed to find out what meaning the children gave to constructing a description of the plane, and how they used their intrinsic schema to interpret the grid's properties in order to construct it. The aim of the second part of the activities was to study the extent to which they used and related their intrinsic schema and the location describing system, in projects of their own choice.

6.4.3 Findings

6.4.3a) Developing a method for naming locations.

After the researcher had drawn fig. 6.4.1a on paper, the children's strategy for constructing the grid was to put a "flag" (POST) on the turtle's position, give it a name in alphabetical order and move on to the next position (fig. 6.4.1b). Their method of naming was by alphabetical order, in the order in which the turtle constructed the grid points, rather than imposing an absolute naming system, e.g. rows or columns, or A at the centre (fig. 6.4.1c). **The actual order in which they placed the points is indicative of a "drawing" schema** (Hillel, 1986), despite the fact that they not only used the orthogonal (90 degree turns and equal lengths of 40) properties, but also combined equal distance with an operation in typing BK 80 to take the turtle from H to the last point, I (fig. 6.4.1c).

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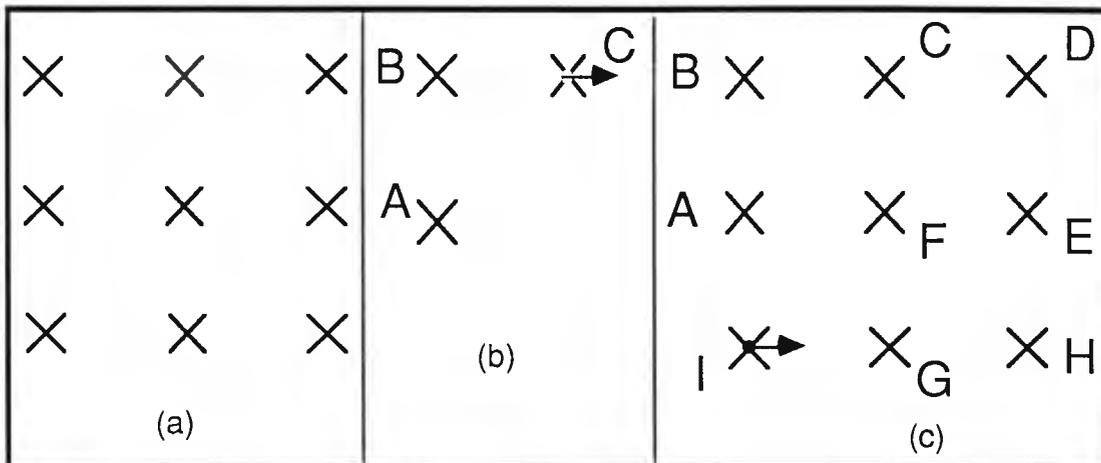
Figure 6.4.1 A. and L.: Constructing the grid

In discussing what the use of the grid might be, after some vague reasons, (e.g. to make shapes, to play the game, to join points) **Loukia implicitly suggested the possibility of a plane location determining a state change.**

L: "Let's say I can here write a command which says, go straight away, say e.g. go to F, go to E..."

However, although she adopted a direct speech mode, "addressing" the turtle, **she had not thought about a logical method to integrate the turtle's characteristics with this location - centred state change.**

L: "... I don't know how it will be, without you having to make it turn, it turns by itself. By going there."

6.4.3.b) Discriminating between length and angle measurements.

The children's first use of the instruments was in the context of taking the turtle to points of their choice in PEN UP mode, i.e. from I to E, to B, to G. However, it is suggested that initial lack of confidence with using the instruments

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influenced the children's strategy. They discussed each change of state, making perceptual and logical efforts to determine quantities, even after having made a conjecture involving an implicit use of a property (symmetry) which was less apparent than that of equal distances:

A (why IE = EB): *"Because it's in the same place as the other one."*

It seemed, therefore, that the turtle's instruments were used by the children as a check to see whether their conjectures were right. **Their initial confusions seemed to reveal a lack of discrimination between the differing meaning of information collected from the ruler and the protractor** (they tried to use length information to turn the turtle) and a lack of a strict correspondence between the words direction and distance and their respective uses for angular and linear notions, i.e. they used the word distance to describe the amount of a turn:

L: *"Yes, the distance from here, to turn."*

Their confusions also suggest an unclear distinction between the heading and the position state of the turtle.

A: *"The degrees it has to turn to go to B."*
L: *"Yes, to look at B exactly."*

The researcher decided to intervene to explain that the turtle protractor gave information about right turns, since this was an arbitrary characteristic of the instrument.

6.4.3c) Using geometrical properties versus carrying out measurements.

The children's first goal was a "well defined", "abstract" (Hoyles and Sutherland, 1988) rhombus. They made a plan before starting, to join up points G, E, C, A, G (fig. 6.4.2) and did not seem to have problems in

developing a strategy for the turtle's actions (right turn and forward move). However, as their quantity conjectures proved to not always be correct, and their confidence with using the instruments grew, **there seemed to be a shift in the importance they attributed to the measurement results.**

B X   X^C   X^D

A X   X_F   X E

I X   X_G   X H
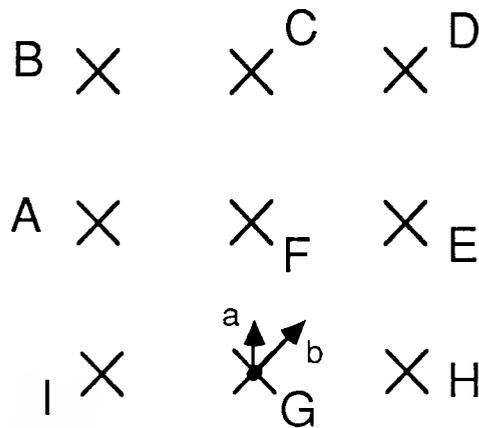
Figure 6.4.2 A. and L.: The "rhombus"

In the process of constructing the rhombus, **they seemed to show the first signs of a distinction between measurement and action and of a distinction between the notion of measurement and the naming of a location, in both angular and linear measurements.** The first of the following examples illustrates this point for the heading and the second for the position:

(turtle at G, heading upwards fig. 6.4.2a)
L: "Hm. Till E, lets find E. Ok... PR... yes... DIRECTION... (A types, L waits for her to finish)... *and till E."* (they wait for the result)
A: "Ok, RT..."

(turtle at G, heading towards E fig. 6.4.2b)
L: "Yes but we don't want to make a line yet. We want to find out how much the distance is."
A: "Yes."
L: "But it will be sure that it's 40."

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A: "Do a PR, can't you see how many we got wrong?"

The children then decided to make another shape. It was a "real world" goal (Hoyles and Sutherland, 1988) which they drew on paper, Anna called it a bow tie, Loukia a "butterfly in a field". Their strategy was to join up the following points: F, D, E, H, F, B, A, I, and F (fig. 6.4.3a). It is suggested that **their increasing fluency in the use of the instruments resulted in a decreasing confidence to commit themselves to making conjectures before reflecting,** the reflection occasionally encouraged by the researcher. The only property they used without a measurement was the equal point distances, e.g. they typed FD 80 to take the turtle from D to H and from B to I. However, the use of two more properties was invoked by the children by differing methods.
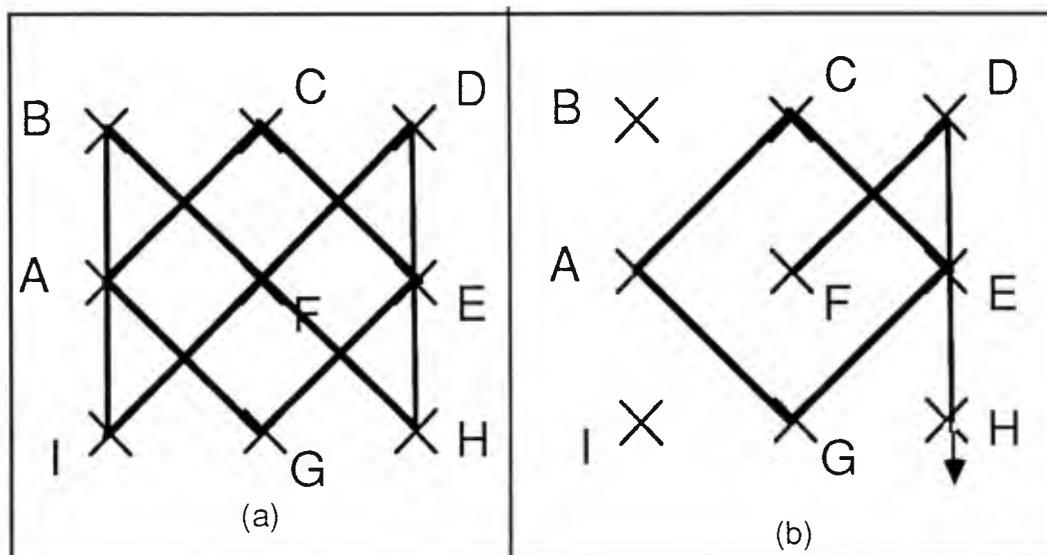
Figure 6.4.3 A. and L.: The "butterfly"

Firstly, they explained that the outcome of 45 degrees obtained from a rotation measurement which they had carried out was due to the rotation being half that of a 90 degree turtle turn. (turtle position: F, heading measurement from C to D). Anna used this half turn theory later in a more complex situation, adding it on to a 90 degree turn to predict the turtle turn at H (fig. 6.4.3b):

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R: "Who can think how much it will be?"

A: "90... 90 plus..."

L: "90 plus 44.999"

A: "90 plus 45."

Loukia's answer seems to refer to their previous measurement (turtle at F), rather than the use of a property. This noticing and matching equal quantities is also the method by which Anna made gradually more confident conjectures about the length quantities. Her first (on HF) and last (on IF) conjectures illustrate this point:

A (on HF): "I say 56 to 57. (result on screen) since it's the same when it was like that... (means FD)"

A (on IF): (before result on screen)"565686." (that was the length of FB)

However, the children did not use the half angle and equal length properties in slightly different contexts, i.e. in a 225 turn at points B and I, and in linking the bow - tie slanted lengths to the rombus slanted lengths.

6.5 SUMMARY OF THE "PLANE DESCRIPTION" SESSIONS

Although the aim of this study was to investigate whether the children could use their intrinsic schema to understand coordinate notions, it is seen as useful, for a moment, to isolate the turtle - action issues in order to highlight those concerning the children's understandings of the systematic description of the plane. The analysis of the data indicates three types of issues which the children did not seem to relate to some specific prior experience, i.e. for them, they were new:

a) the existence of an organised system for naming locations, its usefulness and its nature;

b) the existence of an analytical method of locating points via an origin, i.e. combining the distances of a location from a horizontal and a vertical axis;

c) the "rules" of the coordinate value system, i.e.

- the order of the values;
- the meaning of numbers as names of places;
- the meaning of signs as regions of the plane;

For example, it is suggested that it did not occur to Anna and Loukia to think about and devise some systematic naming for the points in their grid since for them there was only a small discrete number of points to be named. Moreover, Maria and Korina "discovered" the method of combining horizontal and vertical directions and using two labels to form the name of a point, only as the outcome of the conflict created by one - letter labelling, i.e. two points having the same name. The first group, Natassa and Ioanna, did not seem to have much difficulty with any of the "rules" in isolation. Given the children's age, this finding is not inconsistent with Lawler's research (Lawler, 1985). Initially, however, they did not seem to adopt an analytical way of working out the coordinate values of a location, but tried to incorporate all three issues at the same time. It is possible that the "obligatory" interaction between symbolic and graphical representations of locations played a role in the children's developing analytical method.

Another point of interest, highlighted by the study of Natassa and Ioanna's work, is how the children seemed to use a sequentiality schema in a context where seemingly there was no use of an action - quantity schema. It is true that the notion of sequentiality was embedded in the task. The children, however, seemed to use it in a convincing way, by treating the third input to the PLACE command as distinct from the coordinate values and by perceiving it as denoting the order of the point they were placing. They also seemed to use a sequence notion by placing the first point of a figure again at the end to "close" it with the DODOTS command, and by attempting to

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construct a circle (which changed into a bow tie constructed in "circular" formation) in an intrinsic "polygon approximation" way. It is therefore suggested that the sequence schema used in this context could have been, to a substantial degree, a carry over from the children's previous experience with Turtle geometry, i.e. the sequentiality "part" of their intrinsic schema.

Regarding the change of the turtle's state, the children initially seemed to be reluctant to contest, or modify their action - quantity schema and pursued their approximating efforts to make the turtle reach a point in the plane. Using the instruments seemed, on the one hand, to involve difficulties in discriminating action from quantity by focusing on the latter as distinct from the former. On the other hand, the children seemed to have had a rather implicit understanding of the existence of two different states and of the difference in their metric systems. Their schema for changing the turtle's state up till now, seemed to be "move - steps" or "turn degrees". It is suggested that they had been using these notions without really having discriminated them, i.e. analysed them into their component parts (Hoyles and Noss, 1987a). The use of the instruments, in a somewhat paradoxic manner, seemed to invoke a more explicit understanding of the two - state nature of the Logo turtle.

The meaning the children gave to the measurements was that of the method to work out the quantity of an action. They did not seem to question the turtle - centred way of changing the state and consider the notion of an external point determining this change. On the contrary, it seemed quite natural for the children to think in terms of the turtle (or themselves) performing the measurement to find out the quantity needed to change from its present state to the next.

Finally, the studies revealed the children's substantial involvement with the embedded geometrical properties of the orthogonal plane description systems. For example, Natassa and Ioanna used notions of symmetry and built (although in a naive way) "theories" of their own of how to place a point at the intersection of a line segment with the x axis. Anna and Loukia made

conjectures about quantities attempting to use the orthogonal properties of their grid and Maria and Korina went "further", by breaking down the combined action / measurement commands (e.g. FD DISTANCE :C3) and "carrying" properties from one project to the other in order to work out quantities.

6.6.1 Task analysis.

The first set of activities in this category involved taking the turtle to a specific location which was shown on the screen, in the coordinate plane, by using commands which refer directly to the structure of the absolute coordinate and heading systems. Initialising the program drew the coordinate axes on the screen, callibrated in units of ten. The turtle was in the initial position, and a location was denoted by a small cross ("x" - sign, fig. 6.6.1). The children could use the following commands which were given to them on paper: SETH "value", SETX "value", SETY "value", WRITE "name", PU, PD. The WRITE command drew the coordinates of a location on the screen if the turtle was on that location. The children were introduced to the meaning and the syntax of the commands in a way which provided insight into their initial conception of non - intrinsic turtle control, and then asked to take the turtle to the point and label it using the WRITE command.
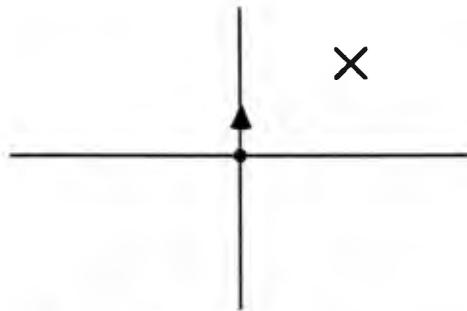
Figure 6.6.1: Initialising the task

The SETH, SETX and SETY commands provide a non - intrinsic control of the turtle, in the sense that they do not embody an action - quantification element on the one hand, and require knowledge of the method of describing locations via absolute description systems (heading and position) on the other. The use of these commands refers directly to the structure of the systems themselves (e.g. SETH 270 refers to a name given to a specific absolute direction rather than a direction relative to another location).

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However, the turtle retains its position / heading state, by imposing the restriction that changes of position can only be done in the direction the turtle is facing.

The aim of this set of tasks was to investigate the extent to which the children used the coordinate notions to control the turtle and the role their intrinsic schema played in how they made sense of non - intrinsic turtle control. An insight was sought, into how they used notions from the locating method, the naming system, the plane description geometrical properties and the dynamic non - intrinsic control of the two - state turtle to cause actions on the screen. The aim was also to throw light on the nature of any misconceptions in their involvement with this complex environment.

The aim of the second set of activities in this category was to provide the children with the experience of changing the turtle state by describing plane locations directly, rather than one coordinate at a time, and to further the investigation into the issues mentioned in the previous set of activities. The activities involved taking the turtle to two locations shown on the screen, in the coordinate plane, by using non - intrinsic commands referring to plane locations. It also involved measuring the distance between them by using the turtle's "ruler", the DISTANCE command. Initialising the program had the same effect as the previous set of activities, but this time there were two points on the screen instead of one. The commands which the children could use, given to them on paper were: SETPOS "name", SETH TOWARDS "name", WRITE "name", DISTANCE "name", PU, PD. A similar method of introducing the meaning and the syntax of the commands was used, as in the previous set. The children were asked to take the turtle to one of the two points, measure the distance between them and then take the turtle to the second point by drawing a line to join the two. They were also asked to label the points as the turtle reached them.

Controlling the turtle in this environment also requires the plane description notions mentioned in the previous set of activities. However, use of these

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notions requires a higher level of abstraction, since change of state is achieved by a direct description of the location of change rather than one coordinate at a time. For instance, changing the heading does not depend on absolute directions (e.g. SETH 90), but on descriptions of locations towards which the end heading should be directed, e.g. SETH TOWARDS [100 0]. Finally, the measurement element was added to the task in order to investigate the meaning the children gave to the numeric output from the measurement.

6.6.2 Findings.

The findings are presented in four main sections (6.6.2a to 6.6.2d). Each of the first two sections (6.6.2a and 6.6.2b) is divided into three subsections, each of which involves findings from the activities of one pair of children.

6.6.2a) Forming a schema to control the turtle heading in relation to an absolute heading system.

- Anna and Loukia: Dissociating from action - quantity and forming a schema for absolute heading changes.

Although in the category 1 activities, the children had had experience with the notion of an external point determining a change of the turtle's heading, their schema was still that of action and quantification of that action. Despite the fact that to work out the quantity they needed to take an external point into account, the meaning the children gave to the result from a measurement seemed, quite logically, to be that of the degrees the turtle had to turn, i.e. the amount of the action. **This section illustrates the nature of the children's initial confusion in changing the heading of the non - intrinsically controlled turtle, and the way they attempted to use their action - quantification schema to develop an understanding of the absolute heading system.**

After the children were introduced to the coordinate system and the SETX, SETY and SETH commands, they did not seem to have problems with taking the turtle to the point in the first quadrant. In attempting the next task (the point was at 80 -60), they typed SETH 90 and SETX 80 (fig. 6.6.2), so their task was to make the turtle face downwards. **The natural way in which Anna imposed an action - quantity characteristic on the SETH command** is illustrated by the fact that she typed her next command and uncharacteristically went on to the next one without even looking at the screen to check the result (fig. 6.2.2).

A: "SETH... 90... (result on screen, Anna confident,
did not even look) *and SET... Y...".*
L: (looking at screen) *"Did it turn?"*
A: "Yes. (looks at screen) *No it didn't..."*
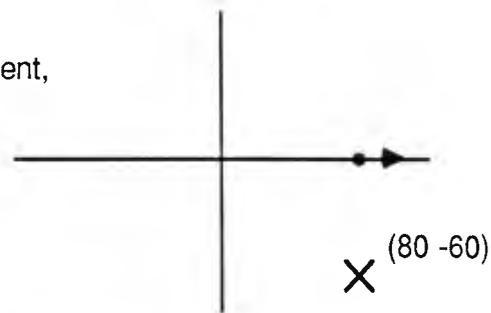
<u>Figure 6.6.2: A. and L. : Discussing how to make the turtle face downwards</u>

Their surprise was so great that they tried the same command again, looked closely this time and were baffled by the result (the turtle wouldn't turn). **When asked what SETH means, Loukia seemingly understood, implicitly dissociating from the action - quantification paradigm by referring to an end direction:**

L: "To lift me up and to turn me where you'll... to put my direction which you like."

However, she had not linked the notion of direction to the absolute system, using the x axis to give the direction.

R: "Where is her nose looking?"
L: "There, at x."
R: "Where's that?"

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L: "It's at x."

R: "x is a ruler."

A: "Forward."

Anna's answer shows how she linked the turtle's direction to the turtle's body, thus stumbling into a circular argument. However, their consequent efforts to answer the question **finally lead to a link between the absolute system and the non - intrinsic control:**

L: "Where its nose is."

A: "Where its nose is looking."

R: "Where is it looking?"

A: "Ah!.... 180. (smiles)

R: "Where is it looking now?"

A: "Now at 90."

Nevertheless, this link was only made for a moment, since in her explanation of why SETH 90 hadn't worked, **Anna seemed to initially impose a sequential characteristic to the two headings by "adding on" 90 degrees to describe the 180 direction:**

A: "Oh, yes 90 this way. Then again 90 its there..."

L: "This is 180...270..."

A: "Yes, 270, and again... 0. No.... yes 0. 90, 180, 270."

In the next task (point at -90 -40), **Anna would not abandon her turn schema, but tried to use it to form a theory about the absolute system.**

A: "From 0 till 90 its... 0. 90 to 180 is 90. From 180 till 270 is 180 and from 270 till 0 its 270."

She did not abandon the sequentiality schema even later

however, when her understanding of absolute heading had developed in that seemingly, she had dissociated from the notion of turn. For example, to explain the result of SETH 90 on a turtle heading towards 270, she said "She's looking at 90, from here till there, where its 90", and later on in the next task, to cause the same heading change she typed: SETH 0 SETH 90.

<u>- Maria and Korina: Dissociating from a sequentiality schema and imposing absolute heading notions on coordinate location - based heading changes.</u>

The first discussion concerning the method of changing the turtle's heading arose in the context of a mistake, i.e. Maria's apparent unclear distinction between the two states and the nature of their metric systems (degrees and length units), resulting in her typing in SETH and then counting on the x axis for an 80 input (turtle at home, point at 80 -60). **The process of discussing the meaning of the SETH command and its input in order to understand the turtle's resulting heading of 80, seemed to favour the development of an awareness of an external direction as the determinant of heading change.** The following extract illustrates the apparent carry - over of this awareness to the next task (turtle at -90 0, heading 270, point at -90 -40, fig 6.6.3):

M: "SETH..."
K: "To show where it's looking, yes..."
(meaning of SETH)
M: "SETH..."
K: "How much... wait... to look downwards..."
(meaning of the input)
M: "SETH 180."

<u>Figure 6.6.3: M. and K. : Discussing the meaning of SETH</u>

However, it seems that this insight in dissociating heading change from action - quantity, did not incorporate a dissociation of what has been referred to as the "sequentiality schema", i.e. the notion the children seem to have built from their turtle geometry experience,

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that a heading change is caused by a turtle action from its previous heading to the new one.

This can be illustrated by the children's attempt, in the task with the invisible axes (point at -100 90, turtle at 0 100, fig. 6.6.4), to make the turtle face downwards, i.e. change its heading from 0 to 180. Although Maria's verbal expression of her plan seemed to indicate an understanding of relating heading change to an absolute direction *("...this is 0 now, if we turn and we say SETH 180...")*, she had not really seen the absolute direction as the only necessary determinant of the change.

(she types in SETH -20, confusing again turtle steps and degrees)

M: "So we should tell it to go 180.
Therefore, 200. Let's see..."
(types SETH 200)
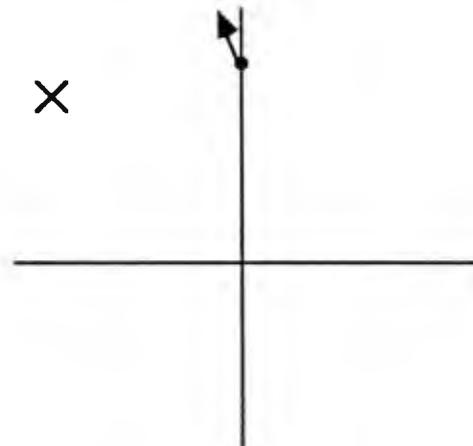
Figure 6.6.4: M. and K. : Discussing how to make the turtle face downwards

It is suggested that Maria's mind focused on the rotational "distance" from -20 degrees to 180, imposing an input which was dependent on the previous heading. This sequentiality schema seemed to have a very strong resistance to change in the children's mind; after discussing the outcome and trying out different inputs to SETH, **Maria did seem to have an insight into the absolute nature of this method of heading change:**

M: "Therefore, however much it is, let's say 5 degrees further, it's not relevant, let's say we mustn't add it to...".

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K: "We should put it normally (she means just the end heading) whatever it is."

M: "Good. Now let's tell her... 10 distance."

Inspite of the different context (change of position) it was seen as important to put in the last phrase of this dialogue, which seems to indicate that **although Maria had just had an insight into the notion of the end direction being the important factor in changing the heading, she did not carry that notion to the change of the turtle's position from (0 100) to (0 90), focusing on the distance from 100 to 90.**

In the next tasks, the children were introduced to the SETH TOWARDS and SETPOS commands, and were asked to draw the line and measure the distance between two points on the screen. Using the heading change command did not seem to be a straightforward task for the children. The schema they had formed for changing the heading depended on directions of an absolute heading system, i.e. the input represented a direction of the absolute heading system which would be the end heading of the turtle.

In their first attempts to change the heading at point (-90 70) towards point (100 -40), the children used an approximation "perceptual" technique concerning absolute directions.

K: "It's not exactly, it's that way eh? It's somewhere... it's a bit above..."
M: "It's a bit further than 180 eh?... in between 90 and 180."

This seems to indicate that **the children imposed this absolute directions schema to the new heading change method in which the input represented a location of the coordinate system.** The children did not seem to find it easy to dissociate from using this schema, since, in the previous example, they had just been introduced to the new method and had used it to change the heading from 0 to face the first point (-90 70). Moreover, they imposed the schema again on two occasions in the

next two tasks.

- Natassa and Ioanna: Contrasting action - quantity, absolute, and coordinate - based heading changes.

With respect to changing the turtle's heading, the children seemed to initially impose an action - quantity characteristic to the SETH command, in trying to make the turtle face downwards in the second quadrant task (80 0 position and 90 degree heading, see fig. 6.6.2). However, from that point onwards, **dissociating heading change from action - quantity did not seem to be a major difficulty for the children.** It is interesting that they were quite ready to use explicit wording and indicated a conception of heading change determined by an absolute, external direction, e.g.

N: "SET... it will be downwards... 180." (turtle at -90 0, head. 270, point at -90 -40, see fig. 6.6.3)
I: "From this way it's... 0." (turtle at -100 0, head. 270, point at -100 90, fig. 6.6.5)
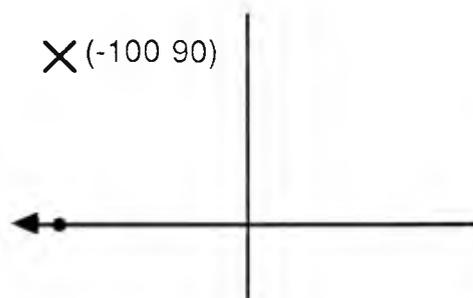
Figure 6.6.5: N. and I. : Discussing how to make
the turtle face upwards

However, it is suggested that Natassa had not thought about relating or contrasting the two ways of changing the heading. In the fifth task of joining up two points and measuring the distance between them, the children were discussing how to change the turtle's heading from a (0 0) / (-90 70) direction (turtle at -90 70) to face the other point (100 -40) (see fig. 6.6.10, section 6.6.2d). Natassa's argument was the following:

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N: "...on her (turtle) 180... her 180 is there...i.e. on her own cross..."

She seemed to confuse three things: Firstly she used the turtle's present heading in a "from here to there" notion, not realising that the only determinant in an absolute heading change is the end direction, i.e. the direction the turtle is going to face.

Secondly, she ignored a very important characteristic of the absolute direction system, i.e. that it is fixed, by imposing the system to match the direction of the turtle.

Thirdly, she ignored completely the difference in the function of the SETH TOWARDS command, i.e. that heading change was now caused by external coordinate locations and not by external directions. This was unexpected since a little previously she had used sophisticated wording to explain this function to Ioanna:

N: "...We'll tell her to turn her head, i.e. to turn towards there so that she's looking... at the point which is 80 80."

6.2.2b) Forming a schema to control the turtle's position in relation to the absolute coordinate location system.

- Anna and Loukia: Dissociating from an action - quantity schema.

For the tasks involving points in the first three quadrants, the children had no serious problems in corresponding intended position changes to the axes and the plus and minus regions, even though (as illustrated in the next section) they did have problems with notions regarding the regions and the axes themselves. This deceiving fluency however, broke down in the fourth task (point at -100 90) where the absence of the axes on the screen led in an "overtaking" of the x coordinate, the turtle taken to -120 instead of -100 (fig.

6.6.6a). **The strength of the action - quantification schema in the children's minds is illustrated by the way they imposed a reverse action notion** (note that SETX -120 was momentarily seen as a FD 120 operation) **and an operation on the quantity of that action** (120 - 100):

L: "Ooh!... it's BK."
A: "20... I think it wants 20. BK, is there a BK?"
L: "Why not?"
(they tried BK 20 and got an error message)
A: "We should turn it first. To go forward 120
to go back turn it and move it 20..."
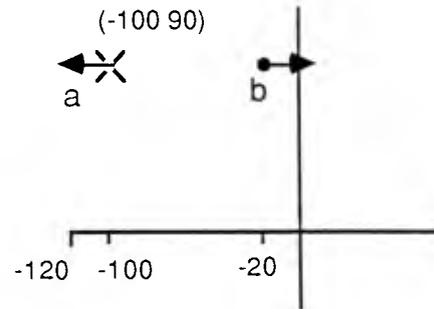
Figure 6.6.6: A. and L. : Changing the turtle's position

They turned the turtle to a 90 degree heading. At that point Loukia demanded a clarification of the plus and minus regions of the axes, which was provided by Anna.

L: "So its -20"

A: "Let's see. (result on screen) -20? God!... (laughter)

Despite their implicit use of the region of the x axis (minus sign), **their striking disregard of the notion of the location determining the change of position was not only illustrated by their surprise, but also by their initial inability to make sense of the result.**

R: (asked where the turtle was on the x axis)

A: "Around the middle."

R: "On which point exactly?"

A: "I don't know."

L: "In the middle."

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The researcher intervened "giving the answer", to find out how and if they would try and make sense of the situation.

R: "What if I said that it's written (on the screen)."

A: "Its on the -20 of the x... I think I've got it, we told it here lift me up and take me to minus 20 and minus 20 in relation to axis x is here (finger on correct spot). Say that axis x is lets say here, -20 is here.

Anna's explanation shows the way she dissociated from the action - quantification schema (lift me up and take me to) and how she saw the number -20 as a location on the x axis. This gave her a new insight into the meaning of number as a name for a place:

A: "We should have taken it... lift me and take me to minus 100. If we wanted to undo the 20... so we have to turn again."

When Loukia was required to explain the same thing, she again wanted to clarify the plus and minus regions on the plane (following section). The explanation came as an outcome of her verbally making sense of the regions, and is indicative of how **she imposed a "distance" notion from the turtle position to the -100 location:**

(turtle in position (b), fig. 6.6.6)
L: "...and from here till there its 100."
A: "No... its 80."
R: "Why?"
A: "Because from the middle its 100 steps, from -20 its 80."

Although Anna had an insight into the relationship between the meaning of numbers as locations and as distances from the origin, **her action - quantification schema remained separate from the idea of locations and position changes.**

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R: "Ok, but she told her lift me up and take me... -100, and she moves 80? How do you explain that?"

A: "That she didn't move 80 she went to the position... -100."

This distinction in her mind, led to a confusion between amount of turtle move and region of the axis.

R: "Ah. And how much did she move?"

A: "80... minus 80.

R: "Ok. Minus 80?"

A: "Since it's on the side of the minus?"

Anna seemed to "extend" the application of the meaning she had given to the minus sign in the coordinate system, onto the notion of distance. It is not clear, however, whether she thought that distances are "negative" when they are in a "negative" region of the plane, or whether she simply assigned the minus sign on the number 80.

- Maria and Korina: Dissociating from a "relative distance", sequentiality notion.

In attempting the task with the invisible axes (point at -100 90), the children took the turtle to a (0 100) point. Their discussion on how to take the turtle to (0 90) (fig. 6.6.7), illustrates how **the children seemed to impose a distance notion on changing the position of the turtle:**

M: "No, it's too much."
K: "Yes... a bit less."
M: "Em... minus 10. Minus 20,
therefore 80."
K: "Yes, I said 80 at the
beginning too."
M: "O.K., minus 20 then."

✕

(-100 90)
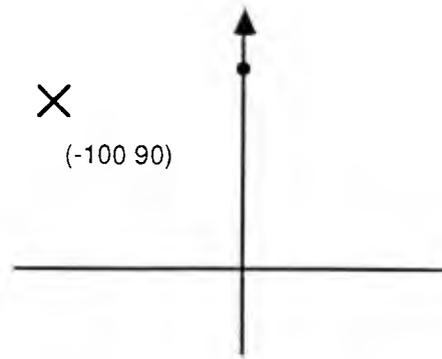
<u>Figure 6.6.7: M. and K. : Changing the turtle's position</u>

The children seemed to be talking about the turtle steps from the 100 to the 80 point, i.e. the distance from the present position, to the position of change. **They also seemed to impose a "reverse action" notion, of "undoing" an apparent forward 100 action by subtracting the distance.**

The strength of this "relative distance" (as opposed to distance from the origin) schema is illustrated by the children's persistance to employ it in their subsequent activities: at first they typed in -20, forgetting about the SETY command. After discussing the error message from the SETY -20 command which led to a turning of the turtle to face downwards, and although Maria had had an insight into the notion of the end direction being the important factor in changing the heading (see above), she did not carry that notion to the change of the turtle's position from (0 100) to (0 90). Focusing on the distance from 100 to 90, she typed in SETY 10, and after the result on the screen, SETY -10, thinking she had failed to include a "reverse action" element.

The children turned the turtle to face upwards again and then took it to (0 80), saying forward 80 and typing SETY 80. **Only then, did one of them (Korina) show some indication of dissociating from the relative distance notion,** expressing an opposition to a proposed SETY 10 command (fig. 6.6.8):

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M: "Now. SET... Y... 10."
K: "10? I say, let's do... 90."

$\times$

(-100 90)
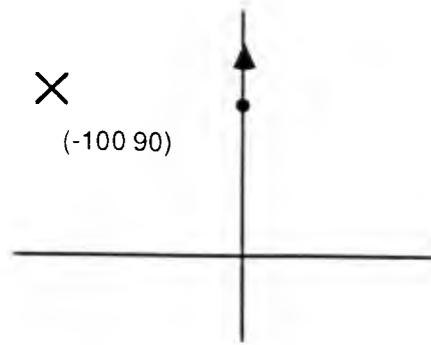
Figure 6.6.8: M. and K. : The meaning of SETY

However, the children did not explicitly use the notion of position change caused by giving the end position as an input, in any of the subsequent tasks in this session.

- Natassa and Ioanna: A "distance from the origin" and a "name for a place" notion of a coordinate value.

Concerning the change of position, the children met the first difficulties in trying to move the turtle from a -100 0 to a -110 0 position in order to decide on the x coordinate of the (-100 90) point in the fourth task (the axes were invisible, fig. 6.6.9). In their effort to explain why their first attempt (SETX -10) did not work while their second (SETX -110) did, **the children constructed a "theory" for the meaning of the number of the x value.**

I: "...we did it again from 0 till 110
and it came out."
N: "...we can't do 10 because we've
done 100 already. Plus 10 we want
to do... 110."
I: "She doesn't go... because we've
past 10."

(-100 90)

$\times$
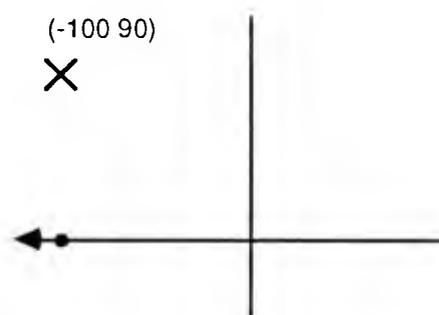
Figure 6.6.9: N. and I. : Changing the turtle's position

Ioanna seemed to suggest two ways of interpreting the meaning of the x

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value: **firstly, the value represents the distance from the origin, and therefore the SETX command operates in such distances, and secondly it represents a name for a place** (*"...we've past* - the place - *10."*). Natassa seemed to take on board the distance from the origin theory. Notice how she used a specific way to talk about a number when it represented an x value (by using the word "do" in front of such numbers), and seemed to implicitly contrast it to the normal meaning of number ("...plus 10, we want to do...110).

6.62c) Making sense of the coordinate system.

At the beginning of the activities, the children were introduced to the "mechanics" of the coordinate system, i.e. the method for giving names to locations, the names of the axes, the order of the coordinates, the calibration units and the plus and minus regions on the axes. This section describes the development of the way they used these "mechanics" in situations of naming or referring to locations.

Not surprisingly, **the children did not seem to have substantial conceptual problems in understanding the above coordinate notions.** However, an issue that emerged from the data was the lack of awareness of the importance of arbitrary "rules", such as the order of the coordinate names, and the plus and minus regions of the axes and of the plane. Regarding this issue, the Logo error messages seemed to have had some effect in increasing the children's awareness of certain arbitrary rules.

- The order of the coordinate names

For the children, **giving a name to a location by stating the x coordinate first, seemed to be perceived as a given, arbitrary rule.** The first confusion arose when they wanted to print the label of the first point (60 50) on the screen with the WRITE command. Although they distinguished between the function of the WRITE command as that of

labelling and the inputs as the name of the location, **Loukia did not immediately perceive that the order of the inputs was important.** The case was not so for Anna however, despite the fact that she got the order wrong and, therefore, an "I'M NOT THERE" Logo message.

A: "Yes, the WRITE and the name we'll name it."
L: "Yes. 60 50 or 50 60. (types)
A: "50 60 because its 50 first and then 60."

From then on they had no problems getting the order right, making it verbally explicit for the next two tasks, and using it implicitly after that. They also did not seem to have a problem assigning the right name to the right axis.

- Plus and minus regions of axes

The children's initially frequent problems regarding this issue were not really due to forgetting which axis region is which, but rather on not using the minus sign when needed for the name of an axis location, in the context of naming a location to change the turtle's state. However, the Logo error messages pertaining to state changes and labelling, gradually drew the children's attention to the importance of the sign being correct.

The children's initial strategy was often that of counting in tens till they got to the respective location. The minus sign was therefore forgotten.

L (point 80 -60): "Lets count first, one, two..... seven eight... eighty.
A: "Eighty there.
L (on y axis): "One two three... sixty. (they both check)
A: "So, 80 60 we'll name it

However, in the context of giving a name to an axis location, the meaning of the minus sign did, for them, seem to indicate an axis region.

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A: "Why -80, -80 is this way, this way its plus 80." (on 80 -60)

<u>- Plus and minus regions of the plane</u>

Although the issue of the regions of a single axis did not seem to be much of a conceptual problem for the children, the case was not so when the understanding of a combination of the regions of the two axes was required, to make sense of the regions of the plane. Although they made several attempts to explain to each other the regions of "both axes at once", **their verbalisations were confined to one axis at a time rather than a quadrant as region.** For example:

A: "When its this way minus, when its that way plus, when its this way minus when its that way plus."
L: "From this side, from the right of Y its 20 30 40 50... and from the left of Y its minus, on X."

<u>6.6.2d) A clash between the action - quantification and the coordinate schema.</u>

In the fifth task, to join up and measure the distance between (-90 70) and (100 -40), **Anna's intrinsic schema seemed to come into conflict with her newly acquired schema for non - intrinsic controlling of the turtle.**

The children took the turtle to the first point, turned it to face the second and used the PRINT DISTANCE command spontaneously, linking it to the notion of measurement (fig. 6.6.10). Their experience of giving names to a location to carry out the measurement was employed in this context quite naturally, despite the fact that the name here was two numbers instead of a letter.

A: (types PR DISTANCE) " What name
shall we put..."
L: "100 -40"
A: "Ah yes 100 and -40."

(-90 70)

(100 -40)
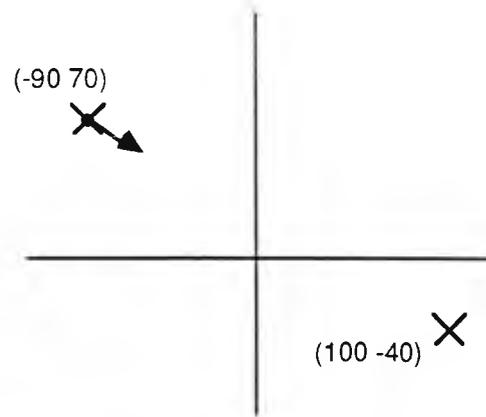
Figure 6.6.10: A. and L. : Discussing the input to SETPOS

The meaning of the numerical output (that of turtle steps) seemed to result in a shift in the children's minds regarding the method by which they subsequently attempted to change the turtle's position (i.e. take it to 100 -40): **they seemed to ignore the location - determined characteristic of the SETPOS command and impose an action - quantity meaning, by spontaneously typing SETPOS 219.545.** The error message encouraged a first verbal attempt to combine the two notions:

A: "To go... to... point 100 -40... to move 219... these steps, going to the point 100 -40. Thats what it means."

However, Anna did not initially realise that only one notion was required for that change of state, and tried to use both in one command typing SETPOS 100 -40 219.545 ("and put her on the point 100 -40 moving 219.545"). Her understanding was prompted by the very fact that she did not, in effect, have a choice, since the turtle action commands were inoperative in this session:

R: "Does this turtle know how to move?"
A: "No..."
L: "No she doesn't. She knows how to go from one point to another."
A: "We found the distance, we don't have to... use it."

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6.6.3 Summary and discussion of the
"coordinate turtle control" activities.

The findings from the three pairs of children in this session seem to fall into two categories. Firstly, the children's developing use of the "mechanics" of the coordinate system is described. Despite the relatively short time they had available to them, they began to show an increase in awareness of the importance of certain "rules" which, to them, were arbitrary, in the sense that they did not seem to have a meaning (i.e. the order of the coordinate values, or the plus and minus axis regions). It is believed that the Logo error messages played a role in this development. Not surprisingly, the children also found difficulties in their attempts to incorporate all the coordinate "rules", in cases where they had to use them to change the state of the turtle. However, what seems interesting is an indication of a developing "breakdown" of the children's accustomed attempts to give quick, factual answers (see preliminary study), in favour of a more analytical approach which methodically took one coordinate rule into account at a time.

The second category of issues in this session refers to the conflict arising from the children's attempts to understand the notions involved in changing the state of the coordinate - controlled turtle. Although the function of the coordinate state change commands had been explained to them at the start of the sessions, the children initially seemed to employ concepts based on their previous turtle geometry experiences. However, in their attempts to control the turtle, they seemed to dissociate from their intrinsic schema and develop new schemas for heading and position changes. This development was not, of course, uniform across pairs or individuals. The children seemed to have "insights" into parts of the coordinate method at various times during the activities but, not surprisingly considering the time and the complexity of the task, no single child seemed to explicitly synthesise the notions into a concise method of state change.

Nevertheless, a model of a "coordinate schema" is proposed, which synthesises the children's insights into the notions involved in the coordinate controlling of the turtle (fig. 6.6.11). The model consists of heading change and position change schemas, which the children seemed to be in the process of building as a result of dissociating from intrinsic notions:

Heading change:

a) It is necessarily and sufficiently determined by the end heading. A dissociation from notions of sequentiality seemed to be involved.

b) The end heading can be described by means of an absolute direction system. A dissociation from action - quantity notions seemed to be involved here.

c) The end heading can be described as the direction towards a location of the coordinate plane. The emerging insights into such changes (the children had more experience on this issue in the third category activities) seemed to involve dissociating from action - quantity notions and relating directions to positions on the plane.

Position change:

a) It is necessarily and sufficiently determined by the end position. A dissociation from notions of sequentiality seemed to be involved.

b) A numerical coordinate value has a meaning of either a name of a place on an axis or the distance from the origin. This involved dissociating from an action - quantity notion and from a "relative distance" notion.

c) The sign in front of a numerical coordinate value has a meaning of a region of an axis. This involved dissociating from a "relative distance" notion.
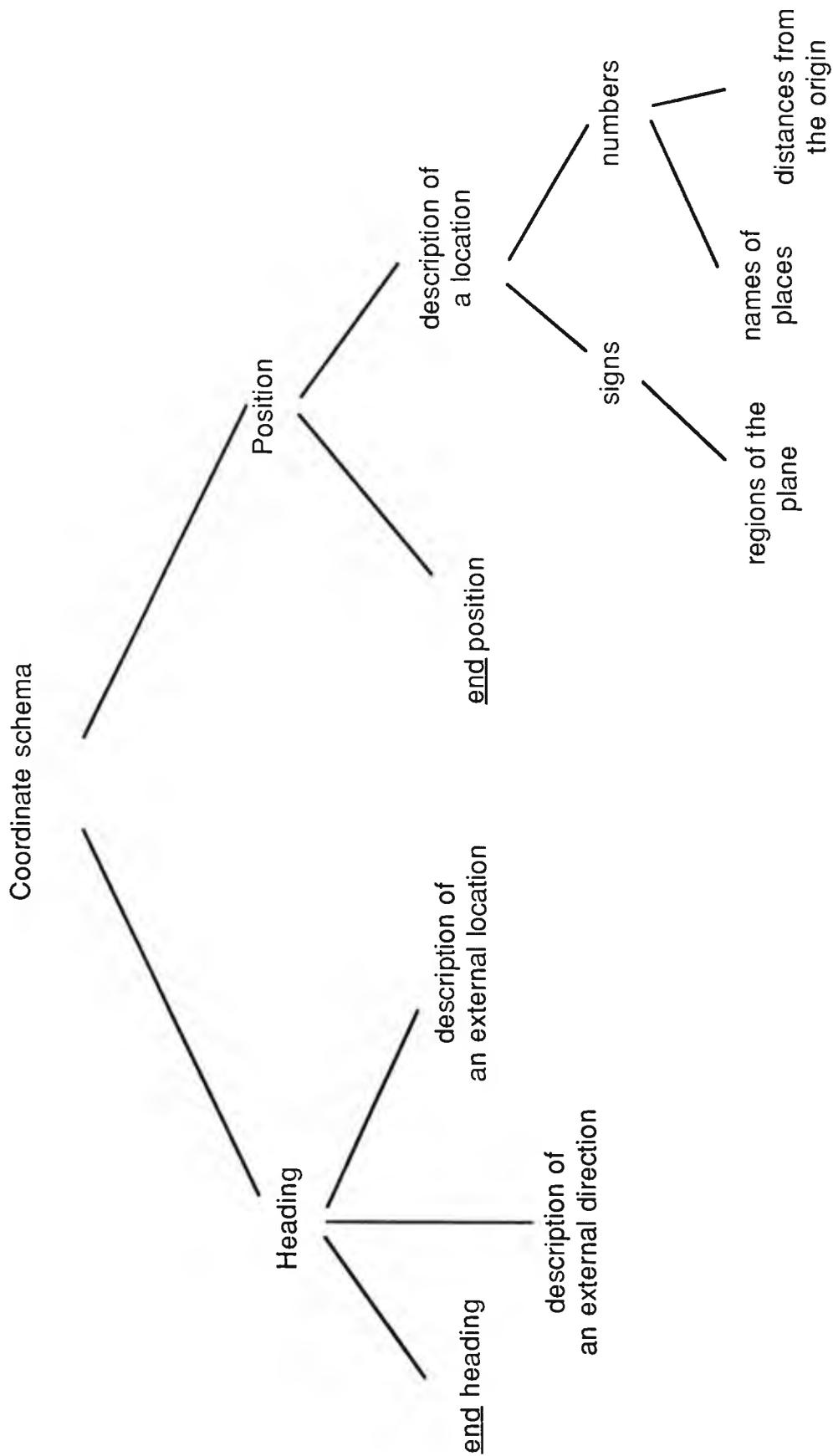
Figure 6. 6. 11: A diagram of the children's "coordinate schema".

Although the development of these schemas could be described as temporally parallel, the children did not seem to relate notions regarding heading and position changes. Moreover, although they did seem to have insights into all the described coordinate issues concerning state change, the children varied in the extent to which they used the new notions in subsequent situations in this category of tasks, often showing a tendency to re - employ their intrinsic schema. It is true that the children in the study had had considerable experience with turtle geometry (50 - 60 hours) and therefore were inclined to employ intrinsic notions to control the turtle. However, the coordinate method of controlling the turtle had been explicitly explained to them at the start of the session and the turle itself was introduced as a "different being" to the Logo turtle (the words used were "the turtle's sister). They were also given time to try the new commands out before attempting the tasks. This finding corroborates Lawler's research which demonstrates the strength of the intrinsic schema in the attempts of a six - year old child, experienced with the Logo turtle, to understand simple coordinate notions. Finally, the process of dissociating from the intrinsic schema and developing another, seemed to throw light on specific notions the children had built for controlling the turtle during their 15 - month - long experience with turtle geometry, thus clarifying components of the intrinsic schema itself. The following model of these components is proposed (fig 6.6.12):

a) an action - quantity notion, which involved a "turn degrees" notion for heading changes and a "move steps" notion for position changes, and

b) a sequentiality notion, which involved a notion of "one change after another" and "a change depends on the immediately previous state".

There is evidence of all six children readily using action - quantity and sequentiality notions during these activities. This, however, does not imply that they did not have difficulties with these notions, such as problems with discriminating between the two states and between an action and its quantity, or with concentrating on one state change at a time.
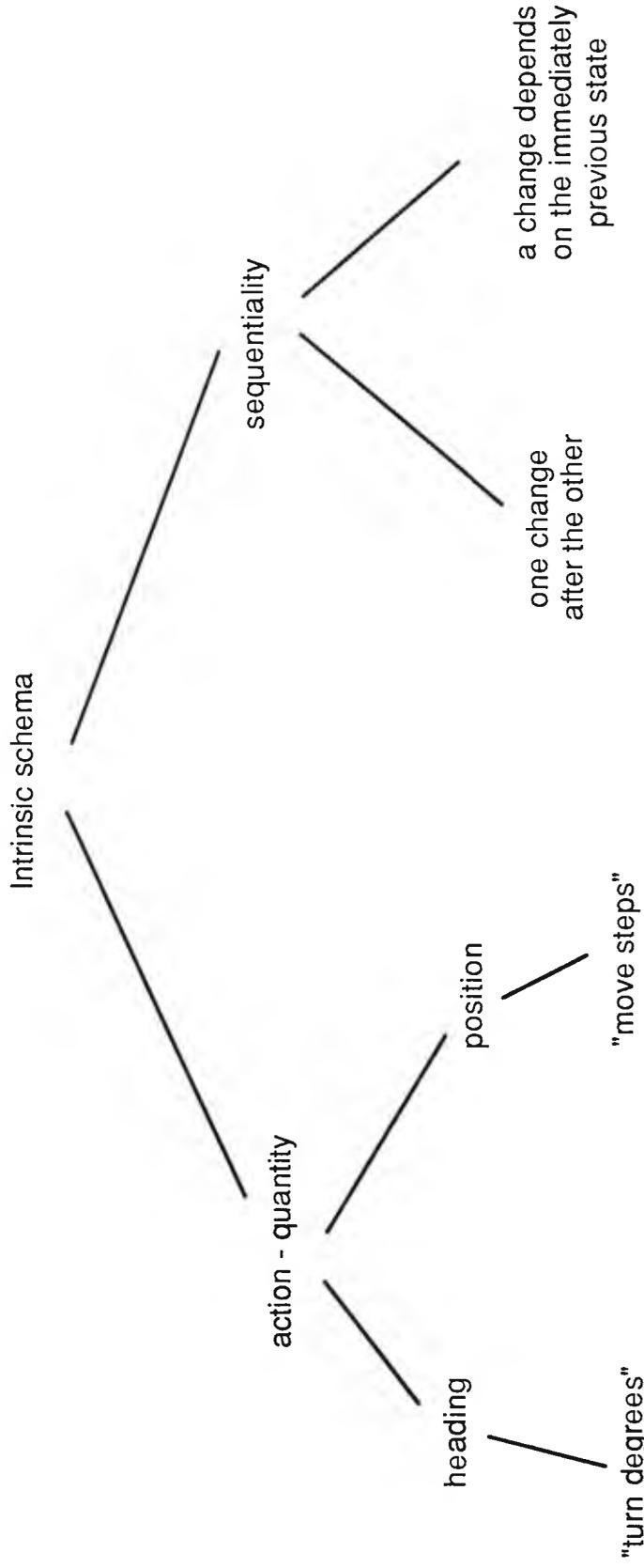
Figure 6.6.12: A diagram of the children's "intrinsic schema"

6.7.1 Task Analysis

These activities involved making decisions on the method of controlling the turtle, and on how and when to use coordinate notions in order to drive the turtle on the coordinate plane and make linear and angular measurements. Initialising the program had the same screen effect as in the category 2 activities, only this time with three points shown on the screen (fig. 6.7.1). The children could use all the commands of the T.C.P. microworld, which were given to them on paper, i.e. FD, BK, RT, LT "quantity", DISTANCE, DIRECTION "name", SETX, SETY, SETH "value", SETPOS, SETH TOWARDS "name", and PU, PD, WRITE "name".
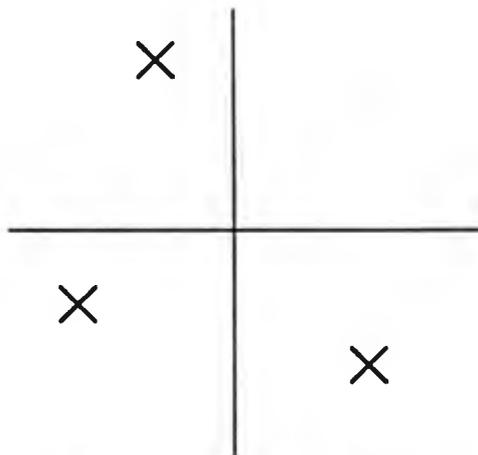
Figure 6.7.1 Initialising the task

The aim of this activity was to investigate the children's strategies in solving a problem which required an interplay between causing actions and collecting information which depended on the turtle's state. The investigation concentrated on two types of notions:

a) on the interplay between intrinsic and cartesian notions used to perform actions and collect information (see conceptual field analysis, section 6.1.3), and

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b) on the interplay between static and dynamic interpretations of lengths and angles.

The researcher made sure the children remembered all the commands, and allowed time for them to try out ones that they were not clear about, before initialising the program and asking them to join the points with lines, label them, and find the lengths of the sides and the sizes of the angles of the formed triangle.

6.7.2 Findings

6.7.2a) The children's strategies for solving the task.

Anna and Loukia's plan, after having been introduced to the T.C.P. microworld and the triangle task, revealed a substantial degree of separation between functions whose primary characteristic was that of an action - i.e. move or turn the turtle, label a point, make a line - and that of a collection of information (lengths and angles). Their actual strategy was even more clear - cut: the first part of their activities consisted of a consistent anticlockwise "labelling and drawing a line" sequence (starting from -20 90, fig. 6.7.2), and the second part consisted of a sequence of an angle measurement, a line measurement and a move to the next (clockwise) point. They measured the angles by giving the turtle "suitable" headings and using the turtle protractor in information mode for the measurement.

Maria and Korina adopted a strategy of "drawing" the triangle first, without labelling the points, by using the measuring instruments in information mode and the outcomes as quantities for the action commands. They then decided to label the points, and did so by raising the pen and taking the turtle around the triangle again by directly using the previous measurement information in a straight forward action - quantity method. Only after the labelling, when they decided to measure the lengths and angles did they explicitly realise that the

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length information was already collected. To measure the angles, they used the turtle's protractor again in information mode, after giving the turtle a "suitable" heading to perform the measurement.

Natassa and Ioanna seemed to prefer to collect all the information they could while they were constructing the triangle in a clockwise direction, also starting from (-20 90) (fig. 6.7.2). For instance, at (70 -70), they labelled the point, measured the lengths of (-20 90) (70 -70) and (-80 -40) (70 -70) and then the angle. They used predominantly the SET commands to change the turtle's heading or position and the measuring instruments mainly to collect the length and angle information.
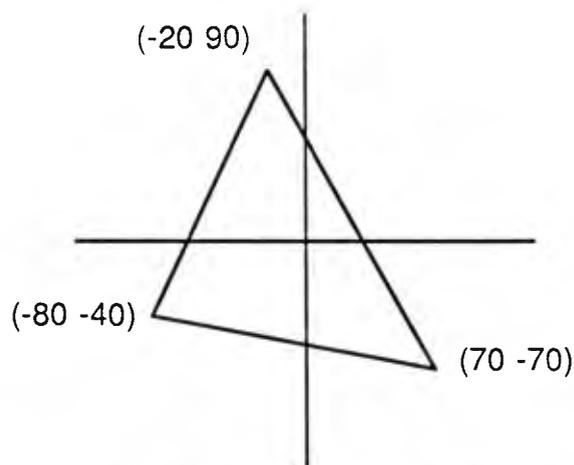
Figure 6.7.2 Drawing and labelling the triangle

6.7.2b) The arbitrariness of coordinate symbols.

The following episode illustrates how **Anna and Loukia at the beginning of the task, perceived each calibrating line as denoting one unit instead of ten.** During their attempt to take the turtle to the first point (-20 90), Loukia counted on the screen to find the coordinate values. Her counting the number of lines, rather than the actual units (of ten steps), illustrates how the notion which was represented by the line calibration (ten units per line), seemed to give way to a more "realistic", concrete meaning of "number of lines".

What is interesting, is **the resistance both children showed in realising the change they had imposed on the meaning of the calibration line symbol.** They both carried on, typing in the command with -2 9 for the coordinate values (RT DIRECTION -2 9), but although Loukia realised, in a perceptual way (Hillel et al, 1986), that the turtle did not have the correct heading, they could not see where they had gone wrong. The researcher's intervention illustrates how the problem seemed that of a change of meaning to a symbol, rather than the children forgetting the size of the calibrating unit.

R: "How much is each little line?"
A: "Oh no..."
L: "Oh, yes."
A: "So, -20 to begin with."

It could be argued that the factor for the children's "error", was the arbitrary nature of the ten - step units, i.e. that in an implicit way, there was no specific reason for the children, why the distances along the axes should be counted in tens and not, which for them was more logical, in units.

6.7.2c) Sequentiality

After typing in RT DIRECTION -2 9 (turtle at 0 0), Anna and Loukia's consequent effort to get the heading right (i.e. towards -20 90), illustrates **their confusion regarding the notion of a turtle turn being determined by an external location, rather than the quantity of a turtle action.** Loukia regarded the problem of the quantity of the next turn as insoluble, since the turn would be dependent on the "unknown" heading of -2 9, and therefore suggested that they take the turtle to a "known" heading (0) first. Her use of the notion of the absolute heading was therefore in the context of overcoming uncertainty due to relative turtle headings. This proved to be a developing strategy of "when the turtle heading is unknown, take it to

an upright position", i.e. use the absolute system directly, and measure angles from there: Loukia suggested it again later, as a strategy for measuring the internal angle at -20 90. However, in the previous problem of changing the heading from -2 9 to -20 90, Anna challenged Loukia's strategy by saying that they should do it directly, without the "intermediary" 0 heading:

L: "Yes, but now?"

A: "Eh, never mind she'll find the direction." (the turtle)

L: "But she's already turned."

A: "So what?"

L: "She will turn more."

A: "She'll find the direction..."

Anna seemed to have realised that the point -20 90 would determine the turtle's turn, i.e. that the turn was independent of the previous state of the turtle. However, Loukia's sequentiality schema seemed resistant to change, since she seemed to impose an action characteristic on changing the heading *("...but she's already turned...")* and a dependency on the previous heading of the turtle *("...she will turn more...").*

6.7.2d) The intrinsic and the coordinate "notional fragments".

The children's strategies for changing the state of the turtle in order to measure distances and angles, seemed to reveal the process by which they formed a non - intrinsic schema in situations which could not be resolved by the action - quantity method. There are also indications of their subsequent implicit or explicit decisions on which schema to use in particular cases as they progressed with the task, and of the problems they met in employing either schema in situations which needed the combined use of both. Episodes from the activities of all three pairs are used to illustrate the issues in this section.

Anna and Loukia completed the "drawing" of the triangle, labelling a point and then using the combined action - information method to turn the turtle and move it to the next vertex. **Their thinking schema seemed predominantly to be that of turtle action and quantity of that action. However, they did use the notions of measurement and naming of locations as a method to determine the quantity of the turtle action.** Anna's fortunate "thinking aloud" illustrates this point:

A (turtle at -20 90, fig. 6.7.2): *"RT how will we know how much... oh, yes, RT...* (counts on the screen to find the coordinates) *RT DIRECTION.... space... 80...* (result on screen) *Now FD... FD DISTANCE..."*

The children decided to start measuring the triangle elements when they had completed the "drawing" of the triangle, i.e. the turtle was at position -20 90, heading from 70 -70 to -20 90, (fig. 6.7.3a). The way Anna explained how she would measure the -20 90 angle reveals **a drastic shift from her so far predominant schema of turtle action and its quantity, to a new method of changing the turtle's state.**

A: "Yes I've got it. Towards where is it looking? Towards there. We can measure... (pause) *make it look firstly at 80 -40... -80 -40, and if it's looking this way it will be able to, from here, see how much it should be to look towards here."*
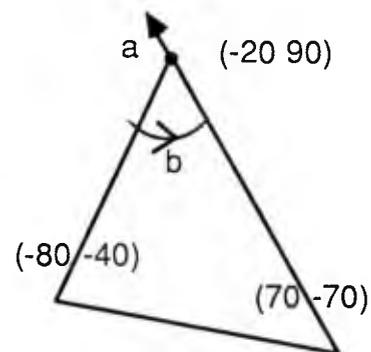
Figure 6.7.3 Anna's "Cartesian view" of the problem

The nature of Anna's new schema for the turtle, firstly involved an identification of the turtle heading via some external direction *("...towards where is it looking? Towards there...").* Secondly, it involved a conception of heading changes relying on descriptions of the locations defining the end turtle headings *("...make it look firstly at 80 -40..."),* restricting the use of the

"from here to there" notion to convey the meaning of measurement. Thirdly, however, Anna's new method did incorporate the notion of the turtle state, and the idea of performing a change of that state so that the measurement would be feasible.

Nevertheless, Anna's strategy did require an intrinsic notion underlying the measurement: the turtle's heading has to be such that the angle to be measured is on the right of the turtle, i.e. the information concerns a potential right turn. To incorporate this notion in her non - intrinsic schema proved to be a difficult task. It is proposed that the fact that she made a mistake (from -80 -40 to 70 -70 is left) **was only the result of her temporary confinement to a notional "fragment" built to understand non - intrinsic concepts.** Notice how she changed level of precision when she had to use the "right turn" concept, i.e. she said *"...see how much it should be to look towards..."* instead of e.g. "see how many degrees it is to turn right to look towards...". Moreover, when she was required to show which angle she was talking about, she moved her finger from one side of the triangle to the other (fig. 6.7.3b) instead of rotating it on the same spot as she had done several times before to show a turtle turn.

The degree of disparity of these two notional fragments is illustrated by Anna's consequent efforts to explain her strategy to Loukia: she expressed the whole process four times without realising that the measurement was towards the left of the turtle. In the end, a researcher's question invoked her intrinsic notional fragment. The purpose was to ensure that she hadn't confused left and right as such, due to the turtle heading downwards, but that she had just not thought of it. Her answer not only clarified this issue, but also resulted in her understanding of her mistake:

R: *"Which way do we turn to do that?"*
A: *"Towards the... left, so it can't be done."*

This realisation invoked a new strategy which was expressed by Anna who

seemed to make an effort to combine the two fragments, in order to provide the correct solution:

A: "Yes I got it. Instead of taking it from here
(she means towards the left) we'll turn her
this way to look at 70 -70 but without measuring
it. And then we'll tell it to turn, to look there.   (-80 -40)"

(-20 90)

(-80 -40)

(70 -70)
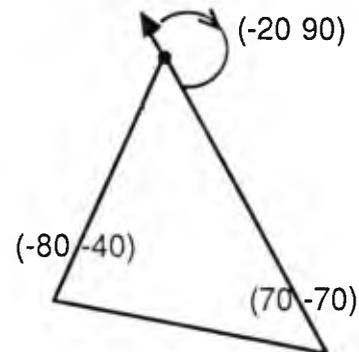
Figure 6.7.4 Anna's "Intrinsic view" of the problem

This combination is apparent in Anna's carefully chosen words; to change the turtle's heading, she uses an action notion ("...we'll turn her this way...") but not a quantity one: the action is determined only by an external location on the coordinate plane ("...to look at 70 -70...").

Notice, however, how fragile this combination was, when in her last phrase, she imposed a turtle action trying to convey the meaning of a simple measurement ("...we'll tell it to turn..."). Moreover, this episode seemed to "bring on" her intrinsic "notional fragment" in deciding firstly to adopt a "circular" strategy for measuring the other angles and secondly in combining the right turn property of the "protractor" with a clockwise (right turn) order for the measurments:

A: "No, from here to there because then we'll have to do... look if we come there we'll have to do LT to get there, while this way we'll do RT and RT and RT. Ok, so now I'm finding the distance till 70 -70. From the oposite side the distance from -20 90 till 70 -70."

In the latter part of the above extract, the first phrase Anna used for length measurement is notionally consistent with her previous argument: it takes for granted that the starting point of the measurement is the turtle, i.e. measure

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from the turtle to an external location *("...so now I'm finding the distance till 70 -70...").* This "turtle centred" measurement, has the intrinsic characteristic that it does not convey information about the location of the turtle's position. **Anna's second phrase shows a spontaneous shift from her intrinsic to her non - intrinsic fragment: she abandons the turtle concept and talks about the distance between two plane locations.** Nevertheless, a "directional" characteristic (from... to) remained and was consistent with the turtle measurement direction.

The children's next decision, however, seems to indicate a shift in the meaning of the distance output. To actually move the turtle, they typed FD 183.569, using the result of a measurement for an action - quantity operation for the first time. The interest in this activity lies in the fact that this was the very first opportunity to use such an operation - they wanted an action from the turtle and they knew the quantity - and the children took it without hesitation. They then did the same thing for the turn, measured first and then turned the turtle: PR DIRECTION [-80 -40] (result 130.665) RT 130.665.

Maria and Korina seemed to keep their action quantity schema separate, in their minds, from the coordinate notions. The emergence of their adopted strategy of making measurements and using their outcome in action quantity commands can be clearly illustrated by the following extract:

(turtle at -20 90, heading towards 70 -70, fig. 6.7.5)

K: "FD..."
M: "No, we want her to give us the distance.
PR   DISTANCE...   (she types that)
right (means O.K.). *To the point 70 and -70*
(Korina types that, they press RETURN
and write the result on paper) *Now... FD... 183
and 256.* (result on screen) *Ah, good."*
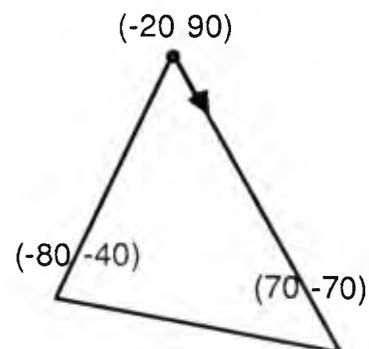
Figure 6.7.5 M. and K.: Measuring how much to move the turtle

It is suggested that Maria's objection to Korina's apparent intention to perform an action straight away, was due to her strategy to perform a measurement in order to find the quantity. It seems that for Maria the method for collecting the distance information was distinct from the turtle's action (turn) and the meaning of the output was the distance from where the turtle was to a certain location. **However, it is proposed that her reason for carrying out the measurement was to find the quantity of a turtle action** in spite of the fact that part of the task required length measurements.

It is also seems likely that she used coordinate notions as part of the measurement, i.e. as separate from the turtle's action. Korina's subsequent strategy for turning the turtle seems to support this argument.

K: "Now we have to turn it... PRINT DIRECTION towards the point (they count on screen)...*-80 -40."* (result on screen (1), they write it on paper)) *M: "Right* (she means O.K.)...,*RT..."* *K: "Yes RT that's right, RT how much do we want to take her...* (they type the degrees, result on screen (2)) *Good."*
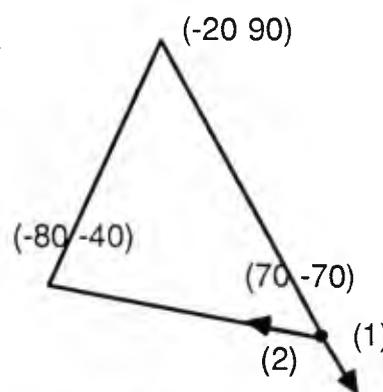
Figure 6.7.6 M. amd K.: Measuring how much to turn the turtle

Notice how she seems to express a global purpose *("...now we have to turn it...")* and then changes to the process of achieving it, which is aparently split into two parts: first carry out the measurement, write down the result and then use it for the quantity of the action. It is suggested that the coherent way in which the children used English for the commands, and continued the same phrase in their own language, is indicative of their understanding of the meaning of the commands.

