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**Authors’ Response**

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**Abstract:** The commentators have raised many pertinent points that allow us to refine and clarify our view. We classify our response comments into seven sections: automaticity; developmental and educational questions; priming; multiple representations or multiple access(?); terminology; methodological advances; and simulated cognition and numerical cognition. We conclude that the default numerical representations are not abstract.

So, do we represent numbers non-abstractly? We appreciate the tone and the quality of the commentaries on our target article in general. The commentators provided us with a mixed view: Only 7 commentators defend the abstract view, 11 commentators are agnostic and their arguments tend toward the non-abstract representations or abstract representation, and 10 commentators support the idea that numerical representations are non-abstract. Clearly, our view has facilitated an important debate. In this response, we integrate the different positions, explain why some of the arguments against the non-abstract view are invalid (mainly based on clarifications of arguments that we provided in the target article), and conclude that the default representations of numbers are non-abstract.

**R1. Automaticity**

Algom raised important concerns, and contentious topics. Before dealing with his main points, we would like to point out several places in his commentary where our perspective was extended to places that we did not state in our article. It might be that we were not clear enough on these topics in our article, and for some of the readers these misinterpretations might be minor, but we would like to state them for the sake of theoretical clarity. We neither said nor believe that numerical magnitude is processed automatically whenever a numeral is presented for view. This is a very strong definition of automaticity, and Algom and others have shown that such a definition of automaticity does not hold. We also did not state that Stroop-like tasks are the best behavioural tasks to reveal the effect of notation on numerical magnitude. We do believe that there are some advantages for using this paradigm (and also some disadvantages).

Cohen and Algom describe findings by Cohen (2009), in which the physical shape, rather than the numerical magnitude, was processed. There is no reason to be surprised by this result. If the physical shape is more salient than the numerical magnitude, it will mask the effects of the numerical magnitude. We expect that the reverse will be obtained if the numerical magnitude is made more salient using the same paradigm.

The other point that Algom mentioned is one raised some years ago, researched extensively, and we believe, refuted. Algom states that, virtually all studies that demonstrated the effect (of task-irrelevant numerical magnitude on judgments of physical size [i.e., size congruity effect]) used a design that favored the numerical over the physical dimension in the first place. Thus, more values of number than values of physical size were typically presented (indeed, most studies used merely two values for size: large, small) [termed variability]. Moreover, the numerals were easier to discriminate from one another than their physical sizes [termed discriminability]. (Our explanations added to Algom’s in square brackets.)
These are potential problems that Algom has raised previously (Algom et al. 1996; Pansky & Algom 1999; 2002), and that were ignored by some researchers, including us (Cohen Kadosh et al. 2007d; Girelli et al. 2000; Henik & Tzelgov 1982; Rubinsteen et al. 2002; Tzelgov et al. 1992). However, recently we examined whether the factors discriminability and variability affected the size congruity effect. We found that modulating these factors does not affect the size congruity effect, even when they are completely biased toward the other dimensions in discriminability or variability, and the size congruity, in contrast to Algom’s arguments, does not disappear (Cohen Kadosh et al. 2008e). Furthermore, a careful examination of Algom and colleagues’ previous studies reveals that the size congruity effect disappears when only two numbers are being presented (Pansky & Algom 1999). This limited amount of stimuli increased the chance for response repetition, thus creating a confound. Cohen Kadosh, Gevers, and Notebaert (submitted a) examined this issue, and found that the size congruity effect disappears when the response sequence of the irrelevant, rather than the relevant dimension, is repeated. In light of the issues that we raised here, we disagree with Algom’s theoretical perspective. Variability and discriminability play little role in the appearance of the size congruity effect, and other factors, such as response repetition (or processing speed; Cohen Kadosh et al. 2005e) that were confounded with variability and discriminability in some experiments, might diminish the size congruity effect.

Our view of automaticity, however, is compatible with Algom. We agree with Algom that automatic processing and intentional processing are not dichotomous, but are end-points of a fine-grained continuum, and that numerical magnitude is not activated in an automatic fashion on an unlimited scale (see also, Schwarz & Ischebeck 2003; Tzelgov & Ganor-Stern 2005).

Algom’s concern from the adaptation paradigm is partly justified (as we mentioned in sect. 11). Namely, he suggests that some features of the experimental situation might encourage numerical processing, and this is totally compatible with our claims in the target article, as we suggested that the specific instructions by the experimenters might lead to different patterns of activation (see also, Piazza et al. 2007). In addition, other non-numerical factors should be controlled, as was done in other studies (Ansari et al. 2006a; Cantlon et al. 2006; Cohen Kadosh et al. 2007b), and preferably the level of activation in the parietal lobes should be modulated by numerical quantity factors (e.g., numerical deviation from the adapted quantity; Ansari et al. 2006a; Piazza et al. 2004; 2007). However, stating that a passive viewing task is a suboptimal tool to explore neuronal specialization is overstating the case. Passive viewing is just another task, and one should not use a single approach to characterize how cognitive processes are operationalized, and how the brain is organized. This seems to be a general problem that many commentators such as Orban, Wiesel, Pauzen, & Dueck (Wiesel et al.; Mayo; and Freeman & Kozma have criticized (e.g., paradigm/technique x is not suitable) or praised (e.g., paradigm/technique y is the solution) (see sect. R6). However, we believe that integration and variety of different paradigms/techniques is the right approach to pursue, and our theory in the target article is not based on a single given paradigm/technique.

