

Measuring and correcting wobble in large-scale transmission radiography

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Abstract

BACKGROUND: Large-scale transmission radiography scanners are used to image vehicles and cargo containers. Acquired images are inspected for threats by a human operator or a computer algorithm. To make accurate detections, it is important that image values are precise. However, due to the scale (~ 5 m tall) of such systems, they can be mechanically unstable, causing the imaging array to wobble during a scan. This leads to an effective loss of precision in the captured image.

OBJECTIVE: We consider the measurement of wobble and amelioration of the consequent loss of image precision.

METHODS: Following our previous work, we use Beam Position Detectors (BPDs) to measure the cross-sectional profile of the X-ray beam, allowing for estimation, and thus correction, of wobble. We propose: (i) a model of image formation with a wobbling detector array; (ii) a method of wobble correction derived from this model; (iii) methods for calibrating sensor sensitivities and relative offsets; (iv) a Random Regression Forest based method for instantaneous estimation of detector wobble; and (v) using these estimates to apply corrections to captured images of difficult scenes.

RESULTS: We show that these methods are able to correct for 87% of image error due wobble, and when applied to difficult images, a significant visible improvement in the intensity-windowed image quality is observed.

CONCLUSIONS: The method improves the precision of wobble affected images, which should help improve detection of threats and the identification of different materials in the image.

1 Introduction

Large-scale transmission radiography has become an essential tool for detecting threats inside vehicles and cargo containers. Threats may be related to customs (drugs, counterfeit goods, banned imports, stowaways, stolen cars) or security (firearms, improvised explosive devices, special nuclear materials, missiles) [8, 22–26]. Transmission radiography systems have become a mainstay of customs and border agencies around the world, and are finding increasing use in areas such as defence and the security of critical infrastructure, ports and events.

Images acquired by large-scale transmission radiography (Fig. 1) are inspected by a human operator or increasingly by computer algorithm [8, 18]. Detection of threats by operators is assisted by intensity manipulation (e.g. windowing, logarithms, histogram equalisation) and pseudo-colouring [3]. Additionally, scanners that acquire images at multiple photon energies permit material separation [14] to be visualised based on differential absorption. On the basis of this visual inspection, the operator will either flag the vehicle for manual inspection or allow it to continue unimpeded.

In order to detect threats, high spatial resolution and accurate image values are required [16]. The former, because threats may be small, and the latter because threats may be shielded by other cargo or only revealed by subtle differential absorption. State-of-the-art transmission systems offer imaging of vehicle contents at resolutions of a few mm/pixel [11] and precisions of 16 bits. In some systems, mechanical instability (i.e. wobble) leads to effective loss of precision. Whilst large-scale X-ray Computed Tomography (CT) could alleviate the issues of wobble and shielding, such systems are not widely deployed because they are too

25 expensive and inefficient to be competitive [2, 27].

26 In our previous work [19] we proposed that wobble can be measured using
27 Beam Position Detectors (BPDs) which are placed perpendicular to the imaging
28 array (Fig. 2). Wobble was estimated by performing Gaussian model fitting to the
29 BPD data to obtain instantaneous beam position estimates. These instantaneous
30 estimates were Bayesian fused with an estimate from an Auto-Regression (AR)
31 to make estimates more robust at scanning moments where the BPD was non-
32 uniformly obscured by an object in the scanned scene. The wobble estimates were
33 then used to make corrections to air-only images in order to quantify performance.
34 We determined that we could correct 70% of image error due to wobble.

35 In this work, we follow a similar approach using BPDs but with several new
36 contributions: (i) a model of image formation in the presence of wobble and
37 other scanner design imperfections such as variable imaging sensor misalign-
38 ments, variable sensor responses, and source fluctuation; (ii) improved wobble
39 estimation using a Random Regression Forest (RRF) model for improved instan-
40 taneous estimation of wobble and its uncertainty; (iii) improved image correction
41 by estimating the relative offsets of sensors; and (iv) estimation of sensor sensi-
42 tivities by Sum of Squared Error (SSE) minimisation model fitting. Furthermore,
43 we extend testing of image correction methods to include qualitative evaluation
44 on images of complicated scenes.

45 In Sec. 2, we set out the technical background and review related work. In
46 Sec. 3, we give a precise description of the effects of wobble and a method of
47 image correction based on using BPD measurements to estimate fixed (i.e. sensor
48 sensitivities, sensor offsets, and beam geometry) and dynamic (i.e. wobble and
49 photon flux) system parameters. In Sec. 4 we propose a method for estimating the

50 dynamic system parameters. Finally, in Sec. 5, we test these methods on images
51 that we have collected from a Rapiscan Eagle®G60, a large-scale transmission
52 X-ray gantry system, modified by the addition of four BPDs.

53 **2 Background and related work**

54 Large-scale transmission radiography scanners operate either in portal or traverse
55 mode, and are sometimes capable of both [17]. In portal mode the scanner is sta-
56 tionary and the scene moves between the source and imaging array at a controlled
57 speed. In traverse mode the detector and source move either side of the stationary
58 scene. Portal mode is most useful in high-throughput scenarios; vehicles can drive
59 through the scanner arch without the driver having to exit the vehicle, or a rail-
60 scanner can scan multiple cargo containers carried by train at up to 60 km/h [18].
61 Traverse mode is useful in security scenarios where an unoccupied vehicle cannot
62 be interfered with, such as if it is suspected to be a car- or truck-bomb, or if it
63 needs to be covertly inspected so as not to raise suspicions. The traverse mode
64 is also useful for scanning lines of stationary cargo containers at ports [17]. The
65 traverse mode has advantages in some cases: (i) the scanned vehicle is unoccu-
66 pied, so higher doses can be used, resulting in higher precision images; (ii) there
67 is greater control over scanning speed and detector-object distance resulting in
68 less spatial warping of the captured image; and (iv) they have a compact scanning
69 footprint [15].

70 In the traverse mode, the imaging array may wobble as it moves across the
71 scene due to uneven ground or vibrations from the engine (truck systems), os-
72 cillations in the boom (truck and rail systems), or due to wind or vibrations from

73 traffic (truck and rail systems). This has a particular impact when operators search
74 for threats placed in dense scenes, since under intensity windowing [3] the wob-
75 ble artefact becomes apparent (Fig. 3). Furthermore, wobble reduces the quality
76 of material separation images [14], since their computation is dependent on pre-
77 cise values. Discrimination of high atomic numbers is particularly important as it
78 can reveal smuggled nuclear materials, or their shielding [4, 15]. Wobble occurs
79 in both truck-mounted and gantry systems. In truck-mounted systems wobble is
80 variable from scan to scan, but in gantry systems it is systematic. In this work we
81 study a gantry system, since it allows determination of the wobble ground truth,
82 but our methods can equally be applied to truck-mounted systems.

83 To our knowledge, other than our previous work [19], there have been no pub-
84 lications on addressing wobble in large-scale transmission radiography. However,
85 wobble leads to artefacts in a range of imaging devices, including micro-CT and
86 C-arm CT. We describe the most relevant work here and how it relates to the
87 wobble effect that we attempt to measure and correct in this work.

88 C-arm CT systems suffer from wobble as the gantry rotates. This means that
89 individual projections are translated relative to those captured by an ideal wobble-
90 free device. Researchers note that the wobble of the C-arm gantry is often repeat-
91 able over periods of up to two years and so wobble artefacts can be corrected by
92 a one-off system calibration [6, 21]. This is similar to some large-scale transmis-
93 sion systems, particularly those that are in fixed deployment and the gantry moves
94 along rails, where the wobble effect tends to be systematic. However, in truck-
95 mounted systems wobble is much more unpredictable due to variable scan speed
96 and variation in the topology of the surface that the truck traverses. Moreover, in
97 C-arm CT wobble artefacts tend to lead to a blurring effect in the reconstructed

98 image due to the misalignment of individual projections, whereas in large-scale
99 transmission systems, wobble mostly leads to image intensity variations as the
100 fan-beam comes in and out of alignment with the detectors. Indeed, geometric
101 image distortions can be observed if wobble is particularly severe, but this will be
102 the focus of later work.

103 Silver *et al.* [21] propose a method for determining and correcting wobble in
104 C-arm CT. The authors assume that the wobbling motion of the C-arm is identi-
105 cal for each image capture process, and so calibrate wobble correction based on a
106 phantom image. The phantom consists of a helical structure of tungsten carbide
107 spheres (pellets). The calibration computes wobble coefficients that are used di-
108 rectly in image reconstruction to obtain a wobble-artefact free image. The wobble
109 coefficients are determined by fitting a mapping from physical space to projec-
110 tion space using least-squares. Fahrig and Holdsworth [6] also adopt a calibration
111 approach to determine projection translations. They use bi-cubic spline interpola-
112 tion to determine translated projections. Since the calibration process determines
113 translations at discrete gantry angles, they linearly interpolate between them to
114 obtain estimates for intermediate projection angles if required.

115 Wobble is also observed in micro-CT systems, but the wobble manifests in
116 the rotation table since the detector and source are kept stationary [20]. In this
117 case, wobble again leads to a blurring effect in the image, quite different to the
118 effect observed in large-scale transmission systems. Authors have investigated
119 image-based, calibration and online methods to correct for wobble.

120 Sasov *et al.* [20] investigate and evaluate an image-based and a calibration-
121 based method. The image-based method is an iterative compensation scheme,
122 which first does an initial reconstruction using filtered back-projection, yielding

123 a blurry wobble affected image. Estimates of projection translations, to compen-
124 sate for wobble, are determined by comparing the original projections with cor-
125 responding forward-projected image estimates. The comparison is done either by
126 cross-correlation or least-squares. Under these translations a new reconstruction
127 is made and the process is iterated until the reconstructed image is satisfactory.
128 The calibration-based method, measures wobble in a short reference scan directly
129 before or after image capture to determine the compensatory translations of indi-
130 vidual projections. They measure the position of the focal spot, relative to a metal
131 pin placed in the scene, by fixing a fine metal mesh to the X-ray source. The au-
132 thors claim that the second method is more suitable for slow drifts (wobble) and
133 that the approach is faster and less computationally demanding than the iterative
134 based method. However, the image-based method has the advantage of working
135 purely on measured image values.

136 Zhao *et al.* [29] propose an online method, which uses capacitive distance sen-
137 sors to measure the wobble of the rotation table in Micro-CT. The measurements
138 are used to translate individual projections to compensate for the displacement of
139 the rotation table due to wobble. The authors report that the methods improve
140 images by 53.1% and 65.5% when calibrating projections in the horizontal and
141 vertical directions, respectively.

