Multiple Metamagnetic Quantum Criticality in Sr₃Ru₂O₇

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Bilayer strontium ruthenate Sr₃Ru₂O₇ displays pronounced non-Fermi liquid behavior at magnetic fields around 8 T, applied perpendicular to the ruthenate planes, which previously has been associated with an itinerant metamagnetic quantum critical end point (QCEP). We focus on the magnetic Grüneisen parameter Γ_M, which is the most direct probe to characterize field-induced quantum criticality. We confirm quantum critical scaling due to a putative two-dimensional QCEP near 7.845(5) T, which is masked by two ordered phases A and B, identified previously by neutron scattering. In addition, we find evidence for a QCEP at 7.53 (2) T and determine the quantum critical regimes of both instabilities and the effect of their superposition.

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Quantum criticality denotes critical behavior that is associated with continuous transformations of matter at zero temperature. Because of the absence of thermal fluctuations at T = 0 it is qualitatively different from classical criticality [1]. In metals the unconventional excitation spectrum near a quantum critical point (QCP) causes the breakdown of Fermi liquid (FL) behavior and its intimate relation to exotic states, such as unconventional superconductivity, adds even more importance to this topic. To date, the influence of quantum critical magnetic excitations on electrons in a metal is far from being understood. For instance, the applicability of the itinerant Hertz-Millis-Moriya theory on f-electron-based Kondo lattice systems has been disproved by several experiments [2] and alternative descriptions are not fully established yet. Quantum criticality related to itinerant metamagnetism is exceptional in the sense that electronic degrees of freedom are irrelevant, and a quantitative application to experimental results should be possible [3].

The generic metamagnetic quantum critical end point (QCEP) arises from the suppression to T = 0 of the end point of a line of first-order metamagnetic transitions in temperature-field phase space by tuning, e.g., composition, pressure, or the magnetic field orientation [3]. Metamagnetic QCEPs have been realized in the f-electron-based compounds CeRu₂Si₂ [4,5] and UCoAl [6], as well as d-electron Sr₃Ru₂O₇ [7,8].

We focus on bilayer strontium ruthenate Sr₃Ru₂O₇. Magnetization of this compound along the tetragonal c axis at low temperature exhibits three successive superlinear, i.e., metamagnetic, rises at μ₀H_M₁ = 7.5 T, μ₀H_M₂ = 7.85 T, and μ₀H_M₃ = 8.1 T [9]. The first one is a metamagnetic crossover (M1). The second and third ones are first-order metamagnetic transitions (M2 and M3), ending at critical temperatures of about 1 and 0.5 K, respectively [10]. A line of second-order thermal phase transitions, connecting the critical end points of M2 and M3, has been discovered in electrical resistivity and thermodynamic experiments [10,11], which recently by neutron scattering has been identified as phase boundary of a spin-density-wave (SDW) “phase A” [12,13] (see Fig. 1.). The lower and upper critical fields of SDW-A correspond, respectively, to H_M₂ and H_M₃. Additionally, another SDW “phase B” has been observed in between H_M₃ and 8.3 T [12,15]. The observed incommensurate ordering vectors in both SDW phases have been related to Fermi surface nesting [12]. Magnetic susceptibility and magnetostriction have revealed the strongest peak at the M2 metamagnetic transition and weaker maxima at M1 and M3 [10]. The critical field has been extrapolated to μ₀H_c₋₋ = 7.845(5) T [11], which is indeed very close to μ₀H_M₂. Non-Fermi-liquid behavior at elevated temperatures was previously associated with a critical field close to H_M₂ [9]. Outside the SDW phases A and B and not too close to the M1 crossover, thermal expansion obeys quantum critical scaling in accordance with the expectations for a two-dimensional (2D) metamagnetic QCEP near 7.845 T [11]. This includes both the predicted divergence upon cooling within the quantum critical regime as well as the magnetic field dependence within the low-temperature FL regime upon tuning the field from both sides towards M2. However, the previous description of the specific heat coefficient C/T by a strong divergence |H_M₂ − H|⁻¹ [8,16] is in clear contradiction to the theoretical prediction C/T ∝ |H_c − H|⁻¹/3 [3].

We solve this discrepancy by proving that Sr₃Ru₂O₇ displays two QCEPs at μ₀H_c₋₋ = 7.53(2) T and μ₀H_c₋₊ = 7.845(5) T, respectively. We determine regimes in phase space where either of the two QCEPs leads to scaling of the
magnetic Grüneisen parameter. We also show where scaling failures due to the superposition of criticality from both instabilities. Multiple quantum criticality as an origin for instabilities, see Supplemental Material.[14]. As shown by the blue solid line in Fig. 2, the data are well described by $C/T \propto (H_c - H)^{-1/3}$, predicted for a 2D QCEP [3,24] with critical field close to $H_M^1$ but significantly smaller than $H_M^2$. This indicates that the previously anticipated scenario with a single field-tuned QCEP near $H_M^2$ [8] is insufficient.

