Robust sliding mode observer design for interconnected systems

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***Abstract*— In this paper, a class of nonlinear interconnected systems is considered in the presence of structured and unstructured uncertainties. The bounds on the uncertainties are nonlinear and are employed in the observer design to reject the effect of the uncertainties. Under the condition that the structure matrices of the uncertainties are known, a robust sliding mode observer is designed and a set of sufficient conditions is developed such that the error dynamics are asymptotically stable. If the structure of the uncertainties is unknown, an untimately bounded observer is developed using sliding mode techniques. The obtained results are applied to a multimachine power system to demonstrate the effectiveness of the developed methods.**

I. INTRODUCTION

The development of advanced technologies has produced corresponding growth in physical systems. Such systems can be expressed by sets of lower-order ordinary differential equations which are linked through interconnections. Such models are typically called large scale interconnected systems (see, e.g.[7], [16]). Large scale interconnected systems widely exists in the real world for example, the energy systems and bilogical systems [1], [7]. One of the most important examples of an interconnected system is the interconnected power system or multimachine power system which consists of multi power generators connected via a power distribution network [13]. Naturally, the model of the power system is inherently nonlinear containing disturbances and uncertainties [8], [13].

Recently, sliding mode controllers have been successfully applied for large scale power systems due to their effectiveness and robustness against various disturbances [11]. Sliding mode controllers for a single machine are proposed in [3] and multimachine power systems are considered in [2]. In all the results mentioned above, it is assumed that all the system state variables are available. However, in practice, only a subset of state variables is accessible/measurable. In order to implement these control schemes, one of the choices is to design an observer to estimate system states, and then use the estimated states to form the feedback loop. Therefore, a state estimation process is very important.

An observer-based controller is proposed in [6] by combining a variable structure control with a reduced-order observer and this is applied to a power system stabilizer. In [10] unknown-input observer-based monitors which can estimate the system states as well as perform fault detection and isolation are proposed and applied to a three-bus power system example, which consists of one generator and two loads. However, observer design in the presence of unknown signals is very difficult in practice. An iteratively re-weighted least squares method for power system state estimation is presented in [9]. An extended complex Kalman filter is used in [4] to enhance frequency estimation of distorted power system signals. A sliding mode observer is presented in [5] to develop a robust observer-based nonlinear controller and then to construct state variables of the system and estimate the perturbation including all the system nonlinearities and uncertainties. In [8], a sliding mode observer is developed for damper winding currents which are modelled as a 5-th order system.

In this paper, a robust sliding mode observer is established for a class of interconnected systems in the presence of uncertainties. Both the known nonlinear interconnections and uncertain nonlinear interconnections are considered. A set of sufficient conditions is developed such that the error dynamics are asymptotically stable if the structure of the uncertainties is known and satisfies the constrained Lyapunov equation. In the case when the structure of the uncertainties is not available but the bounds on the uncertainties are known constants, an ultimately bounded sliding mode observer is proposed to estimate the states of the interconnected system. All the bounds on the uncertainties involed in this paper are nonlinear and are employed in the observer design to reject/reduce the effect of uncertainties. The results obtained are applied to multimachine power systems. Simulation for a two machine power systems is used to demonstrate the effectiveness of the developed results.

II. SYSTEM DESCRIPTION AND PRELIMINARIES

Consider a nonlinear interconnected system composed of N subsystems as follows

$$\dot{x}_i = A_i x_i + B_i u_i + \Delta\phi_i(x_i, u_i) + M_i(x) + \Delta M_i(x) \quad (1)$$

$$y_i = C_i x_i \quad (2)$$

where $x_i \in R^{n_i}$, $u_i \in U \in R^{m_i}$ (U is the admissible control set) and $y_i \in R^{p_i}$ with $m_i \le p_i \le n_i$ are the state variables, inputs and outputs of the i-th subsystem respectively. The matrix triples (A_i, B_i, C_i) are constants with appropriate dimensions and C_i are full column rank for $i = 1, 2, \cdots, N$. The terms $\Delta\phi_i(x_i, u_i)$ and $\Delta M_i(x)$ are the uncertainties in the i-th isolated subsystems and interconnections respectively. The terms $M_i(x)$ are the known interconnections for

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$i = 1, \cdots, N$.

Assumption 1. The uncertainties $\Delta\phi_i(x_i, u_i)$ and $\Delta M_i(x)$ have the decomposition

$$\Delta\phi_i(x_i, u_i) = H_i^a \Delta\xi_i(x_i, u_i), \;\; \Delta M_i(x) = H_i^b \Delta E_i(x) \quad (3)$$

where $H_i^a \in R^{n_i \times k_i}$ and $H_i^b \in R^{n_i \times r_i}$ are the distribution matrices of the uncertainties, and

$$\|\Delta\xi_i(x_i, u_i)\| \le \rho_i(x_i, u_i), \text{ and } \|\Delta E_i(x)\| \le \sigma_i(x) \quad (4)$$

where $\rho_i(x_i, u_i)$ is known and Lipshitz about x_i uniformly for $u_i \in U$, and $\sigma_i(x)$ is known and Lipshitz about x.

