Rayleigh Scattering and Atomic Dynamics in Dissipative Optical Lattices

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We investigate Rayleigh scattering in dissipative optical lattices. In particular, following recent proposals (S. Guibal et al, Phys. Rev. Lett. 78, 4709 (1997); C. Jurczak et al, Phys. Rev. Lett. 77, 1727 (1996)), we study whether the Rayleigh resonance originates from the diffraction on a density grating, and is therefore a probe of transport of atoms in optical lattices. It turns out that this is not the case: the Rayleigh line is instead a measure of the cooling rate, while spatial diffusion contributes to the scattering spectrum with a much broader resonance.

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Light scattering, i.e., the scattering of photons resulting from the interaction with a material medium, is a technique widely used to determine the properties of many different types of media. From the position and the width of the scattering resonances it is in fact possible to identify the dynamical modes of the system and derive the rates of relaxation toward equilibrium. This is well exemplified by the Landau-Placzek relation, valid for light scattering originating from the density fluctuations of a medium at thermal equilibrium, which connects the strength of the different components of the scattering spectrum to the specific heats of the medium at constant volume and constant pressure.

Recently light scattering has been extensively used to study the properties of cold atomic samples, and in particular it turned out to be an essential tool for the understanding of the basic properties of dissipative optical lattices. The same technique may also apply to far-off-resonance non-dissipative optical lattices which are currently investigated by many groups in connection with Bose-Einstein condensation experiments. However to derive the damping rates of the system from light scattering measurements is in general a highly non-trivial task. This is especially true for quasi-elastic (Rayleigh) scattering, which gives access to the relaxation rates of non-propagating material observables. In this work we investigate the mechanism behind the Rayleigh scattering in dissipative optical lattices, and identify the relaxation process which determines the width of the Rayleigh resonance in the scattering spectrum.

The starting point of the present study is the previous claim that Rayleigh resonances may originate from the excitation of the atomic density, and consequently the width of the Rayleigh line would provide a measure of the diffusion coefficients of the atoms in an optical lattice. Following a similar approach, Jurczak et al derived values for the diffusion coefficients from polarization-selective intensity correlations.

In our analysis we first assume, on the lines of these previous works, that the material observable excited in the pump-probe spectroscopy is the atomic density, and derive the expected relation between the width of the Rayleigh resonance and the spatial diffusion coefficients.

Through experimental and theoretical work we show that this relation is actually not satisfied by independent measurements/calculations of the width of the resonance and the diffusion coefficients. Instead, we show that the narrow Rayleigh resonance originates from the atomic velocity damping, i.e., the width of the resonance is a measure of the cooling rate, while spatial diffusion contributes to the scattering spectrum with a much broader resonance.

Consider first the general relation between the width of the Rayleigh line and the relaxation rate of the material observable excited in the optical process. In the basic setup of pump-probe spectroscopy, an atomic sample interacts with two laser fields: a strong pump beam, with frequency \( \omega_p \), and a weak probe beam with frequency \( \omega_p + \delta \). The superimposition of the pump and probe fields results in an interference pattern moving with phase velocity \( v = \delta/|\Delta k| \), with \( \Delta k \) the difference between pump and probe wavevectors. The atomic sample tends to follow the interference pattern and a grating of an atomic observable (typically density, magnetization or temperature) is created. However due to the finite response time of the atomic medium the material grating is phase-shifted with respect to the light interference pattern. Therefore the pump beam can be diffracted on the material grating in the direction of the probe, modifying the probe transmission. It is then clear that it should be possible to derive information about the atomic response time from the transmission spectrum. More precisely if we assume that only one atomic observable is excited in the optical process, and that the time evolution of this observable is characterized by a single relaxation rate \( \gamma \), the probe gain spectrum \( g(\delta) \) has then a dispersive line shape

\[
g \propto \frac{\delta}{\gamma^2 + \delta^2} \tag{1}
\]

with peak-to-peak distance \( 2\gamma \), as derived in [7].

Consider now the specific configuration with linearly-polarized pump and probe beams, the two polarization vectors being parallel. The resulting interference pattern gives rise, via the dipole force, to a grating...
of the atomic density \(n\) of the form

\[
n = n_0 + n_1 (\exp[-i(\delta \cdot t - \Delta \vec{k} \cdot \vec{r})] + \text{c.c.}). \tag{2}
\]