Other commentators are not convinced that intentional processing is inherently unsuitable for testing the effect of notations. We are puzzled by this position, as we showed that several studies (Cohen Kadosh 2008a; Dehaene et al. 2008; Droit-Volet et al. 2008; Ganor-Stern & Tzelgov 2008; Holloway & Ansari, 2009) used intentional numerical processing and still obtained different numerical quantity effects for different notations. To be accurate, we argued that non-intentional tasks are more sensitive to differences in the representations for different notations, and this is also reflected in our model (see target article, Fig. 5).

Algom also provides some experimental evidence that allegedly supports the existence of an abstract representation. However, in the discussed task, both Arabic digits and verbal numbers are presented, and the task is an intentional comparison task. We cannot understand how such a design can overcome the limitations that we mentioned in our review. Moreover, the effect of the Arabic digits on verbal numbers processing was approximately twice as large as the effect of verbal numbers on Arabic digits, although the processing time for Arabic digits and verbal numbers seems to be equal. This finding is not completely in line with the abstract view, and actually is in line with the idea of multiple numerical representations, and our model.

Finally, some authors consider parity a suitable measure for non-magnitude processing, for example, in priming tasks. Tzelgov and Ganor-Stern (2005) noted that the level of triggering (i.e., activation of the irrelevant dimension, in this case magnitude, provided by the experimental task) by numerical parity task is high. This is due to the fact that both dimensions are numerical and require semantic access to numerical information. Therefore, the processing of the relevant parity dimension can trigger the processing of the irrelevant magnitude dimension. This notion of triggering is also important to priming studies that are cited in the priming section.

Cohen argues that numerical representations are neither abstract nor automatic. We agree with the first part of the statement and, to some degree, also with the second part. Numerical representation is not always automatic (see our reply to Algom). Different tasks will lead to different degrees of automaticity. This relates to the notion of triggering that we mentioned in the previous paragraph. The comment made by Cohen that numerical distance is one of several features that are correlated with the order of the numbers on the number line, and that researchers rarely (if ever) consider plausible alternatives to the numerical distance hypothesis is true (for similar views see Cohen Kadosh et al. 2008b; Van Opstal et al. 2008a). For example, the numerical distance effect might be affected by linguistic frequency (Cohen Kadosh et al. 2009; Landauer & Dumais 1997). However, some studies were able to limit the number of other factors that might affect the numerical distance effect (Lyons & Ansari 2009; Tzelgov et al. 2000; Van Opstal et al. 2008), and still observed the distance effect. We believe that numerical information can be processed automatically, but further processing is required for it to affect performance (Cohen Kadosh et al. 2008e). The results by Cohen (2009) are important, and should be examined with other paradigms, and also under conditions in which the physical shape is harder to process.
**Ganor-Stern** raises important points to consider when one finds differences between notations under automatic processing, before concluding that numerical representation is not abstract. We agree with part of her comments, and considered them in previous works. For example, Cohen Kadosh et al. (2008e) found that the processing of verbal numbers differs from digits not only quantitatively, but also qualitatively. In addition, at least in size congruity tasks, slower access to abstract representation (see also Grabner and Santens, Fias, & Verguts [Santens et al.]) should have led to larger size congruity effects with the slower processed notation when it is the relevant dimension, and smaller size congruity effect when it is the irrelevant dimension (Schwarz & Ischebeck 2003), but these patterns of results were not obtained (Cohen Kadosh et al. 2008e; Ito & Hatta 2003). Therefore, speed of access to the representation cannot (fully) explain the interaction between notation and congruity. Ganor-Stern mentions that finding a size congruity for the mixed notations, as in Ganor-Stern and Tzelgov (2008), is evidence for an abstract representation. This might be the case, but a more likely explanation, in our view, is that each notation activated a separate representation and the conflict arose at the response level. This response-related explanation for the size congruity effect has support from recent studies that examined the source of the size congruity effect (Cohen Kadosh et al. 2007c; 2008d; Szucs & Soltesz 2007; Szucs et al. 2007; Szucs et al., in press) (see also our remarks in response to Algom).

Even the argument that the size congruity effect is obtained not only for digits, but also for verbal numbers (although the effect is qualitatively and quantitatively different), does not indicate that numbers are represented abstractly, as size congruity is obtained also for non-numerical dimensions, for example, animals’ names (Rubinsten & Henik 2002); but it will be odd to claim that animal names shared an abstract representation with digits. This type of argument demonstrates our view that similar behavioural results do not indicate shared representation. Even if one assumes that some of the parameters that Ganor-Stern mentioned are correct, it is not apparent why she concludes that automatic numerical processing is based on an abstract representation. We nevertheless agree with Ganor-Stern that not any non-additive difference between numerical processing of the different notations is evidence for a notation-specific representation, and the differences should be theoretically relevant to the issue in question. The results that we reviewed in Section 6 of the target article are in line with this view.

**Núñez** gave some examples from the productive side of cognition. We think that more research on the issue of the productive side in numerical cognition is required, and thank Núñez for pointing out this issue. We nevertheless think that some of the examples might not be suitable for examining automatic processing. The reason, in our view, is that they do not fit with the view of automaticity, that is, they are all task-relevant, and therefore are monitored (e.g., are parts of the conversation, and therefore deliberative; Dulany 1996).

**Tzelgov & Pinhas** suggest that although different populations of neurons are sensitive to difference numerical representations, at the level of brain area (horizontal IPS) numerical representation is abstract. We do not agree with this definition. The question is one of (spatial) resolution, and the better it is, the better one will be able to discriminate between different representations or other processes. For example, looking at Tel-Aviv and Jerusalem on a map with a scale of 1 : 30,000,000 cm one will not find any difference in the location of these cities; however, at a scale of 1 : 10,000,000 cm the differences between these cities are apparent. One would not conclude that at the more crude scale these cities are the same. The same logic can be applied when one needs to detect differences in the human brain.

