

A NUMERICAL APPROACH FOR LIQUEFACTION POTENTIAL DEFINITION

G. Vessia¹, J. Arias Truillo², S. Lopez-Querol³, C. Cherubini⁴, R. Blazquez⁵, D. Foti⁶

¹ Contract researcher, Dept. of Civil and Environmental Engineering, Section of Geotechnical and Geoenvironmental Engineering, Bari. Italy.

² PhD student, Dept. of Civil Engineering, Universidad de Castilla La Mancha, Ciudad Real. Spain.

³ Professor, Dept. of Civil Engineering, Universidad de Castilla La Mancha, Ciudad Real. Spain.

⁴ Full professor, Dept. of Civil and Environmental Engineering, Section of Geotechnical and Geoenvironmental Engineering, Bari. Italy.

⁵ Full professor, Dept. of Civil Engineering, Universidad de Castilla La Mancha, Ciudad Real. Spain.

⁶ Professor, Dept. of Civil and Environmental Engineering, Section of Geotechnical and Geoenvironmental Engineering, Bari. Italy.

Email: g.vessia@poliba.it, juana.arias@uclm.es, mariasusana.lopez@uclm.es, c.cherubini@poliba.it,
rafael.blazquez@uclm.es, d.foti@poliba.it

ABSTRACT :

Liquefaction phenomenon in saturated granular soil is not that frequent as amplification cases but can cause heavy damages on buildings and infrastructures whenever it occurs especially within superficial strata. In fact the lack of shear resistance of soil due to liquefaction affects mostly shallow foundations and road surfaces. Up now, several studies have been addressed to overpass the inadequacy of liquefaction safety factor by means of introducing the liquefaction potential. Nevertheless, the difficulty in (1) defining a scale of damage related to liquefaction potential values and (2) collecting field data from damages caused prevalently by liquefaction makes the punctual factor of safety still popular in engineering practice. In this paper a new approach to liquefaction potential estimation is proposed based on finite element dynamic analyses and on the concept of "significant volume" according to possible effects suffered by shallow foundations. One-dimensional simulation of liquefaction occurrence is performed by means of the Pastor-Zienkiewicz constitutive law. Hence the estimation of liquefaction potential is gained as well as the stress influence factor from Westergaard solution is calculated.

KEYWORDS: Liquefaction potential, Pastor-Zienkiewicz constitutive law, granular soil.

1. INTRODUCTION

Several methods for assessing the liquefaction potential of granular soil have been developed along thirty years. They tried to transform the punctual liquefaction factor of safety into a spatial index which suggests how much buildings can be affected by the lost of shear resistance of soil beneath their foundation. All the contributions can be grouped into two types:

- Deterministic approach
- Probabilistic approach

However, all of them referring to in situ or laboratory tests need to be calibrated by means of field evidences, which show liquefaction occurrence on soil and/or damages caused on buildings, collected after a strong earthquake struck. Although the evaluation of liquefaction potential on free soil is a relevant topic, the main interest in potential index definition should be devoted to relate the liquefaction occurrence to the degree of damage caused. Hence fragility curves for different types of buildings have been proposed in literature for bridges (Shinozuka et al., 2000) and buildings (Bird et al., 2006) but these studies are not able to connect the soil liquefaction extent to the potential damage caused.

In the following the authors propose a new approach to the definition of liquefaction potential by means of comparing the significant volume of a shallow foundation with the liquefied depth in one-dimensional finite element numerical analyses.