The existence of two separate 2D metamagnetic QCEPs is evident from the analysis of the magnetic Grüneisen parameter $\Gamma_H$ given below. In contrast to the specific heat

![FIG. 1. Phase diagram of Sr$_3$Ru$_2$O$_7$ for $H||c$ with color coding of the magnetic Grüneisen parameter $\Gamma_H$. Solid green symbols mark positions of sharp anomalies in $\Gamma_H(T,H)$, related to metamagnetism [10] and the spin-density-wave phases A and B [12]. Open green symbols indicate positions of maxima in the field dependence of specific heat. The dotted black line marks quantum criticality. Below, we report a thorough study of direct access to the critical exponents which characterize QCEP1 and QCEP2 is observed. Within the QC

![FIG. 2. Specific heat divided by temperature of Sr$_3$Ru$_2$O$_7$ as a function of magnetic field applied parallel to $c$ axis at different constant temperatures. The solid blue line indicates $C/T = \alpha/|\mu_0(H_c - H)|^{1/3} + \gamma$ with $\alpha = (0.073 \pm 0.002)$ J/Ru-mol K$^2$ T$^{4/3}$, $\mu_0H_c = 7.57(4)$ T and $\gamma = 0.058(1)$ J/Ru-mol K$^2$, in accordance with a two-dimensional metamagnetic QCEP [3,24].

well as heat capacity measurements performed with the quasiadiabatic heat pulse technique, on a high-quality single crystal of Sr$_3$Ru$_2$O$_7$, grown by the floating zone technique [23], for fields applied along the $c$ axis.

Figure 2 displays the magnetic field dependence of the specific heat coefficient at various low temperatures. Data at 0.2 K display a single peak at 7.85 T. At larger temperatures, this peak is split into two peaks and the respective separation increases with increasing temperature. Qualitatively, such behavior is characteristic of itinerant metamagnetism and has also been found for CeRu$_2$Si$_3$ [25]. For a generic QCEP with a critical free energy $F_c(h) = F_c(-h)$ [where $h = \mu_0(H - H_c)$], symmetric peaks for the heat capacity are expected. Our measurements, however, display more broadened $C/T$ peaks on the high field compared to the low-field sides. As discussed later, this may be related to a slight increase of the effective dimensionality of the critical fluctuations at large fields.

The magnetic field dependence of the 0.2 K data is in perfect agreement with previous data [8,16], see Supplemental Material [14]. As shown by the blue solid line in Fig. 2, the data are well described by $C/T \propto (H_c - H)^{-1/3}$, predicted for a 2D QCEP [3,24] with critical field close to $H_M^1$ but significantly smaller than $H_M^2$. This indicates that the previously anticipated scenario with a single field-tuned QCEP near $H_M^2$ [8] is insufficient.

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coefficient, which has a substantial noncritical background, $\Gamma_H$ is more sensitive to quantum criticality because of a negligibly small noncritical contribution.

Figure 3 shows an isothermal scan of the magnetic Grüneisen parameter at 0.2 K. $\Gamma_H(H)$ increases by more than a factor of 10 in between 6 to 7.5 T. For any field-tuned QCP, the magnetic Grüneisen parameter displays a generic $1/h$ divergence [17]. Thus, the inverse of the Grüneisen parameter vs field must follow a linear dependence and crosses zero at the critical field. As shown in the inset of Fig. 3, this universal dependence is indeed observed, yielding a critical field very close to $H_{M1}$, which confirms our heat capacity analysis.

At fields beyond $H_{M1}$, a cascade of further sign changes and anomalies is found in $\Gamma_H(H)$. They are associated with metamagnetic transitions $M2$ and $M3$ and, respectively, the SDW phases $A$ and $B$ [12,15], as well as (see the green arrows) an anomaly labeled “C” in the phase diagram of Fig. 1, whose magnetic Grüneisen parameter signature is discussed in the Supplemental Material [14].

Each zero crossing of $\Gamma_H(H)$ from negative to positive with increasing field indicates an entropy accumulation which arises either above a QCP or at the boundary of an ordered phase. Although the behavior is very complex, it is qualitatively similar to the field dependence of the low-temperature thermal expansion coefficient [11]. A simpler field dependence with only one sign change of $\Gamma_H(H)$ related to $M2$ is found at elevated temperatures above 1 K [14]. There, the thermodynamic properties are mostly influenced by QCEP2 (cf. Fig. 1).