Natassa and Ioanna seemed to use a non - intrinsic schema for changing the turtle's state. The schema appeared to incorporate the

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notion of action, but not to emphasise it; **the children seemed to often use action words (e.g. turn, move) to express which state they would change, rather than the method by which they would make the change.** This is illustrated by the following extract in which Natassa declared which state she would change first, using the word "turn", and then gave a very different description of heading change:

(turtle at HOME position, aim to take it to -20 90)
N: "Ok. First I'll turn and put the nose of the turtle to look at this point here, on the top... top left. SETHEADING TOWARDS SE... (she types) *em... -20...* (she types -20 90). *Now let's take it up there. With SETPOS."*

Notice how her explanation seems to be dissociated from a "from here to there" notion *("...put the nose of the turtle to look at...")* and to regard a specific location as the only determinant of direction change. It is interesting how she expressed this idea first, i.e. that a location would make the turtle change its heading, then started to type the commands in, and only in the end started to count on the screen to find the coordinate values of the location. **This suggests that Natassa's insight was primarily on the method of state change via a location on the plane.** The argument is supported by the reason Natassa gave for using the protractor in her first decision to do so:

(turtle at -20 90, heading from 0 0 to -20 90)
I: "SETH... how much... 20..."
N: "...protractor to see how much it will turn to go there... Now, RT."

The meaning she gave to the measurement was that it provides the quantity for a turtle action. It is suggested that Natassa may have implicitly compared the two methods and wanted to use one which would reveal the quantity of an action.

However, it is questionable to what extent she was aware of the differences

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between the two methods. For instance, having "forgotten" to measure the (-20 90) (70 -70) distance and having typed SETPOS 70 -70, she imposed a "reverse action" schema to take the turtle back to -20 90 to do the measurement:

(turtle at 70 -70, fig. 6.7.7)

N: "We'll go back... she knows the command BK.
We'll tell her BK 70 -70."
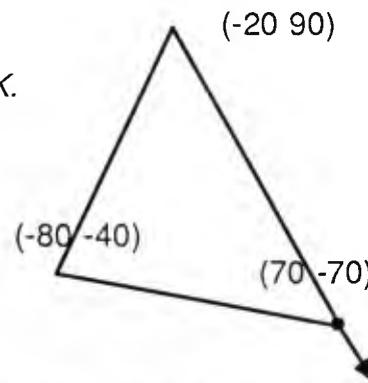
Figure 6.7.7 Natassa's confusion on the meaning of SETPOS

It seems that Natassa confused the meaning of the SETPOS command and the state change method underlying it, and imposed forward - quantity notion on it. That would explain why she used the BK command, i.e. to counter the forward action, and why she put 70 -70 as an input, i.e. to make the turtle go back the same quantity as it had come forward. **The fact that Natassa did not seem to see anything strange in this plan of hers supports the view that the action quantity schema was very easily accessible in her mind.**

6.7.2e) Schemas employed for dynamic and static notions of angle.

The first part of this section illustrates Anna's development of dissociating the notion of an angular quantity from its role as the quantity of a turn. After having measured the first angle (at point -20 90), the children took the turtle to the (70 -70) point (fig. 6.7.8a), typed PR DIRECTION [-80 -40] and then RT 130.665. Their reason for the measurement was to obtain information about the angle, since they had finished with drawing the triangle and labelling its vertices. Anna's comment indicates that

she associated the measurement with the turtle turn.

A: "We'll find the degrees from here till here.
It will do RT."
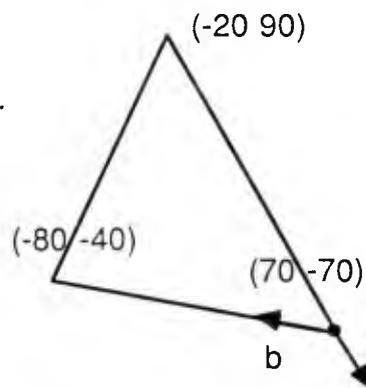
Figure 6.7.8 A. and L.: Measuring an angle

Although she realised the meaning of the result, ("*The degrees it has to turn. Its the degrees of... which are from 70 -70 till -80 -40, the point 70 -70 till the point -80 -40.*"), **the employment of her "intrinsic fragment" did not allow her to discriminate between the external and the internal angle and she carried on as if the matter was solved.** When the researcher called upon her non - intrinsic fragment, by asking if the result was the size of the "inside" angle, Anna's attention seemed to immediately "jump" to an angle notion which was not involved in a turning action.

Initially, her experience with the role of measurements as quantities for potential turns seemed to lead to an incorrect strategy, (similar to that of point -20 90) i.e. one which did not take the intrinsic "right of the turtle" property of the protractor into account.

(turtle at 70 -70, heading towards -80 -40, fig. 6.7.8b)
A: "*Ah!... does she know LT? She does.*
R: "*Why do you ask this?*"
A: "*To do it in a different way to turn it from here to turn it to look at -20 90, then measure it. But it's LT, so it can't be done...*"

The difficulty she had had with the same problem (in measuring the angle at -20 90), i.e. to combine plane locations with the turtle turn, was drastically

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reduced. **It was the first time Anna seemed to verbally express the use of a measurement independent of action. It is suggested that she employed her intrinsic fragment, first realising the bug in her strategy, and then how to correct it.**

L: "So we take it to zero."

A (heading: towards -80 -40, fig. 6.7.8b): "Like it's now... we'll ask it how much to look at -20 90."

Loukia's suggested solution indicated that she hadn't changed her strategy of "when in doubt take the turtle to an absolute "known" heading".

However, Anna's realisation that, in contrast to what she first thought, there was no need for an action (change of state) for this measurement, brought about a new stage in her dissociating action from measurement, as her next activity illustrated:

A: "Oh, yes, why turn? I can count the distance now."

Her strategy for measuring the third angle is split in two parts, the first involving a change of state and the second a measurement. As the following episode illustrates, **Anna used an action schema for the former, implying that the quantity would be determined by a plane location (-20 90).** The command that she typed was RT DIRECTION [-20 90], illustrating how she verbally expressed DIRECTION [-20 90] as "this way" (fig. 6.7.9).

A: "We'll turn this way... (she means towards -20 90) and then from here we'll ask it... we'll say PR DIRECTION... 70 -70 and she'll tell us."
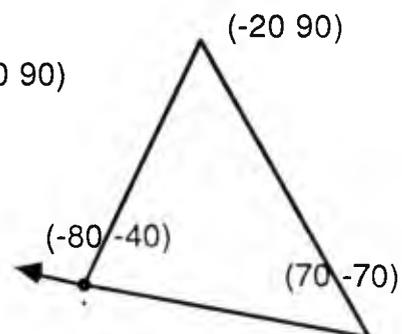
Figure 6.7.9 Anna's strategy for measuring an angle

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The latter part of the strategy, was expressed as a non - action measurement *("we'll ask it... and she'll tell us").* The difficulty Anna had with explicitly expressing what involved a combination of her intrinsic and non - intrinsic notional fragments - although implicitly she used it successfuly - is illustrated by her sudden switch to the summative use of the Logo code *("...we'll ask it...* (she doesn't continue her phrase, but changes to Logo commands) *we'll say PR DIRECTION... 70 -70...").*

Maria and Korina did not seem to talk about angle measurements independently of turns, even though their emerging strategy for measuring the angles was quite sophisticated. Korina's first effort at forming such a strategy suggests that she had an insight into thinking about the angle as a quantifiable entity *("...to find the angles, i.e. what they are..."),* but for her, that entity was still the quantity of a turn:

K: "...and to find the angles, i.e. what they are, i.e. if the turtle was here how much she would turn to go to the other vertex."

Maria's correct strategy for measuring their first angle (-80 -40, turtle facing -20 90) indicates an even closer connection between turn and angular quantity in her mind (fig. 6.7.10).

M: "We'll turn her to find how much it is. And like we're turning her she will give us the angle."

(-20 90)

(-80 -40)

(70 -70)
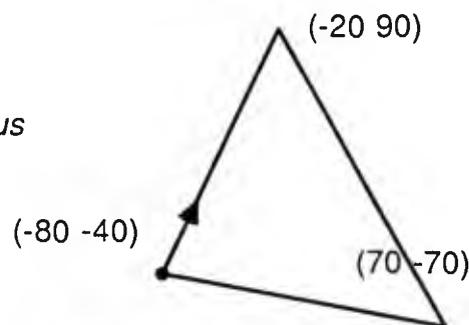
Figure 6.7.10 Maria's stategy for measuring an angle

She seemed to consider the measurement as part of the action *("...turn her to find how much it is...").*

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Natassa and Ioanna adopted a strategy of collecting linear and angular information as they reached each point, starting from (70 -70). However, Natassa had the impression that the turtle's turn at (-20 90) from a (0 0) (-20 90) to a (70 -70) heading was a required angle (fig. 6.7.11). The interest in the following episode is in Ioanna's rationale for disagreeing with her peer:

I: "This isn't an angle because you say PRINT DIRECTION 70 -70... it's not an angle... it's how much she'll turn."
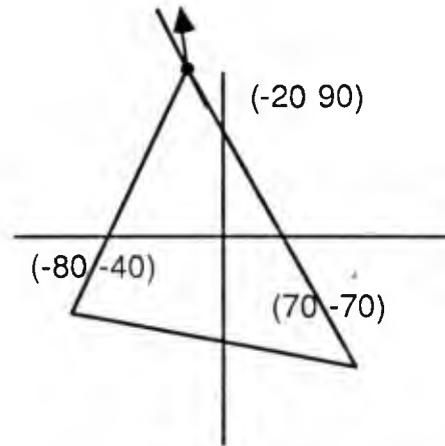
Figure 6.7.11 Ioanna's argument on the meaning of angle and turn

It is suggested that Ioanna had two separate notions in her mind, that of an action (turn) and that of a static angle. She did not seem to consider the notion of a dynamic angle as an angle at all *("...it's not an angle..."),* but rather as part of the turning action, i.e. its quantity *("...it's how much she will turn...").*

In her strategy for measuring the angle at (-80 -40) while the turtle's direction was from (70 -70) to (-80 -40), **Natassa seemed to dissociate the measurement from a turtle action.** In spite of her initial confusion in thinking about employing the absolute heading system rather than a location oriented heading change, her strategy did not involve a turtle action at all (fig. 6.7.12).

(turtle at -80 -40, heading (70 -70) (-80 -40))

N: "It should look upwards, this... ah... I got it... O...
no, what O... (she types SETH TOWARDS -20 90,
result on screen). Now we'll tell the protractor..."
I: "We'll find the angle now..."   (they type PR
DIRECTION 70 -70)
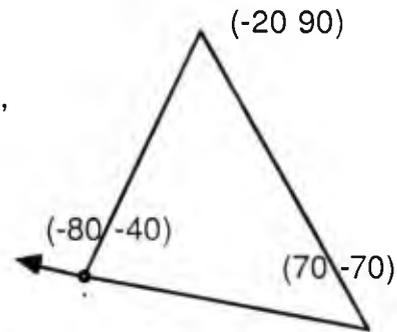
Figure 6.7.12 N. and I.: Measuring an angle

She changed the heading with a non action - quantity method, and then performed a measurement without seeming to relate it to the quantity of an action ("...now we'll tell the protractor..."). Ioanna's comment supports the previous argument of the childrens' conception of a measurement independently of their action - quantity schema.

6.7.2f) The children's choice between the intrinsic and coordinate "notional fragments".

The three groups of children used differing methods to solve the task and differing commands to perform state changes. Natassa and Ioanna seemed to prefer using the SET commands when a change of state was involved and the measuring commands in "information mode" to carry out the measurements. Maria and Korina preferred making the measurements first, initially restricting the use of the outcomes as quantities for the intrinsic commands (FD, BK, RT, LT), and subsequently incorporating a meaning of quantities for the required length and angle information. Anna and Loukia used the combined action - measurement commands (e.g. RT DIRECTION...) at first, when they were in "action mode" (drawing the triangle), split the commands in order to make the measurements and subsequently started to use the intrinsic commands with the measurment outcomes for changing the turtle's state.

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To make sure that their choice was genuine and not due to their having forgotten, being unaware of other options for state changes or finding some difficulty in understanding a particular command, the researcher asked each group, at a time when they were well engaged in the task, whether there was a different way of changing some particular state. In all three cases, there was a near - instant answer, providing the alternative option.

In Anna's and Loukia's case, for instance, the researcher intervened for this reason, before Anna pressed RETURN to the RT DIRECTION [-20 90] command (turtle at -80 -40), when the children were near completing the task:

R: "Tell me something Anna... before you do that... do you know any other way to make her look over there?"
A (straight away): *"SET... SETPOS... up there."*

Discussion soon revealed that she really meant SETH, but had forgotten the syntax for it. **However, the fact that she answered immediately indicates that she was quite aware of the option.** Loukia's explanation of what SETH meant (to Anna) revealed an understanding of a location determining a state change, but also an unclear distinction between the two states of the turtle.

L: "Turn me and make me look towards where you want me to go." A: "Ah, so that's what we want (she types SETH TOWARDS SE -20 90)... *isn't it the same with RT DIRECTION..."*

The discussion arising from this point concerned the choice between action - quantity and coordinate commands. The children were clear about the "equivalence" between the two, and both said that they preferred the former. **The reasons they gave indicate the links they made between the notions required by the "action - quantity" paradigm, and their intuitions:**

A: "Because it explains it better."

R: "Why does it explain it better?"

A: "Because it says... turn... eh... the degrees to go... to look at that point..."

L: "Its explains it in easier words."

R: "Why are these words easier."

L: "Because we say them more... because they come to our mind easier... because the rest of the world is used to listening to them more and so it comes to our mind easier to search and find..."

The main interest here is not that they preferred the intrinsic commands, since it could be argued that they had had much more experience with them than with the coordinate commands, but the reasons they gave for their preference. By means of her statement that the "words" move and turn are "easier", **Loukia seemed to be making a case for the accessibility of the action notions to the children, providing two reasons for it: that the notions are frequently used in their environment** *(..."the rest of the world is used to listening to them...")* **and that they provide useable thinking tools** *("...it comes to our mind easier to think and find...").*

6.7.2g) The meaning of a location.

The first point of interest in this section, is the meaning Anna and Loukia gave to the WRITE command, as that of labelling a point, i.e. giving a name to a point. This seems to relate to the experience they had had of the process of labelling from the first session, i.e. placing a marker on the turtle's path to make future reference to that location possible. The children's first experience with labelling, however, was in an intrinsic geometrical environment, where there was no plane description, more so, it was the label command which provided such a characteristic. Therefore, the label (marker) was "created" by the children, the point had no name until it was labelled, and only when it was marked did it become a "location". The children seemed to transfer this meaning to the WRITE command, **thus missing the point that in the coordinate plane the locations are already there, they have an**

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absolute system of naming, and the WRITE command merely prints that name on the screen given that the turtle is on that location. On the other hand, the children did seem to have a precise meaning for giving a label to a point (unlike the other two pairs) and this meaning, paradoxically involved the use of the coordinate system itself, and **a substantially clear conception of the notion of a location in the plane, and the process of refering to that location to cause actions or collect information.**

The strength of this link is illustrated by the way the children said they were going to use the WRITE command, and by the fact that although they knew its existence and had used it in the previous session, they used the word POST. Furthermore, this happened in two instances of considerable time difference between them, firstly when they were planning their course of action *(A: "...lets go to one of the points first... and to make the POST and then draw the line...")*, and then, in the first time they used it (point -20 90):6

L: "Can I do POST if I want?"
A: "Why don't you write POST, its better, easier."

When they saw that POST was not in the turtle's "repertory", they typed WRITE, and the coordinates without hesitation.

Maria and Korina did not seem to think about labelling until they had completed the "drawing" of the triangle, and that only after a hint from the researcher that this too was part of the task. It is suggested that this was not due to problems they might have had with using the WRITE command, but to **the children's lack of experience with the process of labelling in the previous sessions**. For instance, they seemed quite happy to use the method for locating the triangle points on the coordinate grid while carrying out measurements and the meaning they gave to the coordinate values did seem to be that of a name for a location. **However, labelling a point did not seem to have a particular meaning, it was just "part of the**

task". Although there is no indication to support the view that the children had realised the existence of an absolute locating system, it could be argued that they did not seem to form a schema for the locations such as the previous group, i.e. that they existed after having been labelled.

6.7.3 Summary and discussion of the T.C.P. microworld activities.

The first main issue examined in the study of the category 3 activities, was the nature of the schema or schemas the children used in the process of controlling the turtle in the composite intrinsic / coordinate "environment" of the T.C.P. microworld. There can be no conclusive evidence as to the effect of the differing initial activities on the children's forming of the two schemas, i.e. the intrinsic and the coordinate. It is interesting to consider, however, the case of the "path 2" children (fig. 6.1.1), who had had the longest experience with the integrated action / information use of the measuring instruments (e.g. FD DISTANCE :D3), which lend themselves to a perception of measurements as part of an action; Maria and Korina did not attempt to employ their coordinate schema, maintaining an action - quantity schema throughout the task, and using notions from the coordinate system only in the process of measuring. Moreover, they did not seem to realise until half way through the task, that their measurements offered length and angle information as well as the quantity of an action.

On the other hand, the "path 3" children, Anna and Loukia (fig. 6.1.1), who had had the experience of constructing a system to describe plane locations using an intrinsic method, did not hesitate, initially, to attempt to use their coordinate schema which (in the category 2 activities) they had formed as a "contradiction" to intrinsic notions, to change the turtle's state. However, the conflict created by the composite requirements of controlling the turtle in the T.C.P. microworld seemed to encourage the children's emerging use of two disparate schemas, the intrinsic and the coordinate. The children made only scarce attempts at relating the two and seemed to have a tendency to favour the intrinsic schema.

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Finally, the "path 1" children, who had had more experience with the "mechanics" of the coordinate system, but no experience with using coordinate notions in an action - quantity turtle control method, seemed to favour the use of their coordinate schema for changing the turtle's state, regarding the measurements as a process of collecting information. The evidence indicates that the intrinsic schema was easily accessible to the children, but that they had very little awareness of the differences in the two methods of turtle control.

The second main research issue involves the schemas the children built for the static and dynamic notions of angle required by the task. In the process of turning the turtle and measuring the internal angles, Anna and Loukia seemed to discriminate between angle as the quantity of a turn and angle as an entity in itself. They also discriminated between external and internal angles of the triangle (this issue is investigated in detail in chapter 7). Maria and Korina, however, did not seem to discriminate between the two meanings for angle, assigning a dynamic, "quantity of a turn" meaning to the notion, both in cases of action and measuring. Natassa and Ioanna seemed to consider the two notions as distinct, i.e. that an angle was distinct from a turn. They mainly used the coordinate command for changing the heading and seemed to consider angle measurements as independent of heading changes.

The schemas the children formed for the geometrical notion of angle, seem related to the schemas for controlling the turtle. For instance, Anna and Loukia seemed to develop an understanding of a non - turtle - centred notion of angle via conflicts between their intrinsic schema and the computer environment. This issue was seen as warranting further investigation, which was carried out in the following two studies (chapters 7 and 8). Here, it is also interesting to consider drawing a parallelism between the connections of the children's emerging angle - schemas with their experiences in the initial activities, and the respective connections between their schemas for turtle

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control notions described above.

Finally, the other research issues which emerged from the analysis of the data were:

a) the arbitrariness of the coordinate system "rules", where an example is given, of the children's using what they perceived as a more realistic "number of lines" notion to the callibrating units of ten turtle steps,

b) the children's persisting difficulties in discriminating between two notions of state change, i.e. a sequentiality notion and that of a state change determined by the end state,

c) the "validation" of the children's choice, i.e. whether they were aware of the two alternative methods of state change while they were engaged in the task. This section included a discussion between the researcher and the children, yielding their reasons for their preference of the intrinsic commands, which the researcher interprets as being that the action notions of "move" and "turn" are frequently used in their environment and are useful tools to think with.

d) the effects of the experience concerning the notion of location which the "path 2" (Maria and Korina) and "path 3" (Anna and Loukia) children had in their category 1 activities (fig. 6.1.1), on their perception of this notion in the T.C.P. microworld activities. Maria and Korina did not seem to attribute a particular meaning to labelling locations, while the meaning which Anna and Loukia seemed to have for labelling was that of "constructing" a point, i.e. they seemed to disregard the absolute nature of the coordinate grid.

6.8 OVERALL DISCUSSION AND CONCLUDING REMARKS

Although the researcher is aware of the limitations of the applied methodology (i.e. subjectivity of interpretation, limited grounds for generalisations), it was felt that it was a justifiable means of acquiring the

insight into the children's thinking which was needed in order to investigate the feasibility of their development of understandings of cartesian geometry by employing the cognitive schema they seem to have built from their experience with the Logo turtle. As discussed in chapter 4 (section 4.2.8), a substantial component of the analysis was presented in the form of chosen "significant" episodes during the children's activities which illustrate the nature of their insights or confusions related to the research issues. Throughout this study, the organisation and the choice of the presented episodes provide an overall picture of the balance of events in kind and in time. The same also applies to the two subsequent studies.

On the one hand, it is felt that the study was limited by the relatively high degree of structure of the activities. For instance, the desire to investigate different aspects of the notions involving the coordinate "mechanics" in the category 1 activities, restricted the children's experience to the respective aspects. All three types of notions, however, were important for the children's understandings in the subsequent tasks, although it is arguable whether they should have been introduced sequentially or concurrently. A similar argument applies to the different categories of activities which were presented to the children sequentially, in the specific (and therefore arguably restrictive) order of plane description notions, coordinate turtle control and choice of turtle control.

On the other hand, however, the investigation of detailed aspects of the children's thinking required a research model which would restrict the unavoidable research "noise" of more open - ended activities. Moreover, there was an effort to minimise imposed restrictions, by incorporating some project work (in category 1) and designing the tasks so that they allow flexibility in the employing of strategies to solve them (categories 2 and 3). As a consequence from the previous argument, and as a result from the analysis of the data, it could be argued that the children engaged in learning activities, the potential of which this study set out to investigate.

The study provides a description of the process by which the children apparently began to build a mental schema with dynamic characteristics, i.e. one which would enable them to make controlled changes in the coordinate plane. The schema seemed to emerge in the children's minds, from its "antithesis" to the intrinsic schema, caused by the coordinate nature of the category 2 tasks. In the category 3 activities, the children seemed to use the necessary coordinate notions (labelled "coordinate mechanics" in the study) either by employing their intrinsic schema (e.g. FD DISTANCE 70 -70) or their coordinate schema (SETPOS 70 -70). It seems therefore interesting to consider the potential of the T.C.P. microworld in providing the children with the opportunity of a dynamic interplay between the two geometrical systems by means of the option to employ a method to make changes, based on concepts belonging to either system. The issue of the nature of the criteria children employ in choosing between the use of intrinsic and non - intrinsic notions is investigated in chapter 8, in the context of a microworld embedding intrinsic and euclidean notions.

CHAPTER 7

FROM INTRINSIC TO EUCLIDEAN GEOMETRY

7.1 OVERALL DESIGN OF THE STUDY

7.1.1 Objectives.

As discussed in the review of the literature, Euclidean geometry has mainly been taught in schools as a "tight" deductive system resulting in most children achieving no more than a superficial "rote" mastery of the subject. As a consequence, the role of Euclidean geometry in educational practice has recently been diminished, at least in the U.K.. However, the case has been made that, due to its mathematical nature, geometry has substantial potential as a field within which children can primarily practice inductive inferences from personal experience, while simultaneously being a field inviting engagement in deductive thinking (Freudenthal, 1973). Perceiving geometry in such a role could involve important educational implications since, for instance, it has been argued that deductive abilities in mathematics must be based on the practice of inductive inference (von Glasersfeld, 1985a). However, there is very little evidence of children working in geometrical environments which might fulfill this potential for geometry in education.

The general aim of this study was to investigate the potential for children to use their intrinsic schema in a Turtle geometric environment in the process of forming inductively generated understandings of euclidean geometry.

A pair of children were provided with a microworld enabling them to mark turtle positions on the screen and make linear and angular measurements between those positions (the P.D.D. microworld, see sections 4.2.2. and 7.1.4). The study involved a series of activities consisting of both structured tasks and more open - ended projects with varying degrees of constraint. A detailed observation of the children engaged in these activities was carried

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out, in order to illuminate the process by which they might use their intrinsic schema to develop understandings of euclidean geometrical notions. The children's activities involved the use of the turtle's new tools (adapted from Loethe, 1985 and incorporated in the P.D.D. microworld) which were designed to be applicable in both the intrinsic and the euclidean geometrical systems (chapter 3 and section 7.1.4). In particular, the investigation focused on;

a) how the tools were firstly adapted to the children's existing schema for controlling the turtle,

b) how they were used for representing concepts in a euclidean setting, and

c) the way in which this experience influenced the children's strategies in their own Logo projects.

The children's activities were accordingly split into three groups (the word "categories" is used in the study). The corresponding specific research objectives for each "category" of activities were to investigate:

a) the process by which they integrated concepts involved in using the new tools into their existing knowledge and in particular the extent and the way in which their intrinsic schema was employed during this process;

b) the development of understandings of the nature of euclidean geometry and the extent to which the children employed a turtle geometric schema incorporating the P.D.D. turtle tools for this purpose;

c) how and if their experience in making sense of the tools and using them in a euclidean setting influenced their thinking in Logo projects of their own.

As mentioned above, the children's activities were split into three categories. Since the findings from each category will be presented separately, an analysis of the specific tasks in each category will be given at the beginning of each findings section (i.e. sections 7.2, 7.3 and 7.4). However, it seems useful at this point to present an overview of the activities involved in each category.

The aim of the first category was to investigate the process by which the children integrated the concepts required to use the tools into their existing knowledge. The tasks initially involved the use of the measuring instruments only (resembling a turtle "ruler" and "protractor"), the research aim being to find out how they made sense of linking measurements to turtle actions and how far they distinguished action from measurement. A final set of tasks incorporated the use of the POST command, so that the children would make decisions on which features they wanted to measure in order to construct figures with particular characteristics. The aim here was to investigate their process of incorporating the naming of points on the screen and in particular the criteria by which the children decided to give names to specific points in relation to future use of the points for measurements (as discussed in chapter 3, a measurement could only be done with the use of a marked point).

The aim of the second category was to investigate the development of the children's awareness of the essence of the "logical" nature of euclidean geometry, i.e. the way in which the previous incorporation of the new tools in their turtle geometry schema, played a part in the growth of this awareness and in developing an understanding of certain euclidean concepts. The tasks involved the construction of an isosceles triangle, the investigation of specific properties of the isosceles triangle and the construction of a generalised procedure for the isosceles triangle. The investigation concentrated on how the children used intrinsic and euclidean concepts in the process of solving the tasks.

The third category had as an aim to investigate the influence of the children's experience in more open - ended Logo projects. The research intention in their first project (where they had the constraint of using their generalised isosceles triangle procedure) was to reveal how the children might incorporate a complex tool, based on euclidean concepts, in their familiar (see chapter 5) open - ended Logo projects. The children's final project did not involve the constraint of using the isosceles triangle procedure or the P.D.D. tools. The aim was to find out what reasons the children might have for using the tools (if any), i.e. the effect of their experience in using concepts from both geometries.

7.1.3 Overview of the learning environment.

The learning environment was designed so that the children would be provided with a microworld (Hoyles and Noss, 1987b) constructed so that the use of the intrinsic schema would be applicable for understanding non - intrinsic, euclidean ideas, i.e. the "tools" could be used for both the intrinsic and the euclidean representational systems (see section 7.1.4). In this sense, the structure of the children's activities within this microworld can be described as a "conceptual pathway" from the intrinsic to the euclidean representational system.

The technical component of the microworld was designed so that the tools would be readily usable by the children and they would be seen as "primitives", i.e. an extension of the basic turtle functions. In this sense, there were no concepts embodied within the tools/programs as, for instance in the "parallelogram microworld" (Hoyles and Noss, 1987a), discussed in chapter 2, but rather, the concepts were embedded in the use of the tools as primitive commands. A tool/program which the children would be able to look into and reflect upon its construction could serve as a microworld within the wider P.D.D. microworld. This issue is discussed in section 7.5, in the light of the analysis of the data. The words "post", "distance" and "direction" were chosen

so that they have a meaning tightly related to their function (e.g. distance measures the relation between two positions).

The pedagogical component consisted of the activity structure and the researcher as teacher. The first priority of the intervention strategy as part of the pedagogical component was initially to provide the children with a meaningful way of thinking about the use of the tools, adaptable to their turtle schema. For example, DISTANCE was introduced as the turtle's ruler, and PRINT DISTANCE :M was explained as "write me how much the distance to M is". The aim of the research was to investigate the way in which this was picked up and used by the children from then on.

Moreover, other pedagogical aims of the researcher's interventions were to encourage or provoke reflection on children's actions, and to encourage explicit explanations concerning important points. An important aspect of the intervention strategy was the frequent encouragement for the children to be active in their learning, to feel at ease with making conjectures and saying their thoughts out loud, i.e. to establish the "legitimacy" of a kind of mathematics learning which, due to the educational system, was very unfamiliar to the children (chapter 5).

Care was also taken to ease tensions: not only disappointment and frustration, but also over-enthousiasm leading to lack of concentration; this affective aspect was partly the result of the unfamiliar learning method. The children's perspective of the situations was respected so that it often appeared to the children that what they would do was decided by them, rather than by the researcher.

7.1.4 Analysis of the conceptual field of the P.D.D. microworld.

7.1.4a) Tool description.

POST. This provides the ability to name a position of the turtle (and see the name on the screen). The name can only be given when the turtle is in that position.

DISTANCE. This provides knowledge of the distance between the turtle's current and some previous position. This previous position can only be specified by a name having been given to it by POST.

DIRECTION. This provides knowledge of the angle between the turtle's current heading and the heading required to face some previous position. The angle is measured from the current heading clockwise. The previous position can, again, only be specified by a name having been given to it by POST.

7.1.4b) Change of state.

It is important to analyse the process of changing the state of this microworld's mathematical entity (the turtle) in order to clarify how the process relates to state change in the differential (intrinsic), logical (euclidean) and analytical (cartesian) geometrical systems.

In Turtle geometry, the state is changed by an action (move or turn) followed by a quantification of that action. The distinction between action and quantification plays a crucial part in making Turtle geometry intrinsic. The action describes the nature of the change which is about to take place (e.g. in Turtle geometry, change means change of position or change of heading).

In the P.D.D. microworld, change of state is also caused by an action (move or turn) and a quantification. The difference lies in the method used to

determine quantities of actions, which depends on being able to "compare" two not necessarily adjacent states of the turtle's path by measuring the linear or angular difference between them. This can only be done between the turtle's present state and any previous one provided it had been marked when the turtle was in that position. Comparison between two non - adjacent turtle states contradicts the differential characteristic of turtle geometry, in that, here, growth is not necessarily described by change at the growing tip (Papert, 1980). On the other hand, the action - quantification schema is maintained, and the use of such measurements is optional.

The dynamic "action - quantification" characteristic is absent in the mathematical entity of "logical" Euclidean or "analytical" Cartesian geometry, i.e. the static point. Changing state in these systems is an absolute process, i.e. dependence on previous states is not necessary. The dependence lies rather on the notion of the plane (three dimensional geometries are not considered in the study). In Euclidean geometry, the notion of the plane is defined, although there is no absolute, systematic description of plane locations, as is the case in Cartesian geometry (chapter 6). It is therefore meaningless to talk about changing the state, since points can be placed on the plane either in an absolute way, or in relation to any other point.

An analysis will now follow of the notion of the plane in the different systems, in relation to the P.D.D. microworld.

7.1.4c) Awareness of the plane.

The intrinsic nature of Turtle geometry restricts mathematical awareness of the space around the mathematical entity (i.e. the turtle). In intrinsic geometry, the position and the heading are not determined by their relationship to any absolute or imposed, systematic or otherwise, description of the plane. In fact, mathematically speaking, the turtle "does not know" of any plane at all (Papert, 1980). It can only do operations that are directly dependent on the immediately previous state. The turtle's position and heading are generated

by its own actions.

In the P.D.D. microworld, the dependence on the immediately previous state is not necessary: the turtle's state can change depending on any previous position (note that POST does not mark the heading), providing that the position has been named. Such positions are entirely dependent on the turtle's path, i.e. there is no absolute description of any location. Furthermore, the described positions/points in the plane are discrete (not an infinite number) and (unless imposed as in the T.C.P. microworld study) do not have a systematic code of naming. The turtle is not conscious of any plane, but of certain discrete points somewhere where it has been before. Relating to a position other than the immediately previous implies a notion of relative location. The turtle's two measuring instruments give a quantification of the relationship between two states without the constraint of being temporally adjacent.

7.1.4d) An Example.

The geometry stemming from this microworld has mainly intrinsic characteristics. Those characteristics that are not intrinsic by definition, are constructed so that they constitute a natural (and logical) extension of the intrinsic abilities of the turtle. For instance, in the command FD DISTANCE "name (of POST), the turtle's actions are intrinsic. There is the action command, "move yourself forward" and the quantification command "so much". The non - intrinsic extention, is in the nature of the quantifier. There is no numeric quantity, but a description of the change of position in terms of a "measurement" that the turtle does. This "measurement" involves the point that has been "posted", which contradicts the differential character of Intrinsic geometry.

7.1.5 Methodology.

The pilot study consisted of two phases, the preliminary phase involving trying out the new primitives, the programs for the tasks and the activities (whether or not they involved programs). As a result of the preliminary phase, a trial structure of the learning path was designed and tested out in a detailed main pilot study.

7.1.5a) The main pilot study.

The study was carried out in an English school, with two children aged 13, one of whom had had three years of Logo experience and the other nine months (this was the nearest available situation to the conditions of the main study with respect to age and Logo experience). The children and the researcher had a room to themselves, which made concentration and taping feasible. All the activities in the first two categories were piloted. Data was collected by the following means;

- audio taping,
- transcriptions,
- a record of everything that they typed,
- researcher's notes.

The global changes resulting from the pilot study concern the data collection method, the structuring of the activities and the researcher's interventions. As a result of the main pilot study, it was seen as necessary, due to the study's detailed nature, to implement the following changes in the collection of the data. Three main forms of data were developed, the children's typing (dribble files), their verbalisations (audio taping), and revealing graphics outputs (using the dribble playback facility discussed in chapter 6). It was seen as important to record what programs the children chose to save on disk, and to keep a record of when and what they chose to write on paper (paper and pen were made available at all times). The researcher also developed a strategy

for identifying and taking written notes on important events which would not be captured by the data.

The changes in the activity structure were related to the development of three categories of activities, sensitive to the research issues (section 7.1.1). Certain changes in the order and the way tasks were presented were seen as important for conforming to the notion of a conceptual pathway, and for making the tasks clear to the children. For example, the introduction of POST was seen as meaningful after learning how to make measurements, since that was the reason for which POST would be used. Furthermore, introduction of the new primitives would be made as an answer to situations where they could be used, rather than at the beginning of a session or activity.

A technique for intervening was developed, in order to conform to the dual role of researcher / teacher in a way which would be clear for later analysis, without restricting the research outcome. For instance, the children were often asked to explain their actions verbally, even when the probability of revealing relevant information seemed low at the time.

Finally, several detailed changes were made, with respect to the task programs in category 1, programming complications in the change from Logotron Logo to Apple Logo II (e.g. no connection between graphics and text screen in the latter), clarity in presenting the tasks, and the switch from the English to the Greek language. For instance, the translation of the word used for "procedure" was "concept". This convention was built during preliminary study, and since it was applied in the children's other Logo activities at the time of the research, it was maintained in the research sessions (see chapter 4).

7.1.5b) The main study.

The main study involved two children from the Logo club, Philip and Nikos. The research was carried out after school hours in the research room, in two

90 minute sessions weekly for five weeks. During their school hours, the children participated normally in the Logo club and in the school Logo programme. The machine and the Logo version used for the research were the children's familiar (from their school activities) Apple IIC and Apple Logo II respectively. As in the other case - studies, during the research sessions, they were faced towards the machine, the researcher seated behind them so that their collaboration was unrestricted except in the case of researcher interventions.

The research data consisted of;

- audio taping of everything that was said,
- soft and hard copies of verbatim transcriptions translated into English,
- soft and hard copies of everything the children typed,
- hard copies of graphics screen dumps, acquired by playing back the dribble files and pausing them to print,
- soft and hard copies of all the procedures the children wanted to save on disk,
- researcher's notes and children's prompted and unprompted notes on paper.

During the whole of the research, the children had paper and pens in front of them to use if and when they wanted. The researcher's notes mainly consisted of recording observations that would otherwise slip through the mechanical data collection net (e.g. how the children demonstrated an angle using their fingers on the screen). For this study, the dribble playback facility enabled a session to start at the precise point where the previous finished, or at any other crucial point.

The analysis of the data is presented in three parts corresponding to the three categories of activities, in order to facilitate the reader. However, the structure of the presented findings within each category is based on the research issues, rather than the tasks themselves, and is the outcome of a synthesis of

"significant" episodes, as elaborated in section 4.2.8. Moreover, a further synthesis of the findings is discussed in section 7.5, involving a framework of four perspectives of the children's activities, namely:

a) The nature of their activities from a Logo programming perspective, i.e. their programming and mathematical strategies and the mathematical nature of their thinking;

b) The role of the P.D.D. primitives as a mediating tool between intrinsic and euclidean ideas;

c) The children's use of their intrinsic schema for both intrinsic and euclidean ideas;

d) The nature of the children's thinking in the context of euclidean geometry.

7.1.6 The children

Nikos was characteri sed by his teachers as a bright child with above average grades in mathematics. The Logo club teacher (teacher F) was impressed by his very high motivation regarding computers; *"...he is the only one in the group who takes time between computer sessions to think about what he'd like to do...".* However, he was dominant with his ideas within his group in the Logo club, apparently striving to be *"...looked upon as knowing the answers...".* His answer in the pupil questionnaire on whether he liked to work on his own or not illustrates this point: *"I like to work on my own because I want to make anything I like on my own...".* As discussed in section 7.6, Nikos' dominance did cause some difficulties during the research. On the other hand, Nikos was not hindered by making mistakes, showing a relative confidence with the process of debugging, as illustrated by an extract from his essay, *"...to make the main part of a program... or to make it perfect and to correct the mistakes...".*Moreover, he showed progress in structuring his programming. In the "Four squares" task, for instance, this progress is

illustrated by his shift from D to H, in figure 5.2 (appendix H).

Philip was characterised as an *"average - above average"* student and as a *"strong personality"*, but also a *"cooperative and sound worker"*. The last point is illustrated by his degree of acknowledging the cooperative character of his work within his group, given that his peers showed much less involvement with the group's activities throughout the year; *"Together with my colleagues... we made various and pretty things..."*. During the research, he seemed to weather Nikos' "tantrums", but also showed that his initiative in expressing his thoughts was not unaffected by them. His responses to the questionnaire show that he liked mathematics *"because they are the most enjoyable lesson"* . His essay illustrates that he also enjoyed working with Logo. His programming was characterised by a consistent use of procedures and involved a relatively high degree of structure (shown by groups F and G in figure 5.2, appendix A). However, Philip also showed a persisting resistance to use subpocedures in the process of solving a task.

7.2 CATEGORY 1: MAKING SENSE OF THE TURTLE'S NEW TOOLS

7.2.1 Task analysis

This category of activities consisted of three sets of tasks (fig. 7.2.1), the first of which involved the introduction and use of the measuring instruments as a means of taking the turtle to specific points on the screen, designated by a cross (to show the point) and a letter (to show the name).

The first task (task 1 set 1, fig. 7.2.1) to take the turtle to a point (prior to introducing the instruments) was designed to probe the extent of the children's persistance in trusting a perceptual way of attempting the problem and to investigate their perception of the problem by means of what tools they might require for solving it.

Subsequently, (tasks 2 and 3 set 1, fig. 7.2.1) the children were required to take the turtle from one point to another, after having been introduced to the measuring instrument primitives and their syntax. The first question simply involved two points on the right of the turtle's initial heading (task 3), while the second involved three (task 4), the first being on the left. Thirdly, ten points appeared on the screen (task 5) and the children were asked to find out which point was nearest and furthest from the centre. Finally they were asked to draw a picture by joining up any of the points.

The aim of tasks 2, 3, 4 and 5 (fig. 7.2.1) was to investigate the process by which the children started making sense of the use of the instruments in the context of relating the turtle's state to an external point. The focus in this set was on using the instruments to cause action (change of state) rather than carrying out measurements. The plan was to see whether the children would integrate this action - quantification idea with their intrinsic schema. The following main concepts were required for the task:

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SET 1

1 Take the turtle to point A
(point A shown on screen)

2 Take the turtle to point A
(point A on screen)

3 Take the turtle to point B and then
to point C (B and C on screen)

4 Take the turtle to points D, E and F
(D, E and F on screen)

5 (ten points on screen)
Give the distances of the points from
the centre in order.
Draw a shape by joining points

SET 2

1 (three points on the screen)
Join the points up to form a triangle.
Find the lengths of its sides.
Find the sizes of it's angles.

2 (three other points on the screen)
Join the points up to form a triangle.
Find the lengths of its sides.
Find the sizes of it's angles.

SET 3

1 (given elements drawn on paper)
Make a triangle with one side of 90 steps,
one side of 70 steps and a 30 degree angle
between them.
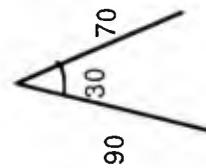
2 (given elements drawn on paper)
Make a quadrilateral with:
a side of 85 - an angle of 40
a side of 60 - an angle of 135 and
a side of 30
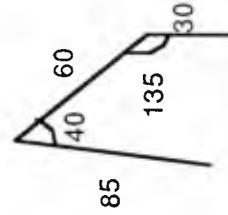
Figure 7.2.1 N and P: The sets of tasks for the first category of activities

- distinguishing the function of the two measuring instruments and understanding the convention of measuring the turn to the right;

- distinguishing the function of an instrument from the "action" turtle moving primitives;

- distinguishing between a perceptual and an analytical way of calculating length or turn.

The second set of tasks in this category is more balanced in the importance of the action or the measurement; two of the three questions require a measurement outcome, the action now being the means to reach that outcome. The tasks resemble those in the third category in the T.C.P. microworld study (fig. 6.1.1). The difference in this case, however, lies in the conceptual field of the microworld which the children were using and in the related research issues.

For each task, three labelled points appeared on the screen, and the children were asked to:

a) join the points up to form a triangle,

b) find the length of its sides and

c) find the size of its angles.

The tasks were designed to find out the strategies that the children would form to combine turtle actions with the non - intrinsic features of the tasks i.e. measuring the internal angles and the lengths, turning or moving the turtle in relation to an external point in the plane. The tasks not only required an integration of a static and a dynamic concept of angle (Kieran, 1986b) to

make an angular measurement but also required moving the turtle to such a position that a measurement could take place, i.e. relating the turtle's state to a non - intrinsic angle in the plane. Such a measurement could be done directly or indirectly (by measuring the suplementary angle) depending on whether the turtle's position revealed the internal or the external angle. Futhermore, more than one measurement could be done without changing the turtle's state (e.g. two linear and one angular). In what way would the children relate action to measurement?

The third set of tasks (fig. 7.2.1) involved two tasks to construct geometric figures, given certain length and angle elements of the figures. The given elements were drawn on paper by the researcher while he was verbally stating the task.

The first task required;

a) the construction of a triangle, given the lengths of two of its sides and the size of the internal angle formed by the two given sides and;

b) finding the size of the remaining elements.

This task was planned as a meaningful context for introducing the marking and labelling of turtle positions (POST). From this task onwards, the imposing of absolute points was abandoned, leaving the decision for marking positions to the children. The aim was to find out, firstly, how and if the children associate the marking of a turtle position to a means of relating to that position directly, i.e. in contrast to the intrinsic method of inversing the turtle's movements. Secondly, the aim was to investigate the strategies they formed in choosing which turtle positions need marking and which do not. Thinking about whether a certain position needs posting required an abstraction of the task in order to determine whether there will be any need for relating to that position in any future point of the task.

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The second task (task 2, set 3, 7.2.1) required;

a) the construction of a quadrilateral, given the size of three of its sides and the two angles formed by the three given sides, and;

b) finding the size of the remaining elements of the quadrilateral.

The aim of this task was to pursue the investigation of the development of children's strategies for using the POST feature in a construction and measurement context, and give time for the children to make sense of using POST and the measuring instruments.

7.2.2 Findings

7.2.2a) Introduction.

The activities in the first two sets (fig. 7.2.1) in this category are, in effect, very much related to certain activities concerning the description of the plane in the previous study. Philip and Nikos did, initially, seem to have the same type of problems as the children in the T.C.P. microworld study, i.e. in integrating the measuring instruments in their action - quantity schema, and in discriminating between i) information from the "ruler" and the "protractor" ii) the angle and length metric sysytems and iii) the position and heading state.

However, the analysis of the data in this study focuses on a different research issue, i.e. the illumination of the process by which it was possible for the children to integrate the POST / DISTANCE / DIRECTION microworld into their intrinsic schema for controlling the turtle and the extent to which this integration encouraged a euclidean interpretation of geometrical ideas.

7.2.2b) Relating perceptual cues to measurement outcomes.

Not surprisingly, before being introduced to the measuring instruments, the children made lengthy and detailed efforts to take the turtle to a fixed and labelled point on the screen, using the HT (hide turtle) command repeatedly as their approximations became "better", so that they could see the end of the turtle's path. When they had finished, **they seemed convinced by their perceptual cues that the turtle was precisely on the spot.** Despite the researcher's subsequent repeated questioning of the precision of their method, the existence of an "accurate" method, or one related to the point away from the turtle, did not seem to occur to them. Moreover, **they did not seem to see the need for more accuracy,** as this dialogue illustrates:

R: "...is it exactly, exactly in the middle?" (of the cross denoting the location)
P: "No, but it doesn't matter."

It is suggested that the children's conviction of the accuracy of their method was not only due to the obvious factor that they were unaware of any means to acheive more accuracy; it may also have been the case that they had not, up till that point, come across the need to make changes in the turtle's state which would require more accuracy than that provided by their perception.

However, the children seemed more convinced of the superior accuracy of the measurement outcomes, in the process of using the DISTANCE and the DIRECTION commands. In their means of deciding on length and angle quantities, they soon seemed to incorporate the treating of the measurement outcomes as confirmation of these quantities; the first indications of this change were during the comparison of the distances of the ten points on the screen from the centre. The children enjoyed measuring to a point where they became almost obsessive with it; in the ten point task, they wanted to measure all the distances and the angles from the centre, and then the relative quantities from point to point.

However, as the subsequent tasks revealed, **performing the measurements did not necessarily imply that the children understood what they were measuring, especially in the case of angle** (e.g. discriminating between the two states, between length and angle measurements). The process by which the children discriminated their new tools will be illustrated in the following sections.

7.2.2c) Creating plane locations relative to the turtle's path.

Although the children were introduced to the POST command, after the set 1 and 2 tasks, they had already effectively asked for a way to "make points" on three occasions, (the first being early on, in task 3, set 1) so that they could carry out some project of their own in the Logo club.

The POST command was introduced after the children's approximating efforts to construct a triangle with two equal sides. This task was used here only as an opportunity to set up a meaningful context for introducing POST. For the purposes of this category, therefore, the task was seen as part of task 1, set 3. However, the children's difficulties in understanding the abstract nature of the given information were of considerable enough interest to invite analysis, which is presented within a more relevant context, at a later stage in the category 2 tasks.

The introduction to POST took place at the end of a session. In the next session, task 1, set 3 was given to the children (a triangle with two sides of 90 and 70 length, and a 30 degree angle between them) without any mention of the POST command. **Nikos then had the idea of using the command, and expressed a plan for constructing the triangle** (see fig. 7.2.1, task 1, set 3) **which incorporated "posting":**

(draws a triangle on paper)

N: "... the POST alpha, the point alpha, can we make it with points? Beta here, e.g., this is redundant... but anyway... no this is not redundant, but... and the gamma, the C. So we'll say, we'll be here (at C), *since we know how much this is* (the given elements), *we'll be here and we'll tell her to turn to point A. She turns to point A, and we tell her to go to point A and to tell us as well, so that we can write it down* (he means measure the distance)... *and she goes here* (point A) *and the triangle is done. And then we can find the angles...* (measure the remaining elements)".

The children used the POST command in their next task (quadrilateral, task 2, set 3, fig. 7.2.1) in a matter - of - fact manner, without talking about it. They forgot to put the post on B and took the turtle back from C to do so. **Nikos, however, discussed the necessity of posting B at that point in time,** since the point was "surrounded" by given elements of the quadrilateral:

P: "Ugh! We didn't put a POST."
N: "Ah, yes? It doesn't matter. We don't need it very much... here, since we know her, what do we need it? When we're passing through (he means again, to measure) *we'll put a POST."*
P: "Yes, but to be sure..."

It is suggested therefore that the children began to develop a use for the POST command in a meaningful way, i.e. they used it for changing the turtle's state in relation to the posted point at a later stage, in order to "close" a figure, something that they now seemed convinced they could not do with "normal" Logo. During the quadrilateral task, **they also seemed to discriminate the command and its input, realising that the input was a name for a place rather than some fixed letter label.** After giving the letters A, B, and C for the first three points of the quadrilateral, for instance, they decided to give the letter N (possibly N

for Nikos) for the fourth, asking the researcher whether he minded.

7.2.2d) Internal and external angle.

During their first experiences with measuring angles in the fixed triangle task (task 1, set 2), **the children did not seem to discriminate between internal and external angles;** during the construction of the triangle **they treated the measurement outcomes from the turtle rotations as the sizes of the angles of the triangle** which were required by the task. The researcher pointed out the internal angles and had to ask whether angle A looked like a 136.705 degree angle before a conflict was created between the children's perception and the measurement (136.705 was the "rotation" measurement outcome, fig. 7.2.2 - a global point about the 3 decimal places of the numerical outputs has been made in chapter 6). Philip's "theory" about this "paradox" seemed to show a lack of understanding of the functioning of the turtle's protractor (i.e. that it measures potential right turns only):

P: "...instead of doing it right, we do it left."

Philip seemed to keep this implicit and unclear "theory" in his mind throughout the tasks in this category (category 1), offering it as an "explanation" when there were difficulties in understanding the meaning of measurements (i.e. what was being measured), or how to measure specific angles. In their first angle measurement (angle A, fig 7.2.2), a discussion with the researcher, who drew the triangle and the turtle's heading on paper led to **the children's mutual decision to employ a method similar to Anna's method in the T.C.P. microworld study** (fig. 6.7.3):

N: "We turn it there, where it can do RT,
and then we do it." (He means state b)
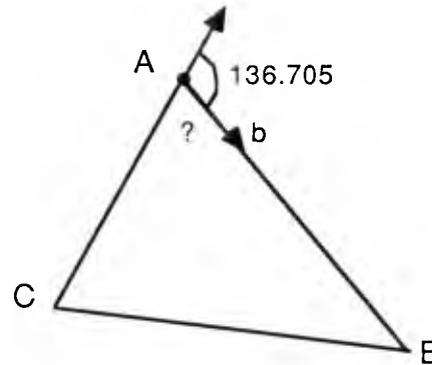
Figure 7.2.2 N. and P. : The first measurement of an internal angle

The children then measured the other angles with the same method, Nikos attempting to express it at the end:

N: "Since we couldn't know the angle when it looked towards AC, we put it to look towards beta, and like that we could find the angle, and we found it." (fig. 7.2.2)

However, from the second fixed triangle task (task 2, set 2) onwards, Nikos developed a different strategy for measuring the internal angles. He measured the external (rotation) angle and then used the computer to subtract it from 180 (fig. 7.2.3).

(turtle in state a)
? PR DIRECTION :E
143.18
? PR 180 - 143.18
36.82
? RT 143.18 (turtle in state b)
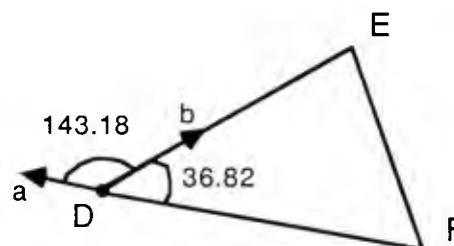
Figure 7.2.3 N. and P. : Using the "supplementary angles" theorem to measure an internal angle

In using this method, Nikos seemed to combine the dynamic notion of turn

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with an operation (partitioning) on static angles. **He therefore seemed to be using an angular measurement outcome for both dynamic and static interpretations of angular notions.** Figure 7.2.4 illustrates the children's use of length and angle measurements to collect information to be used for notions belonging to both the intrinsic and euclidean representational systems.

(turtle in state a)
? PR DIRECTION :D
114.867
? PR 180 - 114.867
65.133
? RT 114.867
? PR DISTANCE :D
111.018
? FD 111.018
(turtle in state b)
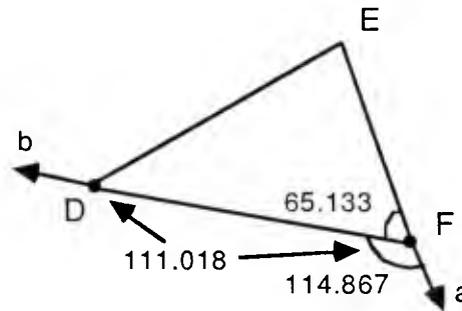
Figure 7.2.4 N. and P. : Using quantity information for "Intrinsic" actions and "Euclidean" measurements

In the 90/30/70 triangle task (task 2, set 3), **the children did not realise at first, that they had the opposite problem of using static angle information to perform a turtle rotation:** they typed RT 30 FD 70 after posting point B (fig. 7.2.5). Philip recognised that the angle was not 30 degrees, and argued that it was obtuse demonstrating his knowledge on angle sizes from school geometry. **Nikos, however, was the one to carry over the supplementary angles strategy from the fixed triangles tasks:**

N: "...shall I tell you (talking to Philip) *how much it is? 150."*
R: "Why?"
N: "Because this here is 180, from here till there it's 30 and
this is 150 (shows with fingers on the screen along the
two directions a and b)"
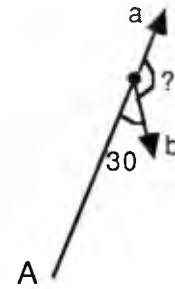
Figure 7.2.5 N. and P. : Using static angle information
to perform a turtle rotation

In the remainder of this task and in the next one (quadrilateral, task 2, set 3), **Nikos seemed to have grasped the relationship between internal and external angles and what was being measured by the turtle's protractor.** An indication of this is in the following episode in the quadrilateral task: the children forgot to post point B and took the turtle at point C, posting the point and turning it right 45 degrees (the internal angle was given, 135 degrees). They decided to take the turtle back to B, and Nikos typed RT 135, using the supplementary angles method to perform an intrinsic turtle turn (fig. 7.2.6).
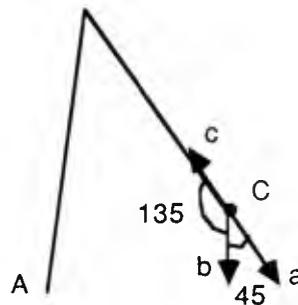
Figure 7.2.6 N. and P. : Turning the turtle back to post point B

However, Philip did not seem to connect notions from the two geometries until the end of the category 1 tasks (fig 7.2.1). Although he had a sound knowledge of school (Euclidean) geometry and was a good Logo programmer compared to the Logo club children, **he seemed to find it difficult to combine intrinsic and euclidean notions.**

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7.3 CATEGORY 2: USING THE NEW TOOLS TO UNDERSTAND CONCEPTS FROM EUCLIDEAN GEOMETRY

7.3.1 Task analysis.

This category consisted of four activities involving the use of the turtle's new tools in the construction of an isosceles triangle, in discovering and generalising some of its properties, and in abstracting properties to write a generalised procedure for an isosceles triangle (fig. 7.3.1).

The first activity consisted of the task of constructing a triangle for which the only information / restriction that was given was that it had two equal sides (fig. 7.3.1, task 1). After the construction the children were required to find the size of the triangle's elements. The aim of this task was firstly to investigate the children's use of the new tools to construct a figure for which they had abstract information and secondly, how they might use the tools to develop understanding of the generalisability of the two equal angles property revealed by their measurements.

The second activity (task 2, fig. 7.3.1) was a task to construct an isosceles triangle in an upright orientation on the screen. The aim here was to investigate the strategy the children developed for such a construction since the task required giving the turtle the correct heading before starting to draw the triangle on the screen and consequently mentally manipulating specific properties.

The third activity involved the use of a procedure which draws an upright isosceles triangle. The task involved measurements inviting the use of certain properties of the figure (task 3, fig. 7.3.1). At first, the children were asked to make a line that split the triangle in half. The aim was to find out the children's perception of what bisection means and to investigate if and how the children used the tools to make a line that would bisect the triangle. A further aim was to gain insights into their processes of building theories concerning the

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The tasks for the category 2 activities

Task 1

Make a triangle with
two equal sides
Measure its sides and its angles
Put it in a procedure

Task 2

Put the triangle in an "upright"
position

Task 3

Split the triangle in half
Compare the formed angles
and lengths

Task 4

Make a procedure for an isosceles
with one angle and one length input

Figure 7.3.1. N. and P.
The tasks for the category 2 activities

following triangle properties: the properties of the bisector of the isosceles triangle, how these properties do not apply for the other two bisectors, the equality of the triangles formed by the bisector, the concept of triangle equality, how these properties may apply to an equilateral triangle, generalisation of the properties for equilaterals and isosceles, and finally generalising the turning of the isosceles trinangle in an upright position. For this purpose, the researcher employed a method of asking the children to compare angles or lengths and challenging their theories by inquiring into the extent of the generalisability of their findings.

At a deeper level, the aim of this task was to illuminate the children's interplay between their intrinsic schema and the euclidean concepts, by using the turtle's measuring tools in the described context.

In their fourth activity (task 4, fig. 7.3.1), the children constructed a generalised procedure for an isosceles triangle with one angle and one length input. The task was based on their attempt to generalise the rule of how much the turtle should turn at the beginning so that any triangle would be upright. The request involved constructing a procedure with a variable input for the size of the equal angles. The aim of the task was two fold; firstly to investigate the nature of the difficulties and confusions arising from the highly abstract elements of the task (e.g. understanding that knowledge of one angle is sufficient while dealing with an abstract variable for that angle). Secondly, to follow the development of their understanding of the ideas embedded in the procedure while they used it to make various triangles.

For reasons of clarity, the analysis of the activities is presented under task headings. It is within each "task" section that findings pertaining to the research issues are analysed.

7.3.2a) Constructing an isosceles triangle.

- Using abstract information for a geometrical construction.

The children were not meeting the task to construct an isosceles triangle for the first time; they had been given the task at an earlier stage, as a means of introducing the POST primitive. Although the task seemed to have had success at conveying a meaningful context for the use of the POST command (see section 7.2.2c), **the children had had difficulties in understanding the abstract nature of the given information.** For instance, they had repeatedly asked the researcher to give them the length of the two equal sides and decided on a length of their own only after a clear statement that they could do so. What is more, the variability of the angles implied by the task seemed even more difficult for the children; they did not seem to explicitly take account of internal angles and turned the turtle by giving fixed, "obvious" quantities:

```
POST "A
LT 45 FD 60 LT 90
POST "B
FD 60
POST "C
LT 90 LT 45
FD DISTANCE :A
```

The previous argument becomes apparent from the children's strategy of turning the turtle after "posting" the third point (C), to face point A. They turned the turtle the same fixed turn of 90 degrees, as in the previous vertex and then in a "perceptual" way decided to turn another 45 (which - unfortunately for the researcher - happened to be the correct turn). **It did not seem to occur to them that the turning quantity was "unknown", despite the fact that they seemed to assign such a characteristic to the length of the third side by using the ruler to measure the distance.** It is interesting to consider, however, whether this characteristic came across to

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the children as being complementary to the given length information, i.e. that they could see that a length quantity was unknown as a contrast to the given length information regarding the other two sides of the triangle. Note that the children attempted this task before tasks 1 and 2 in set 3 (fig 7.2.1), where they were given fixed information, i.e. the division between "knowns" and "unknowns" was "concrete".

It is suggested that the children's subsequent experience with constructing figures by using given restricted information on fixed lengths and angles and measuring to find the (fixed) "unknown" quantities (tasks 1 and 2, fig 7.2.1) was essential for their developing some understanding of the abstract nature of given generalised information, such as "a triangle with two equal sides". For instance, when the task was given again to the children in the fifth session (task 1 fig. 7.3.1), Philip seemed to realise that the angle between the two "given" sides could vary, and in order to construct the required triangle it was he who had to "fix" a value for it:

R: "Right, so a triangle, which has two equal sides."
P: "Ah, O.K. (started typing). And then you want us to measure?"
R: "O.K. afterwards, yes." (Philip has typed: POST "A FD 70 POST "B)
P: "Now it doesn't matter how much we'll turn... let's do it... so that it isn't straight or 90, it will be 135."

This implicit insight *("...it doesn't matter how much we'll turn...")*, followed by Philip's taking of the responsiblity of proposing a fixed value for the turn, **seemed to set the basis for discussing the issue of "bringing in" their knowledge of a Euclidean theorem concerning angular quantities** (equal angles) to challenge the necessity of measuring them. This issue is discussed in the following section.

It was during the construction of the isosceles triangle, after Philip had turned the turtle 135 degrees and made the second side when **Nikos seemed to make a link between the task and the geometry they had done at school** (they had done that particular topic in the last term of the previous year):

N: "Shall I say something? I think that in order for a triangle to be isosceles, it must have two equal angles as well."

The researcher intervened at that point, in an attempt to encourage the children to contest this "factual" knowledge by measuring the angles. Although the children did not seem to doubt the truth of the geometrical "theorem", **they did seem to give a meaning to the measurements they made for the two remaining turns, i.e. that of verifying it.** For instance, before the measurement was done, Nikos expressed a rather elaborate mental calculation providing the size of one of the two equal angles, by apparently "combining" two theorems:

N: "And I say it will be 22.5. Yes because 22.5 and 22.5 is 45, and this is 135 and 135 and 45 is 180, see?" (fig. 7.3.2)

POST "A
FD 70
POST "B
RT 135
FD 70
POST "C
RT DIRECTION :A
FD DISTANCE :A
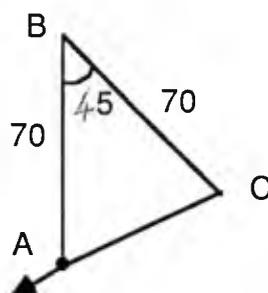
Figure 7.3.2 N. and P. : Constructing the first isosceles triangle

Notice, however, how he did not seem to make the link between the involved "intrinsic" notion of the 135 degree turn and the non

- intrinsic notion of an internal triangle, mistaking one for the other (fig. 7.3.2).

Although the children initially seemed somewhat surprised by the researcher - proposed idea to contest the "truth" of geometrical knowledge they had learned at school, they seemed to engage purposefully in the activity of "verifying" this knowledge by constructing more isosceles triangles for this reason.

- Using Logo programming ideas for a geometrical construction.

The children constructed several isosceles triangles in direct - drive mode and decided they wanted to make one which was "upright". Their strategy for doing this was by a right turn at the start of the construction, the quantity of which they determined in an approximating perceptual way. **Nikos then had the idea of making a procedure for an isosceles triangle.** The reason he gave for doing this seemed to be to avoid having to write the same commands again and again, a reason which had been discussed among the children in the Logo club towards the end of the previous year (see preliminary study). The children called their procedure Y, and included the initial right turn they had used before to turn the isosceles triangle in an upright position (fig. 7.3.3 - 1):

```
TO Y                        TO  Y :NIK
RT 37                       RT  :NIK
POST "A                     POST "A
FD  90                      FD  90
RT  100                     RT  100
POST "O                     POST "O
FD  90                      FD  90
POST "Q                     POST "Q
PR DIRECTION :A             PR DIRECTION :A
RT 130                      RT 130
FD DISTANCE :A              FD DISTANCE :A
END                         END
        1                           2
```

Figure 7.3.3 N. and P. : The first procedure for an isosceles triangle:

Approximating the initial turn

In response to the researcher's taking of this opportunity to set the task of turning the isosceles triangle in a precise upright position (task 2 fig. 7.3.1), **the children, not surprisingly, decided to apply an approximation strategy, i.e. by trying out different inputs to the first RT command of the Y procedure. Nikos then had the idea of using a variable for the input to RT, seemingly as a result of realising the tediousness involved in editing the procedure for each new input** (fig. 7.3.3 - 2).

- An intrinsic and a euclidean argument in an angle problem.

Not surprisingly, for the children, the approximation strategy for finding the "correct" quantity for the right turn, in order to make the isosceles triangle upright, seemed quite valid. This validity was strengthened by the fact that - due to the unsatisfactory quality of the screen resolution - a horizontal line was perceptually distinguishable from one with the slightest slant (it had no "dents"). The researcher intervened at that point, firstly to point out the limitations of their method by drawing links with previous tasks (e.g. task 1, set 1 and task 1, set 2, fig. 7.2.1) and secondly in order to encourage the children to approach the problem in an analytical way.

The discussion which arose from this intervention proved fruitful, in the sense that **although both children devised a sound argument to "prove" that the quantity of 40 degrees (which had been decided upon in a perceptual way) was correct, each child had a fundamentally different approach to the problem.**

Philip's argument, which was based on angle sizes, was that the 90 degree angle (fig. 7.3.4 - 1) consisted of the known angle of the isosceles triangle plus the one they were looking for and therefore the problem was one of subtracting the former angle from 90. **Nikos' argument, however, was based on turtle turns.** Starting from point A, the turtle facing towards point O (fig. 7.3.4 - 2), he verbally added up the quantities of the turns at points O

and Q, and partitioned the turn at point A into a turn of 90 degrees (to make the turtle face upwards), plus "what's left till 360", which, he said, was the total turn. It is interesting to consider at this point, that Philip and Nikos had devised equivalent strategies involving angles and turns respectively in a previous task involving the sum of the angles of a quadrilateral (task 3, set 3, fig. 7.2.1).
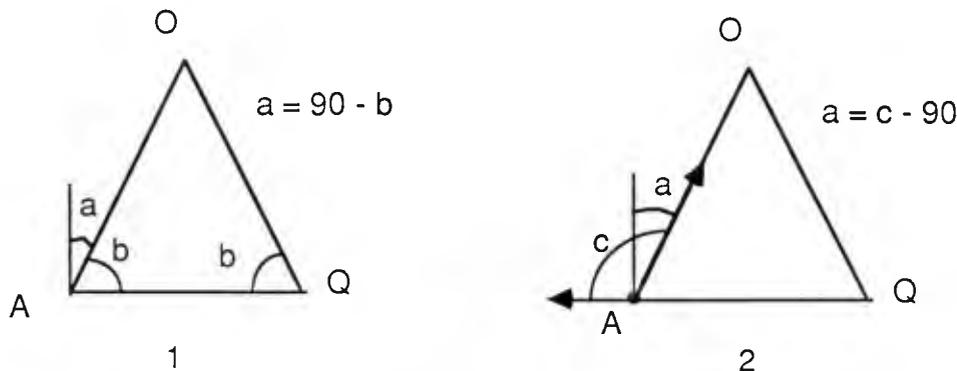
Figure 7.3.4 N. and P. : A "Euclidean" (Philip) and an "Intrinsic" (Nikos) view of the initial turn problem

7.3.2b) Bisecting the isosceles triangle.

- Challenging the "perceptual schema".

In the outset of their task 3 activities (fig. 7.3.1) the children loaded their Y procedure of an isosceles triangle on the computer, typed Y 40 to get the figure "upright" on the screen and were given the task to split the triangle in half. **The perceptual nature of Philip's strategy for solving the task was immediately challenged by Nikos, who did not seem to be satisfied by the fact that they did not "know" a certain quantity:**

P: "Shall I tell you what I think? (talking to Nikos)
     Here we'll go here (he means take the turtle from
     A to O (1))   and there (at point O)   turn it that much to
     go there." (shows middle of the base)
N: "Yes, and how do we know how much that t is? (means
     the turn) From where it is   (turtle at point o)   it could
     be 90 degrees, it could be 200. Shall I say my idea?"
P: "Go on."
N: "We'll see how much it is from A till Q, the distance
     (he means measure (2))   We'll divide it by
     two, that is we'll find half of what it is, we'll turn it
     towards point O, we'll tell it therefore PR DIRECTION O
     or RT DIRECTION O, as you like... and we'll take it to
     point O. "(He then tyrped RT 180, FD (DISTANCE :Q) / 2)
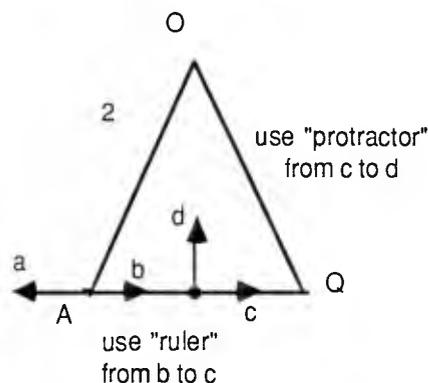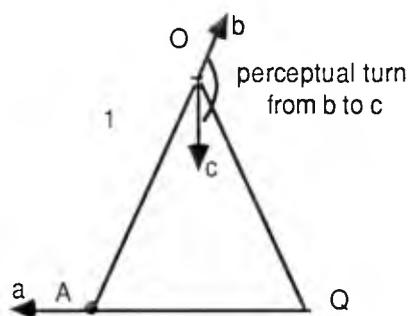
Figure 7.3.5 N. and P. : Niko's use of the measuring instruments
to challenge a perceptual strategy

Nikos' strategy seemed to have predominantly intrinsic characteristics, i.e. it seemed to be concentrating on the turtle performing actions. **The interest however, lies firstly in his apparent respect for the value of employing an analytical method to find an accurate quantity and secondly the discrimination of when this method was needed.** For instance, he typed RT 180 (fig. 7.3.5 - 2, from state a to b) using his knowledge for the turning quantity, but used the ruler for an unknown forward quantity.