2. LIQUEFACTION POTENTIAL INDEXES

An overview of liquefaction potential formulation is a difficult task because several contributions have been developed up now. Past experiences took advantages from the work of numerous researchers at the National Center for Earthquake Engineering Research (NCEER) since 1987 and the Technical Committee 4 (TC4) Earthquake Geotechnical Engineering of ISSMGE (International Society of Soil Mechanics and Geotechnical Engineering) since 1984. The attention to liquefaction potential is devoted, on one hand, to enhance in situ and laboratory tests (Robertson and Wride, 1998; Youd et al., 2001) for evaluating the so called liquefaction safety factor (FSL), defined as:

$$FSL = \frac{CRR}{CSR} \cdot MSF \quad (2.1)$$

where CRR is the cyclic resistance ratio; CSR is the cyclic strength ratio and MSF is the coefficient which adjusts the liquefaction resistance of a soil to the reference magnitude of 7.5 to different magnitude values. Those resistance and loading ratios are commonly related to in situ tests as CPT, SPT and V_s . Moreover, recent studies have focused on the use of the seismic dilatometer (SDMT) at this scope (Maugeri and Monaco, 2006). On the other hand, the exigency of predicting the potential damages that liquefaction occurrence has proven to cause all around the world, led the research interest toward the definition of a liquefaction potential index that can take into account the extension of the phenomenon along depth. Accordingly, since thirty years ago, Iwasaki et al. (1978) developed the following expression for liquefaction potential index (LPI):

$$LPI = \int_0^{20m} FSL \cdot w(z) dz \quad (2.2)$$

in which FSL is the liquefaction safety factor as defined by Eq. (2.1), $w(z)$ is the depth weighting factor and z is the depth:

$$w(z) = 10 - 0.5z \quad (2.3)$$

Such approach is sheared by numerous authors that have been improved the previous expression according to field data (Juang and Chen, 2000; Juang et al., 2000a; Toprak and Holzer, 2003; Nagase et al., 2005) and

probabilistic studies (Juang et al., 2000b; Cetin et al., 2004; Juang et al., 2004; Blazquez and Lopez-Querol, 2003). All of deterministic and probabilistic studies are nowadays enhanced by GIS and geostatistical tools, respectively, for addressing the spatial distribution of liquefaction potential by means of maps of liquefaction susceptibility (Liu and Chen, 2006; Chien et al., 2002).

Another approach to address the liquefaction hazard deals with the definition of annual probability of liquefaction (P_L) based on FSL values (Kramer, 1996; Juang et al., 2001) or on P_L -FSL mapping function (Juang et al., 2005). Within the previous aim, other authors proposed a weighted P_L on the first 20m depth, where P_L is estimated by means of the energy dissipation method (Lee et al., 2007; Juang and Chen, 2000). Such method is based on the assumption suggested by Nasser and Shokooch (1979) that the pore water pressure is directly related to the amount of seismic energy dissipated in the soil and accordingly Davis and Berrill (1982) developed the relationship between the increase in excess pore pressure and energy dissipation:

$$\Delta u = \frac{450}{R^2 \sqrt{\sigma'_0}} \cdot \frac{N_1^2}{10^{1.5M}} \quad (2.4)$$

where Δu = excess pore pressure (kPa); M = earthquake magnitude; R = distance from the epicenter (km); σ'_0 =effective overburden stress (kPa); N_1 =corrected standard penetration blow count according to Liao and Whitman (1986). Thus a general equation for the developed excess pore water pressure may be expressed as:

$$\frac{\Delta u}{\sigma'_0} = \frac{A \cdot 10^{1.5M}}{R^B N_1^C \sigma'^D_0} \quad (2.5)$$

where A , B , C and D are unknown regression coefficients that may be calibrated using case histories of liquefaction where the surface manifestation was observed. By definition, when the ratio from Eq. (2.5) is equal to 1 the liquefaction will occur.

In this paper a new definition of liquefaction potential index has been conceived from the energy dissipation approach.

3. NEW PROPOSAL FOR LIQUEFACTION POTENTIAL DETERMINATION

This paper suggests a new one-dimensional deterministic approach to the estimation of liquefaction potential which is not any more related to a safety factor determined at various depths but involves the extent along depth of the following condition:

$$\frac{\Delta u}{\sigma'_{v0}} \geq 1 \quad (3.1)$$

where Δu is the excess pore water pressure value during the seismic shaking and σ'_{v0} is the initial vertical effective stress. This ratio shows how much the seismic shaking has modified the stress equilibrium between the solid skeleton and the fluid phase. Hence, whenever and wherever pore water pressure equals the static effective stress value the physical soil status changes in viscous fluid material.