In addition to isothermal measurements, we also study the temperature dependence of $\Gamma_H$ at various fields, cf. Fig. 4. At $T > 1$ K, all curves below $H_{M2}$ show a negative $\Gamma_H$, while it is positive for $H > H_{M2}$. Since $\Gamma_H = -(dM/dT)/C$, where the heat capacity $C > 0$, this reflects the change of sign in the temperature dependence of the magnetization associated with metamagnetism (ordinary paramagnetic behavior below $H_{M2}$ and field polarized behavior above $H_{M2}$). The overall symmetric behavior of $\Gamma_H(T)$ with respect to the critical field of QCEP2 is reflecting the Ising symmetry of critical metamagnetic fluctuations [24]. Upon cooling, $|\Gamma_H|$ increases within the critical regime of QCEP2 and passes a maximum upon entering the low-temperature FL state, as seen, e.g., for the 9 T data in Fig. 4. Transitions to phases $A$ and $B$ lead to distinct anomalies indicated by arrows. Particularly interesting behavior is found at 7.5 T where, upon cooling, $\Gamma_H(T)$ passes the minimum at 1.5 K, due to the FL crossover of QCEP2, but subsequently displays a negative divergence as $T \to 0$, related to the nearby QCEP1 (cf. Fig. 1).

We now turn to a quantitative comparison of our data with the theory of metamagnetic quantum criticality [3,24]. The latter predicts $\Gamma_H h \sim h^2/T^{(4+2d)/3}$ in the quantum critical and $\Gamma_H h = (3 - d)/3$ in FL regime, where $d$ denotes the dimensionality and $h = \mu_0(H - H_c)$. This leads to universal scaling in a plot of $\Gamma_H h$ vs $h^2/T^c$, where $c = (4 + 2d)/3 = 8/3$ for $d = 2$. Respective scaling behavior of our data is shown in Fig. 5. Here we fixed the critical field to 7.845 T [11], which is the position of QCEP2. The data collapse over several orders of magnitude, similar as previously found for thermal expansion [11], proves quantum critical behavior and indicates the applicability of the itinerant theory. However, a close inspection provides further information [14]. First, for
behavior for a QCEP with dimensionality $d$.

Critical fluctuations, deduced from our scaling analysis. This could be naturally explained by additional contributions to the free energy arising from classical critical behavior. Furthermore, there is an anomalous depression of $\Gamma_H$ at 9 T below 1 K (cf. Fig. 4), which could not be accounted for by the scaling due to QCEP2. The magnetic field dependence of $\Gamma_H(T)$ (Fig. 3, see also Supplemental Material [14]) indicates low-temperature anomalies in this field regime, labeled C in the phase diagram (Fig. 1). Since heat capacity does not show an anomaly these are rather weak thermodynamic signatures for phase formation. The fields where these anomalies are observed are temperature dependent. Thus, it is unlikely, that these anomalies originate from low frequency quantum oscillations [8].

Our measurements of the magnetic Grüneisen parameter and specific heat coefficient establish the existence of two itinerant metamagnetic QCEPs in bilayer strontium ruthenate Sr$_3$Ru$_2$O$_7$ for magnetic fields applied parallel to the $c$ direction. QCEP1 appears at a metamagnetic crossover near 7.5 T while QCEP2, which has already previously been established, is located at 7.845 T. The phase diagram shown in Fig. 1 indicates the scaling regimes QC1 and QC2 determined from the magnetic Grüneisen parameter behavior (see also Supplemental Material [14]). While QC2 is largely extended at elevated temperatures, QC1 is confined to a narrow regimes close to QCEP1. In between these scaling regimes, there exists a range in phase space, in which scaling fails due to the superposition of criticality from both instabilities. The phase diagram is even richer and contains also two SDW phases A and B [12] and some anomalous yet unidentified regime labeled C. Likely, the observed complexity is related to the complicated electronic structure of this material [26]. The Fermi surface contains several pockets that could give rise to nesting and sheets near a van Hove singularity. From a general perspective, multiple quantum criticality may cause anomalous behaviors in different material classes, including heavy fermions and high-$T_c$ superconductors. The Grüneisen parameter is ideally suited to disentangle multiple quantum criticality.

Stimulating discussions with M. Brando, M. Garst, and C. Stingl are gratefully acknowledged.

Note added in proof—The existence of a QCEP at $\mu_0 H_{M1}$ has most recently also been concluded from heat capacity and (non-adiabatic) magnetocaloric effect measurements [28].

The different regimes where the magnetic Grüneisen parameter displays scaling with respect to QCEP 1 and QCEP2 are indicated in Fig. 1. In between both regimes neither scaling works, because criticality from both instabilities is adding up (see Supplemental Material [14]). Next, we discuss the influence of the ordered phases A and B. In the approach of these phase transitions, $\Gamma_H$ data deviate from the expected quantum critical scaling. This could be naturally explained by additional contributions to the free energy arising from classical critical behavior.

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