Counting-On, Trading and Partitioning:

Effects of Training and Prior Knowledge on Performance on Base-10 Tasks

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ABSTRACT

Factors affecting performance on Base-10 tasks were investigated in a series of four studies with a total of 453 children aged five to seven years. Training in counting-on was found to enhance child performance on Base-10 tasks (Studies 2, 3, and 4), while prior knowledge of counting-on (Study 1), trading (Studies 1 and 3) and partitioning (Studies 1 and 4) were associated with enhanced Base-10 performance. It emerged that procedural knowledge of counting-on, trading and partitioning can lead to improvements in procedural knowledge of the Base-10 system. The findings lend support to the model of iterative development of conceptual and procedural knowledge advanced by Rittle-Johnson, Siegler and Alibali (2001).

INTRODUCTION

Adherence to Base 10 as the system for organizing numbers is near universal in cultures around the world. Widespread subscription to the Base-10 system is underpinned by a long history, arising as it did in ancient times both in Chinese and Indo-European civilizations (O'Connor & Robertson, 2004a; 2004b). The origins of its popularity are not known, though the human endowment of eight fingers and two thumbs provides a plausible stimulus for organizing numbers into groups of ten. In fact, the use of any base system yields the advantage that large numbers can be represented and manipulated more efficiently than collections of single units. At the same time, the task of grasping the concepts underlying the organization of a base system presents children with a substantial cognitive challenge (e.g., Baroody, 1990; Brown & Burton, 1978; Fuson, 1990; Hiebert & Lefevre, 1986; Hiebert & Wearne, 1996; Kouba, Carpenter & Swafford, 1989). Without knowledge of the Base-10 system, children cannot easily broach problems of multi-digit addition and subtraction, less yet complex multiplication and division problems. Furthermore, children require knowledge of the Base-10 system as a pre-requisite for understanding place value, the notational system for recording multi-digit numbers with written marks (Fuson & Briars, 1990; Ross, 1989).

Given its importance, it is not surprising that explicit teaching of the Base-10 system typically begins quite early in the school career of many children. In the U.S., for example, Base-10 concepts figure in the curriculum for Grade 1 children (aged 6 years). In the U.K., instruction on the Base-10 system is part of the National Numeracy Strategy for Year 1 children, aged 5 years (Department for Education and Employment, 1999). Of interest here is the fact that explicit instruction about the Base-10 system occupies a prominent position in formal school curriculums. And yet, as noted above, acquisition of the Base-10 system is often slow and effortful for many children. Not only do many children fail to acquire this knowledge spontaneously, for others, direct teaching also fails (Ross, 1989).

The work reported here examines three mathematical skills which, we predict, may facilitate access to the Base-10 system: (1) counting-on; (2) trading; and (3) partitioning. The term *skill* is used here simply to demarcate discrete aspects of mathematical ability, and serves a purely descriptive function. In using this term, therefore, conceptual and procedural aspects of knowledge are not distinguished (see below), although care is taken to acknowledge this distinction where necessary. The first target skill, counting-on, is deployed by (some) children when finding the sum of two addends (Baroody, 1995; Fuson, 1982; Fuson, Secada & Hall, 1983; Resnick, 1983; Siegler & Jenkins, 1989). To find the sum of 5 + 2 in this way, the child would start from the highest addend (5) and count on two more from there (5 ... 6, 7). The second skill, trading, requires an appreciation that a single item can be equivalent in value to an entire collection of individual units. In consequence, one can exchange, or trade, a group of individual items for a single ('large value') item, with no loss or gain in amount. The third target skill, partitioning, refers to the ability to divide any number (n>1) into separate parts, or subsets. Thus, 13 can be partitioned into one subset of 6 and another subset of 7, or into subsets of 11 and 2, and so on.

We predict that these target skills bear the potential to function as precursors in the acquisition of Base-10 knowledge. To account for the developmental progression envisaged, we draw on the distinction between *conceptual knowledge* and *procedural knowledge* (Rittle-Johnson & Siegler, 1998). This distinction is based on the observation that children sometimes execute mathematical procedures accurately without necessarily understanding the concepts that underlie those procedures (e.g., Bisanz & Lefevre, 1990; Canobi, 2004; Canobi, Reeve & Pattison, 2003; Hiebert & Lefevre, 1986; Xiadong, Tardif, Guoxiong & Fuxi, 2004). For example, Byrnes and Wasik (1991) found that 16% of 4th and 5th grade children solved fraction multiplication problems successfully, yet performed poorly on tests of related conceptual knowledge. In a similar vein, young children are often fluent in basic counting procedures, even though their conceptual understanding of counting may be limited (e.g., Frye, Braisby, Love, Maroudas & Nicholls, 1989). In both of these examples, procedural knowledge outstrips conceptual knowledge. However, examples in the reverse direction are plentiful (e.g., Dixon & Moore, 1996; Gelman & Williams, 1998; Sarnecka & Gelman, 2004).

Evidently, development can take place in both directions: from conceptual to procedural, and vice-versa. To accommodate this variability, Rittle-Johnson, Siegler and Alibali (2001) propose a model of iterative development, whereby gains in conceptual knowledge can lead to gains in procedural knowledge, which in turn, can lead to greater conceptual facility and so on (see Figure 1). The starting point for this iterative cycle can be either conceptual or procedural knowledge, and depends on frequency of exposure and type of experience with a particular skill. As shown in Figure 1, changes in procedural and conceptual knowledge are hypothesized to influence the representation of problems, where *problem representation* is defined as "the internal depiction or re-creation of a problem in working memory during problem solving" (Rittle-Johnson et al., 2001, p.348). On this view, improvements in problem representation can enter into the iterative chain, being both influenced by and serving to influence enhancements in either conceptual or procedural knowledge. In the work reported below, we consider one possible developmental pathway across different mathematical skills. Specifically, we predict that procedural knowledge of each one of the three target skills can enhance procedural knowledge of the Base-10 system.

The first skill, counting-on, depends on prior knowledge of cardinality, that is, knowledge that a number such as five refers to a set containing five items. In the act of counting-on, therefore, the child takes a group of items as the starting point and adds on a certain number of units to this group. There is, then, a parallel with multi-digit numbers organized around a base of ten. For example, the number 14 can be conceptualized as a group of 10 items plus 4 further units. If the child were to solve the addition problem 10 + 4 using counting-on, they would, effectively, be using a base of ten and adding four units to it. To do so would not require prior knowledge of the Base-10 system. Instead, the counting-on procedure yields particular examples that also conform, in part at least, to the demands of some Base-10 tasks. Hence, the child would be implementing a mathematical procedure that can also be applied to Base-10 problems. Procedural knowledge of counting-on is, arguably, less complex than that required for full fluency with the Base-10 system. Hence, training in counting-on is likely to be more tractable than the intensive training regimes that introduce the full panoply of Base-10 and place value concepts (e.g., Fuson & Briars, 1990). At the same time, given the overlap in procedures needed to solve problems in each skill, we predict that training in counting-on procedures will be transferable and lead to enhanced performance on (at least some) Base-10 tasks.

A further issue to consider is the potential distinction between cases where there is a direct parallel between Base-10 and counting-on procedures (as in the 10 + 4 example above), and a more abstract parallel that can be drawn between the two procedures. To solve both 10 + 4 and 7 + 4 via counting-on depends on the child appreciating that the larger addend in each case represents the cardinal value of a set comprising X > 1 units. Evidently, though, the 10 + 4 problem provides a more explicit cue to parallels with Base-10 procedures than the 7 + 4 problem. In the latter case, the child is presented with the general case only of a procedure that entails taking the cardinal value of a whole group of units as the starting point for further counting. In the studies reported below, we train children on ‘general’ counting-on problems like 7 + 4. In this way, we provide a stringent test of our hypothesis that procedural knowledge can be transferred across separate skills. We also avoid the confound whereby training in counting-on with problems like 10 + 4 would be difficult to distinguish from direct training in Base-10 procedures.

