Developing an online cooperative police patrol routing strategy

Huanfa Chen, Tao Cheng *, Sarah Wise 1

SpaceTimeLab, Department of Civil, Environmental & Geomatic Engineering, University College London, London WC1E 6BT, United Kingdom

A R T I C L E   I N F O

Article history:
Received 21 December 2015
Received in revised form 27 October 2016
Accepted 28 October 2016
Available online xxxx

Keywords:
Police patrols
Multi-agent patrol routing
Bayesian-based decision making
Ant colony algorithm
Agent-based modelling

A B S T R A C T

A cooperative routing strategy for daily operations is necessary to maintain the effects of hotspot policing and to reduce crime and disorder. Existing robot patrol routing strategies are not suitable, as they omit the peculiarities and challenges of daily police patrol including minimising the average time lag between two consecutive visits to hotspots, as well as coordinating multiple patrollers and imparting unpredictability to patrol routes. In this research, we propose a set of guidelines for patrol routing strategies to meet the challenges of police patrol. Following these guidelines, we develop an innovative heuristic-based and Bayesian-inspired real-time strategy for cooperative routing police patrols. Using two real-world cases and a benchmark patrol strategy, an online agent-based simulation has been implemented to testify the efficiency, flexibility, scalability, unpredictability, and robustness of the proposed strategy and the usability of the proposed guidelines.

© 2016 The Authors. Published by Elsevier Ltd. This is an open access article under the CC BY license (http://creativecommons.org/licenses/by/4.0/).

1. Introduction

Patrolling is defined as “the act of walking or travelling around an area, at regular intervals, in order to protect or supervise it” (Abate, 1997, p 578). Police patrol occupies a central place in crime control efforts (Koper, 1995). As Cook (1980) stated, a visible police presence can increase the public’s certainty of punishment, and a frequent police presence enhances potential criminals’ perceptions of risk in the local area. In daily operations, approaches to police patrols range dramatically across varying contexts and cultures. Among them, one effective and promising approach is hotspot patrolling, or place-based patrolling, which focuses on crime hotspots, i.e., small geographical units with high crime intensity, such as street segments or small groups of street blocks (Braga, Papachristos, & Hureau, 2012). The effectiveness of hotspot patrolling in reducing crime has been proved by a range of experiments, such as the Minneapolis Hot Spots Patrol Experiment (Sherman & Weisburd, 1995) and the study conducted in Philadelphia (Ratcliffe et al., 2011). In patrolling, when there is more than one hotspot to cover, typically, police officers rotate randomly between hot spots, as in the field trial in Sacramento, California (Telep et al., 2014). However, the randomised strategy cannot be applied to situations where police resources are limited and there are many “hotspots” areas. Rather, the successful operation of patrolling to cover the “hotspots” requires a detailed patrol routing strategy. A relevant topic for policing is determining the efficient spatial distributions of police patrol areas to provide maximal and multiple coverage of incidents (Curtin et al., 2010). However, such strategies are focused on the location of centres of patrol areas, and do not consider a detailed routing strategy for patrol teams.

Designing a routing strategy for police patrolling is never a simple task due to several challenges. First, officers are required to cover hotspots regularly and repetitively (Curtin et al., 2010) as well as responding to emergencies. Thus, the performance of covering hotspots should not deteriorate significantly when emergencies occur and some patrollers are dispatched to handle them. Second, to cover the whole hotspot area effectively, police patrol requires cooperation among patrollers. Third, to confuse criminals and deter crime, the patrol routes should be somewhat difficult to predict. Additionally, hotspots may have different levels of importance and thus require different levels of attention. This problem is called the optimal design of patrol routes (ODPR) problem (Reis et al., 2006) or patrol route planning problem (Chen & Yum, 2010). This work focuses on designing patrol routes for foot patrol, rather than vehicle patrol.

All these challenges are very similar to the multi-agent patrolling problem (Almeida et al., 2004), or multi-robot patrolling problem (Portugal & Rocha, 2011), which focuses on surveillance tasks using multiple mobile robots to frequently visit important places in the environment. Here, we review the routing strategies in both police patrol and multi-agent robot patrolling can benefit police patrol. Distinct solutions have been proposed to design patrol routes, which present different strategies in terms of routing, cooperation, evaluation, and other features. In general, they can be divided into pioneer strategies (Almeida et al., 2004; Machado et al., 2002; Portugal & Rocha, 2013a), operations research strategies (Chevalleyre, 2004; Elmaliach, Agmon, & Kaminka, 2009; Portugal & Rocha, 2010), alternative coordination strategies (Chen & Yum, 2010; Chu et al., 2007; Santana et al., 2004; Sempe & Drogoul, 2003), and interaction strategies (Reis et al., 2006; Tsai et al., 2010).

* Corresponding author.

E-mail addresses: huanfa.chen@ucl.ac.uk (H. Chen), tao.cheng@ucl.ac.uk (T. Cheng), s.wise@ucl.ac.uk (S. Wise).

1 Bartlett Centre for Advanced Spatial Analysis (CASA), University College London.

http://dx.doi.org/10.1016/j.compenvurbsys.2016.10.013

0198-9715/© 2016 The Authors. Published by Elsevier Ltd. This is an open access article under the CC BY license (http://creativecommons.org/licenses/by/4.0/).
Pioneer strategies use simple pioneer architectures to guide patrol- ers to visit places that have been visited less recently, and it would be convenient for them to consider other factors, such as distance and co- ordination. These simple and heuristics-based strategies have been shown to achieve good performance in covering hotspots and coordi- nating patrollers (Portugal & Rocha, 2013a). However, most of the pio- neer strategies are developed in the context of robot patrol, and they neglect the aspects of being unpredictable in patrol routes and being ro- bust to the influences of emergency response.