The children's strategies for the next step, i.e. to turn the turtle towards the vertex at point O, were along the same lines: Philip suggested a left turn of 90 degrees and executed the command on the computer, while Nikos verbalised a doubt for the accuracy of the quantity and decided to "check", by using the protractor:

(Philip typed LT 90, turtle in state d, fig. 7.3.5 - 2)

N: "Let's check too, let's see... PRINT DIRECTION, shouldn't we check? (Philip agrees, Nikos talks while typing PR DIRECTION :O) *To be sure, what are we going to do, non - straight lines? What kind of engineers will we become?* (Nikos presses RETURN, result on screen: 0.0). *Ah, yes, 0 point 0, that's correct, we're something!"*

This was essentially a different use for a measuring instrument, which Nikos seemed to have picked up earlier, in the first category tasks, with the researcher's encouragement; **using the protractor did not seem to convey the meaning of measuring the quantity of a potential turn, or the size of a "static" angle, so much as a notion of verifying a turtle heading by means of it's relation to an external location.**

This time, however, during both the previous episode and the present the researcher made no interventions, i.e. there was no "prompt" to challenge the validity of the perceptual strategy. It could be proposed, therefore, that in this case, **Nikos developed, out of his own accord, this "sensitisation" of the existence of two strategies and of the merits of the analytical strategy, i.e. it's accuracy.** The previous extract illustrates this point by means of Nikos' wording which seems to reveal his engagement in the task and his reasons for challenging the "perceptually" obvious *("...to be sure, what are we going to do, non - straight lines? What kind of engineers will we become?..")*.

- The children's attempt to develop, prove and generalise a "theorem".

Although **Philip did not seem ready at that point in time, to start to change his predisposition to employ a perceptual schema** when he perceived a problem as a turtle task, the case was not so when he thought of a problem as a "geometrical" one (i.e. "belonging" to school geometry), as he seemed to do - eventually - in the next requirement of comparing the angles formed at the vertex at point O (fig. 7.3.6a).
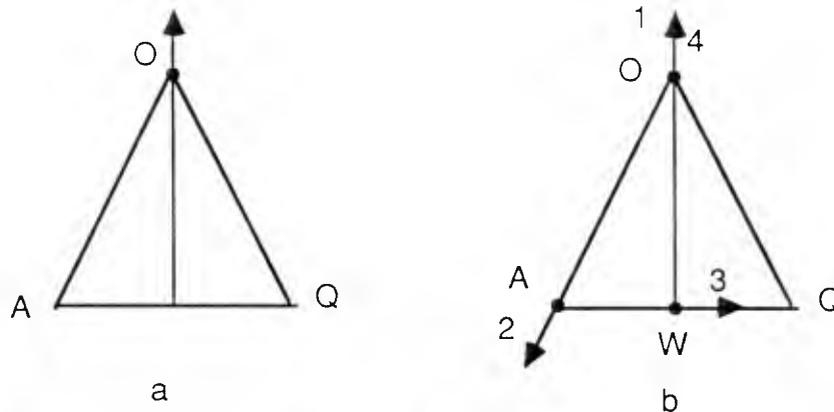
Figure 7.3.6 N. and P. : Posting the mid-point to compare angles AOW and WOQ

The children had decided to post the mid - point of the base, in order to name the angles in question, Nikos realising that they could not take the turtle there (from point O) along the altitude and deciding to take it via point A (fig. 7.3.6b - 1,2,3). After posting the mid - point (W), they decided to compare the angles (WOQ and AOW) by taking the turtle to point O in order to measure them (fig. 7.3.6b - 4). Not surprisingly, Nikos seemed to use the relationship between the angle of 180, and the quantities of the turtle potential turns towards point Q and point A, verbally **demonstrating an understanding of the relationship between turns and angles** (as he had done in previous measurements during the category 1 activities). The developing sophistication of his method lies in **the avoidance to perform a turtle action, by measuring two different angles which he then related to the ones required by the task** (fig. 7.3.7)

?PR DIRECTION :Q
139.999
?PR 180 - 139.999
40.001

?PR DIRECTION :A
220
?PR 220 -180
40
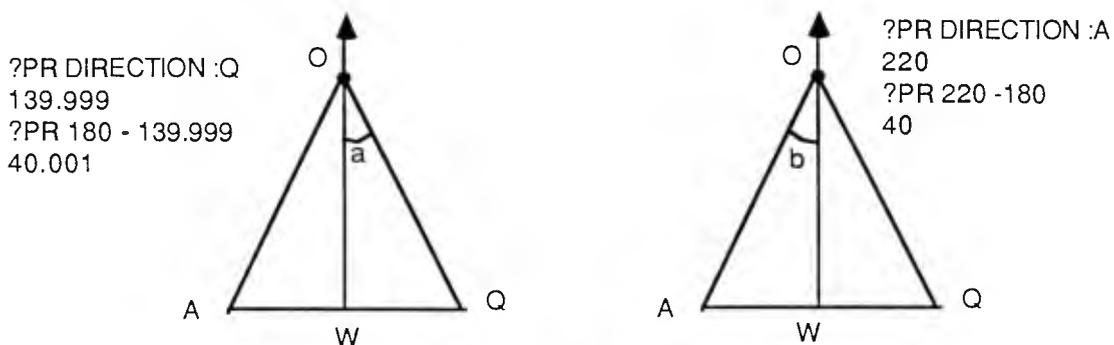
Figure 7.3.7 N. and P. : Comparing angles a and b

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The researcher's probing of the "geometrical" aspect of the task, by asking the children to draw their conclusions from their measurements brought an insightful comment from Philip, who seemed to have been thinking of the problem in a global way:

P: "...if we cut an isosceles triangle in the middle, two equal triangles come out."

It cannot be easily implied that Philip was actually aware of the generalisation he had made with respect to the isosceles triangle, since school geometry was taught in the same "general mode", i.e. general theorems were taught without, apparently, allowing much ground for the children to make the generalisations. However, his following comment could be **an indication of reaching his triangle equality conclusion by an implicit synthesising of the equal quantities they had measured:**

R: "Hm. What does equal triangles mean?"
P: "Equal, completely equal. That is, they have the same sides, the same angles..."

An interesting point, however, is the difficulty that Philip found in analysing his answer by providing precise comparisons of the respective triangle elements. **It is proposed that the difficulty lay in the deductive nature of the argument,** i.e. to start from a generalisation in order to compare specific elements. In support of this argument, when the researcher asked for the "proof" in a very precise way by asking Philip to compare specific elements, he did not seem to have problems with providing it. Philip did so for the angles, and Nikos, who had been taking part in the discussion, seemed to realise the "essence" of the analytical comparisons process, by volunteering to "prove" the equality of the lengths:

N: "They are the same, they are the same (excited). This, isn't the triangle

isosceles? There, these are equal (AO and OQ) *then. Isn't it split in two? So, won't these be equal?* (AW and WQ)."

R: "And the third side?"

N: "Eh, that's common for both of them."

It is therefore suggested that the inductive process of reaching a generalisation as a result of specific comparisons provided a means for the children to form an understanding of the euclidean theorem of triangle equality.

<u>- Philip's attempt to expand the applicability of a "theorem".</u>

The researcher therefore, decided to prompt their understanding by asking the children whether this "theorem" applied to other isosceles triangles, apart from AOQ. During the discussion, after they decided that the "conclusion goes for all the isosceles", Philip attempted to make another generalisation:

P: "Can I say something? This goes in all... in all the isosceles, all, all, all and in the equilateral ones. (researcher asks why) *Because the equilateral is like you've got the isosceles, only... only if it was equilateral all would be equal, and whenever you split it, it would be in the middle of the base because there are three bases in the equilateral, it would come out always equal."*

Philip seemed to have made a verbal attempt to convey the meaning of an isosceles triangle as a synthesis of triangle properties. It could well be that this perspective of an isosceles triangle played a role in his insightful remark, which in effect, seems to convey the notion of the equilateral as a synthesis of properties belonging to the isosceles triangle *("...there are three bases in the equilateral...")*.

The researcher attempted to probe the children's understanding of the notion of refuting a generalisation by asking them if their conclusion applied for all the medians, encouraging them to construct the median from point A (fig. 7.3.8). The children constructed the median with the same strategy as before, i.e. taking the turtle half - way between the O and Q vertices, posting (S) the point and using the "protractor" to turn to face the opposite vertex (A). In the discussion that followed, **the children found several counterexamples to the "triangle equality" theorem by comparing elements** (for instance, angles AOS and AQS, sides AO and AQ), as is illustrated by Nikos' comment:

N: "These angles here, this one and this one (AOS and AQS) are not equal, nor are these two sides equal (AO and AQ), therefore they're not equal (the triangles)."
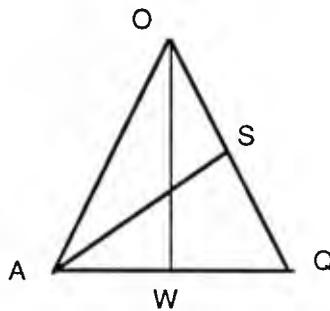
Figure 7.3.8 N. and P. : Refuting a generalistion by a counter example

An apparent factor in their arguments was their use of "known" elements, rather than ones which seemed unequal in a perceptual way. Not surpisingly, however, **the notion that one single counterexample was sufficient did not seem to occur to the children.** This would corroborate the view of Dreyfus and Hadas who argued that this notion is difficult for children even in secondary school (Dreyfus and Hadas, 1987). Nevertheless, there was some indication of an implicit contemplation of a "related" notion, since the children did not proceed with measuring all the elements, but seemed

convinced with one angular and one length inequality.

7.3.2c) Writing a procedure for a generalised isosceles triangle.

The researcher gave task 4 (fig. 7.3.1) to the children in the context of their subsequent writing of procedures for isosceles triangles with fixed elements of different sizes.

- Generalising the relationship between angles and turns

The children decided to call their procedure LASER :N :P, the first input denoting the length of the equal sides, the second denoting the size of the equal angles. During the process of writing the procedure in the Logo editor - after having typed POST "I FD :N POST "H - they came across the problem of working out a relationship between the angle input and the first turtle turn at point H. Initially, however, they did not seem to realise the nature of the problem, i.e. **it did not seem to occur to them to look for an underlying generalisable relationship between the turn and angle quantities,** as Philip's comment seems to indicate:

P: "There, we know that this and that will be equal (the two sides) *these two angles we know them* (equal angles)*and this* (angle IHM) *will be on its own. What can we put up there?"*
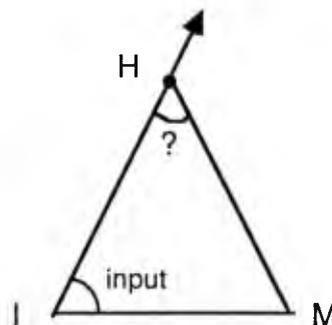
Figure 7.3.10 N. and P. : The problem of the relationship between the angle input and the other angular quantities related to the construction of the isosceles triangle

When the researcher intervened to encourage the children to use a specific example in order to think about the relationship, Philip responded with ease:

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R: "Why don't we try an example."

P: "Ah, yes, like that it's easy. Let's say that the two equal angles are 40 each, the top one would be 100."

R: "The top turn?"

P: "80."

Philip's answer seems to indicate that he could use the relationship for a specific case - note how he seemed to make the calculations mentally and provide an instant, coherent answer involving the results. However, he seemed to be implicitly aware of the difficulty of working out a generalised form for it *("...like that* (i.e. through an example) *it's easy* (implying perhaps the contrary for the generalisation)...").* **When the researcher probed him to reflect on the method he used for the example** *(R: "How did you work that out?"),* **he did not seem to be able to do so, answering that the turn would be different if he tried another angle.** However, the researcher's subsequent attempt to probe Nikos to reflect on this issue was more successful:

N: "These two... these will be equal (the angles HIM and IMH), *you can add them, that is you can multiply them by two, yes, and subtract them from 180, and then, whatever it finds* (the computer) *subtract it from 180."*

Nikos' first argument seems to refer to the relationship between the internal angles, and the second seems to link the internal angle at the H vertex (fig. 7.3.10) with the turn. It is proposed that the sophistication in Nikos' answer lies in the fact that **he seemed to have an insight into a general relationship, i.e. he seemed to make an generalisation from the previous example.** In an attempt to preserve the focus of attention on the relationship problem, the researcher wrote the algebraic / Logo symbols of Nikos' strategy on paper, as Nikos was expressing it (the children had not had experience with such formalisations in school). The researcher then probed the children's ability to distinguish between a calculation and an

action, since Nikos had typed the algebraic expression in as an input to a PRINT command, i.e. PR 180 - (180 - (2 * :P)):

"R: "That (the PR command and its input) will just tell us how much that is."
P: "Ah, yes. But it won't do it for us will it?"
N: "No... RT, can I do it?"
P: "But can we do RT the subtraction?"

The children's response illustrates how they seemed ready to use an action - quantity schema, i.e. how **they seemed to perceive the outcome of the "calculation" as the quantity of the turtle turn, which could indicate that they related the algebraic expression to their strategy. What was not apparently obvious to Philip, however, was the use of the action - quantity schema when the quantity is reached by some method, rather than given as a straight numerical input.** It seems interesting, therefore, to consider Philip's extent of relating his experience of measurements while using the P.D.D. microworld (if one draws a parallel between the previous numerical calculation and a "measuring" calculation), to the intrinsic schema of action - quantity.

```
TO LASER :N :P
POST "I
RT 90 - :P
FD :N
POST "H
RT 180 - (180 - (2 * :P))
FD :N
POST "M
RT DIRECTION :I
FD DISTANCE :I
RT 90
END
```
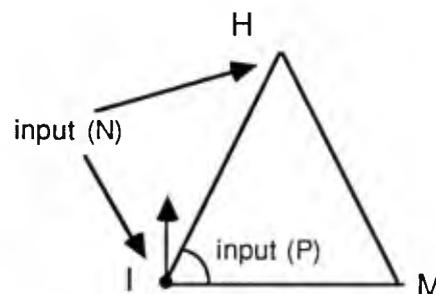

Figure 7.3.11 N. and P. : The children's LASER procedure
for a generalised isosceles triangle

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In spite of the analytical method by which the children constructed the LASER procedure (fig. 7.3.11), **it took them a relatively long time to discriminate the concepts involved in its execution (i.e. use) on the computer. The children seemed to have a difficulty in understanding the nature of the angular input (i.e. angle or turn) and in connecting it to the elements (angles and turns) of the figure which the procedure constructed.** Philip seemed to reveal limitations in his understanding of the relationship between the internal equal angles (e.g. HIM, fig. 7.3.10) and the turn at the H vertex, while Nikos showed bemusement with the result of his (correct) calculations when, thinking that the input was a turn, did the calculations with the complementary quantity.

The children's initial efforts to understand how the LASER procedure "worked", however, seemed product - oriented, i.e. it did not seem to occur to them to use an analytical step - by - step method. It is interesting to consider the consistency of this finding with the findings from the T.C.P. microworld study where the children tended to try to incorporate all the coordinate factors in one go. This finding could be related to the Greek educational system, regarding the relative lack of focus on investigational activities.

The researcher intervened twice taking the role of the teacher, in order to encourage the children to think of the relationships between the angular quantities in an analytical way. The first intervention did not seem to have much effect in the children's discrimination process. However, there was a difference in the nature of the second intervention; **the researcher asked the children to think in terms of turtle actions, attempting to encourage the employment of their intrinsic schema.** Philip's response and Nikos' "debugging" of the response was as follows:

P: "It (the turtle) *puts a POST, it goes forward as much as we've put the*

variable, then it does a POST again, then it does... a subtraction... it does the angle times two, it subtracts it from 180 and what it finds it subtracts it from 180... so that it can find the top angle."

N: "I think Philip said something about the top angle. I think it's the top turn."

It is interesting on the one hand, but perhaps not so surprising, that **adopting a sequential strategy involving the use of the intrinsic schema** (in the T.C.P. microworld study a sequantiality schema is proposed as a component of the intrinsic schema) **seemed to provide the children with the means of discriminating component parts of a generalised conceptual tool, such as the LASER procedure.** On the other hand, however, it may be useful to consider how, **in the process of thinking about sequential turtle actions, the children seemed to use other mathematical notions,** such as a "primitive" notion of variable *("...it goes forward as much as we've put the variable..."),* or a geometrical relationship derived from "euclidean" axioms and theorems (relating two equal angles of the isosceles triangle to the sum of the angles of a triangle). Moreover, during the children's subsequent executions of the procedure with different inputs, and their discussion on the meaning of the inputs in relation to the graphical outcomes (fig. 7.3.11), **Philip seemed to make the connection between turtle turn and internal angle for the first time on his own accord,** i.e. not as an agreement to a comment from Nikos:

(they had just executed LASER 50 89, the discussion starting with the researcher asking why the figure on the screen was a triangle)

P: "Because the top angle isn't zero... look these two are 89. 89 plus 89, 178. 178 minus 180, 2.

N: "178 - 180 is minus 2. You've said it the other way round. But this, why is it like that?" (he means line distortion due to screen resolution)

P: "Look it turns 2 degrees, I mean the angle is 2 degrees, it turns 178 degrees..."

What Philip seemed to realise here was the importance of distinguishing

angle from turn, i.e. **he seemed to discriminate the two notions not as a result of being asked to, but on his own initiative, in an explanation to his peer.**

- Generalising an angle relationship to make the isosceles triangle upright.

The issue of turning the isosceles triangle in an upright position was put forward by the researcher regarding a specific triangle (LASER 50 3), while the children were trying out different inputs to LASER. Philip seemed to apply his strategy of 90 minus the internal angle (see fig. 7.3.4) in this specific case, by actually typing in RT 90 - 1.5 (the fact that the size of the angle - 1.5 - is incorrect is not relevant) in the generalised LASER procedure in the Logo editor (fig. 7.3.11). **It seems, therefore, that Philip had been able to "transfer" the application of his strategy from one specific case to another** - note that there was a considerable length of time (15 days) in between. However, as the following extract illustrates, Philip seemed to have an insight into a genaralised application of his strategy:

R: "So how will we make the triangle straight?"
P: "We'll tell it... 90 minus... 1.5 (he went into the Logo editor and typed RT 90 - 1.5 in the LASER procedure, fig. 7.3.11). *Ah, this thing to make any isosceles?... I've got it sir...* (he deletes the previous and types in RT 90 - :P). *Shall we do it?"* (he means execute the procedure).

It could be argued that Philip's insight into a generalised use of a relationship seemed to be a result of a synthesis of the previous specific applications. It is interesting to consider the role of the LASER procedure in this case, since it could be that the generalised angular input "required" from the children to think in general terms in order to debug or add things in LASER.

Some print cut off
Edge of page
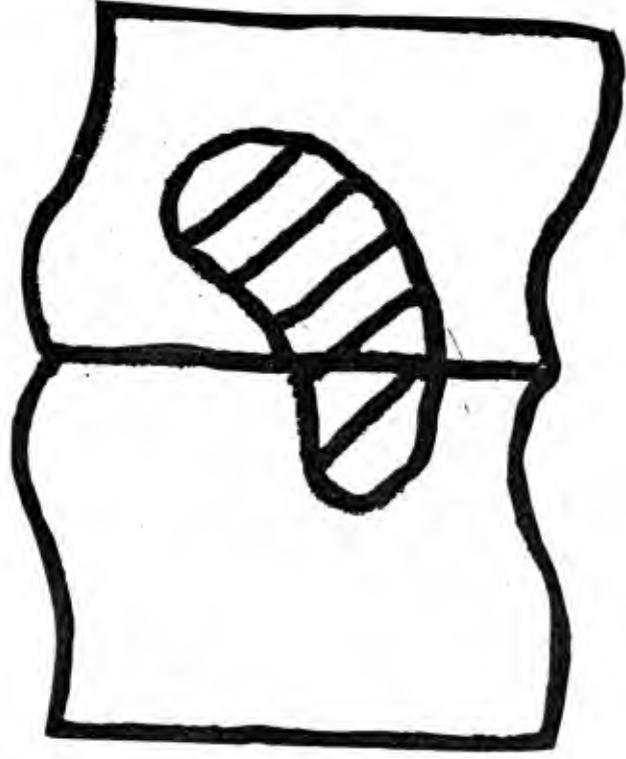
THIS VOLUME HAS A
VERY TIGHT BINDING
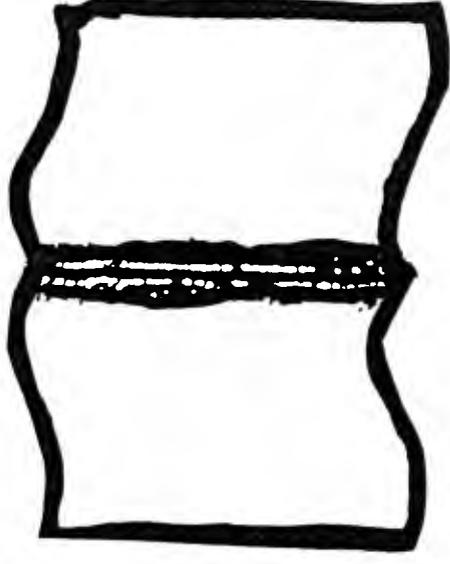
7.4 CATEGORY 3: DECIDING ON THE USE OF THE TOOLS IN PERSONAL PROJECTS

7.4.1 Task analysis

The two types of activities involved in this category are in the form of children's own Logo projects ("investigations" was the name used in the school Logo program, see appendix B). The aim was to gain insight into the role of their experience so far with the microworld on if, how and why they used the tools in "personalised" activities, and on the geometrical ideas they used while they were engaged in such projects.

The first two activities had the restriction of using the children's generalised procedure for an isosceles triangle. The task was presented in the familiar to them way i.e. an "investigation with an initial idea", the "idea" being the procedure (see chapter 5 and appendix B). The aim of the task was to gain insight into the way they integrated the concepts involved in using the procedure with conventional Logo ideas.

The two subsequent "investigations" had no restrictions at all, and this was stressed to the children. The aim was to gain insight into if and how they would use the tools for their own purposes and into the geometrical ideas underlying their activities. The presentation of the results begins with an analysis of how the children built a "naive" geometrical theorem during their first investigation. A summary of the children's activities during the remaining three "investigations" is then presented, in order to allow for a subsequent analysis of research issues across "investigations".

7.4.2a) The first "Investigation": Using Logo to build a personal geometrical "theorem".

After an introductory discussion with the researcher concerning the notions of an "investigation" and an "initial idea" and incorporating examples from the children's Logo club activities, Philip and Nikos decided to construct a shape made of "nested" triangles and drew the figure on paper (fig. 7.4.1). Their strategy, which incorporated a substantial degree of the use of perceptual cues and an organised "approximation" technique, was predominantly from the smallest triangle "outwards", i.e. to larger ones.
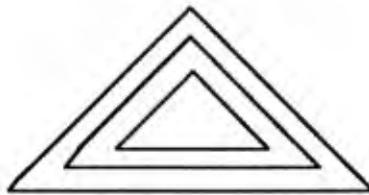
Figure 7.4.1 N. and P. : First "Investigation": "Nested triangles"

The first part of the investigation consisted of the children's attempts to solve the problem of determining the quantities of the interfacing commands between the first and the second triangle, and the length input to LASER for the second triangle. **Their approximating "trials" on the computer seemed to have an implicit modular structure,** i.e. a "module" (trial) involved:

a) near - constant length inputs to LASER for the first triangle,

b) a constant strategy for the interface involving a horizontal and a vertical change of position, using 90 degree turns and changing the quantities of the FD and BK commands (fig. 7.4.2), and finally,

LT 90
FD (changing quantity)
RT 90
BK (changing quantity)
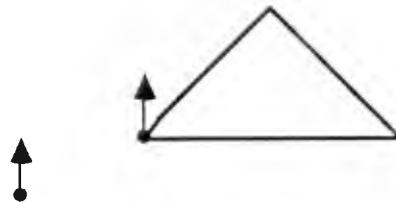
c) changing the length input to LASER to determine the size of the second
triangle. In table 7.4.1, the numerical quantities which the children gave in
each trial are presented.

Trial	Input to FD	Input to BK	Length input to LASER for the second triangle	Length input increment from first to second triangle
1	5	0	55	5
2	5	5	65	15
3	10	5	50	5
4	20	5	65	20
5	20	5	85	40
6	20 b	20	85 a	40
7	20	55	80	35
8	40	20	85	40
9	40	20	125	80
10	40	20	105	60

Table 7.4.1 N. and P. : First "Investigation": "Nested triangles" Approximations
for the interface between the first and the second triangle and for the size of
the second triangle

It is interesting that, although at first the children's trials do not seem to have a
specific organisation, one could argue that there seem to be underlying
decisions on holding at least one of the variables "steady" as, for instance, the
input to LASER from the 5th to the 8th trial (table 7.4.1 a) and the input to FD

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from the 4th to the 7th (table 7.4.1 b). It is useful to consider, however, that the decision for the three quantities was not simultaneous for each trial, i.e. the children had time to see the effects of an input and then make a decision for the next. Although the children were finally satisfied with the first two triangles, they seemed rather frustrated with the tediousness of their method, i.e. having to clear the screen and re - type the commands for another trial. The researcher decided to make an intervention, adopting the role of the teacher, feeling that this was an appropriate moment to suggest the use of a Logo programming idea, in the sense that the context and the timing provided an opportunity for the children to use it in a personally meaningful way. The suggestion was to make a procedure for the two triangles, so that they could carry on with their trials for the third. Although there are limited explicit comments illustrating the children's appreciation of the power of "superprocedures", the children did decide to follow up the suggestion, called their procedure BAM *("...because it makes two triangles with a 'bam'* "- Greek for "bang"), and made comments while using it, suggesting enthusiasm, but also **a rather implicit understanding of the procedure's "summative" power:**

R: "What would happen if we didn't have the 'concept'? (procedure)" *N: "Goodnight...* (laughter). *We would waste all our time..."*

The children, however, did seem to integrate the use of this new tool in their strategy for constructing the third triangle; the strategy retained the organised approximations characteristic, involving the use of perceptual cues and a "modular" structure. However, **the modularity was incorporated in the BAM procedure,** since the children (after checking that BAM had no bugs) went into the editor to add another "module" of interfacing commands and the LASER procedure (fig. 7.4.3). Their trials, therefore, consisted of a process of observing the graphical output from the execution of BAM and entering the Logo editor to change the relevant quantities involving the construction of the third triangle within the BAM procedure.

```
TO BAM
LASER 45 50
PU
BK 20
LT 90
FD 40
RT 90
PD
LASER 105 50
PU
BK 40
LT 90
FD 80
RT 90
PD
LASER 125
END
```
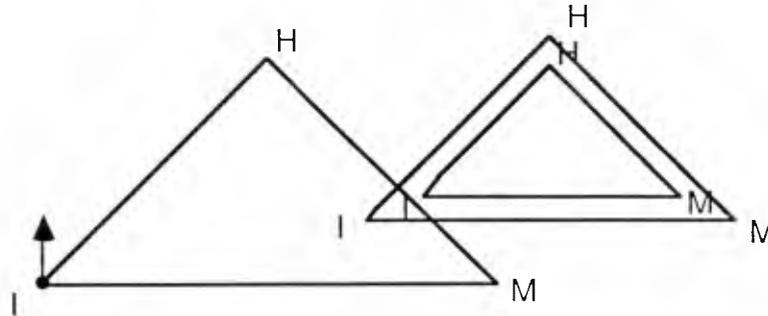

Figure 7.4.3 N. and P. : First "Investigation": "Nested triangles" The first "trial" for the third triangle

When the children were satisfied with their third triangle (the quantities of their trials are presented in table 7.4.2), the researcher - through the encouragement of a relevant discussion - attempted to prompt the extent and the nature of the organisation of the children's approximations trying not to impose his own organisation.

trial	input to BK	input to FD	angle input to LASER	increment from second triangle
1	40	80	125	20
2	20	40	105	0
3	20	40	165	60

Table 7.4.2 N. and P. : First "Investigation": "Nested triangles" Approximations for the interface between the second and third triangle and for the size of the third triangle

The children's response indicates that during the trials of the third triangle, they seemed to have started to look for a "theory",

or a consistent "rule" which would generalise the "transition" from one triangle to the next. The "theory" which they decided on in the end, incorporated all three variable quantities and could summatively be described as follows: there was a constant "displacement" of the position of the turtle for the interface between triangles (i.e. BK 20 LT 90 FD 40 RT 90) and a constant additive increase of 60 in the length input of a "new" triangle. It is interesting that **although** it seemed evident from the children's comments and from their typed approximations (tables 7.4.1 and 7.4.2) that **they had implicitly made a generalisation of a transition from one triangle to the next, they found it difficult to express their theory in a generalised form,** i.e. they did not seem to be able to avoid using numerical examples.

7.4.2b) The children's second, third and fourth "investigations": A summary of their activities.

A summary of the children's activities during the remaining three "investigations" is presented at this point, in order to allow for a subsequent analysis of research issues across "investigations".

- The "Striped Triangle".

Philip proposed the topic of this "investigation", drawing the figure on paper. The children decided to construct a triangle with the LASER procedure and a "stripe", in order to subsequently try out the "FILL" command, newly learned at the "Logo club". After executing the LASER procedure, they took the turtle along side IH and then decided that a POST was needed at the "opposite" side, in order to take the turtle there with accuracy (fig. 7.4.4 - 1,2,3,4). They took the turtle back along side IH, used the measuring instruments to take it to their posted point (D) (fig. 7.4.4 - 6,7), and then followed a similar strategy for the second segment of the stripe: they posted a point on side IM (point E), at a distance of 20 from (D), took the turtle at the same distance from the first segment on side IH, and then took it back to the posted point (E) using the

measuring instruments (fig. 7.4.4 - 8,9,10,11,12,13). The children then filled the stripe in with colour.

The episode analysis involves the children's use of geometrical notions embedded in the LASER procedure, a description of an episode where they meaningfuly used the P.D.D. microworld primitives out of their own initiative and their use of static angle and length notions to work out the quantity of turtle actions.

- The "Pyramid".

The children decided to make a "3 - dimensional" pyramid, using the LASER procedure for the central part and perceptual "projections" of triangles for the other two "visible" surfaces (fig. 7.4.5). They decided to use perceptual cues for the slant and the base of the triangle on the left, after an apparent conflict between reality (i.e. that the surfaces of the pyramid would be equal) and 3 - D representation on a 2 - D surface (fig. 7.4.5). After forgetting to put the pen down when they took the turtle to point H (fig. 7.4.5 - 2) and realising they had not posted the previous point, they decided to take the turtle on a "known" path in order to "draw" the remaining side of the triangle on the left (fig. 7.4.5 - 2,3,4,5,6). They did so by either using the measuring instruments in "action" mode or direct Logo commands. For the triangle on the right, they made a base symmetrical to the one on the left and took the turtle to H.

The episode analysis involves the children's use of static angle relationships to work out the quantity of the turn which was necessary for the symmetrical construction of the base of the right - hand triangle.

- The "Sailing Boat".

The children decided to post the turtle's starting point (point A, fig. 7.4.6), in order to be able to "close" the "bulk" of the ship. After constructing the "bulk", they took the turtle along points A, B and C again (they had forgotten to post

the last vertex of the "bulk") by using the instruments only when they had forgotten a quantity, made the ship's mast and tried out different methods to construct a triangular sail (fig. 7.4.6b, c). The analysed episodes involve the children's initial decision to use the POST command and their methods of constructing the triangular sail.

```
LASER 85 50
RT 40
FD 50
RT 90
RT 90
FD 50
RT 230
FD (DISTANCE :M / 2)
FD (DISTANCE :M) / 2
POST "D
BK (DISTANCE :M) / 2
BK (DISTANCE :M) * 2
PE
FD DISTANCE :I
PD
LT 50
FD 50
RT DIRECTION :D
FD DISTANCE :D
RT DIRECTION :M
FD 20
POST "E
FD DISTANCE :M
RT 230
FD DISTANCE :H
RT DIRECTION :I
FD 15
RT DIRECTION :E
FD DISTANCE :E
```
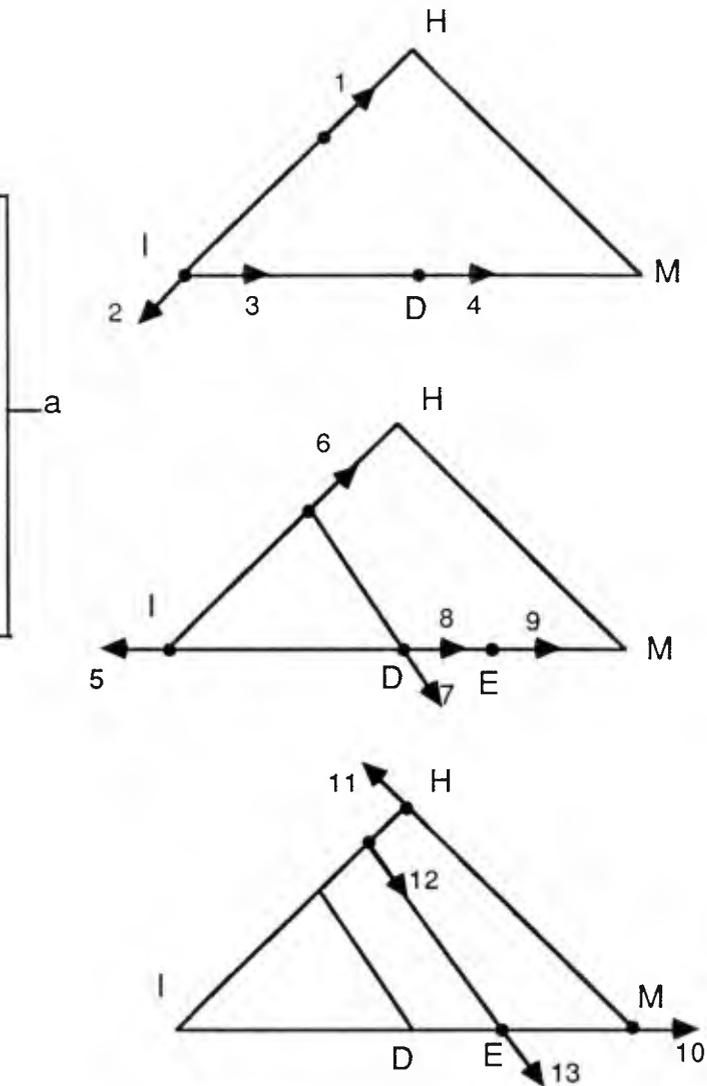

Figure 7.4.4 N. and P. : The second "Investigation": The "Striped Triangle"

```
LASER 50 70
LT 90
PR DISTANCE :M
(34.202)
RT 30
FD 34.202
PE
BK 14.2027
RT DIRECTION :H
FD DISTANCE :H
RT DIRECTION :I
FD DISTANCE :I
PD
BK DISTANCE :H
FD DISTANCE :H
FD DISTANCE :I
RT DIRECTION :M
RT 180
RT 30
FD 20
RT DIRECTION :H
POST "E
FD DISTANCE :H
RT DIRECTION :M
FD DISTANCE :M
LT 100
FD 20
RT DIRECTION :H
FD DISTANCE :H
```
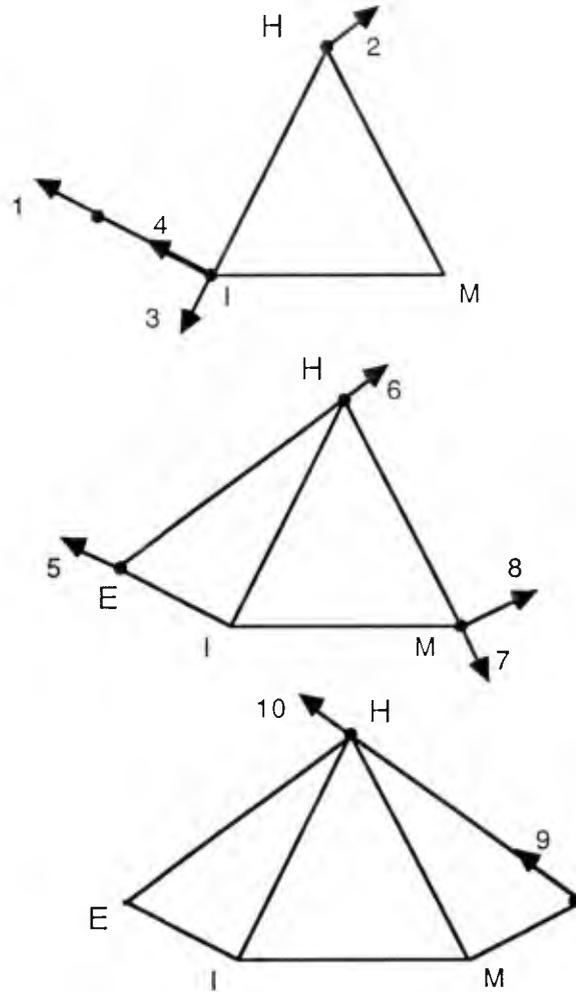

Figure 7.4.5 N. and P. : The third "Investigation": The "Pyramid"
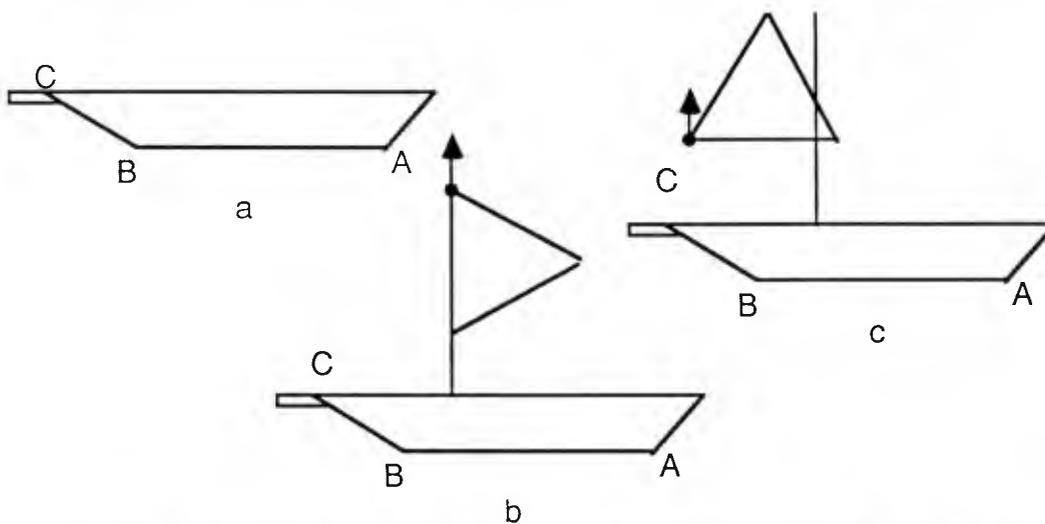
Figure 7.4.6 N. and P. : The fourth "Investigation": The "Sailing Boat"

In the first "investigation" ("Nested triangles"), **the children did not seem to focus on notions embedded within the LASER procedure, but seemed to treat the procedure as an object.** In the "Striped triangle investigation", however, the focus seemed to shift from the start, the children's discussion involving the form of triangle they wanted to construct, i.e. they decided to make it large and non - equilateral. The inputs they gave to the LASER procedure and their subsequent initial commands seem to reveal **the emerging of a coherent use of geometrical properties by the children, as a means of reaching a personal objective.** For instance, the children gave inputs of 85 and 50 for their "large and non - equilateral" triangle and after they decided to take the turtle 50 steps along side IH (fig. 7.4.7), implicitly used the angle properties to turn the turtle to face H:

N: *"85 the side* (of the triangle).*O.K.,*
we'll take 50 on the side, O.K.?"
P: *"Yes, it will go there* (shows with finger)."
N: *"Only..."*
P: *"How we'll turn..."*
N: *"50... is this angle... aren't these two*
angles 50? We'll turn 40."
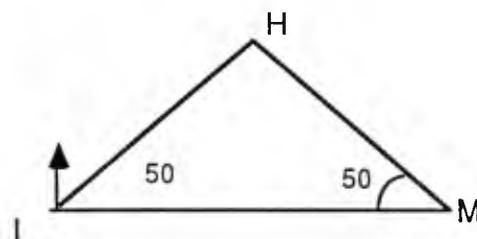
Figure 7.4.7 N. and P. : The second "Investigation": The "Striped Triangle".
The children's first use of the geometrical properties within the LASER
procedure

It seems interesting to contrast the ways in which the children seemed to be engaged in geometrical activities in this and the previous "investigations", i.e. **an investigative conjectural approach, leading to the formation of a naive personal theory involving "nested" triangles versus a more knowledgeable use of geometrical theorems and properties.** This distinction between the two styles of geometrical activities is not meant to question the validity of one or the other, but rather to highlight the versatility of the potential of using more complex tools (such as the LASER procedure) in

investigational work.

Initially, the children did not seem to have a pre - planned strategy to draw the first segment of the "stripe". They took the turtle to a 50 step distance from point I (fig. 7.4.8) and turned it 90 degrees right. **The following two extracts illustrate a difference between Philip and Nikos in the approach of the problem of drawing the segment,** the discussion triggered off by Nikos, who stopped after typing FD - apparently realising the absence of a non - perceptual way to take the turtle to the opposite side (fig. 7.4.8):

N: "FD... Oh!"
P: "What Oh? Put whatever, and then we can...
do FD and then guess about how much...
and if it's more... we'll put PE and when we
hit the spot... (researcher asks how he can
be sure he hit the spot)   *we can tell it to*
look towards M, and if it's..."
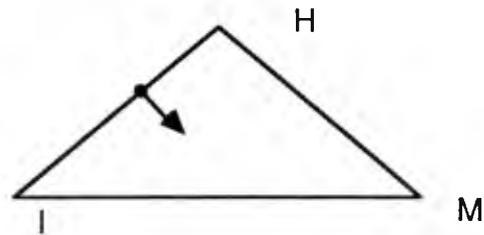
N: "Idea. I found it. We'll go to iota   (I) over here,
we'll tell it to go... from iota to mi   (M) ...
till half way. And from there we'll make a point...
and the rest we will know... we'll take it   (the turtle)
up here   (he means back to where it was when the
discussion started)   *and we'll make... we'll take it to*
the point..."

Figure 7.4.8 N. and P. : The second "Investigation": The "Striped Triangle". A method of approximation (Philip) and a method using "known" factors (Nikos) to solve the problem

Before analysing Nikos' answer, it seems interesting to consider **the resistance of Philip's perceptual schema to change.** Even though Nikos seemed to "provide" the awareness of the possibility of a different option by stopping short before typing an "approximation quantity", Philip seemed to insist on a strategy which, although organised and logical, offering a "checking" technique which brought on the precision offered by the use of

the P.D.D. microworld, was nevertheless based on perceptual cues. **This point is not necessarily made as an indication of a "lack" of understanding of a "knowledgeable" method on Philip's behalf -** on the contrary in a subsequent part of the discussion he verbally agreed at least on the legitimacy of such a method. **It is however proposed that, at that point in time, the perceptual schema seemed to retain a high status of priority in Philip's mind,** i.e. Philip seemed to attribute equivalent degrees of legitimacy in perceptual and "knowledgeable" methods.

Nikos' strategy on the other hand, seems to reveal a preference for the use of "known" factors. In Nikos' verbalisation of a plan of his strategy (fig. 7.4.8), and the subsequent application of the plan (see fig 7.4.4, a), the "known" factors which he seemed to use fall in two categories:

a) use of geometrical properties, theorems and axioms:

In his verbal plan, for instance, the use of geometrical notions was rather implicit, since Nikos' argument centred on the posting of a point on the "opposite" side of the triangle. However, he seemed to refer to "known" geometrical factors in the process of taking the turtle to the point and back *("...and the rest we will know... we'll take it up here...").* The commands he typed for turtle rotations illustrate a more explicit use of angle and turn relations (fig. 7.4.4a,)

b) use of the P.D.D. microworld primitives for "signposting" and quantity measurements

The "essence" of Nikos' strategy involved the use of the P.D.D. primitives in order to ensure the accuracy of a turtle change of position, when the intrinsic method of action - quantity was insufficient and, apparently, Nikos could not use euclidean geometrical notions to determine the quantity of the turtle action.

It could be argued that his idea to measure the required distance by taking the turtle to the "opposite side" to put a marker, and then back again to make the measurement, involved a substantial degree of precise planning and "foresight". In that sense, it seems useful to consider the role of the P.D.D. primitives in processes of planning and the involved thinking about "abstract" notions.

A relevant episode took place at a later session, at the beginning of the children's "Sailing Boat investigation" (fig. 7.4.6). After drawing a plan of the boat on paper and taking the turtle at a "suitable" starting position (bottom right of the screen), Philip suggested that that position should be posted. His argument to Nikos' challenge on the need for a post, was that it would make the closing of the shape of the "bulk" "smoother", since the turtle would be able to refer to the post with the use of the measuring instruments. The first point which seems important here, is that **the suggestion came from Philip, who apparently for the first time initiated the consideration of a non - perceptual method as a priority. Secondly, it was the first time the use of the POST command was suggested before a conflicting situation arose** (posting point E in the "striped triangle" seemed to be a result of posting point D which was discussed after a conflicting situation).

7.4.2e) The role of the P.D.D. commands on understandings of the concept of length.

Nikos' plan for taking the turtle to the "opposite" side of the triangle to place a post, involved moving the turtle from point I to half way between I and M (fig. 7.4.9). Nikos used a FD (DISTANCE :M) / 2 command, apparently adopting the same strategy as in the construction of the median in the isosceles triangle tasks (task 3, fig. 7.3.1).

In attempting to take the turtle back from the newly posted point (D) to point I, however, **the children did not seem to realise the dependence of**

the state - changing method of measuring the quantity of an action (i.e. FD (DISTANCE :M) / 2) on an external point (e.g. point M in this case):

(having typed FD (DISTANCE :M) / 2)
N: "POST... D, that's fun (types POST "D)
Ah, we should have asked."
P: "What?"
N: "How much the distance is, now we
have to go back."
P: "Yes, with BK DISTANCE."
(Nikos types BK (DISTANCE :M) / 2)
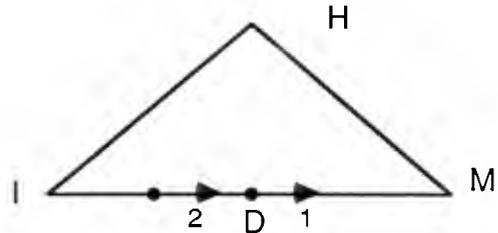
Figure 7.4.9 N. and P. : The second "Investigation": The "Striped Triangle".
Imposing an Intrinsic schema on distance

It could be argued that in this episode, the children used their intrinsic schema of a turtle action and its quantity in order to reverse the outcome of the turtle having moved forward, i.e. "the turtle has gone forward a specific quantity, now it will go back the same quantity" (see also chapter 6, Anna and Loukia). **The children's "mistake" could imply a lack of having discriminated between the geometrical notions embedded in the two methods of determining the quantity of a turtle action, i.e. a "turtle - centred" numerical quantity depending entirely on the turtle's present state and a quantity derived from a measurement involving the turtle's state and an external location.**

Further analysis of the data, however, revealed **another factor which may have played a role in the children's confusion, i.e. the abstract nature of a quantity derived by a measurement, when the measurement is performed in conjunction with the action,** e.g. using the DISTANCE command in a combined "action - information" mode (see category 1 activities) rather than making a measurement, and using the specific numerical outcome in a direct action - quantity command.

The following episode is described in support of the argument, since, in the process of constructing the second segment of the "stripe", and having to deal with specific numbers, **Nikos seemed to combine the notion of a "turtle - centred" quantity with that of a quantity depending on an external point.** The episode took place when, after having posted point E at a distance of 20 units from D (fig. 7.4.10), the children were discussing how much to move the turtle from the point at the H vertex, on the "opposite" side of the triangle, in order to subsequently "use" post E to draw the second segment. Nikos then seemed to make a mental calculation on known length quantities in order to find how much the turtle should move from point H, to be at a 20 step distance from the ("unposted") intersection between IH and the first segment (for convenience named "(A)" by the researcher, fig. 7.4.10).

(turtle at point H, facing towards I,
Nikos addressing Philip)
N: "I'll take it 15 forward... you know why?
Look... this is 50... this is 35... we want it
to go 20 don't we? Since all this is 85,
that's 35, we'll take 20, 15..."
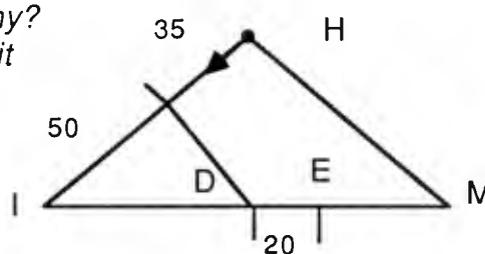
Figure 7.4.10 N. and P. : The second "Investigation": The "Striped Triangle".
Working out length relations to find the input to FD

The apparent objective of Nikos' operation on specific length quantities involved **the relationship of a turtle - centred command (FD 15) to an external point,** i.e. point (A), a relationship **which he had not perceived when having to deal with "unknown" abstract quantities** such as the length of IM and IM / 2.

A broader point could be raised here, therefore, of the potential role of the P.D.D. commands as tools with which the children could address problems involving operations on both concrete and abstract quantities. On the one hand, **the interplay between concrete and abstract values could be**

reinforced in environments such as the above, where the children have to deal with both in subsequent situations. On the other hand, the interplay between using the measuring instruments in information and action - information modes (see category 1) could **enable the children to perceive abstract quantities or relationships between quantities, as abstractions of concrete cases.**

7.4.2f) Using angle axioms and theorems to work out turning quantities.

On the subject of relating static angle notions with rotational quantities of turtle turns, the children showed distinct differences in their approach. **Philip,** on the one hand, with the perceptual approximation method still high in his priority list, **seemed to be lacking in developing connections between his knowledge of angle relationships and the quantities of turtle turns.** Although he agreed on such connections made by Nikos, engaged in discussions involving the relationships and often helped Nikos work out a relationship between angles, his predisposition was either to use perceptual cues or the turtle's "protractor", the DIRECTION command.

Nikos, on the other hand, **seemed to further develop his preference in using the direct action - quantity commands when he perceived the possibility of working out a respective quantity,** the developement referring to the degree of complexity in the respective geometrical problem Nikos had to face. **The following three episodes illustrate this developing complexity in the situations in which Nikos seemed to be prepared to work out a quantity.**

The first two episodes took place during the striped triangle project. Firstly, on the way to place a post (D) in the middle of IM, and in order to turn the turtle at point I, to face towards point M (fig. 7.4.11) Nikos, rather than using the DIRECTION command added the two angular components of the respective right turn (from a to b, fig. 7.4.11), explaining his strategy to Philip:

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(turtle in state a, Nikos addressing Philip)
N: "RT... 230. You know why I'm putting 230?
Because 230... look... from here to here it's
180 isn't it? (P. agrees) *This angle, isn't*
it 50? (P. agrees again) *O.K., 180 and 50*
doesn't it make us... for the turtle to go
woop, woop and it does us 230?"
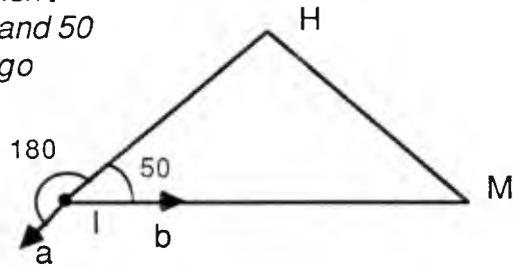
Figure 7.4.11 N. and P. : The second "Investigation": The "Striped Triangle".
Adding two components of an angle

Notice how Nikos made an angle calculation *("180 and 50 doesn't it make us..."),* seemed to pause to think and finally seemed to change to talking about the outcome of the calculation as if it was the quantity of a turtle turn *("...for the turtle to go... and it does us 230.").*

The second episode took place in the process of taking the turtle from point I on segment IH to draw the second stripe. Nikos added the two components of the right turn at point H (fig. 7.4.12), in this case calculating the internal angle involved, by using the respective properties of the triangle:

(turtle in state a)
N: "Turn it... 260. You know why? This is 180
and this is 80, you know why it's 80? 50
plus 50... 100, and 80... 180."
R: "And how much would it turn left?"
N: "Left? Shall I say... I think I got it... 100."
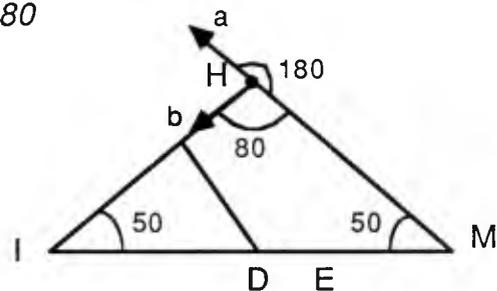
Figure 7.4.12 N. and P. : The second "Investigation": The "Striped Triangle".
Working out a component of an angle

It could be argued that, in using the isosceles and general triangle properties, Nikos seemed to be drawing upon his experience from the category 2

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activities, and in particular the ones involving the construction and use of the LASER procedure. In this case, however, the focus of attention was not on the isosceles triangle but, rather, on the problem in hand, i.e. to turn the turtle and eventually draw the second segment of the "stripe". It is therefore interesting to consider firstly Nikos' use of a theorem and an axiom in a context different to the one he first learned them in, and secondly **the difference between the nature of this activity and the ones in category 2, regarding the control of the learner over the learning situation, since in this case, the children were pursuing personal goals.**

The third episode took place during the "pyramid" project (fig. 7.4.13) in the children's attempt to draw the third surface of the pyramid and while they were discussing how much to turn the turtle at point M (fig. 7.4.14 - 1) so that the base would be symmetrical to EI (they did not use the term or the notion explicitly).
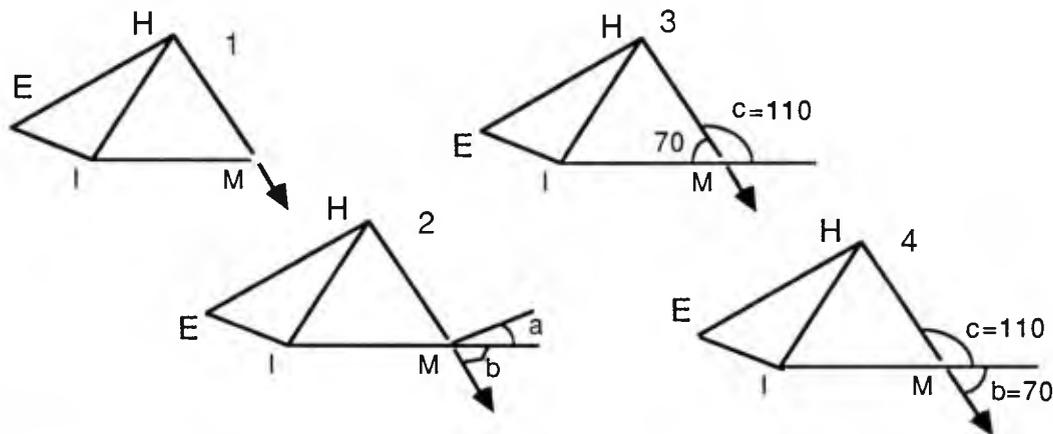
Figure 7.4.13 N. and P. : The third "Investigation": The "Pyramid". Working out the component of a left turn

The complexity in Nikos' solution of the problem he had to face, lay firstly in organising the potential left turn into two components, i.e. in effect, implicitly "extending" the segment IM so that the required angle split into two "soluble" quantities (a and b in fig. 7.4.14 - 2). The second difficulty lay in working out component b (fig. 7.4.14 - 2), since Nikos was apparently not aware of the

relevant theorem of equal corresponding angles. His "explanation" to Philip of how he worked out angle b, could be described as a specific case of a general proof of the theorem:

N: "Look, this we said is 70 and that is 180, so all of that is 110 (fig. 7.4.13 - 3, angle c). *O.K. This here, oop* (rotates pencil on point M on the screen, towards the left, starting from the turtle's state) *is 180* (fig. 7.4.13- 4). *We take away that here...* (means angle c) *the 110 which was like that, wasn't that 110?"*
P: "Yes, it makes us 70."
N: "70. And from here... 70 won't we go here, straight (means horizontal)? *Plus 30 which we had put over here* (means symmetrical angle at I). *70 plus 30 is 100 isn't it?"* (Nikos then typed LT 100)

Setting aside the complexity of Nikos' angle calculations and his apparent implicit use of the sum of the supplementary angles theorem twice, **it seems interesting to consider the learning potential of such an episode in a broader sense.** On the one hand it could be argued that the dialectic between "personalised" and mathematical activities seems to have played a role in Nikos' readiness to involve himself in relatively complicated mathematics, in the context of an open - ended "investigation" with no "teaching" interventions or specific teaching intentions. On the other hand, if Nikos' strategy for solving the problem is perceived as a specific case of a general proof of a theorem, such a situation could be suitable for a "teaching" intervention with the aim of encouraging Nikos to generalise his solution and ultimately to convey the meaning of proof.

7.4.2g) A "Logo" triangle and a "LASER" triangle.

A situation generating similar potential arose towards the end of the children's fourth "investigation", the sailing boat, during their efforts to construct the sail. The discussion on how to make the sail started when the turtle was at the top of the "mast" (fig. 7.4.14 - 1) since, although both children agreed on a triangular shape, they had a different proposition on the method

of constructing the triangle.

- Sensitivity to geometrical relations during personalised activities: a potential role for intervention.

Nikos' idea was to make an equilateral triangle in the "classic" Logo method, apparently drawing from his experience in the Logo club and school Logo program activities (see chapter 4). He tried his idea out first, typing REPEAT 3 [RT 120 FD 50], the turtle being in the state shown in figure 7.4.15 - 1. From his reaction to the result on the screen it became apparent that he had not achieved what he was implicitly aiming, i.e. the triangle to be in an "upright" position, it's median being a part of the "mast". Nikos did not pursue his idea by debugging the triangle but allowed Philip to try out his proposal.

REPEAT 3 [RT 120 FD 50]   LASER 65 60
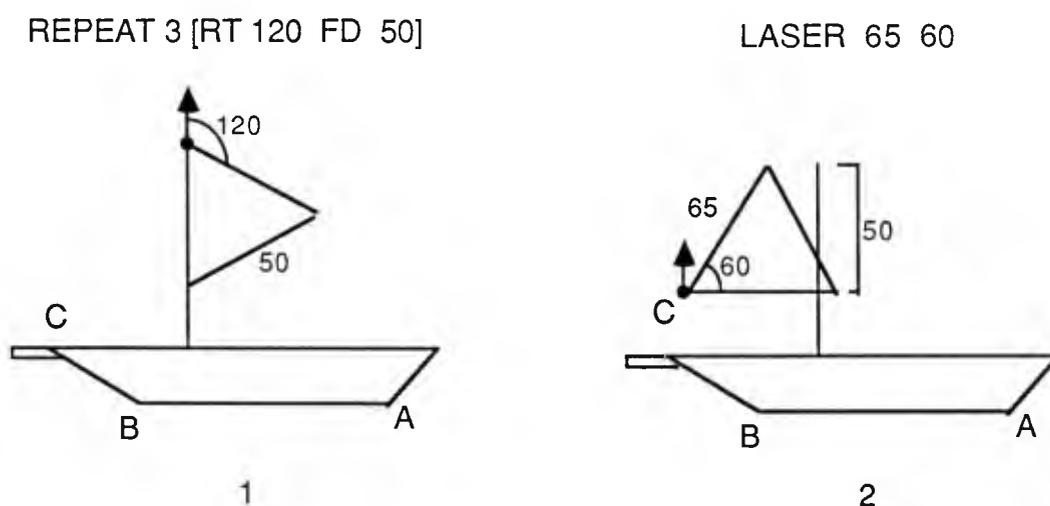
Figure 7.4.14 N. and P. : The fourth "Investigation": The "Sailing Boat". A "Logo" Triangle and a "LASER" triangle

Philip's suggestion was to use the LASER procedure, firstly taking the turtle in a "suitable" starting position (fig. 7.4.15 - 2). He expressed his decision to make an equilateral before giving an input of 60 for the angle. However, although the value of 80 was mentioned for the length input (no reason was explicitly expressed), Philip's opinion was that it would

be too large in comparison with the length of the "mast", which was 50 (fig. 7.4.15 - 2) and so he typed 65. His rationale against an input of 80 was not incorrect, since the top vertice of the triangle would be higher than the top of the mast. It seems, however, to show **his priority for more obvious features - incorporating not only perceptual, but also "logical" conjectures -** rather than ones with a more complicated structure, as for instance, what would happen to the triangle constructed by LASER by altering the length of its side. It could also be argued that Philip had not yet developed the sensitivity to the mathematical nature of the LASER procedure which he would need in order to incorporate less obvious features of the procedure in the process of an open - ended activity, where those features are not focused upon.

It may therefore be useful to consider a "teaching" intervention at that point in time with the purpose of bringing into focus the mathematical notions embedded within LASER, i.e. increasing the awareness of the relationship between mathematics, LASER and the ship's sail. It is not argued that such an intervention would necessarily be specific, since the graphical outcome of executing the procedure with the "wrong" input could play an important role in creating a conflict which in turn could spark a rich mathematical discussion. The intervention could, in such a case, be restricted to encouraging or "legitimising" such a discussion. It is, however, suggested that insensitivity to the situation from a teaching perspective could jeopardise the opportunity for the children to make such relations between mathematical notions and context. For example, the children's reaction to the execution of LASER 65 60 was to "try out" the alternative LASER 80 60 without explicitly discussing or reflecting on the reasons why the latter "worked".

Finally, it seems useful to analyse the geometrical context of a similar situation which had arisen beforehand, when the children seemed to "switch" from Nikos' "classical" Logo triangle to Philip's "LASER" triangle for the sail. In the construction method for the former triangle there are embedded "intrinsic"

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geometrical ideas, such as the notion of turtle state - changes depending on the immediately previous state, the notion of iterating a constant module of move and turn (the triangle could be seen as a "crude" circle approximation), and the notion of total turn. The construction method for the LASER triangle, however, depends on euclidean geometrical notions, such as state changes determined by external points (a parallel could be drawn here of constructing the median, or the altitude of a triangle), and quantities determined by internal relationships of the figure, which in Euclidean geometry are in the form of axioms and theorems.

In that moment in their investigation, however, it did not occur to Philip and Nikos to consider relations or contrasts in the two triangle procedures. **A teaching intervention, perhaps to suggest the debugging of the "intrinsic" triangle procedure regarding the figure's position and the construction of a second sail using the LASER could have proved fruitful in sparking off a powerful mathematical discussion which, although restricted to the context of triangles, would, in effect, consider the relationships between the two geometrical systems.**

The main aim of the category 3 activities, however, was their "open - endedness", in order to investigate the extent and the way in which the children would use Logo, the P.D.D. microworld and the euclidean LASER procedure. A microworld designed to "bring to the surface" the issue of the relationships between intrinsic and euclidean geometry within a specific context was used in the third study of the research (chapter 8), in order to investigate the children's perceptions of such relationships.

7.5 DISCUSSION

This chapter has analysed the children's mathematical thinking primarily within the contexts of intrinsic and euclidean geometry. However, the analysis implicitly concentrated on a broader four perspectives of the children's activities, i.e.;

1) The nature of their activities from a Logo programming perspective, i.e. in relation to:

    a) the programming and mathematical strategies children seem to develop in investigative Logo environments (such as the Brookline, Chiltern and Logo Maths projects - see section 2.1.3 -, but also as in the preliminary phase of the present study, chapter 5), and;

    b) the mathematical nature of their thinking, in the sense of Hoyles and Noss (Hoyles and Noss, 1987a).

2) The role of the P.D.D. primitives as a mediating tool between intrinsic and euclidean ideas.

3) The children's use of their intrinsic schema for both intrinsic and euclidean ideas and;

4) The nature of the children's thinking in the context of euclidean geometry.

Rather than attempting to (artificially) strictly classify the children's activities in the four desribed apsects, it was seen as more meaningful to present an integrated analysis of detailed "significant" episodes and to subsequently attempt a synthesis of the four perspectives of the children's activities, which is presented in this section.

7.5.1 The Logo environment

The analysis has presented evidence that the environment created in the study preserved the general characteristics of Logo microworld environments, such as the ones mentioned above. The children built personalised strategies in the process of making sense of new tools (either primitive commands or newly written procedures) as, for example, in using the P.D.D. instruments for measuring quantities (category 1 activities), or trying out the LASER procedure (category 2 activities). They also engaged in goal - oriented activities, the goals set either by the researcher (e.g. splitting the isosceles triangle in half, in category 2) or by themselves (e.g. deciding to make a "pyramid" for one of their projects). They used programming ideas, such as procedure and variable, either on their own initiative, or after the researcher's suggestion. An example of child - initiated use of procedure and variable is the first procedure they wrote for an isosceles triangle and the variable they added to the procedure to make approximations for the required initial right turn in their attempts to make the isosceles triangle upright (a strategy called "homing in" by Noss, 1985). Examples of prompted use of programming ideas are the generalised procedure for an isosceles triangle (named LASER by the children), and the procedure for the "Nested triangles investigation" (named BAM).

The children seemed to develop programming strategies in a manner familiar from previous work on the issue in terms of the structure in their programming (Leron, 1983, Hillel and Samurcay 1985b, Sutherland and Hoyles 1987). Not surprisingly, the children's programming generally lacked in structure, as for example in the case of their "pyramid investigation", where the modules would involve the turtle going over the same line more than once (Noss, 1985, Hoyles and Sutherland, in press). Moreover, the initial use of a superprocedure in the "nested triangles investigation" involved a "nudge" from the researcher, even in a context where, in the researcher's eyes, there was an evident functional advantage in using this idea. However, there are instances of implicit modular structure in the children's programming, as in

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the case of their nested triangles procedure ("BAM", fig. 7.4.3), where they used a generalised procedure, LASER :N :P, as a fixed subprocedure for the variant module (by giving it different inputs), but did not show a clear perception of the invariance of the interface as an entity in itself (Noss, 1985).

What was the nature of the children's activities from a mathematical perspective? Although due to the inductive nature of the research (see chapter 4) there was no a-priori attempt to employ a theoretical model to interpret children's behaviours, certain notions embedded in the U.D.G.S. model of mathematical learning (see section 2.1.3) provide a useful tool for understanding the mathematical nature of Philip and Nikos' work. For example, the use of concepts such as angle, for functional purposes involving the measurement of an angle quantity via the DIRECTION command, seemed to relate to the discrimination of their components as in the case where the outcome of a measurement could mean the size of an angle and/or the quantity of a rotation. Using a concept also seemed to relate to generalisations involving the concept in question, such as Philip's extending of the relationship between supplementary angles from the "fixed triangle" tasks to the quadrilateral task, in the category 1 activities.

It is interesting here, to consider the role of the children's LASER procedure for a generalised isosceles triangle, as a component of a "Triangle microworld", in the sense of the proportion and parallelogram microworlds proposed by Hoyles and Noss in recent research (Hoyles and Noss, 1986 and 1988) attempting to restrict and specify the mathematical content of children's Logo activities. Although in this study, the LASER procedure was constructed by the children as part of an on - going process of investigating isosceles triangles, having thus started the investigation at an earlier "level", the children's activities with the procedure seem to have a qualitative resemblance to those of the children using the proportion and parallelogram microworlds. After constructing the LASER procedure, the children's use of it enabled them to:

a) discriminate certain embedded concepts, such as the meaning of the angular input in relation to other angles within the figure,

b) generalise concepts embedded within the procedure - such as the angle relationship to make the isosceles triangle upright - and the LASER procedure itself, e.g. by consciously using it to construct equilaterals, and by extending its use to their own projects.

7.5.2 The role of the P.D.D. commands

The main characteristic of the conceptual field of the P.D.D. microworld is that it provides the means to use notions from both the intrinsic and the euclidean geometrical systems. The study, however, concentrated on how the microworld's tools would be used by the children. There is evidence from the study of the children's developing use of the turtle's new tools as:

a) a set of means to change the turtle's state with accuracy, when such accuracy was perceived by the children as unobtainable by the "normal" action - quantity commands;

b) a way of measuring angle or length quantities on the plane, either simply to determine a respective size, or with the purpose of using the quantity for a turtle action.