142 Due to the unpredictable component (e.g. wind, uneven topology, vibrations)
143 of wobble in large-scale transmission radiography, it is not possible to correct
144 wobble purely by calibration. Image-based methods that do not use BPDs or
145 any prior knowledge about large-scale radiography, such as Total Variation (TV)
146 denoising or Translation Invariant Wavelet Shrinkage (TIWS) [12], may be ap-
147 plicable. However, such methods are difficult to use in practice without prior

148 knowledge on the severity of the wobble artefact, which we measure (online) in
 149 this work. In this contribution we use both a calibration procedure and an online
 150 method. The calibration procedure is used to estimate a number of parameters
 151 that are fixed for a given system, including: misalignments of imaging sensors;
 152 the collimated width of the fan-beam; and the sensitivities of individual sensors
 153 due to housing attenuation and their intrinsic response. The online component, is
 154 for the estimation of wobble and estimation of the fluctuation in the photon flux,
 155 which can both vary unpredictably during a traverse mode scan. We describe these
 156 methods in the next section.

157 **3 A model of image formation with wobble**

158 To describe image formation with a wobbling detector, we use three coordinate
 159 systems (Fig. 4). We denote: the coordinates of imaging sensor pixels along the
 160 Γ -shaped imaging array (image vertical) by $y \in \mathbb{Y}$; the time coordinates indexing
 161 each scanning moment during image acquisition (image horizontal) by $t \in \mathbb{T}$; and
 162 the coordinates along the orientation of the BPDs (perpendicular to the beam and
 163 imaging array) by $x \in \mathbb{X}$. The origin $x = 0$ is taken as the vertical midline (dashed
 164 line in Fig. 4) of the imaging array.

165 The formation process of an image $I_{ty} \in \mathbb{R}^+$ is described as follows. The X-ray
 166 source emits a photon flux $A_t \in \mathbb{N}$ at scanning moment t . This flux is collimated
 167 into a fan-beam of width β_y , which has a spatial distribution on the imaging plane
 168 according to

$$\exp\left(-\frac{(b_{ty} - d_y)^2}{2\beta_y^2}\right). \quad (1)$$

169 The parameters $b_{ty} \in \mathbb{X}$ define the displacements of the beam cross-section max-
 170 imum from the vertical midline: when wobble occurs this varies with t and y ;
 171 without wobble only with y . The parameters $d_y \in \mathbb{X}$ are the horizontal offsets of
 172 the imaging sensors from the vertical midline. For a given linear ID with endpoint
 173 offsets $\{\delta_l, \delta_u\}$ we constrain d_y to a linear function

$$d_y := (y_u - y_l)^{-1}((y_u - y)\delta_l + (y - y_l)\delta_u), \text{ where } y_l < y < y_u. \quad (2)$$

174 The X-ray photons pass through the scene and interact via absorption and scat-
 175 tering, and we denote the scene transmission by $S_{ty} \in [0, 1]$. This is dependent on
 176 the thickness and type of material composing the scene. The final measured im-
 177 age is determined according to a sensitivity factor $R_y \in [0, 1]$, which incorporates
 178 (i) the fraction of photons that are transmitted through the sensor housing and not
 179 absorbed or scattered, and (ii) the fraction of photons impinging on the detector
 180 that are counted (the intrinsic response of a sensor).

181 Therefore, the final image, assuming no cross-pixel effects such as photon
 182 scatter or detector cross-talk, is approximated by

$$I_{ty} = A_t \cdot \exp(-(b_{ty} - d_y)^2 / (2\beta_y^2)) \cdot S_{ty} \cdot R_y. \quad (3)$$

183 The scene transmission S_{ty} is the physical quantity that we are trying to mea-
 184 sure, therefore the ideal image is

$$\underbrace{S_{ty}}_{\text{ideal}} = \underbrace{I_{ty}}_{\text{raw}} \cdot \underbrace{(A_t \cdot \exp(-(b_{ty} - d_y)^2 / (2\beta_y^2)) \cdot R_y)^{-1}}_{\text{correction factor}}. \quad (4)$$

185 To obtain the ideal image, one must estimate the different components of the

186 correction factor. In the *portal* scanning mode, correction is straightforward. Ab-
187 sence of wobble means that $b_{ty} = b_y$, so that all that needs to be dealt with is:

- 188 1. image column variations due to fluctuations of the photon source A_t ;
- 189 2. image row variations due to sensitivity R_y , and the fixed position and geom-
190 etry of the beam $\exp(-(b_y - d_y)^2 / (2\beta_y^2))$;
- 191 3. image pixel variations due to Poisson variation in the number of photons
192 that reach an imaging sensor.

193 The image column and row variations (1 & 2) can be corrected by normalising
194 the columns and rows in the image respectively. In this work we do not attempt to
195 correct for Poisson variation (3), however there are several denoising algorithms
196 for Poisson-distributed noise [5, 10] in the literature. Note that Poisson variation
197 can also be ameliorated by increasing the beam intensity or exposure time, but
198 this has implications on safety and cost.

199 In the *traverse* scanning mode, where wobble *does* occur, the correction is
200 complicated. The beam position b_{ty} now varies with t as well as y , and the imag-
201 ing sensor offsets d_y must now also be estimated. These and the other parameters
202 in the correction factor (Eq. 4) can be separated into two classes; (i) *system pa-*
203 *rameters* ($\beta_y, \{\delta_l, \delta_u\}$ and R_y) that are estimated in a one-off calibration which we
204 describe below, and (ii) *dynamic parameters* (b_{ty}, A_t) that are estimated per time-
205 point (online). The source variation A_t is straightforward to address by taking the
206 mean pixel response of an ID close to the source at each timepoint. In the remain-
207 der of this paper we work on A_t -corrected images. In Sec. 4 we describe a method
208 to estimate b_{ty} .

209 In the one-off calibration, for each BPD we estimate β_y (beam width at the
 210 BPD location) and R_x (the sensitivity of the sensors along the BPD). For each ID,
 211 we estimate $\{\delta_l, \delta_u\}$ (the misalignments of the ID at its endpoints) and R_y (the
 212 sensitivity of the sensors along the ID). The estimates are determined by model
 213 fitting to data collected during a traverse (wobbling) scan of an air-only scene.
 214 Although wobble has a detrimental effect on image precision, we benefit from
 215 wobble in these estimations since it allows us to disentangle (i) β_y and R_x , and (ii)
 216 b_{ly} and $\{\delta_l, \delta_u\}$.

217 The calibration is two-step and summarised as follows. First, we perform a
 218 Sum of Squared Errors (SSE) minimisation model fit using a Gaussian model of
 219 the fan-beam incident on the BPD, masked by the sensor sensitivities. In the fitted
 220 model, the Gaussian centre is allowed to vary freely with time but the beam width
 221 and sensitivities are unvarying. Having estimated the unvarying beam widths and
 222 the time-varying beam positions at each BPD, we linearly interpolate these to the
 223 positions of the sensors of the IDs. With these estimated, next we model fit to
 224 determine the ID parameters. We perform a SSE fit to the data from each ID to
 225 jointly estimate $\{\delta_l, \delta_u\}$ and R_y . The SSE is taken between the ideal image (raw
 226 image multiplied by correction factor, as in Eq. 4) and a uniform unit-valued im-
 227 age. The correction factor is composed using the interpolated β_y and b_{ly} estimates
 228 (from the first step), and the estimated parameters $\{\delta_l, \delta_u\}$ and R_y .

229 **4 Wobble estimation algorithm**

230 To estimate wobble for inhomogeneous scenes, we need to estimate b_{ly} at each
 231 BPD and then interpolate these estimates along the IDs. However, the simple

232 model fitting of the previous section is not applicable for inhomogeneous scenes.
 233 At some scanning moments the beam will be distorted from a Gaussian shape, and
 234 at other moments it will be undetectable due to dense loads. To cope with this,
 235 we estimate the beam position at time t by fusing an instantaneous estimate \hat{b}_{inst}
 236 (with uncertainty $\hat{\sigma}_{\text{inst}}$), with an estimate \hat{b}_{prior} (with uncertainty $\hat{\sigma}_{\text{prior}}$), based on
 237 the previous n beam position estimates.

238 4.1 Instantaneous estimation

239 The profile (D_{tx}) measured at each instant by a BPD, is a multiplicative combi-
 240 nation of (i) the beam profile (P_{tx}), (ii) the scene transmission (S_{tx}), and (iii) the
 241 sensitivity (R_x). We estimate the beam profile from the measured profile, fixed es-
 242 timates of the sensitivity (Sec. 3), and dynamic estimates of the scene transmission
 243 estimated from previous timepoints of the BPD signal, according to

$$\hat{P}_{tx} = D_{tx} / (\hat{R}_x \hat{S}_{tx}). \quad (5)$$

244 This estimation works well in cases where the scene is not too dense (Fig. 5, 1b
 245 & 2b); but when it is the estimated beam profile can be inaccurate due to (i) the
 246 low (noise-dominated) sensor signal, or (ii) deviation of photon trajectories due to
 247 scatter (Fig. 5, 3b).

248 We estimate the scene transmission function \hat{S}_{tx} using measurements of the
 249 BPD as it slides across the scene. A given pixel on the BPD samples each point, at
 250 its y -value, in the scene (Fig. 6). Plotting the response of this pixel as a function of
 251 time gives an estimate of the scene transmission function. Since each of the BPD
 252 sensors also sample each point in the scene, we can construct a similar estimate

253 for each sensor. The final estimate of \hat{S}_{tx} is obtained by taking a weighted average
254 of the estimates from each of the sensors. We take the weighted average to reduce
255 noise in the estimate from sensors that are aligned with the low signal tails of the
256 Gaussian cross-section.

257 With the estimate of the beam profile (\hat{P}_{tx}), we can estimate the instantaneous
258 beam position b_{inst} and its uncertainty σ_{inst} . The estimator should be able to deal
259 with non-linear relationships in the data and be able to produce data dependent
260 uncertainty estimates. We have experimented with using Gaussian model fitting,
261 as used in Ref. [19], but find that the non-normal distribution of the errors makes
262 estimation of the uncertainty unreliable.

263 In this contribution, we use a Random Regression Forest (RRF) [1] to con-
264 struct a robust estimator of the beam position from the beam profile estimates. A
265 RRF model is based on an ensemble of decision trees and is capable of modelling
266 non-linear relationships as required. Each tree in the RRF produces an estimate
267 of the beam position. We obtain estimates of the instantaneous beam position \hat{b}_{inst}
268 and its uncertainty $\hat{\sigma}_{\text{inst}}$ by taking the mean and standard deviation of the tree re-
269 sponses, respectively. We observe, for this study, that the standard deviation of
270 the tree responses has a strong correlation with the actual error in the beam posi-
271 tion estimate. Other advantages of RRF is that it is fast to train and deploy, and
272 resistant to overfitting.

