Bailouts, Time Inconsistency, and Optimal Regulation: A Macroeconomic View†

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A common view is that bailouts of firms by governments are needed to cure inefficiencies in private markets. We propose an alternative view: even when private markets are efficient, costly bankruptcies will occur and benevolent governments without commitment will bail out firms to avoid bankruptcy costs. Bailouts then introduce inefficiencies where none had existed. Although granting the government orderly resolution powers which allow it to rewrite private contracts improves on bailout outcomes, regulating leverage and taxing size is needed to achieve the relevant constrained efficient outcome, the sustainably efficient outcome. This outcome respects governments’ incentives to intervene when they lack commitment. (JEL D86, E32, G33, H81, L51)

Recent experience has shown that governments can and will intervene during financial crises. During such crises, many firms are faced with the prospect of costly bankruptcy and liquidation. To minimize these costs, governments bail out firms that are on the brink of bankruptcy by purchasing their debt. Governments then renegotiate the debt contracts to allow such firms to continue operations without having to go through bankruptcy, and pay for such bailouts with taxes.

One view is that costly bankruptcies occur because private markets function poorly and that bailouts are needed to remedy inefficiencies in private markets. Here we formalize an alternative view that bailouts create rather than cure inefficiencies, propose a perspective that builds the incentives of governments without commitment into the efficiency criterion, and evaluate policies that are at the center of the current debate on regulating the financial system.

Our alternative view is that even if private markets are efficient, costly bankruptcies would still occur, and well-meaning governments without commitment would...
bail out firms in order to avoid bankruptcy costs. Such interventions introduce inefficiencies into otherwise well-functioning markets.

In analyzing policy, our perspective is that governments in practice lack commitment and, regardless of the laws in place, will find ways to engage in tax-financed bailouts if they find it desirable to do so. This perspective implies that outcomes which do not respect the incentives of the government to engage in bailouts are unattainable and leads us to label outcomes that do respect these incentives as sustainable. The relevant notion of constrained efficiency, then, is the best outcome that respects sustainability as well as the resource and informational constraints in the economy. We label such outcomes sustainably efficient.

Our analysis leads to three results. The first is that if the bailout temptation is sufficiently strong, then outcomes with bailouts are sustainably inefficient. The second is that an orderly resolution provision which, akin to a provision in the Dodd-Frank Act, grants the government the power to impose losses on unsecured creditors improves on bailout outcomes but is sustainably inefficient. The third is that sustainably efficient outcomes can be implemented by granting the government the regulatory authority to impose limits on the debt-to-value ratios and a tax on firm size.

A key insight of our analysis is that regulation should be most stringent when, absent intervention, the resources lost to bankruptcy would be the largest. We use this insight to determine how regulation should vary over the business cycle as well as across industries.

We begin with a simple one-period model that highlights the time inconsistency problem faced by governments. Our model, which uses some features of Townsend (1979), has information frictions, and we allow private agents to renegotiate contracts. With these features, private agents optimally choose to enter into debt-equity contracts that specify the size of the firm, as well as payments to debt holders, equity holders, and managers. Debt levels are chosen high enough so that the firm enters into costly bankruptcy with positive probability. Such bankruptcy plays a dual role: it is needed both to provide managers with incentives to exert effort and to optimally extract payments to investors from the firm. These contracts lead to ex ante efficient outcomes, and with commitment the government chooses not to intervene. Moreover, the ability of private agents to renegotiate contracts implies that the equilibrium is also ex post efficient.

Even though the equilibrium is ex post efficient, a benevolent government that seeks to maximize the sum of agents’ utilities will find it optimal to eliminate all impending bankruptcies because doing so raises this sum. The government buys up the debt of distressed firms and renegotiates their debt contracts in order to eliminate all bankruptcies. It finances debt purchases by levying taxes on all firms. Such a policy introduces a subsidy distortion by effectively subsidizing payments to the firm in the event of impending bankruptcy. An individual firm has an incentive to increase its likelihood of bankruptcy so as to receive these subsidies because, from its perspective, the taxes it pays are not affected by its choices. Of course, in equilibrium each firm’s expected subsidies equal its expected tax payments. These subsidies thus induce a distortion in effort and size choices even though no external funds are used to finance them. This distortion leads welfare to be strictly lower than it would be under commitment, and thus our economy has a time inconsistency problem.
The one-period model is a useful prelude to our main analysis, but it lacks a key feature of the data, namely that we observe partial rather than complete bailouts. More important, it misses a critical insight into size externalities, which we discuss later. Our dynamic model, an infinitely repeated version of the one-period model, addresses both of these deficiencies.

In our dynamic model, reputational considerations impose ex post costs of unexpected intervention by affecting private agents’ beliefs about future policies. To see how such ex post costs can arise, suppose that an unexpectedly large bailout today leads private agents to expect that all distressed firms will be bailed out in the future. Such expectations imply that a bailout authority who is contemplating an unexpectedly large bailout may be deterred from doing so, because the current gain from reducing bankruptcy may be outweighed by the future losses arising from distortions induced by anticipated bailouts.

We show that such logic implies that equilibrium outcomes respect the incentives of the government to intervene if and only if they satisfy a sustainability constraint. This constraint requires that the current gains from policy deviations must be outweighed by the future losses induced by changes in private expectations from such deviations. We show that if the discount factor is not too high, the sustainability constraint binds and we have partial bailouts in that the bailout authority bails out some but not all firms.

We show that bailout equilibrium outcomes are sustainably inefficient if the sustainability constraint is binding. This inefficiency arises from subsidy distortions and size externalities. The subsidy distortions arise for the same reasons as in the one-period model. The size externality, which is present only in the dynamic model, arises from a free-rider problem generated by the sustainability requirement. When firms in the aggregate increase their size, the resources lost to bankruptcy increase and the bailout authority is more tempted to intervene ex post. Each individual firm is made better off if all firms reduce their size below privately optimal levels so that the government does not intervene, but no individual firm will agree to do so. At a mechanical level, the inefficiency arises because in the sustainability constraint the gains to deviation are increasing in the current size of firms.

One proposed remedy reflects the popular view that bailouts lead to inefficiency because they subsidize firms that would otherwise go bankrupt. The Dodd-Frank Act contains provisions that reflect this popular view. The key feature of this lengthy act, which we refer to as an orderly resolution provision, grants to a governmental authority the power to impose losses on unsecured creditors. Our reading of the act is that, although the additional powers are intended to dampen the incentives of the government to engage in tax-financed bailouts, in practice the government cannot be prevented from engaging in such bailouts in times of severe financial stress if it chooses to do so.

We model the orderly resolution provision by granting the bailout authority the additional power to force debt holders of firms facing impending bankruptcy to accept lower debt payments. We show that armed with this additional power, this authority eliminates the subsidy distortions by effectively making each firm pay for its own bailouts. It does not eliminate the size externality, however, because individual firms do not internalize that the extent of intervention by this authority is determined by the size of firms in the aggregate.
Optimal regulation in the form of a limit on the debt-to-value ratio and a tax on size achieves the sustainably efficient outcome. It eliminates the subsidy distortions by eliminating the incentives of the government to conduct bailouts ex post, and it eliminates the size externalities by inducing firms to reduce their size.

The general implication of our analysis is that regulation should be most stringent when, absent intervention, the resources lost to bankruptcy would be the largest. In terms of the cyclicality of regulation, this insight implies that regulation should be more stringent in aggregate states for which lost resources are higher, but interestingly, this regulation can be either procyclical or countercyclical depending on the details of how aggregate shocks affect the sustainability constraint.

Optimal regulation in our model also addresses a type of a too-big-to-fail problem. Usual discussions of this problem focus on the idea that individual firms in industries such as banking become too large. Since the aggregate loss to bankruptcy determines the incentives of the bailout authority to intervene, our analysis suggests that the too-big-to-fail problem is that the industry as a whole becomes too large rather than that particular firms within an industry become too large.

In this paper we formalize the view that the inefficiency of bailout outcomes arises not from perverse incentives that private agents might have but rather from the incentives of well-meaning governments that cannot commit themselves. To do so, we allow private agents to have access to a rich set of contracts and allow these agents to optimally adjust their contracts as their expectations of government policy change. In particular, we do not restrict our analysis to a fixed set of contracts. We purposely do so because analyses of bailouts that restrict themselves to a fixed set of suboptimal contracts make it hard to distinguish between regulation that is needed to overcome inefficient behavior by private agents arising from restrictions imposed on contracts and regulation that is needed to overcome inefficient behavior by governments without commitment.

In formalizing this view, we have purposely abstracted from any inefficiencies in private markets. These include externalities generated by spillover effects—say, from fire sales. A standard story is that when the financial sector undergoes severe stress, these stresses spill over to other sectors in a way that is not internalized by the market. Such spillovers generate externalities that can be mitigated by regulation. Clearly, these spillover externalities have nothing to do with the externalities generated by lack of commitment by the government; regulation is needed even under commitment. Obviously, we can add such spillovers to our model. Our analysis can be thought of as describing regulation that is needed to address the externalities arising from lack of commitment by the government over and above those needed to cure standard spillover externalities.

We have also purposely abstracted from the inefficiencies that arise from coordination problems by focusing on the best equilibrium in each policy regime. We have abstracted from coordination problems for two reasons. First, we want to focus on
the role of regulation in mitigating distortions and externalities arising from a time inconsistency problem. Second, our dynamic model has multiple Pareto-ranked equilibria, and regulation cannot cure this multiplicity.

In focusing on the best equilibrium, we follow a long tradition in public finance and mechanism design that looks for policies and mechanisms that weakly implement desired outcomes. That is, under the constructed policies and mechanisms, the desired outcome is one equilibrium among many possible equilibria. The idea is that society can somehow find a way to coordinate the desired equilibrium outcome. Note, however, that even if we follow this tradition, a classic time inconsistency problem remains and regulation is desirable. (See Lucas and Stokey 1983 for the classic definition.1)

Related Literature.—The size externality in our model arises because of the feature that the aggregate size of the capital stock appears in the sustainability constraint and, with competitive agents, each individual agent takes this aggregate as given when choosing individual capital stocks. This general feature arises in many macroeconomic models that have competitive agents and limited commitment by the government. See, for example, Marcet and Marimon (1998); Kehoe and Perri (2004); Acemoglu, Golosov, and Tsyvinski (2008); Aguiar and Amador (2011); and Ales, Maziero, and Yared (2014).

We have motivated our modeling of orderly resolution based on our reading of the Dodd-Frank Act. Another motivation comes from a report by 15 academic economists (French et al. 2010). These economists propose that regulators institute ways to expedite resolution of distressed financial institutions. We show that our orderly resolution mechanism captures the spirit of their main proposals.

A paper that is closely related to ours is Gertler, Kiyotaki, and Queralto (2012). They develop a model in which a macroprudential policy that subsidizes equity finance ameliorates a pecuniary externality. As in our work, the debt-equity ratios of banks are determined, in part, by expectations of the extent of government intervention. Unlike our paper, the government intervention policy is not optimally determined given the decisions of private agents. Thus, their paper does not address the central question in our paper: the extent to which regulation is needed to cure the externalities generated by time inconsistency problems.

In interesting related work, Farhi and Tirole (2012) analyze the role of regulation in addressing a coordination problem. Absent regulation, without commitment their model has multiple equilibria, one of which coincides with the equilibrium under commitment. Regulation can uniquely implement the commitment equilibrium. Since the commitment equilibrium is also an equilibrium without commitment, their model does not have a classic time inconsistency problem in the sense of Lucas and Stokey (1983). Hence, if we applied the traditional weak implementation approach to their model, we would conclude that regulation is unnecessary. More important, their model does not have bailouts that involve purchases of debt by the government accompanied by forgiveness. Thus, they cannot analyze the role of policy

1 Recall from Lucas and Stokey (1983) that a Ramsey plan starting at date 0, thought of as the best equilibrium with commitment, is time consistent if, taking as given the history of policies and private allocations from the date 0 plan, a Ramsey planner at any future date \( t \geq 1 \) would choose to continue with the date 0 plan. If the Ramsey plan does not have this property, then the economy has a time inconsistency problem.
interventions, such as orderly resolution, that force unsecured creditors to absorb losses. (It is worth noting that the model in Farhi and Tirole 2012 has the feature that there is a unique equilibrium under commitment, but additional, inferior equilibria appear when the government does not have commitment. Hence, the coordination problem is connected to the lack of commitment by the government.)