Since C_i are full column rank, there exist nonsingular matrices T_{c_i} such that

$$\bar{A}_i = \begin{bmatrix} \bar{A}_{i1} & \bar{A}_{i2} \\ \bar{A}_{i3} & \bar{A}_{i4} \end{bmatrix} := T_{c_i} A_i T_{c_i}^{-1}, \quad (5)$$

$$\bar{B}_i = \begin{bmatrix} \bar{B}_{i1} \\ \bar{B}_{i2} \end{bmatrix} := T_{c_i} B_i, \;\; \bar{C}_i = \begin{bmatrix} 0 & I_{p_i} \end{bmatrix} := C_i T_{c_i}^{-1} \quad (6)$$

where $\bar{A}_{i1} \in R^{(n_i - p_i) \times (n_i - p_i)}$, $\bar{B}_{i1} \in R^{(n_i - p_i) \times m_i}$ and $\bar{B}_{i2} \in R^{p_i \times m_i}$ for $i = 1, \cdots, N$. Then in the new coordinates

$$\bar{x}_i = T_{c_i} x_i \quad (7)$$

system (1)-(2) can be rewritten as

$$\begin{aligned} \dot{\bar{x}}_{i1} &= \bar{A}_{i1}\bar{x}_{i1} + \bar{A}_{i2}\bar{x}_{i2} + \bar{B}_{i1}u_i + \bar{H}_{i1}^a \Delta\bar{\phi}_i(\bar{x}_i, u_i) \\ &\quad + \bar{M}_{i1}(\bar{x}) + \bar{H}_{i1}^b \Delta\bar{M}_i(\bar{x}) \quad (8) \\ \dot{\bar{x}}_{i2} &= \bar{A}_{i3}\bar{x}_{i1} + \bar{A}_{i4}\bar{x}_{i2} + \bar{B}_{i2}u_i + \bar{H}_{i2}^a \Delta\bar{\phi}_i(\bar{x}_i, u_i) \\ &\quad + \bar{M}_{i2}(\bar{x}) + \bar{H}_{i2}^b \Delta\bar{M}_i(\bar{x}) \quad (9) \\ y_i &= \bar{x}_{i2} \quad (10) \end{aligned}$$

where $\bar{x} = \text{col}(\bar{x}_1, \bar{x}_2, \cdots, \bar{x}_N)$, $\bar{x}_i = \text{col}(\bar{x}_{i1}, \bar{x}_{i2})$, $\bar{x}_{i1} \in R^{n_i - p_i}$, $\bar{x}_{i2} \in R^{p_i}$, $\bar{A}_{ij}$ and $\bar{B}_{il}$ are defined in (5)-(6) for $j = 1, 2, 3, 4$, $l = 1, 2$, $i = 1, 2, \cdots, N$, and

$$\begin{bmatrix} \bar{H}_{i1}^a \\ \bar{H}_{i2}^a \end{bmatrix} : = T_{c_i} H_i^a, \quad \begin{bmatrix} \bar{H}_{i1}^b \\ \bar{H}_{i2}^b \end{bmatrix} := T_{c_i} H_i^b \quad (11)$$

$$\begin{bmatrix} \bar{M}_{i1}(x) \\ \bar{M}_{i2}(x) \end{bmatrix} : = T_{c_i} M_i(x) \quad (12)$$

$$\begin{aligned} \Delta\bar{\phi}_i(\bar{x}_i, u_i) &= \Delta\xi_i(T_{c_i}^{-1}\bar{x}_i, u_i) \quad (13) \\ \Delta\bar{M}_i(\bar{x}) &= \Delta E_i(T_{c_i}^{-1}\bar{x}) \quad (14) \end{aligned}$$

where $\bar{H}_{i1}^a \in R^{(n_i - p_i) \times k_i}$, $\bar{H}_{i1}^b \in R^{(n_i - p_i) \times r_i}$, and $\bar{M}_{i1}(\cdot) \in R^{(n_i - p_i)}$ for $i = 1, 2, \cdots, N$.

Assumption 2. The matrix pair $(\bar{A}_i, \bar{C}_i)$ in (5)-(6) is observable for $i = 1, 2, \cdots, N$.

Under Assumption 2, there exists a matrix L_i such that $\bar{A}_i - L_i\bar{C}_i$ is stable, and thus for any $Q_i > 0$ the Lyapunov equation

$$(\bar{A}_i - L_i\bar{C}_i)^T P_i + P_i(\bar{A}_i - L_i\bar{C}_i) = -Q_i \quad (15)$$

has an unique solution $P_i > 0$ for $i = 1, 2, \cdots, N$.

Assumption 3. There exist a matrices $F_i^a \in R^{k_i \times p_i}$ and $F_i^b \in R^{r_i \times p_i}$ such that the solution P_i to the Lyapunov equation (15) satisfies the constraint

$$\bar{H}_i^{aT} P_i = F_i^a \bar{C}_i \quad (16)$$

$$\bar{H}_i^{bT} P_i = F_i^b \bar{C}_i \quad (17)$$

Introduce partitions of P_i and Q_i which are conformable with the decomposition in (8)-(10) as follows

$$P_i = \begin{bmatrix} P_{i1} & P_{i2} \\ P_{i2}^T & P_{i3} \end{bmatrix}, \qquad Q_i = \begin{bmatrix} Q_{i1} & Q_{i2} \\ Q_{i2}^T & Q_{i3} \end{bmatrix} \quad (18)$$

Then, from $P_i > 0$ and $Q_i > 0$ that $P_{i1} > 0$, $P_{i3} > 0$, $Q_{i1} > 0$ and $Q_{i3} > 0$.

The following results are required for further analysis.

Lemma 1. If P_i and Q_i have the partition in (18), then under Assumption 3

(i) $P_{i1}^{-1} P_{i2} \bar{H}_{i2}^a + \bar{H}_{i1}^a = 0$ if (16) is satisfied.

(ii) $P_{i1}^{-1} P_{i2} \bar{H}_{i2}^b + \bar{H}_{i1}^b = 0$ if (17) is satisfied.

(iii) The matrix $A_{i1} + P_{i1}^{-1} P_{i2} A_{i3}$ is Hurwitz stable if the Lyapunov equation (15) is satisfied.

Proof. See Lemma 2.1 in [14].