Another issue, as Tzelgov & Pinhas rightly pointed out, is that single-digit positive integers may be considered as the “primitives” of numerical cognition and are automatically accessed. However, the same is also true for non-symbolic numbers (Gebuis et al. 2009; Roggeman et al. 2007), and for verbal numbers (Cohen Kadosh et al. 2008e; Dehaene & Akhavein 1995).

**R2. Developmental and educational questions**

The commentators raise very important issues about the construction of numerical representation over the course of learning and development. We are grateful for these comments, as they make the discussion much fuller and complete, and we dealt in our review mainly with adults and less with infants and children.

**Ansari** suggests that abstract representations of numerical magnitude are a more plausible outcome of development than non-abstract representations. He claims that, “while the processes that are involved in mapping from external to internal representations may differ between stimulus formats, the internal semantic referent does not differ between representation formats.” We agree with the first part of his claim, but could not understand what is the evidence for the last part, that is, that the internal semantic referent does not differ between representation formats.

**Ansari** further suggests that format-specificity lies in the process of mapping between different external representations, and the mapping between external representation and internal numerical representation. However, this suggestion is invalid given the experimental evidence that we provided mainly in section 6. The differences are not only in general processing speed, and the parameters that reflect the numerical representation differ quantitatively and even qualitatively both for symbolic and non-symbolic numbers. Ansari also discusses the development trajectory for format-independent representation, which Kucian & Kaufmann extend and for which they provide a theoretical framework that hypothesises the creation of increased format-independent representation from format-dependent representation. Kucian & Kaufmann and Ansari might be right, and further research is needed, but we suggest that: (1) this format-independent representation is partly due to maturation of the prefrontal cortex, and (2) that it is a working representation and not the default representation (and therefore it needs a mature prefrontal cortex). The results from children and monkeys (Cantlon et al., in press; Diester &
Nieder 2007), which have a less developed prefrontal cortex (Striedter 2005; Tsujimoto 2008), support this idea.

Another possibility that the commentators did not mention is that infants might have initial shared representation for numbers, but with learning, and interaction with the environment, there is a neuronal specialisation in the brain that leads to multiple numerical representations. This idea is feasible (Johnson 2001), and has been shown in other domains (Cohen Kadosh & Johnson 2007). For example, children do not show cortical specialisation for face processing and other non-facial objects. However, as a function of development and interaction with the environment, their brain becomes tuned to different categories (Johnson et al. 2009). We see no reason why numbers, which depend much more on education, and are acquired later in life, will not follow a similar trajectory of neuronal specialisation. (See also the comment by Szućs, Soltész, & Goswami [Szućs et al.].)

Ansari also bases his suggestions on the recent study by Cantlon and colleagues (Cantlon et al., in press), however, this functional magnetic resonance imaging (fMRI) study involved an intentional comparison task, and therefore has the limitations that we mentioned in section 5. Moreover, the discussed study focused only on what is shared between symbolic and non-symbolic numbers, and neglected the important question of the differences between the notations, and whether children show more evidence of the existence of non-abstract representations than adults. However, as this study suffers from the limitations that we discussed in section 5 (e.g., the insertion of response selection to the experimental task, spatial resolution), we are not sure if it is the most optimal study to shed light on this question.

An important point Ansari mentions is that, “If the proposal by CK&W is indeed correct, then the current models of the development of numerical magnitude representations need to be radically revised,” and that “children cannot use their semantic representation of number words in order to begin understanding the meaning of Arabic digits.” Therefore, this may have educational repercussions and lead to less focus on the relationships between different formats of representations in the classroom. However, cognitive psychologists have shown that humans are able to learn artificial digits to a high level of expertise, and show numerical effects, even without any connection to numerical information, symbolic or non-symbolic (Tzelgov et al. 2000). This might suggest that it is not necessary to map one numerical notation to another in order to have intact numerical understanding. Moreover, it might be that this mapping is even maladaptive. For example, children with visuospatial impairments might suffer from mapping digits to numerosity, or children with dyslexia might have similar problems if required to understand digits by mapping them to verbal numbers. At this stage, our discussion is purely theoretical, but a better understanding might be able to shed light on the connection between visuospatial impairment and dyscalculia (Rourke 1993), as well as dyslexia and dyscalculia (Rubinsten & Henik 2009).

Cantlon, Cordes, Libertus, & Brannon (Cantlon et al.) (see also Núñez) argue that the stipulation that numerical abstraction requires identical responses in identical neurons is potentially impossible to satisfy. We find this statement paradoxical, since Cantlon and colleagues stated recently that, “different quantitative dimensions can be represented by generic magnitude-coding neurons” (Cantlon et al. 2009, p. 89). For other non-numerical features in the ventral stream, it is also possible (e.g., Sawamura et al. 2006). Cantlon et al. argue that even if it is possible to satisfy this criterion (see Diester & Nieder [2007] for fulfilling this criterion for numerical representation in the prefrontal cortex), it is not clear whether it is the appropriate criterion for establishing numerical abstraction.