Other interesting related work is that by Keister (2014), who studies the role of bailouts in a Diamond and Dybvig (1983) model. Keister’s model also has multiple equilibria, and the best equilibrium without commitment coincides with the commitment equilibrium. In this sense, Keister also does not have a time inconsistency problem, and the commitment outcome can be weakly implemented without regulation.

Our paper is closely related to a literature that analyzes the role of bailouts in international sovereign debt markets. (See Aguiar and Amador 2014 for a survey.) Burnside, Eichenbaum, and Rebelo (2001) and Schneider and Tornell (2004) emphasize the role that bailouts by domestic governments play in exacerbating financial crises. Neither of these papers, however, analyzes the role of regulatory policy. A branch of this literature analyzes the role of an international lender of last resort, which, during crises, provides funds at a below-market rate and, in this sense, bails out sovereign borrowers. Cole and Kehoe (2000); Conesa and Kehoe (2014); Lorenzoni and Werning (2013); and Ayres et al. (2015) show that such a lender can play a beneficial role by helping to solve coordination problems. As we have noted, solving such coordination problems is not the focus of our analysis.

A burgeoning recent literature gives a prominent role to regulation as the way to correct pecuniary externalities arising either from lack of commitment by private agents or from hidden trading. (See, for example, the work of Lorenzoni 2008; Farhi, Golosov, and Tsyvinski 2009; Bianchi and Mendoza 2010; Bianchi 2011; and Gertler, Kiyotaki, and Queralto 2012.) In contrast, in our work, a subtle externality arises because of lack of commitment by the government.

Here we have derived these results using a variant of the model in Townsend (1979). We conjecture that similar results could be derived in variants of the models in Harris and Raviv (1990) and Holmström and Tirole (1998).


I. The One-Period Economy

We begin with a one-period economy that sets the stage for our infinite-horizon analysis. We describe the key frictions in the economy, solve for the optimal contracts, and show that the competitive equilibrium is both ex ante and ex post efficient. We then introduce a government and show that without commitment, governments engage in bailouts and the resulting equilibria are sustainably inefficient. Finally, we show that an orderly resolution authority implements sustainably efficient outcomes.

A. With Only Private Agents

We begin by considering a one-period economy without a government. In this economy, managers and investors design optimal contracts that are intended to induce effort and share output in the face of three key frictions. First, the effort of the manager is
privately observed by the manager. Second, although the manager costlessly observes a component of productivity, labeled *private*, investors can observe this component only by putting the firm through bankruptcy. We assume that bankruptcy is costly in that it reduces the output of firms proportionately. Third, the manager and the investors cannot commit to the terms of their contracts; that is, they can renegotiate the terms of a contract after the manager chooses effort. This ability to renegotiate implies that the contract is *ex post efficient* in the sense that there is no incentive feasible way to make all agents better off ex post. We allow for private renegotiation to ensure that the ex post gains from bailouts do not arise from a desire to improve ex post efficiency.

We allow investors and managers to sign any contracts as long as they respect the informational and renegotiation constraints of the environment. We show that the optimal contracts are *debt-equity* contracts, and under sufficient conditions have costly bankruptcy. We also show that the resulting competitive equilibrium is ex ante efficient in that a planner, confronted with the same information and renegotiation frictions, would choose the same outcomes.

**Setup and Characterization of the Competitive Equilibrium.**—In the model, decisions are made in two stages: a first stage at the beginning of the period and a second stage at the end. The economy has two types of agents, called *managers* and *investors*, both of whom are risk neutral and consume at the end of the period. The economy has a measure 1 of managers and a measure 1 of investors.

The technology uses two inputs in the first stage, an investment of $k$ units of goods per manager and effort $p$ by the manager, to produce output at the second stage. The effort level $p$ of any individual manager is unobserved by other agents in the economy.

The amount of output produced in the second stage stochastically depends on the effort level $p$, the amount of investment $k$, two idiosyncratic shocks denoted $A_s$ and $\varepsilon$, and a bankruptcy decision. The shock $A_s$, $s \in \{H, L\}$, determines the average level of productivity and is called the health status. It is publicly observed at no cost. We refer to $A_H$ as the *healthy state* and $A_L$ as the *distressed state*. These shocks satisfy $A_H > A_L$. With probability $p_H = p$ the healthy state is realized, and with complementary probability $p_L = 1 - p$ the distressed state is realized.

The shock $\varepsilon$ is privately observed by the manager and is made public only if the firm declares bankruptcy, as described later. We assume that $\varepsilon$ has density $h_s(\varepsilon)$ and distribution $H_s(\varepsilon)$ with mean 1 and support $[\underline{\varepsilon}_s, \overline{\varepsilon}_s]$. The idiosyncratic shocks $A_s$ and $\varepsilon$ are realized after the effort level is chosen and are independently and identically distributed across firms.

After the shocks have been realized, the firm can choose whether to declare bankruptcy. If it does not declare bankruptcy, output is given by $A_s\varepsilon g(k)$ for $s \in \{H, L\}$, where $g(k)$ is an increasing concave function. If the firm declares bankruptcy, then it is monitored, and the idiosyncratic shock $\varepsilon$ becomes publicly known. Monitoring is costly in that output is then given by $RA_s\varepsilon g(k)$, where $R < 1$.

Let $B_s$ denote the *bankruptcy set*, namely, the set of idiosyncratic shocks $\varepsilon$ such that the firms declare bankruptcy when the health state is $s \in \{H, L\}$. The complementary set in which no bankruptcy occurs is denoted $N_s$.2

2Throughout, we restrict attention to deterministic monitoring. For analyses with stochastic monitoring, see Townsend (1979) and Mookherjee and Png (1989).
Managers have no endowments of goods but do have the specialized skills needed to operate the technology. Investors have $\omega$ units of endowments but do not have these specialized skills. Investors choose how much to invest in the technology and can store the rest of their endowments at rate 1. The only role of storage is to pin down the opportunity costs of funds to be 1. We assume that $\omega$ is sufficiently large so that some amount of the endowment is always stored. We also assume that the technology is sufficiently attractive so that it is always active.

Investors invest with competitive financial intermediaries who enter into contracts with a continuum of managers. Given that all investors are identical, we can restrict attention to a representative financial intermediary who behaves competitively.

A contract between the representative financial intermediary and a given manager specifies the size of the firm, the recommended effort level for the manager, the consumption level of the manager, payments to the financial intermediary, and the bankruptcy set. By symmetry we can restrict attention to a representative contract. Let $C_s(\varepsilon)$ denote the consumption of the managers when the health state is $s \in \{H, L\}$ and the idiosyncratic shock is $\varepsilon$. Managers are risk neutral over consumption. Their disutility from effort depends on $p_H = p$, is proportional to the size of the project $g(k)$, and is given by $v(p_H)g(k)$, where $v(p_H)$ is an increasing convex function. Thus, the manager’s utility function is given by

$$\sum_s p_s \int C_s(\varepsilon) dH_s(\varepsilon) - v(p_H)g(k).$$

The consumption of the managers must satisfy a nonnegativity constraint $C_s(\varepsilon) \geq 0$.

Investors invest their endowment $\omega$ with the representative financial intermediary. This intermediary invests $k$ units with each firm and stores $\omega - k$ units. Let $D_s(\varepsilon)$ denote the payments a firm with idiosyncratic shocks $s$ and $\varepsilon$ makes to the financial intermediary. Since the representative financial intermediary enters into contracts with a continuum of managers, we can invoke the law of large numbers and obtain that the deterministic payoff to the representative financial intermediary, and hence to the investors it represents, is

$$\sum_s p_s \int D_s(\varepsilon) dH_s(\varepsilon) + \omega - k.$$

The financial intermediary maximizes (2).

When the firm does not declare bankruptcy, the firm’s resource constraint is

$$C_s(\varepsilon) + D_s(\varepsilon) = A_s \varepsilon g(k),$$

and when the firm does declare bankruptcy, the firm’s resource constraint is

$$C_s(\varepsilon) + D_s(\varepsilon) = RA_s \varepsilon g(k).$$

Thus, the overall budget constraint of the financial intermediary is that the sum of the consumption of managers and the (deterministic) consumption of investors
associated with the payoffs from the financial intermediary be no more than the output produced including stored endowments:

\[
\sum_s p_s \left[ \int C_s(\varepsilon) dH_s(\varepsilon) \right] + \sum_s p_s \int D_s(\varepsilon) dH_s(\varepsilon) + \omega - k \leq \sum_s p_s \left[ \int N_s A_s \varepsilon dH_s(\varepsilon) + \int B_s R A_s \varepsilon dH_s(\varepsilon) \right] g(k) + \omega - k.
\]

A representative contract consists of \( x = \{k, p, C_s(\varepsilon), D_s(\varepsilon), B_s\} \). Here, effort is not observable so it is not directly contractible, but the contract will be designed to ensure that the manager exerts the agreed-upon level of effort. Note that given a contract, the consumption of the investors is implied by the resource constraint.

The timing is as follows. The financial intermediary and managers first agree to a contract, and then the managers choose their effort level \( p \). After the effort level is chosen, the health status of each firm \( s \) is publicly realized. The intermediary and managers then renegotiate the contract. Finally, the idiosyncratic shocks \( \varepsilon \) are realized, and the bankruptcy decisions are made according to the renegotiated contract.

To be part of a competitive equilibrium, a contract has to satisfy various conditions. Consider the conditions that must be satisfied after \( k \) and \( p \) are chosen. One is that any contract must be incentive compatible with respect to reporting the shock \( \varepsilon \); that is, a manager must prefer to report the idiosyncratic shock \( \varepsilon \) truthfully rather than misreport it. A manager with a shock \( \varepsilon \) in the nonbankruptcy set must not have an incentive to misreport any other shock \( \hat{\varepsilon} \) in this nonbankruptcy set, so that

\[
C_s(\varepsilon) = A_s \varepsilon g(k) - D_s(\varepsilon) \geq A_s \varepsilon g(k) - D_s(\hat{\varepsilon}) \quad \text{for all } \varepsilon \in N_s, \hat{\varepsilon} \in N_s.
\]

This constraint implies that for all \( \varepsilon \in N_s \), payments to the financial intermediary \( D_s(\varepsilon) \) are constant in the nonbankruptcy set at some level, denoted \( D_s \). Using this result, since a manager with a shock \( \varepsilon \) in the bankruptcy set must not have an incentive to misreport any other shock \( \hat{\varepsilon} \) in the nonbankruptcy set, we have

\[
C_s(\varepsilon) = RA_s \varepsilon g(k) - D_s(\varepsilon) \geq A_s \varepsilon g(k) - D_s \quad \text{for all } \varepsilon \in B_s, \hat{\varepsilon} \in N_s.
\]

(Of course, it is not feasible for the agent to claim some \( \hat{\varepsilon} \) in the bankruptcy set, since that report will be monitored.)

A contract is incentive feasible if it is incentive compatible in that it satisfies (6) and (7), and resource feasible in that it satisfies the resource constraints (3) and (4) and the nonnegativity constraint on the manager’s consumption.

Since we allow the financial intermediary and the managers to renegotiate contracts, we can restrict attention to contracts with the property that no renegotiation occurs. Before renegotiation begins, a particular representative contract \( x \) has been agreed to, effort level \( p \) has been chosen, and health shocks \( s \) have been realized for all the firms. Thus, at the time of renegotiation there are managers at healthy firms, managers at distressed firms, and the financial intermediary.

A contract \( x = \{k, p, C_s(\varepsilon), D_s(\varepsilon), B_s\} \) is immune to renegotiation given \( k \) and \( p \) if it is incentive feasible and no alternative continuation contract \( \{\hat{C}_s(\varepsilon), \hat{D}_s(\varepsilon), \hat{B}_s\} \) exists that is incentive and resource feasible, and makes managers of both healthy
and distressed firms and the financial intermediary better off, with at least one of these three groups strictly better off. That is, given $k$ and $p$, a continuation contract $\{\hat{C}_s(\varepsilon), \hat{D}_s(\varepsilon), \hat{B}_s\}$ does not exist that satisfies the resource and incentive constraints (3)–(7) and makes managers of both healthy and distressed firms and the financial intermediary better off:

$$\int \hat{C}_s(\varepsilon) dH_s(\varepsilon) \geq \int C_s(\varepsilon) dH_s(\varepsilon) \equiv \bar{C}_s \quad \text{for } s = H, L$$

$$\sum_s p_s \int \hat{D}_s(\varepsilon) dH_s(\varepsilon) \geq \sum_s p_s \int D_s(\varepsilon) dH_s(\varepsilon) \equiv \sum_s p_s \overline{D}_s,$$

where $\overline{D}_s = \int D_s(\varepsilon) dH_s(\varepsilon)$ with at least one of these inequalities strict.