III. Sliding mode observer design

A. The structure matrices of the uncertainties are known

Consider the system in (8)-(10). Introduce a linear coordinate transformation

$$z_i = \underbrace{\begin{bmatrix} I_{n_i - p_i} & P_{i1}^{-1} P_{i2} \\ 0 & I_{p_i} \end{bmatrix}}_{T_i} \bar{x}_i \quad (19)$$

In the new coordinate system z_i, system (8)-(10) has the following form

$$\begin{aligned} \dot{z}_{i1} &= (\bar{A}_{i1} + P_{i1}^{-1} P_{i2} \bar{A}_{i3}) z_{i1} + (\bar{A}_{i2} - \bar{A}_{i1} P_{i1}^{-1} P_{i2} \\ &\quad + P_{i1}^{-1} P_{i2} (\bar{A}_{i4} - \bar{A}_{i3} P_{i1}^{-1} P_{i2})) z_{i2} + \bar{B}_{i1} u_i + P_{i1}^{-1} P_{i2} \\ &\quad \times \bar{B}_{i2} u_i + \bar{M}_{i1}(T^{-1}z) + P_{i1}^{-1} P_{i2} \bar{M}_{i2}(T^{-1}z) \quad (20) \\ \dot{z}_{i2} &= \bar{A}_{i3} z_{i1} + (\bar{A}_{i4} - \bar{A}_{i3} P_{i1}^{-1} P_{i2}) z_{i2} + \bar{B}_{i2} u_i \\ &\quad + \bar{H}_{i2}^a \Delta\bar{\phi}_i(T_i^{-1} z_i, u_i) + \bar{M}_{i2}(T^{-1}z) \\ &\quad + \bar{H}_{i2}^b \Delta\bar{M}_i(T^{-1}z) \quad (21) \\ y_i &= z_{i2} \quad (22) \end{aligned}$$

where $z_i = \text{col}(z_{i1}, z_{i2})$ with $z_{i1} \in R^{n_i - p_i}$. From Assumption 1, (13) and (14)

$$\|\Delta\bar{\phi}_i(T_i^{-1} z_i, u_i)\| \le \rho_i((T_i T_{c_i})^{-1} z_i, u_i) := \bar{\rho}_i(z_i, u_i) \quad (23)$$

$$\|\Delta\bar{M}_i(T^{-1}z)\| \le \sigma_i((TT_c)^{-1}z) := \bar{\sigma}_i(z) \quad (24)$$

and $\bar{\rho}_i(z_i, u_i), \bar{\sigma}_i(z)$ satisfy the Lipschitz condition

$$\begin{aligned} \|\bar{\rho}_i(z_i, u_i) - \bar{\rho}_i(\hat{z}_i, u_i)\| &\le \ell_{\bar{\rho}_i} \|z_i - \hat{z}_i\| \quad (25) \\ \|\bar{\sigma}_i(z) - \bar{\sigma}_i(\hat{z})\| &\le \ell_{\bar{\sigma}_i} \|z - \hat{z}\| \quad (26) \end{aligned}$$

Here $\ell_{\bar{\rho}_i}$ may be a function of u_i.

For system (20)-(22), consider a dynamical system

$$\begin{aligned}\dot{\hat{z}}_{i1} &= (\bar{A}_{i1}+P_{i1}^{-1}P_{i2}\bar{A}_{i3})\hat{z}_{i1}+(\bar{A}_{i2}-\bar{A}_{i1}P_{i1}^{-1}P_{i2}\\&\quad +P_{i1}^{-1}P_{i2}(\bar{A}_{i4}-\bar{A}_{i3}P_{i1}^{-1}P_{i2}))y_i+\bar{B}_{i1}u_i+P_{i1}^{-1}P_{i2}\\&\quad \times\bar{B}_{i2}u_i+\bar{M}_{i1}(T^{-1}\hat{z})+P_{i1}^{-1}P_{i2}\bar{M}_{i2}(T^{-1}\hat{z}) \quad (27)\\ \dot{\hat{z}}_{i2} &= \bar{A}_{i3}\hat{z}_{i1}+(\bar{A}_{i4}-\bar{A}_{i3}P_{i1}^{-1}P_{i2})\hat{z}_{i2}+\bar{B}_{i2}u_i\\&\quad +\bar{M}_{i2}(T^{-1}\hat{z})+d_i(\cdot) \quad (28)\\ \hat{y}_i &= \hat{z}_{i2} \quad (29)\end{aligned}$$

where $\hat{z}=\mathrm{col}(\hat{z}_1,y)$, and the injection term $d_i(\cdot)$ is defined by

$$\begin{aligned}d_i(\cdot) &= (\|\bar{H}_{i2}^a\|\bar{\rho}_i(\hat{z}_i,u_i)+\|\bar{H}_{i2}^b\|\bar{\sigma}_i(\hat{z})+\|\bar{A}_{i4}\\&\quad -\bar{A}_{i3}P_{i1}^{-1}P_{i2}\|\|y_i-\hat{y}_i\|+k_i)\mathrm{sgn}(y_i-\hat{y}_i) \quad (30)\end{aligned}$$

where $\bar{\rho}_i(\hat{z}_i,u_i)=\bar{\rho}_i(\hat{z}_{i1},y_i,u_i)$ and $\bar{\sigma}_i(\hat{z})=\bar{\sigma}_i(\hat{z}_{11},y_1,\hat{z}_{21},y_2,\cdots,\hat{z}_{N1},y_N)$.

Let $e_{i1}=z_{i1}-\hat{z}_{i1}$ and $e_{y_i}=y_i-\hat{y}_i$. Then from (20)-(22) and (27)-(29), the error dynamical equation is described by

$$\begin{aligned}\dot{e}_{i1} &= (\bar{A}_{i1}+P_{i1}^{-1}P_{i2}\bar{A}_{i3})e_{i1}+[\bar{M}_{i1}(T^{-1}z)-\bar{M}_{i1}(T^{-1}\hat{z})]\\&\quad +P_{i1}^{-1}P_{i2}[\bar{M}_{i2}(T^{-1}z)-\bar{M}_{i2}(T^{-1}\hat{z})] \quad (31)\\ \dot{e}_{y_i} &= \bar{A}_{i3}e_{i1}+(\bar{A}_{i4}-\bar{A}_{i3}P_{i1}^{-1}P_{i2})e_{y_i}+[\bar{M}_{i2}(T^{-1}z)\\&\quad -\bar{M}_{i2}(T^{-1}\hat{z})]+\bar{H}_{i2}^a\Delta\bar{\phi}_i(T_i^{-1}z_i,u_i)\\&\quad +\bar{H}_{i2}^b\Delta\bar{M}_i(T^{-1}z)-d_i(\cdot) \quad (32)\end{aligned}$$

where $d_i(\cdot)$ is given in (30) for $i=1,2,\cdots,N$.