We would like to thank Houde for his suggestion that the initial numerical representation is not abstract, and that abstract numerical representation is gained through inhibition processes. This leads to support for our suggestion that abstraction is created intentionally, but does not exist as a default representation, or, in Tzelgov & Pinhas’s terminology, it is a “working representation.” The involvement of inhibitory operations is suberved by prefrontal cortex maturation (Tsujimoto 2008; Wood et al., in press), and therefore, the involvement of prefrontal cortex in creating an abstract representation is also in line with our dual-code model. Houde provides important evidence that children up to the age of 7 years confuse the layout of the display with the numerical estimation. Kucian & Kaufmann provide another example from 3-year-old children, who seem to rely on perceptual cues if the ambiguity between numerical and non-numerical stimulus properties is overwhelming (Rousselle et al. 2004; cf. Hurewitz et al. 2006, for evidence with adults; but see Gebuis et al. 2009). Wiefel et al. present data on calculation tasks in toddlers showing that number operations strongly depend upon how numerosities are presented at preschool age. Elementary school education teaches the children to flexibly shift between the different numerical notations. Future studies should examine whether this shift is due to maturation of the prefrontal cortex, expertise, and education. However, these results, as well as others that were mentioned in this section, are in contrast to Cantlon et al.’s argument against the existence of non-abstract representations in early developmental stages.

We would like to thank Peters & Castel for highlighting the influence of the nature of numerical representation, whether intentional or automatic, on decision-making. Indeed, this will generate a new area of research that will elucidate the significance of numerical representation in everyday decisions. Another important comment is that, to have a better understanding of numerical representations, researchers need to examine this question in connection with individual use of numbers. Will high expertise with numbers be associated with non-abstract representation, or vice versa? We believe that this question will be of interest for cognitive psychologists and developmental psychologists.

Rosenberg-Lee, Tsang, & Menon (Rosenberg-Lee et al.) highlight the scenario in which various numerical notations exploit magnitude-processing capacities in the IPS to different degrees. More specifically they suggest, based on behavioural, neuroimaging, and single-cell neurophysiology studies, that at a first stage, different numerical notations are encoded in the IPS in a non-magnitude-dependent fashion. As a function of experience these non-magnitude representations become involved in automatic analogue magnitude
representations. This is a powerful prediction, and as suggested by Rosenberg-Lee et al., future studies that will use learning paradigms and longitudinal developmental research will shed light on this developmental hypothesis. One interesting question is how different hemispheres are influenced by these developmental trajectories. Why, in Cohen Kadosh et al. (2007b), did the right IPS not show adaptation for verbal numbers (which is in line with Rosenberg-Lee et al.’s suggestion), while the left IPS did show an adaptation?

Szücs et al. emphasize the educational perspective in numerical cognition. They make a clear distinction between an evolutionarily grounded sense of magnitude and a culturally acquired abstract number concept. They further suggest that developmental and cultural studies do not support the idea of format-independent numerical representation. They also raise another issue that is of high importance: whether numerical representation causes better math skills, and vice versa, or whether there is any correlation between these two abilities at all. We believe that further studies are needed to examine this issue, which at the moment shows more support for the connection between numerical abilities and math skills (Booth & Siegler 2008; Rubinsten & Henik 2009).

In contrast to the nativist approach that is dominant in the field of numerical cognition, Kucian & Kaufmann base their discussion on “neural constructivism” – which suggests that the representational features in the human neocortex are dynamic and influenced by interactions between neural growth mechanisms and environmentally derived neural activity. This view is also in line with the suggestions made by Szücs et al. We are more sympathetic to this approach; numerical skills that are heavily influenced by education and environment (e.g., Hung et al. 2008; Tang et al. 2006b) will probably be modified as a function of development and training. After Kucian & Kaufmann provided evidence for non-abstract numerical representations from studies that include children with typical and atypical development, they presented a model that describes the overlap between different numerical representations as a function of age, experience, and schooling. We found this model stimulating, and it emphasizes the dichotomy in the field of development on numerical representation: Kucian & Kaufmann, Wiefel et al., Ansari, and Houdé suggest that the numerical representation at early developmental stages is non-abstract, whereas Cantlon et al. suggest that the numerical representation de nuce is abstract.

On the whole, it seems that commentators from the field of developmental psychology/neuroscience did not reach a consensus, but most of the commentators supported the existence of non-abstract representations. One should note that the computational model by Verguts and Fias (2004) assumes abstract representation by training digits and non-symbolic numbers together (thus also biases the model from the beginning toward abstract representation). In light of the comments in this section, it seems that this model should examine different methods for learning and development of numerical representations.

In sum, we are happy to trigger such a scientific disagreement and hope that future studies will shed further light on this issue.

R3. Multiple representations or multiple access?

Grabner emphasizes the importance of considering symbol-referent mapping expertise in theories of numerical representation. We agree with his suggestion, and believe that such an approach can provide better understanding of learning, education, and development, and in addition, provide knowledge on how the different representations can be created and modified as a function of symbol-referent mapping. We would like to stress that, in our case, the differences between numerical representations cannot stem from differences in the access to the numerical representation. In this scenario, one would find differences in the overall processing time and/or accuracy, but not different numerical representation-related effects for different notations (e.g., different Weber-ratio: Droit-Volet et al. 2008; mapping of number into space: Dehaene et al. 2008; distance effect: Cohen Kadosh et al. 2008e; Ganor-Stern & Tzelgov 2008; Holloway & Ansari 2009), or size congruity effect (e.g., different facilitation, interference, and differences between incongruent and congruent conditions: Cohen Kadosh et al. 2008e; Ganor-Stern & Tzelgov 2008; Ito & Hatta 2003). Moreover, in some cases, even when the differences in the processing time between the different notations is taken into account, this cannot explain the differential effects for different notations (e.g., Cohen Kadosh et al. 2008e). Lastly, the difference in symbol-referent mapping expertise cannot explain why, in brain imaging studies, left or right IPS is notation-sensitive, while the contralateral IPS does not reach significance (Cohen Kadosh et al. 2007b; Piazza et al. 2007).