To help interpret the implications of immunity to renegotiation, consider the situation of a financial intermediary after $k$ and $p$ have been chosen and the health shock $s$ has been realized for all firms. The original contract $x$ specifies the continuation $(C_H(\varepsilon), D_H(\varepsilon), B_H)$ for healthy firms and $(C_L(\varepsilon), D_L(\varepsilon), B_L)$ for distressed firms. In the renegotiation stage, the financial intermediary can propose any alternative continuation contract $(\hat{C}_H(\varepsilon), \hat{D}_H(\varepsilon), \hat{B}_H)$ for healthy firms and $(\hat{C}_L(\varepsilon), \hat{D}_L(\varepsilon), \hat{B}_L)$ for distressed firms. This alternative continuation contract will be accepted by managers of healthy firms and distressed firms if it satisfies (8) and will be proposed by the financial intermediary if it satisfies (9). Note that this way of modeling renegotiation allows for cross-subsidization between healthy and distressed firms. In principle, the financial intermediary can simultaneously adjust the bankruptcy sets and receipts of both healthy and distressed firms to minimize bankruptcy costs, as long as the total receipts from all firms do not decrease, and the managers accept the renegotiated contracts.

We now turn to the ex ante optimal contract in our economy. We think of managers as offering contracts. The financial intermediary will accept the contract as long as the expected rate of return on their investment is at least 1. Thus, any contract must satisfy the participation constraint

$$\sum_s p_s \int D_s(\varepsilon) dH_s(\varepsilon) \geq k,$$

as well as the resource constraints (3) and (4). The contract must also give the manager the incentive to exert the intended level of effort $p$ and thus must satisfy the effort incentive constraint

$$p = p_H \in \arg \max_{p_H} \sum_s p_s \int C_s(\varepsilon) dH_s(\varepsilon) - v(p_H)g(k).$$

Since all contracts can be renegotiated after the manager has chosen effort, when defining an equilibrium, it suffices to consider contracts that are immune to renegotiation. A contract is implementable if it satisfies the participation constraint, (10), satisfies the manager’s effort incentive constraint, (11), and is immune to renegotiation. Note that in this definition, the requirement that contracts be immune to renegotiation incorporates the incentives to report $\varepsilon$ truthfully, so we do not need to add (6) and (7) as separate constraints.
A competitive equilibrium consists of a contract $x$ that maximizes the manager’s utility over the set of implementable contracts.\footnote{Here, in a competitive equilibrium managers offer contracts to investors. An alternative way of setting up the equilibrium is to have the financial intermediaries offer contracts to managers—contracts that maximize expected profits subject to the incentive constraints, feasibility constraints, and participation constraints on managers. Competition between financial intermediaries drives their profits to zero so that the return on these contracts equals that on storage. By duality, these two setups yield equivalent outcomes.}

We now turn to the ex ante efficiency of a competitive equilibrium. A contract $x$ is ex ante efficient if it is implementable and no alternative implementable contract $x'$ exists that makes all managers and the financial intermediary better off, with at least one manager or the financial intermediary being made strictly better off.

A contract $x$ is ex post efficient if, given that the firm size $k$ and the effort level $p$ have been chosen, no alternative continuation contract exists that is immune to renegotiation.

The following proposition is immediate.

**Proposition 1:** The competitive equilibrium is both ex ante and ex post efficient.

We now turn to characterizing the competitive equilibrium. We do so by working backward from the end of the period in two stages. Once effort and size have been chosen, we show that only contracts with a simple form are immune to renegotiation. Given the simple form of contracts, we then move back to the ex ante stage and solve the contracting problem.

Here, bankruptcy plays a dual role. At the renegotiation stage, bankruptcy is used to induce the manager to pay more than a minimal amount, whereas at the ex ante stage, the level of bankruptcy is used to induce the manager to exert the optimal amount of effort.

At the renegotiation stage, size and effort have been chosen and our framework is, by design, similar to that in Townsend (1979). In Proposition 2 we show that a contract is immune to renegotiation if and only if it has a simple form, labeled a debt-equity contract for reasons discussed later.

Here we describe a debt-equity contract. We do so by considering a contract with expected payments in state $s$ given by $U_s = (C_s - s, D_s - s)$ defined in (8) and (9). In renegotiating a contract, the parties seek to minimize bankruptcy costs while ensuring that the managers and the financial intermediary receive at least the expected amounts promised under the original contract in that state. In the proof, we show that this reasoning implies that if this contract is immune to renegotiation, it specifies a bankruptcy cutoff $\varepsilon_s^*$ in each state. If $\varepsilon_s$ is greater than $\varepsilon_s^*$, then the firm does not declare bankruptcy, the payments to the financial intermediary are constant, and the manager receives all of the residual output. If, instead, $\varepsilon_s$ is less than $\varepsilon_s^*$, then the firm declares bankruptcy, the financial intermediary receives all of the reduced output, and the manager’s consumption is zero.

Specifically, if the expected payments to the financial intermediary are sufficiently low, in that $D_s \leq A_s \varepsilon_s g(k)$, it is incentive feasible to meet these payments without monitoring so that the contract has no bankruptcy. The consumption of the manager is then given by $C_s(\varepsilon) = A_s \varepsilon g(k) - D_s$, and the payoff to the financial intermediary for each $\varepsilon$ is $D_s$. 
If instead $D_s > A_s \varepsilon_s g(k)$, it is not incentive feasible to meet these payments without monitoring so that bankruptcy is necessary. Then a cutoff $\varepsilon^*_s$ exists such that the contract has bankruptcy for $\varepsilon \leq \varepsilon^*_s$, the payments to the financial intermediary are given by

$$D_s(\varepsilon) = \begin{cases} D_s = A_s \varepsilon^*_s g(k) & \text{for } \varepsilon \geq \varepsilon^*_s, \\ RA_s \varepsilon g(k) & \text{for } \varepsilon < \varepsilon^*_s \end{cases},$$

and the consumption of the manager is given by $C_s(\varepsilon) = A_s \varepsilon g(k) - D_s$ for $\varepsilon > \varepsilon^*_s$ and $C_s(\varepsilon) = 0$ for $\varepsilon \leq \varepsilon^*_s$. Given a cutoff $\varepsilon^*_s$, any such contract induces expected payments to the financial intermediary in state $s$ of

$$\bar{D}_s(\varepsilon^*_s) = RA_s \int_{\varepsilon^*_s}^{\varepsilon_s^*} \varepsilon dH_s(\varepsilon) + A_s \varepsilon_s^* \int_{\varepsilon^*_s}^{\varepsilon_s^*} dH_s(\varepsilon) g(k)$$

and expected consumption of the manager in state $s$ of

$$\bar{C}_s(\varepsilon^*_s) = A_s \int_{\varepsilon_s^*}^{\varepsilon^*_s} (\varepsilon - \varepsilon^*_s) dH_s(\varepsilon) g(k).$$

At the renegotiation stage, the model has the following tension: any change in the bankruptcy cutoff drives the payments to managers and the financial intermediary in opposite directions. This tension determines the bankruptcy cutoff $\varepsilon^*_s$. To determine this cutoff, it is convenient to consider the scaled payments to the manager $c_s(\varepsilon_s^*) = \bar{C}_s(\varepsilon^*_s)/g(k)$ and the financial intermediary $d_s(\varepsilon_s^*) = \bar{D}_s(\varepsilon^*_s)/g(k)$. It is immediate from (14) that the scaled payments to the manager are decreasing in the bankruptcy cutoff. We will assume that $d_s(\varepsilon_s^*)$ is single-peaked in $\varepsilon_s^*$ in that it first increases and then decreases in $\varepsilon_s^*$. This assumption holds for a wide variety of distribution functions $H_s(\varepsilon)$ including the uniform distribution, as long as $R$ is sufficiently close to 1. Let $\varepsilon_{s,max}$ be the bankruptcy cutoff that maximizes the scaled payments to the financial intermediary in state $s$, namely $d_s(\varepsilon^*_s)$. We can think of $d_s(\varepsilon^*_s)$ as a debt Laffer curve and display it in panel A of Figure 1.

Immunity to renegotiation implies that the bankruptcy cutoff must be to the left of the Laffer curve’s peak in that $\varepsilon^*_s \leq \varepsilon_{s,max}$. To see why, note that the payments to managers, displayed in panel B of Figure 1, are strictly decreasing in the bankruptcy cutoff. Thus, if the cutoff were to the right of the Laffer curve peak, then it is possible to reduce the cutoff and make both the manager and the financial intermediary better off.

Inspection of Figure 1 shows that if required payments to the financial intermediary are larger than $d_s(\varepsilon_s)$, the only way of meeting these required payments is by having bankruptcy. The reason is that bankruptcy, along with the associated monitoring, is the only incentive compatible way for investors to receive their required payments from the firm. Absent bankruptcy, the manager would always claim that the worst idiosyncratic state $\varepsilon_s$ has been realized.

The following proposition and others, except where specified, are proved in the online Appendix.
PROPOSITION 2: A contract is immune to renegotiation if and only if it is a debt-equity contract, in that it has the form given in \((12)\), where \(\varepsilon^*_s\) is the cutoff for bankruptcy and \(\varepsilon^*_s \leq \varepsilon_{s, \max}\).

The proof is similar to that in Townsend (1979). This proposition says that without loss of generality, we can restrict attention to such debt-equity contracts.

Here we have assumed that the financial intermediary and managers can renegotiate their contracts immediately before the idiosyncratic shock \(\varepsilon\) is realized. It should be clear that if we allowed a second renegotiation phase after the idiosyncratic shock is realized, incentive compatibility implies that no renegotiation will take place.

As with our scaling of our payments to the financial intermediary, we find it convenient to let \(y_s(\varepsilon^*_s)\) denote the scaled value of expected output in state \(s\) given by

\[
y_s(\varepsilon^*_s) = RA_s \int_{\varepsilon_s}^{\varepsilon^*_s} \varepsilon dH_s(\varepsilon) + A_s \int_{\varepsilon_s}^{\varepsilon^*_s} \varepsilon dH_s(\varepsilon),
\]

and note that the scaled expected consumption of the manager in state \(s\) is

\[
c_s(\varepsilon^*_s) = A_s \int_{\varepsilon_s}^{\varepsilon^*_s} (\varepsilon - \varepsilon^*_s) dH_s(\varepsilon)
\]

if \(\varepsilon^*_s > \varepsilon_s\), and if \(\varepsilon^*_s = \varepsilon_s\) the scaled expected consumption in state \(s\) is a constant \(c_s\) that satisfies

\[
c_s \geq c_s(\varepsilon_s) = A_s (1 - \varepsilon_s).
\]

The inequality \((16)\) arises because when there is no bankruptcy in state \(s\), the lenders cannot receive an amount larger than \(A_s \varepsilon_s\) but can receive less than this amount.

There should be no confusion if, for brevity, we simply refer to \(y_s(\varepsilon^*_s)\), \(c_s(\varepsilon^*_s)\), and \(d_s(\varepsilon^*_s)\) as output, consumption, and payments to the financial intermediary, realizing that they are all scaled. The scaled variables in the event of no bankruptcy are defined analogously. For future use, note that \(y_s(\varepsilon^*_s)\) is decreasing in \(\varepsilon^*_s\).
For simplicity only, we will assume throughout that no bankruptcy occurs in the healthy state and discuss sufficient conditions below. When there is no bankruptcy in the healthy state, the expected output in that state is \( A_H \int \varepsilon \_H dH(\varepsilon) = A_H \), since the mean of \( \varepsilon \_H \) is 1. The manager’s expected consumption in this state is \( c_H \) and must satisfy the constraint \((16)\). Notice that we no longer need the notation for \( h_H, H_H, \) and \( \varepsilon \_H \). To conserve on notation, we drop the subscript \( L \) on all variables associated with the distressed state: we let \( h(\varepsilon) = h_L(\varepsilon), H(\varepsilon) = H_L(\varepsilon), \) \( \varepsilon = \varepsilon_L, d(\varepsilon^*) = d_L(\varepsilon^*_L), \) and \( \varepsilon_{\text{max}} = \varepsilon_{L,\text{max}}. \)

Proposition 2 implies that a competitive equilibrium consists of a debt-equity contract that maximizes the manager’s utility subject to the participation constraint, \((10)\), and the manager’s effort incentive constraint, \((11)\). Note that given \((k, \varepsilon^*, p, c_H)\) we can recover the original contract \( x = \{k, p, C_s(\varepsilon), D_s(\varepsilon), B_s\} \) by manipulating and integrating \((3), (4), \) and \((12)\). Thus, we can summarize a contract by \((k, \varepsilon^*, p, c_H)\).

