Enhancing Conventional GNSS Positioning with 3D Mapping without Accurate Prior Knowledge

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BIOGRAPHY

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ABSTRACT

GNSS positioning performance in dense urban areas is severely degraded due to the obstruction and reflection of the signals by the surrounding buildings. A basic GNSS position solution can exhibit errors of tens of metres. This paper contributes towards the goal of real-time metres-level mobile positioning in outdoor urban environments by making use of spatial data in the form of 3D city models. We assess the extent to which the performance of height-aided conventional ranging-based GNSS positioning can be improved by finding the best way of determining which signals are non-line-of-sight (NLOS) or multipath contaminated. In contrast to previous approaches, we do not assume that the position is already known to within a few metres. Instead, we consider the case where the initial position, from a basic GNSS solution, is only accurate to within a few tens of metre. Therefore, which satellites are directly visible will vary across the search area. We thus use the 3D city model to predict the probability across the search area of each satellite signal being directly visible. Practical test results demonstrate improvement in the horizontal and vertical accuracy of conventional ranging-based GNSS positioning in urban areas by 58% and 78%, respectively.

1. INTRODUCTION

Improving upon the relative poor real-time positioning accuracy achievable in dense urban areas can unlock the potential for a host of new positioning applications. Examples include navigation for the visually impaired, tracking people with chronic medical conditions and emergency caller location. For these latter applications, it is important to determine which side of the street a pedestrian is on and which building they are in front of. This is also useful for guiding visitors, meeting friends and business associates and location-based advertising, while augmented reality relies on knowing where the user is. Similarly, to make best use of the space in cities, sustainable transport requires advanced lane control systems for vehicles and advanced railway signalling systems, both of which require accurate positioning. With the emergence of citizen science, low-cost devices to measure noise and pollution are becoming prevalent. As these measurements vary greatly across a street, accurate positioning is required to interpret the results.

The Global Positioning System (GPS) provides metres-level positioning in open environments, but the accuracy and reliability in urban areas is poor because buildings block, attenuate, reflect and diffract radio signals. This has conventionally been a major hindrance to positioning, with errors of tens of metres common and often no position solution available at all [1][2][3]. Using the new Global
Navigation Satellite Systems (GNSS) constellations (GLONASS and, in future, Galileo and Compass) in addition to GPS dramatically increases the number of usable satellites. This improves the availability of a position solution in urban areas, but not the accuracy [4].

One way of improving positioning performance is to integrate GNSS with dead reckoning (DR) sensors, such as low-cost inertial sensors and car odometers [5]. DR sensors measure change in position, so require a good GNSS position solution for initialisation. Following this, their positioning errors increase over time, so they are only useful for bridging short gaps in GNSS coverage. Another approach is to use other widely available radio signals, such as Wi-Fi, phone signals, and television. However, these typically suffer from the same propagation errors as GNSS in urban environments so do not offer better accuracy. Visual techniques are another option. However, they require extensive processing and data storage capacity and can be sensitive to passing pedestrians and vehicles, and variation in lighting.

Reliable meters-level positioning in dense urban areas is almost impossible to achieve cost-effectively using a single method. To achieve this goal, a paradigm shift is needed. Instead of designing a single-technology navigation or positioning system, we need to use as much information as we can cost-effectively obtain from many different sources in order to determine the best possible navigation solution in terms of both accuracy and reliability.

This new approach to navigation and real-time positioning in challenging environments requires many new lines of research to be pursued [6]. These include:
- How to integrate many different navigation and positioning technologies when the necessary expertise is spread across multiple organisations [7];
- How to adapt a multisensor navigation system in real-time to changes in the environmental and behavioural context to maintain an optimal solution [8];
- How to obtain more information for positioning by making use of new features of the environment [9];
- How to use 3D mapping to improve the performance of existing positioning technologies, such as GNSS, in dense urban areas.

The final item is the subject of the present paper. Intelligent urban positioning (IUP) aims to achieve a step change in the performance of real-time GNSS positioning in dense urban areas by combining three key ingredients [10]:
- Multi-constellation GNSS;
- New techniques for detection of non-line-of-sight (NLOS) signal propagation;
- Three-dimensional mapping.

Making use of the signals from all visible GNSS satellites significantly increases the amount of information available to compute a position solution from. It also provides the flexibility to select which signals to use and which to discard. NLOS signals are received only via reflected surfaces and can contribute large ranging errors. If these signals can be identified and excluded [11][12], the accuracy of conventional GNSS positioning may be substantially improved. Therefore, multi-constellation GNSS and effective NLOS detection are both critical components of any initiative to improve GNSS positioning accuracy in challenging urban environments.

There are at least three ways in which 3D mapping can be used to enhance GNSS positioning: detection and mitigation of NLOS reception, shadow matching and height aiding. A full IUP implementation would incorporate all three of these techniques and could also use conventional map matching [13].