The second target skill, trading, has featured in (successful) intervention programs to enhance children's understanding of multi-digit addition and subtraction (e.g., Fuson & Briars, 1990; Hiebert & Wearne, 1996; Resnick, 1982; Resnick & Omanson, 1987). In particular, trading underpins the borrowing (or 'carrying over') procedure needed to solve complex written problems like 302 – 147. Trading is also important in learning about money. For example, children will learn that ten individual cents can be traded for a single dime, because they are equivalent in value. Similarly, with multi-digit numbers, children must learn that a collection of ten units can be traded for a single marker with a designated value of ten. When the specific exemplar of trading involves a group of ten units, to be traded for a single marker with a value of ten, one is, de facto, also supplying the child with an example of ten as the basis for grouping a collection of units, as found in the Base-10 system. As with counting-on, the child would be equipped with a procedure of direct relevance for completing certain Base-10 tasks (e.g., exchanging a conjoined block of ten unit cubes for ten individual cubes). It is therefore predicted that children with prior procedural knowledge of trading will perform better on Base-10 tasks than their counterparts who lack this knowledge.

The third target skill is partitioning, and again, a parallel can be drawn with particular procedures required on certain Base-10 tasks. Multi-digit numbers can be conceived of as a combination of one subset comprising some multiple of ten and another subset comprising a group of units. The simplest case involves numbers in the teens. Thus, one can partition 14 into one group of 10 and another group of 4. Arguably, children who are conversant with partitioning possess a procedure that can be applied to some Base-10 tasks, including the decomposition of a multi-digit number into its component tens and units. It is predicted, therefore, that children who are equipped with procedural knowledge of partitioning will have an advantage when it comes to performing on Base-10 tasks.

In testing for prior knowledge, an important consideration is that children are often inconsistent in revealing the full extent of their knowledge of a given skill. The cognitive demands of particular tasks, together with lack of experience with given procedures, may interfere with the expression of a child's knowledge (c.f. Chomsky, 1965). In addition, it is important to recognize that children typically possess a range of different strategies for solving problems within a given skill (for an overview, see Siegler, 1996; 2003). Children may succeed or fail on a sequence of formally identical problems because they are switching from one strategy to another. These considerations point to the importance of providing children with several trials on a given problem, and, where appropriate, a range of different tasks to test their knowledge. In this way, the chances of tapping into the child's most sophisticated levels of understanding are increased, be that understanding based on procedural or conceptual knowledge, or both. For these reasons, the children in the studies reported below were supplied with a minimum of two different tasks per target skill and, moreover, a minimum of four trials per task.

The influence of the target skills on tasks of Base-10 performance was investigated in a series of four studies. Study 1 provides an initial exploration of the associations between children's knowledge of the three target skills and their performance on Base-10 tasks. In Studies 2, 3 and 4 we investigated the effects of an intervention, namely, training in counting-on, on Base-10 performance. Studies 3 and 4 also examined the influence of prior knowledge of trading and partitioning, respectively, on children's representations of multi-digit numbers.

STUDY 1

This study examined three target skills predicted to facilitate performance on Base-10 tasks: counting-on, partitioning and trading. We predicted that the more of these skills children possessed, the more likely it would be that children would show an appreciation of Base-10 by representing multi-digit numbers with tens-and-units, rather than units only.

Method

Participants

Participants were drawn from a large, state-funded primary school which serves an economically and socially deprived area of London. The original sample comprised 102 children, but five of these were excluded, either due to inattention or because of difficulties in understanding the instructions given. The remaining sample of 97 children consisted of 53 boys and 44 girls (U.S. grade 1, with a mean age of 81 months (*SD* = 2 months, range 78 months to 86 months). A minority of children (36) were of white U.K. origin, with the remaining 61 children coming from a variety of ethnic backgrounds, including Nigerian, Jamaican and Chinese. Inclusion in the study was based on teacher assessments that English was the first language for these children, spoken both at home and in school. Children were not formally screened for any form of cognitive impairments or disabilities, but there was no indication from either teachers, parents or the researchers implementing the study that any of the participants suffered from hearing impairments, subnormal IQ, or speech and language impairments; nor were any of the children identified as recipients of Special Needs Individual Education Plans (U.K.) for learning or behavioral problems.

Materials

Base-10

Box task. Taken from Ho and Fuson (1998), the child counts aloud as the experimenter transfers 10 unit cubes, one at a time, into a box. This procedure is then repeated, with fewer than ten cubes being added to the box, and with the child being prompted to count this second addend from one. In so doing, the child is prevented from counting-on from ten. The box is then closed and the experimenter says: “First I put ten cubes into the box, and then I put x (<10) more cubes in it. How many cubes are there in the box altogether now?” Children’s reaction times were recorded discretely with a stopwatch and

Base-10 understanding was recorded if: (a) the child gave rapid and accurate responses (<2 seconds); (b) no audible counting behaviors were observed; and (c) no visible counting behaviors were observed (e.g., using fingers, or lip movements). Four trials were undertaken, with the second number of cubes being 2, 5, 7 or 9, presented in random order. Sum totals were confined to numbers in the teens to stave off boredom and inattention. Minor errors in following the experimenter's count were ignored to maintain the focus on Base-10 performance.

Ten-blocks task. Taken from Miura, Kim, Chang and Okamoto (1988), children are supplied with individual cubes and blocks of ten cubes, joined together into columns, but with the individual cubes clearly visible ("See these cubes and blocks. You can use them for counting and showing numbers"). The equivalence between the blocks and single cubes is made explicit by asking the child to count out a line of ten cubes before laying a block of ten next to it and saying (with appropriate pointing): "We have got ten here and ten here." Children were then asked to read out Arabic numerals, presented to them on cards, before being asked to demonstrate for a teddy how to "show this number with the cubes and blocks." Over four trials, the numbers 13, 18, 23 and 29 were presented in quasi-random order. Base-10 understanding was recorded if children chose to use one or more blocks of ten to represent the number (e.g., 18 as 1 ten block and 8 units; or 23 as one ten block and 13 units). A Units response was recorded in cases where unit blocks only were used. Minor errors in counting were ignored, since counting accuracy was not the focus here.

Single-cubes task. Adopted from Saxton and Towse (1998), this task is based on the Ten-blocks task, but uses single cubes to represent tens, rather than blocks of ten units. Two sets of individual cubes of different colors are therefore placed at the child's disposal. With orange cubes designated as tens and green cubes designated as units, the experimenter demonstrates the equivalence between a single orange ten-cube and a collection of ten single green unit cubes. The procedure is then as for the Ten-blocks task, using the numbers 14, 16, 22, and 27 for the four trials.

Counting-On

Counting-on task. Following Siegler (1987), counting-on was assessed by providing children with a range of simple addition problems. Use of the counting-on strategy was gauged by observing child responses and by interviewing them about their chosen methods of solving the problems. Thus, the experimenter said: "I'm going to ask you a question, and when you have the answer, tell me what it is. You can do anything you want to get the right answer. You can count in your head or use your fingers or do whatever you want to do." A practice item (2 + 1 =?) was then provided, before giving 12 test items. The test items were presented via written Arabic numerals on cards, but the experimenter also read out the problem for the child. Four test items had sums less than 10 (3 + 2, 4 + 3, 6 + 3, 7 + 2); four had sums between 10 and 20 (9 + 5, 11 + 6, 12 + 4, 13 + 2,); and four had sums between 20 and 30 (15 + 6, 17 + 5, 18 + 7, 19 + 4). The largest addend was supplied first in each problem, thus avoiding any explicit test of commutativity (Siegler, 1987). In cases where the child's response was ambiguous, follow-up probe questions were used like: "Tell me, how did you figure out the answer to the problem?" or "Did you count or did you already know the answer?" or "What number did you start with?" In cases where a counting-on response was not observed initially, children were further asked if they could solve the problem in any other way. The aim, then, was to provide children with as much opportunity as possible to display the counting-on strategy, if it lay within their repertoire.

Concealed-cubes activity. Children were first supplied with a small collection of unit cubes and asked to report on the quantity. Provided their response was correct, the experimenter covered these cubes with his hand. Some more cubes were then introduced and children were asked to report on the total number of cubes now present ("How many cubes are there on the table altogether now?”). If necessary, children were reminded how many cubes were concealed and encouraged to continue counting by adding on from that value.