Given our assumption of no bankruptcy in the healthy state, the contract in a competitive equilibrium solves

\[
\max_{k, \varepsilon^*, p, c_H} \left[ pc_H + (1 - p) c_L(\varepsilon^*) - v(p) \right] g(k)
\]

subject to \( \varepsilon^* \leq \varepsilon_{\text{max}}, \)

\[
c_H - c_L(\varepsilon^*) = v'(p)
\]

\[
[p(A_H - c_H) + (1 - p)(y_L(\varepsilon^*) - c_L(\varepsilon^*))] \] \( g(k) \geq k, \)

and the constraint on \( c_H \), namely \((16)\). Here we have assumed that the first-order approach is valid so that we can replace the global effort incentive constraint \((11)\) by its local counterpart \((18)\). Moreover, \((19)\) is the rewritten participation constraint \((10)\), where we have used the resource constraints and that the contract has the debt-equity form. In what follows, we assume that \((16)\) is not binding and drop it from now on.

To understand the determination of the bankruptcy cutoff, note that raising the bankruptcy cutoff reduces the objective function by reducing the consumption of the manager in the distressed state. Nonetheless, bankruptcy is useful because of its two roles. First, increasing the bankruptcy cutoff raises the payments to the financial intermediary \( d(\varepsilon^*) = y_L(\varepsilon^*) - c_L(\varepsilon^*) \) and thus relaxes the participation constraint. Second, increasing the bankruptcy cutoff increases the manager’s incentives to exert effort. To see the second role, notice from the manager’s first-order condition \((18)\) that effort \( p \) is increasing in the spread in expected consumption between healthy and distressed states \( c_H - c_L(\varepsilon^*) \). Since \( c_L(\varepsilon^*) \) is decreasing in \( \varepsilon^* \), it follows that a higher bankruptcy effort improves effort.\[4\]

\[4\]Note that this force makes bankruptcy in the healthy state undesirable. Such bankruptcies reduce output and, unlike bankruptcies in the distressed state, worsen incentives. Thus, if the incentive effects are sufficiently strong, the optimal contract will have no bankruptcy in the healthy state.
We now turn to simplifying the contracting problem. First, we substitute out for $c_H$ and combine (18) and (19) into a single constraint called the implementability constraint:

\[
(20) \quad f(p, \varepsilon^*) g(k) \geq k,
\]

where $f(p, \varepsilon^*) = p[A_H - c_L(\varepsilon^*) - v'(p)] + (1 - p)[y_L(\varepsilon^*) - c_L(\varepsilon^*)]$. Note that an outcome is implementable if and only if it satisfies the implementability constraint. In the next lemma we use a duality argument to show that instead of maximizing the welfare of the manager, we can equivalently maximize the sum of the utilities of the managers and the investors, referred to as the surplus, denoted $U(p, \varepsilon^*) g(k) + \omega - k$, where

\[
(21) \quad U(p, \varepsilon^*) = pA_H + (1 - p)y_L(\varepsilon^*) - v(p).
\]

The resulting contracting problem, referred to as the contracting problem with no government is given by

\[
(22) \quad \max_{k, \varepsilon^*, p} U(p, \varepsilon^*) g(k) + \omega - k,
\]

subject to $\varepsilon^* \leq \varepsilon_{\text{max}}$ and the implementability constraint (20). The following lemma formalizes the duality argument.

**LEMMA 1:** The competitive equilibrium contracting problem (17) reduces to the contracting problem with no government (22).

It is straightforward to show that if the resources lost to bankruptcy from raising the bankruptcy cutoff above its minimum value are small, then in any solution to the contracting problem, there is bankruptcy in the distressed state in that $\varepsilon^* > \varepsilon$. These lost resources are small if $R$ is sufficiently close to 1 or if $\varepsilon$ is sufficiently close to zero. In what follows, we will focus on economies in which there is bankruptcy in the distressed state. In light of these results, we can refer to a contract $x$ as a three-tuple $(k, \varepsilon^*, p)$, where the corresponding consumption allocation $c_L(\varepsilon^*)$ is obtained from (15) and $c_H$ from (18).

In sum, in the competitive equilibrium, optimal contracts imply that many firms undergo bankruptcies. These bankruptcies are necessary both to provide the needed return to investors and to provide optimal incentives for managers to exert effort.

**Discussion.—** One question arises: after the manager has exerted effort, why don’t the financial intermediary and the managers agree at the renegotiation stage to eliminate all bankruptcies, given that bankruptcies by distressed firms clearly lower total output to be divided between the financial intermediary and the managers? The reason can be seen clearly from Figure 1. Since the bankruptcy cutoff is to the left of the Laffer curve peak, a reduction in the cutoff reduces the payments to the financial intermediaries and, hence, will be rejected by these intermediaries. Even though the output to be shared between the financial intermediary and the managers is increased, the informational rent to the managers arising from the
feature that $\varepsilon$ is private information increases by so much that payments to the financial intermediary are reduced.

One might wonder, is it not possible for the financial intermediary to renegotiate contracts so as to increase receipts from the healthy firms in order to subsidize the reduction in receipts from distressed firms in a way that increases its total receipts? The answer is no, because the only way the financial intermediary can increase receipts in the healthy state is to have managers of healthy firms reduce their consumption. These managers will not agree to such a reduction, so the original contract is immune to such a renegotiation and no such cross-subsidization is possible.

Finally, we explain why we label our contracts debt-equity contracts. One interpretation of our contracts is that they are simply contingent debt contracts. We prefer the following interpretation. Suppose that $A_H$ is sufficiently large so that $d_H > A_L\varepsilon^*$. Then our contract can be implemented by using a combination of uncontingent debt, outside equity, and inside equity, thought of as managerial compensation. Let $A_L\varepsilon^*$ be the face value of debt in both the healthy and distressed states. In the healthy state, debt holders receive the face value of their debt, whereas in the distressed state, they receive the face value in the event of no bankruptcy and all of output $RA_L\varepsilon$ in the event of bankruptcy. The payments to outside equity are given by $d_H - A_L\varepsilon^*$ in the healthy state and are 0 in the distressed state. The payments to inside equity are given by managerial compensation. Given this implementation, for future use note that the market value of the debt in the distressed state before the idiosyncratic shock $\varepsilon$ is realized is $d(\varepsilon^*)$.

In setting up this implementation, we have in mind that the outside equity holders control the firm and have entered into binding contracts with inside equity holders and debt holders. Here the contract specifies that if the firm cannot make the face value payments to debt holders, the firm is forced into bankruptcy, outside and inside equity holders receive zero, and the debt holders become the residual claimants.

B. Adding a Bailout Authority

Here, we introduce a benevolent government in the form of a bailout authority and show that with commitment the bailout authority does not intervene, but without commitment this authority finds it optimal to eliminate all bankruptcies. The difference between these policies with and without commitment implies that the bailout authority faces a time inconsistency problem.

The bailout authority is in many ways symmetric to private agents. It can participate in negotiating alternative contracts, and it faces the same informational constraints as the private agents. Three features of the bailout authority play a key role in our results. First, the objective function of the bailout authority is the sum of utilities of all the agents. Second, negotiations between the bailout authority and firms are voluntary. Third, this authority has the power to levy taxes on payments to the financial intermediary without their consent.

We begin by analyzing outcomes under commitment in that the bailout authority chooses its policies at the beginning of the period and can commit to them. Since the contracting problem with no government (22) is to maximize the sum of utilities of all agents, it follows that a bailout authority with commitment will choose not to
intervene. Thus, the commitment equilibrium outcomes, denoted $k_{CE}, \varepsilon^{\ast}_{CE}, P_{CE}$ with associated utility level $U_{CE}$, are those that solve the contracting problem (22).

We model lack of commitment by having the bailout authority choose its policies after the manager’s effort choice has been made and all the shocks have been realized. We show that the bailout authority eliminates all bankruptcies by buying up the debt of the distressed firms and levying taxes on the financial intermediary. One might wonder why the bailout authority would intervene if the commitment equilibrium is ex post efficient. The reason is that the bailout authority does not restrict itself to Pareto-improving interventions ex post, but rather intervenes if it can raise the sum of utilities of all agents even if it leads some agents to be worse off. Of course, since the commitment equilibrium is ex post efficient, then any intervention necessarily makes some agents worse off ex post.

We show that in equilibrium managers are made better off by the bailout and investors as a whole are made worse off because it is they who have to pay the taxes for the bailout. Of course, if we added a third set of agents, say workers, who are not party to these contracts but who pay all the taxes for the bailout, then a bailout will make both managers and investors better off but the workers worse off because of the taxes. In this sense, it is not important for our results that the financial intermediary is worse off ex post, but rather that even though many agents are better off ex post, at least one group of agents is worse off.

Formally, the timing in the period is that in the first stage, each firm chooses a contract $x$. Next, each manager chooses a probability $p$. Then the health shock $s$ for each firm is realized. After that, the private agents renegotiate the contract. Then the idiosyncratic shocks $\varepsilon$ are realized. Finally, the bailout authority chooses its policy after observing the health state of each firm and using the optimal decision rules of private agents to infer the effort level of managers.

The bailout authority’s policy $\pi$ has three parts: a (scaled) debt purchase offer $d_b$ for distressed firms, a renegotiated debt level indexed by $\varepsilon_b$, and a tax rate $\tau$. Hence, $\pi = (d_b, \varepsilon_b, \tau)$. The representative financial intermediary has a collection of debt-equity contracts with firms. An individual contract is sold to the bailout authority only if both the financial intermediary and the manager involved in that contract agree to accept the bailout offer. If both parties agree to accept the bailout, an action denoted by $\delta = 1$, the financial intermediary receives $d_b$ and the bankruptcy cutoff is set at $\varepsilon_b$. If either party rejects the bailout, denoted by $\delta = 0$, the original contract is implemented.$^5$

Under this policy the bailout authority offers to purchase debt of distressed firms from the financial intermediary for an amount $d_b$ and, for such firms, offers a new debt contract for managers, summarized by a bankruptcy cutoff $\varepsilon_b$, and levies a uniform tax $\tau$ on receipts of investors scaled by the size of the project $g(k)$.

$^5$Here the bailout authority makes its offer based on its predictions of private agents’ behavior from the private agent decision rules. Since all agents are identical, it predicts that all such agents will choose the same contract. This way of modeling private agents captures the idea that private agents are competitive (or anonymous). As we discuss in Section I of the online Appendix, if the bailout authority could base its offer on each agent’s actual decisions rather than on the predicted decisions of the agent, then private agents would not be anonymous and the bailout authority could effectively punish private agents in a way that is not consistent with the spirit of our competitive setup.
It is important to emphasize that the taxes paid by a given financial intermediary are independent of whether it accepts or rejects the bailout. These taxes should be thought of as paying for the bailouts of all financial intermediaries. Since any given financial intermediary has measure zero, its decisions have no effect on its tax payments.

Note that a bailout policy implies that the bailout authority pays $d_b$ to each firm and receives payments $d(\varepsilon_b)$. Thus, each firm that accepts the offer can be thought of as receiving a subsidy given by $d_b - d(\varepsilon_b)$. This subsidy is financed by taxes on all firms. We will show that this subsidy induces a distortion.

We have also assumed that the bailout authority intervenes after the idiosyncratic shocks $\varepsilon$ are realized. It is possible to show both here and in the dynamic version of the model that the results are identical if we instead assume that the bailout authority intervenes before these shocks are realized.

An equilibrium here consists of strategies that are functions of relevant histories and are optimal for the bailout authority and for private agents. The strategy for an individual firm consists of a contract and are optimal for the bailout authority and for private agents. The strategy for an firm intervenes before these shocks are realized.