A number of research groups have shown that 3D city models can be used to mitigate the effects of NLOS GNSS signal reception, a major source of error in dense urban areas. The 3D model can be used to predict which signals are NLOS and exclude these from the position solution. Early implementations assumed that the position was approximately known [14][15]. Using the 3D model to correct the NLOS ranging errors takes this a stage further [16]. Several research groups have taken this concept of 3D-mapping-aided GNSS ranging a step further by using the 3D city model to predict the path delay of the NLOS signals at each candidate position [17][18][19][20]. A single-epoch positioning accuracy of 4m has been reported [19]. However, the path delay must be determined using ray tracing, which is highly computationally intensive and thus an obstacle to real-time implementation. The urban trench approach [21] enables the path delays of NLOS signals to be computed very efficiently, but only if the building layout is highly symmetric. The challenge is therefore to develop a computationally efficient NLOS mitigation technique that can cope with position uncertainties of tens of metres.

The second way of aiding GNSS using 3D mapping is shadow matching. This is a new technique that determines position by comparing the measured signal availability and strength with predictions made using a 3D city model [22]. It is designed to be used alongside conventional ranging-based GNSS positioning in dense urban areas in order to improve the cross-street accuracy. Since 2011, UCL and other research groups have demonstrated shadow matching experimentally, using both single and multiple epochs of GNSS data [23][24][25][26][27]. Cross-street positions within a few meters have been achieved in environments where the error in the conventional GNSS position solution is tens of meters, enabling users to determine which side of the street they’re on. Shadow matching has also been demonstrated in real time on an Android smartphone [28]. The challenge ahead is to improve shadow matching’s reliability and integrate it with other navigation and positioning techniques, starting with conventional ranging-based GNSS [28].

This paper focuses primarily on the use of 3D mapping to aid ranging-based GNSS positioning, namely assisting the signal selection and weighting within the positioning
algorithm and terrain height aiding. This is described in Section 2 with the results achieved using GPS and GLONASS data collected in London described in Section 3 and conclusions provided in Section 4.

2. APPROACH

Our approach uses spatial data in two different ways: Firstly, building boundary data derived from a 3D city model is used to assist the signal selection and weighting within the positioning algorithm. The building boundaries are used to predict if each signal is directly receivable over a range of candidate positions centred on an approximate GNSS position solution [4]. This approach is much faster than using the city model directly though building boundaries can use more memory. From this, the probability of each signal being NLOS or directly received is then estimated. This was combined with consistency checking, signal geometry and signal strength information, based on previous work performed at UCL [12], to predict which combination and weighting of signals produces the best position solution.

Our second technique uses Digital Terrain Models (DTMs) to aid GNSS positioning by effectively providing an additional ranging measurement. Previous research with simulated height aiding showed that this can improve horizontal as well as vertical positioning in dense urban environments through improved solution geometry [12]. We have automated the height aiding approach as described in Section 2.3.

2.1. 3D Modelling

In March 2014 Ordnance Survey (OS) published an alpha release of the much anticipated Building Height Attribute (BHA) dataset, which is an enhancement to OS MasterMap Topography Layer [30]. The first alpha release of BHA included buildings covering approximately 8,000km² of the UK. Subsequent releases have increased the coverage of the dataset which covers major towns and cities in Great Britain and OS publish an interactive map which shows the extents of the areas covered by the alpha release making it possible to check whether an area of interest is included. A number of attributes are provided for each building, as shown in Figure 1, and they are listed below:

- AbsHMin: ground level;
- AbsH2: the base of the roof;
- AbsHMax: highest part of the roof;
- RelHMax: relative height from ground level to the highest part of the roof;
- RelH2: relative height from ground level to base of the roof.

RelH2 was considered in our work as it provides a good representation of the height of buildings relative to one another.

OS publish the data as a single Comma Separated Values (CSV) file containing over 20 million records. This is a very large dataset and can cause data management problems in a desktop environment so Edinburgh Data and Information Access (EDINA) have split the dataset up using the OS 5km grid allowing users to download the data in tiles for their study area. The data is available in CSV, Keyhole Markup Language (KML) and File Geodatabase formats.

The datasets required to generate the 3D model are:

- OS MasterMap® Topography Layer: the format selected was File Geodatabase (FileGDB) format as this format does not require any conversion to use it in Quantum Geographic Information System (QGIS), a GIS package of choice for our work which is free and open source.
- OS Terrain™ 5 DTM: this will be used as the base (surface) heights for the area;
- BHA data, selected as CSV format;
- OS VectorMap® Local Raster or 1:25 000 Scale Colour Raster (used as a background).

The dataset were merged in QGIS generating the 3D model displayed in Figure 2. The 3D model was exploited to generate building boundaries as described in [4]. The boundaries are from a GNSS user’s perspective, with the buildings edge determined for each azimuth (from 0 to 360°) as a series of elevation angles. The results from this step show where the building edges are located within an azimuth-elevation sky plot. Satellites are visible above this edge and blocked below it. The elevation of the building boundary is computed at a range of azimuths. Building boundaries are computed over a grid of candidate user locations. The altitude of these candidate user locations can be set at a certain distance above the ground, e.g. 1-5 m might be assumed for users holding smartphones in front of them. Only outdoor locations are considered.

Figure 1: Building models [30].
Figure 2: 3D city model of the test sites.