Dot-cards activity. This activity employed cards displaying a small number of large dots (to a maximum of five; c.f. Van de Walle, 1990). Children were shown one of these cards and asked to report on the number of dots present. This first card was then placed face down on the table and a second card was introduced. Children were then asked to report on the total quantity of dots on both cards (“How many dots are there on both these cards altogether now?”). If necessary, children were reminded how many dots there were on the first card and encouraged to count on from that value.

Trading

Large-cube task. Children were provided with a collection ($1 < x < 10$) of small unit cubes measuring 2cm in all dimensions and were told that each cube had a value of one (“The green cubes are ones”). A single large wooden cube was then introduced, measuring 4cm in all dimensions, and children were told that its cardinal value was equivalent to the number of unit cubes they had just been given. Thus, if children had been supplied with a set of three unit cubes, the experimenter would introduce the large cube by saying “This one is worth three.” Children were then invited to trade their unit cubes for the experimenter’s single large cube (“These unit cubes are yours and the wooden cube is mine. I want to swap my wooden cube for yours. Shall we?”). Four trials were conducted using collections of 3, 5, 6 and 9 cubes. In each trial, the same large wooden cube was used.

Small-cube task. This task substituted a ‘base’ cube with a designated value >1 that was physically smaller than the unit cubes. The trading relationship was therefore not cued by the symbolic representation of numerical value by physical size. In other respects this task was identical to the Large-cube task, (including use of the same-sized cubes), with four trials being implemented, using collections of 2, 4, 7 and 8 cubes.

Partitioning

Divided-whole task. Children were supplied with a set of unit cubes and were asked to count and report the sum total. Provided they gave the correct response, the cubes were then divided into two separate groups. Children were then asked once more to report on the total number of cubes across the two groups ("Please tell me, how many cubes there are all together now?"). Knowledge of partitioning was attributed in cases where the child reported the answer without hesitating or going back to count the cubes again. Four trials were implemented, with the following sets and their divisions: 4 (2 + 2), 6 (4 + 2), 8 (5 + 3), 9 (6 + 3).

United-parts task. The mirror image of the Divided-whole task, children were provided with two physically separate sets of cubes and asked to count and report on the total number of cubes across both sets. The two sets were then united into a single set and children were once again asked how many cubes there were in total. Assessment of partitioning knowledge was made as in the Divided-whole task. Four trials were implemented, with the following sets and their divisions: 5 (3 + 2), 7 (4 + 3), 8 (6 + 2), 9 (5 + 4).

Procedure

Children were tested individually in a quiet area within the school. Knowledge of Base-10 was assessed first, using all three tasks, before implementing the tests of counting-on, partitioning and trading. Care was taken to counterbalance the order of tasks for each skill being tested. All children participated in the same testing regime, with the average length of testing sessions being 50 minutes. Children's attention was maintained, in part, through the variety of different tasks, presented to them as fun, game-like activities.

Results

The data were analyzed in two different ways. In the first analyses, we wished to establish whether the number of target skills displayed by children (counting-on, trading and partitioning) was positively associated with their degree of success on the Base-10 tasks. The second approach to the data deployed multiple regression to establish the extent to which the target skills could predict success on the Base-10 tasks.

To tackle the first question, children were initially categorized as either succeeding or failing on the various tasks in the battery presented to them. Base-10 responses were recorded in cases where a multi-digit number was represented using tens-and-units rather than units alone. Children were credited with Base-10 understanding on each task if they produced tens-and-units responses in at least three out of four trials on that task. Given this success criterion, 53 (out of 97) children succeeded on the Box task, 45 on the Ten-blocks task and 38 on the Single-cubes task. A one-way within-subjects Analysis of Variance was conducted to examine differences among responding on these three tasks. Since Mauchly's test revealed a slight tendency towards heterogeneity of covariance (Huyn-Feldt estimate = .95, $p < .05$), an F-ratio with corrected degrees of freedom is reported and post hoc tests applied the Bonferroni correction (Maxwell, 1980). A significant effect of task type on levels of Base-10 responding was found ($F(1.9, 181.6) = 7.83$, $p < .001$, $\eta^2 = .075$). Pairwise comparisons revealed that Base-10 responses were significantly more frequent on the Box task when compared with both the Ten-blocks task ($p < .002$) and the Single-cubes task ($p < .004$). No significant difference was found when performance on the Ten-blocks and Single-cubes tasks were compared. Thus, the Box task emerged as the least demanding Base-10 task. At the same time, it was found that performance on the Box task correlates highly with both the Ten-Cubes task ($r = .81$, $p < .001$) and the Single-cubes task ($r = .69$, $p < .001$), while the corresponding correlation between the Ten-blocks and Single-cubes tasks was also strong ($r = .74$, $p < .001$), indicating close associations among the three tasks. Responses from all three tasks were therefore used in the overall categorization of children with regard to Base-10 responding. Children were deemed to show knowledge of Base-10 if tens-and-units responses were in evidence on at least three out of four trials on at least two out of the three tasks. This criterion for success is not based on theoretically prior notions of success or failure. It simply provides an indication of the child's predominant responses. In the event, 46 children (47.4%) were categorized as showing Base-10 knowledge, although the majority of these children (37) scored at least 3 out of 4 on all three Base-10 tasks, not just two of them. In contrast, of the 51 children categorized as predominantly Units responders, only seven produced three or four Base-10 responses on one of the three tasks while failing the other two (scoring two or less). The large majority of Units responders (86.3%) achieved two Base-10 responses or less on all three tasks.

With regard to counting-on, 52 children were classified as possessing this skill (53.6%). To qualify, children had to deploy this strategy on at least one simple addition problem out of 12 trials provided. A record was made of the counting strategy used for each problem, with responses being categorized as one of the following: counting-on; counting-all, guessing; retrieval; decomposition; or no answer. It was found that 62 children used a range of different strategies for solving addition problems (63.9%).

On the tasks of trading, it was found that 71 children succeeded on the Large-cube task (73.2%), while 58 succeeded on the Small-cube task (59.8%). In both cases, success was operationalised as being able to trade accurately on three or more of the four trials given. It was found that the Large-cube task was significantly easier than the Small-cube task, $t(97) = 7.31$, $p < .001$. At the same time, there was a strong positive correlation in performance on the two tasks, $r = .86$, $p < .001$. Data from the two tasks were therefore combined in classifying children as either Trading or Non-Trading, with the criterion for Trading being set as producing three or more (out of four) trading responses on both tasks. In consequence, 58 children were allocated to the Trading group (59.8%).

With respect to partitioning, success on each task was operationalised as three or more correct responses out of four, both for the Divided-whole and United-parts tasks. On these criteria, 59 children succeeded on the Divided-whole task (60.8%), while 56 children succeeded on the United-parts task (57.7%). Performance across the two partitioning tasks did not differ significantly, $t(97) = .65$, ns. There was also a strong positive correlation in performance on the two tasks, $r = .84$, $p < .001$. Data from the two tasks were collated to classify children as either Partitioning or Non-Partitioning, based on three or more correct responses (out of four) for both tasks. On these criteria, 53 children qualified as possessing knowledge of partitioning (54.6%).

Having established children's success rate on each of the three target skills, children were ranked according to the number of target skills they displayed (0, 1, 2, or 3). Each of these four groups was then further subdivided according to children's performance on the Base-10 tasks, with children being designated as either Base-10 or Unit responders, according to the criteria described above (see Figure 2). Kendal's tau-c statistic was applied to measure the agreement between assignments to unequal-sized sets of ordered categories. This statistic showed that there is a significant association between children's Base-10 performance and the number of target skills they possess (Kendall's tau-c (97) = .54, $p<.001$). As both this measure and the data presented in Figure 2 indicate, tens-and-units (Base-10) responses increase in frequency as the number of target skills rises. Associations among the target concepts was further explored via k-means cluster analysis. The most efficient solution comprised three distinct groups of children, who emerged after three iterations of the clustering procedure. The first group of 14 children typically scored zero on all tasks. Children in the second group (N = 42) typically scored 5 out of 12 on the Base-10 tasks, 5 out of 8 on the partitioning tasks and 6 out of 8 on the trading tasks. Of note, these children typically did not display counting-on. The third, high-scoring group (N = 41) typically scored

10 out of 12 on the Base-10 tasks, 7 out of 8 on both trading and partitioning tasks and, moreover, displayed counting-on.