Operations research strategies use graph theory tools to compute low- cost cycles and efficient routes for each patroller. The tools include Travel- ling Salesman or the Hamilton cycle (Elmaliach et al., 2009; Pasqualetti, Franchi, & Bullo, 2012; Smith & Rus, 2010), spanning trees (Fazli & Davoodi, 2010; Gabriely & Rimon, 2001), and graph partitioning (Sak, Wainer, & Goldenstein, 2008; Stranders et al., 2013). These strategies guarantee high visit frequency on targets and efficient cooperation be- tween patrollers, and they scale well with different numbers of patrollers. However, these strategies are naturally deterministic, which would more easily allow intelligent criminals to predict the patrol routes and take ad- vantage of the idle time between the visits of patrollers. Additionally, Hamilton cycles and other algorithms have high computational complex- ity and are difficult to generalise to large numbers of targets. Moreover, these strategies would have to re-compute patrol routes if the number of patrollers were to change because of an emergency response.

Alternative coordination strategies aim to solve the routing problem using approaches such as task allocation (Sempe & Drogoul, 2003), rein- forcement learning (Santana et al., 2004), cross entropy method (Chen & Yum, 2010), and swarm intelligence (Chu et al., 2007). However, strategies like reinforcement learning and the cross entropy method prove to be very complex in nature, so while they are suitable for de- signing patrol routes for a single patroller, it is difficult to extend them to cooperative patrol with multiple patrollers.

Interaction strategies have been derived from the interactions be- tween officers and criminals, using agent-based simulation or game the- ory models. For example, Reis et al. (2006) designed patrol routes based on genetic algorithms and a multi-agent-based simulation, where a set of criminals frequently try to commit crimes and officers try to prevent crimes. Tsai et al. (2010) derived a strategy for police resource allocation based on modelling the interactions between police and terrorists as an attacker-defender Stackelberg game, where a player always predicts his opponent’s behaviour and chooses the best response. These strategies can effectively prevent crime in crime hotspots, but only on the basis of a substantial knowledge of crime mechanisms in the area, and it is difficult to generalise these strategies to guiding police patrol in large areas and preventing multiple types of crimes.

In summary, existing approaches for patrol routing are not applica- ble to guide police patrol, as they omit the peculiarities and challenges of police daily patrol. To facilitate the design of an effective routing strat- egy for police patrol, the challenges mentioned above need to be speci- fied and formulated using clear guidelines and need to be quantified by appropriate evaluation measures. To our knowledge, few studies have dealt with this issue. Therefore, there is a need for a comprehensive study of guidelines and evaluation measures for designing a routing strategy for police patrols.

In this work, we propose a set of guidelines for an effective police pa- trol routing strategy and the relevant evaluation measures, which are based on the characteristics and challenges of practical police patrol. Under such guidelines, we develop an effective routing strategy based on heuristics and Bayesian techniques, and subsequently quantify their effectiveness through realistic simulation tests and in comparison with a graph theory strategy.

This paper is a further development of, and substantial improvement on, a previous work (Chen, Cheng, & Wise, 2015). In addition to the broad background introduced above, the current paper is substantially improved in five aspects. First, only three guidelines were discussed in Chen et al. (2015), namely, efficiency, flexibility, and unpredictability. Here two more guidelines—scalability and robustness—are developed, which measure the general applicability of the routing strategy in different situations including different team size, hotspot areas, and emer- gencies, as this has not been discussed in any previous literature. Furthermore, the guideline of unpredictability is further quantified here, which was only conceptually discussed in Chen et al. (2015). Sec- ond, the Bayesian Ant-based Patrol Strategy (BAPS) is further developed in accommodating these guidelines. Third, an agent-based modelling is now implemented as an online mode that simulates the real-world po- lice patrol with the interaction of the controller and patrollers. Fourth, the strategy is now tested to include the emergency scenario, which was not included in the previous paper. Finally, in order to test its appli- cability to different areas, a new case, namely, South Chicago, is added in addition to Camden. Furthermore, the Camden case is now conducted with different team sizes and more experiments to cover the five measures.

Fig. 1. From guidelines to BAPS.
The remaining sections are organised as follows. In Section 2, we formulate the guidelines and evaluation measures for police patrol routing strategies, which is followed by a Bayesian Ant-based Patrolling Strategy (BAPS) proposed in Section 3. Section 4 develops an agent-based simulation of real-time police patrols, in order to test the effectiveness of the proposed routing strategies. To test the proposed...
strategy vigorously. Section 5 presents two case studies of police patrol using realistic police and crime. Section 6 summarises the major findings and discusses topics for future study.

2. Guidelines and evaluation measures

Before we introduce the guidelines, it is necessary to describe the patrolling procedure. This study uses a simplified procedure of real-world police patrolling. The environment is the road network in an urban area. Certain road segments are identified as hotspots through crime mapping and prediction (Ratcliffe, 2010), and the n hotspots identified are denoted as \( H = \{ h_1, h_2, ..., h_n \} \). The idleness of a hotspot is defined as the time duration between the two consecutive visits, and the average idleness of a hotspot is the average of the idleness sequence. Patrollers have full knowledge of the area and always travel to the next hotspot via the shortest path on the network. A control centre dispatches the patrolling tasks to the patrollers, and receives the response and feedback from patrollers. The control centre can use different routing strategy to guide the movements of patrollers and to affect how the hotspots are monitored.