273 In the RRF, N_t trees are constructed top-down with bagging and random sub-
274 space sampling. Internal nodes are split using standard thresholding, and opti-
275 mised according to the Residual Sum of Squares (RSS). At each split m features
276 (i.e. BPD pixels; elements of \hat{P}_{tx} at fixed t) are randomly sampled. For stop-
277 ping criteria, we do not set a maximum tree depth and enforce a minimum of two

278 samples per split. To tune N_t and m , we first set m to the recommended default
279 ($m = 1/3 \times \# \text{ features} = 5$) for regression. We then vary N_t and assess the RMSE
280 to choose a sufficient number of trees so that the RMSE is stable, but not so many
281 that training and inference are slow. With the N_t fixed, we then vary m from 3 to
282 12 to find the optimal RMSE, before verifying N_t again as before. By this method
283 we determined that $N_t = 500$ was adequate and the default $m = 5$ was optimal.

284 In this work we use the `randomforest-matlab` implementation of RRFs [9].
285 For training, the ground truth values of the beam displacement were obtained by
286 use of a gantry system in traverse mode, described later in Sec. 5. We train a
287 separate RRF for each BPD, using 1.4×10^5 measurements from five independent
288 scans so that there is no overlap with the test images used in Sec. 5.

289 **4.2 Estimation based on previous estimates**

290 In cases where the BPD is heavily obscured (low signal-to-noise), the RRF-based
291 instantaneous estimate will give a poor estimate of the beam position and a high
292 uncertainty. In these cases, we want the beam position estimate to be sensible,
293 and to achieve this we incorporate information about prior beam positions using
294 an Auto-Regression (AR). The wobble of the detector array is partly deterministic
295 (consider a swinging pendulum), but also stochastic due to the variable scanning
296 surface, wind and vibrations. An AR is capable of learning some of the determin-
297 istic wobble whilst allowing for stochastic variation. It is also simple to implement
298 and fast to compute. Additionally, we observe (Fig. 7) that the beam position trace
299 has a high frequency component due to fluctuations of the photon source, possi-
300 bly originating from electronic circuitry; and a low frequency component due to

301 the wobble of the imaging array. The high frequency component makes simple
 302 estimation, based on the previous timepoint, unreliable. An AR, however, allows
 303 incorporation of n previous timepoints, where n can be tuned on data to achieve
 304 best performance. Moreover, the AR approach effectively smooths out erroneous
 305 estimates from previous timepoints, but is beneficial over other smoothing filters
 306 (e.g. median filter) since it is possible to propagate previous errors to be used in
 307 fusion (Sec. 4.3) with the instantaneous estimate.

308 The AR model predicts the current beam position based on a linear combina-
 309 tion of the previous n beam positions with an added, normally distributed, pertur-
 310 bation

$$b_t = \sum_{t'} w_{t'} b_{t-t'} + N(0, \varepsilon^2) \text{ s.t. } \sum_{t'} w_{t'} = 1, \quad (6)$$

311 where $1 \leq t' \leq n$.

312 The Auto-Regression (AR) weights $w_{t'}$ are determined by model fitting Eq. 6
 313 to an independent air scan. The constraint $\sum w_{t'} = 1$ ensures that the model does
 314 not have an unrealistic systematic drift. The uncertainty ε is determined by ap-
 315 plying the model to a second air-only scan and computing the Root-Mean-Square
 316 Error (RMSE). The fitted model is used to generate the prior beam position esti-
 317 mate and its uncertainty according to:

$$\hat{b}_{\text{prior}} = \sum_{t'} w_{t'} \hat{b}_{t-t'}, \quad \hat{\sigma}_{\text{prior}}^2 = \sum_{t'} w_{t'} \hat{\sigma}_{t+t'}^2 + \varepsilon^2. \quad (7)$$

318 Note that the uncertainties from previous timepoints are propagated when forming
 319 this estimate, so that if the AR operates on previous estimates that are highly
 320 uncertain they are incorporated into the AR uncertainty, which is useful in the
 321 fusion step.

322 4.3 Fusion of estimates

323 To incorporate the information from the previous timepoints, we fuse the estimates
324 from the AR and RRF models according to their uncertainties. The fusion should
325 weight the final estimate more towards the AR if the RRF-based estimate is more
326 uncertain (e.g. due to low signal-to-noise). Equally, if the AR uncertainty is
327 high, because many of the previous n RRF-based estimates were also uncertain,
328 but the next instantaneous estimate is very certain, then the fusion should weight
329 more towards the RRF-based instantaneous estimate. To achieve, this we use a
330 Bayesian fusion, which is equivalent to a Kalman Filter [7]. This approach is
331 illustrated in Fig. 8.

332 To estimate the beam position \hat{b}_t and its uncertainty $\hat{\sigma}_t$, we Bayesian fuse the
333 instantaneous estimate \hat{b}_{inst} and its uncertainty $\hat{\sigma}_{\text{inst}}$ (Sec. 4.1) with a prior estimate
334 \hat{b}_{prior} and its uncertainty (Sec. 4.2). This is expressed as:

$$\hat{b}_t = (\hat{b}_{\text{inst}} \hat{\sigma}_{\text{prior}}^2 + \hat{b}_{\text{prior}} \hat{\sigma}_{\text{inst}}^2) / (\hat{\sigma}_{\text{prior}}^2 + \hat{\sigma}_{\text{inst}}^2), \text{ with } \hat{\sigma}_t^2 = (\hat{\sigma}_{\text{prior}}^2 \hat{\sigma}_{\text{inst}}^2) / (\hat{\sigma}_{\text{prior}}^2 + \hat{\sigma}_{\text{inst}}^2). \quad (8)$$

335 This weights the two beam position estimates by their uncertainty. If the un-
336 certainty of an estimate is low then that estimate contributes more to the fused
337 estimate. In particular, if the instantaneous estimate is uncertain because of dense
338 shielding, the prior estimate will be relied on; but when it is certain (the RRF trees
339 agreeing with each other) it will dominate the overall estimate.

340 **5 Results**

341 For the purposes of this study, and to test out our methods, we collected data using
342 a modified Rapiscan Eagle®G60 transmission X-ray scanner. We rotated four of
343 the IDs by 90° to become BPDs. The BPDs were placed at the extremes of the
344 vertical boom and the horizontal boom, so that there were two BPDs per boom.
345 The wobble characteristics are different at each location, for example wobble is
346 most severe at the bottom of the vertical boom. Note that in a commercial im-
347 plementation of BPDs, the system would have a full set of IDs with additional
348 detectors for BPDs, but we have adopted this modification in experiments to re-
349 duce cost. We collected air-only images in portal and traverse modes, and several
350 traverse mode scans of objects (e.g. trucks, forklifts, scissor lifts) were performed.
351 The scanner operates at 90Hz and has a pixel size of 5.6mm, giving an effective
352 spatial resolution of roughly 3mm. The scanner uses a Bremsstrahlung beam with
353 a cut-off energy of 6MeV. This is the same energy used in commercial systems,
354 and gives enough penetration to achieve reasonable signal-to-noise ratio on the
355 BPD for most objects.

356 We adopted a gantry set-up, since it provides a ground truth for wobble. Wob-
357 ble is observed in both gantry and truck-mounted systems, with a similar ampli-
358 tude and frequency composition. However, for a gantry system, wobble is the
359 same (modulo alignment) for each scan, but variable for truck-mounted systems.
360 The gantry system allows us to obtain an accurate ground truth by aligning wobble
361 estimates from an air-scan with the air parts of an object scan.

362 **5.1 System parameter estimation**

363 The system parameters β_y (beam width), R_y (sensitivities) and d_y (imaging sensor
364 offsets) were estimated according to Sec. 3, and are shown in Fig. 9. Small and
365 large y -values correspond to the bottom and top of the image, or the vertical and
366 horizontal parts of the Γ -shaped imaging array, respectively. The gaps in y -values
367 are where an ID has been removed or rotated to form a BPD.

368 The estimate of β_y (Fig. 9.a) increases as you go along the horizontal of the
369 Γ -shaped imaging array and away from the source due to beam dispersion; it then
370 decreases as you go along the vertical of the Γ -shaped imaging array and slightly
371 closer to the source. The sensitivities R_y (Fig. 9.b) have a lot of variation between
372 adjacent imaging sensors due to their intrinsic response and due to variations in
373 the housing of the Γ -shaped array. The estimated offsets of the IDs (Fig. 9.c) are
374 of the order of a few mm, which when compared to their 10cm length is plausible
375 for a human engineer placing them during the construction of the scanner, and is
376 indeed within the manufacturing tolerance of a scanning device of this scale (6m
377 tall). Note that the piecewise-linear nature of d_y is due to the linear constraint
378 places on each ID (Eq. 2).

379 **5.2 Wobble estimation**

380 The AR was trained on an air-only traverse mode image. Fig. 10(a) shows the
381 RMSE performance of the trained AR on an independent air-only test image as a
382 function of the number (n) of previous timepoints considered. As n is increased
383 the RMSE decreases, reaching a minimum at $n = 64$, before the RMSE begins
384 to grow. When n gets too large the model overfits and performance deteriorates.

385 We choose $n = 32$ since the RMSE is near optimal but requires half the number
386 of parameters. The AR weights for $n = 32$ are shown in Fig. 10(b). It shows
387 that more importance is placed on the most recent b estimates as expected. The
388 oscillating structure is the AR system’s way of coping with the high frequency
389 component of the beam movement.

390 To assess the performance of the proposed beam position estimates, we test
391 performance on “easy”, “intermediate” and “difficult” scenarios from the col-
392 lected data. For each, we compare the new RRF-based method for instantaneous
393 estimation to the old Gaussian-based method from Ref. [19]. We also compare
394 the RRF-based method instantaneous method, with the fused estimate which we
395 refer to as RRF-AR.

396 For the “easy” scenario (Fig. 11), the RRF instantaneous estimate (green) is
397 mostly accurate, with most estimates close to the ground truth (black). The old
398 Gaussian-based method (red) gives wildly inaccurate estimates when the BPD is
399 occluded by an object thus resulting in a non-Gaussian BPD profile. However,
400 the RRF yields estimates much closer to the ground truth, in these cases. These
401 estimates are made very accurate when fused with the AR (blue), since the RRF
402 trees give variable responses which results in a larger uncertainty, so the fusion
403 gives more weight to the AR. In particular, in Fig. 11(d) the fused estimate is
404 much closer to the ground truth than the RRF on its own.

