The Antarctic stratospheric sudden warming of 2002: A self-tuned resonance?

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The extraordinary Antarctic stratospheric warming event of 2002 was characterized by a remarkable vertical structure, with the vortex observed to divide at upper levels in the stratosphere but not at lower levels: such ‘partially’ split vortex events are relatively rare. A simple, yet fully three-dimensional, model is constructed to investigate the dynamics of this unique event. Planetary waves are excited on the model vortex edge by a lower boundary forcing characterized by two parameters: an amplitude $h_F$ and a frequency $\omega_{fs}$ measured relative to a stationary frame. For realistic forcing amplitudes, a partial vortex split resembling that observed during the 2002 event is found only within a specific, narrow band of forcing frequencies. Exploiting the relative simplicity of our model, these frequencies are shown to be those causing a ‘self-tuning’ resonant excitation of the gravest linear mode, during which nonlinear feedback causes an initially off-resonant forcing to approach resonance. Citation: Esler, J. G., L. M. Polvani, and R. K. Scott (2006), The Antarctic stratospheric sudden warming of 2002: A self-tuned resonance?, Geophys. Res. Lett., 33, L12804, doi:10.1029/2006GL026034.

1. Introduction

[2] The remarkable Antarctic stratospheric sudden warming of 2002 has attracted great interest amongst the atmospheric science community (e.g., Journal of Atmospheric Science, 2005, 62(3)), primarily because such an event is unprecedented in roughly 50 years of observations. Between September 23 and September 26, above the 600 K isentropic level (~26 km), the stratospheric vortex was observed to split into two parts [Charlton et al., 2005], and the attendant higher polar temperatures had a dramatic impact on subsequent chemistry with substantially reduced ozone depletion [Stolarski et al., 2005]. A detailed understanding of such significant events is therefore necessary in order to assess the likelihood of future occurrences, with the attendant consequences on the Antarctic ozone hole.

[3] The short timescale associated with stratospheric warmings, typically of the order of several days, indicates that they must be essentially fluid dynamical events, with radiative and chemical processes playing a secondary role. On these time scales the dynamics are essentially adiabatic and ‘balanced’, and hence can be understood on the basis of the three-dimensional distribution of potential vorticity (PV). For the 2002 event, the evolution of one isosurface of scaled PV, derived from ECMWF operational analysis data, is shown in the top row of Figure 1. The isosurface and scaling parameters have been chosen to obtain an accurate fit to the vortex edge over a large altitude range, as described further below. Approximately 20 days before the event, on September 5, the vortex is seen to be relatively cylindrical in appearance. However, at 1200 UT on September 23 the vortex has become strongly elongated throughout its altitude range. The vortex is clearly split by September 26, but only at upper levels, while below ~26 km it appears to have recovered its circular shape. This ‘partial’ split is a distinguishing feature of the 2002 event, and is distinct from the vortex behavior observed for most Northern Hemisphere events [Manney et al., 2005], during which the vortex is observed to split near-simultaneously over its entire altitude range [e.g., Manney et al., 1994].

[4] Although the Antarctic 2002 event was forecast accurately [Simmons et al., 2005], little insight is gained as to which specific dynamical aspects are responsible for the unusual partial split structure. Furthermore, other modeling studies [e.g., Mukougawa et al., 2005] have shown that the vortex evolution is highly sensitive to initial conditions, and it remains unclear what ingredients are needed to produce a successful forecast.

[5] The aim of this work, therefore, is to use a relatively simple model to determine the dynamical conditions necessary to generate a stratospheric sudden warming whose three-dimensional structure resembles the partial split of the 2002 event. The model’s relative simplicity allows a thorough exploration of parameter space and, in particular, the delineation of the narrow region over which partial vortex splits occur. More importantly, however, the model’s simplicity allows for contact to be made with analytic results [Esler and Scott, 2005] which yield understanding into the underlying fundamental dynamics.

[6] Specifically, we aim to demonstrate that the Antarctic 2002 event can be understood as a ‘self-tuned’ resonance, in the sense of Plumb [1981] who showed, using a simple model of the stratosphere, that the maximum wave amplification occurs when the system is forced with a frequency that differs by a finite amount from that of a free mode. As the wave grows the system self-tunes toward resonance by nonlinear feedback. Details of the model and the numerical experiments are given in section 2, the model results are discussed in section 3, and conclusions are given in section 4 below.