Police patrol mainly aims at preventing and reducing potential crime. A fundamental question is what makes a good police patrol routing strategy. We claim that a good patrol routing strategy should follow the guidelines that are proposed in this study, which as a minimum, should include efficiency, flexibility, scalability, unpredictability, and robustness. Since the efficiency and flexibility have been discussed in Chen et al. (2015), we will briefly recap some of the main points here, with more focus on the new measures for scalability, unpredictability and robustness.

Several previous studies have provided inspirations for the guidelines and measures discussed here. Two basic concepts, namely, the idleness and global idleness of patrolled targets, were first introduced by Machado et al. (2002), and these are directly used in this study. Portugal and Rocha (2013a) proposed the measure of team scalability to quantify the impact of team size on robot patrolling, and the measure of robustness to consider the influence of communication errors in robot patrolling. These measures are adapted here to account for the impact of team size and emergency response in police patrol. To our knowledge, this is the first research to use the measures for team scalability and robustness in police patrol. The other measures, including flexibility, unpredictability, and spatial scalability, are first proposed in this study.

2.1. Efficiency

Police patrol requires every important place or hotspot to be regularly and repetitively visited. Thus, efficiency is the foremost requirement for police patrol, which means patrols should minimise the time lag between two visits to every hotspot (Chen et al., 2015).

The measure of efficiency has been systematically discussed in Chen et al. (2015), and the concepts are used in this study, except for the different notations. Efficiency is measured by global average idleness (GAI) (Chen et al., 2015). GAI(t) is the global average idleness among all hotspots at time t, and is the defined as:

\[
GAI(t) = \frac{\sum_{i=1}^{n} AIdl(h_i, t)}{n}
\]

This study uses continuous time, and all the measures related to time use the time unit of second. Here, \( AIdl(h_i, t) \) represents the average idleness of a hotspot \( h_i \) at time \( t \), and \( n \) is the number of hotspots. GAI(t) changes with time. When patrolling begins, the idleness of each hotspot is set as 0, as if it has just been visited (Chevaleyre, 2004) and then it gradually converges as the distribution of patrollers becomes stable. Empirically, GAI(t) is regarded as converged if the relative difference of its value in two consecutive patrolling is within 1%. The converged GAI(t) is denoted as GAI, and is used to measure efficiency.

2.2. Flexibility

The flexibility of a patrol routing strategy is to prioritise the more important hotspots so that they have higher visiting frequency or lower average idleness. Such flexibility is measured by weighted global average idleness (WGAI) (Chen et al., 2015), which is the converged

<table>
<thead>
<tr>
<th>Team size</th>
<th>12</th>
<th>18</th>
<th>24</th>
<th>30</th>
<th>36</th>
<th>42</th>
<th>48</th>
</tr>
</thead>
<tbody>
<tr>
<td>GAI,CCPS</td>
<td>5079</td>
<td>3477</td>
<td>2605</td>
<td>2039</td>
<td>1709</td>
<td>1443</td>
<td>1254</td>
</tr>
<tr>
<td>GAI,BAPS</td>
<td>4407</td>
<td>2813</td>
<td>2128</td>
<td>1700</td>
<td>1401</td>
<td>1223</td>
<td>1075</td>
</tr>
<tr>
<td>Relative change (%)</td>
<td>−13.2</td>
<td>−18.5</td>
<td>−18.3</td>
<td>−16.6</td>
<td>−18.0</td>
<td>−15.2</td>
<td>−14.3</td>
</tr>
</tbody>
</table>
2.3. Scalability

A promising patrol strategy should be applicable to different areas and with different numbers of patrollers; this is described as scalability. There are two types of scalability for a patrol routing strategy: team scalability and spatial scalability.

Team scalability is related to how well the strategy performs as the number of patrollers increases (Portugal & Rocha, 2013b). A scalable patrolling strategy can adapt to different team sizes without severe performance degradation.

Team scalability can be evaluated by a classical metric, called Balch’s speedup measure (Balch & Arkin, 1994). In the patrolling problem, Balch’s speedup measure reveals how much more efficient a group of patrollers is than just one patroller in completing the patrolling task, and is defined as follows:

\[ v(R) = \frac{GAI(1)}{|R \times GAI(R)|} \]  

where \( GAI(R) \) and \( GAI(1) \) are the \( GAI \) value of patrolling by \( R \) patrollers and by one single patroller, respectively.

If a group of \( R \) patrollers are more efficient and achieve a low \( GAI \) value, the resultant \( v(R) \) would be >1.0, and this performance is said to be superlinear. Linear performance is equal to 1.0, which means equal performance, and sublinear performance is <1.0, corresponding to the lower efficiency (Balch & Arkin, 1994). Since it is uncommon for one patroller to patrol a large area, the measure is modified using a small size \( S \) as the reference, and the modified measure is named as ST (Scalability of Team Size):

\[ ST(R) = \left| \frac{S \times GAI(S)}{|R \times GAI(R)} \right| \]  

Spatial scalability of a patrol routing strategy concerns its performance in different space areas, including the layout of the area, the density of crime hotspots, and the distribution of police officers. Unlike team scalability, the various factors of spatial scalability are difficult to quantify. Generally, spatial scalability can be measured by comparing the efficiency of the designed strategy with the benchmark strategy. To our knowledge, no previous study has considered the spatial scalability of routing strategies in the context of police patrol.

Here we consider the influence of crime hotspot density level, as one example of spatial scalability. For convenience, the notation crime density level at \( x\% \) represents that the total length of crime hotspots cover \( x\% \) of the total segment length of the road network. The measure to compare the performance in two hotspot density levels is named as SS (Spatial Scalability):

\[ SS(L_x) = \left| \frac{GAI(L_x) - GAI(L_b)}{|GAI(L_b)|} \right| \]  

Here, \( SS(L_x) \) refers to relative change in \( GAI \) performance due to the change of hotspot density from the baseline level \( L_b \) (e.g. level at 5%) to \( L_x \) (e.g. level at 10% or 15%).