Throughout their work, there seemed to be a developing change in their priorities, regarding their use of perceptual cues to change the turtle's state. It seems as if the availability of tools with which they could achieve accuracy raised their level of awareness of the difference between accurate plane figures and products of approximations or "homing in" strategies. It could also be argued that there is evidence of an increasing appreciation for accuracy; for example, in their own projects and without any intervention from the researcher, they seemed to be increasingly prepared to use knowledge of close proximity on the one hand, and on the other, to reflect or analyse a

problem to impressive lengths in order to decide whether to use the instruments or the direct action - quantity commands, as for instance in Nikos' complex manipulation of angle properties in the "pyramid investigation". This argument is an extension of, and supported by, evidence from the T.C.P. microworld study, where the children showed increasing awareness of properties of plane figures during their "plane description" activities which involved the use of the P.D.D. commands (the path 2 and path 3 children, fig. 6.1.1).

From the perspective of the geometrical content, the role of the P.D.D. commands seemed to be that of extending the range of usable concepts available to the children, from those of the strictly defined intrinsic geometry to concepts belonging to euclidean plane geometry. For example, the construction of an isosceles triangle by means of the intrinsic Logo commands is either impossible, if no knowledge of plane geometry is assumed, or can only be achieved if the euclidean angle and length relationships embedded in the figure are already known. The ability to place markers on the plane and measure quantities relating the turtle's state to the markers, opens a "window" onto limited awareness of the plane, for the otherwise "blind" intrinsic turtle. Furthermore, it is the user who determines which plane properties the turtle needs to be aware of.

For the children, this provided the opportunity to measure angles or lengths - instead of using perceptual cues whenever complete knowledge of all the angular and length relationships was unavailable - and therefore construct the figure accurately, by using a limited span of its properties. For example, they constructed an isosceles triangle using as the only property, the equality of two segments. Furthermore, the accurate construction enabled them to make further measurements resulting in "discoveries" of further properties and investigations of the relationships between them. This, in turn, provided them with knowledge of euclidean concepts which rendered the use of the P.D.D. commands redundant. For example, in the beginning of the "striped triangle investigation" the children typed LASER 85 50 and then RT 40,

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instead of using the DIRECTION command to turn the turtle towards point H, (fig. 7.4.4). In the study, however, they were left to decide when to use the instruments according to their perceptions of which properties they knew. They did not realise, for example, the redundancy of the DIRECTION command in their LASER procedure, since the level to which they had generalised the angle relationships permitted an intrinsic turn (i.e. RT 180 - :P, instead of RT DIRECTION :I, fig. 7.3.11).

In this sense, the role of the P.D.D. commands could be described as a "support" for the understanding of euclidean concepts and properties. In a figure such as the isosceles triangle, embedding a complexity of geometrical relations, the instruments provided the opportunity to address a limited number of ideas at a time within the wholeness of the figure. In the structure of the LASER procedure (fig. 7.3.11), for example, the redundancy of the "supporting" role of the DIRECTION command had been "achieved" as discussed above, but the need for support on length relations still remained, providing the potential for further mathematical activities.

7.5.3 The use of the intrinsic schema.

The study provides evidence of the children's use of an intrinsic schema to drive the turtle on the screen in a similar way, and to a similar extent as in children's activities with "standard" Logo. It seems helpful to use the model - description of the intrinsic schema provided by the T.C.P. microworld study (fig. 6.6.12), in order to understand the schema used by Philip and Nikos. In the Brookline project (Papert et al, 1979), the lack of differentiation between angle and length inputs is the main "axis" for the describing of observed child behaviours regarding the qualitative role of number inputs. However, there is no explicit analysis of whether children who realised that angle and length inputs are different, understood the nature of this difference by discriminating the meaning of the inputs. In the preliminary study of the present research (chapter 5), there was evidence of quite deep confusions on this issue, in most of the children.

The "action - quantity" part of the turtle schema (fig. 6.6.12) formed by Philip and Nikos, not only seemed to retain the duality of the turtle's state, but the nature of the P.D.D. instruments - i.e. a meaningfully different command for the "ruler and the "protractor" - seemed to play a catalytic role in further discrimination between:

a) move and turn actions,

b) the equivalent importance of heading and position as turtle states (the preliminary study revealed a relatively limited awareness of the heading as a state - entity);

c) the qualitatively different role of number inputs, as representing quantities in different metric systems, i.e. unit length and degree (the preliminary study also revealed confusions in discriminaring angle from length inputs);

However, there was a development in the "sequentiality" part of their intrinsic schema. Although the notion of "one change after the other" was retained and, in a sense, emphasised by the focus on process encouraged by the P.D.D. commands, the dependency of a change on the immediately previous state did not remain essential. Determining the quantity of an action could involve the end turtle state, if a measuring instrument was used. However, when the quantity was measured, it was used as part of the action, in an intrinsic manner. The children seemed to develop quite quickly a differentiated use of measurements either as part of the process to perform a state change, or as detached from action, with the purpose of obtaining information about a certain size. Supported by the evidence from the T.C.P. microworld study, it could be argued that the clear discrimination of the two described roles for a measurement is an indication of the strength of the intrinsic schema in the children's minds.

A different perspective of the children's use of the schema as a whole seems

helpful at this point. It involves the integration of the intrinsic schema with the perceptual and analytical schemas dimension proposed by Kieran, Hillel and Erlwanger (1986). A useful way of describing the children's conventional Logo activities is that they seem to develop an intrinsic schema for controlling the turtle and use it either in a perceptual or in an analytical way. That is, they may identify with driving the turtle on the screen, but it doesn't follow that they use mathematics to do so. Seen through this perspective, three developments have been observed in the children's behaviour in this study:

a) an increasing level of awareness of the difference in using perceptual or analytical cues;

b) an increasing level of priority in using analytical cues, and;

c) an increasing breadth of concepts (extending from intrinsic to both intrinsic and euclidean geometry) used by the children as a result of using analytical cues.

Indications of the emerging of such developments are also shown in the plane description activities of the children participating in the T.C.P. microworld study (fig. 6.1.1).

7.5.4 The use of Euclidean Geometry

The study provides evidence of the children's developing use of concepts belonging to euclidean geometry. The measuring of angular and length quantities, enabled the children to conjecture, reflect on and manipulate properties and relations within the domain of triangles. Their developing awareness of the existence of geometrical relations in the environment they were working in, encouraged an increase in their readiness to use and reflect on them.

The difficulties of understanding Euclidean geometry as argued by other

researchers and mathematicians (Freudenthal, 1985, Von Glasersfeld, 1985, Dreyfus, 1987) is that it requires a level of deductive thinking usually not achieved by children, even till the end of secondary education. While not claiming that Nikos and Philip engaged in deductive euclidean thinking, this study rather illustrated how euclidean concepts became available for the children to use in an inductive way as part of the development of their general mathematical strategies. It is perhaps worthwhile to speculate on the role of the accessibility of euclidean concepts for inductive thinking on the children's experience vis - a - vis their later understanding of deductive euclidean thinking.

7.6 CONCLUDING REMARKS

Claiming that the children engaged in mathematical thinking of some "higher order" would be both misplaced and irrelevant to the objectives of the study. The researcher not only recognises the limitations of in - depth investigations of the mathematical thinking of a small number of children, but the study itself reveals differences in the thinking of the two participating children. Nikos seemed always ready to attempt links between turtle geometry and plane geometry, while Philip, although possibly a higher, by certainly not a lower achiever in each of the two domains, was in general more reluctant to do so. It is difficult to comment on the reasons, however, since this could be a characteristic of the collaborative environment built by the children, rather than a characteristic of their conceptual abilities - something which has been obseved in other studies (Hoyles and Sutherland, 1986b). For example, it could be hypothesised that the dominant character of Nikos implicitly imposed the formation of such links, Philip being content to leave this issue to his partener rather than face interpersonal conflict.

The study depended on providing evidence of the existence of four aspects of the children's activities, two of which have already been observed and analysed in children programming in conventional Logo, i.e. their developing

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programming / mathematical strategies and their intrinsic schema for controlling the turtle. The other two aspects, i.e. the children's use of the P.D.D. microworld and of the content of euclidean concepts, provide an extension of the conceptual field available to children engaged in conventional Logo activities. It was not an aim of the study to show outstanding achievements in any one of these aspects, but rather, to investigate the potential of an integrated existence of all four aspects. That is, to investigate whether the children would carry on developing their programming and mathematical strategies and using their intrinsic schema, as they would do with conventional Logo, while extending the geometrical content of their activities to concepts belonging to both intrinsic and euclidean geometry. Although the evidence provided by the study answers the question, it does not throw enough light on the children's awareness of working in a dual geometrical context.

A further issue, therefore, addressed in chapter 8, was the need for investigating children's explicit critical perceptions of relationships between the two geometries by illuminating their choices of which geometrical system to use for representing concepts in a functional way.

CHAPTER 8

INTRINSIC VERSUS EUCLIDEAN GEOMETRY

8.1 OVERALL DESIGN OF THE STUDY

8.1.1 Objectives

There has been little evidence from previous research (see chapter 2) that children form disparate microviews for intrinsic and plane representations of specific geometrical concepts, as for example, in the dynamic and static representation of angle (Kieran, 1986b). This evidence has been corroborated by the findings of the present research (chapters 6 and 7). Little is known, however, about the nature of the criteria children use in choosing between intrinsic and non - intrinsic representations of geometrical concepts in Turtle geometric contexts where both representations have been a part of the children's mathematical experience.

Consequently, the general aim of the Circle microworld study was to investigate the criteria children develop for choosing between intrinsic and euclidean representations of geometrical ideas within the context of Turtle geometry.

A pair of children participated in the study which consisted of two phases; the first involved a learning sequence concerning four distinct methods for constructing a circle, each method involving the use of specific intrinsic or euclidean ideas (see section 8.1.3). The sequence involved phases where the children constructed the procedures and phases where they used them in personal projets (8.1.2a). In the second phase, the children were given a sequence of structured tasks, each involving the construction of a geometrical figure consisting of a composition of circles. Within each figure certain geometrical notions were embedded so that the figure could be constructed either by using intrinsic or euclidean notions or combinations of both (fig.

8.3.1, section 8.3.1). The children could choose which of the four circle procedures they would use and were asked to solve the task individually at first and then collaboratively.

In effect, the set of four circle procedures in conjunction with the standard Logo commands embodies the conceptual field of a Circle microworld (see section 8.1.3). Accordingly, a partial aim of the first phase of the study was for the children to construct the new tools of this microworld and use them in meaningful contexts so that the embedded intrinsic and euclidean notions could become part of their mathematical experience.

However, the first phase of the study also provided a context for the researcher to carry out a preliminary investigation of the initial stages of the children's developing choices between intrinsic and euclidean notions. The following two issues were cosequently investigated during phase 1:

a) the nature of the geometrical notions which were implicitly or explicitly used by the children during the phases of construction of the circle procedures;

b) the extent to which and the way the geometrical notions characterising each circle procedure were used by the children during their own projects.

During phase 2 of the research, the following main issues were investigated:

c) the extent to which the children used the geometrical notions embedded in the structured tasks and the nature of the notions they used for constructing the tasks' figures;

d) the nature of the children's implicit or explicit criteria for choosing intrinsic or euclidean geometrical notions in their constructions.

8.1.2 Overview of the study

8.1.2a) Phase 1

As mentioned above, the first phase consisted of a learning sequence involving four distinct methods of constructing a circle in Turtle geometry. Each construction method employed intrinsic and/or euclidean ideas and was embodied within a procedure. The first procedure involved intrinsic notions for constructing the circle (fig. 8.1.1). The second procedure incorporated the notions of the radius and the centre through the variable input which represented the length of the radius (fig. 8.1.2). The third procedure took the radius as input, but also involved the centre as the point of state transparency of the turtle (fig. 8.1.3). Finally, the fourth employed a construction method representing the euclidean definition of a circle as the set of points equidistant to the centre point (fig. 8.1.4). In order to set up a meaningful context for the construction of each of the procedures, a structured task embedding geometrical ideas to be used for the procedure's construction was given to the children (fig. 8.1.6). After each procedure construction and subsequent construction of the task figure, the children were asked to carry out projects of their own choice with the restriction of using their new procedure to make circles of various sizes.

The four circle procedures as constructed by the children, constituted the Circle microworld's special primitives, which were then used in the second phase of the study. The conceptual field embedded in the Circle microworld is analysed in the following section (8.1.3). An analysis of the structured tasks given to the children during the first phase of the study is presented in section 8.1.4a). Finally, an overview of the learning environment for the first phase of the study is given in section 8.1.5.

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8.1.2b) Phase 2

This phase consisted entirely of the administering of structured tasks to construct figures involving compositions of circles. Each figure consisted of circular formations within which certain geometrical notions were embedded so that the figure could be constructed either by using intrinsic or euclidean notions or combinations of both (fig. 8.3.1, section 8.3.1). A detailed analysis of each task is presented in section 8.1.4b). The aim of administering the tasks was firstly to investigate whether the children would use the embedded geometrical ideas at all, in order to construct the figures. Secondly, the aim was to investigate which geometrical notions they used with respect to their intrinsic or euclidean nature and their criteria for choosing these particular ideas in their construction.

8.1.3 Analysis of the conceptual field
of the Circle microworld

8.1.3a) Tool description

The Circle microworld consisted of all the conventional Logo commands and four distinct procedures, each constructing a circle in a different way. The children used the Circle microworld, i.e. an environment where the four circle procedures were primitive commands, in the second phase of the study. A partial objective of the first phase was for the children to construct and use the microworld's new tool's for themselves. Since the tools of this microworld are not simple as, for instance, the POST, DISTANCE and DIRECTION commands in the T.C.P. and P.D.D. microworlds, it was essential for the children to develop an awareness of the geometrical ideas embedded in using the tools, so that they would be in a position to make a meaningful (rather than a random) choice in the second phase of the research.

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<u>Circle procedure 1.</u>

TO ANYCIRCLE :V
REPEAT 36 [FD :V RT 10]
END
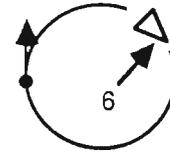
ANYCIRCLE 6

◁ Denotes a turtle state during the execution of the procedure

↑ Denotes the turtle's state of transparency

<u>Figure 8.1.1 Circle procedure 1</u>

ASIDE: The names of the microworld's tools used in this section belong to the researcher and were also used in the document given to the children during the first phase of the study (appendix E.2). In the subsequent "findings" sections, however, the names the children gave to the procedures as they were constructing them are used, since these names were referred to in the collected data and the former ones - not surprisingly - were ignored.

In figure 8.1.1, the procedure ANYCIRCLE, the REPEAT command and the variable input have been used for an intrinsic construction of a circle. There is no reference to any point outside the immediate vicinity of the turtle and each change of state depends on the previous state. The input determines the change of position between turtle turns.

<u>Circle procedure 2.</u>

TO NEWCIRCLE :V
REPEAT 36 [FD :V * 3.14 / 18 RT 10]
END
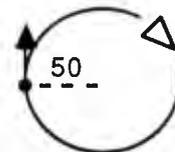
NEWCIRCLE 50

<u>Figure 8.1.2 Circle procedure 2</u>

Procedure NEWCIRCLE seemingly constructs the circle by an intrinsic method, since the turtle repeats a constant change of position and change of heading. However, the quantity of the turtle's change of position does not depend on the action itself, but on information involving a specific point on the plane - the circle's centre - outside the turtle's path. This information involves the relationship between the length of the radius and the length of the turtle's change of position between each turn. This relationship can be derived through the relationship between the circumference and the diameter of the circle, i.e. the number (pi) = 3.14... The length of the radius is given as an input to the procedure and the input to the FD command is calculated so that the constructed circle has a radius of length equal to the numerical input.

In order to provide a context inviting the children to feel in control while investigating the crucial relationship between the radius and the length between each turtle turn, special turtle commands were designed and presented to the children as primitives. The commands represented measuring instruments - one to measure the circumference and one to measure the diameter of the circle. The programming for the commands is given in appendix E.1, the way they were introduced and used by the children is apparent from the worksheets (appendix E.2); the emerging issues related to the research are incorporated in the respective "findings" section (8.2.1).

The visual effects from measuring the two elements of the circle "required" an upright orientation of the figure constructed by the NEWCIRCLE procedure which, in effect, is a polygon approximating a circle. This would imply a "correction" of the turtle's orientation by half the amount of turn in the module generating the curvature (i.e. 5 degrees), before starting to trace the circumference. This could be achieved by the commands RT 5 and LT 5, respectively before and after the construction of the polygon - circle. This correction was applied by the children after they constructed their procedure and following a discussion generated by the researcher, concerning polygon approximations of circles. It is referred to in the respective "findings" section

where relevant to the research issues.

Finally, in relation to this turtle method of constructing the circle, another example of information referring to distant points in the plane and used to determine the quantity of a turtle action is given in chapter 3, where knowledge of euclidean properties is used in constructing the diagonal of a square (section 3.1.1). This procedure, therefore, employs both intrinsic and euclidean ideas, i.e. the "curvature" or "polygon approximation" method of constant position and heading changes and the use of the length of the radius and consequently the reference to the circle's centre. The input to the procedure (:V) determines the length of the radius of the circle.

Circle procedure 3

```
TO CENCIR :V
LT 90 PU FD :V PD RT 90
REPEAT 36 [FD :V * 3.14 / 18 RT 10]
RT 90 PU FD :V PD LT 90
END
```
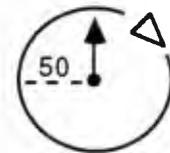

CENCIR 50

Figure 8.1.3 Circle procedure 3

The method for constructing the circle in procedure CENCIR is the same as in procedure NEWCIRCLE. The difference lies in the turtle's state transparency (i.e. the turtle's state before and after execution of the procedure). In this case, the turtle starts and finishes in the centre of the circle. Execution of the procedure, therefore, has the effect of a circle drawn around the turtle. Determining the inputs to the turn and forward commands used to take the turtle to the edge of the circle and back (i.e. 90 degrees and :V steps) requires the use of euclidean properties of the circle (i.e. perpendicularity of tangent and radius).

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```
TO DOTCIR :V
REPEAT 36 [PU FD :V PD DOT PU BK :V RT 10]
END
```
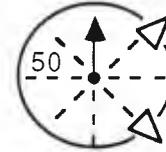

DOTCIR 50

Figure 8.1.4 Circle procedure 4

This procedure uses the euclidean definition of a circle, i.e. that a circle is the set of points which are equidistant to a point in the plane. This point, i.e. the circle's centre, is the turtle's state of transparency. The tracing of the circle's curvature is not embedded in this procedure. Rather, the turtle "jumps" to the edge of the circle and leaves a trace of its position (i.e. makes a dot). The approximation factor in this case does not involve polygons, but the proximity between the points of the curvature, depending on the length of the radius and on the amount of turn.

8.1.3b) An example involving the use of the Circle microworld's tools.

The following example is meant provide some clarification of how the Circle microworld's tools might be used in the construction of a geometrical figure. All four circle procedures can be used to construct figure 8.1.5. In using circle procedure 1, however, there is no reference to any point outside the turtle's immediate vicinity (e.g. ANYCIRCLE 5 RT 180 ANYCIRCLE 5), i.e. the construction method employs intrinsic notions only. The NEWCIRCLE procedure embodies an intrinsic method of constructing the figure, since the turtle traces along the curvatures executing action quantity instructions without the need of a reference to external points for the interface between circles (e.g. NEWCIRCLE 50 RT 180 NEWCIRCLE 50). However, there is an embedded reference to an external point in the plane in determining the input to the procedure. In using the CENCIR procedure, euclidean information is required for the interface between circles, e.g. CENCIR 50 RT 90 PU FD 100

<u>PD LT 90</u> CENCIR 50. Use of the DOTCIR procedure involves a euclidean definition of a circle for the circles' construction and a reference to euclidean information for the interfacing commands.

In using the Circle microworld's tools, therefore, the choice is open regarding intrinsic or euclidean notions to be employed in figures involving circles. However, according to embedded relationships, certain figures invite the use of one microworld tool rather than another. This issue is analysed in relation to the study, in section 8.1.4b).
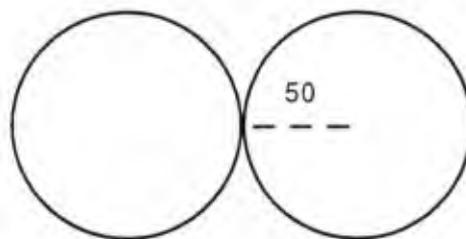
<u>Figure 8.1.5 An example of a figure with circles</u>

<u>8.1.4 Task analysis</u>

<u>8.1.4a) Phase 1</u>

The four tasks in this phase were designed so that they would encourage a focus on the intended geometrical ideas to be used by the children for constructing a circle by means of writing the respective circle procedure. This analysis presents these ideas and the way in which they are embedded in the task figures.

<u>The first task</u>

The task figure consists of five circles of different sizes which could be constructed by executing an intrinsic circle procedure with inputs of different sizes respectively, without any interfacing commands between the procedures (fig. 8.1.6a)). An embedded intrinsic notion is the non -

requirement of reference to any point in the plane away from the turtle's vicinity and the point of state transparency of the turtle for the whole figure, being the circles' point of connection.

The second task

The figure for this task is two concentric circles. A fixed "distance" between the circles was given in the worksheet (fig. 8.1.6b) and appendix E.2). There are two notions embedded in the task which require reference to the plane; the circles' centre and the notion of the distance between two curves. If the distance is perceived as the difference between the lengths of the two radii, then both notions (radius and centre) are euclidean (Abelson and diSessa would say that the notions are intrinsic to the circle, but for the present research this use of the word "intrinsic" has a different meaning, see section 2.2.5). If the distance is perceived as an absolute distance between two curves, the notion of distance is euclidean but not connected to the circles themselves. Constructing this figure could consequently encourage writing a circle procedure which would provide the means of using the radius (and/or the centre) of the circle. Using circle procedure 2 (section 8.1.3) would involve the use of the radius in the form of the input given to the procedure.

ANYCIRCLE 6

Task 1

Circle procedure 1

NEWCIRCLE 50

Task 2

Circle procedure 2

CENCIR 50

Task 3

Circle procedure 3

DOTCIR 50

Task 4
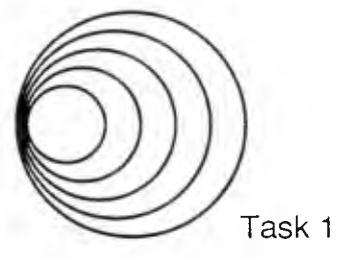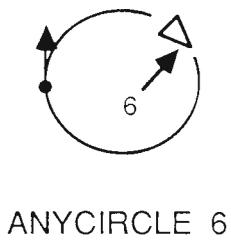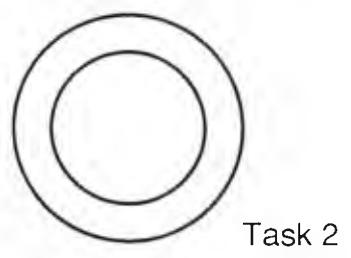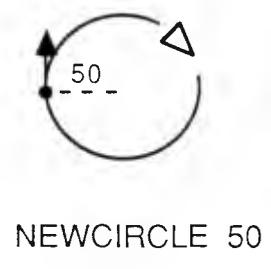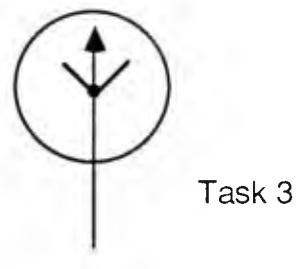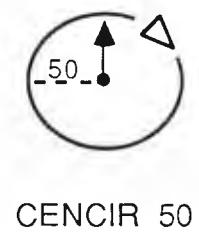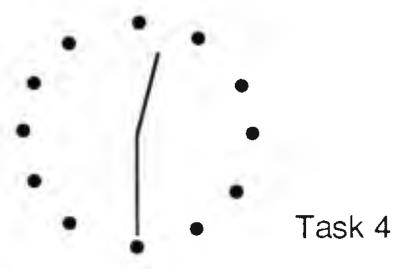
Circle procedure 4

Figure 8.1.6 The four circle procedures
and the respective tasks in the learning sequence

The third task

This task does not depend so strongly on the figure itself, but rather on the process of constructing it. The requirement is to make a 30 - steps long line and from that position of the turtle, to construct various circles of different sizes, erasing them until a circle of a perceptually satisfactory size is found, so that the figure looks like a tree (fig. 8.1.6c)).

In effect, the tasks require the construction of concentric circles with the restriction that the turtle has to start and finish each circle situated in the centre. The embedded idea in the task is the focus on and the use of the centre point of the circle as an integral part of the circle itself. Using circle procedure 3 would facilitate solving the task compared to a laborious use of circle procedure 2 where interfacing with the centre point would have to be carried out in direct drive.

The fourth task

The task is intended to encourage a focus on the centre point - via the construction of the clocks hands - and on points of equal distance to the centre (fig. 8.1.6d)). The two previous circle procedures could be used by instructing the turtle to trace the curvature in PENUP mode and interrupt every total turn of 30 degrees to make a trace of its position. However, the figure can also be constructed by making a module where the turtle "jumps" a fixed distance (i.e. moves in PENUP mode), makes a trace of its position and then "jumps" back the same distance. By repeating the module with the appropriate 30 degree right turn interface, the turtle remains in its state of transparency, the centre, from which the clock's hands can be drawn. The task prompts a reflection on the method of constructing the figure and subsequently suggests constructing it via the latter method.

As mentioned above, each task figure in this phase consisted of circular formations involving geometrical notions embedded so that the figure could be constructed either by using intrinsic or euclidean notions or combinations of both (fig. 8.3.1, section 8.3.1).

An important feature in tasks 1, 2, 4, 6, and 8 is the position of the circles' centres. Figures 1, 4 and 8 do not have linear cues connecting the centres, i.e. there is no explicit reference to the centre points embedded in the tasks. Furthermore, there are specific relationships among the lengths and the positions of "important" radii of the circles of these figures. For instance, the radii on the points of connection of the circles in task 4 determine the length of the sides of the equilateral triangle formed by joinning the three centre points. A discriminatory factor between figure 1 and figures 4 and 8 is the absence in the former of connections among the circles' curvatures.

Tasks 2 and 6 involve line segments joining the centres of the circles' in the figures. The radii in this case are only important for constructing circles of equal sizes. The positioning of a radius or its actual length are not important factors for the construction of the figures. A difference between task 2 and task 6 is that in the latter, the actual length of the segments joining the centres is important for the construction of the figure. This is not the case in the former task.

The figure in task 3 can be perceived as a variation of the figure in the first task in phase 1 of the study (8.1.4a), fig. 8.1.6a)). The difference in that case would be the existence of a right - turn interface between the circles in the figure in task 3. This parallelism is only made to highlight the important factors in constructing the figure, e.g. their curvatures' connecting on a specific common point. Consequently, constructing the figure does not require reference to radii or centres of the circles. A means of altering the size of a circle is required, but there is no need to relate the size to the circle's radius.

Tasks 5 and 9 involve figures positioned in circular formations. In figure 9 there is a linear cue (a circle) embodying the circular placement of eight line segments by connecting their ends. The figure is open to various methods of construction. One set of methods, for instance, could involve a separate construction of the circle and the segments, while another could incorporate both formations in one module. In either set of methods intrinsic or eulcidean (or both) notions can be used. In the latter set, for instance, the turtle could be instructed to trace along the curvature and interrupt to turn and make a line segment. Alternatively, points equidistant to the centre could be drawn for an arc, followed by a segment after an appropriate number of points.

The absence of a linear cue to denote the circular formation of the small circles in the figure in task 5 allows for a choice of which circle to perceive as important for the positioning of the small circles; although the position of the circle's centre would be quite obvious, the radius could connect to any useful point in the small circles' curvatures (e.g. the point closest or furthest to the centre or the point which makes the radius tangent to the small circles). Furthermore, the circle need not be perceived in connection to a centre and a radius at all; the turtle could be instructed to make a large curvature, interrupting at the appropriate intervals to make smaller circles by turning to the right or to the left. An additional feature of this figure with respect to the one in task 9, is that the shapes in circular formation are circles themselves and therefore open to constructions involving intrinsic or euclidean notions. An interesting issue is to what extent the method used to construct one of the circular formations (the large one or the small circles) mathematically restricts the method used for the other. This issue is analysed with respect to the children's constructions in section 8.3.3.

Finally, constructing the line segment in the figure in task 7 is the only factor which requires the employment of euclidean notions in constructing the semicircles; if the segment was absent, the connecting curvatures could be drawn intrinsically. However, in order to join the two edges of the figure,

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reference to the diametrical distance of a semicircle is required in some form or another. For instance, the reference could be achieved through the input signifying the length of the radius in circle procedure 2, or through a centre - oriented circle construction via circle procedure 3 or 4.

It is evident from the above analysis that the 9 figures of the tasks administered during the second phase of the study allow a high degree of flexibility regarding the geometrical notions which can be used in their construction (with the exception of task 3). The tasks were designed in this way so that the children would have the choice of which notions to use in their attempts to construct the figures - in the case, of course, that they would use geometrical notions in the first place.

8.1.5 An overview of the learning environment

In general, the learning environment was designed under the same principles as in the previous study (section 7.1.3). Even though, as discussed above, the children actually used the Circle microworld only in the second phase of the study, for the purposes of describing the learning environment the broader meaning of "microworld" (Hoyles and Noss, 1987b) will be used for both phases.

The pedagogical component in phase 1 firstly consisted of the worksheets (appendix E.2) designed to lead the children to activities on and off the computer. The latter activities involved questions designed to encourage reflection at key points during the construction of the new circle procedures. Secondly, the pedagogical aspect involved the researcher's interventions. During the phases of the construction of new circle procedures the intervention strategy was of a relatively directive nature, in accordance with the principles underlying the corresponding parts of the worksheets. However, the researcher generally attempted to allow the children to take initiatives in their learning, restricting his interventions only to cases where it was necessary. For example, although in the worksheet a procedure for a

circle with a variable is given (page 2, appendix E.2), the researcher did not show the worksheet to the children until they had decided how they would write the procedure themselves. Where relevant, such instances become apparent in the respective "findings" section (8.2.1).

During the children's projects in phase 1, however, the researcher's interventions were restricted to non - directive principles discussed elsewhere (e.g. as in the category 3 activities in chapter 7). Furthermore, during the second phase of the research, interventions were oriented towards illuminating the children's thinking and reasons for their actions according to the research issues.

As in the previous study, during both phases of the present one, an important aim concerning intervention was to encourage or provoke reflection on the children's actions and to encourage their explicit explanations, either to their peer or to the researcher, concerning important research points. A key aspect of the intervention strategy was the frequent encouragement for the children to be active in their learning, to feel comfortable in making conjectures and speaking their thoughts out loud, i.e. to establish the legitimacy of a learning atmosphere which was unfamiliar to the children due to the educational system (see chapter 5).

8.1.6 Methodology

8.1.6a) A teaching experiment used as a pre - pilot study

In 1985, a teaching experiment was carried out by the researcher (Kynigos, 1985), the content of which involved the first two circle procedures and two of the structured tasks (the circles in a tangential triangular and square formation as in tasks 4 and 8, fig. 8.3.1). The experiment involved two pairs of 11 - 13 year old English children with considerable Logo experience and lasted for 2.5 hours in total for each pair. As a result of this study, which was considered as a pre - pilot study for the present research, the following

changes were made and carried out in the main pilot study;

- The learning sequence incorporated the third circle constructing procedure (and the respective task encouraging the construction) as mentioned above, in order to further the employment of euclidean notions involving the definition of a circle.

- Detailed changes in the teaching sequence were made in order to enhance clarity for the children and provide them with the opportunity to use each circle procedure in a project of their own. Consequently, substantially more time was allowed both for the construction of the procedures and for the children's own projects.

- The tasks, investigations and requests for projects were documented (see appendix E.2), in order to provide more scope for the researcher to concentrate on the research issues. The documentation was in the form of "worksheets", with on and off - computer activities. The documentation of the "N" tasks (Hoyles and Noss, 1986) was used as a basis for the worksheets' format and style.

- Additional structured tasks were designed for the second phase of the study (9 in total) in order to probe further the children's criteria for choosing a circle procedure to solve the tasks.

8.1.6b) The main Pilot study.

The main pilot study was carried out in the same English school as the respective studies described in chapters 6 and 7. Two 13 year - old children with considerable Logo experience participated in the study. All the activities of phase 1 and phase two (involving the three first circle procedures) were piloted. Data was collected by the following means;

- audio - taping,

- dribble files of the children's typing,

- the chidren's procedures saved on disk,

- the children's written responses to the "worksheets",

- researcher's notes.

As a result of the main pilot study, the teaching sequence was changed to its final form which consisted of a module for introducing to the children a meaningful context for constructing a circle with each of the methods described above and using each construction in a project of their own choice. Each module involved one of the four methods for constructing the circle. All the modules had the following structure;

a) describing a structured task encouraging the construction of a particular circle procedure;

b) investigating the construction method for the circle and writing a circle procedure incorporating the respective method;

c) solving the task with the use of the new circle procedure;

d) carrying out a project with the use of the procedure.

It was decided to include a fourth circle construction involving a more "extreme" euclidean method (points of equal distance to a specific point in the plane), in order to probe further the children's choices in the second phase of the study. The teaching module for the fourth construction was piloted as in the former part of the main pilot study and detailed changes in the task and the presentation of the worksheet were made.

The primitives for measuring the circumference and the diameter of the circle to be used for the construction of circle procedure 2 were modified so as to increase clarity in their function and to achieve a rigorous consistency with the function of Logo commands in general (see appendix E.1).

Use of the dribble playback files was added to the data collection, in order to acquire screen dumps which proved useful in analysing the children's projects in particular. There was a further increase in the time allowed for the study, in both phases.

Finally, in phase 2, it was decided to present one figure at a time to the children and to allow a whole 90 minute session for each task. Time was allowed for the children to write how they would construct the figure individually on paper, before attempting to solve the task collaboratively. After each construction, time was left for a discussion of the children's choice of procedure.

8.1.6c) The main study

Two children from the Logo club participated in the main study, Valentini and Alexandros. The research was carried out immediately after school in the research room during two 90 minute sessions a week for eight weeks (not including holidays, e.t.c.). As in the case of the children participating in the other studies, Alexandros and Valentini took part, as normal, in the Logo club and in the school program. The computer, the Logo version and the research setting were as in the previous study (section 7.1.5b)).

The research data consisted of;

- audio taping of everything that was said,
- soft and hard copies of verbatim transcriptions translated from the audio tape into English,
- hard copies of graphics screen dumps, acquired by playing back the dribble files and pausing them to make printouts,
- soft and hard copies of all the procedures the children wanted to save on disk,
- the researcher's notes and the children's prompted and unprompted notes

on paper,

- the children's written responses to the worksheets of the learning sequence and,

- their written plans for their strategies in constructing the figures in the structured tasks of phase 2 of the study.

The researcher's notes involved any relevant incidents which would otherwise escape the "net" of data collection, as for instance, the children referring to points on the screen by pointing at them. Printouts of the graphics screen were crucial during the children's own projects, since it was sometimes hard to visualise the effects of superprocedures they wrote, with several layers of subprocedures embedded within them. The printouts were also used in a subsequent "findings" section (8.2.2) in cases where reproduction of the figures by the researcher could not be achieved with accuracy.

During phase 1, the primary factor determining the content of the activities was the children's reading of the worksheets rather than the researcher's verbal comments. The administering of a structured task in phase 2, involved the following procedure:

a) A copy of the figure was given to each child; the figures were drawn on paper by the researcher with the use of compass and ruler in order to avoid biases towards one or the other method of construction due to screen resolution effects or differences in circles constructed by different circle procedures;

b) the children were given time to think and write a plan for constructing the figure; they were requested to write their plan in the forms of commands / procedures and written language;

c) they were then requested to exchange plans and explain them to each other before trying out both plans on the computer;

d) in cases of difficulty with constructing the figure time was allowed for the children to cooperatively discuss and work out how they would solve the task;

e) after constructing the figure, the researcher carried out a semi - structured interview probing further issues concerning the children's solutions and investigating their views on using alternative geometrical notions or strategies to solve the task.

The analysis of the data is presented in two sections (8.2 and 8.3), corresponding to the two phases of the study. In the first section (8.2), it was decided that it was more relevant to the research issues to present the findings from the learning sequence in two parts; the first part reports on the children's activities during the phases of solving the four tasks and constructing the respective circle procedures; the second part presents an analysis of the geometrical notions they employed during their personal projects involving the use of a respective circle procedure.

The structuring of section 8.3 is based on the research issues which emerged from the analysis of the data, rather than on a chronological account of events during the administering of the tasks. The structure is therefore a result of a synthesis of "significant" episodes, as elaborated in section 4.2.8. A further synthesis of the findings is discussed in section 8.4.

8.1.7 The children

Valentini was characterised by her teacher as *"very bright in all subject areas".* Her favourite topic at school was mathematics *"because I like to use my brain"* , as she wrote in response to an interview question at the beginning of the year. However, she showed a tendency to dominate over her peers during the first year of the Logo club where, for instance, she would monopolise the use of the keyboard. She was enth usiastic about her activities in the club and described them with precision - in relation to her

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peers' descriptions - in the essay she wrote at the end of the year. Her perception of what she had learned from participating in the club was *"to use our mind more practically and overall to cooperate"*. Her comment is not only indicative of a progressing self - awareness of her dominant character, but also of a characteristic of her thinking which influenced her projects and her performance in the structured tasks; she would "drift" into unnecessarily complicated routes in her thinking, without trying out things on the computer or standing back to look for simple solutions. In her programming to solve the structured tasks, however, she used procedures and was one of the three children to use subprocedures in the process of constructing the four squares in the third batch of tasks (see fig. 5.1 F and G, appendix H).

Alexandros, on the other hand, was a practically - minded child, perhaps as a consequence of his distinct difficulty in thinking out things in the abstract, as for example, in his writing of procedures in the editor without trying them out. His teacher perceived him as bright, but dominant and self - centred, concious about "failure". However, although during the club activities he took the role of the "leader" among his peers, this was not the case in his partnership with Valentini during the research. Furthermore, his open and likeable character contributed to the investigative atmosphere created during the research sessions. Alexandros' programming in the structured tasks was in the same category as Valentini's (fig. 5.1 F and G, appendix H).

8.2 CONSTRUCTING THE CIRCLE MICROWORLD'S TOOLS AND USING THEM IN PERSONAL PROJECTS

As mentioned in section 8.1.1, a partial aim of the learning sequence in phase 1 of the Circle microworld study was to prepare the children for taking part in phase 2, i.e. to provide them with the opportunity to construct and use the microworld's tools in meaningful contexts so that they might form understandings of the intrinsic and euclidean ideas embedded in the circle procedures. However, the process by which the children formed these understandings and used the geometrical notions in their own projects,

provided the researcher with a context to carry out a preliminary investigation of the initial stages of the children's developing choices between intrinsic and euclidean notions. The two aspects of this investigation are formulated in section 8.1.1 (issues a and b).

In an attempt to keep a balance between the presentation of the findings in the two phases of the study and the importance attributed to the findings with respect to the research issues, it was seen as appropriate to state here only the key issues emerging from the analysis of the data from phase 1. However, a detailed presentation of the analysis is given in appendix G.

The construction phase was characterised by an apparent disparity between the intrinsic and the non - intrinsic schema in the children's minds. In constructing the circle procedure representing the Euclidean definition (circle procedure 4), for instance, the children's insight into the euclidean method came from an experience during a previous project which had no relevance to circles or intrinsic notions embedded within them (appendix G, section 8.2.1d). Furthermore, there were indications of this phenomenon in the children's use of the procedures and not only during their construction. For instance, after a lengthy construction process of a circle procedure employing the euclidean notions of radius and centre, the inputs the children gave in their first executions of the procedure indicate how they apparently ignored the euclidean notions (appendix G., section 8.2.1b).

Not surprisingly, the notions the children used in constructing the task figures seemed to be functional to the task figures rather than the notions embedded in the circle procedures to be constructed. At specific points indicated in the presentations of the findings (see appendix G., sections 8.2.1a, b, c and d), the researcher had consequently to intervene to focus the children's attention on the notions related to the circle procedures. Furthermore, the children seemed to have varying degrees of awareness of the notions they were using. For instance, in constructing the first circle procedure, the important factors became progressively more explicit through discussion (appendix G,

section 8.2.1a).

The children's projects (see section 8.1.1, issue b) were characterised by an infrequent use of the notions embedded in the construction of each circle procedure. It could be the case, that the children's fascination with their progress in programming, e.g. their superprocedure - building strategies (appendix G, figs. 8.2.4, 8.2.7, 8.2.9, 8.2.14) and their progressing familiarity with saving and loading files on disk and using the files in subsequent sessions, influenced their focus on these issues rather than the geometrical ones. However, it could also be the case that the children saw no functional reason to use geometrical notions more often than they did.

In support of the latter argument is that in occasions where using geometrical notions was fuctional for the project, as in the snowman project (appendix G, fig. 8.2.13), the "clocks" project (appendix G, fig. 8.2.15) and the circle of targets project (appendix G, fig. 8.2.9), the children did appear to use the geometry. For instance, during their snowman project, the use of geometry seemed a lot richer than in their circle rotations project with the CIR9 procedure (appendix G, fig. 8.2.6).

A final issue is the children's use of intrinsic and non - intrinsic notions which were not specific to a circle procedure and in certain cases were of a different nature to those embedded in the respective circle procedure. In using the intrinsic CIR4 procedure (circle procedure 1), for example, the children's project involved the use of a non - intrinsic method for constructing their planned figure, i.e. they constructed the four sets of circles in the form of a cross - or two perpendicular directions - rather than using a turle rotation interface (appendix G, fig. 8.2.4a). Conversely, in their first project with the TC procedure (circle procedure 4), they constructed a square formation of "target" figures and used the intrinsic method for constructing the square (appendix G, fig. 8.2.14).

Although it is not within the objectives of the present study to evaluate the

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learning sequence for constructing the four circle procedures, it seems relevant to consider how the sequence might have influenced the children's learning in relation to the research objectives. For instance, the researcher's participation in the children's learning process was very important - the role of the worksheets was only complementary to the researcher's interventions. However, interventions are very difficult to make; the preciseness of the circle procedures to be taught required at some points directive interventions not closely tied to the context, as in the case of correcting the orientation of the "circle - polygon" constructed by the CIR9 procedure (circle procedure 2, see appendix G, section 8.2.1b). However, flexibility was also required so that if the important embedded notions within the procedures were used, room would be allowed for the children's personal ideas, as for instance in their idiosyncratic construction of the DOT subprocedure (see sections 8.1.3a and appendix G, 8.2.1d). During the children's projects on the other hand, the general strategy of relative non - directedness had the drawback that children could have been encouraged to use the circle procedures' embedded notions more than they did.

8.3.1 Introduction

As in the previous two chapters, the primary factor in structuring the presentation of the results from this phase of the study, has been the research issues emerging from the analysis of the data (see section 8.1.1, issues c and d), rather than the way in which the actual research sessions were organised. The episodes used to convey the findings, therefore, are not presented in the chronological order they occurred which is represented by the numerical order given to the tasks in figure 8.3.1. Furthermore, the phase within the solving of a task, during which a presented episode took place, is made obvious during the respective presentation. Finally, a reference to the following two sections might be useful at this point; a) the analysis of the tasks used in this phase of the Circle microworld study, given in section 8.1.4b, and b) an outline of the research procedure following the administering of each task (section 8.1.6c).

Two main research findings are analysed and discussed, namely **the children's use of both intrinsic and euclidean notions within mathematical situations** and secondly **their priorities in choosing the geometrical notions they used.**
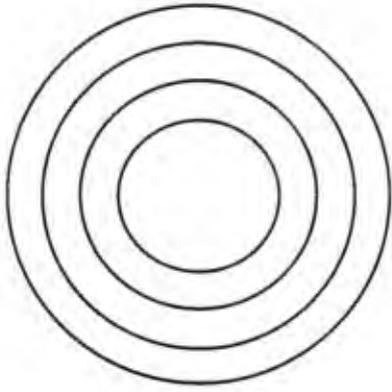
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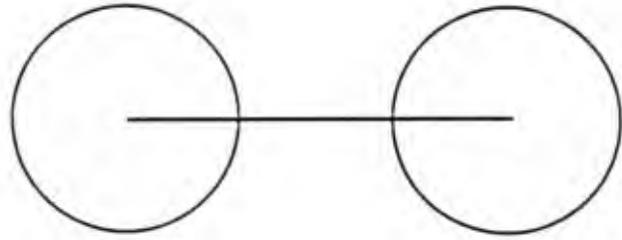
TASK 2
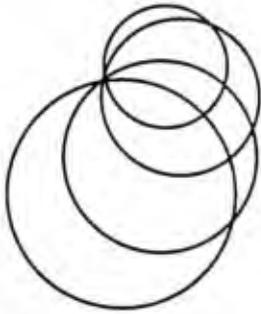
TASK 3
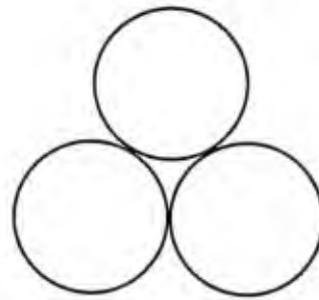
TASK 4
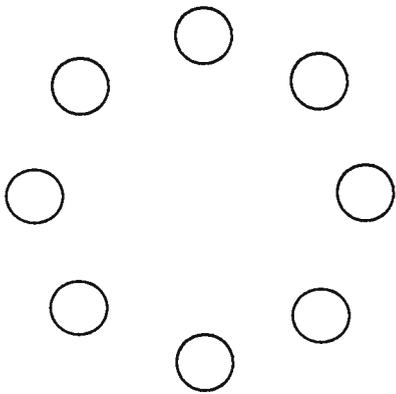
TASK 5
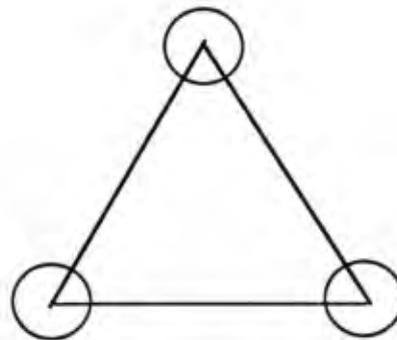
TASK 6
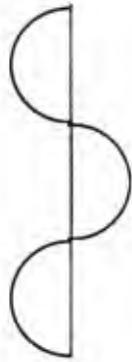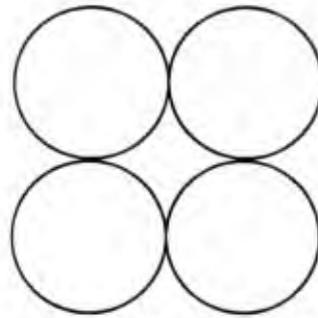
TASK 7   TASK 8
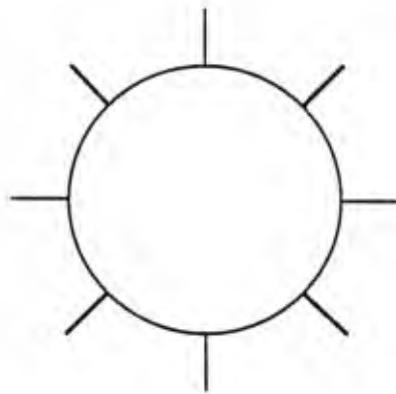
TASK 9

Figure 8.3.1 The structured tasks

8.3.2 The use of intrinsic and euclidean notions within mathematical situations

8.3.2a) Forming theorems in action while not perceiving an embedded geometrical relation

In constructing the figure in task 1 (fig. 8.3.1), the children seemed to use the centre point, focusing on its particular role in this figure by using the CIR19 procedure from the outset, initially with fixed inputs and then incorporating a variable input for their superprocedure (fig. 8.3.1b). **This unquestioning**

use of the centre, however, could have been a consequence of the children's construction of the same figure, in effect, during their own projects.

```
TO C5
CIR19 10
CIR19 20
CIR19 30
CIR19 40
CIR19 50
END
```

```
TO V2 :S
CIR19 :S / 5 * 5
CIR19 :S / 5 * 4
CIR19 :S / 5 * 3
CIR19 :S / 5 * 2
CIR19 :S / 5 * 1
END
```

Figure 8.3.1b The children's programs in task 1

Not surprisingly, **the children's initial strategies for constructing the figure in task 4 (fig. 8.3.1) did not seem to incorporate the use of the geometrical idea embedded in the positioning of the circles' centres.** They both started from constructing the two bottom circles and found difficulties in working out the interface with the third circle, i.e. how to place it in the "correct" position with respect to the other two. **Alexandros adopted a strategy based on perceptual cues.** He used CIR9 (fig. 8.2.2c) to construct the first two circles and the intrinsic idea of a 180 degree rotation for the interface between them (fig. 8.3.2a). Apparently having planned to use CIR9 again for the third circle, he used his perception to take the turtle to the starting point, by typing in FD 10 (fig. 8.3.2b).

```
TO 3CIR
CIR9 20
RT 180 ◀──── a
CIR9 20
RT 180
FD 10 ◀── b
LT 90
CIR9 20
END
```
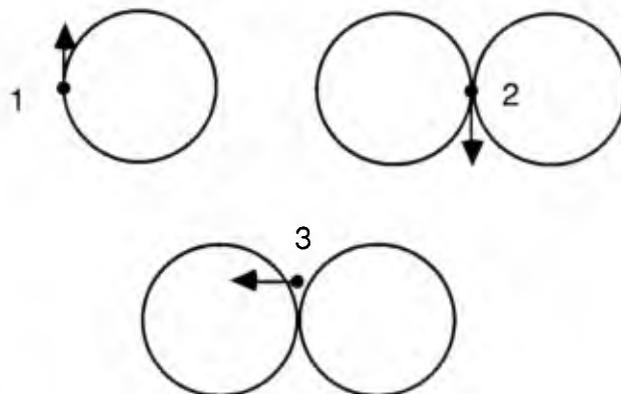

Figure 8.3.2 Alexandros' strategy in task 4

Valentini adopted an analytical strategy. After making the first two circles with the CIR19 procedure and an interface between the circles employing the use of the radii in order to take the turtle to the "correct" position (fig. 8.3.3a), her task was to work out the interface between the second and third triangle. Apparently perceiving the third circle as being in the centre and above the other two, she took the turtle upwards twice the length of the radius and towards the left one length of the radius (fig. 8.3.3b);

V: "It goes forward the radius of the circle, it's looking upwards, and then it does another radius, the equal one, it turns towards the left and it goes to the middle of the future circle..."

```
TO V2          TO V
PU             RT 90
FD 100         PU
LT 90          FD 100
FD 50          LT 90
RT 90          END
END
```
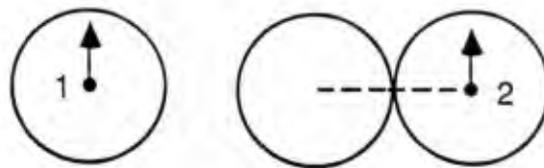
```
TO V.KAIMAKI
CIR19 50
V          ◄─────── a
CIR19 50
V2         ◄─────── b
CIR19 50
END
```
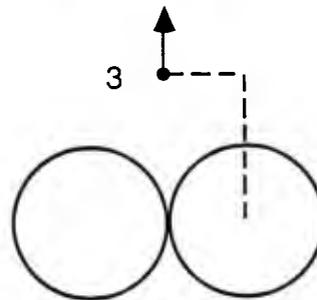

Figure 8.3.3 Valentini's strategy in task 4

The children subsequently discussed and tried out other perceptual and analytical strategies, deciding that they were not "the right ones" perceptually, by looking closely at the screen.

8.3.2b) Strategies involving the use of intrinsic notions

One of their later strategies, i.e. just before they saw the connection between the positions of the three centres, **was interesting vis - a - vis**

the intrinsic ideas they used. Their main idea was to make the top and bottom right circles using the CIR4 command and a 180 degree rotation interface and then trace backwards along the curve of the bottom right circle for the interface between second and third circle (fig 8.3.4).
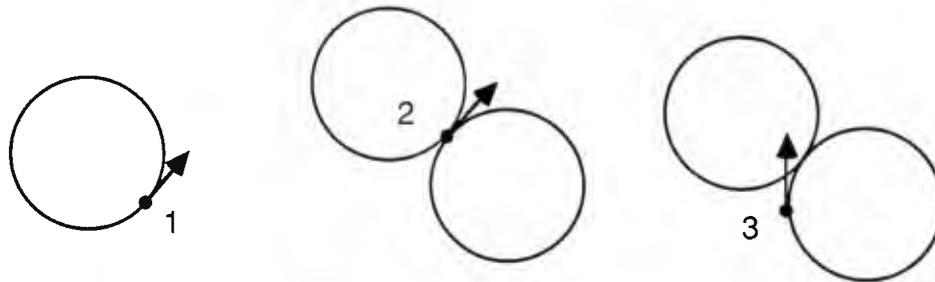
Figure 8.3.4 The children's intrinsic strategy in task 4

Valentini's explanations to Alexandros (the strategy was her initiative) **and her two programming attempts on the computer (fig. 8.3.5a and b) illustrate her strategy;**

(these comments refer to her first programming attempt, fig. 8.3.5a)

V: "...then we'll use CIR4. We do the circle, we then do one sixth of the circle (refers to the interface between second and third circle), we know how much it will go forward (refers to the constant input to FD in the CIR4 procedure), we do one circle..."

A: "Yes, the CIR4, do you know where it finishes?"

V: "Yes, at the place where it started... as it's turning towards there (i.e. to make the first circle, fig. 8.3.4 - 1) it will make another circle turning towards the left... (i.e. the second circle, fig. 8.3.4 - 2) and then it does one sixth of the circle... (i.e. the interface between second and third circles, fig. 8.3.4 - 3)"

A: "How will we tell it to do one sixth?"

V: "Instead of saying 36 we'll say 36 divided by 6." (refers to the input to the REPEAT command)

```
RT 45                                RT 45
CIR4 5                               CIR4 5
REPEAT 36 [FD 5 LT 10]               LT 180
REPEAT 36 / 6 [BK 5 LT 10]           CIR4 5
REPEAT 36 / 6 [FD 5 LT 10]           REPEAT 36 / 6 [FD 5 RT 10]
                                     LT 180
                                     CIR4 5
          a

                                          b
```

Figure 8.3.5 The children's programming attempts
in their intrinsic strategy in task 4

Valentini's first effort involved rewriting curvature commands with left turns for curves extending to the turtle's left. Her perception of the graphical feedback as "incorrect", however, was followed by a second attempt using the same strategy, but only right - turn curves and the appropriate interfaces (fig. 8.3.5b).

It is suggested that Valentini's strategy for the interface between second and third circles employed different geometrical ideas than in the previous strategies. **In her analytical attempt to find some geometrical connection between the position of the second and third circles, she did not use any notions referring to a part of the plane outside the turtle's path.** The shortcoming of her strategy with respect to the outcome, was a result of the only instance where she used her perception, i.e. in deciding that the arc formed by the points of connection between the circles was one sixth of a circle (this may have been geometrically correct, but Valentini did not show signs of using analytical cues).

A similar strategy was employed by Alexandros in task 7 (fig. 8.3.1). He wrote a procedure incorporating three semicircular curves by using the REPEAT command with a suitable input and the appropriate turtle turns (fig. 8.3.6). **He used perceptual cues to decide on the length of the vertical line segment, i.e. his overall strategy did not seem to employ analytical reference to points outside the turtle's path.**

```
TO 3CIR
FD 100
LT 90
REPEAT 180 [FD 1 LT 1]
REPEAT 180 [FD 1 RT 1]
REPEAT 180 [FD 1 LT 1]
END
```

Figure 8.3.6 Alexandros' strategy in task 7

Adopting a strategy of the turtle tracing along a curvature, however, did not necessarily imply lack of analytical reference to points outside of the turtle's path. Valentini's strategy in task 7 illustrates how, on the one hand, she used the notion of curvature in modifying the CIR9 procedure to construct semicircles involving right and left turtle turnings and on the other, how the inputs to her semicircle subprocedures referred to the centres of the semicircles (each input represented the length of the radius) and were such that the sum of the three diameters was equal to the length of the vertical line segment joining the edges of the figure (fig. 8.3.7).

```
TO CIR10 :R
RT 5
REPEAT 18 [FD :R * 2 * 3.14 / 36 RT 10]
LT 5
END
```

```
TO 3C9
RT 90
CIR11 15
CIR10 15
CIR11 15
LT 90
FD 90
END
```

```
TO CIR11 :R
RT 5
REPEAT 18 [FD :R * 2 * 3.14 / 36 LT 10]
LT 5
END
```
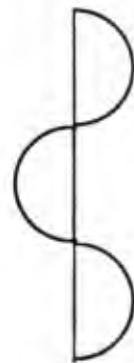
Figure 8.3.7 Valentini's strategy in task 7

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Returning to the children's strategies for constructing the figure in task 4 (fig. 8.3.1), **the following episode illustrates how the children saw the connection between the positions of the centre points of the three circles.** After the children seemed to abandon their intrinsic "curvature - tracing" strategy, the researcher prompted them to think about the interface between the first and second circle. Alexandros then seemed to spot the uniformity of the lengths of the interfaces between the circles by turning the piece of paper with the figure twice, so that a bottom circle would go to the top and vice versa. Although the children seemed enthusiastic about their "discovery" concerning the connections between the radii, they still did not consider the positions of the centres; although they had decided on the length of the second interface, they turned the turtle 45 degrees to the left (fig. 8.3.8), apparently using their perceptual cues.

```
CIR19 40
RT 90
PU
FD 80
LT 90
PD
CIR19 40
LT 45
```
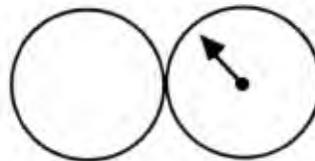
a

Figure 8.3.8 The children's perceptual strategy in task 4

The researcher decided to prompt a focus of attention on the uniformity of the figure they had noticed from Alexandros' turning of the piece of paper. Their dialogue at this point illustrates their first use of the centres of the circles;

V: "You know what I'm thinking? Why should it be 45? (the turn) You know why? Since, if we join the three dots... a triangle is been done (formed)... an equilateral."

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A: "Equilateral."

V: "Eh?"

A: "And the sum of the angles of a triangle... is 180?"

V: "Look. It goes forward. It goes left, you know how much? It goes left 360 divided by 3. So, how much is it? 3... 120. It goes left 120... it goes forward and does the circle... (she observes that the turtle's current heading is zero)... *120... 30 because I was thinking that it's like that, so 90 plus 30...* (she types LT 30)."

This new strategy involves a rather complicated but coherent use of both intrinsic and euclidean notions. The reference to the two radii forming the sides of a triangle and the centres of the circles forming its vertices implies the use of euclidean ideas. On the other hand, deciding on the turtle's turning after constructing each circle was based on a partitioning of a total turtle turn. Furthermore, Valentini's argument for turning the turtle left from a zero heading to face the top vertice of the triangle, was based on partitioning the turtle's turn into a 90 plus 30 degree turn, a strategy which has a striking similarity to an intrinsic strategy used by Nikos in chapter 7 (fig. 7.3.4,- 2).

8.3.2d) The children's differing perceptions of which notions were necessary for the construction of a figure

The children's use of intrinsic or euclidean ideas was, of course, not always related to the ideas embedded in the tasks. Valentini's strategy for constructing the figure in task 3 (fig. 8.3.1) is an example of this point. Although in choosing the CIR9 procedure she used the figure's property of the circles' connecting on a specific point, **her choice also implied a reference to the centre of the circles which was not "necessary" in order to construct the figure (fig. 8.3.9a). Furthermore, she seemed to "impose" an additional property on the figure with the use of her perception, by deciding that the turtle's total turn after constructing the circles was 90 degrees.** In her strategy, she

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partitioned the 90 degree turn into four parts (and later corrected the bug, dividing 90 by 3) before starting to write her program. Alexandros' strategy of using CIR4 with a consistently increasing input for the circles and a 30 degree right turn for the interfaces between them, did not imply the use of euclidean ideas (fig. 8.3.9b). **However, it is not clear whether Alexandros had realised that euclidean ideas were not necessary for constructing the figure.**

TO SHELL
RT 90
CIR9 60
LT 22.5
CIR9 45
LT 22.5
CIR9 30
LT 22.5
CIR9 15
END

a

TO 4CIR
CIR9 20
RT 30
CIR9 40
RT 30
CIR9 60
RT 30
CIR9 80
END

b
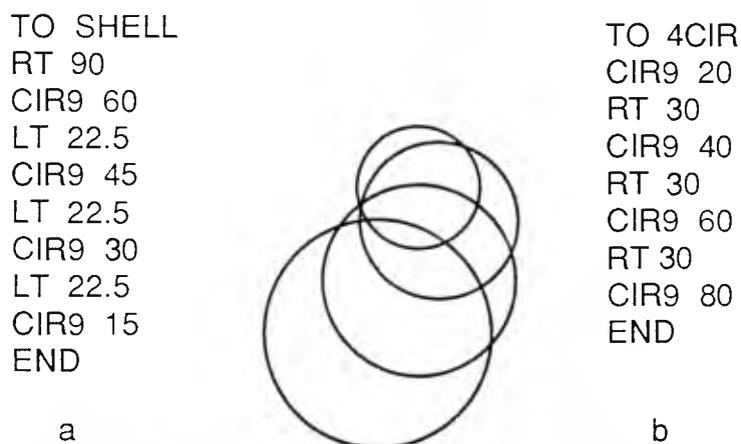
Figure 8.3.9 The children's strategy in task 3

8.3.2e) Discriminating geometrical notions embedded in different methods of circle construction

In the process of constructing the figure in task 5 (fig. 8.3.1), and during the subsequent interview, the children used euclidean ideas concerning the circle and distinguished them from intrinsic ideas.

The programs and the written explanation Valentini wrote on paper illustrate her strategy for constructing the figure;

I will use the CIR9 and a little bit, in a way, (I will use) *the TC, because it suits me to go forward and then to make the circle* (she means the small circles) *from the side. However, the way in which the circles are formed, it's like the TC.* (fig. 8.3.10)

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```
TO C
PU
FD 50
LT 90
PD              TO CLOCK
CIR9 20         REPEAT 8 [C RT 45]
RT 90           END
PU
BK 50
PD
END
```

Figure 8.3.10 Valentini's strategy in task 5

Up until that point, throughout the whole of the study, neither child had made explicit the euclidean method of constructing a circle which was embedded in the TC procedure. During the children's discussion of their programs, however, Valentini verbalised what seemed to be a context - specific version of the euclidean definition of a circle, in the process of explaining to Alexandros why she had chosen to use the CIR9 procedure for the small circles instead of the CIR19. The reason she gave for using CIR9 was that the small circle would be drawn directly from the turtle's position after moving it away from the centre in PENUP mode. The following dialogue illustrates her perception of the euclidean definition of the circle;

V: "...that where the turtle will stop, that's where it will turn and make the circle." (she means the small circle)
R: "Hm. And why does that help you?"
V: "Because I know the distance here will be exactly 50 (50 was the radius of the large circle)"
R: "Ah. Why?"
V: "If I put here 50, there 60, there 70, it won't be a circle because a circle is when we take it from the middle and we measure from all the sides continually round and round, from the same place, if we measure..."
A: "From the centre."

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V: "Yes the centre, round and round and round the same length and we put a little marker we'll get the circle."

It is suggested that although Valentini used the euclidean definition of the circle in explaining her method for constructing the figure, she had not synthesised the notions of the centre and the radius between the contexts in which the children had used them so far and in this new conception of the circle. Notice, for instance, how she seemed to refer to a specific point in the plane rather than make an explicit reference to the centre of the circle before Alexandros drew her attention to the significance of the point she was talking about. Furthermore, although she explained quite clearly about equal distances, she did not mention the word radius, not even after the researcher's direct question on whether this distance had a name; she said *"I don't know"* and continued talking about the radius to complete her argument.

Valentini's case for using the CIR9 procedure and not the CIR19 involved her discriminating of the process by which the circle is constructed in each case. In order to support her argument, she used the similarity between the construction process involved in the CIR4 procedure with that of the CIR9 procedure, in the context of making one of the small circles:

V: "...the CIR4 and the CIR9 are the same, because..."
R: "The same?"
V: "I mean that they are related in this shape in particular. I mean that it goes there, I turn left again, I give it a number, it does the circle again I turn it right and take it back."
R: "So what is it that makes them almost the same?"
V: "Right. That... of course in one we know the precise... in the other one we don't know it, but here in both cases we turn and we make the circle as usual, while if I said that CIR4 and CIR19 were the same... they are not the same because in CIR19 it starts from the middle like TC and in those two it starts

from the edge."

In her argument, Valentini referred to the turtle's action in constructing the circle. **She consequently seemed to refer to the CIR4 and CIR9 procedures as a product of the turtle's action, implicitly de - emphasising how this action is quantified.** Her criterion for distinguishing the CIR4 and CIR9 procedures from the CIR19 and the TC involved the notion of where the turtle started (and ended) constructing the circle, i.e. on the curve itself or on a point away from the curve. **Consequently, there seemed to be an implicit use of the notion of the circle's centre and a de - emphasising of the radius, an interesting contrast with other occasions where the converse occurred** (e.g. in the construction of target figures resembling that of task 1, during the children's projects in the previous phase of the study).

8.7.3f) Discriminating between an intrinsic and a euclidean method for constructing the circle

It seems useful to consider the geometrical ideas used by the children in the two previous episodes in conjunction. In constructing one figure (i.e. in the same context), they seemed to use both intrinsic and euclidean ideas in a coherent way in order to solve the task; Both Alexandros and Valentini used a euclidean method of equal distances from the centre in order to place the small circles in a circular formation, and the latter explicitly referred to a curvature - tracing turtle in the construction of the small circles.

A researcher's probing of the children's perception of the "status" of the centre in connection to the circle at that point in time, seems to indicate, at least, their acknowledgement of its existence as an integral part of the circle.

R: "Which is a nicer part of the circle, the centre or the edge?"
A: "The centre."

V: "The centre."

R: "Why?"

V: "Eh, because the centre is one, the edge is continually 360 times..."

R: "And that's nicer?"

V: "Because there are infinite... we can find edges."

A: "While there's one centre for one circle."

This incident, in connection with the two previous episodes concerning this task, corroborates the argument that **the children did not seem to find a qualitative difference in the geometrical nature of intrinsic and euclidean ideas in the way they were used in this context.** A further probe by the researcher, asking the children which of the three procedures, CIR4, CIR9 or CIR19 was easier resulted in a rather categorical statement that they were of the same difficulty. In the emerging discussion, **Valentini seemed to dissociate from a turtle - oriented method of constructing a circle, accepting that it exists, but not that it is the only valid method;** in explaining why she thought CIR4 and CIR9 were "equally easy", she said:

V: "Because both of them make a circle. A 36 - agon that is. Especially from the turtle's point of view, the turtle would say that 4 is easier. Because 4 is completely clear, you tell her 'go forward turn, go forward turn' while in CIR9 it does all that thing."

R: "So, for the turtle CIR4 is easier. Does that mean that for you CIR9 is easier?"

V: "It's the same."

A: "It's the same."

The above analysis (section 8.3.2) has indicated that the children's criteria for using intrinsic or euclidean notions were not primarily related to inherent characteristics of the notions themselves, but rather on aspects of the broader mathamatical situations generated during the sessions. This issue is further investigated in this section; the focus of analysis is on the nature of the criteria used by the children, i.e. on which aspects of the mathematical situations were important in forming their choices between employing intrinsic and euclidean notions. The titles in this section accordingly refer to these aspects.

8.3.3a) The use of the intrinsic schema

In planning their strategy for constructing the figure in task 6 (fig. 8.3.1), Valentini used the geometrical relation connecting the positions of the centres straight away; she wrote a modular procedure for the figure with a variable input for the radius of the circles and a fixed triangle side (fig. 8.3.11). **The words she wrote illustrate her use of the connection between the three centres:**

I used the CIR19, because at the place where one of the lines of the triangle ends and the other one starts, is the point which is in the middle of the circle.

```
TO  TR :S
RT  30
REPEAT  3  [CIR19 :S  FD  50  RT  120]
END
```

Figure 8.3.11 Valentini's strategy in task 6

Indicative of her perception of the equilateral property of the figure is also the fact that she made the division 360 / 3 on the piece of paper next to her procedure.

Alexandros, however, did not seem to perceive a geometrical connection between the centres, in spite of the fact that he used the CIR19 procedure; his attempts to turn the turtle after each circle were perceptual. Before trying out Valentini's program, the researcher asked Alexandros to explain his strategy;

A: "My idea is that I'll use CIR19. I'll make each circle from those three circles, where's the shape... (picks up paper with figure on it) I'll do one and then I'll move to form the triangle, each side of the triangle."