2. Formulation of Model and Experiments

[7] Arguably the simplest model to capture the fundamental fluid dynamics of stratospheric sudden warmings is
that of a three-dimensional quasi-geostrophic flow in a compressible atmosphere on an f-plane [Dritschel and Saravanan, 1994]. In this model the columnar polar vortex is represented, at each log-pressure height z, by a patch \( S(z) \) of uniformly high PV, which is initially circular with radius \( R(z) \). Outside the vortex, the PV \( q \) is constant; inside \( q \) is a function of \( z \). The flow is adiabatic and frictionless, and hence conserves PV; its dynamics thus obey:

\[
(\partial - \psi_y \partial_x + \psi_x \partial_y) q = 0,
\]

where \( q \) is defined by

\[
q(x,z) = f + \nabla_H^2 \psi + \frac{1}{\rho} \left( N^2 \psi_z \right)_z = \begin{cases} f + \Delta(z) + \Omega & x \in S(z), \\ f + \Omega & \text{otherwise.} \end{cases}
\]

Here \( \psi \) is a streamfunction for the horizontal velocity, \( \mathbf{u} = -\nabla_H \times \psi \mathbf{k} \), \( \nabla_H^2 \) is the horizontal Laplacian operator, \( f \) is the Coriolis parameter, \( N \) is the buoyancy frequency, \( \Omega \) is a constant PV value, and \( \rho = \exp(-z/H) \) is the density, with \( H \) a constant scale height. The function \( \Delta(z) \) denotes the potential vorticity jump at the vortex edge. The lower boundary condition of the model, is

\[
f \psi_z + N^2 h = 0, \quad \text{on } z = 0.
\]

where \( h \) is a ‘topographic’ forcing which excites planetary-scale waves that propagate on the vortex edge.

The values of the model parameters are chosen as given by Waugh and Dritschel [1999]: \( H = 6.14 \text{ km}, f = 4\pi \text{ days}^{-1} \) and \( N = 2.13 \times 10^{-2} \text{ s}^{-1} \), which yields a Rossby radius \( L_R = NH/f = 900 \text{ km} \). The functional forms for \( \Delta(z) \) and \( R(z) \), are obtained by making a crude fit to the observed PV on September 11, 2002, as given by ECMWF operational analysis data sets and plotted in Figure 2. The position of the observed vortex edge, defined as being the location of the maximum gradient in PV with respect to equivalent latitude on each isentropic surface, is marked with a set of crosses on Figure 2. In order to fit this surface, we choose \( \Delta(z) \) and \( R(z) \) as follows

\[
\Delta(z) = \begin{cases} 0 & z < H \\ 0.6f & 7L_R < z < 2H \\ 0.4f & r(z) L_R < 2H < z < 8H \\ 0.4f & 2L_R < z > 8H \end{cases}
\]

As Figure 2 shows, the observed PV is roughly uniform inside the vortex, which has a weak poleward slant with height; we capture this by letting \( r(z) = 4.1 - 0.27z/H \). A PV gradient is also added in the upper troposphere \((H < z < 2H)\), to represent the subtropical tropospheric jet. A choice of \( \Omega = -0.07f \) was found to give the best fit of the model.
respectively, and are then co-located with the
/C0 corresponds to a peak to trough difference in
Kk g a Bessel function. With this choice,
and h
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φt, are
functions of the model, two alternative
interpretations of \( \omega_F \) exist: it is either the frequency of a
transient lower boundary forcing as in (5) above, or it is a
measure of the strength of an anticyclonic solid body
rotation added to the initial flow \( (\Omega \rightarrow \Omega - \omega_F) \) for
the case of a stationary lower boundary forcing. What is
important, therefore, is the angular velocity at the vortex
dge relative to the forcing. Considering the latter
interpretation, it is easily shown that an increase in \( \omega_F \) of 0.01f is
equivalent to reducing the initial stratospheric jet strength
by a mere 1.7 ms \(^{-1}\): this, however, can have dramatic
consequences on the evolution of the flow, as shown by
the regime diagram in Figure 3. In Figure 3 the outcome of
the model experiments, as a function of \( (\omega_F, h_F) \), are
summarized. Vortex splits, denoted by red diamonds, occur
first for \( h_F \sim 0.09 H \), and then only within a vanishing small
range of forcing frequencies around \( \omega_F = -0.01305f \). As \( h_F \)
increases, the range of forcing frequencies over which the
vortex splits broadens considerably: however, in most cases,
the vortex splits over its entire height. Partial vortex splits
(blue squares), such as the one shown in the bottom row of
Figure 1, occur over a much narrow range, and only for 0.09
\( H \leq h_F \leq 0.13 H \).