We assume now, following previous work \cite{5, 6}, that the Rayleigh resonance originates from the scattering on this atomic density grating. It follows that the width of the Rayleigh line is related to the spatial diffusion coefficients. Indeed the relaxation mechanism of a grating of atomic density is spatial diffusion: atoms have to move to destroy the density grating. More quantitatively, if we assume that the dynamics of the atoms in the optical lattice is well described by the Fick law

\[
\frac{\partial n}{\partial t} = D_x \frac{\partial^2 n}{\partial x^2} + D_y \frac{\partial^2 n}{\partial y^2} + D_z \frac{\partial^2 n}{\partial z^2}, \tag{3}
\]

where \(D_i\) \((i = x, y, z)\) is the spatial diffusion coefficient in the \(i\) direction, we find substituting the expression \(\delta\) for \(n\) in \(\delta\) that the relaxation rate \(\gamma_D\) of the atomic density, defined by

\[
\frac{\partial n_1}{\partial t} = -\gamma_D n_1, \tag{4}
\]

is given by

\[
\gamma_D = D_x \Delta k_x^2 + D_y \Delta k_y^2 + D_z \Delta k_z^2. \tag{5}
\]

Under the assumption that the Rayleigh resonance originates from the scattering on the atomic density grating, the half-distance peak-to-peak of the Rayleigh line \(\gamma_R\) is simply equal to the relaxation rate \(\gamma_D\), and therefore measurement of \(\gamma_R\) allows the determination of the diffusion coefficients, as in Refs. \cite{3, 4, 5, 6}. The validity of this approach will be tested by comparing results for the relaxation rate \(\gamma_D\) with measurements of the width of the Rayleigh resonance, as presented below.

In our experiment rubidium atoms are cooled and trapped in a three-dimensional (3D) linear optical lattice \cite{2}. The periodic structure is determined by the interference of four linearly polarized laser beams, arranged as in Fig. 1. The angle \(2\theta\) between copropagating lattice beams is equal to \(60^\circ\). This four-beam configuration is the same, except for the value of the angle \(\theta\), as the one considered in the works of Guibal et al \cite{2, 7} and Jurczak et al \cite{8, 9}.

To determine in a direct way the spatial diffusion coefficients of the atoms in the optical lattice, we observe the atomic cloud expansion by using a Charge Coupled Device (CCD) camera \cite{3, 4, 10}. Since the \(x\) and \(y\) directions are equivalent in our lattice (see Fig. 1), we chose to take images in the \(\xi Oz\) plane, where \(\xi\) is the axis in the \(xOy\) plane forming an angle of \(45^\circ\) with the \(x\) and \(y\) axis. Correspondingly, we determined the diffusion coefficients \(D_x\) and \(D_z\) in the \(\xi\) and \(z\) directions, with results as in Fig. 2.

These values for the diffusion coefficients are not consistent with the value of about \(10 \hbar/M\) determined for the same configuration by Jurczak et al \cite{8} by polarization-selective intensity correlations. As we will show in the following, this inconsistency derives from the unreliability of the determination of the diffusion coefficients by light scattering measurements, as this derivation of the diffusion coefficients is based on the assumption that the narrow Rayleigh resonance originates from the diffraction on an atomic density grating.

We turn now to the measurements of the width of the Rayleigh resonance. The \(y\) polarized probe beam is derived from the lattice beams, with the relative detuning controlled with acousto-optical modulators. This probe beam is sent along the \(z\) axis through the cold atomic sample, (Fig. 1) with its frequency scanned around the lattice beams’ frequency. The probe can interfere with
the different lattice beams, which play the role of the pump.

A typical probe transmission spectrum is shown in Fig. [3]. The lateral resonances have been characterized in great detail in past investigations [11], and we focus here on the resonance at the center of the spectrum (inset of Fig. 3). To determine whether this Rayleigh resonance can be associated with the relaxation mechanism of spatial diffusion, we made a systematic study of the width of the resonance as a function of the interaction parameters (lattice-field intensity and detuning).

The peak-to-peak distance $2\gamma_R$ of the Rayleigh resonance has been determined by fitting the central part of the probe transmission spectrum with the function

$$f(\delta) = a_1 + a_2\delta + \frac{a_3}{\delta^2 + \gamma_R^2} + \frac{a_4\delta}{\delta^2 + \gamma_R^2}. \quad (6)$$

Here the linear term in detuning describes the wings of the sideband resonances [11]. The Lorentzian resonance arises from the radiation pressure, and has the same width of the dispersive line, as discussed in Ref. [3]. Experimental results for $\gamma_R$ are reported in Fig. 4 as a function of the lattice beam intensity, for different values of the lattice detuning.

