Charged-Particle Multiplicity $p \bar{p}$ Collisions at $\sqrt{s} = 1.8$ TeV


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We report on a measurement of the mean charged-particle multiplicity of jets in dijet events with dijet masses in the range 80–630 GeV/c², produced at the Tevatron in p ¯p collisions with √s = 1.8 TeV and recorded by the Collider Detector at Fermilab. The data are fit to perturbative-QCD calculations carried out in the framework of the modified leading log approximation and the hypothesis of local parton-hadron duality. The fit yields values for two parameters in that framework: the ratio of parton multiplicities in gluon and quark jets, r = N_{g\text{-jet}}/N_{q\text{-jet}} = 1.7 ± 0.3, and the ratio of the number of charged hadrons to the number of partons in a jet, K_{\text{LPHD}} \text{ charged} = N_{\text{hadrons}}/N_{\text{partons}} = 0.57 ± 0.11.

Measurement of inclusive charged-particle multiplicities in jets allows testing of the applicability of perturbative QCD methods to the description of the soft process of jet fragmentation. We present here multiplicities measured in dijet events with dijet masses between 80 and 630 GeV/c². These results are compared with perturbative QCD calculations carried out in the framework of the modified leading log approximation, (MLLA) [1], and the hypothesis of local parton-hadron duality, (LPHD) [2]. From this comparison we extract the value of the ratio of parton multiplicities in gluon and quark jets, r = N_{g\text{-jet}}/N_{q\text{-jet}} = 1.7 ± 0.3, as well as the ratio of the number of charged hadrons to the number of partons in a jet, K_{\text{LPHD}} \text{ charged} = N_{\text{hadrons}}/N_{\text{partons}} = 0.57 ± 0.11.

Recent and more accurate solutions [5–7] of the same primary set of QCD evolution equations that forms the basis of the MLLA have resulted in corrections to both N_{\text{partons}} and r. Reported results for the next-to-MLLA correction factor for N_{g\text{-jet}} are F_{n\text{MLLA}} = 1.13 ± 0.02 [5], 1.50 ± 0.08 [6], and 1.40 ± 0.01 [7]. The parameter r takes the values 1.75 ± 0.05, 1.60 ± 0.05, and 1.79 ± 0.07, respectively. For all three calculations, both F_{n\text{MLLA}} and r show little energy dependence and were treated as constants in this analysis. The uncertainties in the numbers quoted above correspond to the range of dijet masses in our sample.

Experimentally, early measurements of r were consistent with 1.0 [8]. More recent results from LEP and SLAC range from 1.0 to 1.5 with typically quite small errors [9]. The spread of the results motivates an independent measurement performed by different methods and in a different environment. Analyses of charged-particle momentum spectra at LEP yield K_{\text{LPHD}} \text{ charged} = 1.28 ± 0.01 [10](about twice the expected value). These measurements are obtained assuming F_{n\text{MLLA}} = 1 and r = 9/4. If one uses F_{n\text{MLLA}} = 1.3 ± 0.2 (the range suggested by [5–7]) and r = 1.5 (the most recent measurements from LEP [9]), one arrives at K_{\text{LPHD}} ~ 0.67.

At the Fermilab Tevatron, dijet events are a mixture of gluon and quark jets. Denoting the fractions of gluon and quark jets as ϵ_g and ϵ_q = (1 − ϵ_g), one can derive an expression for the multiplicity of charged-particles in the mixed jets:

N_{\text{hadrons}}^{\text{charged}} = K_{\text{LPHD}} \text{ charged} \left( ϵ_g + (1 − ϵ_g)\frac{1}{r}\right) F_{n\text{MLLA}} N_{\text{partons}}^{g\text{-jet}}.

The current analysis is based on 95 pb⁻¹ of p ¯p collisions at √s = 1.8 TeV recorded by the Collider Detector at Fermilab (CDF). The CDF detector is described in detail in [11], and references therein. Here, we will focus on those elements of the detector that are directly related to this analysis: the vertex detector (VTX), the central tracking chamber (CTC), and the full set of electromagnetic and hadronic calorimeters.
The VTX is a time-projection drift chamber and determines the $z$ position of the primary vertex (or vertices in the case of multiple $p\bar{p}$ interactions in the same bunch crossing). The CTC is an open-cell drift chamber designed for measuring particle trajectories. Determination of a particle’s momentum is based on the curvature of its trajectory in the solenoidal magnetic field. In our analysis, we considered particles falling in restricted cones around the jet axis and determined the angular parameters of their trajectories with the CTC.