13 Is it possible to understand such complex nonlinear
behavior from the predictions of linear theory? In order to
answer this question, the frequencies of the linear normal
modes of the initial vortex were calculated, using the
eigenvalue method described by Waugh and Dritschel
The three gravest vertical modes were found to have frequencies corresponding to $\omega_F = \{0.0114, 0.149, 0.160\}$, respectively. Those model experiments leading to vortex splits, as described above, occur for forcing frequencies closest to that of the gravest mode (or barotropic mode), highlighted by the vertical dotted line on Figure 3 (0.0114f). Can the vortex splits be associated with the excitation to finite amplitude of the gravest linear mode? A series of model experiments with comparatively low $h_F$ (0.01 $H \rightarrow 0.08 H$) was performed to address this question. For each $h_F$, the value of $\omega_F$ which caused the largest disturbance to the vortex is precisely the cause of both partial and complete vortex splits. Model experiments provide strong evidence that a self-tuning resonant excitation of the gravest linear mode of the vortex is important for wave-2 sudden warmings.

[15] Different initial vortex structures have also been investigated. The initial inward tilt of the vortex was found to be important in allowing partial, rather than complete, vortex splits. For example if the vortex is initially cylindrically forced and resonant excitation occurs, partial vortex splits occur. The model tropopause at $H < z < 2 H$ also appears to have a role in transmitting disturbances to the vortex, although vortex splits also occur in its absence. Each of the vortex structures investigated could be made to split by exciting the gravest linear mode at the correct ‘off-resonant’ frequency.

[16] It is intended that variations of the methods used in this study might be used in practice to assess the likelihood that a specific observed vortex will subsequently undergo a self-tuning resonant excitation, leading to a sudden warming event. However, since the observed vortex is never in a truly undisturbed state it is unclear how one might calculate a priori the frequencies associated with linear resonances. Nevertheless, some progress may be possible. In addition, however, two clear and important qualitative results have emerged from this study, and it is worth emphasizing them. First, it was found that partial vortex splits occur only over a very small range in parameter space: this might explain why such events are relatively rare. Second, it was shown that the vortex evolution in its nonlinear stage is extremely sensitive to characteristics of the forcing: this provides one reason why such events are in practice, rather difficult to forecast.

**Figure 3.** Regime diagram of model behavior as a function of forcing amplitude $h_F$ and forcing frequency $\omega_F$. Red diamonds denote model experiments which result in a complete vortex split, and black triangles denote those experiments in which no split occurs. Blue squares denote experiments where a partial split, such as that illustrated in Figure 1, occurs. The inset is an enlargement of the boxed region. The value of $\omega_F$ causing resonant excitation of the gravest linear mode of the vortex is given by the dotted line. For each forcing amplitude $h_F = 0.01 H, 0.02 H, \ldots , 0.08 H$, a black cross marks the particular forcing frequency $\omega_F$ that causes the greatest disturbance to the vortex.

4. Discussion and Conclusions

[14] In this letter, it has been demonstrated that a simple quasi-geostrophic model is able to capture the unusual ‘partial split’ evolution of the Antarctic vortex observed during the September 2002 sudden warming. The simplicity of the model allows both a thorough exploration of parameter space and a connection to be made with linear theory. In particular it is possible to test the idea that there is a connection between vortex split sudden warmings and the resonant excitation of a linear mode of the flow [Tung and Lindzen, 1979]. Previous theory [Plumb, 1981] predicts that the maximum response should occur when the forcing is initially off-resonant and the flow ‘self-tunes’ toward resonance as the disturbance grows to finite amplitude. The model experiments provide strong evidence that a self-tuning resonant excitation of the gravest linear mode of the vortex is precisely the cause of both partial and complete vortex splits. Model experiments designed to excite other vertical modes of the vortex were also attempted and were not found to cause either partial or complete vortex splits. Hence we conclude that only the gravest linear mode is important for wave-2 sudden warmings.

**References**


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