We now describe the determination of $\gamma_D$. In the examined configuration the probe beam can interfere simultaneously with all lattice beams. Therefore the situation is slightly more complicated than the one analyzed previously leading to Eq. 3 and to derive the link between the width of the Rayleigh resonance and the diffusion coefficients we have to calculate the interference pattern between the probe and the lattice beams. By using the expression for the lattice-beams electric fields for a 3D lin-lin optical lattice [2], we easily find that the intensity-modulation produced by the probe beam is:

$$\delta|\bar{E}|^2 \simeq E_p^*E_0 \cos(Kx) \exp\{i[(K+\xi)z+\delta\cdot t]\} + c.c., \quad (7)$$

with $E_0$ ($E_p$) the amplitude of the lattice (probe) field, $K = k \sin \theta$ and $K_+ = k \cos \theta$. Substituting in Fick's law, Eq. 5, the resulting modulation for the atomic density we find that the relaxation rate $\gamma_D$, defined via Eq. 6, is in the present case

$$\gamma_D = D_x(k \sin \theta)^2 + D_zK^2(1 - \cos \theta)^2. \quad (8)$$

This equation is consistent with the relation derived in Ref. [3] in the limit of small $\theta$. To determine whether the rate $\gamma_D$ of relaxation of the atomic density is equal to the width of the Rayleigh resonance, we calculate from the values $D_x$, $D_z$ of Fig. 2 the relaxation rate $\gamma_D$ of the atomic density, using Eq. 5, with results as in Fig. 4. For the same range of interaction parameters the relaxation rate $\gamma_D$ is four orders of magnitude larger than the half-distance peak-to-peak $\gamma_R$ of the Rayleigh resonance. We therefore conclude that the Rayleigh resonance does not originate from the diffraction on an atomic density grating, and therefore measurements of the width of the Rayleigh line do not allow the determination of the spatial diffusion coefficients.

Our conclusions, based on the presented experimental findings, are supported by numerical calculations. We consider a $J_\theta = 1/2 \to J_r = 3/2$ atomic transition, as customary in numerical analysis of Sisyphus cooling. Taking advantage of the symmetry between the $x$ and $y$ directions (see Fig. 1), we restricted the atomic dynamics to the $xOz$ plane. Through semiclassical Monte Carlo calculations [12, 13], we simulate the dynamics of the atoms in the optical lattice. From the atomic trajectories we determine then the probe transmission spectra and the evolution of the atomic mean square displacements. We calculate the width of the Rayleigh line and the spatial diffusion coefficients. From these diffusion coefficients we then derive through Eq. 5 the relaxation rate of the atomic density. The comparison between the numerically calculated $\gamma_R$ and $\gamma_D$, as shown in Fig. 4 confirms that the width of the Rayleigh line does not correspond to the rate of relaxation of the atomic density.

The final step of our analysis consists in identifying the damping process which leads to the phase shift producing the Rayleigh scattering. Inspired by previous studies of stimulated Rayleigh scattering in corkscrew optical molasses [12], we numerically examined the damping process
FIG. 5: Numerically calculated relaxation rate of the atomic density $\gamma_D$, half-distance peak-to-peak $\gamma_R$ of the Rayleigh line and relaxation rates $\Gamma_{T_x}$, $\Gamma_{T_z}$ of the atomic temperature in the $x$ and $z$ directions. All these quantities are reported as functions of the optical pumping rate $\Gamma_0$, for different values of the light shift per beam $\Delta_0$. Here $\omega_r$ is the atomic recoil frequency.

of the atomic velocity in the optical lattice and calculated the relaxation rates $\Gamma_{T_x}$, $\Gamma_{T_z}$ of the atomic temperature in the $x$ and $z$ directions, with results as in Fig. 5. It appears that the damping rates of the atomic temperature not only are of the same order of magnitude of the width of the Rayleigh line, but that they also display the same linear dependence on the optical pumping rate $\Gamma_0$, at fixed light shift per beam $\Delta_0$, i.e. at fixed depth of the potential wells. More precisely, neglecting $\Gamma_{T_z}$ as $\Gamma_{T_x} << \Gamma_{T_z}$ we find from the data of Fig. 5 that

$$\gamma_R = 0.13(\pm 0.04)\omega_R + 0.25(\pm 0.02)\Gamma_{T_z}. \quad (9)$$

This shows that for an optical lattice the width of the Rayleigh line is a measure of the cooling rate, a behaviour already encountered in corkscrew optical molasses. It is then legitimate to investigate the eventual contribution of the light scattering on the density grating to the probe transmission spectrum. By fitting the broad wings of the numerically calculated spectra with a dispersive function, we found that the corresponding width is approximately equal to the relaxation rate $\gamma_D$. This shows that the scattering on the density grating contributes to the probe transmission spectrum with a resonance much broader than the narrow line observed at the center of the spectra. In other words, the information on the spatial diffusion coefficients is contained in the broad wings ($\omega \gtrsim 10 \text{ MHz}$) of the scattering spectrum, and not in the central narrow resonance.

In summary, in this work we investigated the connection between Rayleigh scattering and the atomic dynamics in dissipative optical lattices. In particular, following recent proposals, we studied whether the Rayleigh resonance originates from the diffraction on a density grating, and is therefore a probe of transport of atoms in optical lattices. It turns out that this is not the case: the Rayleigh line is instead a measure of the cooling rate, while spatial diffusion contributes to the scattering spectrum with a much broader resonance.

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