2.4. Unpredictability

If potential criminals can easily deduce the patrol routes or the visits to hotspots, they would commit a crime within the time between two visits, thus rendering police patrol ineffective (Sak et al., 2008). Therefore, it is important to keep the patrolling strategy unpredictable.

\[ \sum_{i=1}^{n} W(h_i) \times \text{AIdl}(h_i, t) / \sum_{i=1}^{n} W(h_i) \]  

\[ WGA(t), WGA(t) \text{ is the weighted } GAI(t): \]

\[ GAI_R(t) = \frac{1}{C_1} \sum_{i=1}^{n} W(h_i) \times \text{AIdl}(h_i, t) / \sum_{i=1}^{n} W(h_i) \]
There are two kinds of randomness in the patrolling problem: randomness of patrol routes and randomness of visits to hotspots. The former can be evaluated using the entropy of a patrol strategy, as proposed by Chen and Yum (2010). However, the entropy quantifies the dissimilarity of different patrol routes but fails to measure the randomness of police visits to a given place. An experienced burglar waiting around a potential target for a time when no patrols are nearby would be more concerned about predicting the time of the next visit rather than the routes of the patrol team. Here, the randomness of visitations to hotspots is evaluated by the average of the standard deviation of idleness on each hotspot:

$$\text{ASDI}(t) = \frac{1}{n} \sum_{i=1}^{n} SDI(h_i, t)$$

where $SDI(h_i, t)$ is the standard deviation of idleness of hotspot $h_i$ at time $t$, and $ASDI(t)$ is the average of $SDI(h_i, t)$ for all hotspots, with total number $n$.

Similarly, the converged $ASDI(t)$ is denoted as $ASDI$. The higher value of $ASDI$ is favoured, as it means higher unpredictability in the patrol routes and so it is less likely to be predicted by offenders. This is the first time that the unpredictability is quantified for police patrol.

Besides the random visiting time, there are other methods to impart unpredictability to patrolling, such as accessing a long hotspot segment randomly from both ends.

### 2.5. Robustness

Because officers are also responsible for dealing with emergency calls during patrolling, it is necessary to use a robust strategy, i.e., one that remains effective even if some patrollers are dispatched for emergencies. Here, the measure of robustness is the relative increase of the GAI value in emergency scenario in comparison with the GAI value in non-emergency scenario, which is represented as:

$$RI_{GAI} = \frac{(GAI_{Emerg} - GAI_{Norm})}{GAI_{Norm}} \times 100\%$$

where $RI_{GAI}$ is the relative increase of GAI, and $GAI_{Emerg}$ and $GAI_{Norm}$ are the GAI in the emergency scenario and normal scenario respectively. The routing strategy with low $RI_{GAI}$ is preferable, as it is less influenced by emergencies.

To summarise, these guidelines describe five requirements for an effective police patrol routing strategy to minimise the idleness and its unpredictability to patrolling, such as accessing a long hotspot segment randomly from both ends.

### 3. An online Bayesian ant-based patrolling strategy

This section describes how the aforementioned guidelines are transferred into a patrol routing strategy. The strategy to achieve the necessary efficiency and flexibility has been described in Chen et al. (2015); here, we give details of how to turn other guidelines into the same Bayesian Ant-based Patrolling Strategy (BAPS). This will provide a new perspective to understand the routing strategy and an example to transfer the guidelines to a practical strategy.

#### 3.1. Transferring guidelines to strategy

To follow the guidelines, one possible “bottom-up” approach is to turn each guideline into implementable modules, which are then assembled to form a complete routing strategy (see Fig. 1). First, the efficiency requires fair and frequent visits on each hotspot, without any hotspot being neglected for a long time, which requires tracking of the visit history of each hotspot (“history tracking” for short), and decision-making that favours less-visited hotspots. Second, the flexibility requires prioritising important hotspots in the “history tracking”. Third, the unpredictability of patrol routes would prefer irregular or random visiting times to hotspots, rather than the repetition of a predefined route. Fourth, the scalability calls for the cooperation among patrollers, and the cooperation should be adaptable to different hotspot distributions and different team sizes. That is, the route of one patroller should take account of the routes and distribution of other patrollers. Fifth, the robustness requires real-time routing, which considers the current hotspot distribution and activity of patrollers.

The next question is how to implement and assemble these modules to form a strategy. The modules of visit history tracking and prioritising important hotspots can be implemented by the pheromone mechanism, with different decaying rates for different weights if needed. The requirements of preference on less-visited hotspots and cooperation among patrollers can be satisfied by the Bayesian decision-making process. Moreover, the random visiting time and real-time routing is achieved via the one-step routing, which calculates only the next patrol route. Fourth, the scalability calls for the cooperation among patrollers, and the cooperation should be adaptable to different hotspot distributions and different team sizes. That is, the route of one patroller should take account of the routes and distribution of other patrollers. Fifth, the robustness requires real-time routing, which considers the current hotspot distribution and activity of patrollers.

