Spin resonance is observed using microwave electromagnetic and acoustic fields as probes. Interactions between the spins broaden the resonance, whose line shape has been defined elsewhere [1]. The shape may depend on the probe used, and for ions with an effective spin $S' > \frac{1}{2}$ different line shapes result in general [2]. It is proposed to show that the line shape does not depend on the probe for $S' = \frac{1}{2}$; this follows from the possibility of writing any moment of the line shape in a form which does not depend on the details of the probe.

The Hamiltonian for the system of spins is

$$\mathcal{H} = \mathcal{H}_z + \mathcal{H}_{ss} + \mathcal{H}_I,$$

where $\mathcal{H}_z$ is the Zeeman energy for a magnetic field parallel to the $z$ axis, and $\mathcal{H}_I$ describes the effect of the probe. $\mathcal{H}_I$ will be considered small. $\mathcal{H}_{ss}$ is the spin-spin interaction:

$$\mathcal{H}_{ss} = h_{ss} + h_{ss}',$$

where $h_{ss}$ commutes with $\mathcal{H}_z$. The spin-spin interaction is truncated by dropping $h_{ss}'$, which gives subsidiary lines to the main resonance line [3].

If

$$\langle \mathcal{H}_z + h_{ss} | n \rangle = E_n | n \rangle$$

the moments of the line shape for the interaction $\mathcal{H}_I$ are:

$$\langle \omega^N \rangle_I = K \sum_{n,n'} \omega^N_{nm'} \langle n | \mathcal{H}_I | n' \rangle^2,$$

where

$$\hbar \omega_{nm'} = E_n - E_{n'},$$

and $K$ is a normalising factor.

It follows from the definition of an effective spin of $\frac{1}{2}$ that it is always possible to write the probe interaction, $\mathcal{H}_I$, with such spins as:

$$\mathcal{H}_I = \sum_i (a_i S^i_x + b_i S^i_y + c_i S^i_z + d_i t^i),$$

$i$ labelling the spins.

In some cases this can be written

$$\mathcal{H}_I = (A S_x + B S_y) + (C S_z + D t)$$

$$= h_I + h_I',$$

where

$$S_\alpha = \sum_i S^i_\alpha$$

and

$$[\mathcal{H}_z + h_{ss} S_\alpha, h_I] = 0.$$

$h_I'$ can only connect $|n\rangle$ and $|n'\rangle$ if $\omega_{nm'}$ is zero, and so does not contribute to the moments.

The important terms in the Hamiltonian are:

$$\mathcal{H}_z = E S_z$$

$$h_{ss} = \sum_{j,k} [f_{jk} S_j^z S_k^z + g_{jk}(s_j^+ s_k^- + s_j^- s_k^+)]$$

$$h_I = A S_x + B S_y.$$
Choosing $Bc = As$ and writing
\[ K = \left\{ \sum_{n,n'} |\langle n | h_f | n' \rangle|^2 \right\}^{2/3} \]
the $N$th moment is
\[ \langle \omega^N \rangle = \frac{\sum_{n,n'} \omega_{nm}^N |\langle n | a_x | n' \rangle|^2}{\sum_{n,n'} |\langle n | a_x | n' \rangle|^2} \]
and does not depend on $A, B, C, D$ which characterise the probe. The moments (and therefore the line shape) are the same for any probe for which (1) and (2) are equivalent. They are certainly equivalent if the probe interaction does not vary from spin to spin, i.e. if $a_i, b_i, c_i$ and $d_i$ are independent of $i$. In practice there is a complication in that the probe is often a system of standing waves which interact less strongly with spins near nodes than with those near antinodes. Microwave acoustic and electromagnetic fields correspond to wavelengths which are large compared with the spin separations; the $a_i, b_i, c_i$ and $d_i$ are constant over regions containing many spins and (3) holds for each region.

The proof cannot be extended to include spin systems with more complex groups of low lying levels, for their interaction with the probe need not be linear in the spin components. The linearity in spin components was used at several points in this discussion.

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References

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A NOTE ON A PARTIAL SUMMATION OF GRAPHS IN MANY BODY THEORY

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The relation of the Hartree-Fock theory to the graphical perturbation theory of many partial systems was shown by Goldstone [1]. Thouless [2] noted that starting from free particles one can formally do a partial summation of graphs which leads to the replacement of the kinetic energies with the Hartree-Fock energies. In this summation there is a geometrical series and as emphasized several times [2-4] it does not converge in general and the result is therefore only formal.

Attention should be paid, however, to the fact that the rules for graphs are derived by a limiting process and one must always be careful in changing the order of an infinite summation and a limiting process. In this case it proves that doing the partial summation before going to the limit the result will come out quite correctly.

The calculation goes then as follows:

Take an arbitrary "naked" graph, fig. 1, and consider especially the contribution in it of a particle line $j$ which starts at the time $t_1$ and ends at $t_v$. The value of the graph can be written
\[ \ldots \sum \ldots \exp\{-\Sigma T_j (t_v - t_\mu)\} \ldots \]
\[ \ldots j \ldots \]

Next, "dress" this particular line of the graph as shown in fig. 2 with the additional interaction lines at the times $t_1', \ldots , t_k'$. Now, the sum of all graphs with fixed $n_1, n_2, n_3, n_4$ will be
\[ \ldots \sum \ldots \exp\{\Sigma T_j (t_v - t_\mu)\} \{\int_{t_\mu}^{t_1} \Sigma_1 d'\}^m \times \]
\[ \ldots j \ldots \]
\[ \ldots \sum \ldots \exp\{\Sigma_2 d'\}^n_2 \{\int_{t_\mu}^{t_2} \Sigma_3 d'\}^n_3 \{\int_{t_\mu}^{t_3} \Sigma_4 d'\}^n_4 \ldots \]

where
\[ \Sigma_1 = \Sigma_2 = \sum_{r \in k_F} \frac{1}{2} V_{jrr} ; \Sigma_3 = \Sigma_4 = \sum_{r \in k_F} \frac{1}{2} V_{jrr} \]
yielding

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