The jet energy and direction were measured in the central lead-scintillator electromagnetic (CEM) and iron-scintillator hadronic (CHA) calorimeters. The CEM and CHA both have $2\pi$ azimuthal coverage. In pseudorapidity $|\eta|<1.0$. The segmentation of both detectors is $15^\circ$ in $\phi$ and 0.1 unit in $\eta$.

CDF defines jets using a cone algorithm; full details can be found in [13]. The algorithm searches in cones of radius $R = \sqrt{(\Delta \phi)^2 + (\Delta \eta)^2} = 0.7$ around the calorimeter seed towers (any tower with transverse energy $E_T$ above 1 GeV) and adds towers with $E_T$ above 0.1 GeV. If two or more adjacent seed towers are found within $R = 0.7$, they are merged. The coordinates of the jet axis are calculated as $E_T$ - weighted sums of the $\phi_i$ and $\eta_i$ of towers assigned to the jet. Merging continues until a stable set of clusters is found. Corrections were applied to compensate for the nonlinearity and nonuniformity of the energy response of the calorimeter, the energy deposited inside the jet cone from sources other than the parent-parton, and the parent-parton energy that radiates out of the jet cone.

Approximately 100,000 dijet events were accumulated using single-jet triggers with transverse jet energy thresholds of 20, 50, 70, and 100 GeV, the first three triggers being prescaled by 1000, 40, and 8, respectively. To select dijet events, we required the presence of two high-$E_T$ jets, well balanced in the transverse direction: $|E_{T_1} + E_{T_2}|/(E_{T_1} + E_{T_2}) < 0.15$. To avoid biases, 3- and 4-jet events were allowed as well, if the nonleading extra jets were very soft, $(E_{T_3} + E_{T_4})/(E_{T_1} + E_{T_2}) < 0.05$. Only events with both leading jets in the central region of the detector ($|\eta_{1,2}| < 0.9$) were retained for the analysis to ensure that the tracks fell in the fiducial volume of the CTC. The data were further subdivided into nine bins according to dijet mass $M_{JJ}$, as measured by the calorimeters $M_{JJ} = \sqrt{(E_1 + E_2)^2/c^4 -(\vec{P}_1 + \vec{P}_2)^2/c^2}$, where $E_i$ and $\vec{P}_i$ are the jet energies and momenta, and the jets were treated as massless objects, i.e., $|\vec{P}_i| = E_i/c$.

The bin width was uniform in log scale, $\Delta \ln(M_{JJ}) = 0.3$, and was always larger than the resolution errors in the dijet mass determination, $\delta M_{JJ}/M_{JJ} \sim 7\%-11\%$. The mean values of the dijet masses for the nine bins were $M_{JJ} = 82, 105, 140, 182, 229, 293, 378, 488, \text{ and } 629 \text{ GeV}/c^2$.

Charged-particle multiplicities were obtained for tracks lying in three restricted cones with $\theta = 0.17, 0.28, \text{ and } 0.47$ rad around the jet axis, where $\theta_c$ is defined as the angle between the jet axis and the cone side. The analysis was carried out in the dijet center-of-mass frame, so that $E_{jet} = M_{JJ}c^2/2$. All multiplicities quoted below are per jet.

To reconstruct the true charged-particle multiplicities, several cuts and corrections were applied.

First, we required full 3D reconstruction in the CTC and used vertex cuts to ensure that tracks included in the analysis originated in the primary vertex and were not due to secondary interactions, $\gamma$ conversions, $K_S$ and $\Lambda$ decays, or cosmic and other backgrounds.

Second, the data were corrected for CTC track reconstruction inefficiency. To evaluate track losses, we used a procedure based on mixing real data tracks from one jet into the opposite jet in the same event. Tracks were embedded one at a time at the CTC hit level and the full CTC track reconstruction was executed. The parameters of all found tracks were compared to the original parameters of the embedded tracks in order to determine the inefficiency corrections. The average tracking efficiency with the vertex cuts chosen and within the opening angle $\theta_c = 0.47$ was found to be 93% at the lowest dijet masses, decreasing to 78% for the largest dijet masses.