405 In the “intermediate” scenario (Fig. 12). The old Gaussian-based method does
406 even worse, and again the RRF-based method appears relatively robust to non-
407 Gaussian BPD profiles, where the Gaussian-based method fails. In this scenario,
408 fusion with AR, does not give a large change in estimates over just using the RRF,
409 since the RRF trees are confident in their estimate; there is not a large amount of

410 variability in their votes. However, an improvement is seen in Fig. 12(d)

411 For the “difficult” scenario in Fig. 13, the old Gaussian-based method does
412 even worse. The RRF-based instantaneous estimator appears far more robust,
413 with estimates much closer to the ground truth, however the performance is not as
414 great as in the “easy” and “intermediate” scenarios. The fused estimates exhibit
415 a bias (see 13.f) where the RRF performs poorly over a long time period. This
416 happens where the total signal on the BPD is close to the background noise level
417 (it is very heavily occluded by a truck engine), and hence the RRF finds it diffi-
418 cult to make accurate estimates of the beam position. This is reflected in the RRF
419 uncertainty, and so the fused estimate puts full weight on the AR estimate, which
420 results in a constant fused beam position estimate until a good instantaneous es-
421 timate is achieved. So the AR has forced the fused estimate into giving sensible
422 estimates. Since the BPD signal is so low in this object and it occupies a large
423 number of timepoints, we reason that it would be impossible to obtain an accurate
424 instantaneous estimate by any method based on the current BPD set-up.

425 For each of the scenarios (easy, intermediate, and difficult), we have quanti-
426 fied the performance of the methods in terms of: accuracy; bias; precision; and
427 Mean Absolute Error (MAE) for the worst 5%, 1%, and 0.1% of timepoints. We
428 include the worst MAEs since particularly bad timepoints can be lost in the ac-
429 curacy, precision, and bias metrics, particularly if there are many air-only time-
430 points where estimation is straightforward. Moreover, wildly inaccurate wobble
431 estimates could lead to column artefacts in the image after correction so are unde-
432 sirable. The results are given in Table 2. For the “intermediate” and “difficult”
433 scenarios, the RRF-based instantaneous estimation offers roughly an order-of-
434 magnitude improvement across all metrics, over the old Gaussian-based method.

435 For the “easy” scenario, this improvement is approximately 3-fold; the Gaussian
436 method is already quite good at dealing with simple objects. By fusing the RRF
437 with the AR (RRF-AR), the performance increases across most metrics, particu-
438 larly for worst MAEs, however, there is little change (or a slight worsening for the
439 “intermediate” scenario) in the overall accuracy. In the “easy” scenario there is
440 roughly a 15% improvement in the MAE for the worst 5% of timepoints. For the
441 “intermediate” case the improvement drops so about 5%. Finally, for the “diffi-
442 cult” scenario the worst 1% MAE improves by about 3%. Greater improvements
443 are seen for the MAE of the worst 0.1% of timepoints.

444 **5.3 Image correction**

445 We first assess the image correction method on an air-only scene. For air-only
446 images, wobble estimation is straightforward, since the BPD profile is not dis-
447 torted by obscuring objects in the scene. However, air-only images allow us to
448 visualise and fully quantify the improvement from wobble correction. We can
449 assess image quality based on the fact that a perfect (normalised) transmission
450 air-only image would have all pixel values equalling unity. Image precision can
451 therefore be assessed by computing the Root-Mean-Square (RMS) deviation or
452 Peak Signal-to-Noise Ratio (PSNR) from this ideal.

453 In Fig. 14, we show air-only images from traverse and portal mode scans
454 and their full correction split into stages. The stages are: sensitivity R_y cor-
455 rection (Fig. 14.b&f); wobble and ID offset $\exp(-(b_{ry} - d_y^2)/2\beta_y^2)$ correction
456 (Fig. 14.c&g); and source variation A_t correction (Fig. 14.d&h). Images have
457 been intensity windowed so that the wobble effect is visible in (Fig. 14.f). Note

458 the visible difference between images (Fig. 14.b) and (Fig. 14.f), this difference
459 is mostly due to wobble. The PSNR drops from 109 dB to 77.2 dB, from portal
460 image (Fig. 14.b) to traverse image (Fig. 14.f) due to the wobble artefact. After
461 wobble correction, to obtain image (Fig. 14.g), most of the wobble artefact is vis-
462 ibly improved. Indeed, the wobble correction improves the PSNR by 21.3 dB but
463 is unable to achieve the portal mode PSNR.

464 To make quantitative assessment of the effects visible in Fig. 15, the RMS
465 deviations of the traverse and portal mode air-only images, before and after the
466 different corrections, were used to deduce the magnitude of the noise sources
467 before and after correction. Table 1 shows that wobble increases overall image
468 noise, and has also reduced our ability to correct for *sensor sensitivity*, *ID offset*,
469 and *source* variation. Although it is possible to correct for 99% of *sensor sensitiv-*
470 *ity*, the magnitude of *sensor sensitivity* is so large that it is still the most dominant
471 source of noise in the corrected image. Source variation was the least successfully
472 corrected and this is apparent in Fig. 15, since the corrected images (Fig. 14.d&h)
473 have some slightly visible column artefacts. Finally, we are able to correct 87%
474 of wobble, thus outperforming our previous work [19], which did not incorporate
475 sensor offset estimates into the correction.

476 The results for corrections applied to traverse mode images of a scissor lift
477 and a forklift truck are shown in Fig. 15. Images have been intensity windowed,
478 to the same range, to make the wobble artefact visible. The wobble correction is
479 obtained using the Bayesian-fused beam position estimate. The red boxes indi-
480 cate image regions most effected by wobble, and the green boxes show the same
481 regions but after wobble correction. There is a visible improvement in the wob-
482 ble artefact after wobble correction, showing that a good level of correction is

483 obtained even when the BPDs pass through dense objects such as a fork-lift truck.

484 Fig. 16 shows image corrections on a truck image. Since the truck occupies
485 most of the image, it is more difficult to see the effects of wobble and the cor-
486 rections. The most obvious places are the steps up to the driver's cabin and the
487 area surrounding the test object. These are indicated by the red boxes in Fig. 16.b.
488 After wobble correction (green boxes in Fig. 16.c), the artefact is reduced so that
489 the driver's steps and the test object become visible. In Fig. 16.i&ii we plot a col-
490 umn and row of pixels, respectively. In each, the red trace is from Fig. 16.b before
491 wobble correction, and the blue trace is from Fig. 16.c after wobble correction.
492 The pixels are taken from image lines that should have approximately constant
493 (or piece-wise constant) pixel values. However, due to the wobble artefacts they
494 are distorted from constancy. The wobble correction corrects a large part of this
495 distortion.

496 **6 Discussion**

497 We have proposed a series of image corrections to ameliorate detector wobble
498 artefacts in large-scale transmission radiography. The corrections were derived
499 by considering a model of X-ray image formation in the presence of a wobbling
500 detector. The correction relies on the estimation of a number of fixed system pa-
501 rameters and dynamic parameters which vary during a scan. The fixed parameters
502 include sensor sensitivities, sensor misalignments, and the width of the X-ray fan-
503 beam. The dynamic parameters include the position of the beam at different points
504 along the detector array, and the fluctuation of the number of photons emitted by
505 the source. We proposed a method for estimating the fixed system parameters by

506 model fitting to an air calibration image.

507 Wobble is more difficult to estimate, and we adopt a similar approach, using
508 Beam Position Detectors (BPDs), to our previous work [19]. BPDs are placed
509 perpendicular to the imaging array, and measure the cross-sectional profile of the
510 photon beam after interaction with the scene, allowing the position of the beam to
511 be determined and hence detector wobble to be measured. In our previous work,
512 we measured wobble by fitting a Gaussian model to the beam profile to extract an
513 instantaneous estimate of the beam position. This was Bayesian-fused with a prior
514 estimate based on an Auto-Regression (AR). In this contribution, we proposed a
515 new instantaneous estimator based on a Random Regression Forest (RRF). We
516 first estimate the true beam profile, as if the beam had not been attenuated by the
517 scene, and then estimate the beam position and its uncertainty by taking the mean
518 and standard deviation of the responses from the RRF, respectively.

519 To test the wobble estimation and image correction methods, we collected
520 image data of several objects ranging in difficulty from a small scissor lift to a
521 large truck. We used a commercial scanner, which we modified by rotating four
522 imaging detectors by 90° to act as BPDs. Our new RRF-based approach to instan-
523 taneous estimation performs significantly (an order of magnitude in most cases)
524 better than Gaussian fitting [19]. Moreover, its fusion with an AR achieves re-
525 sults close to ground truth, even for difficult objects, and performs better than the
526 RRF by 3-15% in the worst cases. It struggles for cases where the object has a
527 low signal-to-noise ratio for long durations in the scan, and we believe that this
528 problem cannot be solved by wobble estimation based solely on BPD readings,
529 unless one can accurately predict future beam positions from a limited number
530 of accurate prior position estimates. This is unlikely due to the stochastic na-

531 ture of wobble originating from uneven scanning surfaces or wind. Incorporation
532 of measurement devices, such as accelerometers placed along the imaging array,
533 may improve estimates even where there is almost no BPD signal due to object
534 occlusion. This will be a focus of future work.

535 The wobble and system parameter estimates were used to apply corrections
536 to images. We applied corrections to traverse and portal mode air-only images
537 and achieved an 87% reduction in image error due to detector wobble, thus im-
538 proving on our previous work [19]. The wobble correction method was also ap-
539 plied to difficult images of objects and a notable qualitative improvement in the
540 intensity-windowed image quality was observed, clarifying dense regions of the
541 scene and mitigating human error. The method should also allow for improved
542 material discrimination in images captured from dual-energy scanners in traverse
543 mode. State-of-the-art material discrimination, for cargo, is performed by taking
544 the log-ratio (or difference) of images at different energies, and relies on subtle
545 differences between the images [13, 14, 28]. But in commercial traverse-mode
546 systems material discrimination is often inaccurate due to image noise, including
547 from wobble (Fig. 3). And so wobble correction as a pre-processing step could
548 help improve material discrimination accuracy. Testing this, and fully quantifying
549 the effect of wobble on material discrimination, will be left to future work.

550 Future work will include experimenting with other measurement devices, such
551 as accelerometers, to improve the prior estimate of the beam position in cases
552 where the RRF fails to obtain accurate estimates over long time-periods due to
553 large, dense objects such as a truck engine. Potentially, beam position estimation
554 could be improved by using more BPDs or even a 2D imaging array, and this will
555 be explored. Additionally, we will investigate the severity and correction of geo-

556 metric distortions caused by extremely heavy wobble. Such distortions can cause
557 straight lines to become wavy, which potentially impacts on the performance of
558 human operators searching for threats, particularly if their shape is distorted in an
559 unnatural way.

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Tables

Scan mode	Noise source	Symbol	Before	After	Reduction
portal	<i>sensor sensitivity</i>	R_y	0.2305	0.0000	100%
	<i>offset of ID endpoints</i>	$\{\delta_l, \delta_u\}$	0.0013	0.0000	100%
	<i>wobble</i>	b_{ly}	0.0000	0.0000	–
	<i>source variation</i>	A_t	0.0030	0.0000	100%
	<i>photon count</i>	–	0.0029	0.0029	0%
traverse	<i>sensor sensitivity</i>	R_y	0.2305	0.0026	99%
	<i>offset of ID endpoints</i>	$\{\delta_l, \delta_u\}$	0.0013	0.0004	72%
	<i>wobble</i>	b_{ly}	0.0185	0.0054	87%
	<i>source variation</i>	A_t	0.0030	0.0004	74%
	<i>photon count</i>	–	0.0029	0.0029	0%

Table 1: RMS deviation contributions from different noise sources before and after corrections for: sensor sensitivity; Imaging Detector (ID) offsets; wobble; and source fluctuation. We do not attempt to correct Poisson noise in the photon counts.