The analyses reported above are based on the assumption that children can be neatly categorized as either possessing or not possessing a given skill. However, it is also possible to take into account the full range of scores achieved by children on the various tasks and determine the extent to which scores on tasks of counting-on, trading and partitioning predict performance on Base-10 tasks. Table 1 details the level of success on seven of the experimental tasks according to the number of trials passed correctly (range 0 to 4). For the eighth task, the data remain categorical, since children were accorded knowledge of counting-on if they deployed this skill at least once in 12 trials.

Multiple regression analyses were conducted using composite scores across the range of tasks. Thus, child scores for Base-10 were compiled from all three tasks and ranged from 0 to 12. Scores for both trading and partitioning were compiled from the two tasks in each case and thus ranged from 0 to 8. Categorical scores for counting-on were entered in the regression model, because although, in principle, scores could range from 0 to 12, in the event, the full range was not observed, violating the assumption of unbounded data (Field, 2005). A significant model emerged with the entered variables accounting for 73% of the variance, $F(3,93) = 88.85$, $p < .0005$; adjusted $R^2 = .73$. Diagnostic tests pointed to a lack of multicollinearity among the variables. Thus, an initial inspection of correlations among the predictors revealed that none of them exceeded .80 (.39 for trading by counting-on; .72 for trading by partitioning; and .38 for counting-on by partitioning). In addition, the Variance Inflation Factor (VIF) statistics for all three predictors were low (.47 for trading; .47 for trading; and .83 for counting-on). All three variables entered into the model functioned as significant predictors of performance on Base-10 tasks: trading (standardized beta .43, $t(97) =$

5.51, $p < .0005$); partitioning (standardized beta .31, $t(97) = 3.98$, $p < .0005$); and counting-on (standardized beta .32, $t(97) = 5.45$, $p < .0005$).

Discussion

This study demonstrates that children's performance on Base-10 tasks is closely associated with the number of target skills they possess (counting-on, trading and partitioning). The more of these skills children display, the more likely it is that they will represent multi-digit numbers with tens-and-units rather than units only. This finding does not mean that the effects of the target skills are cumulative, but as the cluster analysis also suggests, Base-10 performance is stronger when children possess a combination of the target skills. In particular, strong Base-10 performance is especially associated with possession of counting-on knowledge. Possible interactions among the target skills in their effects on Base-10 performance are explored further in Studies 3 and 4. Of note, we found that children who lacked knowledge of all three target skills showed no ability to represent multi-digit numbers with tens-and-units. We also found that each of these three skills functions individually as a predictor of children's Base-10 performance. These findings suggest clear associations between the mathematical skills examined. They do not, in and of themselves, provide evidence that counting-on, trading and partitioning are *precursors* of Base-10 knowledge. It is conceivable, for example, that acquisition of Base-10 knowledge could facilitate understanding of counting-on, rather than vice-versa. The greater complexity of Base-10 knowledge militates against the likelihood of this interpretation, but does not rule it out. The problem of establishing direction of causality is addressed in the following three studies, where it was decided to screen out in advance those children who showed prior knowledge of the Base-10 system. In this way, we could introduce an intervention, based on training in one of the target skills, and gauge whether it had an effect on subsequent performance on Base-10 tasks. By carefully assessing children's prior knowledge, not only of the Base-10 system, but also of related skills, one can be more confident of establishing that the intervention introduced in the testing situation is responsible for any improvements observed in Base-10 performance.

The cluster analyses performed here lend further support to our decision to focus on counting-on as the target of intervention in Studies 2, 3 and 4. As noted above, children who performed exceptionally well on the Base-10 tasks were distinguished, not only by their mastery of trading and partitioning, but also by their knowledge of counting-on. Children producing either few or no tens-and-units responses were notable for their lack of counting-on knowledge. So although both trading and partitioning were both associated with an ability to produce tens-and-units responses, the additional presence of counting-on seems to provide a powerful fillip, associated with an increase in Base-10 performance towards ceiling levels.

A further finding of note was the variability in success across the three Base-10 tasks. The most likely explanation for this variability is that each task imposes unique task demands on the child, which interfere to varying degrees with the expression of the child's competence. Thus, it would seem that the Box task placed fewer performance demands on children as evidenced by the relatively high success rate. This makes sense when one considers that in this task, unlike the other two, children were not required to construct their own representations of multi-digit numbers. Instead, they are simply asked to report on the representations constructed by the experimenter. Given this pattern of findings, it was decided, in the studies that follow, to use the more demanding Ten-blocks and Single-cubes tests of Base-10 understanding. This approach provides a more stringent test of the prediction that training in counting-on will promote enhanced performance on Base-10 tasks. Children are required to produce their own tens-and-units representations of multi-digit numbers, rather than simply base their responses on the experimenter's behavior. A further advantage is that these two tasks are at an equivalent level of difficulty, as gauged by child performance.

STUDY 2

In Study 2, we tested the prediction that counting-on could function as a *precursor* in the acquisition of Base-10 knowledge. To test this prediction, we trained children in counting-on, in order to gauge the extent to which it enhanced their performance on Base-10 tasks. Our strategy in testing this prediction was to screen out, in advance, all children who displayed any prior knowledge in either of the target skills (counting-on or Base-10). We thus aimed to control for variations in performance that might be caused by differing degrees of knowledge and experience with counting-on. And we also wished to ensure that children had no prior knowledge of Base-10 in order to increase confidence that any post-training effects observed could be attributed to the experimental intervention.

In order to maximize the chances that children would benefit from training in counting-on, all children were required to demonstrate knowledge of cardinality as a prerequisite for inclusion in the study. In counting-on, the cardinal value of one of the addends is used as the starting point for adding on the second addend. Children must first know that the cardinal meaning of a number word refers to the number of objects in a counted set, before being able to use the counting-on strategy as a shortcut to counting (Fuson, 1982).

Given that task demands vary from one task to another, it is possible that children may not always reveal the full extent of their competence. Thus, one can envisage the case of a child who possesses an understanding of Base-10 concepts, but who is inhibited in revealing that competence owing to the performance demands placed on them by a given task or testing situation. In this vein, Saxton and Towse (1998) report that the success of both English and Japanese children on Base-10 tasks increased significantly when performance demands were reduced. Specifically, the experimenter in this study provided a single example of the use of ten-blocks in the representation of multi-digit numbers. Children who appeared to lack Base-

10 knowledge at the outset suddenly revealed this knowledge when given this cue. We decided to explore the influence of this kind of prompt in Study 2, when provided in tandem with training in counting-on. Study 2 thus adopted a two-by-two design, with training in counting-on and the use of a multi-digit (tens) prompt both featuring as between-subjects factors. It was predicted that both training in counting-on and the use of a tens prompt would be associated with enhanced performance on Base-10 tasks.

Method

Participants

The children in this study attended the same school described in Study 1. The same selection criteria were applied and a similar mix of ethnic and social backgrounds was observed. The initial sample comprised 90 children, 40 girls and 50 boys, mean age 70 months (*SD* = 2 months, range 67 to 76 months). Fifty-three of them were from non-indigenous societies (e.g. Jamaican and Nigerian), while the remaining 37 were of white U.K. origin. These children were further assessed for their suitability for inclusion in the study, resulting in a final sample of 56 children, 26 girls and 30 boys (U.S. Kindergarten level), mean age 70 months (*SD* = 2 months, range 67 to 75 months). Of those children who were excluded, four proved unable to attend to the instructions. Four children were excluded because they failed to show knowledge of cardinality. And 25 further children were excluded because they demonstrated prior knowledge of either counting-on (9) or the Base-10 system (4), or both (12). Of the 57 children remaining, one further child was excluded at random to create four groups of equal size (14 in each). Children were then assigned to experimental groups in a quasi-random fashion.