Third, tracks coming from the underlying event and multiple interactions in the same bunch crossing were subtracted. We defined two complementary cones positioned at the same polar angle with respect to the beam line as the original jets and rotated in $\phi$ so that they were at $90^\circ$ with respect to the jet axis. These cones collected statistically the same backgrounds as the cones around jets. The absolute scale of this correction, for the largest opening angle $\theta_c = 0.47$ around the jet axis, was almost independent of the jet energy and amounted to about 0.5–0.6 tracks per jet.

Finally, a small fraction of tracks coming from $\gamma$ conversions that were not removed by the vertex cuts was subtracted. The Herwig Monte Carlo event generator (version 5.6) [14] was used to evaluate the number of remaining $\gamma$-conversion tracks. The scale of this correction was 0.3 (0.8) tracks per jet for the lowest (highest) dijet mass data samples (for cone size $\theta_c = 0.47$).

The major sources of systematic uncertainties were as follows (for $\theta_c = 0.47$): (a) background track removal, 6%–7%, (b) uncertainties in CTC track reconstruction efficiency, 2%–6%, (c) jet energy measurement errors including both resolution and overall scale errors, 0.4%–3%, and (d) errors in the jet direction determination based on energy deposition in the calorimeter, 0.7%–1.2%. The uncertainties from a given source are strongly correlated between different dijet mass samples. These correlations were taken into account in the data analysis.

Table 1 summarizes the multiplicities for the 3-jet opening angles and all dijet mass data samples. Figure 1 shows the charged track multiplicity (per jet) in a cone $\theta_c = 0.47$ as a function of the dijet mass. To show the trends, we also plotted curves corresponding to the function (2)
Therefore, we fixed $Q_{\text{eff}} = 240$ MeV, as obtained in our studies of charged-particle momentum distribution shapes [17], and fitted the data with the function (2) for two free parameters: $r$ and the combination $K_{\text{LPHD}} F_{n\text{MLLA}}$. The fit yielded the following results: $r = 1.7 \pm 0.3 \pm 0.0 \pm 0.0$, for the ratio of multiplicities, and $K_{\text{LPHD}} F_{n\text{MLLA}} = 0.74 \pm 0.04 \pm 0.06 \pm 0.04$.

The first uncertainty comes from statistical and systematic experimental errors (as discussed above and summarized in Table I), the second comes from variations of $Q_{\text{eff}}$ by $\pm 40$ MeV, and the third comes from using different PDFs. The choice of $Q_{\text{eff}}$ and PDFs had little effect on the measurement of $r$. This value agrees well with the three most recent theoretical predictions mentioned above.

Assuming $F_{n\text{MLLA}} = 1.30 \pm 0.20$, the data yielded $K_{\text{LPHD}} = 0.57 \pm 0.06 \pm 0.09$. The first uncertainty includes all statistical and systematic uncertainties discussed above, while the second comes from the theoretical uncertainty in $F_{n\text{MLLA}}$. The result is consistent with the LPHD hypothesis of approximately one-to-one correspondence between final partons and observed hadrons.

Figure 2 shows how the average charged-particle multiplicity in three restricted cones changes with $M_{JJ}$ and how it compares to the Herwig Monte Carlo that uses resummed perturbative calculations similar to MLLA for parton branching and a cluster model of hadronization.

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**TABLE I.** Measured values of inclusive charged-particle multiplicity per jet for tracks falling in restricted cones with opening angles $\theta_c = 0.17$, 0.28, and 0.47. The first error is statistical and the second is total systematic uncertainty. Systematic uncertainties are strongly correlated.