Scenario	Meth.	Acc.	Bias	Prec.	5%	1%	0.1%
Easy	Gauss.	0.105	0.003	0.105	0.300	0.818	2.085
	RRF	0.033	0.004	0.033	0.122	0.168	0.203
	RRF-AR	0.030	0.006	0.029	0.104	0.140	0.167
Intermediate	Gauss.	0.470	-0.010	0.470	1.751	3.523	5.846
	RRF	0.019	-0.001	0.019	0.073	0.125	0.170
	RRF-AR	0.021	-0.001	0.021	0.069	0.111	0.149
Difficult	Gauss.	0.637	-0.135	0.623	2.012	3.329	5.670
	RRF	0.052	-0.008	-0.052	0.188	0.262	0.310
	RRF-AR	0.052	-0.014	0.050	0.191	0.253	0.284

Table 2: Performance metrics for: (i) the old Gaussian-based method of instantaneous estimation (Gauss.); (ii) the proposed Random Regression Forest based method for instantaneous estimation (RRF); and (iii) the Bayesian fusion of the RRF estimates with an Auto-Regression (RRF-AR). The metrics computed are: Accuracy (Acc.); Bias; Precision (Prec.); and the Mean Absolute Error (MAE) for the worst 5%, 1% and 0.1% of estimates. RRF gives over an order-of-magnitude improvement in accuracy over Gauss. for intermediate and difficult scenarios, and 3-fold for the easy scenario. RRF-AR gives a 3-18% improvement (over RRF) in the MAE of the worst 1% and 0.1% depending on the difficulty, with least improvement seen in the difficult scenario.

647 **Figure captions**

648 **Fig. 1:** A raw transmission X-ray image of a cargo container containing vehi-
649 cles and vehicle parts (top) and an intensity manipulated version (bottom). Inten-
650 sity manipulation is often used to reveal details in the image when searching for
651 threats.

652

653 **Fig. 2:** A typical transmission radiography system. Translation of the scene rel-
654 ative to the source and detector produces image columns, whilst each image row
655 corresponds to a single sensor position in the imaging array. The set-up, consid-
656 ered in this paper, has been modified by addition of four Beam Position Detectors
657 (BPDs) which are detector strips oriented at 90° to the imaging array. These
658 allow the intensity profile across the beam width to be measured.

659

660 **Fig. 3:** An X-ray image of a fork-lift truck from a mobile scanner with mechanical
661 instability (left) and the same image with material discrimination applied (right).
662 The image grey-levels have been windowed to make visible the small changes in
663 image value due to wobble. Wobble leads to a rippling curtain effect across the
664 image. Each rectangular test piece corresponds to a single material of uniform
665 thickness. The wobble artefact effects the classification of material type; the clas-
666 sification of a single test piece can change from plastic through to steel due to
667 wobble. This results in a colour change across the test piece (indicated by red
668 arrow) in the material discrimination image, where there should be no change.

669

670 **Fig. 4:** *Left:* Part of the imaging array showing two misaligned Imaging De-
671 tectors (IDs), a Beam Position Detector (BPD), and a wobbling fan-beam. The
672 magnitude of the wobble and the sensor misalignments have been exaggerated in
673 this figure. The offsets d_y for individual imaging sensors are confined to a linear
674 function determined by the offsets $\{\delta_l, \delta_u\}$ of the ID endpoints. The fuzzy bars
675 illustrate the fan-beam incident on the imaging array. The Gaussian (width β_y
676 and position b_{ly}) shows the profile of the fan-beam on the BPD. *Right:* A later

677 timepoint $t_2 > t_1$. Due to wobble, the fan-beam has moved relative to the imaging
678 array so that the intensity recorded by the imaging detectors has changed. This
679 leads to an effective loss of image precision. Correction requires estimation of
680 the beam displacements b_{ty} and the offsets d_y to be estimated. The b_{ty} , dense in t
681 and y , can be interpolated from estimates dense in t but computed at the sparse y
682 values where BPDs are located.

683

684 **Fig. 5:** Examples of the estimated beam profile \hat{P}_{tx} (green) computed by dividing
685 the measured Beam Position Detector (BPD) profile D_{tx} (black) by estimates of
686 the scene transmission \hat{S}_{tx} (red) and the sensor sensitivity \hat{R}_x (blue). *Left:* Example
687 of a homogeneous scene, and thus the estimate \hat{S}_{tx} is flat, resulting in a Gaussian
688 \hat{P}_{tx} . *Middle:* Example of an inhomogeneous scene, the resulting \hat{P}_{tx} is approxi-
689 mately Gaussian. *Right:* Example of a dense inhomogeneous scene, where the
690 resulting \hat{P}_{tx} is non-Gaussian which we attribute to photon scatter.

691

692 **Fig. 6:** *Left:* Illustration of a Beam Position Detector (BPD) being translated
693 across a scene during a scan. At consecutive timepoints $t = \{1, 2, \dots, T\}$ a given
694 sensor (green) samples consecutive points in the scene. *Right:* plotting these sam-
695 ples as a function of t yields an estimate of the scene transmission. Each BPD
696 sensor gives a similar estimate, and we perform a weighted average of them to
697 reduce the noise in the final estimate \hat{S}_{tx} . Sensors towards the ends of the BPD,
698 which receive low signal, are given a lower weighting in the average than those
699 near the Gaussian centre which receive a higher signal.

700

701 **Fig. 7:** *Left:* Beam position b_{ty} as function of time t during a wobbling air scan.

702 *Right:* A zoom so that the high frequency component of the wobble is visible.

703

704 **Fig. 8:** Demonstration of Bayesian fusion of a prior estimate (black) and an in-
705 stantaneous estimate (red) to obtain a fused estimate (blue). The width of the
706 Gaussian correspond to the uncertainty on the estimate, and their centroid to the
707 estimate value. The x -axis can be imagined as the Beam Position Detector (BPD).
708 In (a) the prior has a higher certainty than the instantaneous estimate and so the
709 fused estimate is weighted towards the prior, in (c) the opposite is true. In (b) both
710 estimates have equal certainty and so the fused estimate compromises between the
711 two.

712

713 **Fig. 9:** Estimated system parameters: (a) beam width, β_y ; (b) sensor sensitivities,
714 R_y ; and (c) horizontal imaging sensor offsets from the vertical, d_y . The dashed
715 horizontal line marks the transition from the vertical (below) part of the Γ -shaped
716 imaging array, to the horizontal (above). The black dots indicate the y -positions
717 of the Beam Position Detectors (BPDs). Gaps in y -values are where an Imaging
718 Detector (ID) has been removed or rotated to form a BPD in the experimental
719 set-up. The beam width increases (decreases) as the distance from source to the
720 array increases (decreases), due to dispersion. The sensitivities fluctuate between
721 adjacent sensors due to their different intrinsic responses. The estimated sensor
722 offsets are piece-wise linear because they are grouped by ID, and are of the order
723 of a few mm which is within the manufacturing tolerance of a system of this scale.

724

725 **Fig. 10:** Auto-Regression (AR) model fit: (a) The Root-Mean-Square Error
726 (RMSE) performance of the AR for different numbers of previous timepoints n

727 included in the model (dashed line indicates the standard deviation of the beam po-
728 sition, red circle indicates the near-optimal $n = 32$); and (b) the learnt AR weights
729 $w_{t'}$ when $n = 32$. The RMSE decreases as the number of timepoints n included in
730 the model increases, it reaches an optimum at around $n = 64$, before rising again
731 due to overfitting. The AR weights have larger magnitude for the most recent
732 timepoints ($t' = -1, -2$) as expected, because these are most informative for pre-
733 dicting the next beam position. The oscillating structure in the weights is the AR's
734 way of coping with the high frequency wobble component.

735

736 **Fig. 11:** Beam position estimates for the “easy” scenario. In (a) the new Ran-
737 dom Regression Forest (RRF) based method (green) for instantaneous estimation
738 is compared to the old Gaussian-based method (red), and the ground truth (black).
739 In (c) the RRF-based method (green) for instantaneous estimation is compared
740 to its Bayesian fusion with an Auto-Regression (AR; blue), and the ground truth
741 (black). Plots (c & d) show zooms for the most difficult region. The old Gaussian-
742 based method gives wildly inaccurate estimates ($t/P_x = 400$) where the BPD pro-
743 files are occluded and so measure a non-Gaussian profile. The RRF yields much
744 more accurate estimates, and is improved further (relative to the ground truth)
745 when fused with the AR (see d).

746

747 **Fig. 12:** Beam position estimates for the “intermediate” scenario. In (a) the new
748 Random Regression Forest (RRF) based method (green) for instantaneous esti-
749 mation is compared to the old Gaussian-based method (red), and the ground truth
750 (black). In (c) the RRF-based method (green) for instantaneous estimation is com-
751 pared to its Bayesian fusion with an Auto-Regression (AR; blue), and the ground

752 truth (black). Plots (b & d) show zooms for the most difficult regions. In this case,
753 the old Gaussian-based method behaves very erratically ($t/P_x \in [1100, 1600]$),
754 the RRF method gives estimates much closer to the ground truth. However, some
755 RRF-estimates are inaccurate (d), but these are improved when fused with the AR
756 since the RRF trees give a larger uncertainty than the AR prior estimate.

757

758 **Fig. 13:** Beam position estimates for the “difficult” scenario. In (a) the new Ran-
759 dom Regression Forest (RRF) based method (green) for instantaneous estimation
760 is compared to the old Gaussian-based method (red), and the ground truth (black).
761 In (d) the RRF-based method (green) for instantaneous estimation is compared
762 to its Bayesian fusion with an Auto-Regression (AR; blue), and the ground truth
763 (black). Plots (b, c, e, & f) show zooms for the most difficult regions. The RRF
764 struggles to give accurate estimates in (c), because the BPD is passing across a
765 truck engine, which is very dense and therefore the signal-to-noise is very low.
766 This increases the RRF uncertainty, and so the fused estimate puts full weight on
767 the AR estimate, which results in a constant estimate (f) until a better RRF esti-
768 mates are achieved. So the AR has forced the Bayesian fusion into giving sensible
769 estimates. This also occurs in (b) and (f) but to a much lesser extent.