Such information is directly drawn from a finite element numerical model (Lopez-Querol et al., 2008) which implements the enhancement of the generalized plasticity-based constitutive model for sands known as Pastor-Zienkiewicz (PZ) (Pastor et al., 1990). The description of PZ constitutive law as well as its enhancement and how they were numerically implemented goes beyond the scope of the paper. Nonetheless it must be said that, the model used herein for predicting liquefaction occurrence simulates the dynamic response of saturated sandy soils in one-dimension using displacement fields for coupled solid (u) and fluid (w) phases for both elastic and plastic solid skeleton. Such model has been demonstrated to follow the evolution of sandy soil

response until liquefaction in a more stable way with respect to numerical model in terms of pore pressure (p_w) and σ'_v . Accordingly by means of u-w model it is possible to read stress-strain status nearer the liquefaction than the most commonly used u- p_w formulations.

Hence, in order to determine the liquefaction potential index, the liquefaction occurs wherever the condition (3.1) is satisfied. This means that in a homogeneous soil deposit, the liquefaction is evaluated through the values of pore water pressure compared to the initial effective vertical stress. Once the liquefaction has occurred, the most relevant information for addressing the estimation of possible damages against structures is how much the liquefied soil affects the influence depth of a shallow foundation. To this end the stress influence factor I_σ is considered and calculated by means of Westergaard solution (1938), in the case of a square foundation:

$$I_\sigma = 1 - \left(\frac{1}{1 + \left(\frac{B}{2z_f} \right)^2} \right)^{1.76} \quad (3.2)$$

Hence, the liquefaction potential (LP), is assumed to be the difference between the I_σ related to z_{fini} and z_{ffin} which are depths: z_{fini} is the lowest and z_{ffin} is the highest depth where liquefaction has occurred, that is Eq. (3.1) is satisfied:

$$LP (\%) = I_{\sigma ini} - I_{\sigma fin} \quad (3.3)$$

In such a definition the spatial distribution of liquefaction is considered whereas its time evolution is taken into account up to numerical instability appearance, which typically occurs when the shear strains increase too much, avoiding the initial hypothesis of small strains in the finite element mesh.

Such liquefaction potential is expressed by percentage values varying from 0 to 100% and it can be related to the reduction of bearing capacity value (q_{red}) by means of the following relation:

$$q_{red} = q_{adm} \cdot [(I_{\sigma ini} - I_{\sigma fin})] \quad (3.4)$$

Such definition of liquefaction potential is particularly useful as numerical simulation results are concerned. No assumptions on the maximum depth for liquefaction is assumed because results outside the significant volume of shallow foundation doesn't affect the footing thus doesn't contribute to possible damages. Furthermore, this index doesn't need to be calibrated by field evidences of damages because it depends only on the appropriate experimental characterization and calibration of constitutive law parameters.

4. THE CASE STUDY

Numerical analyses are carried out on three types of granular soils which have been characterized by Pastor et al. (1990) in terms of the advanced constitutive law parameters. On these soils, square shallow foundations represented by stress distributions are applied on the surface in order to predict the soil response until liquefaction. Such materials are here described in terms of the equivalent Mohr-Coulomb failure criterion as reported in Table 4.1 whereas Pastor et al. (1990) in Table II reported PZ model parameters calling them as 15, 16 and 17.