From the structure of the transformation matrix T_i in (19) and the fact that $\hat{z}_i=\mathrm{col}(\hat{z}_{i1},y_i)$, it follows that

$$\|T^{-1}z-T^{-1}\hat{z}\|=\|e_1\| \quad (33)$$

where

$$e_1:=\mathrm{col}(e_{11},e_{21},\cdots,e_{N1}) \quad (34)$$

Therefore,

$$\begin{aligned}\|\bar{M}_{i1}(T^{-1}z)-\bar{M}_{i1}(T^{-1}\hat{z})\| &\le \ell_{\bar{M}_{i1}}\|e_1\| \quad (35)\\ \|\bar{M}_{i2}(T^{-1}z)-M_{i2}(T^{-1}\hat{z})\| &\le \ell_{\bar{M}_{i2}}\|e_1\| \quad (36)\end{aligned}$$

Theorem 1. Under Assumptions $1-3$, the error system (31) is asymptotically stable if the matrix W^T+W is positive definite, where the matrix $W=[w_{ij}]_{N\times N}$, and its entries w_{ij} are defined by

$$w_{ij}=\begin{cases}\lambda_{min}(Q_{i1})-2\big[\|P_{i1}\|\ell_{\bar{M}_{i1}}+\|P_{i2}\|\ell_{\bar{M}_{i2}}\big], & i=j\\ -2\big[\|P_{i1}\|\ell_{\bar{M}_{i1}}+\|P_{i2}\|\ell_{\bar{M}_{i2}}\big], & i\neq j\end{cases} \quad (37)$$

where P_{i1}, P_{i2} and Q_{i1} are given in (18).

Proof. For system (31), consider a Lyapunov function candidate $V=\sum_{i=1}^N e_{i1}^T P_{i1}e_{i1}$. Then, the time derivative of V along the trajectories of system (31) is given by

$$\begin{aligned}\dot{V} &= \sum_{i=1}^N\Big\{e_{i1}^T[P_{i1}(\bar{A}_{i1}+P_{i1}^{-1}P_{i2}\bar{A}_{i3})^T+(\bar{A}_{i1}\\&\quad +P_{i1}^{-1}P_{i2}\bar{A}_{i3})P_{i1}]e_{i1}+2\|P_{i1}\|\|e_{i1}\|\big\{\big[\ell_{\bar{M}_{i1}}\\&\quad +\|P_{i1}^{-1}P_{i2}\|\ell_{\bar{M}_{i2}}\big]\|e_1\|\big\}\Big\}\\ &\le \sum_{i=1}^N\Big\{-e_{i1}^TQ_{i1}e_{i1}+2\|e_{i1}\|\big\{\big[\|P_{i1}\|\ell_{\bar{M}_{i1}}\\&\quad +\|P_{i2}\|\ell_{\bar{M}_{i2}}\big]\|e_1\|\big\}\Big\} \quad (38)\end{aligned}$$

From the definition of e_1 in (34)

$$\|e_1\|\le\sum_{j=1}^N\|e_{j1}\|=\|e_{i1}\|+\sum_{\substack{j=1\\ j\neq i}}^N\|e_{j1}\| \quad (39)$$

Then, from (38) and (39)

$$\begin{aligned}\dot{V} &\le \sum_{i=1}^N\Big\{-e_{i1}^TQ_{i1}e_{i1}+2\|e_{i1}\|\big\{\big[\|P_{i1}\|\ell_{\bar{M}_{i1}}\\&\quad +\|P_{i2}\|\ell_{\bar{M}_{i2}}\big]\big[\|e_{i1}\|+\sum_{\substack{j=1\\ j\neq i}}^N\|e_{j1}\|\big]\big\}\Big\}\\ &\le -\sum_{i=1}^N\Big\{\big\{\lambda_{min}(Q_{i1})-2\big[\|P_{i1}\|\ell_{\bar{M}_{i1}}+\|P_{i2}\|\ell_{\bar{M}_{i2}}\big]\big\}\\&\quad \times\|e_{i1}\|^2-\sum_{\substack{j=1\\ j\neq i}}^N 2\big[\|P_{i1}\|\ell_{\bar{M}_{i1}}+\|P_{i2}\|\ell_{\bar{M}_{i2}}\big]\\&\quad \times\|e_{i1}\|\|e_{j1}\|\Big\} \quad (40)\end{aligned}$$

Then, from the definition of the matrix W in (37) and the inequality above, it follows that

$$\dot{V}\le-\frac{1}{2}X^T[W^T+W]X$$

where $X=[\|e_{11}\|,\|e_{21}\|,\cdots,\|e_{N1}\|]^T$.
Hence, the conclusion follows from $W^T+W>0$. $\triangle$

Remark 1. The proof of Theorm 1 further shows that the stability of the dynamics (31) are actually independent of e_{y_i}. This fact will be used to show the stability of the sliding motion later. From the stability of Theorem 1, it follows that there exists a constant β such that

$$\|e_1\|\le\beta,\quad i=1,2,\cdots,N \quad (41)$$

where β can be estimated using the approach given in [14].

For system (31)-(32), consider a sliding surface

$$\begin{aligned}S &= \{(e_{11},e_{y_1},e_{21},e_{y_2},\cdots,e_{N1},e_{y_N})\big|e_{y_1}=0,\\&\quad e_{y_2}=0,\cdots,e_{y_N}=0\} \quad (42)\end{aligned}$$

From the structure of the error dynamical system (31)-(32), it follows that the system (31) will dominate the sliding motion associated with the sliding surface (42).