Materials

Cardinality

Give-me-X. Cardinality was assessed via a simple "give-me-x" task, in which children were asked to give the experimenter a stated number of unit cubes from a collection (c.f., Schaeffer, Eggleston & Scott, 1974; Towse & Hitch, 1996; Wynn, 1990). Children were assessed as possessing knowledge of cardinality if they succeeded in giving the correct number of items on four trials, where *x* was one of the following numbers: 6, 14, 25, 30. Simple counting errors were not ignored. In cases where counting errors occurred, children were asked to count again. Children who persisted with an erroneous count were assessed as not possessing knowledge of cardinality.

Abstract counting task. Taken from Miller and Stigler (1987), children were first asked to count aloud starting with "1." If necessary, they were prompted at the beginning with "1, 2, 3". If they stopped their counting, they were encouraged to continue by asking "What comes after *x*?" (the last number counted), or by repeating the last three numbers counted, ending on a rising, expectant tone (e.g., "18, 19, 20, …?). Children who still failed to correctly continue the sequence were further prompted by reminding them of the next number beyond the point at which they had stopped. Success on the task was recorded for children who could reach at least 29 in this way.

Both of these tasks provide a measure of children's understanding of cardinality. But it is also clear that they also test children's ability to produce numbers in the count sequence in the correct order. Given the range of skills tested for, therefore, they furnish a useful screening device for ensuring that children will be able to tackle the more complex tasks in our experimental procedure.

Procedure

Pre-training assessment.

Pre-training sessions were conducted individually in a quiet area of the school and lasted roughly 35 minutes. Children were assigned to testing conditions in a quasi-random fashion. In the pre-training assessments, knowledge of cardinality was assessed using the Give-me-X and Abstract-counting tasks. Eight children failed one or both of these tasks. Prior knowledge of Base-10 was assessed using one of two tasks, either the Ten-blocks task or the Single-cubes task (see Study 1 above). Each task was rotated through children. In the post-training phase, Base-10 knowledge was assessed using the task that children had not already experienced. The Counting-on task was used to make an assessment of children's prior knowledge of counting-on (see Study 1 above). This assessment revealed that 58% of the children used multiple strategies for solving addition problems, a figure similar to that found in Study 1 (64%).

Training in Counting-On.

The pre-training phase was conducted seven to ten days prior to the follow-up phase, which involved training in counting-on and an immediate post-training assessment of Base-10 performance. All training and assessment sessions were conducted on an individual basis and lasted roughly 20 minutes. The 56 children who met the experimental criteria were divided into two groups, designated as Training and No Training. Each of these two groups was further sub-divided into two groups, corresponding to Prompt versus No-Prompt in the use of tens, yielding a 2X2 between-subjects design. Children in the Training group were exposed to two activities, supplied in counterbalanced order through children, which were designed to encourage the use of counting-on (the Concealed-cubes activity and the Dot-cards activity, see Study 1 above). Training continued until the experimenter was satisfied that children could perform the counting-on activities successfully. In the event, there was some variability in the amount of training required by children. Of the 28 children in the Training group, 13 applied the counting-on strategy after four trials or fewer. Six children required up to five more trials, while the remaining nine children required further trials. While the number was not counted in these latter cases, it was noted that in no case did training continue for more than ten extra minutes. Thus, all children who took part in the training eventually succeeded in counting-on.

Post-training assessment.

When assessing Base-10 knowledge in the post-training phase, children were assigned either to a Prompt or a No Prompt condition. In both conditions, children were initially shown an Arabic numeral (2) printed on a card and asked to report what it was. Children were then shown how they could make a collection of two cubes corresponding to the number two on the card. In the Prompt condition, children were additionally shown how to make 15 (from one ten and five units). Thus, children were prompted in the use of tens for the representation of multi-digit numbers. The ten was demonstrated by using a collection of ten cubes stuck together end to end in a single block. In the No Prompt condition, children were shown how to make 2 and 5 (both from unit cubes). Following these examples, in the test phase, participants were asked if they could show a teddy how to represent some numbers. The test stimuli, presented on separate cards, consisted of four multi-digit numbers less than 30, as used in the Base-10 tasks described in Study 1 above.

Results and Discussion

The scoring methods adopted for the pre-training assessments were identical to those implemented in Study 1. On this basis, a total of 30 children were excluded from the initial sample, to yield 56 children who were conversant with cardinality, but who failed on tests both of counting-on and Base-10 knowledge. In our first set of analyses, a categorical approach to the data was adopted, with children being designated as showing either Base-10 responses or Unit responses in the representation of multi-digit numbers. The scoring method was identical to that used in Study 1. To summarize, Base-10 understanding was ascribed to children who produced tens-and-units responses on at least three out of four trials. As Table 2 reveals, 48% of children could be categorized as Base-10 responders. It will be recalled that, in addition to testing for the effects of training in counting-on, we also tested for the effects of prompting children in the use of tens, either with or without training in counting-on.

A three-way hierarchical loglinear analysis by backward elimination was carried out in order to determine whether any significant associations obtained between Training in counting-on (Training versus No Training), Prompt (Prompt versus No Prompt) and Base-10 (Tens-and-Units versus Units-Only responding) (Field, 2005). No significant third-order effects emerged (Pearson X^2 (1) = .06, $p > .10$), indicating that there were no reliable three-way associations among the variables. Further analyses, however, revealed evidence for significant second-order effects (Pearson X^2 (3) = 9.31, $p < .05$) indicating that at least one of the two-way associations was significant. The two-way effects indicated by the loglinear model were assessed further by partial associations. These revealed a reliable association between Training and Base-10 (partial X^2(1) = 3.98, $p < .05$), suggesting that training in counting-on led to significant improvements in children's Base-10 performance. Odds ratios indicated that children were 2.78 times more likely to produce Base-10 responses when trained in counting-on. There was also a significant association between Prompt and Base-10 understanding (partial X^2(1) = 6.33, $p < .05$). Base-10 responses were 3.80 times more likely when children were supplied with a prompt. Hence, the practice of giving children a single example of how to represent multi-digit numbers with tens-and-units considerably increased the prevalence of children's own Base-10 representations. In summary, both training in counting-on and prompting in the use of tens enhances children's performance, but no interaction between these two factors was found. Hence, the effects of training are not confined to just those children who have also been prompted in the use of tens on the post-training Base-10 task.

In a second set of analyses, using composite scores from across the range of tasks, multiple regression analyses were deployed in order to predict children's Base-10 responses on the basis of training in counting-on and prompting in the use of tens to represent multi-digit numbers. A significant model emerged, $F(2,53) = 8.63$, $p < .001$; adjusted $R^2 = .22$. Multicollinearity was not problematic among the predictor variables. Both variables entered into the model contributed significantly to the prediction of children's Base-10 responding: counting-on (standardized beta .28, $t = 2.33$, $p < .05$); and prompting in the use of tens (standardized beta .41, $t = 3.44$, $p < .001$).

It will be recalled that two Base-10 tasks were used in this study, namely, the Ten-blocks and Single-cubes tasks. Half the children were provided with the Ten-blocks task in pre-training and then the Single-cubes task in the post-training assessment. The reverse pattern obtained for the remaining half of the children. A comparison of children's performance on the two tasks in pre-training was conducted on the 82 children from the initial sample capable of understanding the instructions. This comparison revealed no significant difference in difficulty level for the two Base-10 tasks, $t(80) = 1.10$, $p > .05$. The equivalent comparison of performance on the post-training assessment tasks also revealed no significant difference in performance, $t(54) = 0.57$, $p > .05$. This latter comparison naturally included only the 56 children who actually took part in the training part of the study. These findings therefore replicate the findings from Study 1, where it was also reported that performance on the Ten-blocks versus the Single-cubes tasks does not differ significantly. We can be more confident, therefore, that these two tasks provide equivalent versions of the same test for assessing Base-10 performance in children.

In this study, we were careful to assess children's prior knowledge of both Base-10 and counting-on, in order better to isolate the effects of our intervention. It is conceivable, however, that other forms of prior knowledge and experience may have influenced children's performance on post-training assessments. In particular, children possessing knowledge of either trading or partitioning or both may have been at an advantage in this respect. In the following two studies, therefore, we assessed children on *all* of the target skills in order to control for the effects of prior knowledge and its influence on Base-10 performance.