Table 4.1 Soil parameters according to Mohr-Coulomb failure criterion.

| Soil type | Mohr-Coulomb parameter |
|-----------|--------------------------------------|
| Soil 1 | $\varphi' = 30^\circ$; $c' = 1$ kPa |
| Soil 2 | $\varphi' = 33^\circ$; $c' = 1$ kPa |
| Soil 3 | $\varphi' = 36^\circ$; $c' = 1$ kPa |

Such double characterization is needed in order to use results from one-dimensional PZ constitutive law for two-dimensional simplified formulation of bearing capacity of shallow foundation by means of Terzaghi expression. For the case of a square footing on granular soil, Terzaghi expression has been used:

$$q_{ult} = 1.3c'N_c + \sigma'_{zD}N_q + 0.4\gamma'BN_\gamma \quad (3.4)$$

where q_{ult} is the ultimate bearing capacity; c' is the effective cohesion; σ'_{zD} is the vertical effective stress at depth D (foundation embedment) below the ground surface; γ' is the effective unit weight of the soil; B is the width of foundation; N_c , N_q and N_γ are bearing capacity factors according to the following expressions:

$$N_q = \frac{1+\sin(\varphi')}{1-\sin(\varphi')} e^{\pi \tan(\varphi')}; \quad N_\gamma = 2(N_q + 1)\tan(\varphi'); \quad N_c = \frac{N_q - 1}{\tan(\varphi')} \quad (3.5)$$

The approximated expression of N_γ is derived by Coduto (1994) fitting a curve to match Terzaghi's. This expression produces N_γ values within about 10 percent of Terzaghi's values.

In this study the design bearing capacity has been calculated according to partial safety factor approach from EC 7 (UNI EN 1997, 2005) for three values of B , D and φ' as shown in Table 4.2-4.4. According to EC7, the effective stress approach has been considered and the combination "M2"+"R3" has been taken into account in order to estimate the geotechnical bearing capacity value only. To this end no seismic actions have been introduced in the bearing capacity estimation.

Moreover, as an example one input earthquake is considered in this study, which is scaled to the peak ground acceleration of seismic zone 1 (0.35g) and zone 2 (0.25g) according to Italian seismic zonation (Gruppo di Lavoro CPTI, 2004). Such earthquake has been drawn from probabilistic hazard seismic study on European earthquake databases.

Table 4.2 Bearing capacity values (kPa) calculated for different B and D values for soil 1.

| Footing width (B) [m] | Footing Embedment (D) [m] | | |
|--------------------------|------------------------------|------|------|
| | 2 | 4 | 6 |
| 1 | 498 | 894 | 1291 |
| 2 | 578 | 975 | 1371 |
| 3 | 658 | 1055 | 1451 |

Table 4.3 Bearing capacity values (kPa) calculated for different B and D values for soil 2.

| Footing width (B) [m] | Footing Embedment (D) [m] | | |
|--------------------------|------------------------------|------|------|
| | 2 | 4 | 6 |
| 1 | 670 | 1200 | 1724 |
| 2 | 787 | 1314 | 1841 |
| 3 | 905 | 1432 | 1960 |

Table 4.4 Bearing capacity values (kPa) calculated for different B and D values for soil 3.

| Footing width (B) [m] | Footing Embedment (D) [m] | | |
|--------------------------|------------------------------|------|------|
| | 2 | 4 | 6 |
| 1 | 920 | 1631 | 2344 |
| 2 | 1093 | 1806 | 2520 |
| 3 | 1267 | 1980 | 2693 |

According to the numerical dynamic analyses carried out, soil 2 and 3 are not liquefiable provided that the ratio in Eq. (3.1) is not always satisfied (the final result corresponds to the end of the earthquake, in several cases, of the time at which numerical instabilities due to extended liquefaction within the soil column appear). On the contrary, soil 1 liquefies in different cases: the extents of liquefaction depth and the related liquefaction potential index values, according Eq. (33), are reported in Table 4.5-4.8.

As can be drawn from Table 4.6 and 4.8, the input signal scaled at 0.35g causes liquefaction in quite all the cases whereas when it is scaled at 0.25g the liquefaction occurs only twice.