Another argument by Cantlon et al. is that the observed interactions are due to some ceiling or floor effects for one dimension but not the other. This might apply to a small fraction of the studies that we presented (e.g., Dehaene & Akhavan 1995), but cannot explain other results. The interactions between different formats and factors that originate from the mental number line include different Weber-ratios for different modalities (Droit-Volet et al. 2008), different mappings of different numerical formats on a physical line (Dehaene et al. 2008), or correlations between math abilities and performance in one numerical format, but not another format. These are all instances of evidence of non-abstract representations that are not due to floor or ceiling effects. The same holds also for the neurobiological evidence that we provided, and especially the case of double dissociation (sect. 6).

Furthermore, the argument that the classification by Dehaene et al. (1999) for approximate and exact can explain our results, is not accurate. Although, we agree that there is overlap between our model and the approximate–exact numerical codes, which we originally mentioned in section 10, our model has more explanatory power. For example, our model presents a continuum rather than a binary classification to approximate and exact systems that are subserved by different brain areas. In addition, our model explains the classification between different symbolic notations, and not only between symbolic and non-symbolic notations. Dehaene’s rebuttal of the non-abstract view dismisses some of the data that we provided – which found differences between a variety of numerical formats in different
Dehaene also discusses the single-cell neurophysiology results from the prefrontal cortex, while not considering the results in the IPS that were observed in the same study (Diester & Nieder 2007). We are said to dismiss these results. However, we focus in our target article on the IPS, the key area for numerical cognition, which is highlighted by Dehaene in many papers (Dehaene et al. 1998a; 2003; 2004). It may be that Dehaene is revising his position, and now suggests that numerical abstraction is in the prefrontal cortex, rather than the parietal cortex. However, before this conclusion can be reached, one has to take into account that these data were: (1) obtained after explicit training of associating digits with numerosity (e.g., 1 is one dot), and (2) in intentional task. Both of these factors might have contributed to the results that were obtained in the prefrontal cortex, as we discussed in the target article.

Dehaene also gives some new unpublished data from his lab (i.e., Eger et al., submitted). It is clear from his description that the task was intentional, and we stressed in our article the limitations of using such tasks. On the other hand, it is unclear if response selection was required in this study, and moreover, the IPS decoder is still limited to the voxel level; and therefore Dehaene ignores our comment that not finding a difference between the notations does not imply that there is an abstract representation: absence of evidence is not evidence of absence; a single demonstration of a dissociation is more compelling than a failure to find evidence of segregation. Nevertheless, if one would like to seriously consider these results as indicative of abstract representation, there are two further analyses that we suggest Eger, Dehaene, and colleagues conduct: First, to show also that when trained with dots, the IPS decoder generalised to digits. Second, to examine the existence of segregation in the multivoxel pattern by using multivariate pattern analysis. Accordingly, in a recent IMR-adaptation paradigm in which subjects processed the colour of the stimuli, we found that the IPS is involved in abstract numerical representation and calculation (Dehaene et al. 2003; 2004), and the putative area for the cortical recycling seems to fall out of the PSPL (see Figure 2 in Dehaene & Cohen 2007). Indeed, in a meta-analysis the hIPS was found by Dehaene et al. (2003) to be involved in abstract mathematical use. As Dehaene puts it: “Those parametric studies are all consistent with the hypothesis that the hIPS codes the abstract quantity meaning of numbers rather the numerical symbols themselves” (p. 492). In a more recent meta-analysis of numerical cognition (Cohen Kadosh et al. 2008f) we found that the middle IPS is involved in numerical representation. In a comprehensive review of the literature, Dehaene identified the PSPL as “being involved in attention orienting in space, can also contribute to attentional selection on other mental dimensions that are analogous to space, such as time, space, or number” (Dehaene et al. 2003, p. 498). The differences in the coordinates of the PSPL and hIPS are too large to be ignored (more than 2 cm on the anterior to posterior axis) (hIPS: x = 41, y = −47, z = 48 [Dehaene et al. 2003]; mIPS: x = 37, y = −46, z = 42 [Cohen Kadosh et al. 2005f]), and PSPL: x = 32, y = −68, z = 46 [Sereno et al. 2001]. Moreover, the behavioural part (Knops et al. 2009) of the cited work was based on several important differences between symbolic and non-symbolic notations that are in line with Campbell & Metcalfe’s view. This part was not considered by Dehaene. Dehaene ignores other results that are not in line with the abstract view. For example, he claims that notation effects “occasionally” affect performance because of numerical precision. Numerical imprecision is observed with non-symbolic numbers (Izard & Dehaene 2008). However, this cannot explain the differential effects between symbolic notations (Arabic digits, Indian digits, Kana, Kanji, verbal numbers in different language), which others, including Dehaene, have observed (see sect. 6 of the target article). Another invalid argument is that the effects – for example, between digits and verbal numbers – are due to speed of processing and perception, or occur at the transcoding level. However, these factors were taken into account in previous studies (e.g., Cohen
Kadosh et al. 2008e), and again some effects that we mentioned cannot be explained by these factors (e.g., Cohen Kadosh et al. 2007b; Dehaene et al. 2008; Droit-Volet et al. 2008; Holloway & Ansari 2009). It is ironic that this comment is made by Dehaene, who based the abstract numerical theory partly on null differences between digits and verbal numbers (Dehaene 1996; Dehaene & Akhavein 1995). If the effects in the studies that we mentioned are due to speed of processing, or are perceptual, or occur at the transcoding level, then his earlier results should have being interpreted as evidence toward the non-abstract view.