STUDY 3

Studies 3 and 4 examine the influence of trading and partitioning, respectively, on Base-10 performance. As mentioned above, the influence of trading and partitioning was gauged from an assessment of children's prior knowledge of these two skills. This approach was partly dictated by pilot work which revealed that training in trading and partitioning was not easily accomplished in a single session for children aged 5 to 7 years. In Study 3, the focus was on trading and accordingly, children were first screened to assess their knowledge of this skill. Further prerequisites were the need to demonstrate prior knowledge of both cardinality and abstract counting. Finally, children were also screened to exclude those displaying prior knowledge of counting-on, partitioning or the Base-10 system. In this way, we could be more confident that any enhancement in performance on the Base-10 tasks was due to our intervention and not due either to prior knowledge of other associated skills (counting-on and partitioning) or of the Base-10 system. Children who met our screening criteria were assigned to one of two experimental groups: Trading versus No Trading, based on the assessment of their prior knowledge of this skill. These two groups were then both further sub-divided to yield two groups: Training versus No Training in counting-on. Hence, a 2X2 between-subjects design was implemented. We predicted that prior knowledge of trading would interact with training in counting-on to yield enhanced performance on tests of Base-10 knowledge. This prediction is based, in part, on the findings from Study 1, where different combinations of target skills were found to enhance Base-10 performance. It is also based on the strong possibility that our method serves to activate both trading and counting-on (in one experimental group), immediately prior to testing Base-10 performance. Thus, our pre-testing for trading may activate this knowledge in children who possess it, while the intervention should activate procedural knowledge of counting-on.

Method

Participants

The initial sample for this study comprised 144 children, drawn from the same U.K. school described in Studies 1 and 2. The same initial selection criteria were also applied, and a similar mix of social and ethnic backgrounds was noted. There were 66 girls and 78 boys (U.S. Kindergarten level), mean age 65 months, (*SD* = 3 months, range 59 to 70 months). 115 of these children were in Year One of primary school (U.K.), while the remaining 29 children had almost completed Nursery Level within the same school. Of this initial sample, 32 children were excluded on the grounds that they either showed no knowledge of cardinality, or alternatively, that they did show prior knowledge of one or more of the following skills: counting-on, partitioning, or the Base-10 system. The final sample comprised 112 children, 52 girls and 60 boys, mean age 65 months, (*SD* = 2 months, range 60 to 69 months).

Procedure

Pre-training.

Children were assessed individually in sessions lasting approximately 40 minutes and were assigned to testing conditions in a quasi-random manner. Task materials and testing procedures for the assessment of children were identical to those presented in Studies 1 and 2. In addition, children were assessed on their knowledge of cardinality and abstract counting ability (see Study 2 above). In the event, nine children were excluded in consequence of failing the tests either of abstract counting or cardinality or both. Twelve children displayed knowledge of counting-on, while 21 children did well on both tests of partitioning. Finally, 13 children were excluded on the basis of prior Base-10 knowledge. The total number of children excluded from the original sample was 32, a figure which reflects the fact that exclusion was based on performance on one or more of the screening criteria. Assessment of trading was made via both the Large-cube and Small-cube tasks (see Study 1 above). It was found that 56 children succeeded on both of these tasks and were assigned to the Trading group. 56 children from the remaining sample were assigned to the No Trading group to produce (between-subjects) experimental groups of equal sizes. Children in each of these two groups were then allocated on a rotation basis to one of two counting-on conditions: Training or No Training.

Training in counting-on and post-training assessment.

Training in counting-on was implemented within seven to ten days of the pre-training assessments. As before, the Concealed-cubes and Dot-cards activities were employed until the experimenter was satisfied that the child was conversant with the skill of counting-on. This training was immediately followed by the post-training assessment of Base-10 performance, in sessions lasting approximately 30 minutes overall. Again, the procedure mirrored that implemented in the previous study.

Results and Discussion

As in Study 1, children were categorized as Base-10 responders if they provided tens-and-units responses on at least 50% of trials (in this case, two out of four trials). Children who produced fewer responses of this kind were categorized as Units responders. Table 3 reveals that almost 28% of children were classified as Base-10 responders in this way. The success criterion reported in Study 2 was more stringent, being three out of four trials. However, the pattern of findings in Study 2 was not significantly affected when success on Base-10 was taken as either two or three trials out of four. We thus reverted to the original scoring criterion from Study 1 (50% success) in Studies 3 and 4. Hierarchical loglinear analysis was used to explore the associations between group membership (Trading or No

Trading), training in counting-on (Training or No Training) and response strategy (Base-10 or Units). No significant three-way association emerged (Pearson $X^2(1) = .90$, $p > .10$). However, there was evidence of second-order effects (Pearson $X^2(3) = 33.84$, $p < .0001$). Exploration of these second-order effects, via partial associations, revealed an association between training in counting-on and Base-10 responding (partial $X^2(1) = 24.22$, $p < .0001$). Odds ratios revealed that tens-and-units responses are 8.84 times more likely to occur in children who have been trained in counting-on. In addition, a significant association was found between trading and Base-10 understanding (partial $X^2(1) = 16.58$, $p < .0001$). Odds ratios indicated that children with a prior knowledge of trading were 5.69 times more likely to display Base-10 understanding than those lacking this knowledge. The association between trading and training in counting-on was not significant (partial $X^2(1) = 3.13$, $p > .05$), indicating that the effects of training were not confined to those children who possessed prior knowledge of trading. In echo of Study 2, training in counting-on alone can significantly enhance Base-10 performance.

Comparisons revealed no significant difference in children's performance on the two Base-10 tasks both in pre-training (t (133) = -.68, $p > 0.05$) and post-training phases (t (110) = -.46, $p > 0.05$). As in Study 2, therefore, child performance was not affected by the particular Base-10 task administered, pre- or post-training in counting-on.

As in the previous two studies, a second set of analyses was conducted to supplement analyses based on a categorical approach to the data. Accordingly, multiple regression analyses were performed to assess how well training in counting-on and also prior knowledge of trading predicted children's performance on Base-10 tasks. These analyses revealed a significant model, $F(2,109) = 25.91$, $p < .0005$; adjusted $R^2 = .31$, while multicollinearity among the variables was not problematic. Both variables entered into the model contributed significantly to the prediction of children's Base-10 responding: counting-on (standardized beta .42, $t = 5.26$, $p < .0005$); and trading (standardized beta .39, $t = 4.92$, $p < .0005$). In confirmation of the first set of analyses, it emerged that these two factors are therefore making separate contributions as predictors of performance on Base-10 tasks.

STUDY 4

Study 4 was designed to follow up on Study 1 and complement the previous study, by exploring possible interactions among the target skills. The focus in this case was on the effects of prior knowledge of partitioning. Following the pattern of Study 3, we also aimed to gauge the effects of training in counting-on on Base-10 performance. As before, children were first assessed for their prior knowledge of partitioning. We also made sure that participants displayed knowledge of abstract counting and cardinality at the outset. Children were then further screened in order to ensure, as far as possible, that they showed no prior knowledge of counting-on, trading, or Base-10, before being assigned to one of two experimental groups: Partitioning or No Partitioning.

Method

Participants

The initial sample comprised 117 children, drawn from the same state primary school described in the previous studies, according to the same criteria. At the start, there were 64 boys and 53 girls (U.S. Kindergarten level), mean age 67 months, (*SD* = 3 months, range 60 to 72 months). The majority of these children (104) were drawn from Year 1 (U.K.), while the remainder (13) had almost completed Nursery level. Of this initial sample, a total of 53 children were excluded from further participation, following pre-training assessments. Only six children failed the tests of abstract counting and cardinality, while 13 children showed prior knowledge of counting-on, 13 children demonstrated Base-10 understanding and 44 children showed an understanding of trading. These figures include several cases of overlap, and therefore, exclusion was based on responses to one or more of the assessment criteria.