Let $x = (k, \varepsilon^*, p)$ and an acceptance policy $\delta(x, \pi)$. Let $x_R = (k_R, \varepsilon_R, p_R)$ denote the representative contract. When the bailout authority makes its decision, its strategy consists of a policy $\pi = (d_b, \varepsilon_b, \tau)$.

The acceptance policy is given by

$$\delta(x, \pi) = \begin{cases} 1 & \text{if } d_b \geq d(\varepsilon^*) \text{ and } \varepsilon_b \leq \varepsilon^* \\ 0 & \text{otherwise} \end{cases}$$

That is, the financial intermediary and the manager accept the bailout offer only if they are both made weakly better off.

Next we construct the payoffs of the bailout authority. Suppose that the bailout offer $(d_b, \varepsilon_b)$ is accepted in the distressed state. Then in the distressed state, the financial intermediary receives $d_b$ and the manager has expected consumption $c_L(\varepsilon_b)$ and, in both states, the firm pays taxes $\tau$. The sum of utilities of the manager and the financial intermediary from the contract is $[\bar{U}(p_R, \varepsilon_b, d_b) - \tau] g(k_R) + \omega - k_R$ where

$$\bar{U}(p_R, \varepsilon_b, d_b) = p_RA_H + (1 - p_R)(d_b + c_L(\varepsilon_b)) - v(p_R).$$

If the bailout offer $(d_b, \varepsilon_b)$ is rejected, the sum of utilities of the manager and the investors from the contract is given by $U(p_R, \varepsilon_R)g(k_R) + \omega - k_R$ where the function $U$ is defined in (21). The budget constraint of the bailout authority is

$$\delta(\varepsilon^*, \pi) = \delta(1 - p_R)[d_b - d(\varepsilon_b)] g(k_R),$$

where $\delta = \delta(x_R, \pi)$. Thus, the problem of the bailout authority is to choose $\pi$, taking the representative contract $x_R$ and the acceptance policy $\delta(x_R, \pi)$ as given, to solve

$$\max_{\pi} \left[ \delta(\bar{U}(p_R, \varepsilon_b, d_b) - \tau) + (1 - \delta) U(p_R, \varepsilon_R) \right] g(k_R) + \omega - k_R,$$

subject to the budget constraint (24). Substituting for $\tau$ from the budget constraint, we can write the problem of the bailout authority as

$$\max_{\pi} \left[ \delta U(p_R, \varepsilon_b) + (1 - \delta) U(p_R, \varepsilon_R) \right] g(k_R) + \omega - k_R.$$
taking as given the acceptance policy \( \delta = \delta(x_R, \pi) \) and the representative contract.

Consider the problem that a contract \( x \) solves for an individual firm. The firm needs to think through what outcomes will occur for the different original contracts it specifies. The firm understands that for certain choices of \( x \), the financial intermediary and the manager will accept the bailout, the financial intermediary will receive \( d_b \), and the bankruptcy cutoff \( \varepsilon_b \) will be implemented, whereas for other choices, they will reject the bailout and the original contract will be implemented.

Thus, we can break the contracting problem into two parts. In the first part, the firm chooses size \( k \) and effort \( p \) anticipating that the bailout will be accepted in the distressed state in that \( \delta(x, \pi) = 1 \), so that the financial intermediary will receive \( d_b \) and the bankruptcy cutoff \( \varepsilon_b \) will be implemented. The implementability constraint with bailouts is then given by

\[
\left( f_{\tilde{p}}(p, \varepsilon, d_b) - \tau \right) g(k) \geq k,
\]

where \( f_{\tilde{p}}(p, \varepsilon, d_b) = p[A_H - c_L(\varepsilon) - v'(p)] + (1 - p)d_b \) is the scaled amount the intermediary receives before taxes. Note that \( f_{\tilde{p}}(p, \varepsilon, d_b) \) coincides with \( f(p, \varepsilon^*) \) except that in the distressed state, the intermediary receives \( d_b \) instead of \( y_L(\varepsilon^*) - c_L(\varepsilon^*) \) and the manager receives \( c_L(\varepsilon_b) \) rather than \( c_L(\varepsilon^*) \).

The contracting problem anticipating acceptance, namely the acceptance contracting problem, is

\[
\max_{k, p} \left[ U(p, \varepsilon, d_b) - \tau \right] g(k) + \omega - k,
\]

subject to (27). Hence, the contracting problem in (28) coincides with that in (22) except that in the distressed state, the firm anticipates that it will take the bailout offer rather than executing the original contract.

Note, for later, that the two first-order conditions for \( k \) and \( p \) for problem (28) can be summarized in a single condition,

\[
\left[ U(p, \varepsilon, d_b) - \tau \right] g'(k) - 1 = \frac{\tilde{U}_p(p, \varepsilon, d_b)}{\tilde{f}_{\tilde{p}}(p, \varepsilon, d_b)} \left[ f(p, \varepsilon, d_b) - \tau \right] g'(k) - 1.
\]

In the second part, the firm determines the best contract among the set of contracts such that the firm will reject the bailout ex post. Let \( X(\varepsilon_b, d_b) \) denote the set of contracts such that the bailout offer \((\varepsilon_b, d_b)\) is rejected in that \( \delta(x, \pi) = 0 \). The contracting problem anticipating rejection, the rejection contracting problem, is

\[
W(\pi) = \sup_{x \in X(\varepsilon_b, d_b)} \left[ U(p, \varepsilon^*) - \tau \right] g(k) + \omega - k,
\]

subject to the associated implementability constraint

\[
\left[ f(p, \varepsilon^*) - \tau \right] g(k) \geq k,
\]

where \( U(p, \varepsilon^*) \) is given in (21) and \( f(p, \varepsilon^*) \) is given in (20).
The individual firm’s problem is to choose the contract that gives the larger surplus of the acceptance contracting problem (28) and the rejection contracting problem (30), and we assume that if these surpluses are tied, the firm chooses (28). We refer to this combined problem as the bailout contracting problem.

When the solution to the acceptance contracting problem is preferred and \( d(\varepsilon_b) \leq d_b \), firms can structure their contracts to make sure that the firm will accept the bailout offer ex post by choosing any \( \varepsilon^* \) such that \( \varepsilon^* \geq \varepsilon_b \) and \( d(\varepsilon^*) \leq d_b \).

It will turn out that in equilibrium, distressed firms accept the bailout offer so that the solution to the acceptance contracting problem (30) dominates the solution to the rejection contracting problem (28). Nonetheless, the rejection contracting problem plays a key role in determining the set of equilibria. This problem implies that the equilibrium bailout offer must be sufficiently generous so that the manager and the financial intermediary do not have an incentive to design a contract under which they will reject the bailout ex post.

A bailout equilibrium consists of strategies for managers, the financial intermediary, and the bailout authority that satisfy (i) representativeness in that the solution to the individual contracting problem \( x \) coincides with the representative contract \( x_R \); (ii) given the policy \( \pi \), the individual contract \( x \) solves the contracting problem; (iii) the policy \( \pi \) maximizes the bailout authority’s objective subject to its budget constraint. Let \((x_R, \tau_b)\) denote an equilibrium outcome.

We turn now to characterizing the equilibrium outcomes. We can always represent an outcome in which the bailout offer is not accepted as one in which it is trivially accepted because it simply implements the original contract by setting \( \varepsilon_b = \varepsilon_R \), \( d_b = d(\varepsilon_R) \), and \( \tau = 0 \).

This representation immediately implies that in any equilibrium, the value of the acceptance contracting problem must be greater than the value of the rejection contracting problem and thus must satisfy the voluntary acceptance constraint given by

\[
\left[ U(p_R, \varepsilon_b, d_b) - \tau_b \right] g(k_R) + \omega - k_R \geq W(\pi).
\]

Consider next the implications of optimality by the bailout authority. Clearly, the payoff of the bailout authority is maximized by setting policy so that in the distressed state the firm accepts a bailout that cancels all bankruptcies. As the acceptance policy (23) makes clear, a bailout offer of the market value of the debt \( d_b = d(\varepsilon_R) \), together with a cancellation of all bankruptcies, \( \varepsilon_b = \varepsilon \), will be accepted and thus induces the desired outcome.

We first show that this economy has a time inconsistency problem. To see why, note that at the time the bailout authority makes its decision, firms have already made their size and effort decisions. Hence, by purchasing all the debt and canceling all bankruptcies, the bailout authority raises the sum of all utilities. Given this policy by the bailout authority, private agents will not find it optimal to choose the same contract as they would under the commitment equilibrium. Thus, this economy has

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6 Of course, in the uninteresting case in which the bailout authority makes a pointless offer by setting the bankruptcy cutoff \( \varepsilon \) such that \( d(\varepsilon) > d_b \), then for any \( \varepsilon^* \) either the manager or the financial intermediary will reject the offer. To see this result, note that if \( d(\varepsilon^*) < d(\varepsilon) \), so that \( \varepsilon^* < \varepsilon_b \), the manager will reject the offer, and if \( d(\varepsilon^*) > d(\varepsilon_b) \), the financial intermediary will reject the offer.
a time inconsistency problem in that any bailout equilibrium yields worse outcomes
than those in the commitment equilibrium.

This economy also has a coordination problem in that, under a mild sufficient
condition, it has multiple equilibria. In all of them, in the distressed state firms
accept a bailout offer that has no bankruptcies and taxes are positive. What differs
across these equilibria is the level of the taxes, the representative firm’s bankruptcy
cutoff $\varepsilon_R$, and the payments to the financial intermediary $d(\varepsilon_R)$.

In particular, the economy always has an equilibrium, referred to as the full bail-
out equilibrium, in which $d(\varepsilon_R) = d(\varepsilon_{\max})$ so that firms choose their debt levels at
the peak of the debt Laffer curve and taxes are relatively high. Under a mild suffi-
cient condition, it has a continuum of other equilibria in which bailout payments and
associated taxes are lower than in the full bailout equilibrium. Any outcome $(x_R, \tau^b)$
such that $x_R$ solves the acceptance contracting problem (28) with $\varepsilon^b = \varepsilon$, satisfies
the government budget constraint, and satisfies the voluntary acceptance constraint
(32) is a bailout equilibrium outcome. Let $\varepsilon_{\min}$ denote the smallest value of $\varepsilon_R$ that
satisfies (32).

We then have the following proposition.

**PROPOSITION 3:** Any bailout equilibrium in the one-period model has no bank-
ruptcy, has positive taxes, and is inefficient. Furthermore, if (32) holds with strict
inequality at $d_b = d(\varepsilon_{\max})$, then the economy has a continuum of equilibria indexed
by $\varepsilon_R$ that satisfies $\varepsilon_{\min} \leq \varepsilon_R \leq \varepsilon_{\max}$, where $\varepsilon_{\min} > \varepsilon$.

The basic idea of the proof that $\varepsilon_{\min} > \varepsilon$ comes from a contradiction argument.
Suppose the equilibrium has zero taxes. Then, from the government’s budget con-
straint and the acceptance policy rule, it must be that $d_b = d(\varepsilon_R) = d(\varepsilon)$, that is,
firms must voluntarily choose contracts with no bankruptcy. Clearly, however, any
individual firm is made better off by instead choosing the commitment contract.
Formally, contracts with no bankruptcy and no taxes violate the voluntary accep-
tance constraint and, thus, cannot be equilibria.

Consider ranking welfare in this continuum of equilibria. In an equi-
librium indexed by $\varepsilon_R$, the bailout authority purchases the associated debt
$d(\varepsilon_R)$. Higher values of $d(\varepsilon_R)$ lead to higher values of the subsidy to distressed firms
d$_b(\varepsilon_R) - d(\varepsilon)$, greater distortions in $k$ and $p$, and lower welfare. Thus, the worst
equilibrium is the full bailout equilibrium and the best equilibrium solves the fol-
lowing problem, referred to as the best bailout problem:

$$\max_{x, \varepsilon} U(p, \varepsilon^*) g(k) + \omega - k,$$

subject to the following requirements: $\varepsilon^* = \varepsilon$ so that there is no bankruptcy, the
voluntary acceptance constraint, the government budget constraint, and that $(k, p)$
solve the acceptance contracting problem. This equilibrium has the lowest taxes
and lowest debt of the bailout equilibria. Importantly, in this problem the voluntary
acceptance constraint is binding.