It is interesting that even though he seemed to perceive of the figure as a triangle he did not think of the geometry involved in turning the turtle to construct it; in continuing with his verbal plan, he stated that he would turn the turtle 45 degrees each time and even after Valentini's protests that *"it's wrong"* and the researcher's prompting for a more careful consideration of the amount of turn, his answers were based on perceptual cues - admitted in the end by Alexandros himself:

A: "It's 30 Mr. Chronis..."
R: "Why should it be 30, because it looks like it, or for any other reason?"
A: (short pause) "It looks like it."

Although children's difficulties with synthesising ideas across contexts has been well researched (for example, see section 2.1.4), it is interesting that the children had essentially solved the same problem in task 4, in a more "difficult" form, i.e. without linear cues between the centres and with the additional "misleading" property of connecting circles. However, Alexandros' lack of synthesis between the two tasks could be attributed to Valentini's initiative in solving the former task, i.e. to the questionable (with the benefit of hindsight) degree to which he had internalised the involved geometry.

In her subsequent explanation of her strategy to her peer, Valentini said that

there were three turns which were "not more than 360" and therefore dividing 360 by 3 gave 120 degrees for each turn. The researcher intervened in an attempt to further investigate the nature of the microview Valentini employed in using the notion of 360 degrees;

R: "Why are they 360 degrees?"

V: (short pause) "...because the three angles, em... (short pause) because if we take the triangle... (short pause) you know, I don't know how to explain it..."

R: "Never mind, we're in no hurry."

V: (looks at the paper with the figure) "I think it's like the circle is where the circle all round is 360 degrees, all its angles are 360 degrees. And like that (pause)..."

R: "What do you mean all its angles are 360 degrees.."

V: "Like in TC... she turns 360 times."

R: "Who does?"

V: "The turtle. Em... I mean if we go one by one degree and we go forward and make a dot, eh, and we do that 360 times we will get a circle. Did you get it?"

R: "So what does the turtle do in total?"

V: "She does one turn around herself."

Valentini's failure to pursue her initial argument may have been a consequence of her attempt to employ knowledge based on triangle properties, an area with which the children had not had much experience, at least in Turtle geometry. In her second attempt, however, she clearly seemed to use her turtle schema - notice how she switched from referring to "the circle" to referring to turtle actions "...she turns 360 times...". Her explanation consists of an interesting coherent combination of intrinsic and euclidean notions. From the beginning, she seemed to be referring to a total turtle turn being 360 degrees. However, in her clarification involving turtle action, she employed the idea of a euclidean construction of the circle using the paradigm of the TC procedure to convey an example of the turtle's total turn.

It is suggested that what made sense to Valentini at that point, was primarily to use the turtle metaphor, i.e. to think of turtle actions rather than to employ intrinsic geometrical ideas because euclidean notions did not make sense. The geometrical notions she employed were a combination of intrinsic and euclidean ideas, which in this context she seemed to find "compatible" with her turtle schema.

8.3.3b) Inductive versus deductive thinking

The researcher was interested at that point to probe further the nature of the children's criteria for using intrinsic or euclidean ideas. If the indications up till then were that within the Turtle geometric context of the Circle microworld, the children were quite prepared to use both intrinsic and euclidean notions in planning and explaining turtle actions, what was their view on employing knowledge of euclidean properties acquired in different contexts (e.g. in the classroom). For instance, the researcher had established that earlier in the school term, the children had been told about the sum of the angles of a triangle during their geometry lesson in their classroom. **What criteria would they employ for choosing between the total turtle trip theorem and the euclidean internal angles theorem?**

The children's experience with the former theorem up till that point, however, had only involved context - specific applications of a geometrical rule. It could not be therefore assumed that they would be aware of the existence of a generalised total turtle turn theorem. **Asking the children to make critical remarks on the two theorems could consequently have very little meaning at that point in time.**

The researcher therefore intervened to encourage the children to reflect on the generalisability of the total turn rule which they had applied in the context of the equilateral triangle during the previous incident; at the end of the

discussion, the children were required to express the general rule. After an attempt from both of them, the researcher asked them to explain how this rule applied to the triangle. Valentini's attempt is presented as an example, since their verbalisations were of equivalent coherence.

V: "If the turtle starts looking e.g. upwards and she makes a shape, eh... turning both right and left and she stops looking again upwards. Then... definitely, the turns... the degrees... the sum of the degrees is zero." (the children had already engaged in discriminating between zero and 360 degree rotations during the discussion)
R: "O.K., and this conclusion, why is it of use to us in this shape?"
A: " That the sum of the angles (apparently means quantities of turns and not internal angles) *of this triangle will have to be either 360 or zero."*

The logical sequence imposed by the researcher in this case was of a deductive nature, i.e. from the general theorem to the particular application. As discussed above, the aim was for the next question to have meaning for the children; checking that they remembered about the internal angles theorem, the researcher then asked the children which of the two theorems they thought was more powerful. The aim of the question was to investigate the meaning they would give to the word powerful (in an attempt to probe their criteria), rather than try and impose the researcher's view of the word's significance.

R: "Which of these two rules seems to you more strong. More powerful. And why."
V: "What do you mean more powerful?"
R: "I don't know. You'll tell me."
A: "For me, it's the 360. Because for me it makes more sense that starting at the same place where it finished is 360... I think that geometry forces you to believe that it's 180. Nobody says that the triangle necessarily has to be 180 degrees, otherwise you're dead. The best thing for me is that since she started and she finished there, it's like she hasn't done... zero. or she's done a

turn around herself."

R: "So... I understand what you're trying to say, you're trying to say that the 360 rule... what does it have?"

A: "It's more convincing."

R: "Hm. And you Valentini?"

V: "I'm with the 360 too but for another reason."

R: "Go on."

V: "Because I think that in a square, the sum of the angles of the square isn't 180, i.e. only in the triangle it's 180...while in any (stressed) shape...eh... if we make that turning, i.e. that thing with the 360... it's 360, over and out."

A: "It's geometry that forces you to say 'the triangle's it will be 180, otherwise it's not a triangle'."

Valentini's answer seems to indicate an appreciation of the notion of generality; she preferred the total turtle turn rule because it was more general, which for her seemed to mean more widely applicable, than the internal angles rule. **It is suggested that the fact that she seemed to consider generalisability as a powerful property, could be attributed to experience she had had in turtle geometry in using the total turn rule in different occasions. Her conception, therefore, of the generalised total turtle turn theorem seemed to have been a consequence of inductive thinking.** In support of this argument, her answer incorporated examples where the "superiority" of a more general rule was evident (e.g. the construction of a square being possible only with the use of the more general theorem).

Alexandros' answer seems to support this argument more clearly. The employment of the turtle schema seemed to make intuitive sense to him; furthermore, his answer incorporated a general statement rather than some specific application of the rule. **It is suggested that this general statement emerged from inductive thinking, i.e. in connection to previous specific applications of the rule. The internal angles theorem did not make so much sense because Alexandros did not**

have specific examples of applying the theorem available to him.

8.3.3c) Using knowledge based on personal experience

An incident providing similar **indications of the children's priority of preference in doing geometry based on personal experience rather than using notions from the intrinsic or the euclidean geometrical system** occurred in an interview after the construction of the figure in task 8 (fig. 8.3.1).

Valentini's strategy was similar to the one she adopted in task 6, i.e. a fixed modular procedure for a square incorporating CIR19 as a subprocedure with the "correct" input (half the size of the input to the FD command). Alexandros did use the geometrical relation connecting the four centres of the circles even though it is not clear whether he perceived of the relation as connected to a square figure. Furthermore, it could be the case that either child may have drawn upon their experience with a similar figure during their project involving the use of the TC procedure (fig. 8.2.14).

During the subsequent interview, the researcher attempted to investigate the issue of the nature of the criteria the children used to choose between using intrinsic and euclidean notions once more, by probing which method for determining the turtle's turn in constructing the square made more sense to them and why. It seems useful to present the emerging discussion in full in view of the subsequent analysis.

R: "How can we know that she turns 90?"
V: "How can we know? 4 times 90 makes us 360, equals zero. What we had said last time..."
R: "What was that?"
V: "Which says that when she makes a turn around herself, it's like she's done either 360 or zero."
R: "Hm. Isn't there another way? (to figure out that she turns 90 each time)?"

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V: "Hm... (long pause)."

R: "What's 90?"

V: "A right angle. Yes, 90 degrees is a right angle. And since a square is all right angles..."

R: "Ok, this is another way..."

V: "Yes."

R: "Why did the first way come to your mind?"

V: "Because we're used to it more, because from all tis geometry and all these things... they say 90, because it's 360 divided by 4 and it should be 360 and..."

R: "I don't understand."

A: "Like the other time."

V: "I mean that they tell us, that definitely it's 360 and that's it, you can't say anything, it's definitely 360 I know and you can't ask, you can't do a thing."

A: "It's like I told you the other time. That geometry forces us, we can't ask her... this, since it's been discovered that this is that much, that much we'll write it. We can't ask why is it like that and why is it like this because they'll tell us because that's what it want's to be."

R: "Ok, for you what's the best way."

A: "For me it's the first one."

R: "Why?"

V: "Because it's more natural... yes it's more natural, now I thought of that... anybody can understand it, that..."

A: "Even if he doesn't know turtle at all."

R: "Tell me something. What does someone have to know to understand this thing?"

V: "Nothing."

In this discussion, the children were effectively asked two questions: **why did the total turn method come to their minds instead of the internal angles and which method did they prefer and why?**

Before the first question, the researcher had probed the children's perception

of the analytical cues they had used to determine the turtle turnings after each circle. Valentini's answer suggests a reference to a generalised theorem in order to justify a specific application of the theorem. Notice how she mentions the specific application first *(..."4 times 90 makes us 360, equals zero.")* and then refers to the theorem, synthesising from another situation - that of solving task 6, which was actually two sessions before - *("...what we said last time...")*. The researcher's probing of whether she was actually referring to the generalised theorem was followed by her answer which confirmed the point. Her answer, which seemed to be explicitly referring to a turtle action and involved the use of intrinsic notions only, was mathematically rigorous *("...when she makes a turn around herself, it's like she's done either 360 or zero.")*.

Not surprisingly, the researcher had to subsequently impose a focus on the internal angles of the square before Valentini perceived the euclidean notions of angle embedded in the figure. The aim of the first question was to probe the children's perception of why the intrinsic method had come to their mind. A part of the answer was, of course, an implicit acknowledgement that they had had more experience with the intrinsic method. However, the interest lies in the other argument in which both children contributed and reflects on the kind of geometry they were doing in their normal mathematics lessons. **Their criterion for rejecting euclidean notions was the lack of personal experience and meaning in the way they had been taught; both children's utterances seem to refer to the Euclidean theorem as if it belongs to others,** e.g. *("...they tell us that definitely it's 360 and that's it, you can't say anything..."), (... since it's been discovered that this is that much, that much we'll write it. We can't ask why is it like that and why is it like this because they'll tell us because that's what it wants to be....")*.

Finally, the children's response to the second question seems to reveal indications of the intuitive nature of their intrinsic schema. Their justification of the statement that the intrinsic method is "more natural" (they used the word without any hint or intervention by the researcher during that situation or in

any other part of the study) involved the criterion that *"...anybody can understand it... even if he doesn't know turtle at all...".* It could be that what the children meant with this statement is that **what they perceived as formal geometrical knowledge was not a prerequisite for understanding an idea based on turtle action - i.e. the idea is based on experiences which already existed not as an immediate result of instruction.** In support of this last point is Valentini's view that in order to understand the total turn theorem one does not need to "know" anything.

8.3.3d) The use of structured programming techniques

Up till this point, the presented findings suggest that the children's criteria for choosing intrinsic or euclidean notions in solving the circle tasks were based on issues which did not seem directly relevant to the geometrical nature of the notions themselves, but rather, on issues related to the broader mathematical situations the children were in. For example, in task 6, the children's preference of using an intrinsic theorem was based on the situation within which this theorem was derived which involved inductive thinking. Furthermore, their case for rejecting the Euclidean theorem was related to the nature of Euclidean thinking rather than the use of the euclidean notions themselves. An example of another aspect of a mathematical situation which played a role in the children's choice of intrinsic or euclidean notions - **the aspect of programming** - is presented through the episode of the children's solving of task 9.

The children's strategies for constructing the figure in task 9 (fig. 8.3.1) illustrate a diversity vis - a - vis the intrinsic or euclidean notions they used. Valentini initially thought about adopting a strategy of the turtle tracing the curvature and interrupting after each total turn of 45 degrees to make a line segment perpendicular to the turtle's current heading (as she explained during the interview). In her written plan, however, while preserving the same modular structure of iterating an arc followed by a ray, she used the euclidean method of equal distances from the centre for both arc and ray (fig. 8.3.12a).

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In her apparent lack of clarity of how her program would "work",
Valentini had another idea which she tried out straight away on the computer,
abandoning her written plan; **her idea involved changing the modular
structure of her program altogether,** i.e. she decided to use the method
of constructing a circle with TC in order to make the 8 rays and then use the
CIR19 procedure in order to make the circle (fig. 8.3.13a).

```
TO M :S              TO M2 :S
PU                   PU
FD :S                FD :S
PD                   PD
FD 1                 FD 20
PU                   PU
BK :S + 1            BK :S + 20
PD                   PD
END                  END


TO SUN :S
REPEAT 8 [REPEAT 44 [M :S RT 1 M2 :S RT 1]]
END
```

a

```
TO MOVE1
PU
FD 40
PD          TO TIC
FD 20       REPEAT 360 [REPEAT 8 [MOVE1 RT 45]]
PU          END
BK 20
PD
END
```

b

Figure 8.3.12 The children's strategies in task 9

Like Valentini, Alexandros attempted to combine circle and rays into one
module in his written plan. The procedure he wrote on paper, however,
illustrates his difficulty in thinking about the construction process and in
perceiving of the precise modules without feedback from the computer;
although the main ideas seem to be present, such as the global plan beeing

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the construction of a circle (i.e. the input of 360, fig. 8.3.12b) and the module consisting of iterations of a subprocedure for the ray followed by the "correct" turn, he seemed to employ intrinsic notions for the former (the circle) and euclidean for the latter which clashed vis - a - vis the construction method. Inspite of the geometrical notions involved in Alexandros' confusion, **it is suggested that the main cause of the problem was his well established difficulty in constructing modular programs in the abstract, i.e. without trying them out on the computer.**

```
TO M2 :S
PU
FD :S              TO SUN :S
PD                 REPEAT 8 [M2 :S RT 45]
FD 20              CIR19 :S
PU                 END
BK :S + 20
PD
END
```

Figure 8.3.13 Valentini's programming in task 9

In fact, both the children's written plans imply some difficulty with the modular structure of incorporating the curvature and the rays in the same module. In the interview that followed, Alexandros seemed to focus on the problem in a way which supports the above suggestion; comparing one modular structure with the other, he said:

A: "...by making these lines, repeating them and afterwards the circle too, that's easier than making both lines and circle together... I mean that on the big shape you have to have both the circle and the lines... to get them in a thingie, where they are both together. So, in this way Valentini said now, it separates them a bit."

Furthermore, the researcher's prompt into **the geometrical notions** involved in the "curvature - ray" iteration structure was followed by a coherent answer by both children, each using different geometrical notions:

V: "We tell her 36 divided by 8, we tell her the 36 divided by 8... therefore the 45... every 45 we tell her to go 90 and make a line too, tak tak. " (shows with finger)

A: "She makes this shape with CIR4. To... she makes the circle and each... she'll divide the 8 with the 360 and each... yes it's 45 and each time she will turn 45, she will turn 90 and make the line..."

Valentini's answer seems to be referring to one of the curvature tracing procedures (either CIR4 or CIR9) interrupted by a 90 degree turn after every 45 degree arc in order to construct the ray *("...every 45 we tell her to go 90 and make a line too...").* Alexandros' answer explicitly refers to the intrinsic procedure (CIR4) and to the turtle action of turning - his explanation in this case is more accurate than Valentini's *("...each time she will turn 45 she will turn 90 and make the line...").*

It could therefore be the case that the children's difficulty with this structure was the nature of the structure itself rather than the involved intrinsic or euclidean notions. In separating rays from circle construction, the children could think of each module of their program as an object which they could understand - they had had experience with constructing both shapes before (the ray - type circle had been a part of their own projects with the TC procedure). Valentini's remark seems to support this argument:

V: "I think that first of all it's much simpler. You make two separate things. You use both CIR19 and TC. With the TC you make only one thing to make these rays and then with a very simple... way it makes the circle...".

This episode has therefore been an example of the difficulties related to an aspect of the mathematical situation the children were in, i.e. the modularity of their programming, which

influenced the geometrical notions they used to construct the figure. After discussing the structure of the curvature - tracing strategy, the children wrote a procedure for the figure based on CIR4 (fig. 8.3.14). In their subsequent discussion comparing circle procedures in the context of this particular task, they pointed out the similarity in the construction method between CIR4 and CIR9, de - emphasising the nature of the input:

A: "Of course it's only the variable which changes. In CIR4 it's the size... the distance from a step, that's the variable, and in the other one we take its radius."

```
TO  BLA
LT  90
FD  20          TO  4SUN
BK  20          REPEAT  8 [REPEAT  45 [FD  1  RT  1]  BLA]
RT  90          END
END
```

Figure 8.3.14 The children's intrinsic strategy in task 9

8.3.3e) The use of analytical cues

The following final episode is used to illustrate the existence of another aspect of the mathematical situations which the children generated, which also seemed to influence the geometrical notions they used, namely **their progressing appreciation of using analytical cues for their constructions,** a phenomenon which was also observed during the previous studies, as for example in chapter 7 during Philip's and Nikos' own projects within the P.D.D. microworld. The episode took place during the interview following the construction of the figure in task 2 (fig. 8.3.1), where both children had used analytical cues in constructing the line segment between the two circles. In discussing the possibility of using CIR4 for the figure, the researcher probed the strength in the children's justification for using analytical cues;

R: "What about CIR4?

V: "CIR4... we wouldn't know where the middle is with CIR4 so that we can start to make... (Alexandros makes agreeing sound)"

R: "O.K., so I take the turtle more or less in the middle."

V: "Eh, we want to be exactly in the middle."

A: "Exactly in the middle... because it doesn't look pretty too..."

V: "That is, it could be pretty if we go more or less in the middle here, but we wouldn't go to the middle there either (means centre of second circle). *And so it will be a mess."*

In agreement with the findings from the other two studies, it would be reasonable to suggest that although the use of perceptual cues seemed entirely legitimate to the children, through their experience with the microworld environments in which they were working, they progressively incorporated the use of analytical cues in their strategies - at least to an equivalent "status" with the use of perceptual cues with respect to their priorities in deciding which of the two to employ. Although the general aspects of this issue are discussed in chapter 9, it seems relevant here to suggest that the children's criteria for choosing intrinsic or euclidean notions in their strategies for constructing the task figures were not influenced in favour of one kind of geometrical notions or the other. However, **the more frequent use of analytical cues seemed to encourage more frequent use of the geometrical notions embedded in the circle procedures and in the task figures, than that which was observed during the children's own projects in the first phase of the study.** In the previous extract, for instance, the children perceived the line segment to connect the centres of the two circles by looking at the figure drawn on paper. Furthermore, their criteria for using the geometrical notions involved seemed to be of a functional, personalised character *("...we want to be exactly in the middle... because* (otherwise)*it doesn't look pretty... and so it will be a mess...").*

8.4 DISCUSSION AND CONCLUSIONS

8.4.1 Discussion

The first phase of the study partially functioned as a learning sequence for the children to construct and use specific circle procedures. However, investigating the situations within which the children started to develop an integrated use of intrinsic and euclidean notions resulted in the illumination of issues related to the research (issues a and b, section 8.1.1).

For instance, the indications of the disparity between the intrinsic and the non - intrinsic schema in the children's minds which characterised the findings in the previous two chapters, were also present in the case of the present study, both during the phases of constructing the circle procedures and during the children's own projects (see section 8.3.2). During the construction phases the children seemed to focus on notions functional to the specifics of each task, which were not always the intended geometrical notions related to the respective circle procedure. Moreover, although during their own projects the children did use the notions embedded in the respective circle procedures, they did so in varying degrees of density over time, in relation to the functionality of the notions for the specific project the children were engaged in. Furthermore, there were indications of their use of intrinsic and non - intrinsic notions which were not specifically related to the notions embedded within the circle procedures.

In the presentation of the findings in phase 2 of the study, episodes concerning the mathematical situations within which the children used intrinsic and/or euclidean notions to solve the structured tasks were analysed with a particular focus on the actual geometrical notions used or ignored by the children. Indications of the children's varying degrees of use of the geometrical ideas embedded in the tasks were given. On the one hand, for instance, both children found difficulty in perceiving the relationship between the positions of the centre points in task 4 and Alexandros ignored the

diametrical properties of the line segment in task 7. On the other hand, both children used the notions embedded in the figure in task 5 in a relatively rigorous way. Instances were also elaborated where the children imposed "unnecessary" geometrical properties, or where it was not clear whether they were aware of which notions were necessary for constructing the figure and which were not, as in the episodes during the constructon of the figure in task 3.

Furthermore, instances of the children's coherent use of both intrinsic and euclidean notions in the construction of a figure were elaborated. For instance, the children's final construction of the figure in task 4 involved the employment of an intrinsic perception of the "global" structure of the task, i.e. an intrinsic method for constructing the embedded equilateral triangle, and the use of the euclidean notions of the radius and the centre of a circle in order to construct the three circles of the figure. A contrasting strategy was adopted in task 5, where the children perceived the global structure of the figure as a circle defined as points equidistant to the centre, and used arguments employing intrinsic notions for the construction of the small circles of the figure.

The first section in the presentation of the findings of phase 2 of the study yielded indications that the children's criteria for using intrinsic or euclidean notions were not primarily related to inherent characteristics of the notions themselves, but rather on aspects of the broader mathematical situations generated during the sessions. This issue was further investigated and presented in more detail in section 8.3.3, where the focus of analysis was on the nature of the criteria used by the children, i.e. on which aspects of the mathematical situations were important in forming their choices between employing intrinsic and euclidean notions. The relationship between these aspects and the children's choices is further discussed at this point.

Firstly, the children seemed to see sense in using their intrinsic schema to represent both intrinsic and euclidean notions. Whether a situation invited the

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use of the intrinsic schema or not seemed to be an important criterion for using a geometrical notion in the first place. However, there were strong indications that they did not seem to favor one kind of notion or the other as a consequence of having used their intrinsic schema to represent it. An example of the children employing their intrinsic schema in order to use geometrical notions of a contrasting nature was their turtle - oriented verbalisations of their plan during the figure in task 6, which incorporated both intrinsic notions (e.g. using the total turtle turn rule to determine the amount of turn at the vertices of the triangle) and euclidean notions (e.g. perceiving of the centres of the circles as the vertices of the triangle and consequently using the appropriate circle procedure).

A second aspect concerning the employment of the intrinsic schema was illuminated by the children's verbal opinion about its nature, expressed in the context of the interview after having constructed the figure in task 8. Although researchers have made insightful contentions concerning the intuitive nature of the intrinsic schema (Papert, 1980, Lawler, 1985), there has been very little hard evidence of children's perceptions on this issue. It could be suggested that Valentini's and Alexandros' opinion that "...it's more natural... anybody can understand it... even if he doesn't know turtle at all... (he would need to know) ...nothing", indicates that they considered thinking with the turtle schema not to require knowledge coming from the outside, i.e. that the prerequisites for using the schema were already there, as part of experience the children would acquire irrespective, for instance, of their schooling. It is recognised that this would be a very strong claim in relation to the presented evidence in isolation. However, it is suggested that the context of the specific mathematical situation within which this dialogue took place and the more general context of the children's considerable experience with the turtle strengthen the significance of the children's comments and consequently support the argument, rather than contradict it.

Consequently, two factors concerning the role of the intrinsic schema in the children's criteria for choosing between the use of intrinsic and euclidean

notions emerged from the analysis of the data; firstly, they saw sense in employing the schema and did not seem to favour one kind of geometrical notion or the other as a consequence of having employed it; secondly, their criteria for using the schema tended to relate to its intuitive nature rather than to the use of geometrical notions.

It seems worthwhile at this point, to draw the reader's attention to the distinction between employing the intrinsic schema and using geometrical ideas. For instance, it does not necessarily follow from the above argument that using the intrinsic schema implied using geometrical notions; from the beginning of the study, there is strong evidence that the children considered the use of perceptual cues as a valid method to decide on inputs to turtle action commands. However, it could be argued that their increasing appreciation of using analytical cues through their experience with the Circle microworld's tools seemed to strengthen the relationship between using the intrinsic schema and using geometrical ideas. In support of this argument is the episode which took place after the construction of the figure in task 2, where the children gave personalised reasons for using analytical cues, i.e. that the shape would be "pretty", whereas in the converse case it would be "a mess". As discussed in the "findings" section, however, this aspect of the progressive use of analytical cues did not seem to relate more to one kind of geometrical notions or the other.

Another aspect of the mathematical situations influencing the children's choices was the programming and modularity involved in the construction of the tasks' figures. Although the structuring of the programs for a figure seemed, in general, to influence the nature of the geometrical notions used, the children's choice of strategy seemed to relate more closely to the involved programming rather than to whether the notions to be used were intrinsic or eulcidean. The episode during the solution of task 9 illustrates the children's change of strategy from a procedure incorporating an integrated module of the two perceived "elements" of the figure (the rays and the circle), to a procedure consisting of two separate modules for each element. The children

seemed to have decided to change their program on the basis of the clarity of the involved modularity, since they coherently used both intrinisc and euclidean notions both before and after changing their strategy.

The children's criteria for criticising generalised rules involving intrinsic and euclidean notions seemed to be based on how this generalisation had been derived, rather than on inherent characteristics of the notions themselves. The interviews after the construction of the figures in tasks 6 and 8 illustrate how the children's criteria for perceiving the respective intrinsic theorems as more powerful than the eulcidean were mainly based on the inductive method with which the former were derived. The arguments against the latter did not seem to be related to difficulties in understanding the involved geometrical notions, nor to possible differences in the amount of experience the children had had with one kind of notions or the other. Rather, the children referred to the way in which the theorems had been derived, i.e. by generalising context - specific applications of a rule, or by being presented with the generalised theorem from the outset.

Finally, a further aspect related to the children's arguments for or against the intrinsic and Eulcidean theorems, was the extent to which they perceived that the notions involved in the theorems had been "personalised". Alexandros expressed the argument better in his statement that *"...geometry* (meaning formal geometry) *forces us, we can't ask her... since it's been discovered that this is that much, that much we'll write it...).* It is suggested that through this argument, the children were not referring to the Euclidean notions as such, but rather to the way they had been presented to them through the school system. In support of this argument is their relatively rich and coherent use of euclidean notions in the turtle geometric context of the study and their comments in other occasions (e.g. during the interview after constructing the figure in task 5), stating that euclidean and intrinsic notions are equally easy to understand.

8.4.2 Concluding remarks

The investigation in the present study concentrated on the nature of the criteria the children developed for choosing between the use of intrinsic and euclidean notions within a Circle microworld. The children had already had experience in using such notions to construct circle procedures and employ them in personal projects.

The children's use of both intrinsic and euclidean notions was identified and elaborated within mathematical situations generated during the solving of the tasks in phase 2 of the study. Investigation of the children's criteria for employing a geometrical notion provided evidence that the children did not seem to perceive qualitative differences between the nature of intrinsic and euclidean notions. However, their decisions on which notion to use were influenced by broader aspects of the mathematical situations which were identified and discussed.

Although the present study provides further indications of the intuitive nature of the intrinsic schema, it also provides evidence that within the Circle microworld, employment of the schema did not seem to be tightly connected to the use of geometrical notions belonging to intrinsic geometry, or to a particular geometrical system. The study therefore provides a support to the argument that there is rich educational potential in creating environments which on the one hand invite children to use their intrinsic schema and on the other consist of microworlds embedding a range of geometrical ideas substantially wider than the one provided by intrinsic geometry.

CHAPTER 9

OVERALL DISCUSSION AND CONCLUSIONS

9.1 SUMMARY

The aim of the present study was to investigate the potential for children to use the turtle metaphor in developing understandings of intrinsic, euclidean and cartesian notions. Four aspects of the problem were investigated:

a) the nature of the schema children form when they use the turtle metaphor in order to drive it on the screen;

b) the possibility for them to use the schema in order to gain insights into certain basic principles of cartesian geometry;

c) how they might use the schema to form understandings of euclidean geometry developed inductively from specific experiences;

d) the criteria they develop in order to choose between using intrinsic and euclidean notions;

Three case - studies were carried out to investigate the above issues, each involving the use of a turtle microworld by (a) pair(s) of children from the total of ten who participated in the main research. The findings from each case - study are summarised below. A synthesis of the findings is undertaken and discussed in section 9.2. The subsequent sections will consider the limitations of the research and propose some implications for further research.

Issues a) and b) (section 9.1) were investigated by means of the T.C.P. microworld study (chapter 6) in which three pairs of children took part. The specific objectives for each of the three categories of activities in the study (each pair of children took part in all three categories - see section 6.1) were:

1) aim of category 1: to illuminate the process by which the children formed understandings of a systematic description of the plane (the activities here were different for each pair of children);

2) aim of category 2: to illuminate the nature of the children's understandings of the absolute coordinate and heading systems, while using a coordinate method to control the turtle;

3) aim of category 3: to investigate if and how they used their intrinisic schema in order to relate intrinsic and coordinate notions while choosing a method of changing the turtle's state in the coordinate plane.

9.1.1a) Findings from category 1

The analysis of the data from the category 1 activities indicates three types of notions which the children did not seem to relate to prior experience;

i) the existence, usefulness and nature of an organised system for naming locations,

ii) the existence of an analytical method for locating points and;

iii) the rules of the coordinate value system, i.e. the order of the values, the meaning of numbers as names of places and the meaning of signs as regions of the plane.

The analysis of the category 1 activities indicates that the children began to make sense of these notions in the context of changing the turtle's state.

9.1.1b) Findings from category 2

As a result of the analysis of the findings of the category 2 activities, the notions the children used in the process of discriminating between intrinsic and cartesian methods of controlling the turtle were identified. Action - quantity and sequentiality were the two important facets of the children's "intrinsic schema", i.e. the set of theorems-in-action the children seemed to have formed in using the intrinsic turtle commands. This finding was specifically related to issue a), section 9.1.

The notions the children used in controlling the turtle via the coordinate commands available in the category 2 activities were similarly analysed. An important aspect of the children's "coordinate schema" was that of changing the turtle's state by means of describing the position or the heading in which the turtle would end up after the change, i.e. the end state. Heading changes involved the description of a location or a direction. Changes of the turtle's position involved the description of a location. Signs and numbers were used for location descriptions and were given special meanings related to the coordinate system.

The discussion of the findings from category 2 in section 6.6.3, concentrated on evidence of a disparity of notions belonging to one schema from notions belonging to the other. That is, the children - at least initially - seemed to employ ideas derived from distinctly different sets of previous experiences in order to control the turtle with the use of the intrinsic or the coordinate commands respectively.

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9.1.1c) Findings from category 3

During the final T.C.P. microworld activites (category 3), there were indications of a balanced use of notions between those belonging to the intrinsic and those belonging to the coordinate schema, i.e. no pair of children seemed to have a predominent preference between using one set of notions or another. However, there was very little trace of children making links between these two kinds of notions. Where the forming of such links was identified, it was of a context - specific nature, as for example, in the case of Anna and Loukia discriminating between a dynamic and a static perception of angle (section 6.7.2e). The findings suggest that the main factor influencing the forming of such links may have been the amount of opportunity each pair of children had had to be in control of the turtle in the outset of their activities; the identification of instances of an integrated use of intrinsic and coordinate notions by Anna and Loukia, may have been related to the fact that their initial activities (category 1) were the most turtle - centred of the three pairs of children.

9.1.1d) Overview of the findings from the T.C.P. microworld study

The analysis of the data from the whole of the T.C.P. microworld study provides a description of the process by which the children began to form a dynamic coordinate schema (i.e. one with which they made changes to the environment) in order to make controlled changes of the turtle's state in the coordinate plane. The analysis suggests that the conflict created between the intrinsic schema and the use of coordinate notions was catalytic to the children's forming of their coordinate schema. Intrinsic and coordinate schemas remained separate in the children's minds for the most part, throughout their activities. There were some indications, however, of the forming of notional links between the two schemas; use of the intrinsic schema seemed to encourage such links.

The P.D.D. microworld study (chapter 7) focused on the investigation of issue c) (section 9.1) in the context of a pair of children's activities within the P.D.D microworld which enabled angle and distance measurements between turtle states (section 7.1.4). The specific objectives for each of the three categories of activities in the study (section 7.1.2) were to investigate:

1) aim of category 1: how the children integrated concepts involved in using the new tools of the P.D.D. microworld into their existing knowledge, and in particular, the extent and the way in which they employed their intrinsic schema in doing so;

2) aim of category 2: the process by which the children developed understandings of euclidean notions and the way in which they incorporated the use of the P.D.D. turtle tools into their intrinsic schema during this process;

3) aim of category 3: how and if their experience in making sense of the tools and using them in a euclidean setting influenced their thinking in Logo projects of their own.

9.1.2a) Findings from category 1

The analysis of the data indicates that the children's initial use of the microworld's tools was characterised by activities involving the discrimination between the two forms of turtle action (i.e. to move and to turn) and the metric systems for their quantification (i.e. turtle steps and degrees). Furthermore, the children began to discriminate between measurement and action and to develop an awareness of the quantity which was being measured on the plane. Their intrinsic schema seemed to retain its characteristics of action - quantity and sequentiality, the measurements partly perceived as a means to quantify turtle actions.

9.1.2b) Findings from category 2

The children's activities in the initial stages of category 2 consisted of constructing a procedure for an isosceles triangle with fixed dimensions, placing it in an upright orientation on the screen, drawing the bisector and measuring resulting internal relations of angles and lengths. The analysis suggests that the children's activities were characterised by an increasing use of analytical cues to decide on turtle actions involving either;

i) the use of geometrical properties (involving intrinsic and/or euclidean notions) perceived by the children or,

ii) the use of the tools to measure quantities which they perceived as otherwise unobtainable.

Their former use of analytical cues (i) began to progressively substitute the latter (ii) in cases where the children generalised from their measurements.

The children's initial attempts to construct their generalised procedure for an isosceles triangle (the LASER procedure, fig. 7.3.11) were hindered by their difficulty in generalising from using the relevant internal properties in specific cases. Furthermore, even after they made the relevant generalisations and incorporated them in their procedure, their first executions of the procedure were characterised by a lack of discrimination of angular properties related to the respective input.

9.1.2c) Findings from category 3

During the category 3 activities, the children engaged in four projects of their own choice, titled "Nested triangles", "Striped triangle", "Pyramid" and "Sailing boat". The analysis of the data indicates that during their first project, the children used the LASER procedure as an object / building block and certain structured programming techniques, in the process of developing a

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personalised geometrical rule. During their second project, the analysis highlighted cases of their meaningful use of the P.D.D. tools out of their own initiative and their use of static angle and length notions to work out the quantity of actions. This last issue was elaborated by means of the findings from the "Pyramid" project. Finally, analysis of the data from the "Sailing boat" project focused on the children's personalised reasons for using the POST command and their use of triangle properties in order to construct the ship's sail.

9.1.2d) A synthesis of the findings from the P.D.D. microworld study

The specific findings from the three categories of activities were related to the general issue investigated by the study (i.e. issue c, section 9.1) by means of putting forward the case that the analysis of the data throughout the P.D.D. microworld study provided evidence of an integrated existence of four aspects of the children's activities;

i) Their developing programming and mathematical strategies.

"Making sense of" and goal - oriented activities were identified. There is evidence that the chidren's programming strategies, such as their use of procedure and modularity, were not a-typical of the strategies observed in studies involving similar Logo environments. Furthermore, it was found that their mathematical thinking could meaningfully be characterised by activities of using, discriminating and generalising ideas. Such activities have also been observed in other studies of children working within Logo environments.

ii) The use of their intrinsic schema for controlling the turtle.

The schema which the children used to control the turtle seemed to preserve the characteristics of action - quantity and sequentiality described in the previous study. The children's use of the P.D.D. tools in the process of determining quantities of actions involved further discrimination of ideas

embedded in the intrinsic schema (see findings of category 1). Furthermore, the employment of the schema involved the use of intrinsic and euclidean notions (see findings of gategory 2).

iii) The use of the microworld's tools to develop understandings of euclidean concepts.

The findings indicate that the children mainly used the P.D.D. commands as mediating tools for the developing of understandings of euclidean notions. This process involved the children's progressive substitution of the employment of the tools in order to determine quantities of turtle actions, by the use of geometrical notions which had been generalised by means of earlier measurements.

iv) The children's use of euclidean geometry.

The analysis of the data suggests that the children's use of intrinsic and euclidean notions was of an inductive nature, i.e. that they used and generalised notions from within the context of their activities (such as the properties of the isosceles triangle, section 7.3), rather than starting from generalised hypotheses.

9.1.2e) Overview of the findings from the P.D.D. microworld study

The analysis of the data focused on the four aspects of the children's activities outlined above, in section 9.1.2d. The integrated existence of all four aspects of the children's activities suggests that a turtle geometric environment involving the use of the intrinsic schema was generated, it was of a dynamic mathematical nature, predominantly involving inductive thinking, and the geometrical content available to the children within this environment was extended from intrinsic to both intrinsic and euclidean geometry. The first two aspects have been previously observed and analysed in conventional Logo environments. The latter two aspects provide an extension of the conceptual

field available to children in conventional Logo activities.

9.1.3 The findings from the Circle microworld study

The Circle microworld study (chapter 8) focused on the investigation of issue d), section 9.1. The study involved a pair of children and consisted of two phases. In phase 1, the children participated in a learning sequence involving the construction and use of four circle procedures, each of which embedded specific intrinsic and/or euclidean notions. In phase 2, the children were given structured tasks involving the construction of figures consisting of compositions of circles. They had the choice of which of the four circle procedures to use in constructing the figures (section 8.1). The specific objectives of the research in phase 1, which was of a preliminary nature, were to investigate:

phase 1, a) the nature of the geometrical notions which were implicitly or explicitly used by the children during the phases of construction of the circle procedures;

phase 1, b) the extent to which and the way the geometrical notions characterising each procedure were used by the children during their own projects.

The objectives in phase 2 were to investigate:

phase 2, a) the extent to which the children used the geometrical notions embedded in the structured tasks and the nature of the notions they used for constructing the tasks' figures;

phase 2, b) the nature of the children's implicit or explicit criteria for choosing intrinsic or euclidean geometrical notions in their constructions.

9.1.3a) Findings from phase 1

The findings from phase 1 highlighted the disparity between the intrinsic and non - intrinsic schema in the children's minds observed in the previous studies. They seemed to focus on notions functional to the specifics of each task rather than the intended geometrical notions. During their projects they used the geometry of the circles in varying degrees, often focusing more on the use of structured programming techniques, rather than the use of geometrical ideas. At the end of the learning sequence they seemed aware of the functioning of the four circle procedures but rather lacked in explicit awareness of which geometrical notions were embedded within each circle construction.

9.1.3b) Findings from phase 2

In phase 2, the analysis of the children's choices between using intrinsic and euclidean notions in constructing figures consisting of circle compositions provided evidence of a balance in their use of both kinds of notions, i.e. the children seemed to be quite prepared to use both intrinsic and euclidean notions in planning and explaining turtle actions. The process by which the children used geometrical notions was illuminated within a perspective of the broader aspects of the mathematical situations generated by the children. Evidence was provided of a relatively rich use of the notions embedded in the figures. However, evidence was also given of cases where the embedded notions were ignored by the children, or where they imposed notions which were not intended by the researcher to play a part in the figure's construction. Furthermore, the children also used personal naive strategies in their constructions.

The indications that the children's criteria for using intrinsic and euclidean notions were not primarily related to inherent characteristics of the notions themselves, but rather on aspects of the broader mathematical situations generated during the research, lead to a further prompting of which of these aspects were important in the forming of the children's choices and why.

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Two factors concerning the role of the intrinsic schema in the children's choices emerged from the analysis. Firstly, they found employing the schema meaningful and did not seem to favour one kind of notion or the other as a consequence of having employed it. Secondly, their criteria for using the schema tended to relate to its intuitive nature rather than to the use of geometrical notions. The programming and modularity involved in the children's strategies also influenced their choices on which notions to use. However, their priorities in their decisions lay with the programming rather than with what kind of geometrical notions to use. The children's critical remarks on generalised rules involving intrinsic and euclidean notions referred to whether the rules had been derived via an inductive method or not, rather than on which kind of generalised rules were easier to understand. Finally, the children expressed a preference of employing notions which they had previously used in personally meaningful contexts than those presented to them through the school system. Their distinction between "personalised" and "impersonal" notions, however, did not seem to be related to the distinction between intrinsic and euclidean notions.

9.1.3c) Overview of the findings from the Circle microworld study

The analysis of the data from the Circle microworld study suggests that the employment of the intrinsic schema by the children did not seem to be tightly connected to the use of geometrical notions belonging to intrinsic geometry or to a particular geometrical system. The children did not seem to perceive qualitative differences between the nature of intrinsic and euclidean notions. However, their decisions on which kind of notions to use were influenced by the broader aspects of the mathematical situations generated in the study.

9.2 DISCUSSION AND IMPLICATIONS

9.2.1 The context of the research

The present study is located within the context of recent research focusing on both the geometrical content learned by children doing Turtle geometry and on their learning process (Hillel, 1985b, Kieran, 1986b, Lawler, 1985, Hoyles and Sutherland, in press, Hillel et al., 1986). Furthermore, from a methodological perspective, it is situated within a series of studies involving the detailed observation of pairs of children working collaboratively with the computer (Hillel and Samurcay, 1985, Kieran, 1986a, Kieran, Hillel and Gurtner, 1987, Hoyles and Sutherland, in press).

The summary of the findings given in the previous section suggests that within each specific microworld environment generated in the research, there is a strong potential for children to use their intrinsic schema to learn intrinsic and cartesian, or intrinsic and euclidean geometry. The discussion which follows attempts a synthesis of the issues emerging from the research across the three studies.

9.2.2 The children's use of analytical and perceptual cues

Throughout the three studies, there is strong evidence that the children considered the use of perceptual cues as at least an acceptable and valid method to make decisions on turtle commands and their inputs. This finding corroborates the results from Hillel et al's research (Hillel et al., 1986).

However, in the present research, the children progressively incorporated the use of analytical cues in their priorities. Moreover, there is evidence that the children in the P.D.D. and Circle microworld studies, which were of longer duration than the first, developed personalised reasons for using analytical cues instead of their perception, such as an appreciation of the accuracy of their constructions.

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This could be attributed to more than one factor. Although there is no conclusive evidence that the researcher's interventions encouraged the use of analytical cues, there is also no evidence of the contrary. It is suggested that there were two important factors influencing the use of analytical cues. Firstly, the nature of the microworlds, i.e. it could be argued that the embedded geometrical ideas were dense and more specific compared to conventional Logo, both because of the geometrical nature of the new primitives and due to the designed activities for the children. For instance, although during the first phase of the Circle microworld study, the children were asked to carry out four projects of their own, they were required to use the respective circle procedure / microworld tool, which in turn embedded specific geometrical ideas.

Secondly, the microworlds incorported a wider range of geometrical notions (than just those belonging to intrinsic geomerty) made available to the children via the use of the new tools. For instance, the P.D.D. microworld tools enabled them to measure quantities on the plane without requiring knowledge of geometrical properties in advance in order to decide on the quantities of turtle commands. Nikos and Philip's perception of using the measuring instruments was that they provided a tool with which to achieve accurate turtle actions. However, noticing the outcomes of their measurements led to conjectures about quantities, which the children tested by subsequent measuring. Confirmation of their conjectures was often followed by abandoning the instruments and using the geometrical notions involved in order to decide on the quantity of an action.

It is suggested that, on the one hand, the conceptual field of conventional Logo may lack in density of embedded geometrical notions and on the other, the available intrinsic notions are too restrictive. It may be the case, therefore, that finding it difficult to synthesise the required geometrical knowledge from their normal curriculum and also finding the process of using geometrical properties to make shapes on the screen not functional, children prefer to use their perception. It could be argued that these impediments enhance the use of their

perception and could be a catalyst for their use of a "drawing schema", observed by Hillel (Hillel, 1986). The children from the study had had considerable experience with conventional Logo and this not surprisingly influenced their use of perceptual cues, especially during the initial phases of the three studies.

It is therefore suggested that the use of analytical cues was not conceptually beyond the children's thinking, but rather, involved the use of a different framework of knowledge than the one which they had developed in using Logo. The argument that children's "naive" thinking is often a matter of the framework of knowledge which they use (Booth, 1981) is therefore corroborated by the present study.

Furthermore, the children's increasing use of analytical cues within the three geometrically rich microworlds of the study would corroborate Vergnaud's contention that there is a need to design conceptual fields where the children would be able to form and refine theorems-in-action (Vergnaud, 1982).

9.2.3 The children's intrinsic schema

Previous research has tended to perceive children's schema for controlling the turtle as integrated with the use of geometrical notions (Papert, 1980, Lawler, 1985). Furthermore, the notions which have been associated with the use of the intrinsic schema were predominantly those of intrinsic geometry (Papert, 1980).

The indications from the present research showed that although the children often seemed to identify with the turtle during their Logo activities, their use of geometrical notions was infrequent, as discussed above. This finding would support the argument that it is far from obvious that when children identify with the turtle to drive it on the screen, they are necessarily engaging in geometrical activity. On the other hand, the study shows that there is ample potential for children to use embedded geometrical notions in geometry - rich microworld

environments such as the ones in the three studies in the research.

The findings from the three studies show that the children seemed to see sense in identifying with the turtle and using experience based on bodily motion in order to control it on the screen. The theorems in action they formed, elaborated in the T.C.P. microworld study are corroborative evidence for this view; it could be argued that their use of action - quantity and sequentiality notions was not perceived by the children as using geometry, but as using experience which they had acquired irrespective of intended imposition of knowledge from the outside. This argument would be further supported by specific findings in the Circle microworld study, where the children themselves expressed the view that no knowledge is required in order to understand the intrinsic total turtle turn theorem.

The findings therefore corroborate Lawler's view that the intrinsic schema is built on intuitions related to bodily movement which, as Lawler states, can be traced back to the sensori - motor period. Moreover, the disparity between intrinsic and non - intrinsic schemas in the minds of the children in the study (this issue was given special attention in chapter 6) seems to support Lawler's contention that intrinsic and coordinate ideas originate from disparate microviews (Lawler, 1985). The links between notions from the two domains made by the children in the present study and their progressive use of non - intrinsic notions within their employment of the intrinsic schema, could be attributed to their age with respect to the six - year old child in Lawler's study; it could be argued that 11 year old children have had much more experience at making links between the different elements of the sensori - motor system.

The model of the intrinsic schema proposed in the T.C.P. microworld study provides a synthesis of the theorems in action used by the children during their activities. As discussed in chapter 6, no one child was aware of a synthesised set of rules to control the turtle; furthermore, the children's difficulties in discriminating between the elements of the model and synthesising notions across the two turtle states supports, rather than contradicts, the view that using

their intrinsic schema did not necessarily mean that they were explicitly aware of using geometrical notions.

9.2.4 The employment of the intrinsic schema and the use of geometrical notions

As mentioned above, the research indicates that using the intrinsic schema did not necessarily imply the use of geometrical notions. This argument, however, does not in turn imply that the children did not use the embedded geometry before and during the research. Furthermore, the analysis indicates that their increasing use of analytical cues was accompanied by an increasing use of geometrical notions. The geometrical nature of these notions is discussed in this section.

The findings suggest that initially, the set of experiences which the children seemed to employ in order to control the turtle seemed disparate to the experiences used to think about notions refe ring to the plane. The children's activities with the microworlds, where notions refe ring to the plane were embedded in controlling the turtle, enabled them to begin to incorporate such notions in their intrinsic schema and in turn to modify the schema itself. In the case of coordinate geometry, the analysis of the data from the T.C.P. microworld study showed that this modification was rather extreme, since the two primary notions of action - quantity and sequentiality were abandonned by the children. It is suggested, however, that the antithesis between the intrinsic and the coordinate schema as portrayed in chapter 6, rather helped the children to discriminate between intrinsic and cartesian notions, than hindered them. It could be argued that what the children found in common in the two schemas, and therefore used their experience from one schema to form the other, was the ability to fall back to enactive symbolising by means of the same vehicle, i.e. the turtle.

The schema the children formed in order to use euclidean notions did not involve such drastic changes to their intrinsic schema. Action - quantity and

sequentiality were preserved, but the euclidean notions were used in deciding on quantities and in referring to parts of a figure away from the turtle's position. Here too, however, the common basis for using intrinsic and euclidean notions was the ability to use the turtle schema, i.e. to think in terms of the turtle changing its state on the screen.

In this context of employing the intrinsic schema to use intrinsic, euclidean and cartesian notions, the study has shown that it is possible for children to generate learning environments akin to the ones generated in studies of children using conventional Logo. This issue, which was in focus during the P.D.D. microworld study in the context of euclidean geometry, gives credence to the argument that turtle geometric environments need not be restricted to intrinsic geometry if they are to preserve their dynamic characteristics. In support to this case, the findings from the Circle microworld study indicate that euclicean and intrinsic notions did not seem to have inherent qualitative differences when the children used them in the context of employing their turtle schema.

Furthermore, the study has elaborated that the children did not seem to find inherent qualitative differences in using intrinsic, euclidean and cartesian notions to control the turtle in the respective microworld environments. Employing the intrinsic schema did not necessarily imply that the children were aware of using geometrical notions. Furthermore, when geometrical notions were used, they were not necessarily intrinsic. This finding contrasts with Papert's implications that using turtle geometry invites using intrinsic geometry and not eulcidean and cartesian.

9.2.5 Employing the intrinsic schema to do inductive geometry

As discussed in chapter 2, educational practice has so far mainly involved the teaching of geometry as a tight deductive system, or in a reduced role of a practical topic, with not so much emphasis on its mathematical nature. It has been contended that an important factor in children's superficial learning of

geometry has been their difficulty in mastering the deductive structure of geometry as it was taught (van Hiele, 1959, Freudenthal, 1973). There have been arguments, however, for the potential of geometry as a means for children to do mathematics if it is seen through a wider perspective, i.e. as a field for engaging in both inductive and deductive thinking. Freudenthal has stressed the potential for such a role for geometry due to its mathematical nature, while von Glasersfeld has argued that a means to master deduction is through wealth of experience with induction (von Glasersfeld, 1985).

Although there was an initial enthusiasm regarding the potential of Logo for the learning of geometry, evidence from children's activities with the turtle has not been very encouraging in that the children did not seem to use geometrical notions embedded in Turtle geometry (Hillel, 1986, Hoyles and Sutherland, in press).

The present study has indicated that under certain circumstances, such as, for instance, widening the span of geometrical notions embedded in Turtle geometry and creating microworld environments inviting children to use geometry in a functional way, Logo may have an important role to play in providing children with the tools to engage in inductive geometrical thinking. The investigation of this potential in the P.D.D. microworld study provided evidence of the children's incorporation of euclidean notions into their developing use of geometry during their activities with the microworld. The children predominantly engaged in inductive thinking by using geometrical notions which they had generalised through specific observations made while measuring distances and angles on the plane and using the information to decide on quantities of turtle actions.

9.2.6 Relationships between the intrinsic schema and programming and geometrical content

In the present research, the geometrical aspect of the children's activities has been analysed in distinction to the programming aspect. Earlier studies of

children engaged in Logo activities have varied in the degree to which they have perceived those two aspects separately. It is not within the scope of this study to take a position on a general strategy of perceiving programming and geometrical activities in separation or not. Differentiating from the Brookline research (Papert et al., 1979), Noss argued that activities with Logo can generate integrated programming / mathematical environments and that programming is an essentially mathematical activity (1985). While not in contention with this point, it is argued here that there is still a need for the illumination of subtle features concerning the programming and the mathematical aspects of children's Logo activities, with emphasis on the latter. Recent research seems to support this view (Hillel, 1986, Kieren, 1987).

In the present study, the distinction between the children's use of geometrical and programming notions, served as a means to focus on geometrical issues of the children's activities. The children's programming strategies were analysed through the perspective of how they related to the children's use of geometrical notions.

During the P.D.D. and Circle microworld studies, the children not surprisingly showed an increasing fluency in using procedures, subprocedures and variables. However, the relationship between their increasing mastery of structured programming and their use of geometry was not always positive. For instance, Nikos' and Philip's use of geometry and programming during their own projects in the later stages of the P.D.D. microworld study seem, at least in some cases, to contrast. For example, the striped triangle project, while not involving the use of modular programming, did involve a relatively sophisticated use of the geometrical notions embedded within the triangle. On the other hand, their building of the BAM superprocedure by means of adding executions of the LASER procedure with fixed inputs involved very little use of the geometry related with the isosceles triangle constructed by the LASER procdure (see section 7.4.2). Furthermore, Valentini and Alexandros' fluent superprocedure - building strategies during their projects in the Circle microworld study, often involved very little use of the geometry embedded in

the circle procedures. In contrast, their project involving the most use of geometrical notions (the "Snowman" project) did not involve structured programming (see section 8.2).

A factor which might have influenced contesting uses of geometry and programming for Nikos and Philip, could be the nature of the P.D.D. microworld tools. Measurements on the plane in order to decide on inputs to specific direct - drive action commands may initially have encouraged direct - drive programming. A further factor may have been the relationship between structured programming and regular figures, such as a square or an equilateral triangle. In mathematical terms, regular figures can be constructed via consistent intrinsic rules (such as REPEAT 4 [FD 50 RT 90]) which do not require reference to the plane. The children's use of euclidean notions enabled them to write procedures for a wider span of geometrical figures (such as the LASER procedure for an isosceles triangle), which however, could not involve internal programming structure (section 7.3.2). Furthermore, when using procedures for figures involving euclidean notions during their projects, the children often focused on the inside of the figures, possibly since they were now able to use the internal geometry to make accurate shapes. A consequence was that in several cases they did not use the figure procedures as an object / building block for structured programming (section 7.4.2).

On the other hand, this may have been a consequence of the children's limited investigations of external geometrical properties of the figures in question. An instance when this issue arose was, for example, during Nikos' and Philip's efforts to place an isosceles triangle on a bisecting line segment in order to make the sail of their ship (section 7.4.2g).

The findings from this research highlight the complexities involved in the relationship between the programming and geometrical aspects of children's Logo activities and consequently support the view that we need to know more about ways in which children can be helped to make links between the two aspects and to use them in a complementary way for the development of their

mathematical thinking.

9.2.7 The learning environment

The crucial role of interventions made by researchers and teachers in Logo environments involving small croups of children working with a computer, has become increasingly obvious in recent research (Hillel and Samurcay, 1985a, Noss, 1985, Hoyles and Sutherland in press, Sutherland, 1988). In the present study, the researcher's attempt to maintain a balance between directive interventions and interventions which left control to the children highlighted the difficulty of this task. Inspite of an explicit intervention strategy outlined in the research design sections, the analysis of the data revealed subtleties in specific situations leading, in some cases, to interventions which were inconsistent with the researcher's intentions. The most frequent type of such intervention was over - directedness in cases where there was ground for the children to find out things for themselves.

There was a consistent attempt, however, throughout the study, to maintain awareness of the type of interventions made. It is suggested that awareness of the geometrical issues which were of interest to the study provided a valuable means of holding back over - directive interventions involving the learning of these issues. A broad distinction of the interventions made during the research, which proved a useful tool for deciding on when and how to intervene, was between those perceived by the researcher as part of the teacher component of the respective microworld and those aiming to clarify the research issues by asking the children to express what they were doing and why. With the benefit of hindsight, however, it is suggested that the latter influenced the children's learning. Without this type of intervention, the children may have restricted their verbal communication, thus keeping ideas which became explicit, at a level of implicit awareness. On the other hand, there were instances where persistent questionning of the children on their activities was counterproductive for their learning. There were also occasions, however, where if such an intervention had been made, a potentially important issue

may have been revealed during the interpretation of the data and/or the children might have made some idea more explicit than they did during the research.

The researcher attempted to maintain explicitness about the perceived influences of his interventions on the children's learning, particularly with respect to illustrated episodes used as examples of research findings. However, it is recognised that the presented episodes were within a context of vivid interrelation among the researcher, the children and the computer feedback. On the other hand, it is also argued that this research involved microworld environments incorporating a teacher component. The difficulties of distinguishing between teacher and researcher have been highlighted in previous research (Hoyles and Sutherland, in press, Weir, 1987).

9.2.8 The role of the microworlds and the designed activities within the context of the microworlds

The designed activities within the three microworlds played the dual role of learning sequences for the children and a means for creating a research environment, i.e. a setting encouraging the relevance of the children's thinking to the research issues. The study indicates the importance of activities involving a balance among children's own projects, activities in the form of solving structured or semi - structured tasks and teaching episodes with specific objectives. The findings support Noss' contention that children should be given ample time to explore ideas for themselves (Noss, 1985), but also indicate the importance of focusing the children's activities on mathematical and/or programming ideas so that they progressively incorporate them in their strategies.

The children's use of the primitives of the three microworlds in the study illustrated the importance of designing such primitives to be conceptually consistent with the Logo language. That is, the syntax, the error messages and the functioning of the new primitives was consistent with the principles

underlying turtle commands, by involving ideas easily understood by the children and consistent with the turtle metaphor. Furthermore, it is suggested that it was important for the introduction and the use of the new primitives to be carried out in a meaningful context for the children, so that they would find a functional purpose to use them.

9.3 LIMITATIONS OF THE RESEARCH

The limitations of the present research can be grouped into two types:

a) limitations related to general issues concerning the qualitative methodology of the research;

b) limitations which involve issues specific to this research only.

a) The context - specificity of the research has the obvious limitation in the generalisability of the findings. There were different aspects to the context in which the research took place, each one limiting a respective type of generalisability. Firstly, the participating children were from within a particular educational system. In spite of the fact that the researcher attempted to organise and take part in a year long "preparation" period, so that the children might participate actively in their learning during their activities with the microworlds of the main studies, there are limits to which this experience affected the children. On the other hand, it is argued that an acceptable informal atmosphere could have been almost impossible if the children had learned Logo in a directive manner during the preliminary phase. Secondly, the school in which the research took place was particularly priviledged and the family backgrounds of its children of above - average socio-economic status; such technology in the time the research was carried out was in any case not available in schools from the state system. Thirdly, the generated microworld environments involved a specific intervention strategy, a tightly designed set of activities for the children and restricted conceptual fields embedded within the use of the primitives of each microworld. It was not

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possible therefore within the scope of the study to investigate children's activities with the same microworlds, but in more open - ended settings. An issue, for example, which was not investigated, was if and how children might use the microworlds' primitives in the absence of the researcher. Furthermore, the unavoidably subjective nature of the interpretation of the results adds to the studies' limitations. Finally, the small number of children (ten) participating in the study, on the one hand limited the generalisability of the results, but on the other, enabled an in-depth analysis of the children's learning not easily achievable by quantitative research methods.

b) The model of the intrinsic schema proposed in chapter 6 was the outcome of a synthesis by the researcher of the notions which the children seemed to have adopted as a result of their previous experience with Logo, since the intention was to identify the theorems-in-action the children actually used. The issue of the children's difficulties or misunderstandings regarding the notions required to control the turtle would therefore warrant further investigation.

A further point regarding the limitations of the model of the intrinsic schema is that it refers to a rather specific environment where the children found themselves in conflict with their experience so far. Although this conflict contributed to the explicitness of the children's ideas about controlling the turtle, thus making things clearer for the researcher to interpret, it may also have limited the validity of the model; the situations of conflict may have influenced the children by encouraging them to discriminate ideas further than they would have done by using the intrinsic commands only. It was not within the aims of the study, however, to examine the extent to which children are unaware of the geometrical ideas embedded in conventional Logo environments.

The model resulting from the children's use of coordinate notions to control the turtle is limited by the time allowed for the children to develop understandings of these notions. It was not, however, within the aims of the study to investigate the question of which coordinate notions were only temporarilly difficult for the

children to understand, maybe as a result of their relatively overwhelming experience with the intrinsic commands, and with which notions the children would find pursuing difficulties.

In the P.D.D. microworld study, limited time was allowed for the children to carry out their own projects during the first two categories of activities. Although this was in line with the objectives of the research, i.e. to investigate children's learning in a euclidean setting, there was no investigation of how the children might use the microworld's tools in unstructured environments from the outset. This in turn could limit the strength of the argument that the children adopted the use of the P.D.D. tools in a natural way.

Furthermore, the question of whether the children would increase their use of euclidean notions within the microworld environment but not in the context of structured activities remains open. Although the children seemed to find the use of the tools relatively straightforward, it is not clear how much they would use them to measure euclidean figures had they not been asked to do so early on in their activities.

A final point concerning the P.D.D. microworld study is the relatively limited geometrical content covered. Although the researcher suspects that similar activities could be designed and similar results may be obtained from the emerging environents (given the limitations pertaining to the qualitative methodology of the research), this question remains open to investigation.

The above point would also apply to the Circle microworld study. The tools of the Circle microworld consisted of constructions of the same figure by methods employing different geometrical notions. Although there was a relatively rich generating of mathematical activities within this environment, it is not obvious whether this could be achieved in the case of similar microworlds involving different methods of constructing figures other than the circle. The researcher suspects that the geometrical content of such microworlds would have to be restricted to figures which can be constructed via an intrinsic method, i.e.

regular figures, such as a square, an equilateral triangle, regular polygons e.t.c.

The learning sequence in the Circle microworld study depended on the extent to which the children would perceive the important geometrical notions in order to construct a new circle procedure facilitating the construction of a task - figure. Not surprisingly, however, interventions were in some cases essential to encourage the children to perceive or use the important notions. This could limit the applicability of such a learning sequence without the participation of a teacher/researcher.

The children's choices between using intrinsic and euclidean notions to solve the structured tasks in the second phase of the Circle microworld study were related to the notions embedded in the task figures. This was in line with the objective of the study which was to investigate the nature of these choices. However, an investigation of the children's choice of notions in unstructured environments was not within the scope of the present study.

9.4 IMPLICATIONS FOR FURTHER RESEARCH

It is suggested that the generally diminishing role of geometry in mathematics curricula limits the importance for research into ways of creating Logo environments where the children might synthesise notions between the contexts of Turtle geometry and an established geometry curriculum, as for example in the case of algebra (Surtherland, 1988). Nevertheless, there is need to investigate ways of creating geometrical environments within the context of the clasroom, which encourage a synthesis between the dynamic and procedural aspect of children's learning and geometrical content in ways similar to the ones generated in the present research.

This study has provided examples of Logo microworld environments where Logo - experienced children have had the opportunity to use their cognitive schema for controlling the screen turtle in order to develop understandings of

geometrical notions belonging to intrinsic, euclidean and cartesian geometry. Further research is needed in order to map out content areas which could be catalytic to the creation of such environments. There is ground for more work on designing generalised microworlds offering a wide span of geometrical ideas for children to explore, such as the P.D.D. microworld. However, work also needs to be done in creating more specialised microworld environments for the children to develop understandings of "tighter" sets of geometrical ideas (an example from the study was the microworld generated with the use of the LASER procedure for the isosceles triangle). The latter microworlds could be perceived within the former ones, as in the case of the above example, or independently, as in the case of the Circle microworld.

There is also need to investigate the potential for generating geometrical microworld environments among younger children, possibly from the beginning of their primary schooling since computer technology might become increasingly available in schools in the near future. The indications from the present research that the children found the process of using their intrinsic schema not to require knowledge provided from some outside source support the argument initially put forward by Papert and Lawler that use of the schema bears upon children's intuitions related to enactive representations of ideas. It is therefore suggested that there is a need to find ways of exploiting these intuitions through turtle microworlds, so that children who are trying to make sense of ideas within mathematical situations have a means of employing enactive representations of these ideas.

The children in the present study engaged in inductive activities involving the use of geometrical notions previously perceived as only belonging to a deductive system. It is suggested that an area for further investigation would be to find ways in which children with considerable experience in using geometry in an inductive way, might use the same geometrical ideas in environments requiring deductive thinking. Computer environments allowing a dynamic manipulation of geometrical ideas within conceptual fields characterised by a deductive structure have recently been designed (e.g. the "CABRI" software by

Laborde and Laborde) and could consequently be used in such an investigation.

Certain specific points raised within the microworld environments of the present research warrant further investigation. As discussed in the previous section, the study illustrates how it is possible for children to use their intrinsic schema in order to form a schema for controlling the turtle via the coordinate commands. It was conjectured that microworld environments such as the T.C.P. microworld, i.e. which provide a choice between controlling the turtle with the intrinsic commands or via reference to coordinate systems, may enhance children's understandings of intrinsic and cartesian geometrical ideas. Such a conjecture warrants further investigation which could profitably employ a research design similar to that of the P.D.D. microworld study.

It is suggested that the P.D.D. microworld could be used to generate mathematical environments with similar characteristics to the one in the present study, over a considerably wide range of geometrical content. However, there is still much work to be done in finding out about ways of encouraging children's use of geometrical ideas within functional mathematical activities. It is felt that designing activities which allow a balance of directed and open-ended work for the children and finding out about efficient ways of intervening so as to stimulate them to form their own understandings leaves ample ground for further investigation. With respect to these two factors, there is need for both in-depth analysis of their effects on children's learning and for the longitudinal tracing of such learning within classroom settings. It is suggested that the present study has contributed to the argument that such research should be now seen as integrated with a systematic mapping out of geometrical content within microworld environments.

The P.D.D. microworld study consisted of a learning sequence involving the construction of a procedure embedding powerful geometrical properties of an isosceles triangle and the use of that procedure in personal projects. In effect, the Circle microworld study consisted of a learning sequence incorporating

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four such learning modules respectively involving four procedures. Furthermore, the structured tasks given in the second phase of the study encouraged the children to discriminate and generalise ideas embedded within the circle. An area for further research which could be seen as a specific case of the issue suggested above, is the design of such learning sequences in connection to generalised procedures embedding properties of geometrical figures. When, how and how often should such activities take place within Logo activities in a classroom setting?

To make a more general final point, it is not suggested that children's activities with Turtle geometry should be seen as related to geometry and programming only. Even in the present study, where the focus was on the geometrical nature of the children's work, there was ample use of other mathematical notions, such as decimal and negative numbers arising from the outputs to distance and angle measurements, arithmetical operations carried out by the children throughout the study and the notion of variable in relation to variable inputs to procedures. It is therefore suggested that research should be carried out into finding ways of encouraging children to synthesise ideas within Turtle geometry, but across mathematical domains.

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APPENDIX A

The data collected during the preliminary phase

Note 1: The following section provides samples of the data collected during the preliminary phase of the research. The "Pupil profile" questionnaires and the "record sheets" were filled in by teacher F in English. The "Pupil questionnaires" and the "Logo logs" are translated here from Greek to English. An original sample of a "Logo log" is given. With the exception of "Structured task 5", the items were adapted from the Logo Maths Project and translated into Greek - where applicable - by the researcher. The following reference was used:

Hoyles, C., Sutherland, R. and Evans, J., (1985), A Preliminary Investigation of the Pupil - Centred Approach to the Learning of Logo in the Secondary School MAthematics Classroom (1983 -1984), The Logo Maths Project, University of London Institute of Education.

Note 2: The analysis of this data was carried out on an informal basis, in line with the methodilogical placing of the preliminary phase within the whole of the present study. The present appendix is referred to in chapters 4 and 5.

Pupil Profile

Form Tutor *A.G.GELIS* ... ...

Please write briefly about the named pupil covering the following
points:

Your general view of the pupil as a person.
Your view of the pupil's ability.
Your view of the pupil's general attitude to work and school.
Any information you have on pupil's ability in different subject areas.
Any evidence that you have that the pupils discuss their Logo
programming activity outside their mathematics lessons.

Pupil ... *MARIA KOKKOU* ... ...

Is said to be a weak student
with a 'confused' self concept.
Nevertheless she is not a passive
member of the groups.

I, however, do not find Maria to
have a 'confused' self concept
but rather 'nervous' in initiating
activity / conversation because
of [fear] possible criticism / negative
reinforcement (which has probably
been / is experienced, in her life encounter

A questionnaire was given to each child after their first five sessions with the Logo club.

The following questions were asked:

- Which was your best moment with Logo so far? Why was it the best?
- What would you like to be able do with Logo in the furture?
- Do you prefer to work on your own or with others in the club? Why?
- What do you like to do in your free time?
- Which lesson do you like best in school? Why?
- Which lesson do you like less in school? Why?
- When you are doing mathematics:
  a) what do you like most and why?
  b) what do you like least and why?
- What would you prefer to design with Logo: a difficult but pretty house, or an easier one, but less pretty?