770

771 **Fig. 14:** Images of portal (a-d) and traverse (e-h) mode air-only scans at different
772 stages of correction, including: the raw image; the image after correction for sen-
773 sor sensitivities; the image after correction for wobble (and sensor offsets); and
774 the final image after after source correction. Corrected images have been intensity
775 windowed so that the wobble artefact is visible in (b). For each image, the Peak
776 Signal-to-Noise Ratio (PSNR) is given in decibels (dB). A noiseless and artefact-

777 free air-only image should be uniform. The wobble artefact is clearly visible in the
778 traverse image after the sensitivities have been corrected (f), and it is not visible in
779 the portal mode image (b) since this mode is not effected by wobble. The PSNR
780 is reduced by 31.8 dB by the wobble artefact. After wobble correction (g) there is
781 a visible improvement in the artefact, and the PSNR has improved by 21.3 dB.

782

783 **Fig. 15:** Images from traverse mode scans of a scissor lift (a-e) and a fork-lift
784 truck (f-j), after a series of corrections for: sensor sensitivities (b & g); wobble
785 and Imaging Detector (ID) offsets (c & h); and source variation (d & i). Cor-
786 rected images have been intensity windowed so that the wobble artefact is visible
787 in (b) and (g). The final images (e & j) are the non-windowed versions of (d & i).
788 The red boxes indicate regions where wobble is particularly visible, and the green
789 boxes indicates the same regions after wobble correction. There is a clear visible
790 improvement in the wobble artefact after correction, and so wobble measurement
791 and correction works well even when BPDs are heavily occluded by dense object.

792

793 **Fig. 16:** Images from a traverse mode scan of a truck, after a series of corrections
794 for: sensor sensitivities (b); wobble and Imaging Detector (ID) offsets (c); and
795 source variation (d). Corrected images have been intensity windowed so that the
796 wobble effect is visible in (b). The final image (e) is the non-windowed version
797 of (d). The red boxes indicate regions around the driver's steps and a test object,
798 where wobble is particularly visible. The green boxes indicate the same regions
799 after wobble correction and one can see a visible improvement in the wobble
800 artefact. The plots (i) and (ii) show traces of the pixel intensities across a column
801 and row in the image, respectively. The red traces are uncorrected for wobble and

802 taken from (b), whilst the blue traces are corrected for wobble and taken (c). The
803 red traces should be approximately (piece-wise in ii) constant, however they are
804 distorted by wobble. The wobble correction corrects most of this distortion.