The performance of team scalability in South Chicago (GAI values in seconds).
the posterior possibility of patrolling hotspot \( h_i \) is defined as:

\[
P(\text{patrol}(h_i)|G(h_i), S(h_i)) = \frac{P(G(h_i)|\text{patrol}(h_i)) 
	imes \left[ \frac{P(S(h_i)|\text{patrol}(h_i))}{P(S(h_i))} \right]}{P(G(h_i))}
\]  

(10)

For simplification, the term of time is omitted in Eq. (10). \( P(\text{patrol}(h_i)|G(h_i), S(h_i)) \) and \( P(\text{patrol}(h_i)) \) are the posterior and prior possibility of patrolling \( h_i \) \( (i = 1, n) \), respectively. \( G(h_i) \) represents the gain of patrolling \( h_i \), \( P(G(h_i)) \) is the prior probability of the gain, and \( P(G(h_i)|\text{patrol}(h_i)) \) is the probability of gain \( G(h_i) \) on the condition that \( h_i \) is patrolled. \( S(h_i) \) represents the number of patrollers that are going to patrol \( h_i \), \( P(S(h_i)) \) is the prior probability of \( S(h_i) \), and \( P(S(h_i)|\text{patrol}(h_i)) \) is the probability of \( S(h_i) \) on the condition that \( h_i \) is patrolled. In this study \( P(\text{patrol}(h_i)) \) is defined as uniform among every hotspot, and is omitted in the computation for simplification.

The gain of patrolling \( h_i \), \( G(h_i) \) is defined as \( G(h_i) = 1/[\text{Phe}(h_i(t)) \times \text{NormDist}(p, h_i)] \), where \( \text{NormDist}(p, h_i) \) is the normalised distance from \( p \), the current position of the patroller, to the hotspot \( h_i \). The normalisation is done to avoid local optima in which patrollers repeatedly patrol hotspots in a small cluster and neglect other hotspots. The distribution of \( G(h_i) \) needs to be defined. Without loss of generalisation, \( G(h_i) \) is defined as a continuous random variable with a probability density function \( f(g) \):

\[
f(g) = \frac{1}{M} \times \ln \left( \frac{1}{T} \right) \times e^{\ln (T) + \frac{g}{T}}
\]  

(11)

where \( L \) and \( M \) are constants, and \( L > 0, M > 0 \). \( L \) controls the probability values for zero gain and \( M \) is the gain saturation. Empirically, the value of \( L \) is selected as close to 0, and \( M \) is the maximum of gain when the

### Table 7

<table>
<thead>
<tr>
<th>HotspotMap_TeamSize</th>
<th>Camden_10p_30</th>
<th>Camden_15p_30</th>
<th>SouthChicago_10p_20</th>
<th>SouthChicago_15p_20</th>
</tr>
</thead>
<tbody>
<tr>
<td>GAL_CCPS</td>
<td>2806</td>
<td>3415</td>
<td>6492</td>
<td>8035</td>
</tr>
<tr>
<td>GAL_BAPS</td>
<td>2489</td>
<td>3132</td>
<td>6358</td>
<td>8392</td>
</tr>
<tr>
<td>Relative change (%)</td>
<td>−11.30</td>
<td>−8.29</td>
<td>−2.96</td>
<td>4.44</td>
</tr>
<tr>
<td>SS_CCPS (%)</td>
<td>37.62</td>
<td>67.48</td>
<td>81.87</td>
<td>114.25</td>
</tr>
<tr>
<td>SS_BAPS (%)</td>
<td>46.41</td>
<td>84.24</td>
<td>62.32</td>
<td>114.25</td>
</tr>
</tbody>
</table>
lower bound of the pheromone level and the lower bound of the normalised distance are used.

In Eq. (10), \( P(G(h_j)) \) is treated as a normalisation factor and omitted in the computation for simplification (Jensen & Nielsen, 2007). \( P(G(h_j)) \) is defined as \( P(G(h_j)) \) = \( f(G(h_j)) \).

\( P(S(h_j)) \) and \( P(S(h_j)) \) are used to coordinate the multiple patrollers. The idea behind is that a patroller should avoid patrolling the same hotspots as other teammates. The distribution of \( S(h_j) \) can be defined as \( f(s) = \frac{e^{-\alpha s}}{\sum_{s} e^{-\alpha s}} \), where \( m \) is the number of patrollers. Like \( P(G(h_j)) \), \( P(S(h_j)) \) is a normalisation term and can be omitted in the computation for simplification, and \( P(S(h_j)) \) equals \( f(s = \text{patrol}(h_j)) \).

Overall, the hotspot to patrol next is the one with the highest posterior probability:

\[
\text{hotspot} = \arg \max_h P(\text{patrol}(h_j) | G(h_j), S(h_j))
\]  

(12)

If more than one hotspot has the equal and highest probability, the hotspot is randomly selected from these candidates.

Notice that BAPS is a greedy strategy, as it searches only for the optimal choice of the next patrol target, instead of building the optimal patrol route for over a long period. A greedy strategy is used to find the locally optimal choice at each step, in the hope that these steps will lead to a globally optimal solution (Thomas et al., 2009). It is useful when obtaining a globally optimal solution is infeasible in a reasonable time.

4. Agent-based modelling of cooperative police patrols

This section presents a multi-agent modelling framework to test the effectiveness of the routing strategy. Agent-based modelling (ABM) is a simulation technique that seeks to capture how individual behavioural units interact with each other and with the environment, allowing higher-order behaviours to emerge from these interactions (Epstein & Axtell, 1996). Chen et al. (2015) tested BAPS in an agent-based simulation, but only in a non-emergency scenario. This study extends the framework to incorporate the emergency response.

In this ABM framework, the environment is a street network in the urban area, and crime hotspots are the street segments with a high crime risk. There are two types of agents, namely, patrollers and the control centre. Foot patrols with uniform skills and speed are dispatched either to patrol or to deal with an emergency. The control centre records the system state (idleness and visiting history of hotspots, etc.), communicates with patrollers, calculates patrol routes, and sends tasks to patrollers.