The pre-testing phase left a sample of 65 children, 33 of whom displayed knowledge of partitioning. To create equal sized (between-subjects) experimental groups, one of these 33 children was excluded at random. Thus, the final sample who progressed on to the intervention phase comprised 64 children (31 girls and 33 boys), mean age 67 months (*SD* = 2 months, range = 62 to 72 months). Children were assigned to experimental groups in a quasi-random fashion.

Procedure

Pre-training assessment.

Children were assessed individually in sessions lasting approximately 40 minutes with the same testing regime as reported for Study 3. In this case, though, the results were used to allocate children to experimental groups on the basis of prior knowledge of partitioning (rather than trading). As mentioned, children with prior knowledge of counting-on, Base-10 and/or trading were excluded from further inclusion in this study.

Training in counting-on and post-training assessment.

As before, training in counting-on was followed immediately by a post-training assessment of Base-10 performance. The procedure followed was identical to that implemented in Study 3.

Results and Discussion

As above, in the initial set of analyses, children were categorized as either Base-10 or Unit responders, with the threshold for Base-10 responding set at two or more multi-digit responses out of four. Table 4 reveals that 28% of children were classified as Base-10 responders in this way. Hierarchical loglinear analysis was deployed to examine the relationship between group membership (Partitioning or No Partitioning), training in counting-on (Training or No Training) and response strategy (Base-10 or Units responses). The three-way association among the variables did not reach significance (Pearson $X^2(1)$ = .

06, $p > .10$), but significant two-way associations did emerge (Pearson $X^2(3) = 9.31$, $p < .05$). Further analyses revealed that training in counting-on was significantly associated with Base-10 performance (partial $X^2(1) = 8.82$, $p < .01$). Odds ratios indicated that tens-and-units (Base-10) responses were 5.44 times more likely in cases where training in counting-on had been supplied. In addition, it was found that prior knowledge of partitioning was associated with an increased frequency of tens-and-units responses (partial $X^2(1) = 5.82$, $p < .05$). Prior knowledge of partitioning rendered Base-10 responses 3.70 times more likely than cases where this knowledge was lacking. The association between training in counting-on and partitioning did not reach significance (partial $X^2(1) = .74$, $p > .10$). These findings mirror those reported in Study 3, since the lack of interaction between training and partitioning suggests that prior knowledge of trading is sufficient, in itself, to enhance Base-10 performance.

Comparisons of performance on the two Base-10 tasks revealed no significant differences, either in the pre-training ($t(109) = -.22$, $p > .05$) or post-training phases ($t(62) = .62$, $p > .05$). Hence, further confirmation is supplied that the two Base-10 tasks deployed here present children with challenges of equivalent difficulty.

In a second set of analyses, aggregated scores across tasks were used (where appropriate), in contrast to the categorical approach to the data reported above. These scores were used in multiple regression analyses, conducted to assess how well training in counting-on and also prior knowledge of partitioning function as predictors of Base-10 performance. A significant model emerged, $F(2,61) = 8.28$, $p <.01$; adjusted $R^2 = .19$, with both variables acting as significant predictors: counting-on (standardized beta .35, $t = 3.07$, $p <.01$); and partitioning (standardized beta .30, $t = 2.67$, $p <.05$). Multicollinearity among the variables was not problematic.

GENERAL DISCUSSION

The studies reported here investigated children's ability to represent multi-digit numbers according to the precepts of the Base-10 system, using tens-and-units collections, rather than units only. It was found that children's performance on Base-10 tasks is enhanced under certain conditions: (1) the possession of prior knowledge of certain target skills (counting-on, trading and/or partitioning); and (2) direct training in one of the target skills (counting-on). With regard to the effects of training, it is noteworthy that a relatively brief exposure to the counting-on procedure can be followed by an immediate improvement in performance on (particular) Base-10 tasks. The consistent replication of this effect across Studies 2, 3 and 4 lends confidence in asserting the reliability of this finding.

Effects of Prior Knowledge and Training

With regard to prior knowledge, two main findings emerged from Study 1. First, tens-and-units responses were entirely absent in children who possessed none of the target skills. And second, most children in possession of all three target skills (84%) produced tens-and-units responses. The effects of prior knowledge were also demonstrated in Studies 3 and 4, where it emerged that Base-10 responses were more likely in those children who possessed prior knowledge of either trading or partitioning. Of interest, we found that prior knowledge of these skills exerted an influence only after training in counting-on. This observation raises the possibility that the training in counting-on activates children's prior knowledge of the target skill in a way that exposure to Base-10 problems alone does not. Activation of prior knowledge (of trading or partitioning) would help explain why those children who possess such knowledge subsequently do better on Base-10 tasks in the post-training phase.

Development Across Different Skills

The findings reported here support the idea that development can take place across different mathematical skills, not just within particular skills. Previous work demonstrates that both developmental pathways are possible. Development within the confines of a single mathematical skill has been shown, for example, with decimal fractions (Rittle-Johnson et al., 2001) and multi-digit addition (Hiebert & Wearne, 1996). However, examples can also be found where knowledge of one skill, be it conceptual or procedural, may well contribute to enhancements in knowledge of another skill. For example, children who display conceptual understanding of the commutativity principle are more likely also to show procedural knowledge of counting-on (Baroody & Gannon, 1984; Cowan & Renton, 1996). Similarly, conceptual knowledge of cardinality and order irrelevance are associated with enhanced procedural ability in counting (Cowan, Dowker, Christakis & Bailey, 1996). The work we report is also concerned with development across different mathematical skills. Thus, children who possess knowledge of one or more of the three target skills are more likely to perform well on Base-10 tasks, even where (as in Studies 2, 3 and 4) they have been assessed as lacking Base-10 knowledge beforehand.

It is of interest to consider whether the knowledge transferred across skills was conceptual or procedural in nature. Conceivably, improvements in conceptual knowledge may have arisen in the interventions reported here, but the methodology and the tasks employed do not allow one to draw any strong conclusions in this regard. Tests of conceptual knowledge typically use novel tasks that force children to generate solutions in cases where they do not already have a solution procedure (e.g., Bisanz & Lefevre, 1992; Rittle-Johnson, in press). One might argue that this was the case for the Base-10 tasks used in Studies 2, 3 and 4, since prior testing confirmed that children did not have the relevant procedures for representing multi-unit numbers with tens-and-units. However, we did not probe explicitly for conceptual understanding, either of Base-10 or of the three target skills. It is unlikely we would have found such understanding, given the brief exposure to Base-10 problems experienced by children. The same observation applies to our intervention in counting-on. This method for solving addition problems was introduced to children and practiced with them in a single training session. In the process, there was no emphasis on the conceptual underpinnings of the procedure. In consequence, it seems likely that child performance in our studies relied on procedural, rather than conceptual, knowledge. Success thus relied on the child's ability to identify the structural parallels between procedures required for trading or partitioning problems and those required for representing multi-unit numbers with tens-and-units.

Procedural knowledge is often viewed as not widely generalizable, because it tends to be tied to specific problem types (Kornilaki & Nunes, 2005; Rittle-Johnson et al., 2001). However, our studies have shown that generalization across skills is not confined exclusively to conceptual knowledge. In addition, we provide evidence that generalization of learning from one aspect of mathematical skill to another is not always slow and effortful (c.f. Siegler & Jenkins, 1989). The formal similarities in procedures between the target skills and our Base-10 tasks provides a likely clue in explaining the relative ease observed here in children's ability to generalize *across* mathematical skills.

It is of interest to consider how children achieve the transfer of knowledge from one skill to another. Of relevance in this regard is the distinction between *taught* versus *spontaneously acquired* knowledge (Siegler, 2003). In the studies reported here, children are not explicitly taught anything about the Base-10 system. Instruction in counting-on did provide the child with experience of procedures necessary to solve counting-on problems. Even here, though, there was no explicit instruction on the conceptual basis of counting-on. And no instruction of any kind was given on the critical process of knowledge transfer from one skill to another. Hence, those children who succeed in this task would seem to be doing so spontaneously, albeit following key experiences that clearly contribute to enhanced performance. It is sometimes possible to observe "eureka" moments of spontaneous learning as they occur (Siegler & Jenkins, 1989). But identifying these moments does not thereby yield an explanation of how such learning takes place.