As liquefaction potential values in Table 4.6 are concerned, for the case of 2m footing embedment, passing from 1m to 3m footing width, they rapidly decreases. This fact can be explained by information from Table 4.5: the nearer the depth of liquefaction beginning is to the base of the footing the larger the liquefaction potential values although the depth lag of liquefaction is narrow.

This is a relevant result: the potential index value is here related to both vertical liquefaction extension and its vicinity to the base of the footing (where the most part of the overburden stress is transmitted to the soil). Similar results can be seen for the case of 4m footing embedment (Table 4.5- 4.6), where the liquefaction occurs only for higher overburden values (as expected) but for the liquefaction potential value the extension of liquefaction is less important than its vicinity to the base of the footing.

Table 4.5 Extension of liquefied zones for earthquake 1 for zone 1.

| Footing width (B) [m] | Footing Embedment (D) [m] | | | | | |
|--------------------------|------------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|
| | 2 | | 4 | | 6 | |
| | $Z_{f\text{ini}}$ [m] | $Z_{f\text{fin}}$ [m] | $Z_{f\text{ini}}$ [m] | $Z_{f\text{fin}}$ [m] | $Z_{f\text{ini}}$ [m] | $Z_{f\text{fin}}$ [m] |
| 1 | 0 | 6 | 0 | | 0 | |
| 2 | 2 | 15 | 0 | 20 | 0 | |
| 3 | 12 | 28 | 0 | 27 | 0 | 20 |

Table 4.6 Liquefaction potential index for earthquake 1 for zone 1.

| Footing width (B) [m] | Footing Embedment (D) [m] | | |
|--------------------------|------------------------------|------|-----|
| | 2 | 4 | 6 |
| | 1 | 99% | 0 |
| 2 | 32% | 100% | 0 |
| 3 | 2.2% | 100% | 99% |

Table 4.7 Extension of liquefied zones for earthquake 1 in zone 2.

| Footing width (B) [m] | Footing Embedment (D) [m] | | | | | |
|--------------------------|------------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|
| | 2 | | 4 | | 6 | |
| | $Z_{f\text{ini}}$ [m] | $Z_{f\text{fin}}$ [m] | $Z_{f\text{ini}}$ [m] | $Z_{f\text{fin}}$ [m] | $Z_{f\text{ini}}$ [m] | $Z_{f\text{fin}}$ [m] |
| 1 | 0 | 11 | 0 | | 0 | 4 |
| 2 | 0 | | 0 | | 0 | |
| 3 | 0 | | 0 | | 0 | |

Table 4.8 Liquefaction potential index for earthquake 1 in zone 2.

| Footing width (B) [m] | Footing Embedment (D) [m] | | |
|--------------------------|------------------------------|------|---|
| | 2 | 4 | 6 |
| | 1 | 100% | 0 |
| 2 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 |

From Table 4.7 and 4.8 the liquefaction phenomenon not frequently appears and only for the case of 1m width. Also in these two cases, the liquefaction potential value is strictly related to the position of liquefaction occurrence with respect to the base of the footing: in the case of 1m width, 4m or 11m depth lag of liquefaction slightly affects potential values because the “significant depth” of that footing is about 4m.

5. CONCLUDING REMARKS

This paper addresses the definition of a liquefaction potential index (LP) which can be related to possible damages suffered by structures resting on shallow foundations. Accordingly a new expression for LP is found based on results from one-dimensional dynamic numerical simulation of soil response subjected to different values of uniform stresses. These stresses correspond to different footing width in a simplified simulation. For defining LP the ratio between pore water pressures and initial vertical effective stress has been employed. Moreover this new formulation of LP takes into account the extension of liquefaction occurrence and its distance from the footing base.

The example performed shows the advantages of using this liquefaction potential index whenever numerical simulations are performed although further development in liquefaction potential definition toward two-dimensional analyses is needed.

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