The framework is used to model BAPS and a benchmark strategy Christofides Cyclic Patrolling Strategy (CCPS), which is a deterministic and cyclic patrolling strategy from graph theory (Chen et al., 2015). CCPS is used as benchmark, as the real-world patrol strategy is confidential and difficult to obtain. Moreover, the cyclic strategies are classic algorithms for patrolling problems and perform well in different situations (Chevaleyre, 2004).

CCPS is fundamentally different from BAPS. CCPS firstly compute the shortest cyclic route that covers every hotspot at least once. This problem is known as the Rural Postman Problem, which can be solved by the Christofides Algorithm (Christofides et al., 1981). Then, the patrollers are distributed evenly on the route, and they begin to patrol following the same direction on the cycle. Thus, CCPS strives to achieve a regular and fair visit on each hotspot. In contrast, patrol routes in BAPS are built in real time, which requires patrollers to communicate with the control centre after they finish the patrol task, and to wait for the command of the next patrol target.

In the emergency scenario, patrollers in both BAPS and CCPS would be interrupted if an emergency occurred. The officers in the neighbourhood of the emergency would stop patrolling and head for the emergency site. They would resume patrolling after dealing with the emergency. To our knowledge, this is the first time that a cyclic patrolling strategy has been tested in the emergency scenario. We believe this will give a fair comparison for both strategies.

Processes of the two strategies are presented in Fig. 2. The control centre consists of the patrol route scheduler and the emergency scheduler. The patrol route scheduler calculates the next patrol target for BAPS patrol (Fig. 2a), or the cycle for CCPS patrol (Fig. 2b), and the emergency scheduler sends out the emergency task to patrollers close to the emergency sites. Patrollers are assigned to patrol or to respond to emergency calls. Whenever a patroller receives the emergency task, s/he stops patrolling and responds to the emergency. In BAPS, whenever a patroller finishes the emergency or patrolling task, s/he sends out “Task completed” to the control centre and awaits the next task. However, in CCPS, patrollers follow the cycle in their patrolling. The spatial differences of the two strategies will be demonstrated in the case studies. The normal scenario is a simplification of the emergency scenario in which no emergency occurs and the emergency scheduler is not used.

5. Case studies

To test the applicability of the guidelines and BAPS, two case studies were conducted, one in the London Borough of Camden, London, United Kingdom, and the other in South Side, Chicago, Illinois, the United States of America. For convenience, they are called Camden and South Chicago. The Camden case in this study is distinguished from that of the previous study (Chen et al., 2015) by different team size and more experiments to cover the five measures.

The data used are provided by various agencies. Details of the Camden data can be found in Chen et al. (2015), and some original crime incidents were aggregated to the centre of grids when they were recorded. Emergency calls data are added here to test the robustness. In the South Chicago case, the street network data are obtained from OpenStreetMap. The locations of police stations and the crime dataset from 2001 to present are from the City of Chicago data portal (https://data.cityofchicago.org). The time duration of crimes used is from 2011-03-01 to 2012-03-01, and the crime types include theft, burglary, homicide, battery, arson, motor vehicle theft, assault, and robbery. Crime counts of each segment are computed by the same method as the Camden case. Due to the lack of emergency call data and police dispatch data for South Chicago, the robustness is tested using hypothetical emergency calls, whose location and time is generated from a uniform distribution in the area and time period. Crime hotspots are identified as the street segments with the highest crime density and covering 5% of the total road length (Chen et al., 2015). There are 311 and 289 crime hotspots in Camden and South Chicago, respectively. Fig. 3 shows the hotspot maps.

The agent-based framework is built using Java and the MASON simulation toolkit (Luke et al., 2005). The simulation is updated on a temporal scale of 5 s per simulation step. Simulation results are analysed using R language and environment (R Core Team, 2015). BAPS and CCPS were tested with different sizes of patrollers (2–8 officers per police station).

### Table 8

<table>
<thead>
<tr>
<th>Team size</th>
<th>12</th>
<th>18</th>
<th>24</th>
<th>30</th>
<th>36</th>
<th>42</th>
<th>48</th>
</tr>
</thead>
<tbody>
<tr>
<td>AStddl_CCPS</td>
<td>939</td>
<td>1233</td>
<td>1599</td>
<td>497</td>
<td>753</td>
<td>583</td>
<td>493</td>
</tr>
<tr>
<td>AStddl_BAPS</td>
<td>4349</td>
<td>3368</td>
<td>2590</td>
<td>497</td>
<td>1649</td>
<td>1422</td>
<td>1197</td>
</tr>
</tbody>
</table>

### Table 9

<table>
<thead>
<tr>
<th>Team size</th>
<th>8</th>
<th>12</th>
<th>16</th>
<th>20</th>
<th>24</th>
<th>28</th>
<th>32</th>
</tr>
</thead>
<tbody>
<tr>
<td>AStddl_CCPS</td>
<td>2218</td>
<td>1309</td>
<td>1843</td>
<td>1441</td>
<td>955</td>
<td>1132</td>
<td>1231</td>
</tr>
<tr>
<td>AStddl_BAPS</td>
<td>7836</td>
<td>5696</td>
<td>4735</td>
<td>4148</td>
<td>3696</td>
<td>3410</td>
<td>3010</td>
</tr>
</tbody>
</table>
**Table 10**
Robustness performance in Camden in emergency scenario.