Theorem 2. Under Assumptions 1-3, system (31)-(32) is driven to the sliding surface (42) in finite time and remains on it if

$$k_i \geq (\|\bar{A}_{i3}\| + \ell_{\bar{M}_{i2}} + \|\bar{H}_{i2}^a\|\ell_{\bar{\rho}} + \|\bar{H}_{i2}^b\|\ell_{\bar{\sigma}})\beta + \eta \tag{43}$$

where β is determined by (41) and η is a positive constant.
Proof. From (32)

$$\begin{aligned}
&\sum_{i=1}^{N} e_{y_i}^T \dot{e}_{y_i} = \sum_{i=1}^{N} e_{y_i}^T \Big\{ \bar{A}_{i3} e_{i1} + (\bar{A}_{i4} - \bar{A}_{i3} P_{i1}^{-1} P_{i2}) \\
&\quad \times e_{y_i} + [\bar{M}_{i2} - \hat{\bar{M}}_{i2}] + \bar{H}_{i2}^a \Delta\bar{\phi}_i(T_i^{-1} z_i, u_i) \\
&\quad + \bar{H}_{i2}^b \Delta\bar{M}_i(T^{-1} z) - d_i(\cdot) \Big\} \\
&\leq \sum_{i=1}^{N} \Big\{ \|\bar{A}_{i3}\| \|e_{i1}\| \|e_{y_i}\| + \ell_{\bar{M}_{i2}} \|e_{y_i}\| \|e_1\| \\
&\quad + \|\bar{H}_{i2}^a\| \bar{\rho}_i(z_i, u_i) \|e_{y_i}\| + \|\bar{H}_{i2}^b\| \bar{\sigma}_i(z) \|e_{y_i}\| \\
&\quad + \|(\bar{A}_{i4} - \bar{A}_{i3} P_{i1}^{-1} P_{i2}) \|e_{y_i}\|^2 - \|e_{y_i}\| \\
&\quad \{ \|\bar{H}_{i2}^a\| \bar{\rho}_i(\hat{z}_{i1}, y_i, u_i) + \|\bar{H}_{i2}^b\| \bar{\sigma}_i(\hat{z}) \\
&\quad + \|\bar{A}_{i4} - \bar{A}_{i3} P_{i1}^{-1} P_{i2}\| \|e_{y_i}\| + k_i) \mathrm{sgn}(e_{y_i}) \} \Big\}
\end{aligned} \tag{44}$$

From (41), $\|e_{i1}\| \leq \beta$. Applying (41) to (44), it follows that

$$\begin{aligned}
\sum_{i=1}^{N} e_{y_i}^T \dot{e}_{y_i} \quad &\leq \quad \sum_{i=1}^{N} \Big\{ \{ (\|\bar{A}_{i3}\| + \ell_{\bar{M}_{i2}} + \|\bar{H}_{i2}^a\| \ell_{\bar{\rho}_i} \\
&\quad + \|\bar{H}_{i2}^b\| \ell_{\bar{\sigma}_i}) \beta - k_i \} \|e_{y_i}\| \Big\}
\end{aligned} \tag{45}$$

Applying (43) to (45)

$$e_{y_i}^T \dot{e}_{y_i} \leq -\eta \|e_{y_i}\| \tag{46}$$

This shows that the reachability condition is satisfied.
Hence the conclusion follows. △

Theorems 1 and 2 show that (27)-(29) is an asymptotic observer of system (20)-(22).

B. The structure of the uncertainties are unknown

Now, if the structure of the uncertainties $\Delta\phi_i(x_i, u_i)$ and $\Delta M_i(x)$ in the system (1)-(2) are unknown, which implies that Assumption 1 does not hold, then an asymptotic observer usually is not available. An ultimately bounded observer will be designed. The following Assumption is required.
Assumption 4. The uncertainties $\Delta\phi_i(x_i, u_i)$ and $\Delta M_i(x)$ in system (1)-(2) satisfy

$$\|\Delta\phi_i(x_i, u_i)\| \quad \leq \quad \varepsilon_i \tag{47}$$

$$\|\Delta\bar{M}_i(x)\| \quad \leq \quad \Upsilon_i \tag{48}$$

where ε_i and Υ_i are positive constants.
In this case, in the new coordinate z the system (1)-(2) is described by

$$\begin{aligned}
\dot{z}_{i1} &= (\bar{A}_{i1} + P_{i1}^{-1} P_{i2} \bar{A}_{i3}) z_{i1} + (\bar{A}_{i2} - \bar{A}_{i1} P_{i1}^{-1} P_{i2} \\
&\quad + P_{i1}^{-1} P_{i2} (\bar{A}_{i4} - \bar{A}_{i3} P_{i1}^{-1} P_{i2})) z_{i2} + \bar{B}_{i1} u_i \\
&\quad + P_{i1}^{-1} P_{i2} \times \bar{B}_{i2} u_i + \bar{M}_{i1}(T^{-1} z) + P_{i1}^{-1} P_{i2} \\
&\quad \times \bar{M}_{i2}(T^{-1} z) + \Delta\tilde{\phi}_{i1}(T_i^{-1} z_i, u_i) + \Delta\tilde{M}_{i1}(T^{-1} z)
\end{aligned} \tag{49}$$

$$\begin{aligned}
\dot{z}_{i2} &= \bar{A}_{i3} z_{i1} + (\bar{A}_{i4} - \bar{A}_{i3} P_{i1}^{-1} P_{i2}) z_{i2} + \bar{B}_{i2} u_i \\
&\quad + \bar{M}_{i2}(T^{-1} z) + \Delta\tilde{\phi}_{i2}(T_i^{-1} z_i, u_i) + \Delta\tilde{M}_{i2}(T^{-1} z)
\end{aligned} \tag{50}$$

$$y_i = z_{i2} \tag{51}$$

where

$$\begin{bmatrix} \Delta\tilde{\phi}_{i1}(T_i^{-1} z_i, u_i) \\ \Delta\tilde{\phi}_{i2}(T_i^{-1} z_i, u_i) \end{bmatrix} = T_i^{-1} \begin{bmatrix} \Delta\phi_{i1}(T_i^{-1} z_i, u_i) \\ \Delta\phi_{i2}(T_i^{-1} z_i, u_i) \end{bmatrix} \tag{52}$$