Proposition 3 makes clear that our economy has a classic time inconsistency
problem in that the best equilibrium without commitment has strictly lower welfare
than does the best equilibrium with commitment. In addition, without commitment
the economy typically has a coordination problem in that there is a continuum of equilibria with differing levels of welfare. In contrast, Farhi and Tirole (2012) do not have a classic time inconsistency problem but rather have only a coordination problem. In their economy, the best equilibrium without commitment coincides with the best equilibrium with commitment.

C. Improving on Bailout Equilibria

The conventional benchmark for evaluating policies in the time inconsistency literature is the ex ante efficient outcome that arises when the government has commitment. We argue that in environments without commitment, a different benchmark called sustainable efficiency is appropriate. We show that the bailout outcome is sustainably inefficient but that if we equip a government with additional powers, it can achieve sustainable efficiency.

Our argument for a different benchmark builds on the idea that the bailout policies are always available to the government and that the government will use these policies if it finds it optimal to do so. We show that this idea is captured by a sustainability constraint, as described later. From this perspective, the sustainability constraint represents just as much of a constraint on achievable allocations as incentive constraints do. Since any allocation must also respect the incentives of private agents and resource constraints, which are summarized by the implementability constraint, the relevant notion of constrained efficiency is the best outcome that satisfies both the sustainability constraint and the implementability constraint. We label such outcomes sustainably efficient.

Confronted with an arbitrary allocation \((k, \varepsilon^*, p)\) with associated surplus and armed with the bailout policies, we assume the government can always intervene by bailing out firms and attain the surplus associated with canceling all bankruptcies. Thus, to respect the incentives of such an intervention, an allocation must satisfy \(\varepsilon^* = \varepsilon\). In order to make this condition parallel to the analogous condition in our dynamic model, we write it as requiring that the value of surplus in the equilibrium must be higher than the value of surplus under the best deviation of canceling all bankruptcies. That is, for fixed levels of \(k\) and \(p\), the cutoff \(\varepsilon^*\) must satisfy

\[
U(p, \varepsilon^*)g(k) + \omega - k \geq U(p, \varepsilon)g(k) + \omega - k,
\]

and refer to it as the sustainability constraint. Clearly, since the left side of this inequality is decreasing in \(\varepsilon^*\), the sustainability constraint is equivalent to \(\varepsilon^* = \varepsilon\).

A sustainably efficient outcome maximizes surplus subject to the implementability constraint (20) and the sustainability constraint (33), which is equivalent to \(\varepsilon^* = \varepsilon\). Thus, the sustainably efficient outcome solves

\[
\max_{k, p} U(p, \varepsilon)g(k) + \omega - k,
\]

subject to \(f(p, \varepsilon)g(k) \geq k\). No policy can achieve higher welfare than this outcome, so it is the relevant upper bound for policy evaluation. (Obviously, a sustainably efficient outcome produces lower welfare than the ex ante efficient outcome, but this latter outcome is irrelevant because no policy could ever achieve it.)
We claim that any bailout equilibrium is not sustainably efficient and that the source of this inefficiency is the presence of the voluntary acceptance constraint. To do so, note that any sustainably efficient outcome has no bankruptcy and no taxes. In our discussion following Proposition 3, we have already shown that any outcome with no bankruptcy and no taxes violates the voluntary acceptance constraint and hence cannot be a bailout equilibrium. Thus, the sustainably efficient outcome cannot be a bailout equilibrium.

To better understand the distortion that renders bailout equilibria sustainably inefficient, note that in any bailout equilibrium, the voluntary acceptance constraint implies that debt is inefficiently high, in that $d_b > d(\xi)$, and taxes $\tau$ are positive. To see how these two features induce distortions, consider the scaled surpluses in the bailout outcome and the sustainably efficient outcome, respectively given by

\begin{align}
&PA_H + (1 - p)(d_b + c_L(\xi)) - \tau - v(p) \\
&PA_H + (1 - p)(d(\xi) + c_L(\xi)) - v(p),
\end{align}

and $\tau = (1 - p)[d_b - d(\xi_b)]$. Note that a marginal increase in effort in the bailout outcome increases surplus by $[A_H - (d_b + c_L(\xi))]$ while in the sustainably efficient outcome, it increases surplus by $[A_H - (d(\xi) + c_L(\xi))]$. Since $d_b > d(\xi)$, the increase in surplus is smaller in the bailout outcome so that the incentives to design contracts with high levels of effort are lower in the bailout outcome. Of course, since these subsidies are simply a form of redistribution across firms, in the sense that subsidies to distressed firms are paid for by taxes on all firms, all that happens is that decisions get distorted. This distortion is akin to subsidizing one consumption good relative to another and financing the subsidies by lump-sum taxes on these same consumers: all that happens is that relative consumption is distorted.

We now turn to asking whether certain policies can achieve sustainable efficiency. Consider first expanding the powers of the bailout authority by allowing it to forcibly impose losses on debt holders in addition to its bailout powers. This additional power has the effect of eliminating the voluntary acceptance constraint because managers will always voluntarily accept a reduction in the bankruptcy cutoff and the financial intermediary is simply forced to accept the associated reduction in payment.

Specifically, after the manager has exerted effort and all shocks have been realized, this authority can reduce the bankruptcy cutoff from $\xi_R$ to a lower level, say, $\xi_O$ and, rather than levy taxes, force the financial intermediary to accept the lower associated payments. After that, if it so desires, this authority can purchase the debt at its reduced value $d(\xi_O)$, reduce the bankruptcy cutoff to $\xi_b$, and levy the needed taxes, so that the government can always engage in bailouts if it chooses to do so. We refer to a government with these powers $(\xi_O; d_b, \xi_b, \tau)$ as an orderly resolution authority.

It is immediate that by reducing the bankruptcy cutoff to $\xi_O = \xi$ and forcing the financial intermediary to accept the lower payments, this authority can implement the sustainably efficient outcome.

Consider next expanding the powers of the government above those of the bailout authority in a different way that allows for ex ante regulation. Specifically, before private contracts are chosen, the authority can set an upper bound on the ratio of the
market value of the debt relative to the value of the firm and a tax on size. We refer to a government with these expanded powers as a *regulatory authority*. It is immediate that by setting the upper bound of the debt-to-value ratio at \(d(\xi) g(k_{SE})/k_{SE} \), it can uniquely implement the sustainably efficient outcomes. Here the power to tax size is irrelevant, but this power will be critical in the dynamic model discussed later.

Here, the regulatory authority does not improve on orderly resolution outcomes. We argue that this result holds because the extent of intervention by the orderly resolution authority is independent of the debt levels and sizes of firms in the sense that this authority sets the bankruptcy cutoff at a given level independent of these choices. In our dynamic model, reputation effects imply that the bankruptcy cutoff of this authority depends on debt levels and sizes. This dependence induces a subtle size externality that orderly resolution cannot cure but regulation can.

II. The Dynamic Model

Here we extend our one-period model to a dynamic model and use it to study bailouts, orderly resolution, and regulation. This extension is motivated by both theoretical and empirical considerations.

From a theoretical perspective, bailouts are sustainably inefficient in our one-period model solely because of subsidy distortions, so that removing the subsidy distortions with orderly resolution yields sustainable efficiency. The key insight from the dynamic model is that bailouts have an additional distortion that arises from size externalities. As firms in the aggregate increase their debt and size, they increase the extent of intervention by the bailout authority. Since no individual firm internalizes the effect of its decisions on the bailout authority’s decisions, this dynamic economy has a size externality that is not present in the one-period economy.

The size externality implies that an orderly resolution authority does not attain sustainable efficiency either. Of course, this authority improves on the best bailout outcome by removing the subsidy distortion.

From an empirical perspective, we observe partial but not complete bailouts. Our one-period model has complete bailouts. Our dynamic model generates partial bailouts. It does so because reputational considerations generate ex post costs that depend on the size of the bailout. Also, from an empirical perspective, we observe bailouts only during crises. We show that a stochastic version of our dynamic model generates partial bailouts only during crises. The reason is that lost resources arising from bankruptcy are greater during crises, so that the bailout authority has stronger incentives to intervene in crises.

We show that a regulatory authority equipped with the power to decree maximal debt-to-value ratios and a tax on the size of firms can achieve the sustainably efficient outcome. The maximal levels of debt-to-value ratios eliminate the subsidy distortion by ensuring that no bailouts occur and the tax on size eliminates the size externality by inducing firms to reduce their size.

Important policy questions are how such regulation should vary over the course of the business cycle and how it should vary across industries. The main insight from our dynamic model is that regulation should be tighter when, absent intervention, the lost resources arising from bankruptcy are larger. We provide simple examples to show that this general insight is consistent with either countercyclical or procyclical
regulation. We also show that this insight implies that firms in industries with high debt-to-value ratios, such as banking, are most in need of regulation.

A. Bailouts

The dynamic model is an infinite repetition of the one-period model. In particular, no physical state variables link periods.

Setup and Definition of Bailout Equilibrium.—The timing within each period is the same as in the one-period model: the firm chooses a contract \(x_t\), the manager chooses \(p_t\), the idiosyncratic shocks \((s_t, \varepsilon_t)\) for each firm are realized, the bailout authority chooses its policy \(\pi_t\), and the firm chooses whether to accept the bailout offer.

To focus attention on the dynamic incentive problem of the bailout authority, we assume that past policies of the bailout authority are observable but that past actions of private agents are not. In particular, we assume that managers are anonymous in the sense that their identities cannot be recorded from period to period. Hence, current contracts cannot be conditioned on the past track record of individual managers, and long-term contracts are infeasible. These assumptions imply that the only links between periods are strategic ones in which the bailout authority’s policies are affected by its forecasts of the responses of private agents in the future to its current actions. To capture these strategic links, we allow strategies to depend on the histories faced by agents when they choose actions. (Technically, we focus attention on perfect public equilibria.) The histories needed to describe strategies evolve as follows. Let \(H_t\) be the history at the beginning of period \(t\) and let \(H_{t+1} = (H_t, \pi_t)\).

The strategies are functions of the histories and are denoted by \(x_t(H_t)\) and \(x_{Rt}(H_t)\) for an individual and the representative contract, by \(\pi_t(H_t)\) for the bailout authority, and by \(\delta_t(x_t, \pi_t, H_t)\) for the acceptance policy.

The payoffs of the bailout authority given a history \(H_t\) are the sum of its period payoffs and continuation values and are given by

\[
\delta U(p_R, \varepsilon_b) + (1 - \delta) U(p_R, \varepsilon_b) g(k_R) + \omega - k_R + \beta V_{t+1}(H_{t+1}),
\]

where \(\delta = \delta_t(x_{Rt}(H_t), \pi_t, H_t)\) is the acceptance policy, \(V_{t+1}(H_{t+1})\) is the continuation payoff given by the present value of period payoffs for the bailout authority starting from period \(t + 1\) induced by the strategies, and \(\beta < 1\) is the discount factor. Note that we have dropped the subscripts for period \(t\).

Given the anonymity assumption, the contracting problem is static and solves the analog of our earlier contracting problem for all histories. Specifically, given a history \(H_t\), firms predict a bailout policy from the bailout authority’s strategy \(\pi_t(H_t)\) and, for any contract \(x_t\) that a given firm chooses, that firm predicts its acceptance policy \(\delta_t(x_t, \pi_t, H_t)\). Given these predictions, the individual contract is the solution to the bailout contracting problem, namely, the larger of (28) and (30).

A bailout equilibrium is a collection of strategies for private agents and the bailout authority, \(\{x_t(H_t), x_{Rt}(H_t), \pi_t(H_t), \delta_t(x_t, \pi_t, H_t)\}\), such that for all histories \(H_t\), (i) given the history \(H_t\), the contract \(x_t(H_t)\) solves the contracting problem;
given the strategies of the private agents, the policy $\pi_t(H_t)$ maximizes the payoff for the bailout authority (37); (iii) the acceptance policy $\delta_t(x_n, \pi_n, H_t)$ is given by (23); (iv) the individual and representative contracts coincide.

The outcomes associated with a bailout equilibrium are sequences $\{x_t, \pi_t\}$ and associated continuation utilities for the bailout authority $\{V_t\}$.

Characterization of Bailout Equilibrium Outcomes.—To characterize bailout equilibrium outcomes, note that optimality by private agents is captured by the requirement that the contracts solve the contracting problem for firms in each period given the policies of the bailout authority. Next, we show that the optimality of the bailout authority is captured by a sustainability constraint.