A.3: The Logo logs

A Logo log was given to each child during each Logo club session, in the last five minutes of the session. The three questions were:

a) What did you like and what didn't you like during this Logo hour?
b) What did you do during this Logo hour?
c) What would you like to do in the next Logo hour?

The (typical) answers in the sample given bellow are as follows:

a) In this Logo hour I played very nicely with my friends.
b) I gave commands to the turtle to go forward 9,999.
c) I would like to play again.

LOG-O-LOG
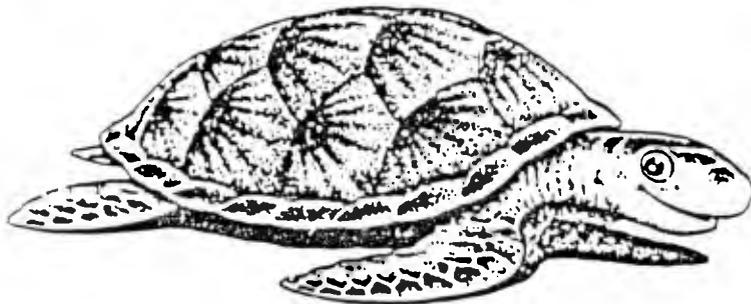
Τι σου άρεσε και τι δεν σου άρεσε σ'αυτή την Logo-ώρα;

Σ'αυτήν την Logo-ώρα έπαιξα πολύ ωραία με τους φίλους μου

Τι έκανες σ'αυτή την Logo-ώρα;

Έξερα διάβασες στη χελώνα να πηγαίνει μπροστά, 9.999

Τι θα 'θελες να κάνεις στην επόμενη Logo-ώρα;

Θα ήθελα να ξαναπαίζω.

Logo Club Record Sheet..

Date: _____

Names of pair:

_____

Number of procedures written: [ ]

Working at syntactical level:

YES NO
[ ] [ ]

Level of Motivation
[ ][ ][ ][ ][ ]
low   *high*

Level of Collaboration
[ ][ ][ ][ ][ ]
low   *high*

Making Sense of..

YES NO
[ ] [ ]

Level of Motivation
[ ][ ][ ][ ][ ]
low   *high*

Level of Collaboration
[ ][ ][ ][ ][ ]
low   *high*

If **YES**, which process? _____

_____

_____

Io obvious goal

Yithin goal directed activity

Iuilding up to a goal

[ ]
[ ]
[ ]

Brief description:_____

_____

_____

Goal Directed.

Initiated by: Pupils   Teacher   Copying

Pupils	Teacher	Copying

YES   **NO**

YES	NO

'el of Motivation

high

Type of goal:

	Picture	Abstract
Precise		
Vague		

Level of Collaboration

low   *high*

How far do you think the pupils understood what they were doing?

Very little	Partially	Very well

pupils achieve original/modified goal?   **YES**   **NO**

YES	NO

[O, why not?

Pupil decision	External decision	Other

Interventions.

	Not Requested	Requested
ivation		
lection		

Directional:

	Not Requested	Requested
Nudge		
Factual		
Powerful/ m/al ideas		

: spontaneous feelings about the session:

_____

_____

_____

_____

A.5: The "Four squares" task

In the second administering of the task, the four squares were in a vertical formation.
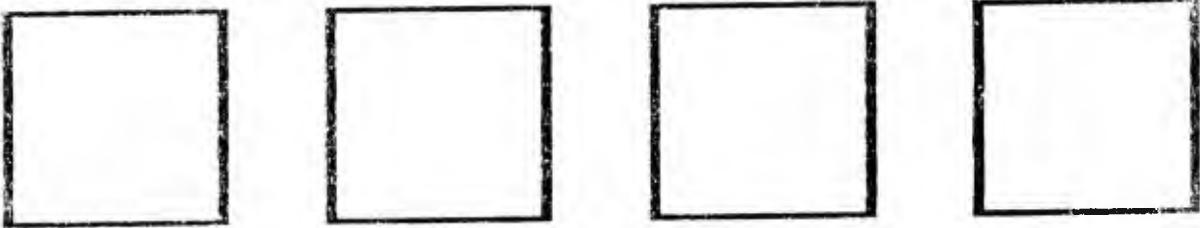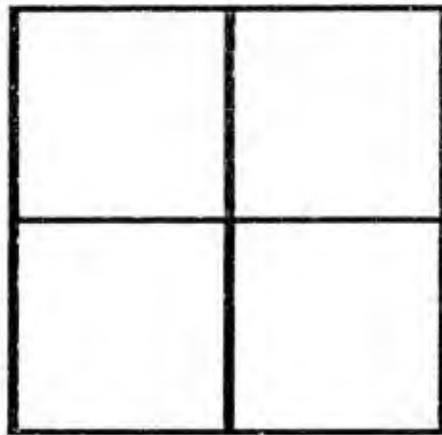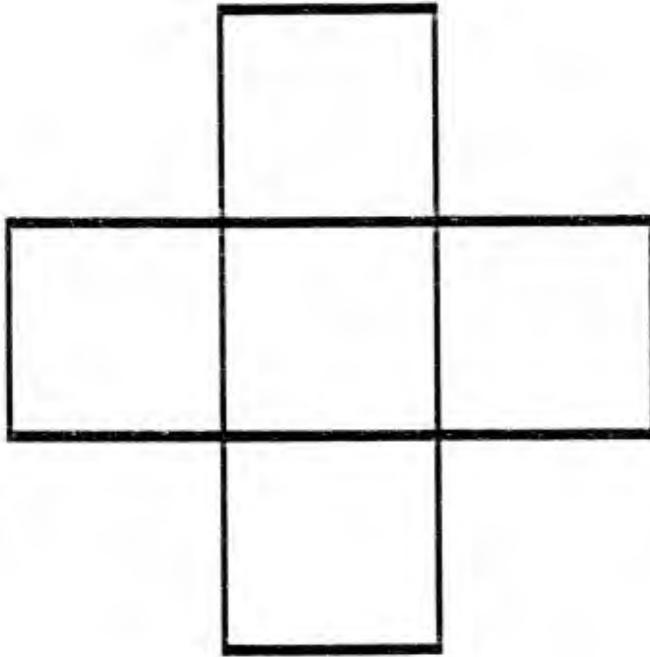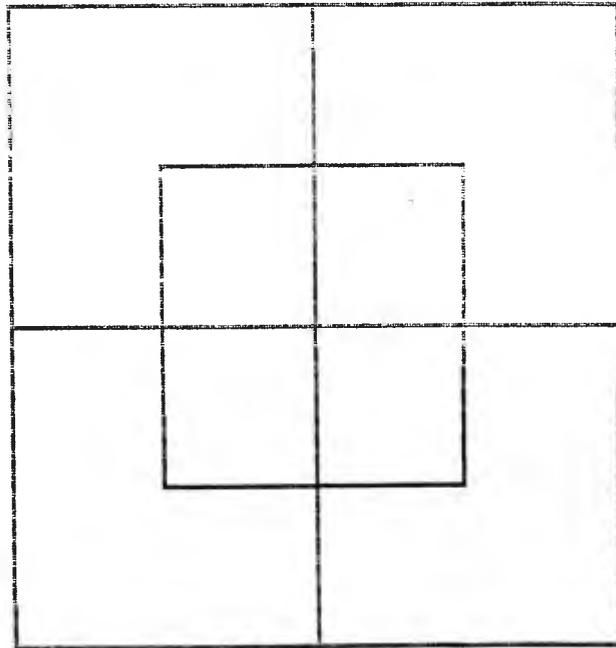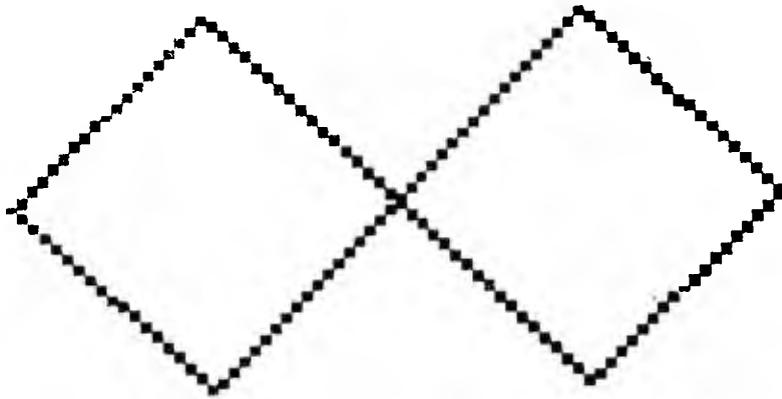
Structured task 1

APPENDIX B

The progress of the school Logo program

The present document is a report of the so far three - year progress of the Psychico College computer program whose general aim is to integrate the use of new technology into the school culture with the purpose of cultivating powerful ways of meeting existing and future educational needs. One of the primary aims of the program is to foster environments where the children take active control of their learning, to encourage original and independent thinking and to develop their mathematical thinking.

Before fully implementing the program, two years were spent in preliminary studies and applications in order to establish the integration of the technology into the present culture of the school. Full implementation of the program started in 1987/88 and may last for four academic years, i.e. till the year 1990/91. I would now like to outline the progress of the program to date.

The preliminary year (1985/86)

The year 85/86 was devoted to studying the feasibility of such a program. For that reason, the teachers had ample access to the computers and were encouraged to have some hands - on experience with Logo whenever they could. They also had meetings discussing their reactions to this "novelty" of computers, and preparatory seminars. Moreover, during that year, 20 eleven year old children participated in a Computer Club outside the regular curriculum. The club partially served as a "pilot" to determine existing educational needs not emphasised by the curriculum. I visited the club three times, took the role of the teacher for 15 hours in total and carried out some detailed research on the children's progress.

Conclusions from the teachers' experience

- they had a lot of "computer anxiety"
- many of them were not interested to learn how to program
- there was concern about the utility of teaching programming to young children
- there was grave concern about whether the teachers were qualified to teach

programming

- there was worry about the mathematics involved in turtle graphics

- Logo was often regarded as a "body of knowledge" (a new language)   to be learned by the teachers so that they could teach it

- most of them were well disposed to learn about this "novelty"

Conclusions from the Logo club experience

- the children would expect the teachers to provide answers to everything

- they did not realise at the beginning that almost all the problems could be solved if they thought about and discussed them

- they were conditioned to treat everything as right or wrong

- the members of the group could not communicate ideas and suggestions

- they would not take initiatives to undertake new projects, often asking the teachers to provide new ideas

- they would not pursue an idea but would abandon it at the first difficulty

- they would not use their creativity to elaborate or expand a project

- they would show signs of "improvement" in taking control of their learning when the teacher did not play up to their game

The preliminary application of the program (1986/87)

The results from the preliminary year's experience of the teachers' reactions and the Computer Club, enabled me to pin - point and clarify the educational objectives and the design of the program. The central aim of the program is for the computers to become a classroom tool in the hands of both teachers and students, for encouraging and actively exploring the social and cognitive aspects of learning. This implied using Logo and an informal, investigative, group - work type classroom setting, in order to achieve an atmosphere encouraging:

a) **active thinking (e.g. to solve own problems)**

b) **initiative (in thinking, creativity and decisions)**

c) **cooperation (cognitive, effective, social)**

The setting is as follows. There is one computer room with ten Apple IIC's, each linked to one of three printers. One computer period a week is allocated for each class of 30 children, during which they work in freely formed but permanent groups of three, with their own teacher encouraging an informal educational atmosphere according to the aims described. Each group uses one machine, disk and writing book. There is free collaboration and the groups are responsable for presenting results. The role of the teacher is to provide an educational environment rich in opportunities for encouraging the development of the educational aims.

For this purpose, in September 1986, I gave a series of intensive seminars in which all the teachers participated. From the beginning of 86/87, all the children were introduced to the program, each class by its own teacher. The program extends from the third to the sixth year inclusive, i.e. it involves around 500 children and 16 (24) teachers. The program depends on a working framework within which a free educational atmosphere is sought. As you know, the Greek educational system is highly centralized and extremely formal compared, for instance, to England; so it will come as no surprise that the informal atmosphere we seek has not arisen automatically.

A major component of this framework is based on collaborative "investigations" carried out by the children, aimed at developing a question and encouragement technique (no "answers" or formal teaching / passive learning). These investigations consist typically of a 4 lesson project, which is either totally up to the group or is based on an initial idea or drawing set by the teacher. Thoughts, activities, results and manner of collaboration are recorded by the children. Every four or five weekly periods, each group is responsible for giving a "presentation" of their investigation, i.e. a report consisting of a printout of drawings and commands / procedures and a group essay on activities, thoughts, collaboration, further ideas and conclusions.

A major advantage of these presentations of the children's work is, on the one hand, the opportunity they give to both the children and the teachers to reflect on what they learned / taught and how they learned / taught it. This is proving to be a powerful way of increasing the awareness of an active, investigative and cooperative kind of learning. It is also a sophisticated means for the children to express their work, enabling them to adress the classroom and hold fruitful discussions. On the other hand, the presentations are advantageous from a point of view of dissemination, due to quantity (there are 160 presentations every 6 weeks, roughly 640 per year), quality and clarity for anyone concerned: parents, educators, other children, authorities. Finally, they serve as a powerful data base for research into a multitude of issues concerning the use of technology in primary education, and also other related educational problems (e.g. a teacher is interested in carrying out research into the sociological aspects of learning in small groups).

Informal analysis of this data base, together with research regarding the teachers' difficulties during the preliminary year and their view on what the children carried out from the program into their normal classroom activities, provided useful information for reformulating the program for the first year of the program's full application. The most important conclusion from the preliminary year experience, was that the process of assimilating the legitimacy of an informal classroom atmosphere and the teaching objectives of cooperation, initiative and active thinking, was much slower than anticipated (for teachers and children). However, such an "atmosphere" was achieved very often, in all the classes.

Other conclusions from the preliminary year of application

- there was a certain confusion between the planned teaching objectives and the objective of teaching a programming language for   both teachers and children
- most teachers were hesitant to comment on the children's reports
- the idea of an investigation was often not put across clearly to the children
- the younger children found it difficult to realise that a project lasted for four periods often treating each period as a new project

- more time was needed to get the idea of investigations across, especially at the beginning
- time was needed after each investigation for groups to discuss their presentations with their peers in the classroom context
- presentations could not be prepared in the computer room
- the program needed to show more sensitivity to the age and experience to the children
- initial ideas for investigations would sometimes be confused with teacher required tasks

The first year of the program's application (1987/88)

In the light of the experience of the preliminary year, the program was reformulated firstly to clarify to the teachers the activities, the emphases and a framework of techniques for establishing the desired classroom atmosphere. Secondly, the uniformity of the program across year groups was broken down to year level; the program was made more sensitive to the age of the children, providing the younger ones with more time to clarify the idea of investigative work. Thirdly, the content regarding the technology (e.g. the programming aspect, the use of the printers) was clarified for each year. Also, documents were prepared for the teachers regarding technical difficulties encountered during the first year of application, and the syntax of certain commands. However, the fourth and most important aim of the reformulation was to stress that the emphasis remains on using the technology for existing educational needs (active thinking, cooperation, initiative) which were also the first priority in reformulating the program. In September 1987 I gave an intensive seminar to the teachers, analysing the content of the program for every year. All the teachers were present in the seminar.

I visited the school in late March 1988 and observed all the teachers in action; I spent one teaching period with each teacher and class in the computer room during their normal Logo program activities. Analysis of my observations revealed the following points.

Conclusions from the first year of the program (1987/88)

- in all the classes the open - ended, child-in-control atmosphere was satisfactory
- the degree of the teachers' awareness and control over the program was generally higher than that of the previous year
- the children's presentations were supervised in a much more satisfactory manner
- the suggested reformulation of the program was implemented by all the teachers
- in general, I was impressed by the increasing level of confidence amongst the teachers considering their original lack of expertise
- several teachers have already started to to develop their own personal style of intervention
- much more work needs to be done on the teachers method of commenting on the children's presentations
- the structure of the presentations was not always satisfactory
- although classroom management has improved there are still some problems
- there are certain problems with the maintenance of the hardware
- the teachers now seem ready for an advancement of their understanding of the mathematics involved in the children's activities
- they are consequently ready to receive further training on how to use the program to improve the children's performance in mathematics

In this way, from the year 1987/88, the design of a specific curriculum for each year has started to develop, maintaining and refining the original pedagogical objectives, but also shaped by the increasing experience of both teachers and children.

APPENDIX C

The T.C.P. microworld

Category 1, path 1.

PLACE :(number x) :(number y) :(number a)

Execution of this command places a point on the plane, denoted by an "X" sign. The inputs signify the two coordinate values and the numerical order of this particular point.

DODOTS

Execution of this command joins the points up in the order they were placed.

The following screen dump was taken from the children's activities.
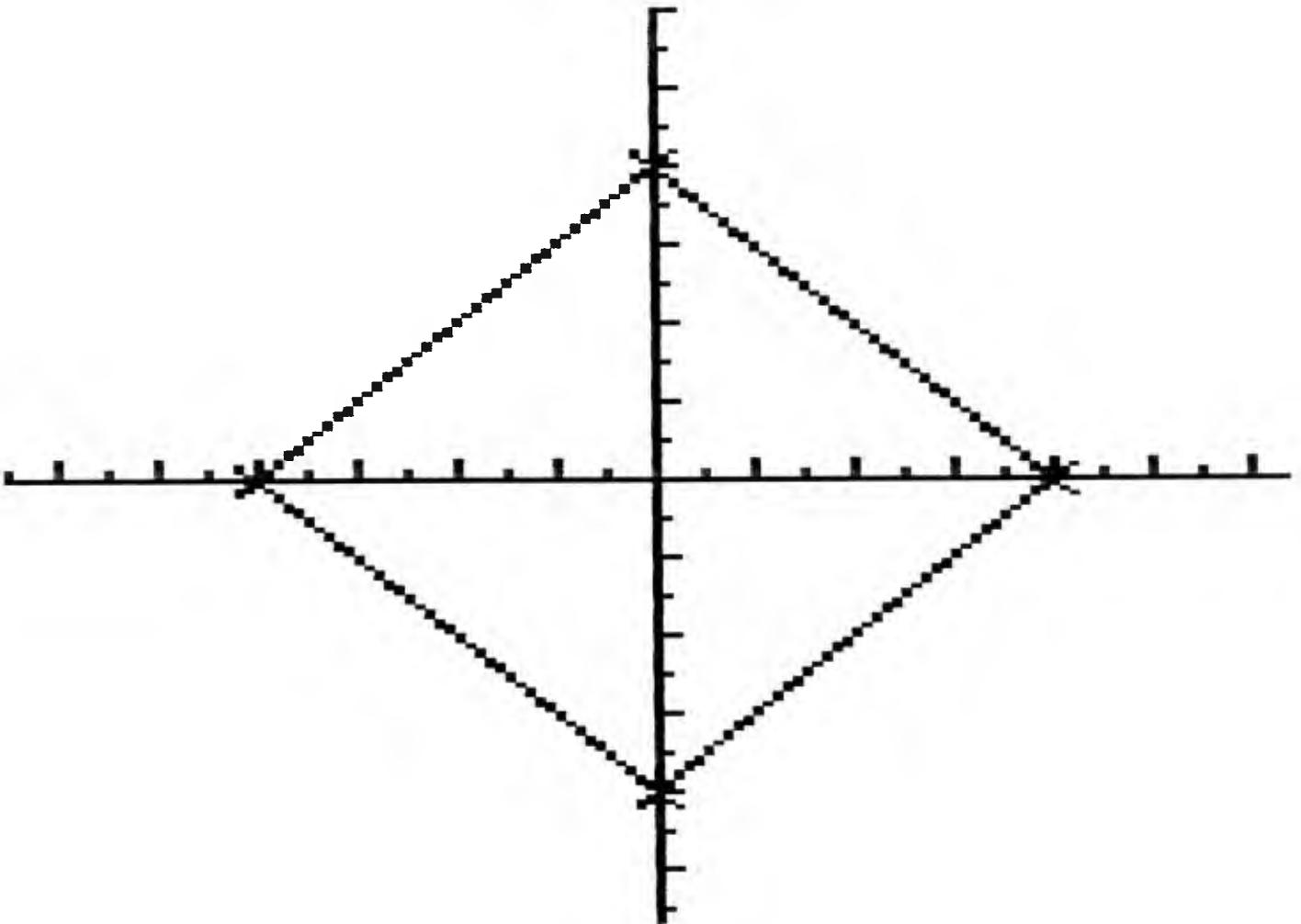
The following commands were available:

PR DISTANCE (name)

PR DIRECTION (name), as in the P.D.D microworld, and

FD, BK, LT, RT, PU, PD as in conventional Logo

The following screen dump was taken from the children's activities.
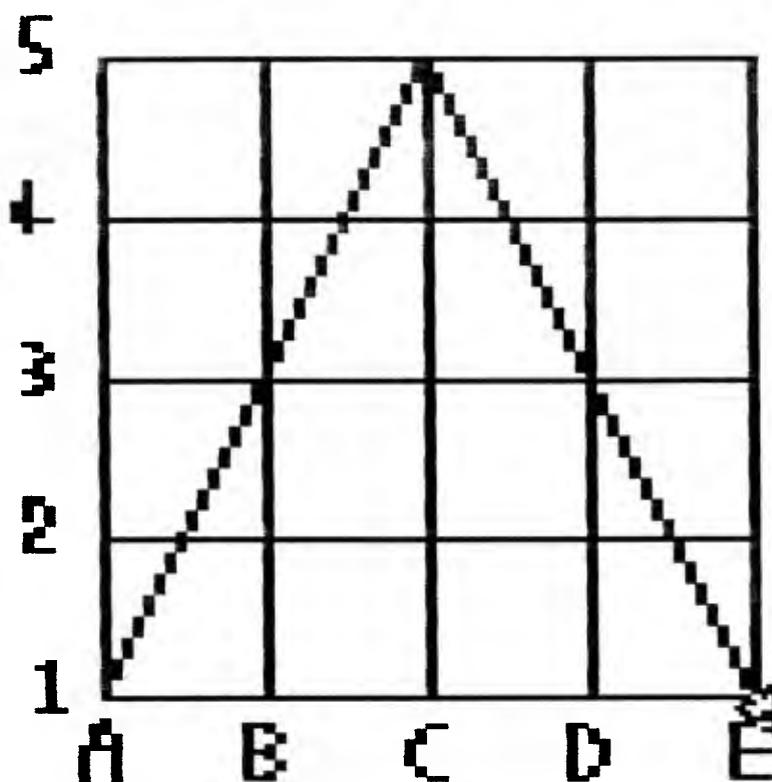
The following commands were available:

POST (name), PR DISTANCE (name), PR DIRECTION (name), as in the P.D.D. microworld, and

FD, BK, RT, LT, PU, PD, as in conventional Logo.

The following screen dump was taken from the children's activities.
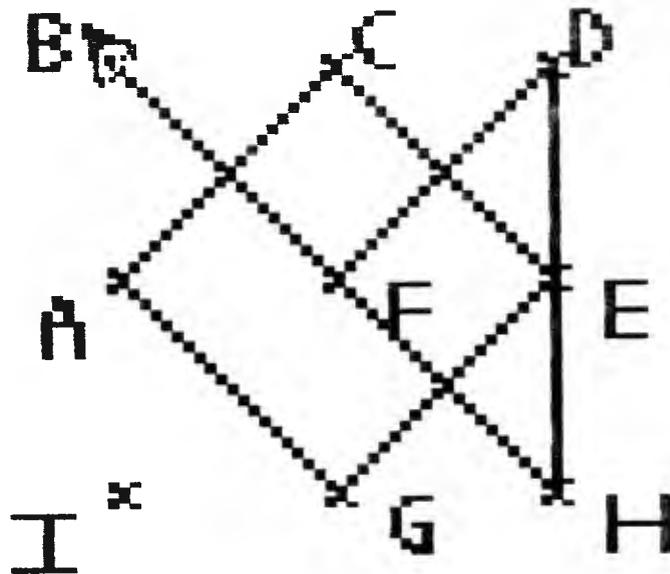
The following commands were available for group of tasks D (fig. 6.1.1):

SETH (value), SETX (value), SETY (value), WRITE (name). The first three commands as in conventional Logo, **except that execution of either SETX or SETY sends the following error message where applicable:**

- I'M NOT HEADING TOWARDS THE PLACE YOU WANT TO TAKE ME

From this point onwards, the SETY, SETX, and SETPOS commands give the same error message where applicable.

Execution of the WRITE command draws the coordinates on the screen if the turtle is on the respective position. Otherwise, it gives the following error message:

- I'M NOT THERE

The following commands were available for group of tasks E, fig. 6.1.1:

SETPOS (name), SETH TOWARDS (name), WRITE (name), PU, PD. The SETPOS command provides the above error message if the turtle's orientation does not coincide with the orientaton of its trajectory. The SETH TOWARDS command sets the turtle's heading towards the position given via the coordinate inputs. The WRITE command as above.

In the activities of category 2, execution of the conventional intrinsic Logo commands, i.e. FD, BK, RT, LT, outputs the error message used in the Logo language for execution of non-primitive and non-defined commands, i.e.:

- I DON'T KNOW HOW TO (whatever has been typed)

The following screen dumps were taken from the children's activities in group D and E of figure 6.1.1 respectively.

406

(50 50)

(-90 70)

x

407

The following commands were available:

Conventional intrinsic Logo commands: FD, BK, RT, LT, PU, PD

Commands from the P.D.D. microworld: DISTANCE, DIRECTION

Coordinate commands, as defined above: SETH, SETY, SETX, SETPOS, SETH TOWARDS, WRITE

The following screen dump was taken from the children's activities.
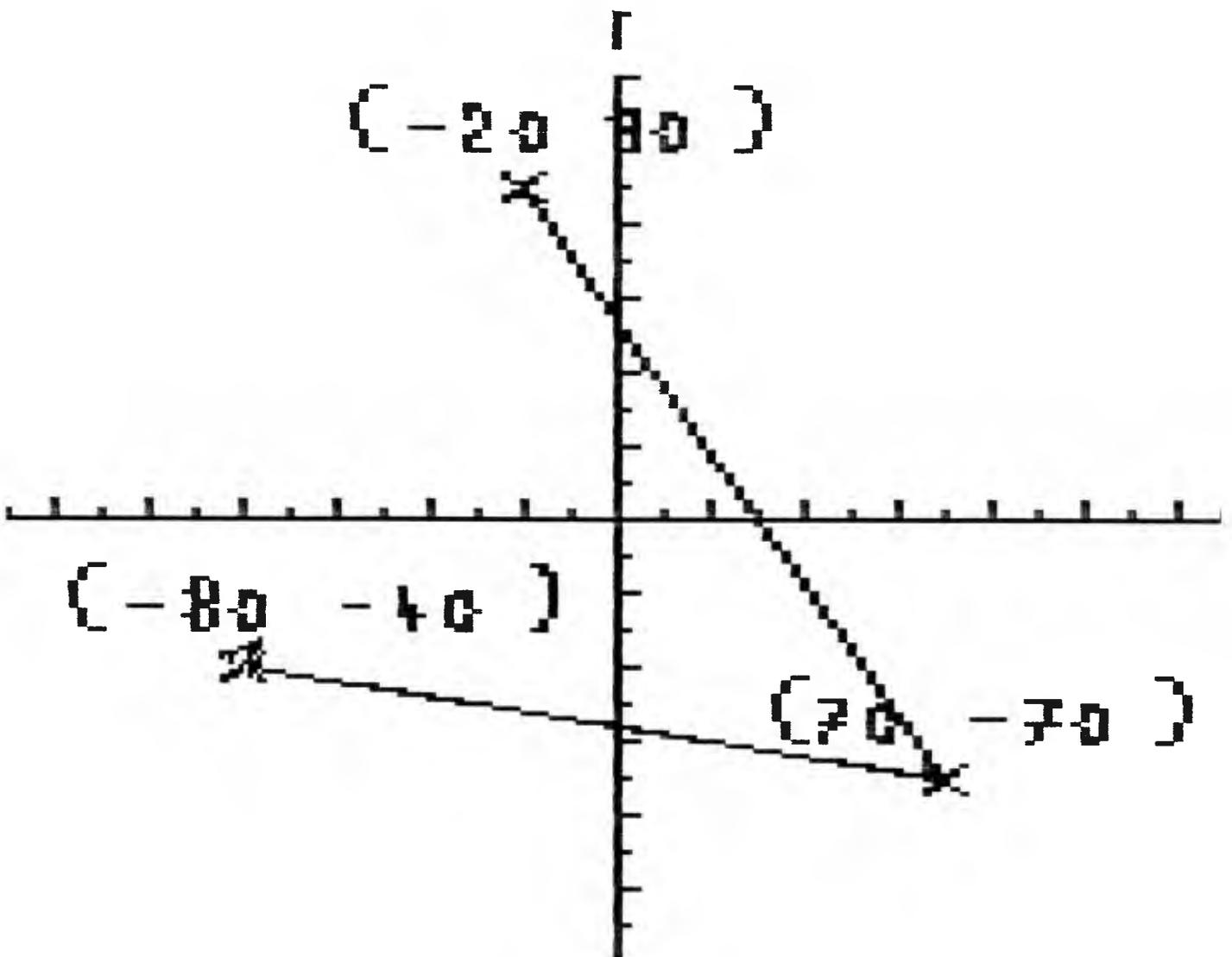
```
TO START
SETPC 0
GRID 120 140
END


TO BEGIN
HT
SETBG 1
SETPC 0
END


TO GRID :A :B
HT
M1 :A
LT 90
M1 :B
LT 90
END


TO M1 :D
BK :D
REPEAT :D * 2 / 20 [M01]
RT 90 FD 5 BK 5 LT 90
BK :D
END


TO M01
RT 90 FD 5 BK 5 LT 90 FD 10
RT 90 FD 2.5 BK 2.5 LT 90 FD 10
END


TO MKDS
HT MAKE "D []
PU
END


TO DODOTS
PU SETPOS THING FIRST :D
SETPC 2
DDT
END


TO DDT
PD SETPOS THING FIRST :D
WAIT 30
MAKE "D BF :D
IF ( COUNT :D ) = 0 [STOP] [DDT]
END


TO POST2 :N
MAKE :N LIST XCOR YCOR
END


TO PLACE :A :B :C
HT PU SETPOS SE :A :B
POST2 :C
MAKE "D SE :D :C
```

```
SETPC 4 PD
RT 45 REPEAT 2 [FD 5 BK 10 FD 5 RT 90] RT 180 LT 45 PU SETPC 0
END

TO CHECK
PLACE 90 90 1
PLACE -90 90 2
PLACE -90 -90 3
PLACE 90 -90 4
PLACE 90 0 5
PLACE 0 90 6
PLACE -90 0 7
PLACE 0 -90 8
PLACE 90 0 5
END

TO FILUP
PU HOME PD
REPEAT 4 [RT 45 PU FD 5 PD FILL PU BK 5 PD LT 45 RT 90]
SETPC 0
END

TO SQ :X
OP :X * :X
END

TO DISTANCE :N
OP SQRT ( SQ ( XCOR - FIRST :N ) ) + ( SQ ( YCOR - LAST :N ) )
END

TO DIFF.A :A :B
IF :A - :B < 0 [OP 360 + :A - :B]
IF :A - :B > 360 [OP :A - :B - 360]
OP :A - :B
END

TO DIRECTION :P
OP DIFF.A TOWARDS SE FIRST :P LAST :P HEADING
END
```

```
TO START :W
LABGR :W
DRGRID :W
VISLAB :W
END


TO AR :S
LT 30 FD :S BK :S
RT 60 FD :S BK :S
LT 30
END


TO LMM :S
LM :S
RT 180 JUMP 15 RT 180
END


TO LNN :S
LN :S
RT 180 JUMP 15 RT 180
END


TO LOO :S
LO :S
RT 180 JUMP 15 RT 180
END


TO LCC :S
LC :S
RT 180 JUMP 15 RT 180
END


TO LPP :S
LP :S
RT 180 JUMP 15
RT 180
END


TO TRY
RUN [LA 13 LB 20 LC 5 LD 5 LE 10 LF 10 LG 5 LH 10 LI 10 LJ 5 LK 10 LL 10 LM 10 LN
5 LP 20 LQ 5]                                                                     10
RUN [RT 90 JUMP 20 LT 90 LR 20 LS 10 LTT 10 LU 5 LV 13 LW 10 LX 13 LY 13 LZ 10]
END


TO LABGR :Z
PU SETPOS SE 0 15
MAKE "DO [A5 B5 C5 D5 E5 A4 B4 C4 D4 E4 A3 B3 C3 D3 E3 A2 B2 C2 D2 E2 A1 B1 C1 D1
HT FD 2 * :Z LT 90 FD 2 * :Z RT 180                                               E1
FIVEL :Z 4
ST PD
END


TO HFIVEL :Z
BK 4 * :Z RT 90 FD :Z LT 90 MAKE "DO BF :DO
END


TO HPOST :NAME
```

411

```
MAKE :NAME LIST XCOR YCOR
END


TO FIVEL :Z :CO
HPOST FIRST :DO
FD :Z
MAKE "CO ( :CO - 1 )
MAKE "DO BF :DO
IF :CO < 1 [HPOST FIRST :DO HFIVEL :Z IF ( COUNT :DO ) < 1 [LT 90 FD 3 * :Z RT 90
 :Z LT 90 STOP] [FIVEL :Z 4]] [FIVEL :Z :CO]
END
```

FD2*

```
TO DRGRID :X
HT SETPC 0 PU SETPOS SE 0 15 PD MS :X
REPEAT 4 [UNIT :X LT 90]
RT 180
MS :X
RT 180
ST
END


TO UNIT :X
REPEAT 2 [FD 4 * :X RT 90 FD :X RT 90 FD 4 * :X LT 90 FD :X LT 90]
END


TO MS :X
BK 2 * :X LT 90 FD 2 * :X RT 90
END


TO VISLAB :X
HT SETPC 5 PU SETPOS SE 0 15 UNJ 3 PU RT 180 FD 2 * :X RT 90 FD 2 * :X LT 90 PD
UJ 15 RT 180 .LABEL "A
JUMP :X - 15
.LABEL "B
JUMP :X - 15
.LABEL "C
JUMP :X - 15
.LABEL "D
JUMP :X - 15
.LABEL "E
PU LT 90 FD 15 FD 4 * :X RT 90 UNJ 7 UJ 13
N1 7 UNJ 10 UJ :X
N2 10 UNJ 10 UJ :X
N3 9 UNJ 10 UJ :X
N4 10 UNJ 10 UJ :X
N5 10 UNJ 10
RT 180
PU
SETH 0
SETPOS SE 0 15
PD ST SETPC 2
END


TO SETL
MAKE "ALPHA [A B C D E F G H I J K L M N O P Q R S T U V W X Y Z]
MAKE "BETA ( LIST [LA 13] [LB 20] [LC 5] [LD 5] [LE 10] [LF 10] [LG 5] [LH 10] [LI
LJ 5] [LK 10] [LL 10] [LM 10] [LN 10] [LO 5] [LP 20] [LQ 5] [LR 20] [LS 10] [LTT 1
U5] [LV 13] [LW 10] [LX 13] [LY 13] [LZ 10] )
```

```
END

TO KLIK :MEMBER :LIST
IF NOT MEMBERP :MEMBER :LIST [OUTPUT 0]
IF :MEMBER = FIRST :LIST [OUTPUT 1]
OUTPUT 1 + KLIK :MEMBER BF :LIST
END

TO CONV :LTR
OP ITEM KLIK :LTR :ALPHA :BETA
END

TO .LABEL :LTR
RUN ( RUN [CONV :LTR] )
END

TO UJ :D
PU FD :D PD
END

TO UNJ :F
RT 180
JUMP :F
RT 180
END

TO RBLOB :S
RT 90 FD :S BK :S LT 90
END

TO N1 :S
.SETSCRUNCH 1.3
RE :S / 4 FD :S
LT 110 FD :S / 4
BK :S / 4 RT 110
BK :S
.SETSCRUNCH 1
JUMP 10
END

TO N2 :S
.SETSCRUNCH 1.3
RT 180 JUMP 2.5 RT 180
PU
FD 2 * :S / 4
PD FD :S / 4 BK :S / 4 RT 90
C :S / 4 1 RT 90
PU
BK :S / 4
PD BK :S / 4
RT 90 FD :S / 3
BK :S / 3 LT 90
JUMP 2.5
.SETSCRUNCH 1
JUMP 10
END

TO N3 :S
```

```
RT 180 JUMP 2.5 RT 180
PU FD 3 * :S / 4 PD
RT 90 C :S / 4 1 RT 90
PU BK :S / 2 PD
RT 90 C :S / 4 1 RT 90
PU BK :S / 4 PD
JUMP 10
END


TO N4 :S
FD :S / 2 BK :S / 4
LT 90 BK :S / 4 FD :S / 2 RT 90
RT 5 FD 2 * :S / 3 BK 2 * :S / 3 LT 5
LT 90 BK :S / 4 RT 90 BK :S / 4
JUMP 10
END


TO N5 :S
PU FD :S / 4 PD
RT 90 C :S / 4 1 RT 90
PU FD :S / 4 PD
FD :S / 2 RT 90 FD :S / 3 BK :S / 3 LT 90
PU BK :S PD
JUMP 10
END


TO RE :S
REPEAT 2 [RBLOB :S RT 180]
END


TO N7 :S
RT 180 JUMP 2.5 RT 180
RT 30 FD :S LT 120 FD :S / 2 BK :S / 2
RT 120 BK :S / 2 LT 30 RE :S / 4 RT 30
BK :S / 2 LT 30
JUMP 2.5
JUMP 10
END


TO N8 :S
PU FD :S / 4 PD
C :S / 4 2
PU FD 7 * :S / 12 PD
C :S / 4 2
PU BK 10 * :S / 12 PD
JUMP 10
END


TO N6 :S
PU FD :S / 4 PD C :S / 4 2
PU LT 90 FD :S / 4 RT 90 PD
FD 2 * :S / 3 RT 90 FD :S / 3 BK :S / 3 LT 90 BK 2 * :S / 3
PU RT 90 FD :S / 4 LT 90 BK :S / 4 PD
JUMP 10
END


TO N9 :S
PU FD 11 * :S / 12 PD RT 180
```

```
N6 :S
RT 180 PU BK 11 + :S / 12 PD
JUMP 20
END

TO N0 :S
.SETSCRUNCH 1.6 PU FD :S / 4 PD
C :S / 4 2
PU BK :S / 4 PD
.SETSCRUNCH 1
END

TO N0B :S
.SETSCRUNCH 1.4
PU FD :S / 2 PD
LT 90 C :S / 2 1 LT 90
PU BK :S / 2 PD
.SETSCRUNCH 1
JUMP 7
END

TO NCB :S
.SETSCRUNCH 1.4
JUMP 5 PU FD :S / 2 PD
RT 90 C :S / 2 1 RT 90
PU BK :S / 2 PD
.SETSCRUNCH 1
END

TO JUMP :S
RT 90 PU FD :S PD LT 90
END

TO MIN :S
PU FD :S / 2 PD RT 90 FD :S / 2 LT 90
PU BK :S / 2 PD
JUMP 10
END

TO C :R :P
PU LT 90 FD :R RT 90
RT 5 PD
LOCAL "M
MAKE "M 2 * :R * 3.1416 / 36
REPEAT 18 * :P [FD :M RT 10]
LT 5
PU RT 90 FD :R LT 90 PD
END

TO LZ :S
.SETSCRUNCH 1.5
LT 90 BK :S
LNN :S
FD :S RT 90
.SETSCRUNCH 1
JUMP 15
END
```

```
TO LT :S
PU RT 90 FD ( :S / 2 ) * COS 60 LT 90 PD
FD :S / 2 AR :S / 2 BK :S / 2
PU LT 90 FD ( :S / 2 ) * COS 60 RT 90 PD
JUMP 15
END


TO LX :S
RT 30 FD :S / 2 LT 30
AR :S / 2
RT 180 AR :S / 2
RT 30 FD :S / 2 RT 150
JUMP 15
END


TO LW :S
PU FD :S RT 90 FD 2 * ( :S * SIN 30 ) RT 90 PD
LMM :S
PU FD :S RT 90 FD 2 * ( :S * SIN 30 ) RT 90 PD
JUMP 15
END


TO LV :S
.SETSCRUNCH 0.9
PU RT 90 FD ( :S / 2 ) * COS 60 LT 90 PD
LT 30 FD :S / 2 RT 30 FD :S / 2 RT 90 PU FD :S * COS 60 RT 90 PD
FD :S / 2 RT 30 FD :S / 2 RT 60
PU FD ( :S / 2 ) * COS 60 PD
RT 90
.SETSCRUNCH 1
JUMP 15
END


TO LU :S
.SETSCRUNCH 2
PU REPEAT 2 [FD :S RT 90]
C :S 1
RT 180
PU REPEAT 2 [FD :S RT 90]
.SETSCRUNCH 1
JUMP 15
END


TO LTT :S
FD :S
RE :S / 2
BK :S
JUMP 15
END


TO LS :S
PU REPEAT 2 [FD :S / 4 RT 90] LT 90 PD
REPEAT 2 [C :S / 4 1 RT 90 PU FD :S / 2 LT 90 PD]
PU RT 90 REPEAT 2 [FD :S / 4 RT 90] PD
JUMP 15
END


TO LR :S
```

416

```
LPP :S
.SETSCRUNCH 0.5
FD 1.2 * :S / 2
RT 155
FD :S / 1.6
BK :S / 1.6
LT 155
BK 1.2 * :S / 2
.SETSCRUNCH 1
JUMP 15
END


TO LQ :S
LOO :S
PU RT 90 FD 2 * :S LT 90 PD
LT 45 FD :S / 3 BK :S FD 2 * :S / 3 RT 45
PU LT 90 FD 2 * :S RT 90 PD
JUMP 15
END


TO LP :S
.SETSCRUNCH 0.5
FD 3 * :S / 4
RT 90 C :S / 4 1
RT 90
FD :S / 4
BK :S
.SETSCRUNCH 1
JUMP 15
END


TO LO :S
PU REPEAT 2 [FD :S RT 90] PD
RT 180
C :S 2
RT 180 PU REPEAT 2 [FD :S RT 90] PD
JUMP 15
END


TO LN :S
FD :S RT 150
FD :S / COS 30
LT 150 FD :S
PU LT 90 FD ( SIN 30 / COS 30 ) * :S
RT 90 BK :S PD
JUMP 15
END


TO LM :S
FD :S RT 150 FD :S
LT 120 FD :S RT 150 FD :S
RT 90 PU FD 2 * ( :S * SIN 30 ) PD
RT 90
JUMP 15
END


TO LL :S
FD :S BK :S
```

417

```
RBLOB :S / 2
JUMP 15
END

TO LK :S
FD :S BK :S / 2 RT 90
AR ( :S / 2 ) / COS 60
LT 90 BK :S / 2
JUMP 15
END

TO LJ :S
.SETSCRUNCH 2
PU FD :S RT 180 PD
C :S 0.5
BK :S * 2 FD :S * 2
RT 90 PU BK :S PD
.SETSCRUNCH 1
JUMP 15
END

TO LI :S
FD :S RE :S / 2 BK :S
RE :S / 2
JUMP 15
END

TO LH :S
FD :S / 2 RT 90
REPEAT 2 [RE :S / 2 FD :S RT 180]
LT 90 BK :S / 2
JUMP 15
END

TO LG :S
LCC :S
PU RT 90 FD :S LT 90 PD
FD :S RT 90 FD :S / 4 BK :S / 2 FD :S / 4 LT 90 BK :S
PU LT 90 FD :S RT 90 PD
JUMP 15
END

TO LF :S
REPEAT 2 [FD :S / 2 RBLOB :S / 2]
BK :S
JUMP 15
END

TO LE :S
REPEAT 2 [FD :S / 2 RBLOB :S / 2]
BK :S
RBLOB :S / 2
JUMP 15
END

TO LD :S
FD 2 * :S BK :S RT 90
C :S 1
```

```
RT 90 BK :S
JUMP 15
END


TO LC :S
PU REPEAT 2 [FD :S RT 90] PD
RT 90
C :S 1
RT 90
PU REPEAT 2 [FD :S RT 90] PD
JUMP 15
END


TO LB :S
LPP :S
.SETSCRUNCH 0.5
FD :S / 4
RT 90 C :S / 4 1
RT 90
BK :S / 4
.SETSCRUNCH 1
JUMP 15
END


TO LA :S
.SETSCRUNCH 0.9
FD :S / 2 RT 30 REPEAT 3 [FD :S / 2 RT 120]
RT 60 FD :S / 2 RT 90 FD :S / 2 PU RT 90 FD :S / 2 RT 90 PD
.SETSCRUNCH 1
JUMP 15
END


TO DIRECTION :P
OP DIFF.A TOWARDS SE FIRST :P LAST :P HEADING
END


TO DISTANCE :N
OP SQRT ( SQ ( XCOR - FIRST :N ) ) + ( SQ ( YCOR - LAST :N ) )
END


TO SQ :X
OP :X * :X
END


TO DIFF.A :A :B
IF :A - :B < 0 [OP 360 + :A - :B]
IF :A - :B > 360 [OP :A - :B - 360]
OP :A - :B
END


MAKE "5E [60.0 -60.0]
MAKE "5D [30.0 -60.0]
MAKE "5C [0.0 -60.0]
MAKE "5B [-30.0 -60.0]
MAKE "5A [-60.0 -60.0]
MAKE "4E [60.0 -30.0]
MAKE "4D [30.0 -30.0]
```

```
TO BEGIN
SETBG 1
SETPC 2
PU
END


TO RBLOB :S
RT 90 FD :S BK :S LT 90
END


TO AR :S
LT 30 FD :S BK :S
RT 60 FD :S BK :S
LT 30
END


TO LMM :S
LM :S
RT 180 JUMP 15 RT 180
END


TO LNN :S
LN :S
RT 180 JUMP 15 RT 180
END


TO LOO :S
LO :S
RT 180 JUMP 15 RT 180
END


TO LCC :S
LC :S
RT 180 JUMP 15 RT 180
END


TO LPP :S
LP :S
RT 180 JUMP 15
RT 180
END


TO SETL
MAKE "ALPHA [A B C D E F G H I J K L M N O P Q R S T U V W X Y Z]          [LT 10]
MAKE "BETA ( LIST [LA 13] [LB 20] [LC 5] [LD 5] [LE 10] [LF 10] [LG 5] [LH 10] [
LJ 5] [LK 10] [LL 10] [LM 10] [LN 10] [LO 5] [LP 20] [LQ 5] [LR 20] [LS 10] [LTT
 5] [LV 13] [LW 10] [LX 13] [LY 13] [LZ 10] )
END                                                                        [LTT 10]


TO KLIK :MEMBER :LIST
IF NOT MEMBERP :MEMBER :LIST [OUTPUT 0]
IF :MEMBER = FIRST :LIST [OUTPUT 1]
OUTPUT 1 + KLIK :MEMBER BF :LIST
END


TO CONV :LTR
OP ITEM KLIK :LTR :ALPHA :BETA
```

420

```
END

TO TRI2
HT
PU SETPOS SE -40 -20 PD POST "D
PU SETPOS SE 50 30 PD POST "E
PU SETPOS SE 70 -35 PD POST "F
PU HOME PD
ST
END

TO TRI
HT
PU SETPOS SE -75 50 PD POST "A
PU SETPOS SE 60 35 PD POST "B
PU SETPOS SE 10 -50 PD POST "C
PU HOME PD
ST
END

TO PLONK10
PLONK1
PLONK2
PLONK3
HT
PU SETPOS SE -10 -10 PD POST "G
PU SETPOS SE -100 -20 PD POST "H
PU SETPOS SE -50 -60 PD POST "I
PU SETPOS SE 60 -40 PD POST "J
PU HOME PD ST
END

TO PLONK3
HT PU SETPOS SE 40 70
PD
POST "D
PU SETPOS SE -40 70 PD
POST "E
PU SETPOS SE 40 -50 PD
POST "F
PU HOME PD ST
END

TO PLONK2
HT PU SETPOS SE -50 50
PD
POST "B
PU
SETPOS SE 20 60
PD
POST "C
PU
HOME
PD ST
END

TO PLONK1
HT PU SETPOS SE 50 50
```

421

```
PD
POST "H
PU
HOME
PD ST
END


TO DIFF.A :H :B
IF :H - :B < 0 [OP 360 + :H - :B]
IF :H - :B > 360 [OP :H - :B - 360]
OP :H - :B
END


TO DIRECTION :P
OP DIFF.A TOWARDS SE FIRST :P LAST :P HEADING
END


TO RE :S
REPEAT 2 [RBLOB :S RT 180]
END


TO POST :LTR
MAKE :LTR POS
LOCAL "WQ
MAKE "WQ HEADING
PD SETPC 4
HT SETH 0 RT 45 RE 3 LT 90 RE 3 RT 45
SETPC 5
LABEL1 :LTR
SETPC 2
ST PU
END


TO .LABEL :LTR
RUN ( RUN [CONV :LTR] )
END


TO LABEL1 :LTR
PU
IF YCOR > 0 [IF XCOR > 0 [SETH 0 JUMP 5] [SETH 180 JUMP 15 SETH 0]] [IF XCOR > 0
5 JUMP 15 SETH 0] [SETH 135 JUMP 20 SETH 0]]
.LABEL :LTR
PU
SETH :WQ
SETPOS SE FIRST THING :LTR LAST THING :LTR
PD
END


TO C :R :P
PU LT 90 FD :R RT 90
RT 5 PD
LOCAL "M
MAKE "M 2 * :R * 3.1416 / 36
REPEAT 18 * :P [FD :M RT 10]
LT 5
PU RT 90 FD :R LT 90 PD
END
```

422

```
TO LZ :S
.SETSCRUNCH 1.5
LT 90 BK :S
LNN :S
FD :S RT 90
.SETSCRUNCH 1
JUMP 15
END

TO LY :S
PU RT 90 FD ( :S / 2 ) * COS 60 LT 90 PD
FD :S / 2 AR :S / 2 BK :S / 2
PU LT 90 FD ( :S / 2 ) * COS 60 RT 90 PD
JUMP 15
END

TO LX :S
RT 30 FD :S / 2 LT 30
AR :S / 2
RT 180 AR :S / 2
RT 30 FD :S / 2 RT 150
JUMP 15
END

TO LW :S
PU FD :S RT 90 FD 2 * ( :S * SIN 30 ) RT 90 PD
LMM :S
PU FD :S RT 90 FD 2 * ( :S * SIN 30 ) RT 90 PD
JUMP 15
END

TO LV :S
.SETSCRUNCH 0.9
PU RT 90 FD ( :S / 2 ) * COS 60 LT 90 PD
LT 30 FD :S / 2 RT 30 FD :S / 2 RT 90 PU FD :S * COS 60 RT 90 PD
FD :S / 2 RT 30 FD :S / 2 RT 60
PU FD ( :S / 2 ) * COS 60 PD
RT 90
.SETSCRUNCH 1
JUMP 15
END

TO LU :S
.SETSCRUNCH 2
PU REPEAT 2 [FD :S RT 90]
C :S 1
RT 180
PU REPEAT 2 [FD :S RT 90]
.SETSCRUNCH 1
JUMP 15
END

TO LTT :S
FD :S
RE :S / 2
BK :S
JUMP 15
END
```

423

```
TO LS :S
PU REPEAT 2 [FD :S / 4 RT 90] LT 90 PD
REPEAT 2 [C :S / 4 1 RT 90 PU FD :S / 2 LT 90 PD]
PU RT 90 REPEAT 2 [FD :S / 4 RT 90] PD
JUMP 15
END


TO LR :S
LPP :S
.SETSCRUNCH 0.5
FD 1.2 * :S / 2
RT 155
FD :S / 1.6
BK :S / 1.6
LT 155
BK 1.2 * :S / 2
.SETSCRUNCH 1
JUMP 15
END


TO LQ :S
LOO :S
PU RT 90 FD 2 * :S LT 90 PD
LT 45 FD :S / 3 BK :S FD 2 * :S / 3 RT 45
PU LT 90 FD 2 * :S RT 90 PD
JUMP 15
END


TO LP :S
.SETSCRUNCH 0.5
FD 3 * :S / 4
RT 90 C :S / 4 1
RT 90
FD :S / 4
BK :S
.SETSCRUNCH 1
JUMP 15
END


TO LO :S
PU REPEAT 2 [FD :S RT 90] PD
RT 180
C :S 2
RT 180 PU REPEAT 2 [FD :S RT 90] PD
JUMP 15
END


TO LN :S
FD :S RT 150
FD :S / COS 30
LT 150 FD :S
PU LT 90 FD ( SIN 30 / COS 30 ) * :S
RT 90 BK :S PD
JUMP 15
END


TO LM :S
```

424

```
FD :S RT 150 FD :S
LT 120 FD :S RT 150 FD :S
RT 90 PU FD 2 + ( :S * SIN 30 ) PD
RT 90
JUMP 15
END

TO LL :S
FD :S BK :S
RBLOB :S / 2
JUMP 15
END

TO LK :S
FD :S BK :S / 2 RT 90
AR ( :S / 2 ) / COS 60
LT 90 BK :S / 2
JUMP 15
END

TO LJ :S
.SETSCRUNCH 2
PU FD :S RT 180 PD
C :S 0.5
BK :S * 2 FD :S * 2
RT 90 PU BK :S PD
.SETSCRUNCH 1
JUMP 15
END

TO LI :S
FD :S RE :S / 2 BK :S
RE :S / 2
JUMP 15
END

TO LH :S
FD :S / 2 RT 90
REPEAT 2 [RE :S / 2 FD :S RT 180]
LT 90 BK :S / 2
JUMP 15
END

TO LG :S
LCC :S
PU RT 90 FD :S LT 90 PD
FD :S RT 90 FD :S / 4 BK :S / 2 FD :S / 4 LT 90 BK :S
PU LT 90 FD :S RT 90 PD
JUMP 15
END

TO LF :S
REPEAT 2 [FD :S / 2 RBLOB :S / 2]
BK :S
JUMP 15
END

TO LE :S
```

```
REPEAT 2 [FD :S / 2 RBLOB :S / 2]
BK :S
RBLOB :S / 2
JUMP 15
END


TO LD :S
FD 2 * :S BK :S RT 90
C :S 1
RT 90 BK :S
JUMP 15
END


TO LC :S
PU REPEAT 2 [FD :S RT 90] PD
RT 90
C :S 1
RT 90
PU REPEAT 2 [FD :S RT 90] PD
JUMP 15
END


TO LB :S
LPP :S
.SETSCRUNCH 0.5
FD :S / 4
RT 90 C :S / 4 1
RT 90
BK :S / 4
.SETSCRUNCH 1
JUMP 15
END


TO LA :S
.SETSCRUNCH 0.9
FD :S / 2 RT 30 REPEAT 3 [FD :S / 2 RT 120]
RT 60 FD :S / 2 RT 90 FD :S / 2 PU RT 90 FD :S / 2 RT 90 PD
.SETSCRUNCH 1
JUMP 15
END


TO SQ :X
OP :X * :X
END


TO DISTANCE :N
IF AND HEADING - ( TOWARDS SE FIRST :N LAST :N ) < 0.5 HEADING - ( TOWARDS SE FIRS
AST :N ) > -0.5 [SETH TOWARDS SE FIRST :N LAST :N OP SQRT ( SQ ( XCOR - FIRST :N )
 SQ ( YCOR - LAST :N ) )] [OP [I'M NOT HEADING IN THAT DIRECTION] STOP]
END


TO JUMP :S
RT 90 PU FD :S PD LT 90
END
```

426

```
TO DIRECTION :P
OP DIFF.A TOWARDS SE FIRST :P LAST :P HEADING
END


TO DIFF.A :H :B
IF :H - :B < 0 [OP 360 + :A - :B]
IF :H - :B > 360 [OP :H - :B - 360]
OP :A - :B
END


TO DISTANCE :N
IF HEADING = ( TOWARDS SE FIRST :N LAST :N ) [OP SQRT ( SQ ( XCOR - FIRST :N ) +
( YCOR - LAST :N ) )] [OP SE [I'M NOT HEADING TOWARDS] :N STOP]
END                                                                        (SQ


TO SQ :X
OP :X * :X
END


TO BEGIN
NSETL
SETL
FIX
END


TO NSETL
MAKE "GAMMA [0 10 20 30 40 50 60 70 80 90 100 110 120 130 140]
MAKE "DELTA ( LIST [ON] [N10] [N20] [N30] [N40] [N50] [N60] [N70] [N80] [N90] [N10(
10] [N120] [N130] [N140] )
END                                                                        ]N1


TO SETL
MAKE "ALPHA [A B C D E F G H I J K L M N O P Q R S T U V W X Y Z]              10]
MAKE "BETA ( LIST [LA 13] [LB 20] [LC 5] [LD 5] [LE 10] [LF 10] [LG 5] [LH 10] [LI
LJ 5] [LK 10] [LL 10] [LM 10] [LN 10] [LO 5] [LP 20] [LQ 5] [LR 20] [LS 10] [LTT 1(
 5] [LV 13] [LW 10] [LX 13] [LY 13] [LZ 10] )                              ][LU
END


TO START
SETBG 1
GRID 120 140
END


TO GRID :A :B
UNFIX HT SETPC 0
M1 :A
LT 90
M1 :B
RT 90
ST
SETPC 2
FIX
END


TO M1 :D
BK :D
REPEAT :D * 2 / 20 [M01]
```

427

```
RT 90 FD 5 BK 5 LT 90
BK :D
END

TO M01
RT 90 FD 5 BK 5 LT 90 FD 10
RT 90 FD 2.5 BK 2.5 LT 90 FD 10
END

TO RBLOB :S
RT 90 FD :S BK :S LT 90
END

TO RE :S
REPEAT 2 [RBLOB :S RT 180]
END

TO N140
N1 7
N4 10
N0 10
END

TO N130
N1 7
N3 9
N0 10
END

TO N120
N1 7
N2 10
N0 10
END

TO N110
N1 7
N1 7
N0 10
END

TO N100
N1 7
N0 10
N0 10
END

TO N9 :S
PU FD 11 * :S / 12 PD RT 180
N6 :S
RT 180 PU BK 11 * :S / 12 PD
JUMP 20
END

TO N90
N9 10
N0 10
END
```

```
TO N8 :S
PU FD :S / 4 PD
C :S / 4 2
PU FD 7 * :S / 12 PD
C :S / 4 2
PU BK 10 * :S / 12 PD
JUMP 10
END


TO N80
N8 10
N0 10
END


TO N7 :S
RT 180 JUMP 2.5 RT 180
RT 30 FD :S LT 120 FD :S / 2 BK :S / 2
RT 120 BK :S / 2 LT 30 RE :S / 4 RT 30
BK :S / 2 LT 30
JUMP 2.5
JUMP 10
END


TO N70
N7 10
N0 10
END


TO N6 :S
PU FD :S / 4 PD C :S / 4 2
PU LT 90 FD :S / 4 RT 90 PD
FD 2 * :S / 3 RT 90 FD :S / 3 BK :S / 3 LT 90 BK 2 * :S / 3
PU RT 90 FD :S / 4 LT 90 BK :S / 4 PD
JUMP 10
END


TO N60
N6 10
N0 10
END


TO N5 :S
PU FD :S / 4 PD
RT 90 C :S / 4 1 RT 90
PU FD :S / 4 PD
FD :S / 2 RT 90 FD :S / 3 BK :S / 3 LT 90
PU BK :S PD
JUMP 10
END


TO N50
N5 10
N0 10
END


TO N4 :S
FD :S / 2 BK :S / 4
```

```
LT 90 BK :S / 4 FD :S / 2 RT 90
RT 5 FD 2 * :S / 3 BK 2 * :S / 3 LT 5
LT 90 BK :S / 4 RT 90 BK :S / 4
JUMP 10
END

TO N40
N4 10
N0 10
END

TO N3 :S
RT 180 JUMP 2.5 RT 180
PU FD 3 * :S / 4 PD
RT 90 C :S / 4 1 RT 90
PU BK :S / 2 PD
RT 90 C :S / 4 1 RT 90
PU BK :S / 4 PD
JUMP 10
END

TO N30
N3 9
N0 10
END

TO N2 :S
.SETSCRUNCH 1.3
RT 180 JUMP 2.5 RT 180
PU
FD 2 * :S / 4
PD FD :S / 4 BK :S / 4 RT 90
C :S / 4 1 RT 90
PU
BK :S / 4
PD BK :S / 4
RT 90 FD :S / 3
BK :S / 3 LT 90
JUMP 2.5
.SETSCRUNCH 1
JUMP 10
END

TO N20
N2 10
N0 10
END

TO N1 :S
.SETSCRUNCH 1.3
RE :S / 4 FD :S
LT 110 FD :S / 4
BK :S / 4 RT 110
BK :S
.SETSCRUNCH 1
JUMP 10
END
```

```
TO N10
N1 7
N0 10
END

TO N0 :S
.SETSCRUNCH 1.6 PU FD :S / 4 PD
C :S / 4 2
PU BK :S / 4 PD
.SETSCRUNCH 1
JUMP 10
END

TO ON
N0 10
END

TO KLIK :MEMBER :LIST
IF NOT MEMBERP :MEMBER :LIST [OUTPUT 0]
IF :MEMBER = FIRST :LIST [OUTPUT 1]
OUTPUT 1 + KLIK :MEMBER BF :LIST
END

TO NCONV :N.0
OP ITEM KLIK :N.0 :GAMMA :DELTA
END

TO MIN :S
PU FD :S / 2 PD RT 90 FD :S / 2 LT 90
PU BK :S / 2 PD
JUMP 10
END

TO NCB :S
.SETSCRUNCH 1.4
JUMP 5 PU FD :S / 2 PD
RT 90 C :S / 2 1 RT 90
PU BK :S / 2 PD
.SETSCRUNCH 1
END

TO N.LABEL :N.OM
IF :N.OM < 0 [MIN 10 RUN ( RUN [NCONV - :N.OM] )] [RUN ( RUN [NCONV :N.OM] )]
END

TO NOB :S
.SETSCRUNCH 1.4
PU FD :S / 2 PD
LT 90 C :S / 2 1 LT 90
PU BK :S / 2 PD
.SETSCRUNCH 1
JUMP 7
END

TO JUMP :S
RT 90 PU FD :S PD LT 90
END
```

431

```
TO WRITE :A :B
UNFIX HT
LOCAL "WQ
MAKE "WQ HEADING
S4POST "Q
SETPC 5
IF AND :A = XCOR :B = YCOR [SETH 0 PU LT 90 FD 30 JUMP 10 RT 90 PD NOB 10 N.LABEl
P 10 N.LABEL :B NCB 10] [PR [I'M NOT THERE] FIX ST PD SETPC 2 STOP]          :JU
PU .SETSCRUNCH 0.8
SETPOS :Q
SETH :WQ
ST SETPC 2 PD
FIX
END


TO S4POST :NAME
MAKE :NAME LIST XCOR YCOR
END


TO GAME1
PLCROSS 60 50
END


TO FIX
FIX1
FIX2
FIX3
FIX4
FIX5
FIX6
FIX7
END


TO CROSS :Q
RT 45 RE :Q LT 90 RE :Q RT 45
END


TO UNFIX
UNFIX1
UNFIX2
UNFIX3
UNFIX4
UNFIX5
UNFIX6
UNFIX7
END


TO GAME2
PLCROSS 80 -60
END


TO GAME3
PLCROSS -90 -40
END


TO GAME4
PLCROSS -100 90
END
```

```
TO UNFIX7
COPYDEF "OLDLT "LT
END


TO UNFIX6
COPYDEF "OLDRT "RT
END


TO UNFIX5
COPYDEF "OLDBK "BK
END


TO UNFIX4
COPYDEF "OLDFD "FD
END


TO UNFIX3
COPYDEF "OLDSETPOS "SETPOS
END


TO UNFIX2
COPYDEF "OLDSETY "SETY
END


TO UNFIX1
COPYDEF "OLDSETX "SETX
END


TO HELP3 :C :B
OLDSETPOS SE :C :B
END


TO PRMES
PR [I'M NOT HEADING TOWARDS THE PLACE YOU WANT TO TAKE ME]
END


TO C :R :P
PU LT 90 FD :R RT 90
RT 5 PD
LOCAL "M
MAKE "M 2 * :R * 3.1416 / 36
REPEAT 18 * :P [FD :M RT 10]
LT 5
PU RT 90 FD :R LT 90 PD
END


TO HELP2 :D :E
OP TOWARDS SE :D :E
END


TO NEWLT :S
PR [I DONT KNOW HOW TO LT]
END


TO FIX7
COPYDEF "LT "OLDLT
COPYDEF "NEWLT "LT
```

```
END

TO NEWRT :S
PR [I DONT KNOW HOW TO RT]
END


TO FIX6
COPYDEF "RT "OLDRT
COPYDEF "NEWRT "RT
END


TO NEWBK :S
PR [I DONT KNOW HOW TO BK]
END


TO FIX5
COPYDEF "BK "OLDBK
COPYDEF "NEWBK "BK
END


TO NEWFD :S
PR [I DONT KNOW HOW TO FD]
END


TO FIX4
COPYDEF "FD "OLDFD
COPYDEF "NEWFD "FD
END


TO NEWSETPOS :C :B
IF POS = SE :C :B [STOP] [IF ( TOWARDS SE :C :B ) = HEADING [OLDSETPOS SE :C :B]
STOP]]                                                                    [PRMES
END


TO FIX3
COPYDEF "SETPOS "OLDSETPOS
COPYDEF "NEWSETPOS "SETPOS
END


TO NEWSETY :S                                                                   P
IF YCOR = :S [OLDSETY :S] [IF YCOR > :S [IF HEADING = 180 [OLDSETY :S] [PRMES STO
 HEADING = 0 [OLDSETY :S] [PRMES STOP]]]
END


TO FIX2
COPYDEF "SETY "OLDSETY
COPYDEF "NEWSETY "SETY
END


TO NEWSETX :S
IF XCOR = :S [OLDSEX :S] [IF XCOR > :S [IF HEADING = 270 [OLDSETX :S] [PRMES STOF
HEADING = 90 [OLDSETX :S] [PRMES STOP]]]
END


TO FIX1
COPYDEF "SETX "OLDSETX
COPYDEF "NEWSETX "SETX
END
```

```
TO GAME5
PLCROSS -90 70
PLCROSS 100 -40
END

TO GAME6
PLCROSS 20 90
PLCROSS -30 -10
END

TO GAME7
PLCROSS 90 0
PLCROSS 70 0
END

TO GAME8
PLCROSS 30 80
PLCROSS 30 -70
END

TO PLCROSS :A :B
UNFIX HT PU SETPOS SE :A :B
SETPC 4
PD CROSS 5 PU
SETPOS SE 0 0
SETPC 2
ST
FIX
END

TO GAME9
PLCROSS 0 -110
PLCROSS 0 -70
END


MAKE "BETA [[LA 1
```

```
TO GAME.2
PLCROSS -40 80
PLCROSS -100 -110
PLCROSS 60 -90
END


TO GAME.1
PLCROSS -30 90
PLCROSS -90 -40
PLCROSS 70 -70
END


TO GAME9
PLCROSS 0 -110
PLCROSS 0 -70
END


TO GAME8
PLCROSS 30 80
PLCROSS 30 -70
END


TO GAME7
PLCROSS 90 0
PLCROSS 70 0
END


TO GAME6
PLCROSS 20 90
PLCROSS -30 -10
END


TO GAME5
PLCROSS -90 70
PLCROSS 100 -40
END


TO NEWSETX :S
IF XCOR = :S [OLDSEX :S] [IF XCOR > :S [IF HEADING = 270 [OLDSETX :S] [PRMES STOP
IF HEADING = 90 [OLDSETX :S] [PRMES STOP]]]
END


TO NEWSETY :S
IF YCOR = :S [OLDSETY :S] [IF YCOR > :S [IF HEADING = 180 [OLDSETY :S] [PRMES STO
 HEADING = 0 [OLDSETY :S] [PRMES STOP]]]
END


TO NEWSETPOS :C :B
IF POS = SE :C :B [STOP] [IF ( TOWARDS SE :C :B ) = HEADING [OLDSETPOS SE :C :B]
STOP]]                                                                [PRMES
END


TO NEWFD :S
PR [I DONT KNOW HOW TO FD]
END


TO NEWBK :S
```

436

```
PR [I DONT KNOW HOW TO BK]
END

TO NEWRT :E
PR [I DONT KNOW HOW TO RT]
END

TO NEWLT :E
PR [I DONT KNOW HOW TO LT]
END

TO HELP2 :D :E
OP TOWARDS SE :D :E
END

TO PRMES
PR [I'M NOT HEADING TOWARDS THE PLACE YOU WANT TO TAKE ME]
END

TO HELP3 :C :B
OLDSETPOS SE :C :B
END

TO GAME4
PLCROSS -100 90
END

TO GAME3
PLCROSS -90 -40
END

TO GAME2
PLCROSS 80 -60
END

TO UNFIX7
COPYDEF "OLDLT "LT
END

TO UNFIX6
COPYDEF "OLDRT "RT
END

TO UNFIX5
COPYDEF "OLDBK "BK
END

TO UNFIX4
COPYDEF "OLDFD "FD
END

TO UNFIX3
COPYDEF "OLDSETPOS "SETPOS
END

TO UNFIX2
COPYDEF "OLDSETY "SETY
END
```

437

```
TO UNFIX1
COPYDEF "OLDSETX "SETX
END


TO CROSS :Q
RT 45 RE :Q LT 90 RE :Q RT 45
END


TO FIX7
COPYDEF "LT "OLDLT
COPYDEF "NEWLT "LT
END


TO FIX6
COPYDEF "RT "OLDRT
COPYDEF "NEWRT "RT
END


TO FIX5
COPYDEF "BK "OLDBK
COPYDEF "NEWBK "BK
END


TO FIX4
COPYDEF "FD "OLDFD
COPYDEF "NEWFD "FD
END


TO FIX3
COPYDEF "SETPOS "OLDSETPOS
COPYDEF "NEWSETPOS "SETPOS
END


TO FIX2
COPYDEF "SETY "OLDSETY
COPYDEF "NEWSETY "SETY
END


TO FIX1
COPYDEF "SETX "OLDSETX
COPYDEF "NEWSETX "SETX
END


TO PLCROSS :A :B
UNFIX HT PU SETPOS SE :A :B
SETPC 4
PD CROSS 5 PU
SETPOS SE 0 0
SETPC 2
ST
FIX
END


TO GAME1
PLCROSS 60 50
END
```

```
TO S4POST :NAME
MAKE :NAME LIST XCOR YCOR
END


TO WRITE :H :B
UNFIX HT
LOCAL "WQ
MAKE "WQ HEADING
SETPC 5                                                                    O PU
IF ( AND :H - XCOR < 0.5 :H - XCOR > -0.5 :B - YCOR < 0.5 :B - YCOR > -0.5 ) [SETH
LT 90 FD 30 JUMP 10 RT 90 PD NOB 10 N.LABEL :H JUMP 10 N.LABEL :B NCB 10] [PR [I'M
HERE] FIX ST PD SETPC 2 STOP]                                               NOT
PU .SETSCRUNCH 0.8
SETPOS SE :H :B
SETH :WQ
ST SETPC 2 PD
FIX
END


TO NOB :S
.SETSCRUNCH 1.4
PU FD :S / 2 PD
LT 90 C :S / 2 1 LT 90
PU BK :S / 2 PD
.SETSCRUNCH 1
JUMP 7
END


TO N.LABEL :N.OM
IF :N.OM < 0 [MIN 10 RUN ( RUN [NCONV - :N.OM] )] [RUN ( RUN [NCONV :N.OM] )]
END


TO NCB :S
.SETSCRUNCH 1.4
JUMP 5 PU FD :S / 2 PD
RT 90 C :S / 2 1 RT 90
PU BK :S / 2 PD
.SETSCRUNCH 1
END


TO MIN :S
PU FD :S / 2 PD RT 90 FD :S / 2 LT 90
PU BK :S / 2 PD
JUMP 10
END


TO NCONV :N.O
OP ITEM KLIK :N.O :GAMMA :DELTA
END


TO KLIK :MEMBER :LIST
IF NOT MEMBERP :MEMBER :LIST [OUTPUT 0]
IF :MEMBER = FIRST :LIST [OUTPUT 1]
OUTPUT 1 + KLIK :MEMBER BF :LIST
END


TO N5 :S
PU FD :S / 4 PD
```

439

```
RT 90 C :S / 4 1 RT 90
PU FD :S / 4 PD
FD :S / 2 RT 90 FD :S / 3 BK :S / 3 LT 90
PU BK :S PD
JUMP 10
END


TO N7 :S
RT 180 JUMP 2.5 RT 180
RT 30 FD :S LT 120 FD :S / 2 BK :S / 2
RT 120 BK :S / 2 LT 30 RE :S / 4 RT 30
BK :S / 2 LT 30
JUMP 2.5
JUMP 10
END


TO N8 :S
PU FD :S / 4 PD
C :S / 4 2
PU FD 7 * :S / 12 PD
C :S / 4 2
PU BK 10 * :S / 12 PD
JUMP 10
END


TO JUMP :S
RT 90 PU FD :S PD LT 90
END


TO N6 :S
PU FD :S / 4 PD C :S / 4 2
PU LT 90 FD :S / 4 RT 90 PD
FD 2 * :S / 3 RT 90 FD :S / 3 BK :S / 3 LT 90 BK 2 * :S / 3
PU RT 90 FD :S / 4 LT 90 BK :S / 4 PD
JUMP 10
END


TO N9 :S
PU FD 11 * :S / 12 PD RT 180
N6 :S
RT 180 PU BK 11 * :S / 12 PD
JUMP 20
END


TO N2 :S
.SETSCRUNCH 1.3
RT 180 JUMP 2.5 RT 180
PU
FD 2 * :S / 4
PD FD :S / 4 BK :S / 4 RT 90
C :S / 4 1 RT 90
PU
BK :S / 4
PD BK :S / 4
RT 90 FD :S / 3
BK :S / 3 LT 90
JUMP 2.5
.SETSCRUNCH 1
```

```
JUMP 10
END


TO N3 :S
RT 180 JUMP 2.5 RT 180
PU FD 3 * :S / 4 PD
RT 90 C :S / 4 1 RT 90
PU BK :S / 2 PD
RT 90 C :S / 4 1 RT 90
PU BK :S / 4 PD
JUMP 10
END


TO N0 :S
.SETSCRUNCH 1.6 PU FD :S / 4 PD
C :S / 4 2
PU BK :S / 4 PD
.SETSCRUNCH 1
JUMP 10
END


TO N4 :S
FD :S / 2 BK :S / 4
LT 90 BK :S / 4 FD :S / 2 RT 90
RT 5 FD 2 * :S / 3 BK 2 * :S / 3 LT 5
LT 90 BK :S / 4 RT 90 BK :S / 4
JUMP 10
END


TO N1 :S
.SETSCRUNCH 1.3
RE :S / 4 FD :S
LT 110 FD :S / 4
BK :S / 4 RT 110
BK :S
.SETSCRUNCH 1
JUMP 10
END


TO RE :S
REPEAT 2 [RBLOB :S RT 180]
END


TO RBLOB :S
RT 90 FD :S BK :S LT 90
END


TO M01
RT 90 FD 5 BK 5 LT 90 FD 10
RT 90 FD 2.5 BK 2.5 LT 90 FD 10
END


TO M1 :D
BK :D
REPEAT :D * 2 / 20 [M01]
RT 90 FD 5 BK 5 LT 90
BK :D
END
```

441

```
TO UNFIX
UNFIX1
UNFIX2
UNFIX3
END

TO GRID :A :B
UNFIX HT SETPC 0
M1 :A
LT 90
M1 :B
RT 90
ST
SETPC 2
FIX
END

TO START
SETBG 1
GRID 120 140
END

TO C :R :P
PU LT 90 FD :R RT 90
RT 5 PD
LOCAL "M
MAKE "M 2 * :R * 3.1416 / 36
REPEAT 18 * :P [FD :M RT 10]
LT 5
PU RT 90 FD :R LT 90 PD
END

TO N140
N1 7
N4 10
N0 10
END

TO N130
N1 7
N3 9
N0 10
END

TO N120
N1 7
N2 10
N0 10
END

TO N110
N1 7
N1 7
N0 10
END

TO N100
```

```
N1 7
N0 10
N0 10
END

TO N90
N9 10
N0 10
END

TO N80
N8 10
N0 10
END

TO N70
N7 10
N0 10
END

TO N60
N6 10
N0 10
END

TO N50
N5 10
N0 10
END

TO N40
N4 10
N0 10
END

TO N30
N3 9
N0 10
END

TO N20
N2 10
N0 10
END

TO N10
N1 7
N0 10
END

TO 0N
N0 10
END

TO FIX
FIX1
FIX2
FIX3
```

```
END

TO SETL
MAKE "ALPHA [A B C D E F G H I J K L M N O P Q R S T U V W X Y Z]            10
MAKE "BETA ( LIST [LA 13] [LB 20] [LC 5] [LD 5] [LE 10] [LF 10] [LG 5] [LH 10] [LI
LJ 5] [LK 10] [LL 10] [LM 10] [LN 10] [LO 5] [LP 20] [LQ 5] [LR 20] [LS 10] [LTT 1
 5] [LV 13] [LW 10] [LX 13] [LY 13] [LZ 10] )                                [LU
END

TO NSETL
MAKE "GAMMA [0 10 20 30 40 50 60 70 80 90 100 110 120 130 140]
MAKE "DELTA ( LIST [ON] [N10] [N20] [N30] [N40] [N50] [N60] [N70] [N80] [N90] [N10
N[ 10] [N120] [N130] [N140] )
END

TO BEGIN
NSETL
SETL
FIX
END

TO SQ :X
OP :X * :X
END

TO DISTANCE :N                                                              :NL
IF AND HEADING - ( TOWARDS SE FIRST :N LAST :N ) ( 0.5 HEADING - ( TOWARDS SE FIRS
AST :N ) ) -0.5 [SETH TOWARDS SE FIRST :N LAST :N OP SQRT ( SQ ( XCOR - FIRST :N )
 SQ ( YCOR - LAST :N ) )] [OP SE [I'M NOT HEADING TOWARDS] :N STOP]
END

TO DIFF.A :A :B
IF :A - :B < 0 [OP 360 + :A - :B]
IF :A - :B > 360 [OP :A - :B - 360]
OP :A - :B
END

TO DIRECTION :P
OP DIFF.A TOWARDS SE FIRST :P LAST :P HEADING
END
```

APPENDIX D

The P.D.D. microworld

POST "(letter)

Execution of this command places a list consisting of the coordinates of the turtle's current position into the computer memory, labels the list according to the letter given and displays the letter on the screen in the proximity of the turtle's position. It also signifies the exact position by placing an "x" sign on it.