Dehaene concludes that considerable evidence points to a notation-independent representation in the monkey IPS. We ask which evidence? The only evidence is for notation-dependent representation in the monkey IPS (Diester & Nieder 2007; see section 8 of the target article and Fig. 4).

We agree with Dehaene that the IPS in humans and monkeys is not a module for representation, and it includes highly distributed neurons in the IPS that are intermingled with other representations of time, space, and other continuous dimensions, including numbers as proposed by Walsh (Walsh 2003), and has been tested and confirmed later by others (Cohen Kadosh et al. 2005; Pinel et al. 2004; Tudusciuc & Nieder 2007; for a review and meta-analysis, see Cohen Kadosh et al. 2008f). We do not see any reason why the principles of these distributed magnitude neurons should not be extended also for different numerical notations.

Santens et al. suggest that the differences between notations in behavioural and neuroimaging studies can occur because of some divergence between the input pathways to this common representation. One should notice that the model by Verguts and Fias (2004) does not include any different pathways for different symbolic numbers, but only differentiates between symbolic and non-symbolic numbers. Moreover, none of the models (including Verguts & Fias 2004) can explain the interaction between effects that stem from numerical representation and different symbols for numbers. Therefore, we do not see any support for Santens et al.’s suggestions, even from their own studies (Santens et al., in press) and model (Verguts & Fias 2004).

Some commentators argued that the effects we discussed might occur prior to the level of numerical quantity representation. Therefore, some clarification is needed. We did not intend to challenge the idea that number words and digits are processed differently at the perceptual stage—and it would be wrong to do so, since there are many studies that showed this difference in processing, including Dehaene (1996) and Schwarz and Ischebeck (2000). Therefore, we did not base our conclusions on the overall difference in reaction times (RTs) between number words and digits, which stems also from differences at the perceptual stage. Rather, the crucial point is the interaction between notation and effects that are post-perceptual and stem from the level of the numerical representation or even later. For example, many studies have shown that the distance effect is independent of the perceptual stage since it takes place at a post-perceptual stage, whether at the level of numerical representation (e.g., Barth et al. 2003; Cohen Kadosh et al. 2007c; Dehaene 1996; Pinel et al. 2001; Schwarz & Ischebeck 2000) or response selection (Cohen Kadosh et al. 2008b; Link 1990; Van Opstal et al. 2008a; Verguts & Fias 2004). Note that in the case of an interaction between distance effect and notation, it does not matter whether the locus of the distance effect is at the level of numerical representation or response selection, because response selection follows the level of the numerical representation. All studies that examined the differences between notations involved different visual displays for their notations, and there were differences in the overall RTs. However, the distance effect is the key effect for examining the question of abstract numerical representation because it taps post-perceptual stages (as reflected by event-related potential (ERP) (e.g., Dehaene 1996; Libertus et al. 2007; Pinel et al. 2001). MRI (e.g., Eger et al. 2003; Pinel et al. 2001), and behavioural results, which have been shown specifically and convincingly by the sequential paradigm (Cohen Kadosh 2008a; Schwarz & Ischebeck 2000). Another piece of evidence is that in ERP studies, the number of letters (but not the distance effect) modulates the N1 component (perceptual component) (Dehaene 1996). However, the distance effect affects only later post-perceptual components (P300: Cohen Kadosh et al. 2007c; Schwarz & Heinze 1998; P2p: Dehaene 1996). In addition, we are not familiar with any findings in the neuroimaging literature (or any other method) that have shown modulation of the perceptual areas by numerical distance and notation when words and digits were used (e.g., Pinel et al. 2001).

Falter, Noreika, Kiverstein, & Molder (Falter et al.) support the non-abstract view for numerical representation, and extend it to other domains such as time. They show that not only numbers are represented non-abstractly, but also other representations that involve the IPS, such as time. In our view, this idea should generate further experiments that will examine the representation of time, similar to our suggestions for numbers.

Campbell & Metcalfe support our theoretical view, and extend it to basic arithmetic. They provide evidence that basic arithmetic is not abstract in two ways. First, it is based on discrete, format- and operation-specific processes. Second, calculation efficiency is format-specific. Our view is very close, and indeed, Campbell was one of the few who has supported the idea of non-abstract numerical representation in the last 20 years (Campbell 1994; Campbell & Clark 1988; Campbell & Epp 2004; 2005). Moreover, our view that strategies might play a role in numerical representation is similar to his view that arithmetic is affected by subjective strategies. We believe that future studies should examine the issue of strategies on numerical representations, as it will clarify why some labs reveal non-abstract representations while others do not find any differences between the different formats. We need to take into account what exactly the researcher tells the subject. This has been shown to affect the results in some studies that reported these instructions (Piazza et al. 2004; 2007).