<table>
<thead>
<tr>
<th>Patrollers per emergency</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>BAPS_18</strong></td>
<td>0.0%</td>
<td>1.8%</td>
<td>4.5%</td>
<td>4.7%</td>
<td>10.2%</td>
</tr>
<tr>
<td><strong>BAPS_30</strong></td>
<td>0.0%</td>
<td>3.0%</td>
<td>3.3%</td>
<td>3.9%</td>
<td>5.4%</td>
</tr>
<tr>
<td><strong>BAPS_48</strong></td>
<td>0.0%</td>
<td>−0.2%</td>
<td>0.3%</td>
<td>1.0%</td>
<td>2.4%</td>
</tr>
<tr>
<td><strong>CCPS_18</strong></td>
<td>0.0%</td>
<td>1.1%</td>
<td>0.9%</td>
<td>1.6%</td>
<td>4.1%</td>
</tr>
<tr>
<td><strong>CCPS_30</strong></td>
<td>0.0%</td>
<td>3.2%</td>
<td>4.1%</td>
<td>3.4%</td>
<td>5.5%</td>
</tr>
<tr>
<td><strong>CCPS_48</strong></td>
<td>0.0%</td>
<td>4.7%</td>
<td>4.8%</td>
<td>5.7%</td>
<td>6.1%</td>
</tr>
</tbody>
</table>

GAI or WGAi is considered to converge when its value after any patrol cycle converges with no ~1% difference from the previous cycle. Each simulation ran for at least 11 patrol cycles, after which, each hotspot had been visited at least 11 times. This number was selected experimentally to guarantee the convergence of GAI or WGAi. The parameters of BAPS were selected to minimise the GAI in the trial experiments. For example, in selecting the pheromone decaying rate, different values (0.9999, 0.99991, 0.99992, 0.99993, etc.) were tested in a typical simulation (Camden case, 30 patrollers, 311 hotspots) with other parameters fixed, and 0.99993 was selected, as it resulted in the lowest GAI. The parameter settings of Camden were directly applied to South Chicago without further experimenting, as they led to good performance in South Chicago.

The computational efficiency of BAPS was tested. The simulations were run on a Dell machine, with a 3.60 GHz Intel Core i7-4790 processor, 32.0 GB RAM and 64-bit Windows 7 operating system. In the experiment of 48 officers covering 311 hotspots in Camden, the simulation lasted 82 s, with 7000 times of determining the next patrol target and thus about 0.01 s cost for each determination. The computational efficiency in the large-scale problem or dynamic situations is subject to further experiments.

We used the Camden case to demonstrate the spatial differences of the two strategies. Fig. 4 shows the coverage and route of six patrollers using BAPS and CCPS after patrolling for 5760 steps, corresponding to 8 h in the real-world, which is a typical shift for policing. The colour of the route is consistent with the station where the patroller started, except that the blue colour represents the route overlap among patrollers. Under BAPS, each patroller had its distinct route, and mainly patrolled a small and different area. The level of route overlap, which is the ratio of segments that had been traversed by over one patroller to segments that had been traversed, is about 35%. However, under CCPS, patrollers travelled on the same cycle that traverses every hotspot, and the level of route overlap is 92%. This indicates that BAPS provides different routes for different patrollers, while CCPS enforces the same patrolling route on all the patrollers involved. The evaluation and comparison of BAPS and CCPS are presented in order according to the guidelines.

**5.1. Efficiency**

Efficiency is measured by GAI. Tables 1 and 2 present the result of GAI and the relative change (Bennett & Briggs, 2005) of GAI:

\[
\text{Relative Change} = \frac{\text{GAI}_{\text{BAPS}} - \text{GAI}_{\text{CCPS}}}{\text{GAI}_{\text{CCPS}}} \times 100\% \quad (13)
\]

In Camden, BAPS has lower GAI and consequently better performance than CCPS. The relative change varies slightly with the team size, reaching the maximum when team size is 18 and 24. In South Chicago, the GAI values in BAPS are lower than in CCPS by around 10%, except for the team size of 8. This might be because with a smaller patrol team, officers have to travel longer distances to cover the whole area, which results in the degeneracy of BAPS efficiency.

**5.2. Flexibility**

The flexibility of routing strategies is measured by Weighted GAI (WGAI). Here, the hotspots are evenly divided into five levels, with crime density and weight decreasing from Level 5 to Level 1. Fig. 5 shows the hotspot map with different risk levels in both cases.

The decay rates are selected experimentally (Chen et al., 2015). The decay rates from Level 5 to 1 are: 0.99998, 0.99990, 0.99993, 0.99992, and 0.99993. Table 3 compares the performance of three strategies (BAPS, WBAPS, and CCPS) in patrolling hotspots of multiple levels using 30 patrollers in Camden. BAPS or WBAPS have a superior performance to CCPS in terms of WGAI, GAI and GAI of each risk level. In comparison with BAPS, WBAPS reduces the WGAI by about 1.8%, at the cost of a slight rise (2.4%) in GAi. Moreover, the GAi at Level 4 and Level 5 hotspots is reduced moderately by 4.7% and 9.4% when WBAPS is used. WBAPS provides an easy and effective approach to highlighting hotspots of higher levels, which shows the advantage of BAPS in that it can be tuned for specific aims. A similar trend is observed in Table 4 in the South Chicago case with 20 patrollers. WBAPS and BAPS have lower GAi and WGAI compared with CCPS. WBAPS has slightly lower GAi and higher WGAI than BAPS, as well as lower GAi in the prioritised hotspots of Level 4 and 5. The result verifies the flexibility of BAPS to patrol hotspots of varied levels by using varied decay rates.

**5.3. Scalability**

To test the team scalability, the ST metric (see Eq. (4)) is calculated for different sizes (see Tables 5 and 6). Table 5 reveals that in Camden, for all tested team sizes, BAPS systems present a superlinear performance as the speedup is >1.0, while the performances of CCPS systems are sublinear when team size is between 18 and 36. On every tested team size, the speedup performance of BAPS outperforms CCPS, indicating the better scalability of BAPS. Moreover, the scalability of CCPS is achieved by setting all patrollers as evenly distributed in time and space (Pasqualetti et al., 2012; Smith & Rus, 2010), which means the starting positions have to be recalculated for each size, while in BAPS starting positions have little influence on its performance. Similarly, in South Chicago, BAPS has a superlinear performance and outperforms CCPS on every team size (see Table 6).