DISTANCE :(letter)

Execution of this command outputs the distance in turtle steps betwen the current position of the turtle and the position signified by (letter).

DIRECTION :(letter)

Execution of this command outputs the number of degrees the turtle would have to turn towards the right from its current heading in order to face the position signified by (letter).

The procedures PLONK1, PLONK2, PLONK3, PLONK10, cause the turtle to move to a number of positions on the screen (the numbers at the end of PLONK show how many for each procedure) in PENUP and HT mode, label the positions and return to the starting position, changing the mode to ST and PD. These procedures were used in the activities labelled SET1, in figure 7.2.1.

The procedures TRI and TRI2, cause the turtle to place three points on the screen (as in the PLONK procedures), in the formation shown in figure 7.2.1. The two procedures were used for the SET2 activities in category 1, fig. 7.2.1.

```
TO .LABEL :LTR
RUN ( RUN [CONV :LTR] )
END


TO CONV :LTR
OP ITEM KLIK :LTR :ALPHA :BETA
END


TO SETL
MAKE "ALPHA [A B C D E F G H I J K L M N O P Q R S T U V W X Y Z]          10
MAKE "BETA ( LIST [LA 13] [LB 20] [LC 5] [LD 5] [LE 10] [LF 10] [LG 5] [LH 10] [LI
LJ 5] [LK 10] [LL 10] [LM 10] [LN 10] [LO 5] [LP 20] [LQ 5] [LR 20] [LS 10] [LTT 1(
LU 5] [LV 13] [LW 10] [LX 13] [LY 13] [LZ 10] )
END


TO KLIK :MEMBER :LIST
IF NOT MEMBERP :MEMBER :LIST [OUTPUT 0]
IF :MEMBER = FIRST :LIST [OUTPUT 1]
OUTPUT 1 + KLIK :MEMBER BF :LIST
END


TO LR :S
LPP :S
.SETSCRUNCH 0.5
FD 1.2 + :S / 2
RT 155
FD :S / 1.6
BK :S / 1.6
LT 155
BK 1.2 * :S / 2
.SETSCRUNCH 1
JUMP 15
END


TO LB :S
LPP :S
.SETSCRUNCH 0.5
FD :S / 4
RT 90 C :S / 4 1
RT 90
BK :S / 4
.SETSCRUNCH 1
JUMP 15
END


TO LG :S
LCC :S
PU RT 90 FD :S LT 90 PD
FD :S RT 90 FD :S / 4 BK :S / 2 FD :S / 4 LT 90 BK :S
PU LT 90 FD :S RT 90 PD
JUMP 15
END


TO LQ :S
LOO :S
PU RT 90 FD 2 * :S LT 90 PD
LT 45 FD :S / 3 BK :S FD 2 * :S / 3 RT 45
```

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```
PU LT 90 FD 2 * :S RT 90 PD
JUMP 15
END


TO LD :S
FD 2 * :S BK :S RT 90
C :S 1
RT 90 BK :S
JUMP 15
END


TO LZ :S
.SETSCRUNCH 1.5
LT 90 BK :S
LNN :S
FD :S RT 90
.SETSCRUNCH 1
JUMP 15
END


TO LS :S
PU REPEAT 2 [FD :S / 4 RT 90] LT 90 PD
REPEAT 2 [C :S / 4 1 RT 90 PU FD :S / 2 LT 90 PD]
PU RT 90 REPEAT 2 [FD :S / 4 RT 90] PD
JUMP 15
END


TO LW :S
PU FD :S RT 90 FD 2 * ( :S * SIN 30 ) RT 90 PD
LMM :S
PU FD :S RT 90 FD 2 * ( :S * SIN 30 ) RT 90 PD
JUMP 15
END


TO LA :S
.SETSCRUNCH 0.9
FD :S / 2 RT 30 REPEAT 3 [FD :S / 2 RT 120]
RT 60 FD :S / 2 RT 90 FD :S / 2 PU RT 90 FD :S / 2 RT 90 PD
.SETSCRUNCH 1
JUMP 15
END


TO LV :S
.SETSCRUNCH 0.9
PU RT 90 FD ( :S / 2 ) * COS 60 LT 90 PD
LT 30 FD :S / 2 RT 30 FD :S / 2 RT 90 PU FD :S * COS 60 RT 90 PD
FD :S / 2 RT 30 FD :S / 2 RT 60
PU FD ( :S / 2 ) * COS 60 PD
RT 90
.SETSCRUNCH 1
JUMP 15
END


TO LU :S
.SETSCRUNCH 2
PU REPEAT 2 [FD :S RT 90]
C :S 1
RT 180
```

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```
PU REPEAT 2 [FD :S RT 90]
.SETSCRUNCH 1
JUMP 15
END


TO C :R :P
PU LT 90 FD :R RT 90
RT 5 PD
LOCAL "M
MAKE "M 2 * :R * 3.1416 / 36
REPEAT 18 * :P [FD :M RT 10]
LT 5
PU RT 90 FD :R LT 90 PD
END


TO LJ :S
.SETSCRUNCH 2
PU FD :S RT 180 PD
C :S 0.5
BK :S * 2 FD :S * 2
RT 90 PU BK :S PD
.SETSCRUNCH 1
JUMP 15
END


TO LX :S
RT 30 FD :S / 2 LT 30
AR :S / 2
RT 180 AR :S / 2
RT 30 FD :S / 2 RT 150
JUMP 15
END


TO LY :S
PU RT 90 FD ( :S / 2 ) * COS 60 LT 90 PD
FD :S / 2 AR :S / 2 BK :S / 2
PU LT 90 FD ( :S / 2 ) * COS 60 RT 90 PD
JUMP 15
END


TO AR :S
LT 30 FD :S BK :S
RT 60 FD :S BK :S
LT 30
END


TO LK :S
FD :S BK :S / 2 RT 90
AR ( :S / 2 ) / COS 60
LT 90 BK :S / 2
JUMP 15
END


TO LE :S
REPEAT 2 [FD :S / 2 RBLOB :S / 2]
BK :S
RBLOB :S / 2
JUMP 15
```

```
END

TO LF :S
REPEAT 2 [FD :S / 2 RBLOB :S / 2]
BK :S
JUMP 15
END


TO LTT :S
FD :S
RE :S / 2
BK :S
JUMP 15
END


TO LI :S
FD :S RE :S / 2 BK :S
RE :S / 2
JUMP 15
END


TO RE :S
REPEAT 2 [RBLOB :S RT 180]
END


TO LH :S
FD :S / 2 RT 90
REPEAT 2 [RE :S / 2 FD :S RT 180]
LT 90 BK :S / 2
JUMP 15
END


TO RBLOB :S
RT 90 FD :S BK :S LT 90
END


TO LL :S
FD :S BK :S
RBLOB :S / 2
JUMP 15
END


TO LP :S
.SETSCRUNCH 0.5
FD 3 * :S / 4
RT 90 C :S / 4 1
RT 90
FD :S / 4
BK :S
.SETSCRUNCH 1
JUMP 15
END


TO LPP :S
LP :S
RT 180 JUMP 15
RT 180
END
```

```
TO LO :S
PU REPEAT 2 [FD :S RT 90] PD
RT 180
C :S 2
RT 180 PU REPEAT 2 [FD :S RT 90] PD
JUMP 15
END


TO LOO :S
LO :S
RT 180 JUMP 15 RT 180
END


TO LC :S
PU REPEAT 2 [FD :S RT 90] PD
RT 90
C :S 1
RT 90
PU REPEAT 2 [FD :S RT 90] PD
JUMP 15
END


TO LCC :S
LC :S
RT 180 JUMP 15 RT 180
END


TO LN :S
FD :S RT 150
FD :S / COS 30
LT 150 FD :S
PU LT 90 FD ( SIN 30 / COS 30 ) * :S
RT 90 BK :S PD
JUMP 15
END


TO LNN :S
LN :S
RT 180 JUMP 15 RT 180
END


TO JUMP :S
RT 90 PU FD :S PD LT 90
END


TO LM :S
FD :S RT 150 FD :S
LT 120 FD :S RT 150 FD :S
RT 90 PU FD 2 * ( :S * SIN 30 ) PD
RT 90
JUMP 15
END


TO LMM :S
LM :S
RT 180 JUMP 15 RT 180
END
```

```
MAKE "BETA [[LA
TO SQ :X
OP :X * :X
END


TO DIFF.A :H :B
IF :H - :B < 0 [OP 360 + :H - :B]
IF :H - :B > 360 [OP :H - :B - 360]
OP :H - :B
END


TO DISTANCE :N
OP SQRT ( SQ ( XCOR - FIRST :N ) + ( SQ ( YCOR - LAST :N )
END


TO DIRECTION :P
OP DIFF.A TOWARDS SE FIRST :P LAST :P HEADING
END


TO POST :LTR
MAKE :LTR POS
LOCAL "WQ
MAKE "WQ HEADING
HT SETH 0 RT 45 RE 3 LT 90 RE 3 RT 45
LABEL1 :LTR
ST
END


TO LABEL1 :LTR
PU
IF YCOR > 0 [IF XCOR > 0 [SETH 0 JUMP 5] [SETH 180 JUMP 15 SETH 0]] [IF XCOR > 0
5 JUMP 15 SETH 0] [SETH 135 JUMP 20 SETH 0]]
.LABEL :LTR
PU
SETH :WQ
SETPOS SE FIRST THING :LTR LAST THING :LTR
PD
END


TO PLONK1
HT PU SETPOS SE 50 50
PD
POST "A
PU
HOME
PD ST
END


TO PLONK2
HT PU SETPOS SE -50 50
PD
POST "B
PU
SETPOS SE 20 60
PD
POST "C
PU
HOME
```

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```
PD ε,
END

TO PLONK3
HT PU SETPOS SE 40 70
PD
POST "D
PU SETPOS SE -40 70 PD
POST "E
PU SETPOS SE 40 -50 PD
POST "F
PU HOME PD ST
END

TO PLONK10
PLONK1
PLONK2
PLONK3
HT
PU SETPOS SE -10 -10 PD POST "G
PU SETPOS SE -100 -20 PD POST "H
PU SETPOS SE -50 -60 PD POST "I
PU SETPOS SE 60 -40 PD POST "J
PU HOME PD ST
END

TO TRI
HT
PU SETPOS SE -75 50 PD POST "A
PU SETPOS SE 60 35 PD POST "B
PU SETPOS SE 10 -50 PD POST "C
PU HOME PD
ST
END

TO TRI2
HT
PU SETPOS SE -40 -20 PD POST "D
PU SETPOS SE 50 30 PD POST "E
PU SETPOS SE 70 -35 PD POST "F
PU HOME PD
ST
END
MAKE "S 91.7898
```

APPENDIX E

The Circle microworld

The screen effect in using the measuring commands was that of the turtle carrying out the measurement by tracing the distance to be measured and pausing to print the distance covered to the current position. The children firstly drew a circle on the screen, using their CIR4 procedure and an input of their choice (for the number of turtle steps between each turn). They then measured the circumference using the COUNT.LENGTH command with the same input; execution of the command caused the turtle to re-trace the curvature and pause after each change of position, printing the distance covered from the beginning to the current position. The children then measured the diameter of the circle via the COUNT.WIDTH command and the same input; execution of the command caused the turtle to re-trace half the curvature and then turn towards the starting position and move towards it in PEN - UP mode, pausing every ten steps to print the distance covered for the diameter (where applicable) and giving the final number for the distance in the end. If the children whished to see the diameter drawn on the screen, they could use the SHOW.WIDTH command with the same input, which caused the turtle to go through the same procedure as in the COUNT.WIDTH command with the difference that there was no counting and the pen was down.

1) To measure the circumference:
COUNT.LENGTH (input)
2) To measure the diameter:
COUNT.WIDTH (input)
3) To show the diameter:
SHOW.WIDTH (input)

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```
TO COUNT.WIDTH :STEP
LOCAL "M
POST2 "M
SEMCIR2 :STEP 10
POST2 "T
PU
RT 90
COUNT.W :M :T
PD
END


TO SHOW.WIDTH :STEP
RT 90 FD DISTANCE2 :T LT 90
END


TO COUNT.L :C :S
IF :C > 36 [STOP]
FD :S RT 10
PRINT :S * :C
WAIT 20
COUNT.L :C + 1 :S
END


TO .PRINT :Q
PRINT ( ROUND ( 100 * :Q ) ) / 100
END


TO COUNT.LENGTH :STEP
RT 5
COUNT.L 1 :STEP
LT 5
END


TO COUNT.W :M :T
LOCAL "S
MAKE "S DISTANCE2 :M
IF :S < 10 [WAIT 20 FD :S RT 90 PRINT DISTANCE2 :T STOP]
WAIT 20 FD 10 WAIT 10 PRINT DISTANCE2 :T COUNT.W :M :T
END


TO SEMCIR2 :STEP :ANGLE
RT :ANGLE / 2
REPEAT 18 [FD :STEP RT :ANGLE WAIT 10]
LT :ANGLE / 2
END


TO POST2 :N
MAKE :N LIST XCOR YCOR
END


TO SQ2 :X
OP :X * :X
END


TO DISTANCE2 :N
OP SQRT ( SQ2 ( XCOR - FIRST :N ) ) + ( SQ2 ( YCOR - LAST :N ) )
END
```

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1) Write a **procedure** to draw this circle on the computer
   clue: imagine you're the turtle and you want to go round a circle
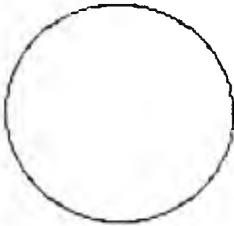
My procedure:

Now try it out on the computer.

2) The circle below is larger than the one you have just drawn.
Without using the computer, write down what you think you
should change in your procedure to draw a larger circle.
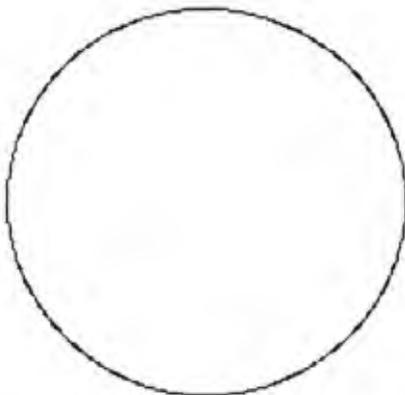
To draw this circle, I had to
change: _____
_____
_____
_____

Explain your answer to question 2 here:
_____
_____
_____
_____
_____
_____

3) Write a **procedure** to draw a smaller circle.
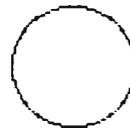
My procedure:

Try your procedure out on the computer.

4) Now, using your circle procedure, try to write a **procedure** to draw this shape:
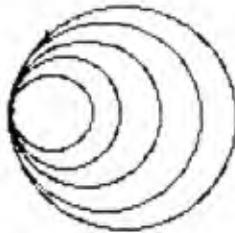
clue: In Logo, instead of writing a different procedure to draw a different circle, you can have the same procedure with a variable input for the value you want to change, for instance: TO ANYCIRCLE :SIDE
          REPEAT 36 [FD :SIDE RT 10]
          END
When you want to use this procedure to draw a circle with, say, a SIDE of 50 units, you just type in : ANYCIRCLE 50

Now try, using the ANYCIRCLE procedure, to draw the shape.

5) Write a **procedure** to draw a shape with circles of different sizes using the ANYCIRCLE as your tool.

My procedure:

Try out your procedure on the computer.

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6) Try to draw this shape on the computer, using ANYCIRCLE
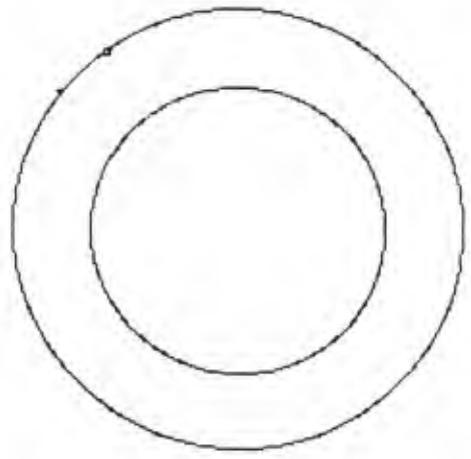
The two circles are at a distance of 30 units from eachother.

You may have managed to draw a shape looking like this, or you may not

6a) **Do you think it is possible to draw the shape accurately if you use ANYCIRCLE?**

6b) **What tool (procedure) would you like to have so that you know your solution is accurate?**

Explain your answer to question 6a) here:

_____

_____

_____

_____

_____

_____

Explain your answer to question 6b) here:

_____

_____

_____

_____

_____

_____

7) How about this tool:
                    A procedure, call it NEWCIRCLE, that takes
        as an input, the radius of the circle. Using this tool would mean
        that NEWCIRCLE   100 would draw a circle with radius 100 units.

7a) Would this tool solve the problem?

Explain your answer to 7a) here:
_____
_____
_____
_____
_____
_____

8) Compare these two procedures:

TO ANYCIRCLE :SIDE   TO NEWCIRCLE :RADIUS
REPEAT 36 [FD :SIDE RT 10]   REPEAT 36 [FD   ?   RT 10]
END   END

8a) What do you need to find out to complete NEWCIRCLE?
     Make your answer as accurate as possible.

Give your answer to 8a here:
_____
_____
_____
_____
_____

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9) There must be a connection between the SIDE and the RADIUS.
Lets collect more information about these two quantities.

9a) What connection does the side have to the length (perimeter)
of the circle?

Give your answer to 9a) here:

9b) What connection does the RADIUS have to the width (diameter)
of the circle?

Give your answer to 9b) here:

10) Let's try to discover a connection between the length and the
     width of a circle.

10a) Draw a circle on the computer, using the ANYCIRCLE procedure,
      say ANYCIRCLE 30.
   i) **The computer can count the length of this circle:**
      If you press the orange key "f_0", you will see
                COUNT.LENGTH.OF.CIRCLE
      written on the screen
      Type in the SIDE of the circle you have drawn (in this example
      you have a SIDE of 30, so type in 30) and press RETURN
      The computer will give you the length of ANYCIRCLE 30,which is 1080

  ii) **The computer can count the width of this circle:**
      If you press the orange key "f_1", you will see
                COUNT.WIDTH OF CIRCLE
      written on the screen.
      Type in the SIDE of the circle you have drawn (in this exampl
      you have a SIDE of 30, so type in 30) and press RETURN.
      The computer will give you the width of ANYCIRCLE 30 , which is 344.21

     10b) You need to find a connection between the length and the
           width. You need to **compare** them. How much bigger is
           the length than the width? Can you think of a number
           operation that will tell you how many times the length
           is bigger than the width?

     ╭───╮
     │ Give your answer to the second question of 10b) here   │
     │   │
     │   │
     ╰───╯

     10c) Press the orange key "f_2" on the keyboard. The word
                     CONNECTION
           will appear on the screen.
           Now type in the operation that will tell you how many
           times the length is bigger than the width. For example,
           if your answer to 10b) was "division", type in the length
           that you found from i) divided by the width that you found
           from ii). This means that for ANYCIRCLE 30, you type in:
           1080 / 344.21

       10d) Do the same thing for more circles and fill in the table on
             the next page.

11) Investigation to find a connection between the length and
the width of a circle.

Table of results

SIDE	length	width	connection
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			

11a) Can you find the connection by looking at the last columr
If not, try another connection rule.
If yes, write in words what you have found.

Give your answer to 11a) here:
_____
_____
_____

Turn now and look at your answers to 9a) and 9b).
Copy your answers here:

My answer to 9a):

My answer to 9b):

11b) Can you think of a way to **combine** these three results,
to find out how many RADIUS's (radii is the right word)
go into one SIDE?

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Give your answer to 11b) here

12) Look at 8) and compare the two procedures again Can you complete NEWCIRCLE now?

12a) Write down the complete NEWCIRCLE procedure

Give your answer to 12a) here:
   TO NEWCIRCLE

13) Try NEWCIRCLE out on the computer.

14) Now draw the shape in page 3, using your NEWCIRCLE procedure

15) Write a **procedure** to draw a shape with circles of different sizes, using the NEWCIRCLE as your tool.

My procedure:

Try out your procedure on the computer.

16) Let's try to draw a tree, starting from its trunk, say, 30 turtle steps length:

Now draw a circular bush whose centre is at the top of the tree trunk (where the turtle is now). Make your circle with a radius of, say, 25 turtle steps.
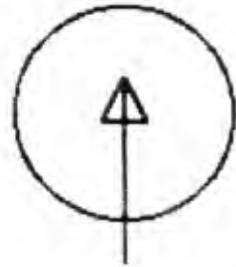
17) Erase this circle (hint. use PE and PD) and draw another one with a smaller radius so that it realy looks like a tree. Try out different sizes for your bush, until it looks right.

18) Now try to think a bit about the circle procedure that you are using. Are you satisfied with it? Can you change it so that this task becomes easier? Write down your thoughts on how to do this.

9) Try to write a procedure that draws a circle **around the turtle.** (imagine the turtle is at the centre of the circle before and after it starts drawing it). Clue:

```
TO CENCIR :RADIUS
GOTOEDGE :RADIUS
NEWCIRCLE :RADIUS
GOTOCENTRE :RADIUS
END
```

Can you write the **GOTOEDGE** and **GOTOCENTRE** procedures?

20) Now draw a tree of any size you like, using your CENCIR procedure
You can make it look nicer by drawing two branches at the top
of the tree trunk, like this:

) Is it easier to draw the tree in this way using the CENCIR
procedure? Give reasons for your answer here:

) Carry out your own project drawing circles of different sizes
using your CENCIR procedure. Write down the procedure(s)
you used to do your project.

23) Try to write a procedure to draw a clockface, like this:
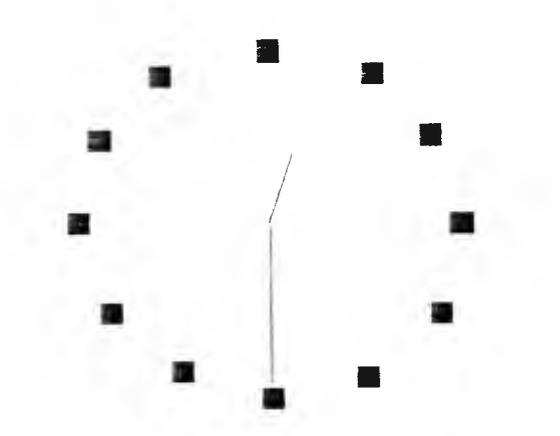
Clue: Write a procedure that draws a dot and use it for your clockface.

24) Is it easier for the turtle to start at the centre of the clockface or at the edge? Give your reasons below:

_____

_____

25) Think about this procedure:

```
TO DOTCIR :RADIUS
REPEAT 12 [ KEY :RADIUS RT 30]
END
```

Write what you think KEY :RADIUS should be so that DOTCIR :RADIUS draws the clockface. Give your answer here:

_____

_____

_____

_____

26) Draw the clockface using your DOTCIR procedure and then carry out your own project using DOTCIR. Write down the procedures you wrote for your project.

_____

_____

_____

_____

1) Φτιάξε μìα ἐννοια που να ζωγραφἰζει ἐνα κυκλο στην οθὸνη
Σημ. φαντὰσου πως εἰσαι η χελὼνα καὶ θἐλεις να προχωρησεις
γυρω απὸ ἐνα κυκλο.

H ἐννοιὰ μου:

KEPEAT 120 [FD 1 RT 3]
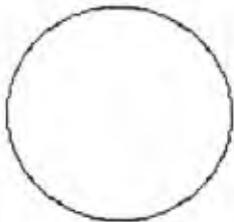
Δοκìμασὲ το τωρα στòν
                    υπολογιστη

2) Ο κυκλος αυτὸς εìναι μεγαλυτερος απò αυτòν που ἐφτιαξες
Χωρìς να χρησιμοποιεìς τòν υπολογιστη, γρὰψε τι
χρειὰζεται ν' αλλὰξεις στην ἐννοιὰ σου γιὰ να φτιὰξεις ἐνα
μεγαλυτερο κυκλο.
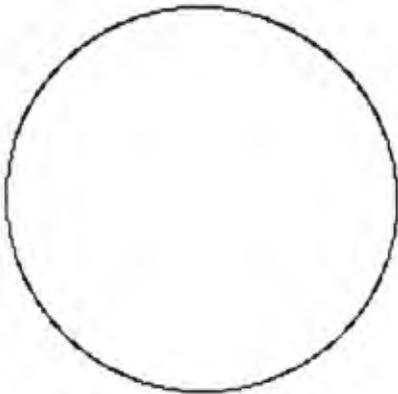
Γιὰ να φτιὰξω αυτòν τον κυκλο,
χρειὰστηκε ν' αλλὰξω: τις φο-
ρὲς που πρὲπει να επα-
ναλάβει η χελὼνα αυτὰ
που εìναι γραμμἐνα στις
                    να τις κὰνουμε, περισσὸτερες
φτιἀξει, και τα πὸσο δε-
ξιἀ θα παει να το κανουμε λιγὸτερο

Εξηγησε την απὰντηση σου στο 2:

Πρὲπει ν' αλλἀξουμε τα προηγούμενα
(γιατι) δηλαδὴ το REPEAT και το RT, γιατι
ἐχοντας τις λιγὸτερες μοὶρες, η χελὼνα
στριβει λιγὸτερο κι ο κὺκλος γινεται
μεγαλὺτερος.

An A4 piece of paper was given to each child, with the respective figure drawn on it by means of a pen and via a "compass and ruler" construction method. A computer printout of the figures constructed in Logo or otherwise was avoided because of the inaccurate representation of curvatures. As a result of the pilot study, it was decided that a methodological shortcomming could arise from the children either guessing which circle procedure had been used (if the circles of a task figure were constructed in Logo) by the researcher, or not perceiving intended circle shapes as circles, due to the "jagged line" effect on the screen. The sample figure given here was the one in task 4.
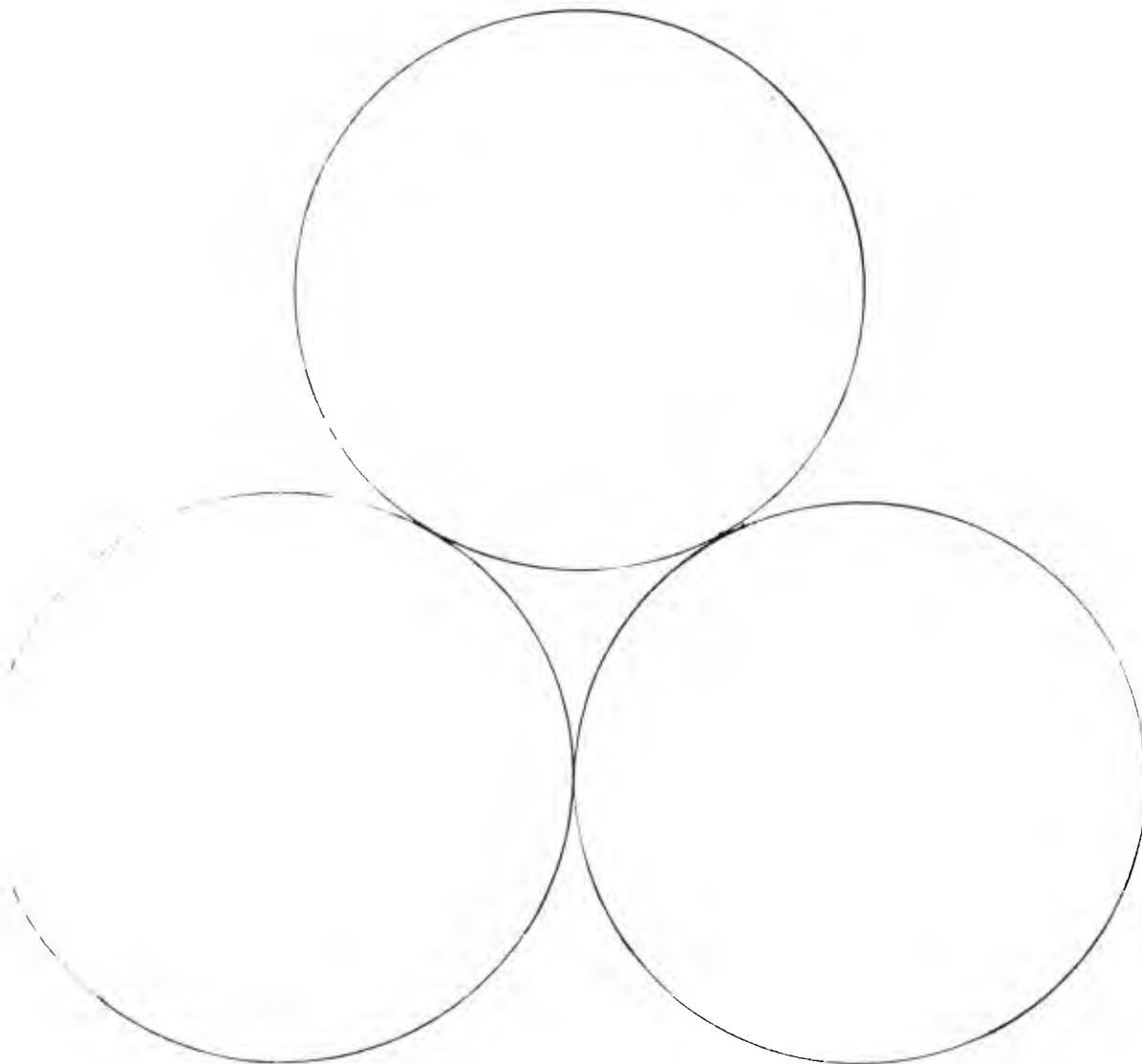
APPENDIX F

A sample of the collected data during the main research

Nikos Anastasopoulos - Philip Chrysikopoulos

S3T10NAPC08.12

Project starts page 6

R: "Tell me, ~ very few words, what you ve done so far, but say that I dont know much about Logo, and that I've no clue of what you ve done so far.

N: "No clue? Triangles!

P: laughs

N: "Whats so funny, didnt we do triangles?

R: "Ok, who '' have a first go at it. It isnt easy.

P: "Yes. We made triangles we find its sides, its angles... and in each angle (corner) we put it lets say a flag, a point... then we made, we made some triangles, i.e. not you... and... we made triangles and we counted the sides too... then we started to make... oh, yes then we startde to count... we counted all the... we found that all the turns are 360 degrees, that all the angles of a triangle are 180... yes, and... we made a quadrangle, to find out how much its turns were... and in the end we made em... with a very strong program, a very strong concept, we made a shape with three triangles one on top of the other lets call it, and then we made another one, that was an isosceles and in the middle were two lines, that... we put it a special command (FILL) and it filled up.

R: "Good. Shall we go? (N)

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N: "Yes, ... at I want to ask something, do you mean when you said ... th a new words like P said it, ... less.

R: "No that ... as ok, about like P said it. What you've done so far, what were the important things.

N: "OK. First, ... learned, mainly about triangles, we first learned how to find the sides and the angles of a triangle with two special commands, wasnt t DIRECTION and DISTANCE... ...es, em, then, em... (short pause) this we did for many lessons, and then we made a concept that can make a (continuous tense) ... even... an isosceles or equilateral triangle, straight. After we had done that many times and we found out how to make it straight. Em... and then we did some, we split some triangles in the middle...

P: "Before.

N: "Eh?

P: "Before.

N: "OK, yes before, sorry I didnt say it in the right order, we split some triangles in the middle and we kept finding the other triangles equal... and... and in the end we made... we learned a command that can make... (means FILL)

R: "Ok. Now tell me something. In all those times, what was, take Logo, OK? What was that, which gave us the ability to do all those things. I.e. which were the most important things we learnt.

N: "I think that the most important ones were how we can investigate a triangle... and... and to find smaller

... bigger ones and to make ... straight... and to split
... middle and these things.

R: For you P, what was the most important thing.

P: The most important was... like Nikos said...

R: "Lets not care about what Nikos said, for you, ok?

P: Yes, yes, yes. That... then em... we kept finding
every angles and sides, we kept finding the elements — the
triangle with the commands, that this helped us do all that.

R: "So what was the important thing?

P: "That all around... first how to find the angle, with
command and to be able to find the distance from one point
to the other. And also to be able to... if we dont know the
distance, to make... to make, lets say with a command and
the turtle does it. I.e. from there...

N: "I.e. to learn how to investigate the instruments...

R: "What, what did you say Philip?

P: "From there, I believe that we started... was the
most important (basic).

R: "I.e. which?

P: "That, to measure the angles. I.e. and the sides.

R: "Is that what you think was the most important?

P: "No, that wasnt the most important but that was what
helped us start... understand them more.

R: "Hm.

N: "can I say something? I think P means that, and I
think ..., that when I said before, em... that (with) which
we can investigate the triangles, I meant, what P said i.e.

now we can use the instruments of the turtle. Is an
important...

R: "the most important?

N: "Not the most important.

R: "What was that for you?

N: "It was important, very important. Perhaps the most
important, I dont know.

R: "What instruments.

N: "The rotameter, and the ruler.

R: "Did we use anything else?

N: "No.

R: "Just the ruler and the rotameter? Any word that
wasnt an instrument. N said something about flags.

N: "Ah, yes, you mean the POST? P said that, not me.

R: "OK. So, if we didnt have the flags, the ruler and
the rotameter, i.e. normal Logo, could the turle have done
all that?

N: "No. I.e. she could with great great great
difficulty.

P: "She wouldnt be able to.

N: "I say that she could.

P: "I.e. to make a triangle, of course she would be
able to... lets say if it was simple sides... i.e. with
great difficulty, if you wrote FD 1... 90 and id did you...
It fell exactly on the point (spot)... It wasnt shune of
course if it was on the spot, and then you had to do back,
forward... whatever you like...

N: "Ah, yes correct, + asnt... how can you know how many degrees exactly...

P: ".....e it could be 90 comma 9°...75, lets say...

N: "How can we know that.

P: "How can one find that.

R: "So?

N: "So, we could do anything withoput the instryments, so this is the most important.

P: "Yes, that we said.

R: "See what 3 words can do? because you can do whatever you like... we did triangles, you could do whatever with those. Whenevr she wants, the turtle can put a flag, and after that if she wants to measure the distance or the direction she does it, yes?

N: "Yes, but can I ask you something? This though, isnt in normal Logo... so what use is it to us, to learn geometry?

R: "What use was it to you?

N: "To learn a bit about triangles, or rather not a bit, a lot, special trigonologists (laughter)...

R: "Did you enjoy it, did you have a good time?

—

the headaches!

P: "I didnt have time to do my homework sometimes, but I did as much as I could.

N: "In the bus.

R: "Ok, what we re going to do no, ok?

N: Yes.

R: "The rest of the session, you'll do an investigation, in which you'll discuss what you're going to do in the bewginning, etc., it doesnt have an initial idea, but you have the ability and the choice, to work with simple Logo, or with ruler or with rotameter, or with POST, whatever you like, eh?

N: Whatever we like.

R: Yes.

N: "A car. A Porche.

P: "We can do, lets say something that has to do with triangles.

N: "Yes, but the car has got tyres.

p: "Yes, well, you can do it a REPEAT...

N: Yes, thats right. (pause)

P: "What do you want, a Porche?

N: "What are you laughing for? What do you want?

P: "I want something that is related to...

N: "Triangle.

P: "Yes.

N: "Lets make a pyramid. Yes, I did that once...

P: "Of Egypt.

N: "Yes!

p: "Go on then.

N: "Not Egypt. I mean not a pyramid, I mean taller than a pyramid. (they pick up paper) Like the obelisks, like

pyramid starts.

those Obelix makes, but more... more square. Not like that,

square, let me show you..

P: "Rectangle.

N: (paper, draws) "Like that, like that, like that, so

that you can see itrs an obelisk, like thatyou cant, but...

I mean someway that you can. But I dont know how to do it.

Not how to do it in Logo, on the paper so that I know mhow

to make it in Logo, i.e. thats very important too.

R: "Shure!,, pause)

N: Look, like that, like that... on paper, e ...

something like that, i.e.. somethhing like that.

P: 'There this.

N: "Shall we do it?

P: 'shall we do it?

R: "Yes, and that line is nice too (cutting triangle)

Hm?

P: "Its not because its difficult. (laughs)

N: "It isnt. No, e'll do the line.

P: "As you like.

N: "Ok.

F: "But we'll use the LASER. We havent done SETPREFIX,

have we? (they loaded their file (KITSOS)) (res) BAM...

there's LASER. Now... (types) Lets make it 40... and...

N: "And... and... can I say my idea? 50. No... 40, 40

and 40.

P: "55.

N: "No, 40 and 40.

P: "50. Oh, its got POSTS aswell. (problem for picture)

R: "Nevermind, say the POSTS are not there.

P: "I say lets do it more like that...

N: "Yes, so put it 30... 40 30.

P: "No, long and thin, i.e. we'll put it 50, the least.

N: "But 30. Now it will be good. Oh, sorry. Ah, yes its the angle, but what am I saying, I thought it was the turn. Put a 70 there. (res)

P: "Oh! (80).

N: "This is too much. Put 65.

P: "70. (res) Ok. Now we'll do that eh? the less. And then lie that...

N: "look... this and... sorry a pyramid has got 3 or 4 thingies?

P: "4 (laughs)

R: "Some of them have 3 sides some of them 4.

P: "4. Its a 4 sided.

N: "Wait. If we're going to make a big one and then a small one, isnt it the same if we make a big one first and here a small one? (means a big one covering pyr primeter, and the nested smaller one) (paid no notice)

R: "Right. So is this one the big one or the small one?

P: "The small one.

n: 'The small one sir.

P: "Now lets turn it...

N: "RT 90. Ah, no, we'll put a... ah, no.

P: "Lets do an LT 45, do you agree?

N: "No. 45 you know how that will go? Here its 90, here
is 45, do e want it to go here?

P: "We're here. And then we want it to go here. To go
45. to go half...

N: "OK, put whatever you want, put 45. Put 50.

P: "OK (types LT 45 FD 20)

N: "Its nice.

P: types with no hasitation, like it was a routine
thing, RT DIRECTION nad FD DISTANCE.

N: (res) Nice.

P: "For the time being... Its terrific, do you know
what it looks like? Like that thing is form behind. (3 dim)

N: "Shall I tell you what it looks like? I dont like
it, its like a jet. Its like a jet isnt it? So, look...

R: "Why dont nyou finish off the pyramid?

N: "Yes, but it isnt. Its doing the pyramids not right
(straight?) Look, one side is like that and the other one do
you know how much it is? Like that. (shows at line) Smaller.
I suggest we measure E till Mi, and to make as much as that
is, this. (left line)

P: "Yes I agree.

N: "Erase it then. We've nothing to loose, have we?
(they did it) (P types) 34 comma (said it) We'll put 34, ok?

P: "No, lets be acurate. Lets put RT somehow... not
very much, I.e. less than what we had put... so that its a
bit higher. It was 50, lets make it RT 30. (res) (they
laugh)

start mgin pyramid.

P: "So the other case was ... ber, eh?

N: "Yes.

P: raises screen accuracy effect, line looks bigger
than straight line, they measured with their ruler then they
erased the line, P types. Then N types, for somewhile, bug,
then does BK... then suddenly: "We dont have a POST there.

R: "What did you say?

P: "I said that there isnt a post over there. And we
cant go BK.

N: 'And what do we mind if there's a post? Do we mind?

P: 'OK, go BK.

N: "And how much will I go?

P: "Eh, that's what I'm telling you. → accurate

N: "Yes, but I want to see... can I see something? We
put LASER 40 50 50 70, LT 50...

P: "Here, say from here.

N: 'From there?

P: 'Yes.

N: 'LASER 50 70 ... (says something) 50?

F: "Hm... this one is 20.

N: 'Eh?

P: "This one is 20. 20. →

N: "What do we do now?

P: "Shall I tell you what we do?

N: "No, dont put CS please.

P: 'Shall I tell you? We can take it back to I etc. → strategy same as Nitsa's

R: "Wait wait, P had an idea, you're not listening (N). Tell us P.

P: "I said lets do RT DIRECTION towards I and lets go there... and then we know how much that side is...

N: "Right.

R: 'He's in great form today. (P types) (they make mistake, they go Oh, _tc, then they correct. (While P types PR DIRECTION :H)

N: "Why didnt you put a PR DISTANCE... you would gain in letters. they kept on typing) (they got to I)

P: "Oh! now how do we turn? —> non - *?*.*.*.*

N: 'How much will we turn?

P: "Oh, no...

R: "Nice eh? (laugh)

P: "Very easy, we ate you! (N laughs) How much is this here... how much does it look towards M, and we'll tell it 180, and then I remember how much... I've got brains... 30. Thats why. Do you agree? (P types, till FD 20) "Shall we better make....

N: 'Yes, thats what I think too, we'd better make a POST.

P: "Eh, ok. (P continues) "Mr if we did PU, would it write the POST? (continues) (he talked about pyramid image, they called it polygon pyramid) (N types , turtle head to M)

N: "Towards M its looking.

R: "Ok, I didnt see.

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N: (continues) (stops at "" they pause to think)
Right... (pause) We do a... 120. An LT 120, put. 120.

P: "Why, how do you know is 120.

N: "Wanna bet?

R: "Tell us why.

N: (pause) "I'm not very sure. I'm mixed up in something, can I do a ctrl T? Because if I see I'll know how much to do. With a bit of a complicated way but it doesnt matter. (He looks, he cant see as far back as he wants CS) Oh! Eh, then, wait, just a minute... I found... I'm finding it. (pause) Good. So... this is... 110... this is 70... this 100! (he means answer)

R: "Why?

N: "This here, from what I saw is 70, because we had put... yes. From here to there its 180, so this here is 110. 110, so from 180 which is all of that, we take away 110... look, this is 180, this is 110, therefore... and it comes out 70. 70 this over here. And here the 30 that we had done over there, 100.

R: "Ah, so you're doing it directly? Did you get what he said?

P: "Not very much.

R: "Go on then tell us again.

P: 'I got it. That the whole of the angle is 70, and all this is 180... we take away and it comes out 110...

R: "Whats 110.

P: "If we take away from 180...

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N: Yes, right, but what is 110?

P: "The turn. But then I didnt get the 10 where e found it.

N: "Look this we said is 70 and that is 180, so all of that is 110. Right. This here, ooop (look at notes) is 180. We take away that here...

P: 'Which.

N: "This, the 110 that was like that, wasnt that 110?

P: "Yes, it makes us 70.

N: "70. And from here... 70 wont we go here, straight? And 30 that we had put over here. 70 plus 30 is 100 isnt it?

P: "Yes yes yes. (N types)

N: "FD 20 how much did you put, 20. (P types) FD DISTANCE H. (they enjoyed res and looked at it for a while) "It could be a vedalia, couldnt it? It could be a motorway too, look. Here is the road...(then they wanted to fill central shape, they did it) (N called it an aeroplane, P called it a greek sweet, N called the turtle the pilot, and its got a mustache - inverted colors)

P: "Ok, then something else. (they fooled around for a while, making suggestions, portrait, computer, video, chip...)

N: "So, what are we going to make?

P: "A... an old ship.

N: "Yes. Draw it so that we can see how....

P: (draws on paper side A)

N: "What will it be, pirate, mearchant...

P: "I dont know.

N: What are you drawing it for then.

P: draws "Something like that, you know, with th sails
on the top...

N: "Yes, why not.

P: "Yes, but simple sails, not make a thingie that goes
phiiiou, and...

N: "Why not do that?

P: "Ok, then why dont you start.

N: "Right, look how the sails will be. (draws)

P: "Triangular.

N: "Eh?

P: "Triangular, the sails will be. And the mast, make
it with FILL.

R: "Wait, we'll think about the FILL in the end.

N: draws.

P: "Look, Niko, lerts make it like that, like that,
like that (repeats that several times, drawing)... and here
two sails as you did. Right, shall we start? First I think
lets take it a bit back so that it fits on the screen.

N: "Yes. (typing)

P: "Ok.

N: "No. Here, arent we going to take it? Will the ship
be like that?

P: "Ok (typing)

N: "Right, lets start. Right, look what we're going to
tell her...

P: "Listen. Can I tell you? To make thjings easier for us... do you mind very much if we put POST (to me), do you mind if its not pretty?

R: "No.

N: "Why should we put POST?

P: "Here, to... because afterwards when we'll get here, we dont have to search for....

N: "Sorry, can I say something? Will the ship be like that?

P: "Yes, the back wont be straight.

N: "Will it be like that? Or will it be anyway... i.e. the front more pointed (symmetry?)

P: "A bit more pointed the front and a bit more like that the back.

N: "This you mean presumably. I.e. and the back will be... like that, straighter.

P: "Just a bit. OK?

R: "Explain to N why you need POST.

P: "Because, the minute we get here... and we make a quadrangle, we can turn it comfortably (smoothly) here. And I think, to make these supposedly (means hor lines), to do these we should put POST)'s) here too.

N: "Yes but these that we learned about the turns, cant we make it without... POST?

P: "What we learned about?

N: "The turns...

P: "Ah, about the 360? Ok, as you like.

N: "What"

R: "Wait, how do you mean, Niko, because P said something concrete.

N: "OK, lets put POST.

P: "You agree?

N: "Yes I agree. typing)

P: "Lets put them order, yes, because we'll need several POST('s) (typing)

N: Mr C does the screen extend right to the end, because then we can make the back. (they decided to make it 150 in the end)

P: (200) "This is too much.

N: "Yes because we cant make the nose. PE...

P: typed He did 2. (typing)

N: "RT, how much, 130?

P: "Not 130, that will go there. Put it 50. No 40.

N: "No 50, 50. FD 20?

P: "No 20 is too little. No.

N: "Look, 20 is enough. Look, it wont be nice like that, its like crude.

P: "Yes, but afterwards it will have the sails and it wont show that much.

N: "Yes and if we do that it will go there and... ok, do as you like. (typing)

P: "This, we have to do now.

N: "RT...

P: "Yes, you're right. Lets put 30, and then we can change it.

N: "I think you should put RT 73, RT 70. To see if it will go straight, we can put an LT 90 and it wil be done.

P: "Is that how we'll make it, straight?

N: "I think straight.

R: "Its straight in a ship.

P: "Ah, OK. FD put it... not very big, 40.

N: "LT, arent we going to put it a bit...

P: "Yes, 10. Oh, it looks very crude.

N: "I know thats what I'm telling you.

P: "Put BK 5.

N: "I say 6. Look, here its good.

P: "A little bit more. Lets put...

N: "5 more, 4 more.

R: "More than what?

P: "To get there. More than the straight line.

R: "Ah, how much is that?

N: "Thats 40.

R: "Right.

P: "Its good, its good (they had forgot PD, they did it) Now do... now its... to come down there.

N: "Yes, and how will I do that? (pause)

P: "Lets tell it to show towards R and then turn 180.

N: "I think it goes 135 LT. I dont know...

R: "Ah, we want to make it head topleft.

N: "Yes, To look to go here and see?

put DI... right, because here it was like that again,
straight down it looked, and we went like that and we did
that and this here goes like that, the same thing that is,
but I have another them. That here, where it is... from
here to here is 90, and from here, till here this is half,
half a right angle, therefore its 45, plus 90 135, isnt it?
Lets see. We'll check of course.

R: "How will you do that?

N: "PR DIRECTION. (he types, it comes out wrong) Oh, it
did 7... I nade a mistake by 7 degrees.

R: "OK, whats wrong here.

N: "I'm wrong.

R: "Why?

P: "Because he asks... when Niko asks he asks how much
it did it from the left. While, normally, how it would do it
from the right... (not shure)...

N: "This doesnt matter, I think.

R: "Why?

N: "Because both from the left and from the right we
should have got 00.

R: "Ah. So, why is it wrong, there must be a reason.

N: "Can I say? Because here its more advanced from here
I think. From here to go there, this here is more advanced

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from that there, because its smaller the thingie. This side
is smaller than that side here I think... (types HT)

    R: "So, are we shure that the turtle is exactly on the
line?

    N: "Not at all.

    P: "Of course not. No.

    R: "So thats why.

    N: types RT 353.66 "ST and FD DISTANCE... (types)

    P: "O. Now that is, .

    N: "RT... RT 130? I think we should put 130.

    R: "Why?

    P: "Because, here we had turned 50... the turn and the
angle is 40...

    R: "Which angle is 40?

    P: "This one here.

    R: "How can we name it?

    P: "ABC.

    N: "ABC yes.

    R: "All thats 40?

    N: "No, the turn it makes...

    P: "The turn is 50.

    N: "No, right, yes what you say is right, but I'm
saying it for another reason. This here is on the same
straight line as this, and this we want to make it be on the
same straight line as this, like that, to be parallel I
think, something like that. Eh, this then, and this is
130...

P: "They were but we put them just like that.

N: "FD... not DISTANCE, and how much had we put do you remember? (trapezium)

P: "Yes, but, its angle is smaller.

N: "Oh, yes, correct. (typing) Now the sails. Will we have time to make them?

R: "Yes.

: "FD how much? Mmmm, we should have kept a PRINT. We dient think of it. (types) O'....

P: "Ok, lets go this way and make the sail like that.

N: "Yes thats what I say too. FS DISTANCE... (he types) I tóhought of that too, but I didnt say it because I was thinking that perhaps there's another way... (cont typing, P) FD 50?

P: "But we said it will make the first sail.

N: "20, 30 how much?

P: "30. (type) (res, mast) Neat!

N: "Neat.

P: "Lets make it double. (pause)

N: "Eh... shall we use the LASER?

P: "Eh, we cant it will do it on the top. Shall we make it like this one but smaller?

N: "I think that we can make an equilateral...

P: "No because it will make one sail on top of the other.

N: "No, look, one can go ther, make one like that like thazt and like that...

R: "So what are you going to do now?

n: "An equilateral. An equilateral and two isosceli. Equilateral. Wait I've got an idea. We can do... if we make an equilateral, which e will split and it will become two isosceli, wont it... if we split an equilateral... look if its equilateral, each angle will be 120. If its 120, this will be 60 and 60... why dont we put an RT 60, here an RT 120... eh... RT 120... eh? Right shall I do it? Can you go E. C? (they brought turtle back to top of mast)

P: "OK, Nikos idea was to make an equilateral.

N: "Yes.

R: "What was P idea.

P; "I dont agree with that, like this was here, one sail will be made like that... then, if this goes like that too, it will go like that (he doesnt like it st notes side B bottom right)

R: "OK, so what do you say?

P: "Lets go back, I dont know how much... Lets go about there (on mast, midway) and tell it... LASER...

N: "Can I say something? If the sails join like that I think the shape will be nicer becqause if the sails arent joined look how it will be. It will be like a chinese... Shall I start? RT 120...

P: "I say it should look to the other side, so that it looks there and does an equilateral.

n: "It will do the same now....

P: "Yes, but the LASER starts... starts from the right.
.., you've got to put it there

N: "Never mind... goes in

R: "Wait, you're not listening to P, what suggestion do
you have?

N: "I'll put... Oh! I could make it with REPEAT too.

P: "Yes, but we can make it with the LASER.

R: "Whatever you like.

N: "What do you say? (pause)

R: "Make one sail with REPEAT and another with LASER.

P: "I'll make the one with LASER though.

N: "OK.

P: "But if he does it now sir... and he should put
forward first and then turn... ah, nothing.

N: "Shall I do it? (res) Oh!

P: "Like a flag.

N: "I made a mistake, I put REPEAT, thats a mistake.

R: "Why?

N: ... (he types, erases)Yes and now BK...

P: "Put it 50.

N: (types) "And from here we'll start, dont move it
forward. From here we'll start. (P types to left) Nice!

R: "Wait, lets see what he's thinking... (types) He's
got something in his mind...

p: "I know, and I've forseen it. (types) Isnt it this?
But here I dont know the angle.

N: "So, of course, dont you want an equilateral, or an isosceles?

P: "OK, put it 60.

R: "Why equilateral?

P: "Isosceles.

R: "So, now we have to decide, equilateral, or isosceles, and why?

N: "Isosceles.

P: "equilateral.

R: "Why equilateral?

P: "To make life easier.

R: "Why will it be easier?

P: "Em.. I dont know.

N: "Shall I tell you? Becaus if we make an equilateral, we we have to go both here like that like that, and like that, like that. i.e. I think that if we make an equilateral, it will go like that I think...

R: "How will it go, if its equilateral?

N: "I dont know... If it ndoes an equilateral, do you know where it will start from? It will go either like that, or if he puts RT first it will go like that.

R: "Yes but thats the equilateral that you made. P makes it in another way.

N: "Ah, show me how he does it...

R: "There, dont you see?

N: "Ah, yes, 60?

P: "Yes, but if we put it isosceles...

R: "Thats the problem. If we put it isosceles now, or equilateral, what difference will it make, will it make a difference?

N: "No. (R I probed problem, but there was no time to do it right. N said something about the top angle, P says that if they make th sail 80 it will be bigger than the mast, because thats 50, said about 60... 65. They did it wrong, they erased, did it right, not very much came out. N said: if we make that 65, then the side should be 32.5... they were too tired to continue I thought, time was 5.55)

Philip types RT DIRECTION H straight away.
Also DISTANCE.

Philip types still   (LT 90 DIST M)
Nick sits by computer starts getting involved
Philip all the way -   ⌐⌐

Nick starts typing at POST G.
Takes over. typing.

Nick finger. at begining picks up pencil.
Rotates pencil around point M to demo angles
+ turns
        types LT 100.
    then Ph picks up typing.

Ph about POST :

means there
or then there
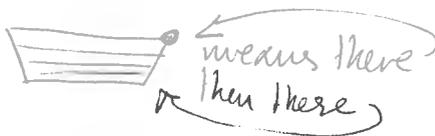
My thought on letter position while posting is that it would
be outside of closure of any figure. !! nice way of putting
it.