$$\begin{bmatrix} \Delta\tilde{M}_{i1}(T^{-1} z) \\ \Delta\tilde{M}_{i2}(T^{-1} z) \end{bmatrix} = T_i^{-1} \begin{bmatrix} \Delta\bar{M}_{i1}(T^{-1} z) \\ \Delta\bar{M}_{i2}(T^{-1} z) \end{bmatrix} \tag{53}$$

and $z_i = \mathrm{col}(z_{i1}, z_{i2})$ with $z_{i1} \in R^{n_i - p_i}$. From (47)-(48), there are constants ε_i^a, ε_i^b, Υ_i^a and Υ_i^b such that

$$\|\Delta\tilde{\phi}_{i1}(T_i^{-1} z_i, u_i)\| \quad \leq \quad \varepsilon_i^a \tag{54}$$

$$\|\Delta\tilde{\phi}_{i2}(T_i^{-1} z_i, u_i)\| \quad \leq \quad \varepsilon_i^b \tag{55}$$

$$\|\Delta\tilde{M}_{i1}(T^{-1} z)\| \quad \leq \quad \Upsilon_i^a \tag{56}$$

$$\|\Delta\tilde{M}_{i2}(T^{-1} z)\| \quad \leq \quad \Upsilon_i^b \tag{57}$$

Now consider dynamical systems

$$\begin{aligned}
\dot{\hat{z}}_{i1} \quad &= \quad (\bar{A}_{i1} + P_{i1}^{-1} P_{i2} \bar{A}_{i3}) \hat{z}_{i1} + (\bar{A}_{i2} - \bar{A}_{i1} P_{i1}^{-1} P_{i2} \\
&\quad + P_{i1}^{-1} P_{i2} (\bar{A}_{i4} - \bar{A}_{i3} P_{i1}^{-1} P_{i2})) y_i + \bar{B}_{i1} u_i \\
&\quad + P_{i1}^{-1} P_{i2} \bar{B}_{i2} u_i + \bar{M}_{i1}(T^{-1} \hat{z}) \\
&\quad + P_{i1}^{-1} P_{i2} \bar{M}_{i2}(T^{-1} \hat{z})
\end{aligned} \tag{58}$$

$$\begin{aligned}
\dot{\hat{z}}_{i2} \quad &= \quad \bar{A}_{i3} \hat{z}_{i1} + (\bar{A}_{i4} - \bar{A}_{i3} P_{i1}^{-1} P_{i2}) \hat{z}_{i2} + \bar{B}_{i2} u_i \\
&\quad + \bar{M}_{i2}(T^{-1} \hat{z}) + d_i(\cdot)
\end{aligned} \tag{59}$$

$$\hat{y}_i \quad = \quad \hat{z}_{i2} \tag{60}$$

where $\hat{z} = \mathrm{col}(\hat{z}_1, y)$. The injection term $d_i(\cdot)$ is defined by

$$\begin{aligned}
d_i(\cdot) \quad &= \quad (\|\Delta\tilde{\phi}_{i2}(T_i^{-1} \hat{z}_i, u_i)\| + \|\Delta\tilde{M}_{i2}(T^{-1} \hat{z})\| + \|\bar{A}_{i4} \\
&\quad - \bar{A}_{i3} P_{i1}^{-1} P_{i2}\| \|y_i - \hat{y}_i\| + k_i) \mathrm{sgn}(y_i - \hat{y}_i)
\end{aligned} \tag{61}$$

Let $e_{i1} = z_{i1} - \hat{z}_{i1}$ and $e_{y_i} = y_i - \hat{y}_i$. Then from (49)-(51) and (58)-(60), the error dynamical equation is described by

$$\begin{aligned}
\dot{e}_{i1} &= (\bar{A}_{i1} + P_{i1}^{-1} P_{i2} \bar{A}_{i3}) e_{i1} + [\bar{M}_{i1}(T^{-1} z) - \bar{M}_{i1}(T^{-1} \hat{z})] \\
&\quad + P_{i1}^{-1} P_{i2} [\bar{M}_{i2}(T^{-1} z) - \bar{M}_{i2}(T^{-1} \hat{z})] \\
&\quad + \Delta\tilde{\phi}_{i1}(T_i^{-1} z_i, u_i) + \Delta\tilde{M}_{i1}(T^{-1} z)
\end{aligned} \tag{62}$$

$$\begin{aligned}
\dot{e}_{y_i} &= \bar{A}_{i3} e_1 + (\bar{A}_{i4} - \bar{A}_{i3} P_{i1}^{-1} P_{i2}) e_{y_i} + [\bar{M}_{i2}(T^{-1} z) \\
&\quad - \bar{M}_{i2}(T^{-1} \hat{z})] + \Delta\tilde{\phi}_{i2}(T_i^{-1} z_i, u_i) \\
&\quad + \Delta\tilde{M}_{i2}(T^{-1} z) - d_i(\cdot)
\end{aligned} \tag{63}$$

Theorem 3. Under Assumptions 2 and 4, the system (62) is an ultimately bounded stable if the function matrix $W^T + W$ is positive definite, where the matrix $W = [w_{ij}]_{N \times N}$, and its entries w_{ij} are defined by

$$w_{ij} = \begin{cases} \lambda_{min}(Q_{i1}) - 2\big[\|P_{i1}\| \ell_{\bar{M}_{i1}} + \|P_{i2}\| \ell_{\bar{M}_{i2}}\big], & i = j \\ -2\big[\|P_{i1}\| \ell_{\bar{M}_{i1}} + \|P_{i2}\| \ell_{\bar{M}_{i2}}\big], & i \neq j \end{cases} \tag{64}$$

where P_{i1}, P_{i2} and Q_{i1} are from (18).