Spatial scalability (SS) was tested by changing the crime hotspot density level from 5% to 10% and 15%. The higher density level requires better cooperation between patrolling to cover all hotspots. The corresponding hotspot maps are shown in Fig. 6, and the results are presented in Table 7. For example, Camden_10p_30 represents the experiment of covering the Camden hotspot map of 10% of the total road length with 30 patrollers. The SS values in Table 7 use 5% as the baseline density level, with other factors fixed, including the patrol area, strategy, and team size. Overall, BAPS outperforms CCPS in all hotspot density levels, except for the 15% South Chicago hotspot map with 20 patrollers. Furthermore, the SS value of BAPS is consistently larger than CCPS, indicating that BAPS is more affected by the hotspot density level. In summary, regarding SS, BAPS has better performance on different hotspot density levels, but it is more sensitive to the high hotspot density levels.

**5.4. Unpredictability**

As$\text{AdIdl}$ is measured to evaluate the unpredictability of patrolling. In Camden, for different team sizes, the As$\text{AdIdl}$ values of BAPS are higher than those of CCPS (see Table 8). The low standard deviation in CCPS can be explained by the even distribution of patrollers on the cycle.
and the same patrolling cycle used by all patrollers. In contrast, the high deviation of idleness in BAPS and the high randomness of patrol routes would create a perceived “omnipresence” of the police that would deter crime in crime hotspots (Sherman & Eck, 2002). Likewise, in South Chicago, for every team size, BAPS has higher Asdill values, compared with CCPS (Table 9).

5.5. Robustness

The experiments of robustness were conducted using the real-world emergency records in Camden (March of 2011) and hypothetical emergency calls in South Chicago. When an emergency is reported, the nearest m patrollers stop patrolling and respond. Due to insufficient details of the emergency responses (time length, number of patrollers dispatched, etc.), different settings were attempted, including the total number of patrollers and the number of patrollers per emergency. Tables 10 and 11 show the robustness performance in Camden and South Chicago. For example, BAPS_18 represents the BAPS simulation with 18 patrollers. The percentages represent relative changes in comparison with the non-emergency scenario (0 patrollers per emergency). In Camden (Table 10), the performance of both BAPS and CCPS deteriorated slightly or moderately as the number of patrollers required per emergency increased. Evidently, the higher number of patrollers needed by an emergency, the more affected the patrolling performance is. Further, holding constant the patrolling strategy and the number of patrollers per emergency means the emergency response has a more prominent impact on the performance when the patrolling group is smaller. Comparatively, with the team size of 18, the BAPS patrol was more affected by the emergency response than was the CCPS patrol. However, when the group size increased to 48, the influence on the BAPS patrol was less prominent than that on the CCPS patrol. A similar comparison exists in South Chicago (see Table 11), where the BAPS patrol was more influenced than the CCPS patrol when the group size was 12 or 20, and was more robust than CCPS when the group size increased to 32. The result supports the robustness of BAPS against emergency responses.

6. Conclusions and future work

This research developed a set of guidelines for real-world police activities, in particular a real-time cooperative police patrol routing. Five quantitative measures have been developed for the guidelines: efficiency, flexibility, unpredictability, scalability, and robustness. Under these guidelines, an online Bayesian Ant-based Patrolling Strategy (BAPS) has been developed. This strategy accounts for multiple factors that affect patrol, and it adopts a probabilistic computational framework, resulting in effective patrolling. As illustrated in the two real-world case studies, BAPS generally outperforms CCPS in terms of multiple measures. Thus, BAPS has great potential for real-time cooperative police patrol and other related applications.

The major contributions of this study include the developments of the relevant guidelines and measures for a police patrol routing strategy, the development of the BAPS routing strategy following these guidelines, and the verification of the strategy using agent-based simulations.

Future work will aim at including the relevant dynamics of police activity, such as the coordination between foot patrol and vehicle patrol, which will provide insight into a more practical patrol routing strategy. To carry out the research, it is necessary to automatically derive realistic police-patrol behaviours from GPS tracks of patrols and combine them into the patrol strategy, which will be another challenge. Other interesting directions would include customising patrolling strategy for alleviating specific crime types or focusing on improving the visibility of policing at public places, which will need to combine multiple sources of data, such as geodemographics.

Acknowledgements

This work is part of the Crime, Policing and Citizenship (CPC): Space-Time Interactions of Dynamic Networks Project (www.ucl.ac.uk/cpc), supported by the UK Engineering and Physical Sciences Research Council (EP/J004197/1). The first author’s PhD research is jointly funded by China Scholarship Council (grant No. 201406010296) and the BEAMS Dean’s Prize from the University College London.

The authors would like to acknowledge the Metropolitan Police Service (MPS) for provision of the crime data in the London Borough of Camden. They are also grateful to Trevor Adams for many valuable discussions about the manuscript and related work. The results presented and views expressed in this manuscript are the responsibility of the authors alone and do not represent the views of Trevor Adams or the MPS.

The authors gratefully acknowledge David Portugal (Gird Services Ltd, Cyprus, Portugal), and team members of the CPC projects, in particular, John Shawe-Taylor, Toby Davies, Kate Bowers and Gabriel Rosser for their feedback on this work.

References


R Core Team (2015). *R: A language and environment for statistical computing*. (Available at: https://www.r-project.org/).