In our infinite horizon model, we focus attention on equilibria that can be supported by trigger-type strategies that specify reversion to outcomes that are no worse than the full bailout outcomes. This set of equilibrium outcomes is analogous to the set of equilibrium outcomes in repeated games that are supported by reversion to the one-shot Nash equilibria. (Of course, following the work of Abreu 1988, more sophisticated strategies could possibly support a larger set of equilibria. The results are similar, but the analysis is more cumbersome.) Specifically, we focus on equilibria in which for every history, even those after deviations by the bailout authority from a given policy plan, the continuation values of the bailout authority satisfy

\[
V_{t+1}(H_{t+1}) \geq V_{FB} = \frac{U_{FB}g(k_{FB}) + \omega - k_{FB}}{1 - \beta},
\]

where $V_{FB}$ is the discounted value of the full bailout outcome. This condition restricts the severity of the trigger strategies to be no worse than that of the strategies implicit in the infinite reversion to the full bailout equilibrium.

To set up our sustainability constraint, we use a standard result that outcomes are sustainable if and only if the payoff is at least as large as the payoff from the best one-shot deviation in the current period, followed by infinite reversion to the worst outcome (see, for example, Chari and Kehoe 1990). To that end, consider a period $t$ in which a contract $x_t$ has been chosen. The best one-shot deviation for the bailout authority is to buy all the debt in the distressed state and then renegotiate with the managers so as to eliminate all bankruptcies by setting $\varepsilon_b = \bar{\varepsilon}$. This deviation yields payoffs given by $U(p_t, \varepsilon_t^*)g(k_t) + \omega - k_t$ in the current period followed by the full bailout outcomes from period $t + 1$ onward. The sustainability constraint of the bailout authority is then given by

\[
U(p_t, \varepsilon_t^*)g(k_t) + \beta V_{t+1} \geq U(p_t, \varepsilon)g(k_t) + \beta V_{FB},
\]

where $V_{t+1}$ is the continuation payoff for such outcomes and we have canceled the $\omega - k_t$ term on each side. Substituting for the surplus function and simplifying, we can rewrite this constraint as

\[
\left[ (1 - p_t)(1 - R)A_L \int_{\bar{\varepsilon}}^{\varepsilon_t^*} \varepsilon dH(\varepsilon) \right] g(k_t) \leq \beta (V_{t+1} - V_{FB}).
\]
This constraint requires that the value of the resources saved by canceling all bankruptcies in the current period is smaller than the present value of losses induced by the reversion to the full bailout outcome in all future periods.

We then have the following proposition.

**PROPOSITION 4:** Under our reversion assumption (38), a sequence \( \{x_t, \pi_t\} \) is the outcome of a bailout equilibrium in a dynamic model if and only if this sequence (i) solves the contracting problem; (ii) satisfies the bailout authority’s budget constraint (24); and (iii) satisfies the sustainability constraint (39).

As part of our characterization of bailout equilibria, we note that the best bailout outcomes \( (x, \pi) = (k, \varepsilon_R, \varepsilon_b, d, \tau) \) solve a programming problem referred to as the *best bailout problem*,

\[
\max_{x, \pi} U(p, \varepsilon_b) g(k) + \omega - k,
\]

subject to the following requirements: \( (k, p) \) solve the acceptance contracting problem (28) given \( \pi \), and the outcome \( (x, \pi) \) satisfies the voluntary acceptance constraint (32), the government budget constraint (24), and the sustainability constraint (39).

We turn now to developing the appropriate notion of efficiency in our dynamic model. Since the government always has the option of using its bailout instruments, no outcomes that violate the sustainability constraint (39) are attainable. Since an attainable outcome must also satisfy the implementability constraint (20), we say that an outcome is *sustainably efficient* if it solves the problem of maximizing surplus subject to both of these constraints. Let \( (k_{SE}, \varepsilon_{SE}, p_{SE}) \) denote such an outcome. Clearly, there is a discount factor, say \( \beta^- \), such that for \( \beta < \beta^- \) the sustainability constraint in this problem is binding and for \( \beta \geq \beta^- \) it is slack.

Obviously, if the sustainability constraint is slack, the sustainably efficient outcome coincides with the commitment outcome. Thus, \( \beta^- \) is defined as the discount factor such that the sustainability constraint holds with equality at the commitment outcome

\[
U(p_{CE}, \varepsilon_{CE}) g(k_{CE}) + \beta^- V_{CE} = U(p_{CE}, \varepsilon) g(k_{CE}) + \beta^- V_{FB},
\]

where \( k_{CE}, \varepsilon_{CE} \), and \( p_{CE} \) are the commitment equilibrium outcomes and \( V_{CE} \) is the continuation equilibrium payoff from the commitment equilibrium.

We then have the following proposition.

**PROPOSITION 5:** If \( \beta < \beta^- \), then any equilibrium allocation has bailouts in that \( \varepsilon_{bt} < \varepsilon_R \) and \( \tau_t > 0 \) for each \( t \) and the equilibrium is sustainably inefficient.

The bailouts equilibrium has two distortions relative to a sustainably efficient outcome: a subsidy distortion similar to that in the one-period model and a size externality that does not arise in that model. Ignoring subsidies for a moment, the only difference between the contracting problem in a bailout equilibrium and the sustainably efficient outcome is that the contracting problem lacks the sustainability
constraint. This difference induces an externality because firms do not internalize that their choices of size affect the sustainability constraint.

To understand the nature of this externality, imagine starting at a bailout equilibrium and suppose that the size of all firms is reduced. The fall in aggregate size reduces the resources lost to bankruptcy and, hence, reduces the temptation of the bailout authority to undertake a full bailout. Hence, the sustainability constraint is now slack. All firms can now increase their bankruptcy cutoffs to some extent and induce managers to exert greater effort without inducing the bailout authority to undertake a full bailout. An individual firm is made even better off if it does not participate in the joint size reduction. These incentives to not participate lead to a free-rider problem that manifests itself as an externality.

Mechanically, the size externality arises because the first-order condition for size \( k_t \) in the sustainably efficient outcome includes a term that contains the derivative of the right side of the sustainability constraint \((39)\), whereas the first-order condition for size \( k_t \) in the contracting problem contains no such term.

**Bailouts and Crises.**—In practice, we typically observe bailouts only during crises. So far we have abstracted from crises to save on notation. Here we show that we can include them in a very simple way that keeps the key implications of our model unchanged while generating partial bailouts only during crises. In this model, referred to as the stochastic incentive model, the severity of the incentive problem varies over the business cycle.

In this model, an aggregate shock \( S \) is realized at the beginning of the period before contracts are agreed to. This shock is i.i.d. over time. For ease of notation only, we let this shock take on two values, \( S \in \{S_N, S_C\} \), with probabilities \( \mu_N \) and \( \mu_C \). We refer to the state \( S_N \) as normal times and the state \( S_C \) as crisis times. This shock affects the probability of the healthy idiosyncratic state. Specifically, if the manager chooses \( p \), then in crisis times the probability is \( p \), but in normal times the probability of the healthy state is \( p + \gamma \) for some positive \( \gamma \) (with the understanding that this probability is 1 if \( p + \gamma > 1 \)).

To make our point simply, we assume that \( \gamma \) is sufficiently high so that the contracting problem under commitment in normal times has no bankruptcies. Given this assumption, it immediately follows that in normal times the efficient outcome is sustainable. The following proposition is the immediate generalization of Proposition 5.

**Proposition 6:** There exists a critical discount factor \( \bar{\beta} < 1 \) such that if \( \beta < \bar{\beta} \), then any equilibrium allocation has no bailouts in normal times and bailouts in all crisis times and is sustainably inefficient.

**B. Orderly Resolution**

Here we equip the government with powers motivated by a key provision of the Dodd-Frank Act in addition to the bailout instruments. This key provision allows regulators to impose losses on creditors without going through bankruptcy. We capture this provision in our model by giving the government the power to reduce debt payments to the financial intermediaries without their consent. After reducing the debt payments, the same authority can then engage in a bailout if it so wishes by
purchasing the debt at its reduced value, reducing bankruptcy further, and levying the appropriate taxes. We refer to a government equipped with these powers as an orderly resolution authority. We show that the best equilibrium with orderly resolution removes the subsidy distortion associated with a bailout equilibrium but does not remove the size externality.

Here the timing of intervention by the orderly resolution authority is similar to that for the bailout authority. Briefly, after private agents have entered into a contract \( x = (k, \varepsilon^*, p) \) with a representative contract \( x_R = (k_R, \varepsilon_R, p_R) \) and the idiosyncratic shocks have been realized, the orderly resolution authority intervenes by setting a bankruptcy cutoff \( \varepsilon_O \) and associated market value \( d(\varepsilon_O) \). For a particular firm, if its bankruptcy cutoff \( \varepsilon^* > \varepsilon_O \), then its cutoff is reduced to \( \varepsilon_b \), whereas its cutoff is left unchanged if \( \varepsilon^* < \varepsilon_O \). In terms of bailout instruments, this authority can then purchase the debt at market value \( d(\varepsilon_O) \), reduce the bankruptcy cutoff to \( \varepsilon_b \), and levy the appropriate taxes. It is notationally convenient and without loss of generality to assume that the orderly resolution authority always intervenes, perhaps trivially by setting \( \varepsilon_O = \varepsilon_R \). Thus, a policy here consists of the policy of the orderly resolution authority \( \varepsilon_O \) together with the bailout instruments \( \pi = (d_b, \varepsilon_b, \tau) \).

An orderly resolution equilibrium is defined analogously to a bailout equilibrium. In particular, any such equilibrium must satisfy the sustainability constraint (39) so that the orderly resolution authority does not have an incentive to deviate from its policy.

The best orderly resolution outcomes \((x; \pi) = (k, \varepsilon_O, p; d_b, \varepsilon_b, \tau)\) solve a programming problem referred to as the best orderly resolution problem,

\[
\max_{x, \pi} U(p, \varepsilon_b)g(k) + \omega - k,
\]

subject to the requirements: \((k, p)\) solve the acceptance contracting problem (28) given \( \pi \), and the outcomes \((x, \pi)\) satisfy the government budget constraint (24) and the sustainability constraint (39).

Note that this problem is identical to the best bailout problem except that we have dropped the voluntary acceptance constraint (32). To see why the voluntary acceptance constraint can be dropped, recall that, in a bailout equilibrium, firms have the option of choosing debt levels so high that, ex post, the financial intermediary will reject the bailout. With orderly resolution, firms do not have this option.

Since the orderly resolution authority has the option to trivially intervene by setting \( \varepsilon_O = \varepsilon_R \) and reproduce the best bailout outcome, the best orderly resolution outcome weakly dominates the best bailout outcome. Under fairly general conditions, it is possible to show that this dominance is strict. Here we provide sufficient conditions under which, in the best orderly resolution outcome, the authority does not use its bailout instruments so that taxes are zero. This result is not obvious. To see why, recall that the theory of the second best implies that in environments with multiple distortions, removing one distortion may not raise welfare. We have both

\( ^7 \) We can think of the orderly resolution procedure as converting some of the firm’s debt into equity, which in the distressed state has a value of zero. This conversion takes place only if the regulator desires it to happen and the firm is unable to meet its debt payments. In this sense, this instrument resembles the hybrid securities proposed in French et al. (2010).
subsidy distortions and size externalities, so removing subsidy distortions alone may not necessarily raise welfare. Our sufficient conditions are

\[ g(k) = k^\alpha \text{ and } v(p) = vp^{1+a}, \text{ where } a \geq 1. \]

The following proposition is proved in the online Appendix.

**PROPOSITION 7:** Under (42), if \( \beta < \bar{\beta} \), then the best orderly resolution outcome has zero taxes and strictly higher surplus than the best bailout outcome. Moreover, if \( 0 < \beta < \bar{\beta} \), any orderly resolution outcome is sustainably inefficient.

The argument that the best orderly resolution equilibrium has zero taxes is by contradiction. Suppose that taxes are positive, so that the bailout instruments are used in equilibrium. Consider a deviation policy under which the orderly resolution cutoff is reduced slightly, leaving the bailout cutoff unchanged and thereby lowering taxes. Under our sufficient conditions, effort rises and size falls. The increased effort and reduced size relax the sustainability constraint and allow surplus to be improved.