Philip types at B
+ types RT 50.   RT DIR. then steps
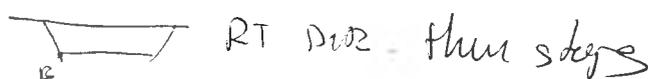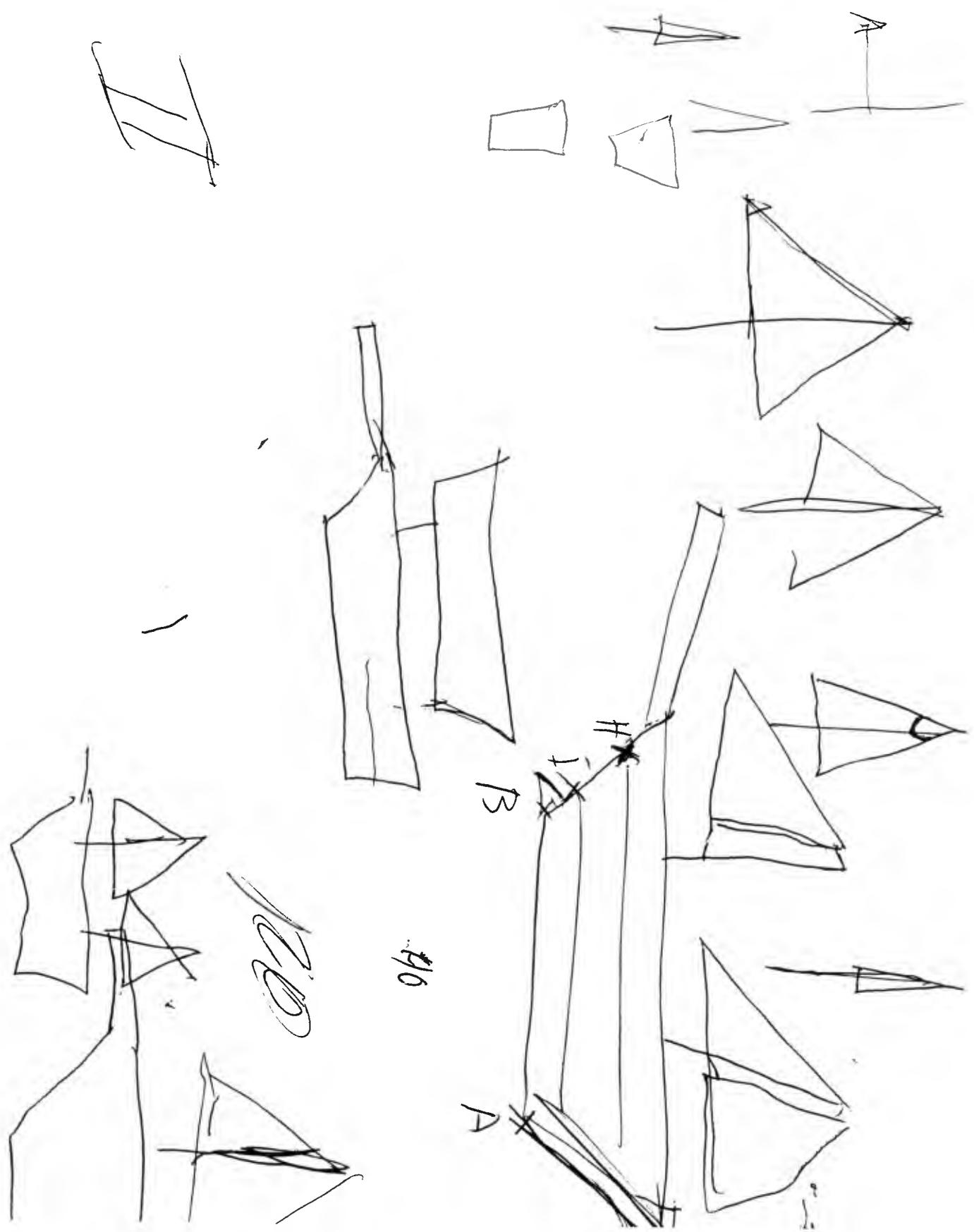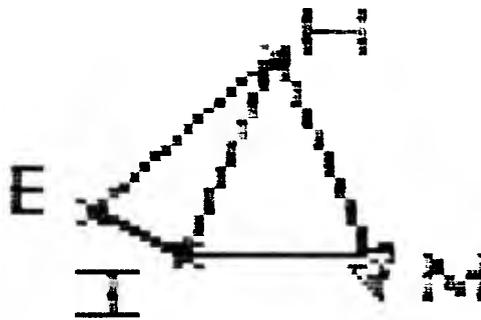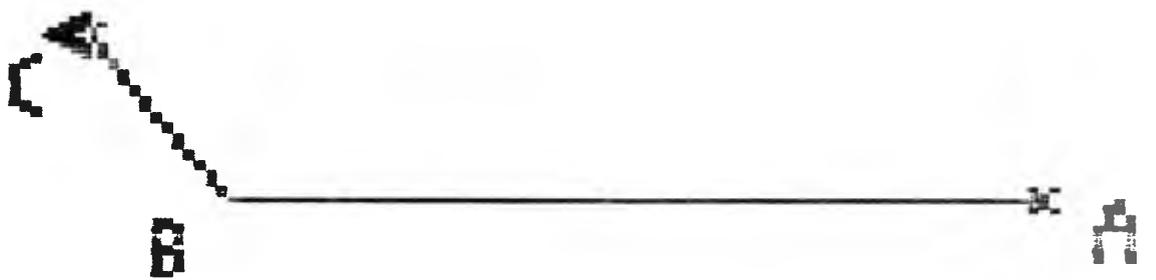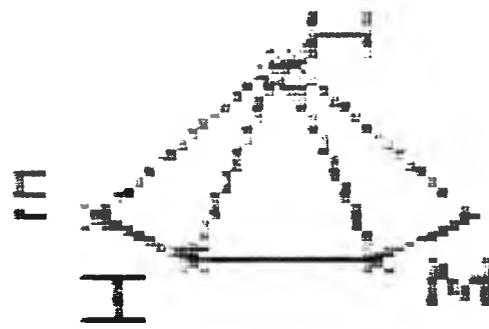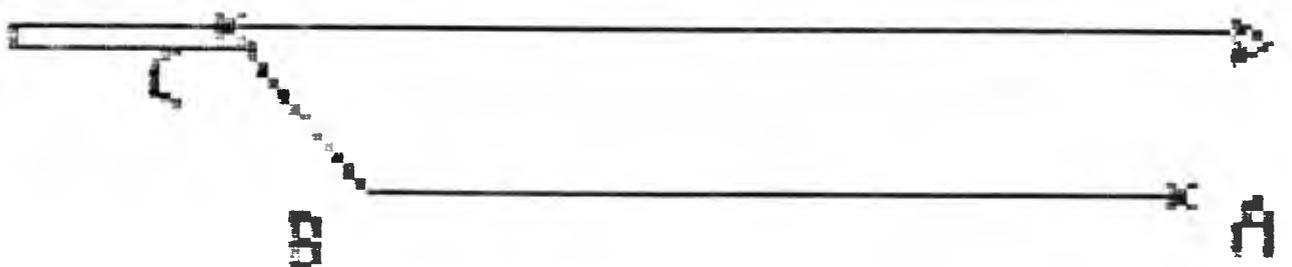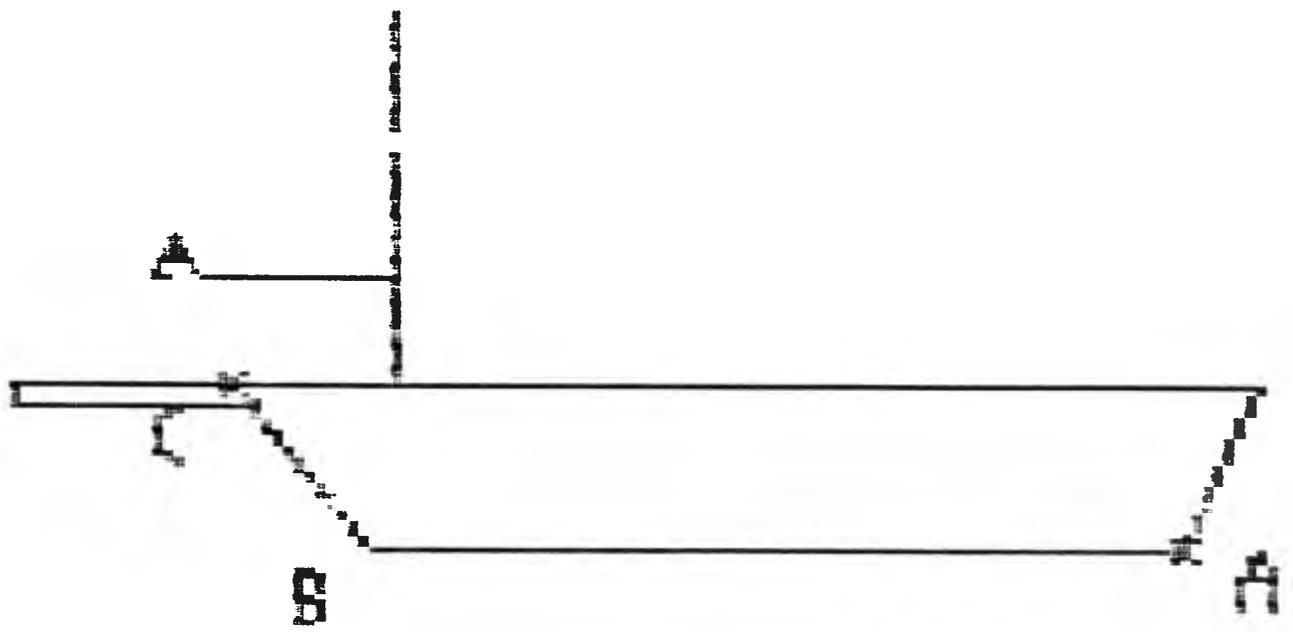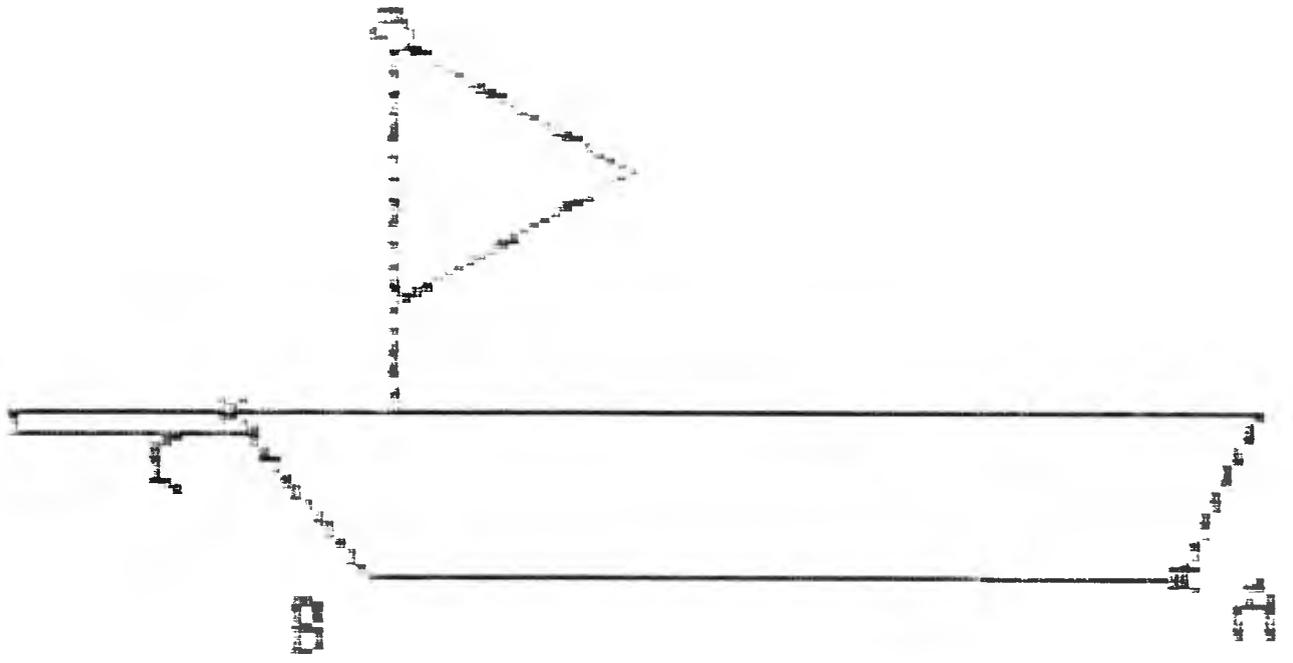
```
?SETPREFIX  "/RCPHD17
?LOAD "KITSOS
BAM DEFINED
LASER DEFINED
?LASER 40 50
?CS
?LASER 50 30
?CS
?LASER 50 80
?CS
?LASER 50 70
?LT 50
?FD 20
?RT DIRECTION :H
?FD DISTANCE :H
?CS
?LASER 50 70
?LT 90
?PR DISTANCE :M
34.202
?RT 30
?FD 34.202
?PE
?BK 14.202 =
?RT DIRECTION :H
?FD DISTANCE :H
?RT DIRECTION :I
?FD DISTANCE :I
?PD
?BK DISTANCE :H
?FD DISTANCE :H
?FD DISTANCE :I
?RT DIRECTION :M
?RT 180
?RT 30
?FD 20
?RT DIRECTION :H
?POST "E
?FD DISTANCE :H
?RT DIRECTION :M
?FD DISTANCE :M
?LT 100
?FD 20
?RT DIRECTION :H
?FD DISTANCE :H
?RT 200
?RT 20
?PU
?FD 10
?PD
?FILL
?RT 180
?FD 10
?CS
?PU
?BK 50
?RT 90
```


start-up pyramid

(FD 20 perc.

turn to I

turn

post E

go to M
H2M

go to H

i.s.

establ. turn.

episode

no Post
episode

gill

ship starts

505
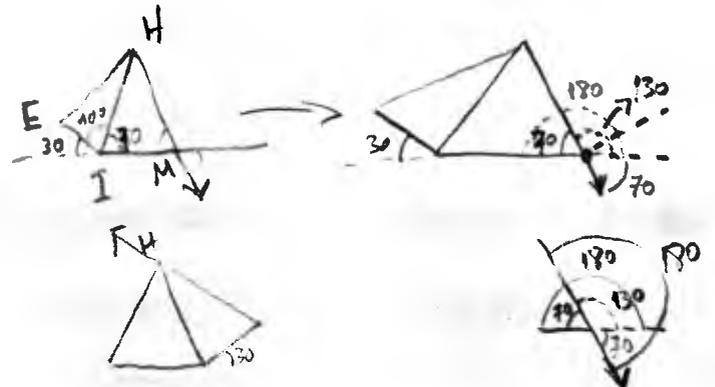
```
?FD 70
?LT 180
?PD
?POST "A
?CS
?POST "A
?CS                                    ≈picture?
?PU
?BK 50
?LT 90
?BBK 100
I DON'T KNOW HOW TO BBK
?BK 100
?PD
?POST "A  →  episode  guess P's reason for use        :-5pieces?
?FD 60
?FD 60
?FD 80
?PE
?BK 50
?POST "B
?PD
?RT 50
?FD 20
?FD 20
I DON'T KNOW HOW TO 20
?FD 20
?POST "C
?LT 30 ⎫
?RT 70 ⎬ relative turn
?LT 90 ⎭
?FD 40 ⎫
?LT 90 ⎪
?FD 10 ⎪      intrinsic vs
?PE    ⎬      ext - re-label -
?BK 6  ⎪      describe mistakes
?LT 90 ⎪      explain it
?FD 45 ⎪
?PD    ⎪
?BK 45 ⎪
?FD 45 ⎭
?
?
?LT 135  →  perceptual.
?PR DIRECTION :C
353.66
?RT 353HT
I DON'T KNOW HOW TO 353HT
?HT
?RT 353.66
?ST
?FD DISTANCE :C
?RT  130
?FD 190
?PE
?BK 190
?RT DIRECTION :R
?RT DIRECTION :B ⎤
```
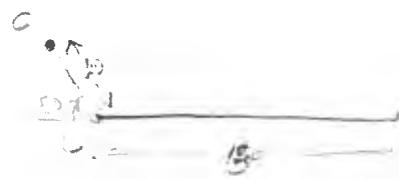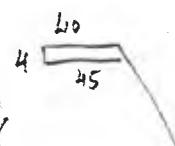
506

```
?RT 120
?RT 120
?PE
?PE 12?
?RT DISTANCE ?
?FD DISTANCE :H
?RT 12?
?RT DIRECTION :B
?FD 150
?RT 50
?FD DISTANCE :I
?RT 120
?PE ??
?LT 90
?FD 7?
?FT 9?
?FD ?
?RT 90
?LT 90
?PE
?BK ?
?PD
?LT 90
?RT 120
?LT 120
?REPEAT 3 [RT 120 FD 50]
?PE
?REPEAT 3 [RT 120 FD 50]
?
?PD
?BK 50
?HT
?ST
?LT 90
?FD 40
?RT 90
?LASER 65 60
?RT DIRECTION :H
?PE FD DISTANCE :H
?RT DIRECTION :M
?FD DISTANCE :M
?RT DIRECTION :I
?FD DISTANCE :I
?RT 90
?PD
?LASER 80 60
?LT 90
?BK 40
?RT 90
?FD DISTANCE :H
?PR DISTANCE :H
0.000017
?NODRIBBLE
```

APPENDIX G

CONSTRUCTING THE CIRCLE MICROWORLD'S TOOLS AND USING THEM IN PERSONAL PROJECTS

CONTENTS OF APPENDIX G

G.1a) (8.2.1.a)) Circle procedure 1

Before being given the task, the children were required to construct a procedure for a circle drawn on paper and think about how they would alter the procedure to change the circle's size (appendix E.2). Not surprisingly, the first procedures they subsequently wrote and tried out were fixed - they did not have a variable (fig 8.2.1a). They decided, however, to make a circle procedure with a length variable when they were shown the task figure, and they called their procedure CIR4 (fig. 8.2.1b). They constructed the task figure by writing a superprocedure (CIR5, fig. 8.2.2b) consisting of a sequence of fixed CIR4 procedures with increasing inputs. **The main relevant geometrical ideas raised during the process of constructing CIR4 involved the factors changing the size of the circle and the issue of polygon approximation.**

```
TO CIR
REPEAT 120 [FD 1 RT 3]
END
```

```
TO CIR2
REPEAT 90 [FD 1 RT 4]
END
```

```
TO CIR4 :S
REPEAT 36 [FD :S RT 10]
END
```

```
TO CIR3
REPEAT 36 [FD 10 RT 4]
END
```

b

```
TO CIR3
REPEAT 36 [FD 10 RT 10]
```

a

Figure 8.2.1 The children's first circle procedures

Perceiving the quantity of the turtle's changes of position as a determinant of the size of the circle was not a triviality for the children; they initially suggested

the input to the REPEAT command as a factor for changing the size, before deciding on the amount "the turtle goes forward between each turn" and explicitly stating the positive relationship between "forward move" and size.

```
TO CIR4 :S
REPEAT 36 [FD :S RT 10}
END
```

a

```
TO CIR5
CIR4  1
CIR4  3
CIR4  5
CIR4  7
CIR4  9
END
```
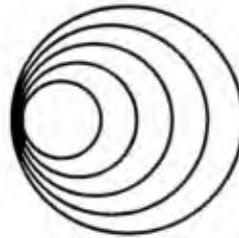
b

```
TO CIR9 :R
RT 5
REPEAT 36 [FD :R * 2 * 3.14 / 36 RT 10]
LT 5
END
```

c

```
CIR9  50
PU
RT 90
FD 5
LT 90
PD
CIR9  45
```
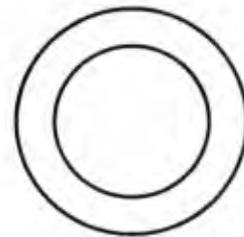
d

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```
TO CIR17 :S        TO CIR18 :S        FD  30          TO CIR21 :S
LT  90             RT  90             CIR19  35        FD  30
PU                 PU                 CIR20  35        CIR19 :S
FD  :S             FD  :S             CIR19  18        RT  45
RT  90             LT  90             CIR20  18        FD  :S / 2
PD                 PD                 CIR19  45        BK  :S / 2
END                END                CIR20  45        LT  90
                                      CIR19  25        FD  :S / 2
                                      CIR20  25        BK  :S / 2
                                      CIR19  19        RT  45
                                                       END

TO CIR19 :S        TO CIR20 :S
CIR17 :S           CIR17 :S
CIR9 :S            PE
CIR18 :S           CIR9 :S
END                CIR18 :S
                   PD
                   END
```

e   f


```
TO TC :S                           TO TIME :S
REPEAT 360 [MOVE :S RT 1]          REPEAT 12 [MOVE :S RT 30]
END                                END


                                   TO H :S
TO MOVE :S                         RT  15
PU                                 FD (:S / 8) * 5
FD :S                              BK (:S / 8) * 5
PD                                 LT  15
FD :S / :S                         BK (:S / 8) * 7
PU                                 FD (:S / 8) * 7
BK :S + :S / :S                    END
PD
END
```
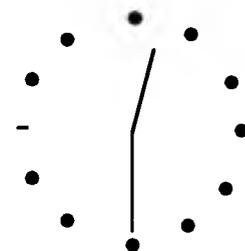
g   h

Figure 8.2.2 The children's procedures for the Circle microworld and their
solutions of the respective tasks

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Although after the researcher's question for other means of changing the size, the children were quite coherent in verbalising the inverse relationship between turtle turn and size of circle, they explicitly juxtaposed the two "rules" only after the researcher's prompting. Furthermore, it seemed that although they verbalised and used the relationship between the input to the REPEAT command and the turtle turn in specific instances, they found it hard to think of the relationship's generality; an indication of this is that when asked which factor they would like to use as input - the length or the turn - they preferred *"moving the steps forward more"* because *"...there isn't a lot of confusion...* (otherwise) *we have to do all the operations* (means each time) *to find... how much we'll turn."* **This comment seems to imply that the children thought they would have to make the operation to find the relationship between the two inputs each time they changed the value of the turn input.**

After they completed the task, **the researcher prompted a conversation on the polygonal nature of the figures constructed by circle procedures,** by drawing the children's attention to the relationship of the number of sides of the polygon and the input to the REPEAT command. In order to explain what happens when by increasing an input of 4 a square shape "becomes" a circle, the children used an intrinsic notion of turtle turn in their implicit attempt to convey the meaning of curvature;

A: "Because we round up the angles..."
R: "Hm. What does that mean."
A: "...we reduce the degrees in the turn of the circle..."
V: "Yes, that is, it turns less and this it does more times... it becomes bigger."

The researcher's questioning of the figures drawn by increasing inputs to REPEAT seemed to lead the children to **conclude that the figure drawn by the CIR4 procedure was "a 36 - agon" which "looked like a circle".**

The construction of the second procedure of the Circle microworld involved a relatively lengthy investigation, described by the relevant worksheets (appendix E.2). Not surprisingly, **the children initially made perceptual attempts to construct the task figure using the CIR4 procedure.** In their attempt to specify the problem, Valentini made an insightful remark, i.e. that they needed some means of identifying the "size" of the circle. It took some time, however, before the children decided on a new meaning for the notion of "circle size", i.e. that of the length of its diameter. They subsequently re-formulated the problem as that of finding a relationship between the "side" and the "radius", Valentini's insightful perception of the meaning of "relationship" being "how much one fits into the other". They consequently seemed to use the turtle's instruments for measuring the perimeter and the diameter of a circle in a meaningful way, enjoying the process of recording the outcomes of their measurements (see appendix E.2). The researcher intervened to help the children with the manipulation of the parameters of the relationship (the children had not had any experience with equations), so that it would "fit" the input to FD in their new procedure. He also intervened to generate a discussion leading to the children's adding of the turns before and after the construction of the circle in order to "correct" its orientation (see also sections 8.1.3a), 8.2.3 and fig. 8.2.2c).

Two issues of direct relevance to the study's objectives arose during the children's first executions of their new procedure (which they named CIR9, fig. 8.2.2c) and during their solving of the task.

During the investigation, the children often showed explicit awareness of the objective of constructing a procedure which would take the radius as an input. However, **in their first execution of the new (CIR9) procedure, they gave an input of 3, i.e. the kind of input which they would have given to the CIR4 procedure.** Their genuine surprise with the outcome was followed by more trials, slowly increasing the input, but not

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understanding what was going on.

V: (types CIR9 50)

A: "Now it made it normal... (means the size)"

V: "Now it made it like, let's say we would make the 5, the 6 (means the input if they used CIR4)"

The incident could be attributed to a disparity of intrinsic and non - intrinsic notions (in the context of constructing a circle) in the children's minds; although they had spent two and a half hours constructing a circle procedure with the radius as input, the notion they used for the input when executing the procedure was the intrinsic quantity of turtle steps. It could well be that the context within which the children were used to constructing circles with the turtle involved intrinsic constructions. However, incorporating the use of the radius in that context was far from automatic. Furthermore, such an interpretation would corroborate the difficulties the children found in linking intrinsic and non - intrinsic notions in other contexts (e.g. angle and turn, triangle) described in the previous two chapters.

The children made the connection of the situation at hand (the circle drawn by executing CIR9 50) with the problem they had solved, when the researcher focused their attention to the input:

R: "So, what on earth is that 50 folks?"
A: "The radius! Of course!"
V: "Eh, yes."

It is interesting, however, that their next action was to turn the turtle to the right and in PENUP mode move it forward 50 steps to "confirm" their answer, by **using their perceptual cues to check whether the turtle was in the centre of the circle. In their consequent solving of the task, however, their use of the length of the radius of the two required circles was quite coherent;** after typing in CIR9 50 PU RT 90 FD 5 LT 90

PD, fig. 8.2.2d) Valentini said;

V: "How much will we do it?" (she typed CIR9 45)

R: "Why 45?"

V: "Because this is 45." (means segment from centre to turtle's position)

R: "So?"

V: "Eh... the radius... the 45 is the radius, since before it was 50, minus 5 is 45."

G.1c) (8.2.1c)) Circle procedure 3

In the third task, the children soon perceived a need to write procedures which would make the process of drawing and erasing circles with a fixed centre less laborious. It was not surprising, however, that **their initial attempts involved writing procedures which were functional to the task and not to the intended geometrical hidden agenda;** since the problem arose at the moment when a circle had been drawn, the function of their first "task - facilitating" procedure (CIR16, fig. 8.2.3) was to erase the circle drawn by the CIR9 procedure and move the turtle to the fixed centre.

```
TO  CIR16  :SIDE
PE
CIR9  :SIDE
RT  90
PU
FD  :SIDE
PD
LT  90
END
```

Figure 8.2.3 The children's first
"task - facilitating" procedure

At this point, the researcher intervened and suggested they make a circle whose centre would be at the turtle's present position. Valentini then decided to make the following procedure:

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V: "...to make a program that tells the turtle to turn left... to go to the place where we want to start the circle."

She then wrote the CIR17 procedure in the editor (fig. 8.2.2e), and typed CIR17 25 CIR9 25. She then went into the editor and wrote the CIR18 procedure (fig. 8.2.2e), subsequently typing CIR18 25. (In Apple Logo II, entering the editor leaves the graphics screen unaffected). The children subsequently used the three procedures typing CIR17 25 PE CIR9 25 CIR18 25 to erase the previous circle and started to make one with a different radius (typing CIR17 18 CIR9 18). At this point, **they seemed to perceive the modularity embedded in the use of their three procedures,** deciding to make two superprocedures (CIR19 and CIR20, fig. 8.2.2e) which for them were equally functional for the task; one - CIR19 - caused the turtle to make a circle and the other - CIR20 - to erase a circle. In both procedures, the state of transparency was the circle's centre. The children solved the task, appreciating the **usefulness of their** two superprocedures in making and erasing circles until **they decided** on a size which they liked (fig. 8.2.2f).

G.1d) (8.2.1d)) Circle procedure 4

On administering the fourth task, the children initially held a relatively long discussion on solving the problem. **The plans they verbalised to each other involved using their familiar intrinsic method to take the turtle along the curve** in PENUP mode and interrupt to make the dots for the clock. They implicitly referred to using the relationship between the input to FD and the radius in order to be able to then take the turtle to the centre to make the clock hands. Inspite of the researcher's suggestion to try and think of another way to make a circle with which they could make the clock very easily, the children took a long time proposing ideas which involved the turtle tracing the curvature.

The interest in their subsequent insight into using the notion of

equal distances from the centre is that their idea did not involve the circle at all but rather, a recent incident which had taken place during their previous project involving the use of the CIR19 procedure, i.e. the "Snowman" (fig. 8.2.13). In making the snowman's broom (fig. 8.2.13a), the children used a sequence of moving the turtle forward and back a fixed length and turning a fixed angle. In the clock task, they specifically stated that they would use the same idea to make the dots for the clock:

A: "...to make what we did in the snowman..."

R: "What did you do in the snowman?"

V: "The broom."

A: "The broom... it had a centre."

V: "Yes, only it drew that time, now it won't draw..."

A: "We'll put a REPEAT that is..."

V: "Yes it will lift the pencil, it will go to the top, it will do the dot, it will go down, then again from the middle it will start... do 12 times lift your pencil go forward, do the dot go back and turn as much as I tell you..."

A: "30."

This incident could be interpreted as an indication of the disparity in the children's minds, of notions involved in the intrinsic and the euclidean construction of a circle. Their previous experience with circles in turtle geometry predominantly involved intrinsic constructions of turtle move and turn sequences along a polygon approximation of the curvature. Although their recent experience during the study involved non - intrinsic notions such as reference to the radius and the centre, it did not involve the use of the euclidean definition of the circle for its construction. It is not surprising, therefore, that their insight into a euclidean construction did not seem to come from their microview of circles, but from a different experience: that of making five "rays" to complete their broom. The notions of "centre" and "radius" were implicitly used both for the broom and for the clock. However, the children initially did not seem to perceive those notions as part of

a circle.

G.2a) (8.2.2a)) Circle procedure 1

For their project with the use of the CIR4 circle procedure, the children used CIR5 - the procedure which made the task figure (fig. 8.2.2b) - as the main module for making their shapes. In constructing CIR6 (fig. 8.2.4a), **Valentini used an intrinsic notion related to the CIR5 shape as a whole,** in order to explain the turtle's turning between the first and second CIR5 shape:

V: "Yes, there's no problem. We'll do one, (CIR5 shape) *then we'll do 180 left or right, then the same..."*
A: "For what reason?"
V: "Say it stops here. (after the first CIR5 shape). *And it has done it like that. It starts like that, it finishes like that* (state transparency). *Then, it goes, it turns downwards and does the same thing. Exactly."*

Implicitly, Valentini used the notion of the turtle's state - transparency in the CIR5 shape and the notion of the right hand side of the turtle after a 180 degree rotation. **However, the overall construction of the CIR6 shape implies the use of a non - intrinsic notion,** i.e. that of perpendicularity in the plane; the first two CIR5 shapes were in a horizontal formation and the next two in a vertical formation. The children subsequently wrote a superprocedure (CIR7, fig.8.2.4a) consisting of two executions of CIR6 with a 45 degree right turn interface between them. **The researcher attempted to probe the children's readiness to perceive a global intrinsic construction** of the CIR6 figure by suggesting that they could do what they usually did in their classroom and Logo club activities: "tidy - up" their program, i.e. the term used in the sense of reflecting on the modularity of a procedure. The children consequently changed their method for constructing the CIR6 and CIR7 shapes (fig. 8.2.4b). **In their new method,**

the only underlying geometrical notion linking the CIR5 shapes was the turtle rotation.
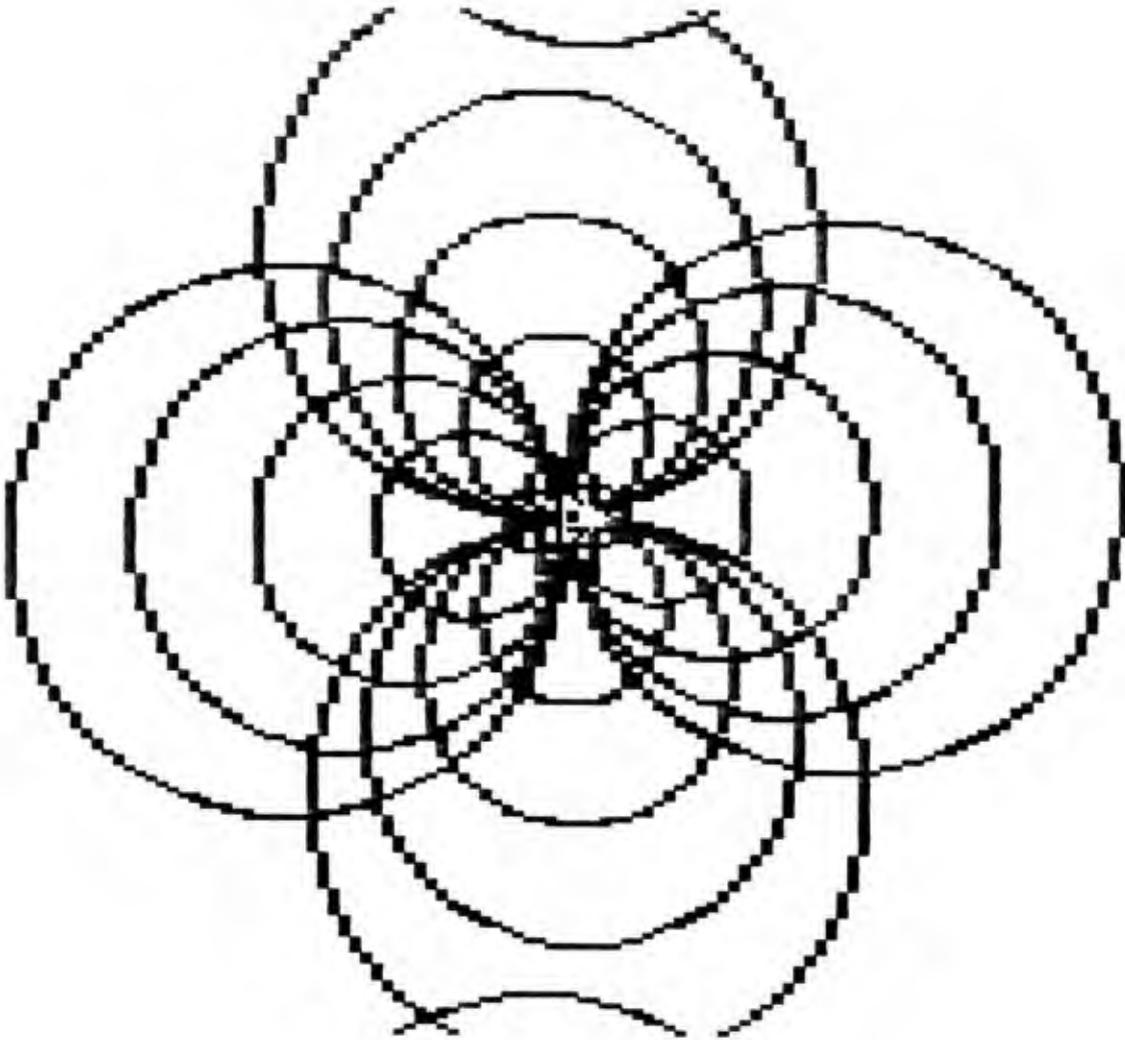

```
TO CIR6
CIR5
RT 180
CIR5
LT 90
CIR5
RT 180
CIR5
END
```

```
TO CIR7
CIR6
RT 45
CIR6
END
```

```
TO CIR6
REPEAT 4 [CIR5 RT 90 ]
END
```

```
TO CIR7
REPEAT 2 [CIR6 RT 45]
END
```

a

b

<u>Figure 8.2.4 The children's projects</u>

<u>involving the use of circle procedure 1</u>

The children's programming indicates the beginning of an increasing appreciation of the power of subprocedure and modularity; as mentioned below, they predominantly used a bottom - up approach throughout their projects, starting from the circle procedure and building several levels of superprocedures involving circle combinations. **However, it does not necessarily follow that there was an increase in the degree to which they used geometrical ideas related to the circle,** as will be discussed below.

<u>G.2b) (8.2.2b)) Circle procedure 2</u>

The first of the next set of projects involving the use of the CIR9 procedure was based on a real - life object, a "koulouri", i.e. a circular bun covered with sesame seeds. Alexandros took the initiative (it was his idea) and used the CIR9 procedure to make two concentric circles, making correct operations for the inputs to CIR9 and the distance between the circles. He then started to take the turtle to different points between the circles with the use of perceptual cues, and to make small circles (the sesame seeds) with the use of the previous circle procedure, i.e. CIR4 (fig. 8.2.5). **This could be an indication that at that point, he had not yet discriminated the difference between using CIR9 and CIR4 in geometrical terms, but used CIR9 for the large circles only because the procedure was functional for that specific figure.**
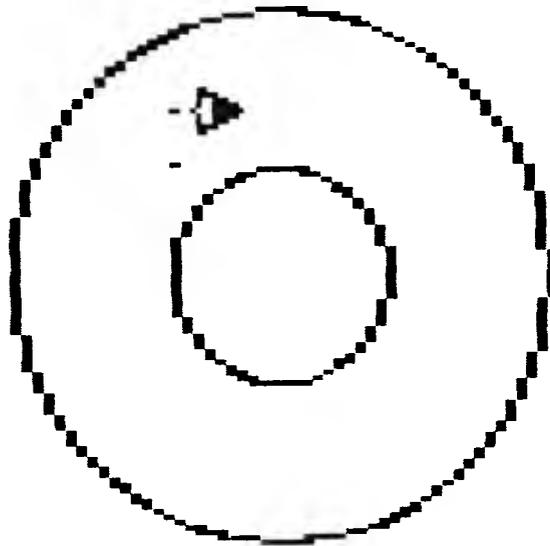
```
CIR9  50          CIR4  0.0004
PU                PU
ST                FD  10
RT  90            PD
FD  30            CIR4  0.0004
LT  90            RT  90
PD                PU
CIR9  20          FD  5
PU                PD
FD  1             CIR4  0.0004
FD  10            END
FD  10
PD
```

Figure 8.2.5 The children's first project

involving the use of circle procedure 2

The second project involving CIR9 was similar to the first project with CIR4, in the sense that the children **used the circle procedure as the bottom level of a sequence of superprocedures.** A difference from a programming point of view is that they made a first - level superprocedure with a variable input - CIR10 :S (fig. 8.2.6) - consisting of a CIR9 procedure and a fixed interface. In their first attempts to make the "target" shape, Valentini tried to work out the proportional decrease in the length of each radius in relation to the larger length. Her laborious attempts, however, did not involve trying the procedure out - she tried to figure the relationship out in her mind, while they were in the editor. This could be attributed to her experience with normal classroom practices, focusing more on deductive methods rather than building on personal experience (see chapter 5). The researcher's intervention, suggesting that they try out each circle first was followed by their trying out circles of a fixed decreasing radius in direct - drive, and then writing the CIR10 and CIR11 procedures (fig. 8.2.6).
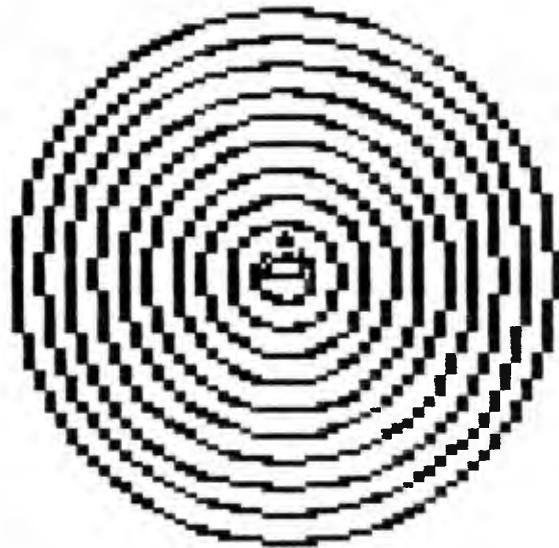
```
                          TO  CIR11
                          CIR10  50
TO CIR10  :S              CIR10  45
CIR9  :S                  CIR10  40
RT  90                    CIR10  35
PU                        CIR10  30
FD  5                     CIR10  25
LT  90                    CIR10  20
PD                        CIR10  15
END                       CIR10  10
                          CIR10  5
                          END
```

Figure 8.2.6 The children's second project

involving the use of circle procedure 2

Figuring out length relationships between radii, however, seemed to be the only instance of the children's use of a non - intrinsic notion involving the circle. With respect to the radius, the children seemed to focus on its length as a line segment, rather than perceiving the radius as "belonging" to a circle. Furthermore, they did not seem to refer to the centre of a circle, either in connection to the radius or not. In their subsequent programs, they showed more interest in the idea and the screen effects of superprocedures, rather than in specific geometrical ideas concerning the circle (fig. 8.2.7). Implicitly, however, they used the relationship between the input to the REPEAT command and the turtle turn, as for example in their CIR12 procedure (fig. 8.2.7).
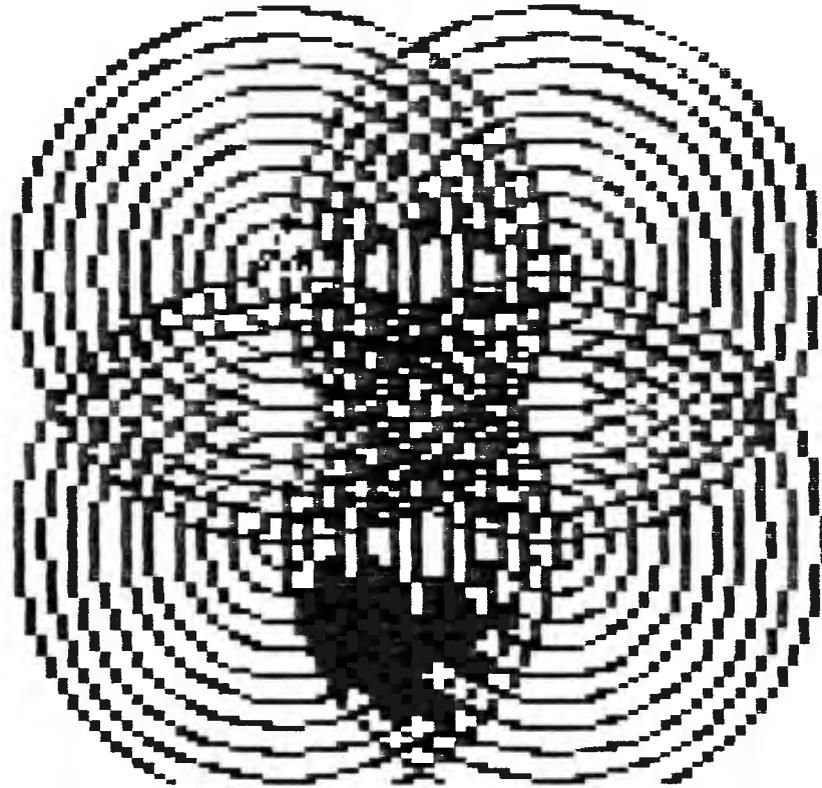
TO CIR12
REPEAT 4 [CIR11 RT 90]
END

TO CIR14
REPEAT 8 [CIR12]
END

TO CIR13
REPEAT 2 [CIR12 RT 45]
END

TO CIR15
REPEAT 2 [CIR12 LT 45]
END

Figure 8.2.7 The children's third project

involving the use of circle procedure 2

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Their first project with the CIR19 procedure also involved a target figure repeated in a circular formation. This time, however, the children's plan was for the outer circles of the target shapes to connect tangentially. **Their previous lack of perceiving a radius and a circle's centre as connected to a circle could be the reason for the first "bug" in their CIR23 procedure,** consisting of the target figure and the interface for the next target (fig. 8.2.8). After completing the first target, the children moved the turtle to the edge of the outer circle and typed the procedure for another target. The figures' overlapping on the screen focused the children's attention to the centre and the radius of the outer circle of the target.

V: "Right look (to Alexandros). *We'll make this circle which is... here in its middle, here the turtle stops here and the radius is 7."* (about target figure)
A: "Why is it 7?... yes yes."
V: "Because the largest circle is 7. So, we'll tell it to go forward 7 but also to turn and another 7 to go to the middle of the circle again... so that afterwards with the program we've made it will go there and it will make other circles, and then the same again..."

```
TO CIR22
CIR19  7
CIR19  3.5
CIR19  1.5
END
```



```
TO CIR23
CIR22
PU
RT  10
FD  7
PD
END
```
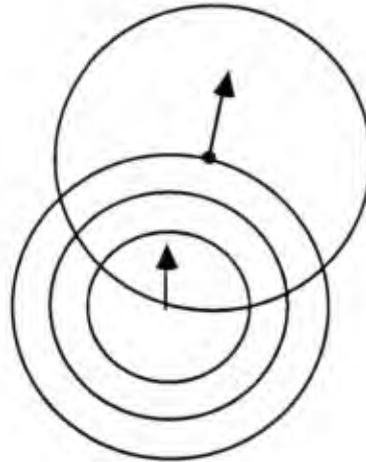
a

```
TO CIR23
CIR22
PU
RT  10
FD  7
RT  10
FD  7
PD
LT  10
END
```
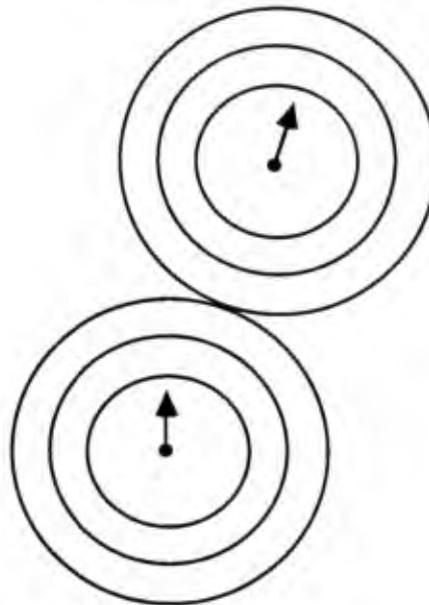
b

Figure 8.2.8 The children's first project
involving the use of circle procedure 3

It is interesting how Valentini seemed to separate the radius of the outer circle of the first target from that of the second which had not yet been drawn on the screen. **It could be suggested that this was the first instance where there was a use of the notions of radius and centre as tightly connected to a circle. However, the children implicitly used the intrinsic curvature construction of a circle with respect to their**

last procedure - **CIR24 (fig. 8.2.9)** - where the input to the REPEAT command was in accordance with the turtle's turn in the interface between targets, so that a circle of target shapes would be formed on the screen.
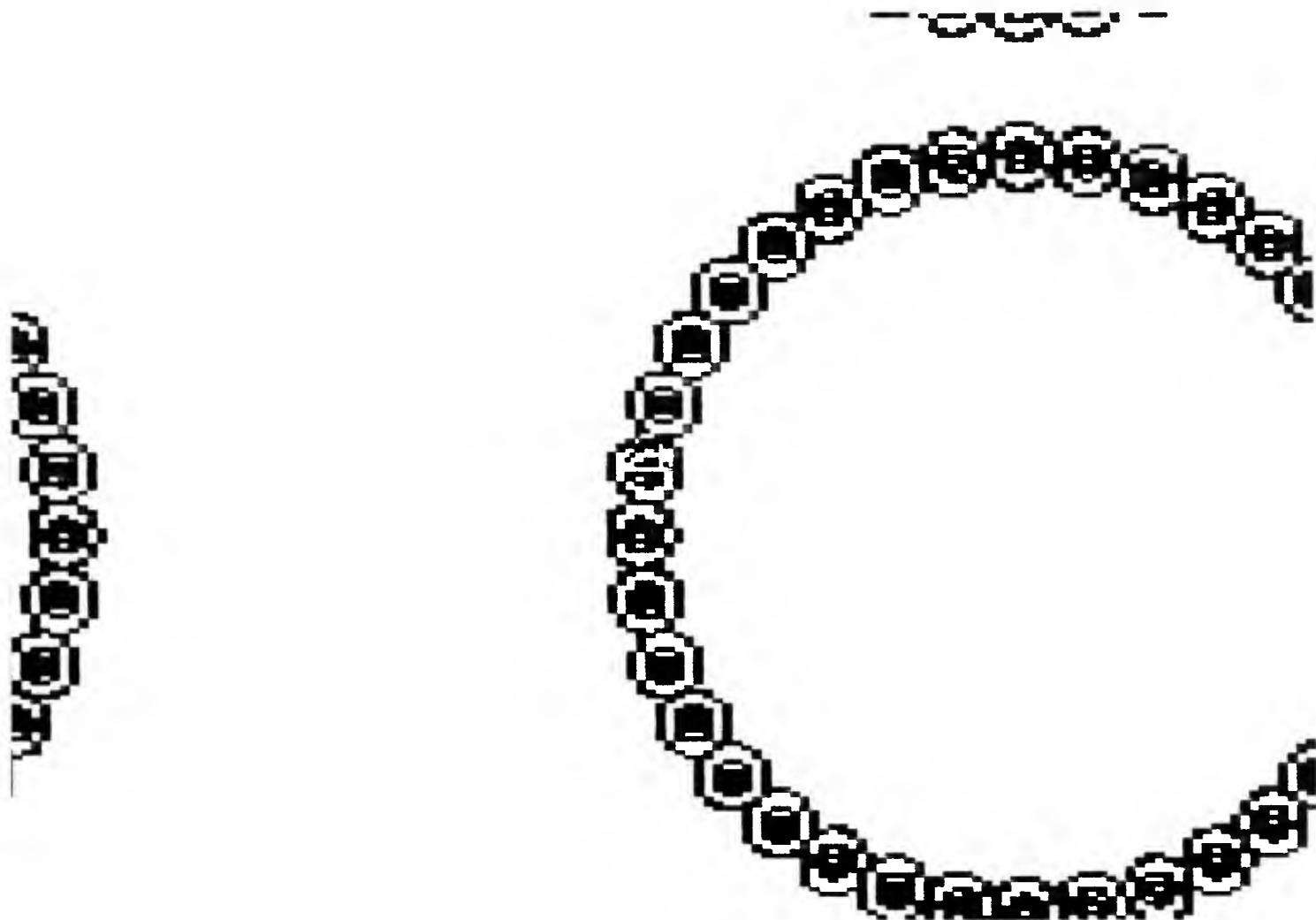
TO CIR24
REPEAT 36 [CIR23]
END

Figure 8.2.9 The final figure resulting from
the children's first project with circle procedure 3

The children carried out three projects with the CIR19 procedure. In their second project they decided to make an apple tree, possibly inspired by the previous task figure. Their first procedure, T (fig. 8.2.10), made a rectangular trunk and a large circle tangentially connected to the middle of the top side of

the rectangle. During the typing of the interface between rectangle and circle, the children made it verbally clear that this was what they intended, i.e. for the circle to "touch" on the top centre of the tree trunk. Moving the turtle forward 40 steps above the rectangle, therefore, was part of the plan to then make a circle with a radius of 40. **In this case, the children seemed to implicitly use the notion of a circle's radius and centre in order to construct the circle.**

From then on, however, the children changed turtle positions in direct drive and in a perceptual way, using CIR19 and a small numerical input, made circles for the apples on the apple tree (fig. 8.2.10). **In this case, employing the CIR19 procedure did not seem to involve using geometrical ideas for constructing the circles,** resembling the way in which Alexandros had used the CIR4 procedure to make circles for the sesame seeds in his "koulouri" project.
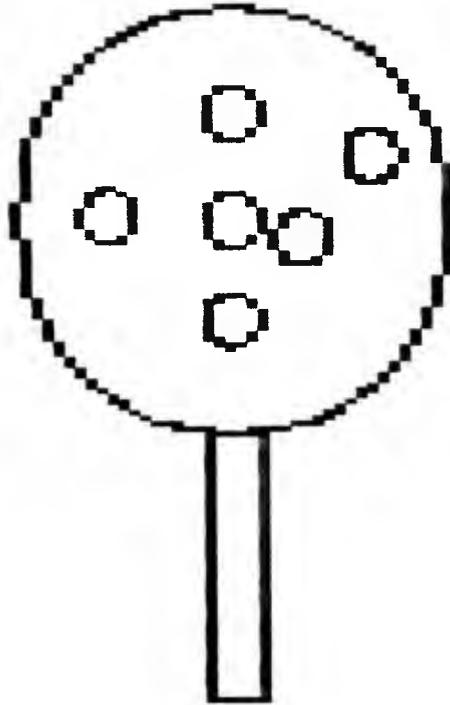
```
TO T
PU
BK 50
PD
REPEAT 2 [FD 50 RT 90 FD 10 RT 90]
FD 50
RT 90
FD 5
LT 90
PU
FD 40
PD
END
```

```
TO TC            PU
T                FD 30
CIR19 40         CIR19 5
CIR19 5          LT 90
PU               PU
FD 20            FD 20
CIR19 5          CIR19 5
LT 130           PU
PU               FD 20
FD 30            CIR19 5
CIR19 5          END
LT 100
```

<u>8.2.10 The children's second project</u>

<u>involving the use of circle procedure 3</u>

The most interesting project, however, was the next one, where the children decided to make a snowman. **The interest lies in the children's construction of the two outer circles, i.e. the torso and the face of the snowman, and in each case in their use of the radius to drive the turtle knowledgeably inside the outer circles to complete the project.** For instance, the children first made the torso and the snowman's buttons. They typed in CIR19 40 and then took the turtle to the bottom edge of the circle. Alexandros had the idea of using the REPEAT command to make the buttons and the interfaces between them. After executing the REPEAT command in direct - drive, the children's plan to take the turtle to the centre of the next circle for the face indicates an implicit use of the radius in two tangent circles of different size:

(turtle at point 3, fig. 8.2.11, after running the REPEAT command)

V: "It will be 20, the other one. Therefore, FD 40. The other radius will be 20. Therefore FD 40, CIR19 20 and its done that and that..."

A: "And it's in the middle. And then CIR19 5."

V: "Why 5?"

A: "To make the nose."

```
CIR19 40
PU
BK 40
PD
REPEAT 3 [ PU FD 20 PD CIR19 5]
PU
FD 40
```
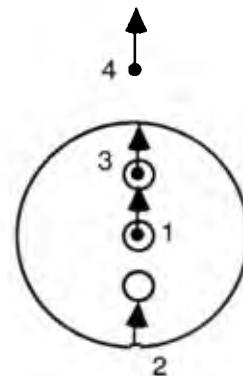
Figure 8.2.11 The children's initial procedure
for their "Snowman" project

Alexandros' explicit acknowledgement that the turtle would be in the centre seems to indicate a meaningful use of the centre point

of the circle, i.e. that they could make the nose straight away, without interfacing commands. The children explicitly decided to move half the length of the radius in both cases of the following interface before making the left eye (fig. 8.2.12a). The interface between the right eye and the mouth involved the turtle moving back to the central axis of the figure (8.2.12b). In the next interface between mouth and broom, however, they did not go back to the central axis, taking the turtle downwards 50 steps:

V: "Right look what I say we should do. LT 90...PU, FD how much... this is 40... (means the radius of the circle for the torso) *50."* (correctly implies that the distance between mouth and edge of face is 10, fig. 8.2.12c)

Nevertheless, the input to the next FD command (fig. 8.2.12d), so that the turtle would go to mid - way along the implied horizontal radius seems to indicate that the children were aware that the turtle was not in the central axis of the figure; they typed in the difference between half the length of the radius and the turtle's distance from the central axis, i.e. FD 15.
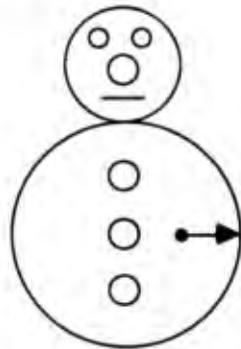
```
TO X
CIR19 40
PU
BK 40
PD
REPEAT 3 [ PU FD 20 PD CIR19 5]
PU
FD 40
CIR19 20
CIR19 5
PU
FD 10
LT 90
FD 10
RT 90
CIR19 3
RT 90
PU
FD 20
PD
```

```
CIR19 3
PU
LT 90
FD 10
LT 90
FD 20
RT 90
PD
FD 5
BK 10
LT 90
PU
FD 50
LT 90
FD 15
END
```

a = interface between nose and left eye
b= interface between right eye and mouth
c= interface between mouth and beginning of broom
d= input to FD for beginning of broom

Figure 8.2.12 The children's progress

in their "Snowman" project

Finally, in the construction of the broom, Valentini used a geometrical idea and Alexandros an idea involving modularity. In the former case, Valentini explicitly partitioned a 90 degree angle into three, in order to decide how much to turn the turtle between the rays of the broom. The researcher's suggestion that the broom might look nicer with five rays instead of three was followed by her attempt to devide 90 by 5 in her mind. Alexandros, however, suggested they "make the turtle do the division" and typed PR 90 / 5. He also

suggested they use the REPEAT command and have a ray and a turn as a module. It was the process of the turtle's construction of the rays of the broom that seemed to have brought their subsequent insight into a euclidean method for constructing the circle, as discussed above (fig. 8.2.13a).


```
TO X                                    CIR19  3
CIR19  40                               PU
PU                                      LT  90
BK  40                                  FD  10
PD                                      LT  90
REPEAT  3 [ PU  FD  20  PD  CIR19  5]    FD  20
PU                                      RT  90
FD  40                                  PD
CIR19  20                               FD  5
CIR19  5                                BK  10
PU                                      LT  90
FD  10                                  PU
LT  90                                  FD  50
FD  10                                  LT  90
RT  90                                  FD  15
CIR19  3                                LT  45
RT  90                                  PD
PU                                      FD  40
FD  20                                  LT  45
PD                                      REPEAT  5 [FD  15 BK  15 RT  18]
                                        END
```

a= commands for the broom

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Figure 8.2.13 The full version of the children's "Snowman" project

The snowman project was presented in some detail in order to provide an overall picture of the children's increasing use of the notions of radius and centre of a circle. In this case, the children did not use the programming - oriented bottom - up superprocedure building technique adopted in previous projects. They started off by defining a procedure (X) to make the torso and the buttons, and then programmed in direct drive for large parts of the figure while keeping a written record of their commands. They added a new set of commands on to the X procedure only in two instances, i.e. after making the mouth and after completing the broom (fig. 8.2.13).

G.2d) (8.2.2d)) Circle procedure 4

The following two projects involved using the TC procedure, which embodied a euclidean construction of a circle. **The first project mainly involved superprocedure - building** from their initial procedure TIME, a variation of the TC procedure used for their clock task (fig. 8.2.2g and h). The first figure consisted of a set of concentric "circles" - the TIME procedure made twelve dots on the screen in a circular formation. The children's subsequent programs consisted of superprocedures of combinations of target figures, as in their projects with the CIR9 and CIR4 procedures. **An interesting sub - project** was their procedure F7, **involving "circles" in a square formation achieved by an intrinsic method of repeated turtle moves and turns** REPEAT 4 ["circle" RT 90 FD 41], (fig. 8.2.14). **In this case, the children seemed to implicitly use the notion of the radius in deciding on the length of the side of the square,** since the radius of the target shape was 41.
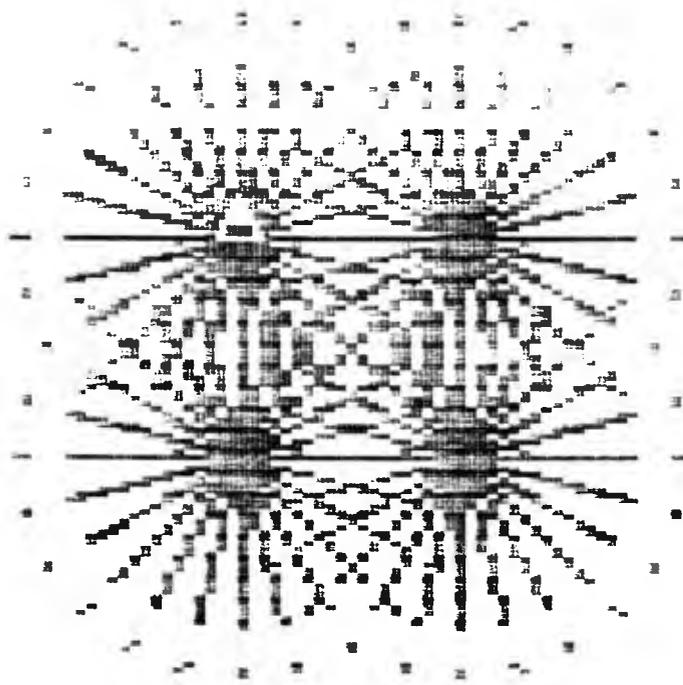
```
TO  F4          TO  F5
PU              REPEAT  24 [F4  RT  15]        TO  F8
FD  40          END                           F7
PD                                            RT  45
FD  1                                         FD  82
PU              TO  F7                         LT  45
BK  9           REPEAT  4 [F5  RT   90  FD  41]  F7
PD              END                           END
BK  32
PU
END
```

Figure 8.2.14 The children's first project

involving circle procedure 4

The final project involved a more explicit use of the radius and the centre in two occasions. The shape the children wanted to make was a rectangular frame the size of the screen, a large "clock" in the middle of the frame and four smaller "clocks" tangentially connected to the four "corners". Their programming method was similar to that adopted in their snowman project in the sense that they would try out a set of commands in direct drive and then put them in a procedure. In this case, however, they wrote a new procedure for each set of commands. In two instances, they "collected" their procedures into a superprocedure, resulting in an "untidy" (from a programming point of view) superprocedure consisting of subprocedures of different levels (fig. 8.2.15).

```
                              TO  MO
          TO  MOV             PU                          TO  MO2
          PU                  FD  130                      MOV
          BK  115             RT  90                        MO
          RT  90              FD  115                      TIME  60
          FD  130             PD                            END
          RT  180             END
          PD
          REPEAT  2 [FD  260 RT  90 FD  230 RT  90]
          END
TO  MO4
REPEAT  2 [TIME  20 PU FD  188 LT  90 TIME  20 PU PD  218 LT  90 PD]
END
                              TO  MO3
                              PU                           TO  MO5
                              BK  115                        MO2
                              RT  90                         MO3
                              FD  109                        MO4
                              LT  90                         END
                              FD  21
                              PD
                              END
```

Figure 8.2.15 The children's second project with circle procedure 4

The geometrical notions used involved the relationship between the dimensions of the frame - which the children had decided by trial and error to be 260 x 230 - and the clocks' radii, in connection to their position in the four corners. In constructing the first clock in the bottom right hand corner of the frame, the children wanted the dots to touch the perimeter with precision. In their attempt to place the turtle at the centre of the clock, they took into account the centre's distance from the vertical right hand side of the frame and the horisontal bottom side, (fig. 8.2.16). **In doing so, they seemed for the first time to use the radius in two positions in order to identify the position of the centre.** In their attempt to achieve more accuracy in their shape, they took into account the length of the dots of the TC figure so that it would be tangential to the sides of the frame. **Furthermore, it was Alexandros' idea to construct a modular subprocedure (MO4, fig. 8.2.15) in order to make the four small clocks at the edges of the frame.** The children had previously carefully calculated the respective lengths, so that the turtle would move forward from clock centre to clock centre. They did so by subtracting the total length of the side of the frame by twice the length of the radius.
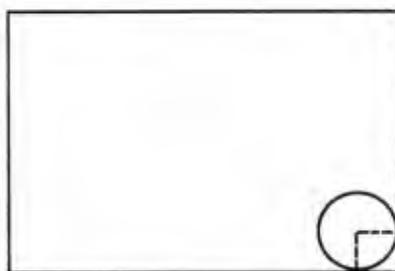

Figure 8.2.16 Using the centre and radius
during the children's "Clock" project

APPENDIX H

<u>The children's programming strategies in solving the "Four squares" tasks</u>

In general, the researcher found it difficult to analyse the data so that the findings would represent the progress of the group of 20 children in a collective and organised manner. This was due to the wide diversity of the children's processes of working at the task, an issue which is corroborated by related research findings (Papert et al, 1979, Hoyles and Sutherland, in press).

At the time of the first occasion on which the task was set, where the children worked in groups, they had not yet been introduced to the REPEAT command or to procedures. Their direct - drive collaborative attempts to construct the figure showed that most groups, although starting off with no clear global plan, became progressively aware of the geometrical properties of the figure. For instance, five out of the eight groups constructed squares of equal size and showed evidence of realising that the distances between the squares were equal.

In analysing the data from the subsequent two administerings of the task, the researcher found it useful to focus on the children's programming strategies, i.e. on the process by which they attempted to construct the figure in connection with the programming method they used. Even then, diversity characterised the children's efforts, even when they used the same "programming techniques" (a procedure, for example) to solve the task. Figure 5.2 represents a summary of the children's strategies regarding their programming, for the second and third administering of the task.

The first two categories of the children's strategies involved direct - drive programming. Evidence of using the geometrical properties and some awareness of the structure of the task distinguished the second category from the first. A substantial proportion of the children (9 in the first occasion and 6 in the second) did not choose to use REPEAT or procedures, even though all the children had used these techniques in their group projects.

The third category consisted of the children who used the REPEAT command

to construct the squares, but otherwise programmed in direct - drive.

The subsequent three categories represented in figure 5.2 involved children writing one procedure to construct the whole of the figure. Three children in the first occasion and one in the second, completed the task in direct drive first, and then wrote a procedure, mainly consisting of the commands used in direct drive (fig. 5.2, D). Two children in each occasion started the construction in direct - drive and at some point, apparently acquiring confidence from the emerging figure on the screen and/or developing an awareness of the structure of the figure, abandoned their direct - drive efforts and started anew within a procedure (fig. 5.2, E). However, two children in the first occasion and seven in the second, started off by defining a procedure, either attempting to construct the whole of the figure from the start and then debug the procedure, or constructing a small part (e.g. one square), running the procedure on the screen, and then "adding on" another feature of the figure, trying it out on the screen, etc. (fig. 5.2, F)). A substantial proportion of children wrote a procedure for the figure (7 in the first occasion and 10 in the second) and there was a substantial increase in the children confident enough to start off by defining a procedure (from 2 to 7).

No children wrote subprocedures or more than one procedure for the task during the first of the two occasions. The strategies of the 4 children who did so during the second, however, are presented in some detail. All four of these children participated in the main case studies (see chapters 7 and 8). One child (Philip) wrote one procedure to construct the task (fig. 5.3 - 1, 2 and 3), his programming initially appearing to be under figure 5.2, F. After completion, however, he wrote one procedure for the interface (fig. 5.3 - 4) and one for the square (fig. 5.3 - 5) and then a superprocedure initially consisting of a "linear" iteration of the two subprocedures (fig. 5.3 - 6 and 7). Finally, he used the REPEAT command for the two subprocedures (fig. 5.3 - 8) and "added" direct - drive commands for the initial positioning of the turtle (fig. 5.3 - 9). This was the only instance of a developing use of subprocedure and modularity (fig. 5.2, G).

Three children, however, adopted a modular approach from the beginning. Valentini wrote one subprocedure incorporating both square and interface (fig. 5.4 - 1 and 6), and a separate subprocedure for the initial positioning of the turtle (fig. 5.4 - 4 and 5). Figure 5.4 illustrates the order in which she wrote the procedures and consequently how she de-bugged them. Alexandros began by perceiving three modules, the initial turtle positioning (fig. 5.5 - 2), the square (fig. 5.5 - 3) and the interface between squares (fig. 5.5 - 4), writing a subprocedure for each of the three and using the REPEAT command for the two latter subprocedures in his superprocedure for the task (fig. 5.5 - 5). Nikos defined only one subprocedure for the square (fig. 5.6 - 1), but seemed to perceived the modular structure of the task in writing the superprocedure (fig. 5.6 - 2, 3 and 4).

Since the task was not completed on all occasions, the researcher had some difficulty in "classifying" incomplete efforts. When it was felt that a pupil using a specific programming strategy (for instance, starting off with a procedure) was well on the way to overcoming a difficulty or completing the task, the effort was classified according to the used programming strategy. More often was the case, however, when a pupil would start off by defining a procedure, but due to confusion involving the programming technique or the geometry (or both), finally revert back to direct - drive.

Although the data indicated some correspondence between children using the geometrical properties involved in the construction of the figure and the sophistication of their programming strategy, it was not always the case that one followed from the other. Some efforts indicated a clear idea of the geometry involved, but did not go beyond direct - drive programming as, for instance, in Ioanna's strategy (fig. 5.7). In some cases, a child would be ready to define a procedure and build or debug it in overcoming confusions due to the geometrical structure of the task, as for example, is illustrated by the changes Nafsika made to her SUPERWOMEN procedure (fig. 5.8).

direct – drive

	A little use of properties using properties	B using properties	C using REPEAT
second administering	7	2	4
third administering	5	1	1

writing a procedure

	D direct – drive and procedure when task is complete	E procedure developing from direct – drive	F starting by defining a procedure
second administering	3	2	2
third administering	1	2	7

writing subprocedures

	G subprocedures developing after a procedure is written	H subprocedures are part of the construction process
second administering	–	–
third administering	1	3

<u>Figure 5.2: The children's programming strategies in the second and third administering of the "Four Squares" task</u>

1

```
TO  GREAT
REPEAT 4 [FD 30 RT 90]
RT 90
FD 40
REPEAT 4 [FD 30 RT 90]
RT 90
FD 40
END
```

2

```
TO  GREAT
REPEAT 4 [FD 30 RT 90]
RT 90
PU
FD 40
PD
LT 90
REPEAT 4 [FD 30 RT 90]
RT 90
PU
FD 40
PD
LT90
REPEAT 4 [FD 30 RT 90]
RT 90
PU
FD 40
PD
LT 90
REPEAT 4 [FD 30 RT 90]
END
```

3

```
TO  GREAT
PU
RT 90
BK 50
PD
LT 90
REPEAT 4 [FD 30 RT 90]
RT 90
PU
FD 40
PD
LT 90
REPEAT 4 [FD 30 RT 90]
RT 90
PU
FD 40
PD
LT90
REPEAT 4 [FD 30 RT 90]
RT 90
PU
FD 40
PD
LT 90
REPEAT 4 [FD 30 RT 90]
END
```

4

```
TO  GREAT1
RT 90
PU
FD 40
PD
LT 90
END
```

5

```
TO  GREAT2
REPEAT 4 [FD 30 RT 90]
END
```

6

```
TO  SUPERGREAT3
GREAT1
GREAT2
END
```

7

```
TO  SUPERGREAT3
GREAT1
GREAT2
GREAT1
GREAT2
GREAT1
GREAT2
GREAT1
GREAT2
END
```

8

```
TO  SUPERGREAT3
REPEAT 4 [GREAT1 GREAT2]
END
```

9

```
TO  SUPERGREAT3
RT 90
PU
BK 60
PD
LT 90
REPEAT 4 [GREAT1 GREAT2]
END
```

(the procedures are numbered in the order in which Philip wrote them)

Figure 5.3 Philip's programming strategy for the
four squares task

547

1

```
TO SQUARES
PEPEAT 4 [FD 40 RT 90]
LT 90
PU
BK 60
PD
END
```

2

```
TO SSQUARES
REPEAT 4 [SQUARES]
END
```

3

```
TO SSQUARES
MOVE
REPEAT 4 [SQUARES]
END
```

4

```
TO MOVE
LT 90
FD 120
END
```

6

```
TO SQUARES
REPEAT 4 [FD 40 RT 90]
LT 90
PU
BK 60
PD
RT 90
END
```

5

```
TO MOVE
PU
LT 90
FD 120
RT 90
PD
END
```

(the procedures are numbered in the order in which Valentini wrote them)

Figure 5.4 Valentini's programming strategy
for the Four Squares task

548

```
        1                2                3                        4

     TO  MOVE1       TO  MOVE1      TO  SQUARE                   TO  MOVE2
     LT 90           LT 90          REPEAT 4 [F 20      RT 90]RT 90
     PU              PU             END                          FD 20
     FD 30           FD 60                                       PU
     RT 90           RT 90                                       FD 10
     PD              PD                                          LT 90
     END             END                                        PD
                                                                 END


                            5

                     TO SQUARES
                     MOVE1
                     REPEAT 4 SQUARE MOVE2              ]
                     END
```

(the procedures are numbered in the order in which Alexandros wrote them)

Figure 5.5 Alexandros' programming strategy

for the Four Squares task

549

1

```
TO  N
REPEAT 4 [FD 30 RT 90]
END
```

2

```
TO  NIK
REPEAT 4 [   LT 90 PU FD 60 RT 90 PD]
END
```

3

```
TO  NIK
RT 90
PU
FD 30
REPEAT 4 [N LT 90 PU FD 60 RT 90 PD]
END
```

4

```
TO  NIK
RT 90
PU
FD 30
PD
LT 90
REPEAT 4 [N LT 90 PU FD 60 RT 90 PD]
END
```

(the procedures are numbered in the order in which Nikos wrote them)

<u>Figure 5.6 Nikos' programming strategy
for the Four suares task</u>

550

```
PU
BK 10
BK 20
BK 10
PD
REPEAT 4 [FD 20 RT 90]
PU
FD 30
REPEAT 4 [FD 20 RT 90]
PD
REPEAT 4 [FD 20 RT 90]
PU
FD 30
REPEAT 4 [FD 20 RT 90]
PU
FD 30
REPEAT 4 [FD 20 RT 90]
```

Figure 5.7 Ioanna's programming strategy
for the Four Squares task

1

```
TO SUPERWOMEN
REPEAT 4 [FD 10 RT 10]
END
```

2

```
TO SUPERWOMEN
REPEAT 4 [FD 10 RT 90]
END
```

3

```
TO SUPERWOMEN
REPEAT 4 [FD 25 RT 90]
END
```

4

```
TO SUPERWOMEN
REPEAT 4 [FD 25 RT 90]
PU
RT 90
FD 35
REPEAT [FD 25 RT 90]
END
```

5

```
TO SUPERWOMEN
REPEAT 4 [FD 25 RT 90]
PU
RT 90
FD 35
LT 90
REPEAT [FD 25 RT 90]
END
```

6

```
TO SUPERWOMEN
REPEAT 4 [FD 25 RT 90]
PU
RT 90
FD 35
LT 90
PD
REPEAT [FD 25 RT 90]
PU
RT 90
FD 35
LT 90
PD
REPEAT 4 [FD 25 RT 90]
PU
RT 90
FD 35
LT 90
PD
END
```

7

```
TO SUPERWOMEN
LT 90
FD 80
RT 90
```

(the procedures are numbered in the order Nafsika wrote them)

Figure 5.8 Phases of Nafsika's attempts to solve
the Four Squares task