805 **Figures**

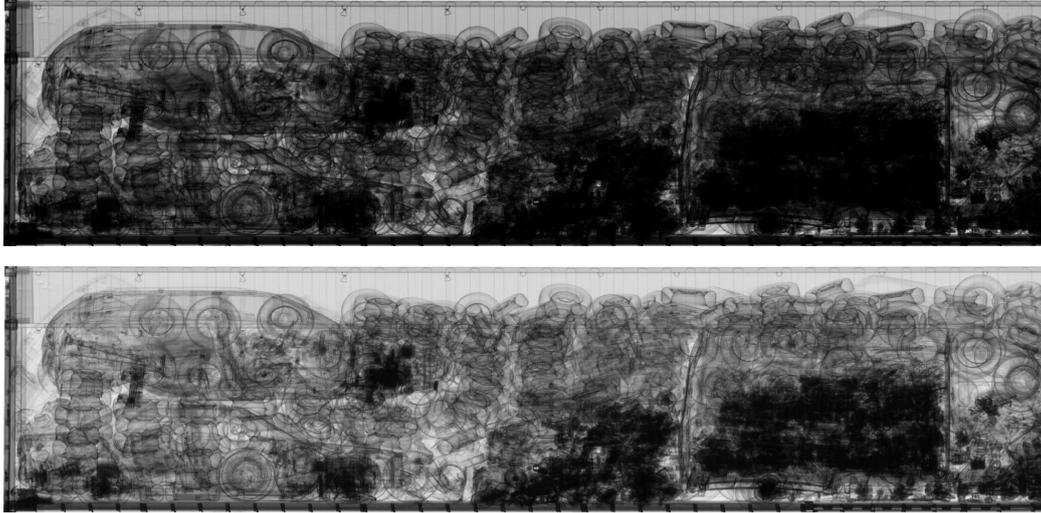


Fig. 1

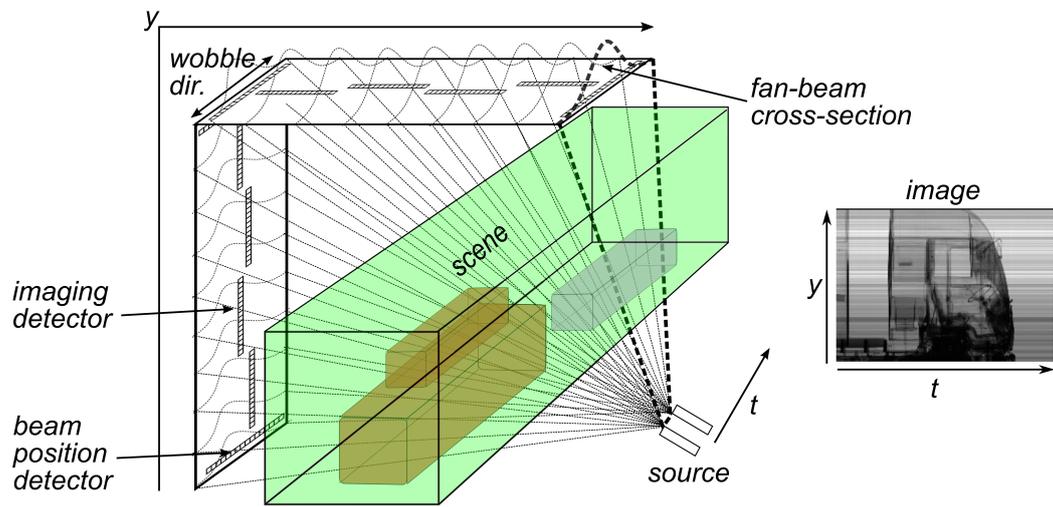


Fig. 2

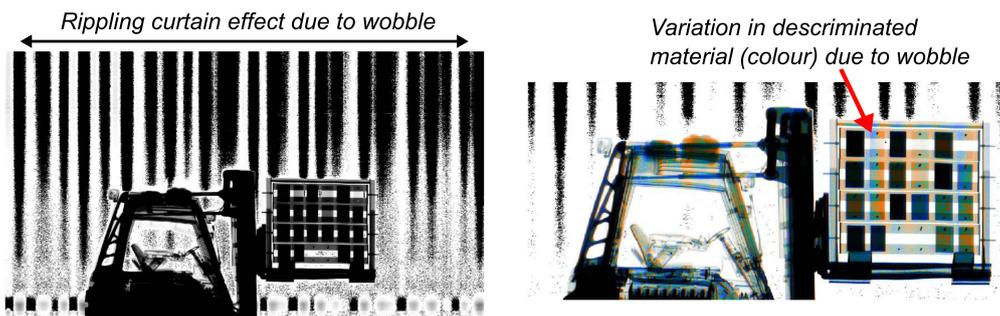


Fig. 3

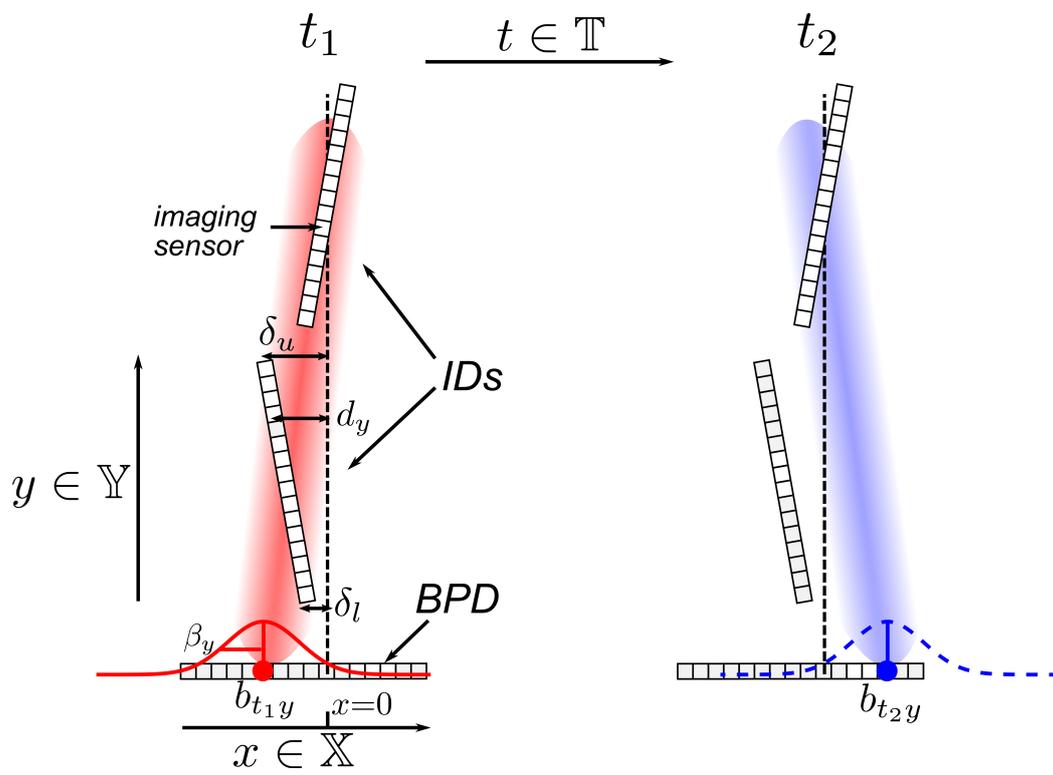


Fig. 4

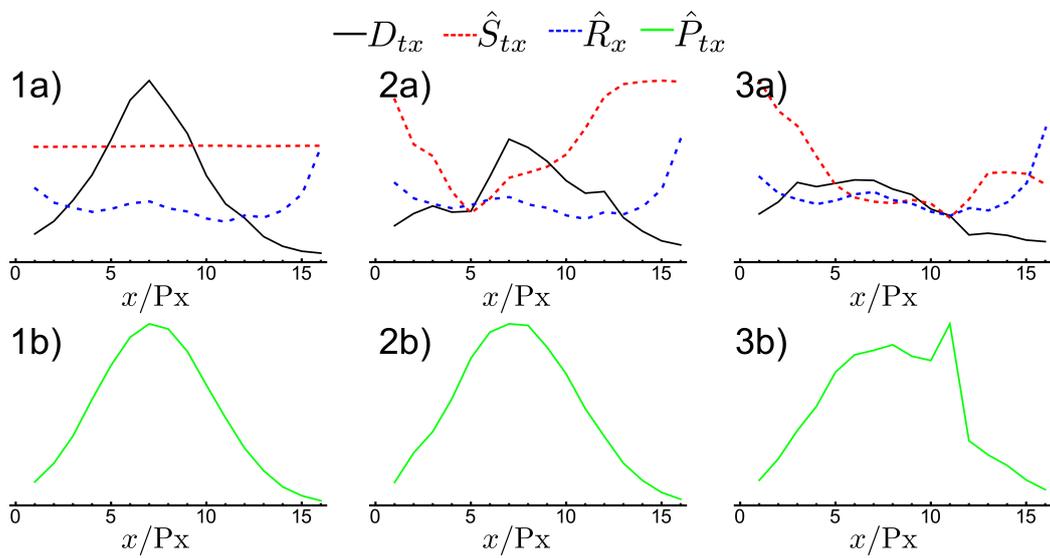


Fig. 5

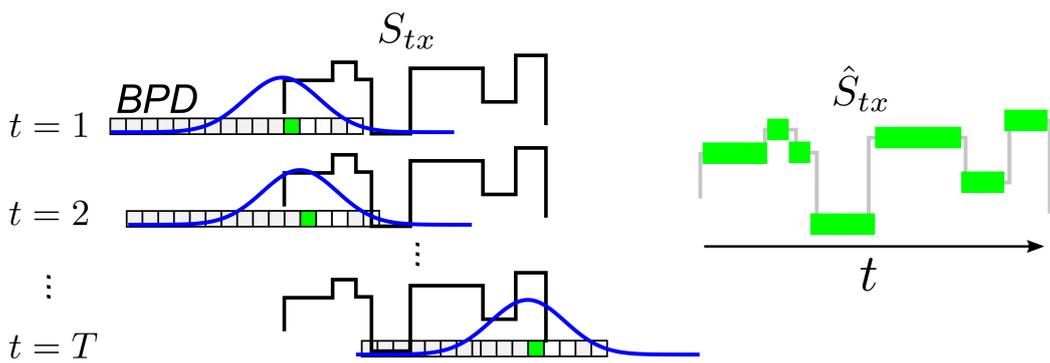


Fig. 6

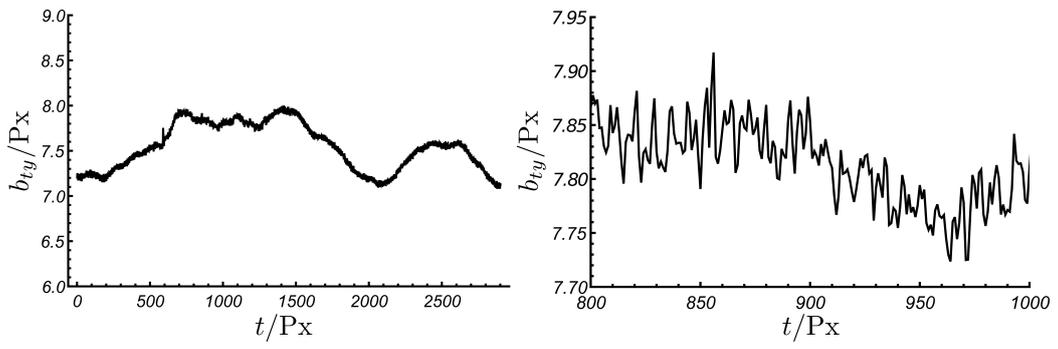


Fig. 7

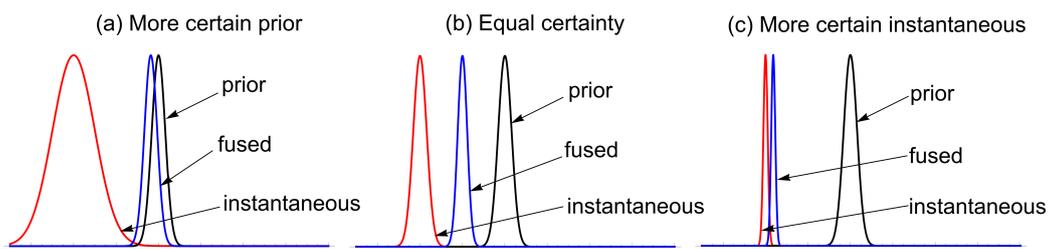