We turn now to demonstrating that if \( 0 < \beta < \bar{\beta} \), the best orderly resolution outcome is sustainably inefficient. To do so, note first that the constraint that \((k, p)\) solve the contracting problem (28) is summarized by the combined first-order condition (29). Using the result that taxes are zero, we can simplify (29) to be

\[ U(p, \varepsilon_O)g'(k) - 1 - \frac{U_p(p, \varepsilon_O)}{f_p(p, \varepsilon_O)} [f(p, \varepsilon_O)g'(k) - 1] = 0. \]

The best orderly resolution problem thus reduces to choosing a contract \((k, \varepsilon_O, p)\) to maximize surplus subject to the implementability constraint (20), the sustainability constraint (39), and the first-order condition (43).

Note that the only difference between the problem that defines the sustainably efficient outcome and the best orderly resolution problem is that the best orderly resolution problem has an additional constraint, (43). This extra constraint is present because firms choose their capital freely under orderly resolution.

The logic of the proof that any orderly resolution outcome is sustainably inefficient if \( 0 < \beta < \bar{\beta} \) is as follows. If \( \beta < \bar{\beta} \) the sustainability constraint binds. If \( 0 < \beta \) then a positive amount of resources is lost to bankruptcy. When both of these conditions hold, the combined first-order condition in the sustainably efficient outcome does not satisfy (43), so that the orderly resolution outcomes are sustainably inefficient.

Next, if \( \beta = 0 \), no resources are lost to bankruptcy and our dynamic model effectively collapses to our one-period model. Here the best orderly resolution equilibrium is sustainably efficient. The reason is that here, altering the size does not change the incentives of the orderly resolution authority to intervene. Hence, in this case there is no size externality and the best orderly resolution equilibrium is sustainably efficient.

In Figure 2, we compare the properties of the best bailout equilibrium, the best orderly resolution equilibrium, the sustainably efficient outcome, and the
commitment outcome. Panel A illustrates our theoretical ranking of welfare. The order from highest to lowest is commitment, then sustainably efficient, then best orderly resolution, then best bailout. Panels B and D illustrate that the rankings for the bankruptcy cutoff and effort follow that of welfare. Panel C shows that although size is largest under commitment, the rankings of size for the three economies without commitment are in reverse order compared with the rankings for welfare. That is, in the best bailout equilibrium, firms are the largest; in the sustainably efficient outcome, firms are the smallest.

C. Optimal Regulation

Here we equip the government with the power to regulate firms ex ante. This government retains its ex post bailout powers. Clearly, if a government could simply dictate the terms of private contracts, it could trivially achieve sustainable efficiency. Instead, we limit the government’s ex ante powers to an upper bound on the debt-to-value ratio and a tax on size. We refer to a government with such powers in addition to its bailout instruments as a regulatory authority. We will show that the ex ante powers alone implement sustainably efficient outcomes and the bailout instruments are not used.
We model the upper bound on the debt-to-value ratio as a constraint on contracts of the form

\[ \frac{d(\varepsilon^*)}{k} \leq v. \]  

We model the tax on size as follows. A financial intermediary that enters into a contract that specifies a size of \( k \) for the firm is required to pay a tax \( \theta k \) to the regulatory authority. The tax proceeds are then distributed in a lump-sum fashion to managers, each of whom receives \( T \).

We will show that for a suitably chosen regulatory policy \((v, \theta, T)\), the bailout instruments \((d_b, \varepsilon_b, \tau)\) will not be used. Assuming that the bailout instruments are not used, consider the contracting problem for a given regulatory policy \((v, \theta, T)\),

\[
\max_{k, \varepsilon^*, p} U(p, \varepsilon^*)g(k) + \omega + T - (1 + \theta)k,
\]

subject to (44) and an implementability constraint with taxes and transfers

\[
f(p, \varepsilon^*)g(k) + T \geq (1 + \theta)k.
\]

To understand the objective function in (45), note that together the financial intermediary and the manager pay a tax \( \theta k \) to the regulatory authority and receive a lump-sum transfer of \( T \). To understand (46), recall that the left side of this constraint is the output of the firm minus the payments to the manager, and the right side is the amount invested by the intermediary including taxes. Since the manager receives a lump-sum payment of \( T \) from the regulatory authority, the information rents of the manager can be reduced by \( T \). (See the online Appendix for details.) Since the regulatory authority must redistribute the tax proceeds to the managers, in equilibrium the policy must satisfy the regulatory budget constraint \( T = \theta k \).

A regulatory equilibrium without bailouts consists of a contract \( x \) and a policy \((v, \theta, T)\) such that (i) \( x \) solves (45) and (ii) the policy satisfies the regulatory budget constraint.

In the next proposition, we provide sufficient conditions under which the sustainably efficient outcome can be implemented as a regulatory equilibrium without bailouts.

**PROPOSITION 8:** If productivity in the healthy state, \( A_H \), is sufficiently large, \( g(k) = k^\alpha \), and \( v''(p) \) is bounded above, the sustainably efficient outcomes can be implemented as a regulatory equilibrium without bailouts.

**D. Regulation over the Business Cycle**

Here we ask whether ex ante regulation should vary over the course of the business cycle. To do so, we consider a stochastic model and allow for a variety of aggregate shocks. We establish the general result that regulation should be tighter when, absent intervention, the lost resources arising from bankruptcy are larger. Translating this general result into specific implications for the cyclicality of policy,
however, depends on the detailed specification of how the shocks affect outcomes. We show that with one specification, lost resources are highest in recessions, so that countercyclical regulation is optimal; that is, the optimal ex ante debt and size limits become tighter during recessions. In another specification, lost resources are highest in booms, so that procyclical regulation is optimal; that is, the optimal ex ante debt and size limits become tighter during booms.

Let the aggregate shock \( S \) be i.i.d. over time and let the probability of shock \( S \) be \( \mu(S) \). In each period, the aggregate shock is realized before all other decisions, and the rest of the timing is the same as before. This shock can affect the probability of the healthy idiosyncratic states, now given by \( p(S) \), as well as the productivities in the healthy and distressed states, now given by \( A_H(S), A_L(S), g(k(S), S) \) and on the return to the storage technology \( \rho(S) \).

Let \( U(p(S), \varepsilon^*(S), S) \) denote the surplus in state \( S \). The best sustainable outcome solves the analog of that in the deterministic model. The pattern of regulation in response to aggregate shocks clearly depends on the binding pattern of the sustainability constraint given by

\[
U(p(S), \varepsilon^*(S), S) + \frac{\beta \sum_{S'} \mu(S') U(S')}{1 - \beta} \geq U(p(S), \bar{\varepsilon}, S) + \frac{\beta \sum_{S'} \mu(S') U_{FB}(S')}{1 - \beta}.
\]

Here \( U(S) \) denotes the surplus in state \( S \) and \( U_{FB}(S) \) denotes the surplus in the full bailout outcome in state \( S \). Rearranging the sustainability constraint and using the expressions for surplus, we obtain that

\[
\frac{\beta \sum_{S'} \mu(S') [U(S') - U^*(S')]}{1 - \beta} \geq (1 - p(S))(1 - R)A_L(S)g(k(S)) \int_{\varepsilon^*(S)}^{\bar{\varepsilon}} \varepsilon dH(\varepsilon).
\]

The left side of (48) is a constant independent of the current state \( S \), and the right side of (48) equals the lost resources due to bankruptcy. Let \( U^* \) denote the value of the left side at this optimum. This value is the dynamic gain from sticking with the prescribed policy.

Let \( k_{CE}(S), \varepsilon_{CE}(S), p_{CE}(S) \) denote the competitive equilibrium without government. Let \( G_{CE}(S) \) denote the static gain from eliminating bankruptcy in state \( S \), namely, the right side of (48). Clearly, the bailout authority has the greatest incentive to intervene in states for which the static gain from eliminating bankruptcy is the highest. In particular, if \( G_{CE}(S) > U^* \), it is then optimal to have ex ante regulation in state \( S \), whereas if this inequality is reversed, regulation in state \( S \) is not optimal. This logic immediately implies the following proposition.

**PROPOSITION 9:** Regulation is most desirable in states in which the competitive equilibrium has the most resources lost to bankruptcy.

*Countercyclical Regulation.*—It is immediate that in the stochastic incentive model introduced earlier, it is optimal to have regulation only during crises.
Procyclical Regulation.—Consider a proportional shock model in which the aggregate shock \( S \in \{ S_N, S_C \} \) affects only the production function and the return to storage. In particular, \( g(k, S_N) = \gamma g(k, S_C) \) and the return to storage is no longer 1 but given by \( \rho(S_N) = \gamma \rho(S_C) \) with \( \gamma > 1 \). Under these assumptions, it follows immediately that in the competitive equilibrium, the size of the project, the bankruptcy cutoff, and the effort of the manager are all the same in the two states in that \( k(S_N) = k(S_C), \varepsilon^*(S_N) = \varepsilon^*(S_C) \), and \( p(S_N) = p(S_C) \), so that the static gain from eliminating bankruptcies is greater in normal times than in crises times because \( G_{CE}(S_N) = \gamma G_{CE}(S_C) > G_{CE}(S_C) \). Under the assumption that the dynamic gain \( U^* \) lies between these static gains, it is optimal to have regulation only in normal times. In this sense, optimal regulation is procyclical.

E. Which Industries Should Be Regulated?

We use the general implication of our analysis that regulation should be most stringent when the bailout authorities have the strongest incentive to intervene in order to analyze which industries should be regulated most stringently.

We capture heterogeneity across industries by allowing the severity of the incentive problem to differ across industries. The idea is that when incentive problems are more severe, optimal contracts imply higher bankruptcy cutoffs as a way of providing incentives. Such bankruptcy cutoffs are typically associated with higher debt-to-value ratios. Of course, when bankruptcy cutoffs are higher, the losses due to bankruptcy are typically higher, as are the incentives of a bailout authority to intervene. This reasoning suggests that regulation may be most desirable in industries in which firms have the highest debt-to-value ratios.

We capture heterogeneity in the incentive problem by allowing the curvature of the disutility of effort function \( v(p) \) to vary across industries. From the first-order condition for a manager’s effort (18), we see that the elasticity of the manager’s effort in response to a change in the spread \( c_H - c_L(\varepsilon^*) \) falls as \( v(p) \) becomes more convex, so that industries with more convex disutility of effort functions face more severe incentive problems.

Specifically, we let \( v(p) = vp^{1+a} \) and increase the curvature parameter \( a \) to make the incentive problems more severe. We use the face value of the debt relative to the value of the firm as our measure of the debt-to-value ratio. Thus, this ratio is given by \( A_L \varepsilon_R g(k)/k \). With bailouts, the bailout authority purchases the outstanding debt and pays the market value \( d(\varepsilon_R) g(k) \).

Now consider the equilibrium outcomes in crisis times in our stochastic incentive model. Let \( r_D \) and \( r_k \) denote the debt-to-value ratio and the size in the best bailout equilibrium relative to their counterparts in the sustainably efficient outcome. It is easy to show that if the curvature parameter is less than a threshold, \( \hat{a} \), the best bailout outcome is sustainably efficient in that \( r_D = r_k = 1 \). Thus, for industries with \( a \leq \hat{a} \), no regulation is needed. Moreover, \( r_D \) and \( r_k \) increase as \( a \) increases above \( \hat{a} \), so that industries with high values of \( a \) need more stringent regulation on both debt-to-value ratios and size. (See Figure 3 in the online Appendix.)

Next, we apply this theory to analyze which industries should be regulated. Our theory implies that industries with more severe incentive problems have higher debt-to-value ratios in normal times. Indeed, we can show that debt-to-value ratios
in normal times increase with $a$. Thus, our theory implies that industries that have higher debt-to-value ratios in normal times should be regulated more stringently in crisis times.

In practice, firms with high debt-to-value ratios are disproportionately located in financial industries. The classic example, of course, is the banking industry in which firms tend to have much higher debt-to-value ratios than in essentially all other industries. Our analysis thus suggests that industries such as banking should be the most highly regulated and those with sufficiently low debt-to-value ratios should not be regulated at all.

### III. Conclusion

We have formalized an alternative view about the source of inefficiencies from bailouts: even when private markets are efficient, costly bankruptcies will occur and benevolent governments without commitment will bail out firms to avoid bankruptcy costs. Bailouts then introduce inefficiencies where none had existed. We have used our model to evaluate policies that are at the center of the current debate on regulating the financial system.

**REFERENCES**


