

Bose *et al.* Reply: Our papers [1,2] propose an experiment in which the observation of Ramsey fringes would evidence a spatial superposition. We analyzed this as a magnetic effect creating a Stern-Gerlach (SG) like spin dependent separation of the center of mass (c.m.) states in conjunction with a gravitational effect imparting a relative phase between the states. The preceding Comment [3] points out that this could be interpreted in a different way. It contends that the interference manifested in the spin states is not due to the spatial separation as it can also be interpreted as a Zeeman effect. To support its contention, the comment splits the Hamiltonian into parts H_1 and H_2 , where only H_1 couples the c.m. with the spin states, while H_2 imparts the phase factor. However, the periodic factorizability of the c.m. and the spin states requires the action of H_1 as well. It is this factorizability that makes the phase detectable by a measurement on the spin alone. For instance, if the c.m. and spin states are not entangled at $T/2$, the evolution by H_1 alone for an additional time $T/2$ will not be able to factorize them. This will lead to the Ramsey interference pattern being suppressed. Thus, the visibility of the phase due to H_2 hinges on the interference brought about by H_1 . Both treatments (our's and the Comment's) are valid as they use the same Hamiltonian. In both cases, the absence of coherence in the c.m. motion (which could be due to decoherence from air molecules, for example) would lower the visibility.

In the absence of decoherence, an arbitrary initial coherent state $|\beta\rangle$ of the c.m. and an initial spin state $(|+1\rangle + |-1\rangle)/\sqrt{2}$ evolves jointly as

$$(e^{-i\phi_+(t)}|\beta(t, +1)\rangle|+1\rangle + e^{-i\phi_-(t)}|\beta(t, -1)\rangle|-1\rangle)/\sqrt{2},$$

where $|\beta(t, \pm 1)\rangle$ are c.m. coherent states with the time-varying separation of $\Delta z(t) = 8\lambda\delta_z/\hbar\omega_z(1 - \cos\omega_z t)$ with $\delta_z = \sqrt{\hbar/2m\omega_z}$ being the ground state position spread of the oscillator. Despite the fact that $|\beta(t, \pm 1)\rangle$ oscillate about centers $-g\cos\theta/\omega_z^2 \pm 4\lambda\delta_z/\hbar\omega_z$, where there are finite magnetic fields, in our approach, the entire inhomogeneous magnetic field term of the Hamiltonian is “used up” to accomplish the SG-like separation $\Delta z(t)$, and is thereby not available any more to impart a Zeeman phase between the separated states. The integrated gravitational phase shift $\int_0^T [mg\cos\theta\Delta z(t)dt/\hbar]$ gives exactly the phase shift $\phi = \phi_+(T) - \phi_-(T) = \phi_{\text{grav}}$ of Refs. [1,2].

The spin dependent spatial splitting of the c.m. states in an external magnetic field gradient is essentially the well-verified SG effect. Evidencing the *coherence* between the split states is the challenge. Now consider a case where only the c.m. motion decoheres: the off diagonal terms $|\beta(t, +1)\rangle\langle\beta(t, -1)|$ are damped by a factor of $e^{-\gamma(t)}$. Then the evolved state at $t = NT$ is $\rho(NT) = |\beta\rangle\langle\beta|_{\frac{1}{2}}\{|+1\rangle\langle+1| + |-1\rangle\langle-1| + e^{-\gamma(NT)}(e^{-iN\phi}|+1\rangle\langle-1| + e^{iN\phi}|-1\rangle\langle+1|)\}$. We see that the spin density matrix has

also decohered despite the fact that the decoherence was exclusively for the c.m. state [4,5]. In particular if the c.m. state is completely decohered [$\gamma(NT) \rightarrow \infty$] the phase to be measured disappears from the density matrix. Thus the visibility of the phase is evidence of the coherence (interference) between $|\beta(t, +1)\rangle$ and $|\beta(t, -1)\rangle$. Note that if one insists on an independent verification of the SG effect through spin-position correlation experiments, then the position splitting can be enhanced by a lower ω_z or free flight [6]. The time varying spatial separation between $|\beta(t, +1)\rangle$ and $|\beta(t, -1)\rangle$ can also be inferred from the spin state alone through a time modulation of the visibility of $\phi_+(t) - \phi_-(t)$ [7,8].

The pitfalls of a purely Zeeman interpretation of the relative phase development between $|\beta(t, +1)\rangle|+1\rangle$ and $|\beta(t, -1)\rangle|-1\rangle$ in the presence of gravity can be highlighted by considering the following case. Suppose we start with $\theta = \pi/2$ so that there is no gravitational term in the Hamiltonian and evolve till time $t = T/2$ to obtain a spatial separation $\Delta z(T/2)$ between the superposed coherent states $|\beta(T/2, \pm 1)\rangle$. At time $t = T/2$ we instantaneously switch off the magnetic field (for practical purposes by mapping electronic spin states to nuclear spin states) and then apply a gravitational pulse by changing θ from $\pi/2$ to 0 for a very short time $\delta t \ll T$. The off diagonal component of the spin part of the density matrix evolves as [9]

$$\langle+1|\rho_S(T/2)|-1\rangle \rightarrow e^{-i[mg\delta t\Delta z(T/2)/\hbar]}\langle+1|\rho_S(T/2)|-1\rangle.$$

We still see that a phase $[mg\delta t\Delta z(T/2)]/\hbar$ develops although there is no Zeeman term.

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- [1] M. Scala, M. S. Kim, G. W. Morley, P. F. Barker, and S. Bose, *Phys. Rev. Lett.* **111**, 180403 (2013).
- [2] C. Wan, M. Scala, S. Bose, A. C. Frangskou, ATM A. Rahman, G. W. Morley, P. F. Barker, and M. S. Kim, *Phys. Rev. A* **93**, 043852 (2016).
- [3] F. Robicheaux, preceding Comment, *Phys. Rev. Lett.* **118**, 108901 (2017).

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- [4] S. Bose, K. Jacobs, and P.L. Knight, *Phys. Rev. A* **59**, 3204 (1999).
- [5] S. Bose, *Phys. Rev. Lett.* **96**, 060402 (2006).
- [6] C. Wan, M. Scala, G.W. Morley, ATM A. Rahman, H. Ulbricht, J. Bateman, P.F. Baker, S. Bose, and M.S. Kim, *Phys. Rev. Lett.* **117**, 143003 (2016).
- [7] A.D. Armour, M.P. Blencowe, and K.C. Schwab, *Phys. Rev. Lett.* **88**, 148301 (2002).
- [8] W. Marshall, C. Simon, R. Penrose, and D. Bouwmeester, *Phys. Rev. Lett.* **91**, 130401 (2003).
- [9] K. Jacobs, R. Balu, and J.D. Teufel, [arXiv:1612.07246v1](https://arxiv.org/abs/1612.07246v1).