Fig. 8

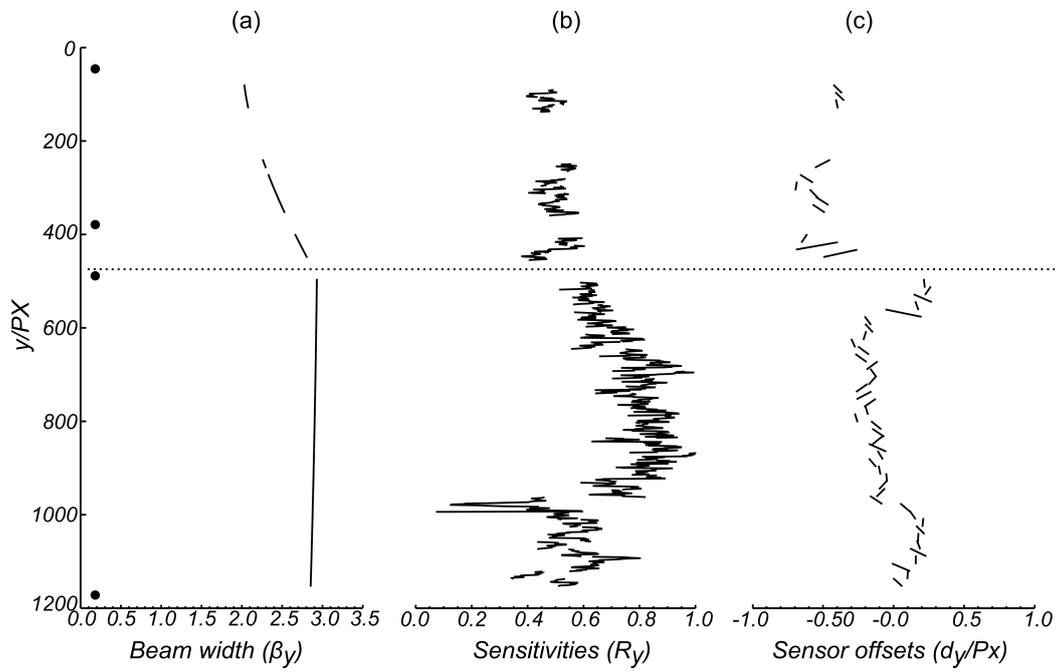


Fig. 9

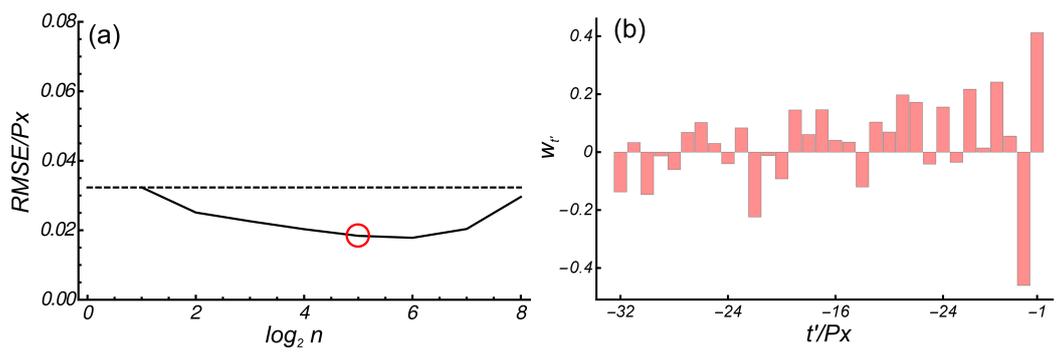


Fig. 10

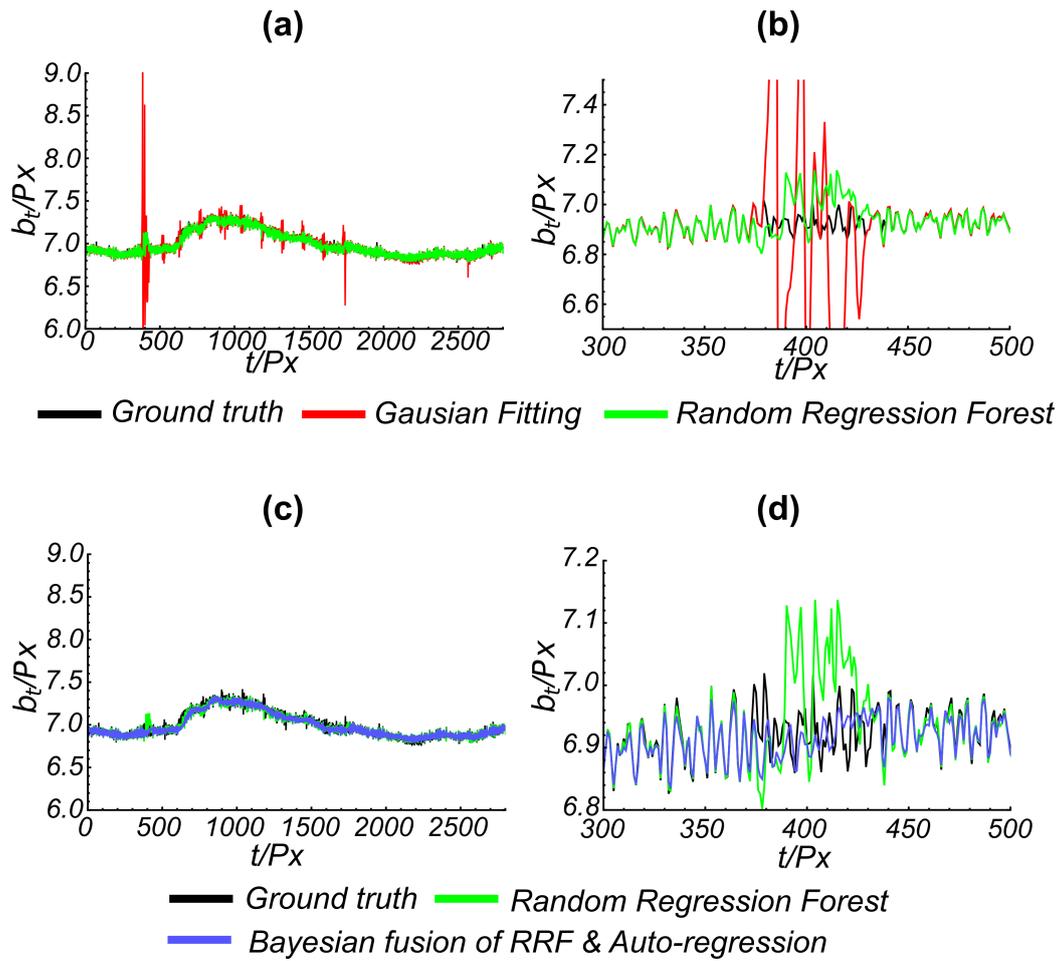


Fig. 11

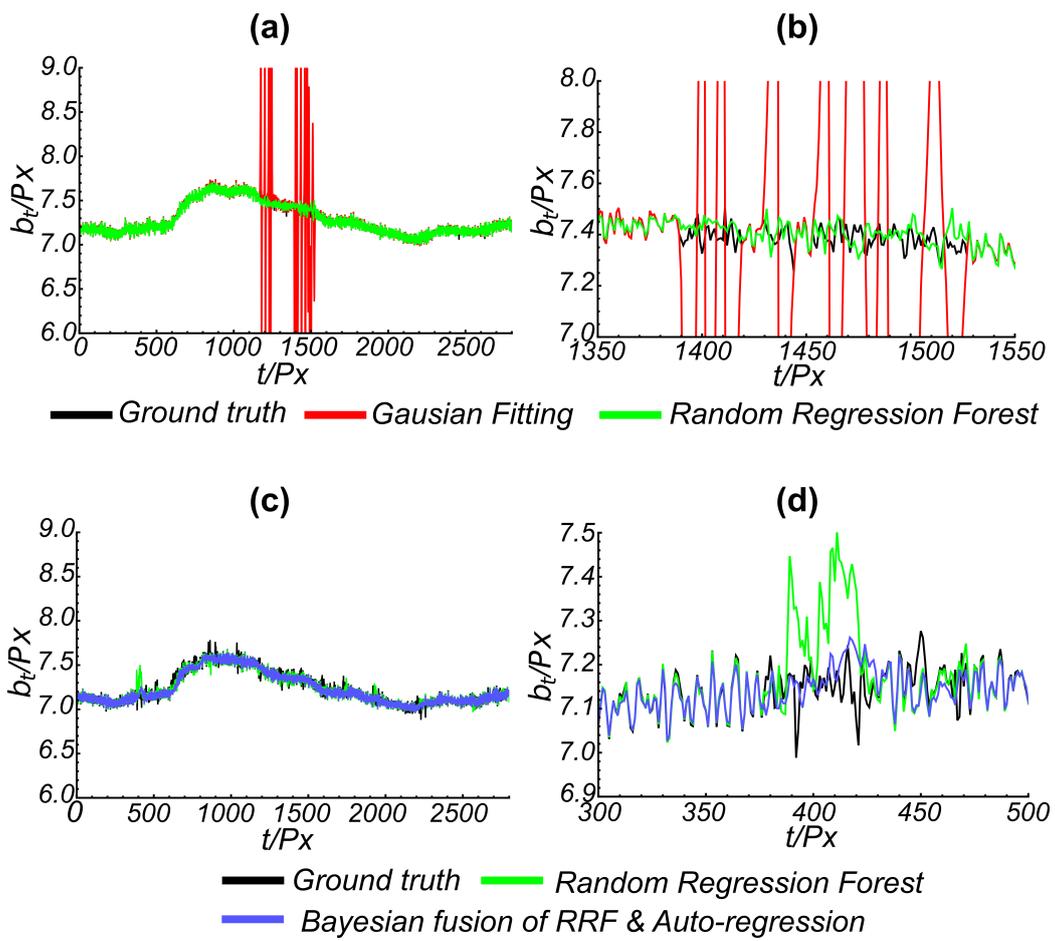


Fig. 12

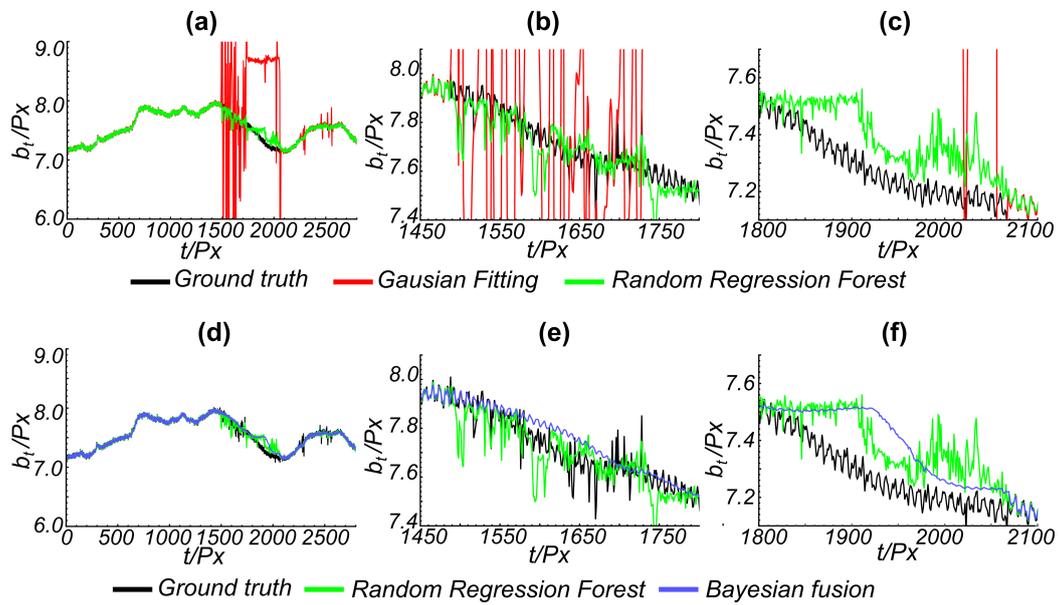


Fig. 13

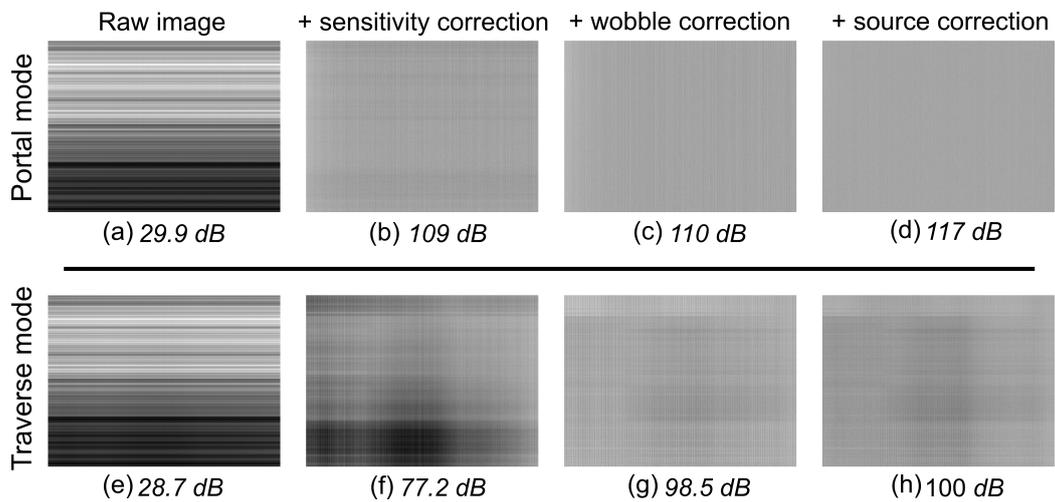


Fig. 14

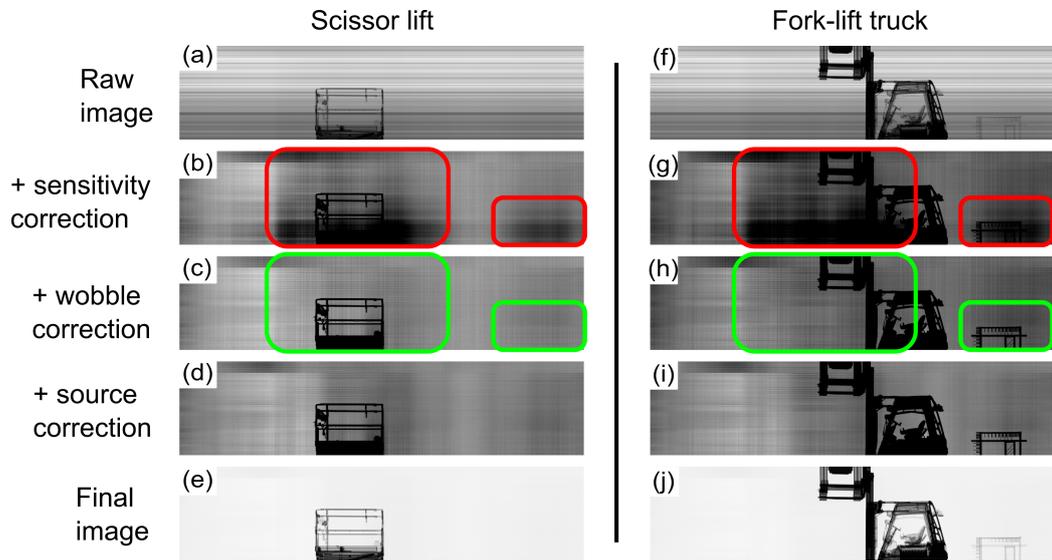


Fig. 15

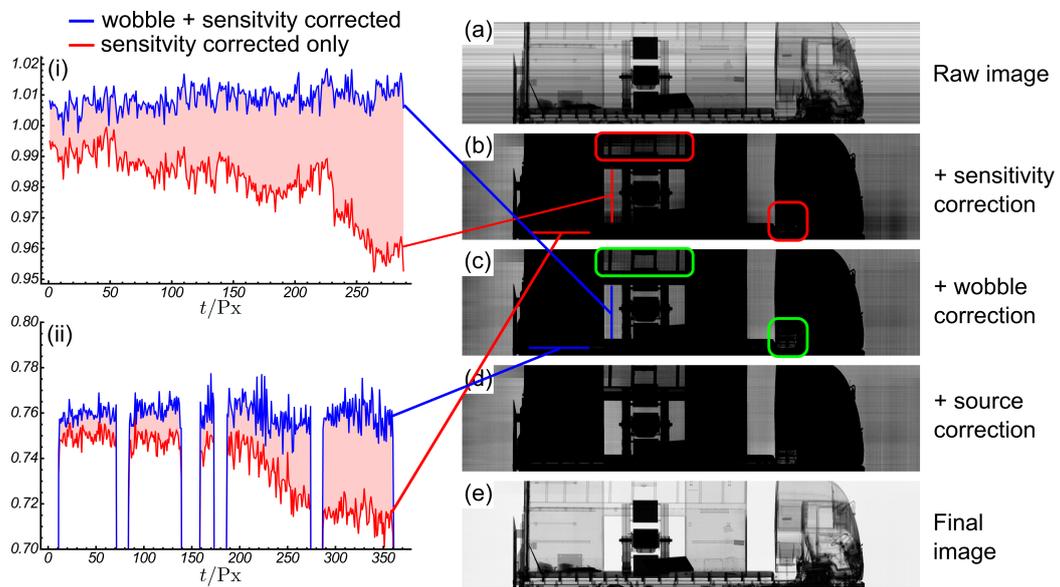


Fig. 16