Proof. For system (62), consider the same Lyapunov function as in the proof of Theorem 1. Following a similar proof as in Theorem 1, it is obtained

$$\begin{aligned}\dot{V} \leq & -\sum_{i=1}^{N}\Big\{\big\{\lambda_{min}(Q_{i1}) - 2\big[\|P_{i1}\|\ell_{\bar{M}_{i1}} + \|P_{i2}\|\ell_{\bar{M}_{i2}}\big]\big\} \\ & \|e_{i1}\| - \sum_{\substack{j=1\\ j\neq i}}^{N} 2\big[\|P_{i1}\|\ell_{\bar{M}_{i1}} + \|P_{i2}\|\ell_{\bar{M}_{i2}}\big]\|e_{j1}\|\Big\}\|e_{i1}\| \\ & +2\sum_{i=1}^{N}\|P_{i1}\|\big[\varepsilon_i^a + \Upsilon_i^a\big]\|e_{i1}\| \end{aligned} \tag{65}$$

Then, from the definition of the matrix W in theorem 2 and the inequality above, it follows that

$$\begin{aligned}\dot{V} &\leq -\frac{1}{2}X^T[W^T + W]X + \mu X \\ &= -(\frac{1}{2}\lambda_{min}(W^T+W)\|X\| - \mu)\|X\| \end{aligned} \tag{66}$$

where $\mu = 2\sqrt{\sum_{i=1}^{N}(\|P_{i1}\|\big[\varepsilon_i^a + \Upsilon_i^a\big])^2}$ and $X = [\|e_{11}\|, \|e_{21}\|, \cdots, \|e_{N1}\|]^T$.

It is clear to see that $\dot{V} < 0$ if $\mu < \frac{1}{2}\lambda_{min}(W^T + W)$. Therefore system (62) is ultimately bounded. △

For the system (62)-(63), consider the same sliding surface S given in (42). It is straightforward to see that Theorem 3 implies that the sliding mode of the system (62)-(63) associated with the sliding surface S given in (42) is ultimately bounded.

Theorem 4. Under Assumptions 2 and 4, the system (62)-(63) is driven to the sliding surface (42) in finite time and remains on it if

$$k_i \geq (\|\bar{A}_{i3}\| + \ell_{\bar{M}_{i2}} + \ell_{\Delta\tilde{\phi}_{i2}} + \ell_{\Delta\tilde{M}_{i2}})\beta + \eta \tag{67}$$

where β is determined by (41) and η is a positive constant.

The proof can be obtained directly from Theorem 2.

Remark 2 The sliding mode observer in z coordinates is provided in (27)-(29) or (58)-(60). Therefore the estimate $\hat{x}_i$ for x_i can be given by $\hat{x}_i = (T_iT_{c_i})^{-1}\hat{z}_i$, where T_{c_i} and T_i are given in (7) and (19) respectively and $\hat{z}_i$ is given in (27)-(29) or (58)-(60).

IV. Simulation example

In this section, the excitation control problem for a multimachine power system is considered. Let $x_i = [x_{i1}\ \ x_{i2}\ \ x_{i3}] = [\delta_i - \delta_{i0}\ \ \omega_i\ \ \Delta P_{ei}]$ with $\Delta P_{ei} \equiv$: $P_{ei} - P_{mi0}$ for $i = 1, 2, \cdots, N$. It is assumed that, $P_{mi} = P_{mi0} =$ constant since only excitation control is considered and δ_i is the generator power angle [rad], P_{ei} is electrical power [p.u.], and ω_i is relative speed [rad/s]. All terms are explained in [15]. Then by using direct feedback linearsation compensation for the power system as in [12], the multimachine power system can be described by the system $(1)-(2)$ with

$$\begin{aligned} A_i &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\frac{D_i}{2H_i} & -\frac{\omega_0}{2H_i} \\ 0 & 0 & -\frac{1}{T'_{doi}} \end{bmatrix}, B_i = \begin{bmatrix} 0 \\ 0 \\ T'_{doi} \end{bmatrix} \\ C_i &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

The following uncertainties are added to the isolated systems

$$\Delta\phi_1(x_1, u_1) = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0.5 \end{bmatrix}}_{H_1^a} \underbrace{|x_{11}|\ |\sin u_1|}_{\Delta\xi_1(x_1,u_1)} \tag{68}$$

$$\Delta\phi_2(x_2, u_2) = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0.2 \end{bmatrix}}_{H_2^a} \underbrace{|\sin^2(x_{21} + x_{23})|}_{\Delta\xi_2(x_2,u_2)} \tag{69}$$

where $|\Delta\xi_1(x_1, u_1)| < |x_{11}|\ |\sin u_1| = \rho_1(x_1, u_1)$ and $\Delta\xi_2(x_2, u_2) < |\sin^2(x_{21} + x_{23})| = \rho_2(x_2, u_2)$.

The input control variables, interconnection and its uncertain terms are chosen the same as in [15]. Choose

$$T_{c_i} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{for} \quad i =, 1, 2, \cdots, N. \tag{70}$$

The system matrices after transformation $\bar{x}_i = T_{c_i}x_i$ with comparing $(5)-(6)$ are

$$\begin{aligned} \bar{A}_{i1} &= -\frac{D_i}{2H_i}, \qquad \bar{A}_{i2} = \begin{bmatrix} 0 & -\frac{\omega_0}{2H_i} \end{bmatrix} \\ \bar{A}_{i3} &= \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \qquad \bar{A}_{i4} = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{T'_{doi}} \end{bmatrix} \\ \bar{B}_{i1} &= 0, \qquad \bar{B}_{i2} = \begin{bmatrix} 0 \\ \frac{1}{T'_{doi}} \end{bmatrix} \\ \Delta\bar{M}_{i1} &= 0, \qquad \Delta\bar{M}_{i2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}\Phi_i(x) \end{aligned}$$

In order to illustrate the results obtained in this paper, consider two machine power systems where all the parameters are chosen as in [15]. Then, let $Q_1 = Q_2 = I_3$.