**R4. Priming**

Part of the evidence that Dehaene, Reynvoet & Notebaert, and Santens et al. focus on is subliminal priming. Surprisingly, they all ignore the good evidence that subliminal priming can originate at the level of response (for behavioural evidence, see Kiesel et al. 2007; Kunde et al. 2003; 2005; for fMRI and ERP
evidence, see Dehaene et al. 1998b). More surprising is that Dehaene uses the cross-notation subliminal priming data from Naccache and Dehaene (2001a) in his commentary (see Dehaene’s Fig. 1) to argue that digits and verbal numbers have a shared representation. However, these data are based on the same data in earlier work by Dehaene et al. (1998b). In this work, Dehaene et al. showed that an unconscious prime (digit or verbal number) is processed up to the response level (see Figs. 3 & 5 in Dehaene et al. 1998b). Therefore, the results by Naccache and Dehaene (2001a) can be explained perfectly by response selection rather than shared numerical representation, which is in line with other behavioural evidence in the field of subliminal priming (Kiesel et al. 2007; Kunde et al. 2003; 2005). Therefore, both digits and verbal numbers were processed up to the response level, a result that is in line with the non-abstract view (e.g., the digit 1 activated response by the left hand and the verbal number NINE activated response by the right hand), and can explain the subliminal priming effect (Kiesel et al. 2007; Kunde et al. 2003; 2005). Support for our view also comes from another study that used functional connectivity analysis. It was shown that the IPS and the frontal eye field, that are involved in response selection, are also coactivated with the motor cortex, when numerical magnitude is processed up to the response level (Cohen Kadosh et al. 2008d). Therefore the activation in the IPS in Naccache and Dehaene (2001a) and the motor cortex activation in Dehaene et al. (1998b) that were observed in the same data, can be argued to be functionally connected and involved in response selection, rather than shared representation.

Another issue is that some of the results from the cited subliminal priming studies actually support the non-abstract view, for example, by suggesting a preferred format to which the different numbers are mapped (e.g., digit; Noël & Seron 1993). Unfortunately, alternative explanations are given to explain these effects that are not compatible with the abstract view, rather than mentioning the additional support for the non-abstract view (see, e.g., Santens et al.).

Reynvoet & Notebaert also raised the issue that some of the evidence in favor of a notation dependent magnitude representation is based on a null effect in a particular condition. It is true that in some results a null effect was observed for one notation and not for another, but this is only a small fraction of the data, and other studies show that the numerical representation depends on the notation both qualitatively and/or quantitatively (e.g., Cohen Kadoshi et al. 2007b; Cohen Kadoshi et al. 2008c; Dehaene et al. 2008; Droit-Volet et al. 2008; Ganor-Stern & Tzelgov 2008; Holloway & Ansari 2009; Nuerk et al. 2002; Piazza et al. 2007).

The researchers who are working on priming suggested that subliminal priming is automatic. However, they would need to take into account the view that the priming distance effect is not evidence for automatic processing, but rather of incidental processing (Tzelgov & Ganor-Stern 2005).

R5. Terminology

Several commentators raised the issue of the definition of abstract. We based our review on a previous and well-accepted definition in the field of numerical cognition, and we are grateful for their contributions that will provide us with a better definition for the abstract representation.

Nuñez criticizes our characterization of abstraction. He mentioned that this definition is specific and unnecessarily restrictive, thus making the extension to other non-numerical areas of cognition hard. We are sympathetic to this concern, but see no reason why the terminology of abstract in the field of numerical cognition cannot be applied to other domains. It is interesting to note that numbers as a concept do not clearly fall into abstract or concrete categories. For example, chair is more concrete than truth but 2 does not fall clearly into one of these categories, and can vary among these dichotomies.

Pease, Smaill, & Guhe (Pease et al.) also comment on the definition of abstraction in the field. We agree with their view that a binary distinction between abstract and non-abstract is not the optimal way to conceptualise the problem, and our model reflects this view. Pease et al. suggest also that multi-modal representations of mathematics, such as diagrammatic or algebraic reasoning, are assumed to abstract to a common domain. We do not agree with this claim, and several researchers argued that the deeper knowledge of experts facilitates the ability to integrate the different representational formats (Ainsworth et al. 2002; Kozma et al. 2000; Tabachneck et al. 1994) (see Peters & Castel, for some support with this view in numerical cognition). This idea is similar to the one made by Lakoff (2008) that Pease et al. cite, and is similar to our suggestion, that the numerical representation is composed from multiple representations, and that a strong association can be created between them by the subject as a working representation.

Piazza & Izard raise many questions that will be of interest for future studies. We agree with them that abstract representation has become the default theory of the mathematical brain; indeed the need for our target article is partly predicated on the fact that it has become an unhealthily unquestioned default. However, in contrast to their claim, we do not offer a dichotomy (see Fig. 5), and our focus on non-abstract representations was done in order to shake the foundations of the prevailing orthodoxy, which leading researchers have ignored to some degree (Piazza & Dehaene 2004; Piazza et al. 2007; Pica et al. 2004).

In contrast to Piazza & Izard’s claims, we also do not view numerical representation as a module, and we stated in our review our divergence from such a view (see sect. 5). Neuronal populations that code numerical quantity can be modality-sensitive, but they can still be sensitive to other non-numerical features. As for the issue of serial processing: albeit that there is ample evidence that supports the idea that numbers are processed serially (Blankenberger & Vorberg 1997; Dehaene 1996; Schwarz & Ischebeck 2000), the interaction between modality and numerical effects, such as the distance effect, does not depend only on serial processing, because the additive factor method analysis is also valid in most of the cases of cascade processing (Sanders 1998).

It is worrying that researchers in the field of numerical cognition, such as Piazza & Izard and Santens et al., consider interactions between modality and the representation-related effects as an indication of abstract
representation. We have explained in this section, and in section R3, why this view is wrong. Nevertheless, these commentators’ view is a new view, but a risky one. If additive or interactive effects indicate abstract representation, the abstract view is immune to falsification – the death knell for any scientific idea.

We like very much the idea of selectivity of neurons to numbers and other features, a view that is partly in line with previous works by us (Cohen Kadosh et al. 2008f; Walsh 2003). Piazza & Izard gave an example of how when one examines single neuron spiking activity or the fMRI signal, some neurons that respond both to number and length, or number and motion, or length and motion, will not show selectivity when averaged across populations. Extrapolating their idea, the same can also hold when one examines dots and digits, digits and verbal numbers, and dots and verbal numbers. Given that all the studies so far were confined to only two modalities, the chance that abstract representation was concluded mistakenly is increased.