<table>
<thead>
<tr>
<th>Dijet mass (GeV/c^2)</th>
<th>$N_{\text{events}}$</th>
<th>Mean charged-particle multiplicity per jet</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(Cone $\theta_c = 0.17$)</td>
<td>(Cone $\theta_c = 0.28$)</td>
</tr>
<tr>
<td>82</td>
<td>4148</td>
<td>2.9 ± 0.03 ± 0.2</td>
</tr>
<tr>
<td>105</td>
<td>1968</td>
<td>3.4 ± 0.04 ± 0.3</td>
</tr>
<tr>
<td>140</td>
<td>3378</td>
<td>4.0 ± 0.04 ± 0.3</td>
</tr>
<tr>
<td>182</td>
<td>12058</td>
<td>4.9 ± 0.04 ± 0.3</td>
</tr>
<tr>
<td>229</td>
<td>31406</td>
<td>5.2 ± 0.04 ± 0.4</td>
</tr>
<tr>
<td>293</td>
<td>23206</td>
<td>6.0 ± 0.05 ± 0.4</td>
</tr>
<tr>
<td>378</td>
<td>7153</td>
<td>6.7 ± 0.06 ± 0.5</td>
</tr>
<tr>
<td>488</td>
<td>1943</td>
<td>7.4 ± 0.08 ± 0.6</td>
</tr>
<tr>
<td>629</td>
<td>416</td>
<td>7.5 ± 0.14 ± 0.7</td>
</tr>
</tbody>
</table>

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**FIG. 1.** Average multiplicity of charged-particles per jet within a cone of size $\theta_c = 0.47$ in dijet events (points with error bars) vs dijet mass. A set of MLLA curves (normalized to the first data point) correspond to different values of $r$ (from top to bottom $r=$1, 1.2, 1.4, 1.6, 1.8, 2.0, and 2.25). The two-parameter MLLA fit is represented by the solid line in the inset.

**FIG. 2.** Average multiplicity of charged-particles within cones $\theta_c = 0.17$, 0.28, and 0.47 in dijet events (symbols with error bars) compared to the Herwig predictions including detector simulation (lines), scaled by a factor of 0.89. Data errors are dominated by systematic uncertainties.
The error bars are statistical and the systematic uncertainties are added in quadrature. Herwig was found to be above the data by approximately 11%. A fit of the data to the Herwig predictions, where the overall Herwig normalization was treated as a free parameter $N$, and which took into account all systematic errors and their correlations, resulted in $N = 0.89 \pm 0.06$ (illustrated in Fig. 2).

In summary, we have measured the inclusive charged-particle multiplicity in dijet events for a wide range of dijet masses 80–630 GeV/$c^2$. The data were compared to calculations carried out in the framework of the modified leading log approximation complemented with the hypothesis of local parton-hadron duality. Assuming that multiplicity evolves with energy as prescribed by MLLA, we have fit two parameters of the model and found the ratio of parton multiplicities in gluon and quark jets $r = N_{\text{partons}}^{g,jet}/N_{\text{partons}}^{q,jet} = 1.7 \pm 0.3$ and the LPHD conversion constant $k_{\text{LPHD}} = 0.57 \pm 0.11$. The Herwig Monte Carlo was found to reflect the major trends observed in data, although an overall scaling of the Monte Carlo multiplicities by a factor of 0.89 is preferred.

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[1] Yu. Dokshitzer and S. Troyan, in Proceedings of the XIX Winter School of LNPI (LNPI, Leningrad, 1984), Vol. 1, p. 144; A.H. Mueller, Nucl. Phys. B213, 85 (1983); B241, 141(E) (1984); Yu. L. Dokshitzer, V.A. Khoze, and S.I. Troyan, Int. J. Mod. Phys. A 7, 1875 (1992); Z. Phys. C 55, 107 (1992). In MLLA, the parton multiplicity in a gluon jet is given by $N_{\text{partons}}^{g,jet} = \Gamma(B)(z/2)^{b+1}B(z),$ with $z = \sqrt{16N_e/b\ln(E_{\text{jet}}\sin\theta_e/Q_{\text{jet}})}$, and where $I_{b+1}(z)$ is the modified Bessel function of order $B + 1$. For the number of colors $N_c = 3$ and the number of flavors of light quarks $n_f = 3$ used in this analysis, $B = 101/81$ and $b = 9$.


[12] The pseudorapidity $\eta$ is defined as $-\ln[\tan(\theta/2)]$, where $\theta$ is the polar angle measured relative to the outgoing proton beam. The transverse energy $E_T$ is defined as $E \sin \theta$.