By direct computation, The solutions of Lyapunov equation (15) P_1 and P_2 can be found and under the transformation $x_i = (T_iT_{c_i})^{-1}z_i$ with T_{c_i} and T_i defined in (70) and (19), the two machine power systems can be described in z coordinates as in the form of $(20)-(22)$ with

$$\begin{aligned} \|\bar{\rho}_1(z_1, u_1)\| &\leq |z_{121}||\sin u_1| \\ \|\bar{\rho}_2(z_2, u_2)\| &\leq |\sin^2(z_{221} + z_{222})| \end{aligned}$$

and

$$\begin{aligned} |\bar{\sigma}_1(z)| &\leq (\gamma_{11}^I|\sin z_{121}| + \gamma_{11}^{II}|(z_{11} + 0.2311z_{121}|)) \\ &\quad +(\gamma_{12}^I|\sin z_{221}| + \gamma_{12}^{II}|(z_{21} + 0.4412z_{221}|)) \\ |\bar{\sigma}_2(z)| &\leq (\gamma_{21}^I|\sin z_{121}| + \gamma_{21}^{II}|(z_{11} + 0.2311z_{121}|)) \\ &\quad +(\gamma_{22}^I|\sin z_{221}| + \gamma_{22}^{II}|(z_{21} + 0.4412z_{221}|)) \end{aligned}$$

Therefore,

$$|\bar{\sigma}_1(z) - \bar{\sigma}_1(\hat{z})| = \begin{bmatrix} \gamma_{11}^{II} & \gamma_{11}^{I} + 0.2311\gamma_{11}^{II} & 0 & \gamma_{12}^{II} & \gamma_{12}^{I} + 0.4412\gamma_{12}^{II} & 0 \end{bmatrix} \times [\|z - \hat{z}\|]$$

$$|\bar{\sigma}_2(z) - \bar{\sigma}_2(\hat{z})| = \begin{bmatrix} \gamma_{21}^{II} & \gamma_{21}^{I} + 0.2311\gamma_{21}^{II} & 0 & \gamma_{22}^{II} & \gamma_{22}^{I} + 0.4412\gamma_{22}^{II} & 0 \end{bmatrix} \times [\|z - \hat{z}\|]$$

where $\gamma_{11}^{I} = 0.9$, $\gamma_{12}^{I} = 0.7355$, $\gamma_{11}^{II} = \gamma_{12}^{II} = 1.4$ and $\gamma_{21}^{I} = 0.966$, $\gamma_{22}^{I} = 0.788$, $\gamma_{21}^{II} = \gamma_{22}^{II} = 1.5$. Thus $\ell_{\bar{\sigma}_1} = 2.69224$ and $\ell_{\bar{\sigma}_2} = 2.88532$.

By direct computation, it follows that the matrix $W^T + W$ is positive definite. Thus, all the conditions of Theorem 1 are satisfied. Therefore the dynamical system $(27) - (29)$ is an asymptotic observer of the system $(20) - (22)$ which is well defined and $\hat{x}_i = (T_i T_{c_i})^{-1}\hat{z}_i$ is an estimate of $x_i = [x_{i1} \;\; x_{i2} \;\; x_{i3}] = [\delta_i - \delta_{i0} \;\; \omega_i \;\; \Delta P_{ei}]$. The simulation results are presented in Figs 1 and 2, which show the effective of the designed observer.

V. Conclusion

In this paper, a robust sliding mode observer has been designed for a class of interconnected systems in the presence of uncertainties. Both the known nonlinear interconnections and uncertain nonlinear interconnections have been dealt with separately to reduce the effects of the interconnections. Sufficient conditions have been provided such as that the error dynamics are asymptotically stable if the structure of the uncertainties is known. An ultimately bounded sliding mode observer is proposed to estimate the states of the interconnected system if the structure of the uncertainties is not available. All the bounds on the uncertainties involved in this paper are nonlinear and are employed in the observer design to reject/reduce the effect of uncertainties. The obtained results have been applied to a multimachine power system to show the feasibility of the proposed approach.

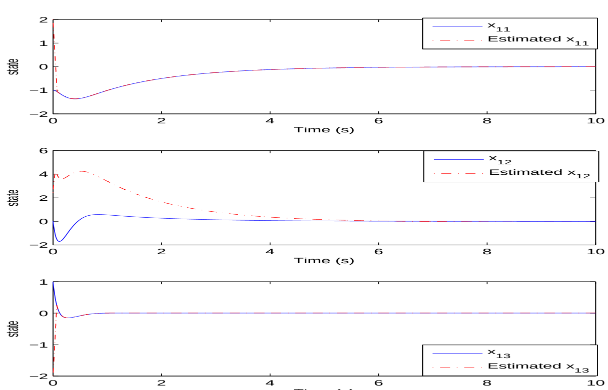

Fig. 1. The time response of 1st subsystem states $x_1 = \text{col}\,(x_{11}, x_{12}, x_{13})$ and their estimation $\hat{x}_1 = \text{col}\,(\hat{x}_{11}, \hat{x}_{12}, \hat{x}_{13})$

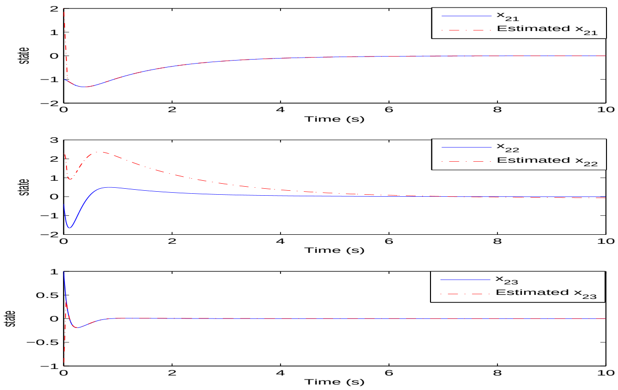

Fig. 2. The time response of 1st subsystem states $x_2 = \text{col}\,(x_{21}, x_{22}, x_{23})$ and their estimation $\hat{x}_2 = \text{col}\,(\hat{x}_{21}, \hat{x}_{22}, \hat{x}_{23})$

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