Vallar & Girelli pointed out that the dichotomy between abstract and non-abstract is too general to capture the variety of possible supramodal numerical representations. We agree with their argument, and would like to stress that our view is not that there is a dichotomy between abstract and non-abstract, but that both are endpoints of a continuum, and may interact with spatial codes. However, we believe that spatial codes do not affect different numerical notations to the same extent (see Dehaene et al. 2008, for support for our view), and therefore, that spatial codes are notation-dependent.

Pesenti & Andres raise very important points as to the definitions of abstract and non-abstract representations that are used by many authors in the field, including us. These definitions prevail also in the current issue (e.g., our target article review, the commentaries by Dehaene, Cantlon et al., etc.). Some of Pesenti & Andres’ comments are thought-provoking, such as that analog representations cannot be abstract. We are grateful for their comments, and agree that the researchers in the field of numerical cognition need to use more accurate definitions. However, we believe that their commentary raises more concerns for the existence of abstract representation. It seems that differential effects at the semantic level as a function of notation (e.g., Dehaene et al. 2008; Diester & Nieder 2007; Droit-Volet et al. 2008; Tudusciuc & Nieder 2007) cannot be compatible with abstract representation, whereas results that support the existence of shared representation do not necessarily indicate abstract numerical representation. Nevertheless, even if one abandons the definition of abstract/non-abstract representations and adopts instead the idea of shared/multiple representations, or alternatively, modality-(in)dependence, the weight of evidence seems to support the existence of multiple representations (or modality-dependence).

R6. Methodological advances

Freeman & Kozma, Mayo, and Orban offer several suggestions to advance our understanding of numerical cognition in humans and nonhuman primates. Freeman & Kozma suggest that aside from single-cell neurophysiology, and fMRI, additional techniques such as electroencephalography (EEG) or magnetoencephalography (MEG) are required to examine the nature of numerical representations, and that these techniques will enable one to uncover the involvement of wide regions in intermittent spatially coherent oscillations. We entirely agree with their suggestions and mentioned some of them toward the end of our article.

Mayo suggests two manipulations in single-neuron recording: reversible inactivation and adaptation of apparent numerosity. We agree that these new manipulations, which to date have not been used in this context, will be of help and can provide a causal understanding of the neuronal basis of numerical processing. It is worth asking, however, whether in principal this could be established in neurons in the rat brain that responded to numerosity, or by using TMS adaptation techniques in humans.

Orban suggests that monkey fMRI provides a solution to the limitations of neuroimaging studies that we raised (which, according to him, we grossly underestimated). He correctly mentions the study by Sirotin and Das (2009) – which appeared after our target article had been accepted – to stress the idea that, compared to intentional tasks, passive tasks (e.g., adaptation paradigm) provided a better link between the haemodynamic response and neuronal activity. Indeed, adaptation paradigms have some limitations, but this is clearly a better tool to explore the theoretical question at hand, as also implied by Orban. Orban further mentions the important study by Sawamura et al. (2006) (also cited in our review), which shows that cross-format adaptation (e.g., adaptation for two consecutive trials for pigs, or hammers, vs. pig follows a hammer or vice versa) overestimates neuronal response selectivity. This point should be taken into account when the conclusions toward abstract/non-abstract are based on the level of cross-notational adaptation. After mentioning several possible methodological problems, Orban suggests the use of fMRI in the awake monkey as the solution to the theoretical question. We would suggest a note of caution in using monkeys as a model for human numerical cognition. Human numerical cognition cannot be studied independently of language (Carey 2004; see also Ansari). We must also take into account the large hemispheric asymmetries in different numerical functions, as well as deal with the human tendency to represent numbers spatially. Monkeys do not speak, do not show our pronounced lateralisation, do not represent numbers from left to right, or right to left, as far as we know, and they do not learn about numbers and quantity in the same way as humans. Technical muscle may therefore not be the answer to these conceptual questions.

R7. Simulated cognition and numerical cognition

Lindemann, Rueschmeyer, & Bekkering (Lindemann et al.) provide a new point of view on numerical cognition, by suggesting an action-based number semantics to provide new insights into the way that we represent numerical magnitude. They suggest that abstract representation might emerge from association between the numerical information and action. We agree with this view, which is in line with Walsh (2003), and is in
accordance with our suggestion in the target article (i.e., abstraction requires intention).

Similarly, Myachykov, Platenburg, & Fischer (Myachykov et al.) extend our theory to the simulated cognition framework. We appreciate their innovative thinking, which suggests that understanding non-abstract representations within the framework of simulated cognition provides a theoretical platform for real-life numerical representation. Their view provides a hierarchy of features of numerical representation, which includes embodied, grounded, and situated cognition, and can explain effects in the field of numerical cognition, and provide support for some of the effects that we discussed and for our theoretical view. Furthermore, Myachykov, et al. provide the first independent experimental data to examine our dual-code model (sect. 10).

R8. Conclusions

We are grateful to the commentators for their valuable comments that helped us to refine and clarify our theoretical perspective. We have shown that even if one takes into account factors that might affect numerical representation, numerical representation is primarily non-abstract. Many questions were raised by the commentators and we are sure that new questions will come from this interaction. It is now time to return to the lab and generate new data on the ways that humans represent numbers.

References

The letters “a” and “e” before author’s initials stand for target article and response references, respectively.

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