Explaining how development takes place presents a fundamental problem for theorists. In this regard, Siegler (1996, p.177) has observed that "despite the obvious importance of constructing new ways of thinking, we know little about how the process occurs." Acts of spontaneous learning seem to present an especially difficult challenge in this regard, in part because the connections are not obvious between the child's experience and the mental processes that lead to new learning. Nor are the mental processes themselves directly observable or easily described and explained. That said, when children are deliberately shown new mathematical procedures, one can at least identify key experiences associated with change. In the studies reported here, instruction in counting-on seems to be associated with changes (enhanced performance) on Base-10 tasks.

Performance on the Single-cubes and Ten-blocks tasks

Although not related directly to the main aims of this research, a finding that stands out, if only because it was replicated in Studies 2, 3 and 4, is that children perform equally well on both the Single-cubes and Ten-blocks tasks. At face value, one might expect the Ten-blocks task to be easier, if only because the size of each block, when compared with individual cubes, provides a physical clue to the relationship between tens and units. Of note, block size influenced child performance on the trading tasks (see Study 1). In the case of the ten-blocks, moreover, the ten component cubes in each block are clearly demarcated and the child's attention is explicitly drawn to these sub-units when demonstrating the equivalence between the block and a collection of ten units. The demarcations could thus provide the child with a cue concerning the internal composition of ten-blocks. Demarcations aside, though, the Single-cubes task also embodies an explicit demonstration of the equivalence between a single (ten) cube (or block) and a collection of ten individual cubes. Hence, two clear differences remain between the two tasks. First, there is the symbolically potent cue provided by the sheer size of each ten-block, relative to individual unit cubes. And second, there is the demarcation of the block into ten sub-parts. Evidently, these differences are not sufficient to create significant differences in group performance across the two tasks. As it stands, our findings suggest that the most salient cue for children aged five to seven years is provided by the demonstration of equivalence between, on the one hand, the item denoted as being worth ten, and, on the other, a collection of ten individual unit cubes. Concomitantly, one might predict that the physical demarcations within the ten-block do not provide an especially strong cue to the designated numerosity of the ten-blocks. More broadly, future work might consider the extent to which children attend to and assimilate the various cues made available to them on each task.

Precursors in the Acquisition of Base-10 Knowledge

These studies were based initially on the prediction that certain skills can act as *precursors* in the acquisition of Base-10 (procedural) knowledge. In support, Studies 2, 3 and 4 showed that training in counting-on has an immediate (beneficial) effect on children's propensity to give tens-and-units responses. But this conclusion rests on the assumption that the pre-training assessments of children unfailingly provide an accurate picture of children's mathematical knowledge. Given these assessments, it was assumed that participating children possessed no prior knowledge of Base-10. In the event, however, a significant minority of children (13%) performed well on Base-10 tasks in the post-training phase, despite their prior allocation to the No Training (control) groups.

An explanation is required for this sudden demonstration of Base-10 knowledge in some children in the control conditions. One possibility is that children do not deploy their mathematical knowledge in a consistent manner. Nor do they necessarily perform to the maximum possible level of their competence on every occasion. In a similar vein, children may have switched from an unsuccessful to a more successful strategy on the Base-10 tasks from pre- to post-testing phases (Siegler, 1996). Moreover, the simple exposure to numerous mathematical tasks and activities during the course of the study may increase the likelihood of prompting recall of Base-10 knowledge that was not in evidence at the start. This notion of prompting recall, and thereby enhancing performance, was lent credence in Study 2. Children in the Prompt condition outperformed those in the No Prompt condition on Base-10 tasks. Prompting consisted of demonstrating (once only) the use of tens in the representation of a multi-digit number. Again, despite being assessed as showing no prior knowledge of Base-10, many children who were given this single demonstration were subsequently able to produce tens-and-units responses of their own. Moreover, this effect was observed independent of the effects of training in counting-on. In the event, relatively few children demonstrated a sudden, seemingly *ab initio*, knowledge of the Base-10 system in the control conditions. At the same time, it is evident that strong conclusions about training in counting-on as a *precursor* to children's understanding of Base-10 are unwarranted.

Immediate versus Longer-term Effects

It is worth emphasizing that the observed effects of training in counting-on are confined to *immediate* effects only on Base-10 performance. One should exercise caution, therefore, in estimating the impact of training in, and prior knowledge of, the target skills on long-term learning about the Base-10 system. However, the study of immediate effects is not only tractable, but also seems a natural place to start when initiating a research program of this kind. That said, it is of considerable interest to examine also longer-term effects on child understanding and performance. In particular, future research might aim to establish the conditions under which knowledge, transferred from one mathematical skill to another, can be established in long-term memory and, further, accessed under a range of different conditions. In this particular case, it is of interest to examine how newly acquired procedural knowledge of Base-10 might provide the basis for further advances in conceptual knowledge of Base-10. In brief, can interventions of the kind described here provide a basis for developing not just procedural, but (ultimately) *conceptual* knowledge, of the Base-10 system? The answer to this question is of considerable theoretical interest. And given the extensive difficulties experienced by many children in apprehending the Base-10 system, there may well be an important pedagogical motivation for exploring these issues further.

Concluding Remarks

The studies reported here highlight three mathematical skills implicated in success on Base-10 tasks. It has been argued that procedural knowledge about each of these skills can contribute to enhancements in the child's knowledge about the Base-10 system. The nature of the Base-10 knowledge acquired in this way might be viewed as fragile in at least two ways. First, we have, at this stage, evidence only for immediate effects. And second, it is likely that procedural knowledge only is being acquired. Nevertheless, these studies provide an indication that counting-on, trading and partitioning may well prove useful, as a bridge, in permitting access to the complexities of the Base-10 system. These findings are thus of potential importance in pedagogical settings, given the well-documented problems many children experience in acquiring this complex aspect of mathematical knowledge. Research on interventions has demonstrated that a range of instructional approaches can facilitate children's learning about the Base-10 system (e.g., Fuson, Fraivilling & Burghardt, 1992; Fuson, Smith & Cicero, 1997; Hiebert & Wearne, 1992; 1996; Resnick, 1982; Resnick & Omanson, 1987). It may well be of interest, in designing future interventions, to consider more closely the potential roles of counting-on, trading and partitioning.

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Table 1

Study 1: Number of Children out of 97 Succeeding on 0 to 4 Trials for Seven Experimental tasks

	Number of Trials Completed Successfully				
	0	1	2	3	4
Base-10 tasks					
Box	16	8	20	26	27
Ten-blocks	16	18	18	29	16
Single-cubes	17	17	25	22	16
Trading tasks					
Large-cube	14	0	12	22	49
Small-cube	16	13	10	35	23
Partitioning tasks					
Divided Whole	14	8	16	36	23
United Parts	14	8	19	21	35

Table 2

Study 2: Effects of Prompting in the Use of Ten-Blocks and Effects of Training in Counting-On on Frequency of Base-10 and Units Responses

	Prompt		No Prompt	
	Base-10	Unit	Base-10	Unit
Training in Counting-On				
Training	11	3	6	8
No Training	7	7	3	11
Total	18	10	9	19

Table 3

Study 3: Effects of Prior Knowledge of Trading and Effects of Training in Counting-On on Frequency of Base-10 and Units Responses

	Trading		No Trading	
	Base-10	Unit	Base-10	Unit
Training in Counting-On				
Training	19	9	7	21
No Training	5	23	0	28
Total	24	32	7	49

Table 4

Study 4: Effects of Prior Knowledge of Partitioning and Effects of Training in Counting on Frequency of Base-10 and Units Responses

	Partitioning		No Partitioning	
	Base-10	Unit	Base-10	Unit
Training in Counting-On				
Training	9	7	5	11
No Training	4	12	0	16
Total	13	19	5	27

Figure Captions

Figure 1. Model of iterative development of conceptual and procedural knowledge (Rittle-Johnson, Siegler & Alibali, 2001).

Figure 2. Study 1: Frequency of Base-10 and unit responses in relation to the number of target skills demonstrated (counting-on, trading, partitioning)

(Improved)
Representation of
Problem
(Improved)
Conceptual
Knowledge
(Improved)
Procedural
Knowledge