2.2. Signal Selection and Weighting

With a multi-constellation GNSS receiver, the number of measurements available will normally greatly exceed the minimum number required for a position solution. Therefore, measurements contaminated by NLOS reception or multipath can be downweighted (or in some cases rejected) in order to obtain the best position solution from the measurements available. The challenge is to identify which are the best signals. In benign reception environments, this can be done using consistency checking techniques. However, this is unreliable in dense urban environments, even using a more robust algorithm. Here, we use 3D data to extend the work presented in [12] where combinations of three techniques for mitigating the impact of NLOS and multipath interference on positioning accuracy were investigated, namely: consistency checking, elevation-based weighting and signal-strength-based weighting.

As demonstrated in [5], a position solution may be computed from a set of pseudo-range measurements using least-squares estimation. This is given by

\[
\hat{x}^+ = \hat{x}^- + (H_g^T W_p H_g)^{-1} H_g^T W_p (\tilde{z} - \hat{z}^-), \tag{1}
\]

with \(\hat{x}^-\) representing the estimated state vector, comprising the position and time solution, \(\hat{x}^-\) is the predicted state vector, \(\tilde{z}\) is the measurement vector, \(\hat{z}^-\) is the vector of measurements predicted from \(\hat{x}^-\), \(W_p\) is the weighting matrix and \(H_g^T\) is the measurement matrix. For GPS and GLONASS measurements with unknown interconstellation timing offset, the state vector and measurement vector are

\[
x = \begin{pmatrix} r_{ea}^e \\ \delta p_{a,c}^e \\ \delta p_{c,gl}^e \end{pmatrix}, \quad z = \begin{pmatrix} \rho_{a,1,c} \\ \rho_{a,2,c} \\ \vdots \\ \rho_{a,m,c} \end{pmatrix}, \tag{2}
\]

where \(r_{ea}^e\) is the Cartesian position, resolved about and with respect to an Earth-centred Earth-fixed (ECEF frame), \(\delta p_{a,c}^e\) and \(\delta p_{c,gl}^e\) are, respectively, the receiver clock offset and GLONASS-GPS timing offset, expressed as ranges, \(\rho_{a,m,c}\) is the pseudo-range from satellite \(j\) and \(m\) is the number of satellite used. The measurement matrix is given by

\[
H_g = \begin{pmatrix} -u_{a1,x} & -u_{a1,y} & -u_{a1,z} & 1 & -\delta_{1eGL} \\ -u_{a2,x} & -u_{a2,y} & -u_{a2,z} & 1 & -\delta_{2eGL} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -u_{am,x} & -u_{am,y} & -u_{am,z} & 1 & -\delta_{meGL} \end{pmatrix}, \tag{3}
\]

where \(u_{aj}^-\) is the line-of-sight vector from the user antenna to satellite \(j\) and \(\delta_{ieGL}\) is 1 where satellite \(j\) is a GLONASS satellite and zero otherwise. The line-of-sight vectors and predicted pseudo-ranges, \(\hat{\rho}_{a,c}\), are given by

\[
u_{aj}^- \approx \frac{r_{aj}^- - r_{ea}^e}{|r_{aj}^- - r_{ea}^e|}, \tag{4}
\]

\[
\hat{\rho}_{a,c}^- = \sqrt{|r_{aj}^- - r_{ea}^e|^2 + \delta \hat{\rho}_{c}^{ie} + \delta_{ieGL}\delta \hat{\rho}_{c}^{ie} + \delta \hat{\rho}_{e,a}^-}, \tag{5}
\]

where \(\hat{\rho}_{aj}^-\) is the predicted pseudo-range from satellite \(j\), \(\hat{\rho}_{ea}^-\) is the predicted user position, \(\delta \hat{\rho}_{c}^{ie}\) is the predicted receiver clock offset, \(\delta \hat{\rho}_{c}^{ie}\) is the predicted GLONASS-GPS timing offset, \(\delta \hat{\rho}_{e,a}^-\) is the satellite \(j\) Sagnac correction and \(\delta_{ieGL}\) is 1 for GLONASS satellite and 0 otherwise [12].

The different weighting schemes considered are: conventional elevation-based weighting, \(C/N_0\)-based weighting and no weighting. \(W_p\) is given by

\[
W_p = \begin{pmatrix} \rho_{p1}^{-2} & 0 & \cdots & 0 \\ 0 & \rho_{p2}^{-2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \rho_{pm}^{-2} \end{pmatrix}, \tag{6}
\]
where, for the elevation-based weighting,

\[ \sigma_{pj} = a + b \exp(-\theta_{nj}/\theta_0), \]  

(7)

where \( \theta_{nj} \) is the elevation angle of the \( j^{th} \) satellite and the constants are \( a = 0.13m, b = 0.56m \) and \( \theta_0 = 0.1745 \text{ rad} \) [31] while, for \( C/N_0 \)-based weighting,

\[ \sigma_{pj} = \sqrt{c \times 10^{-(C/N_0)_j/10}}, \]  

(8)

where \( (C/N_0)_j \) is the measured carrier-power-to-noise-density ratio of the \( j^{th} \) satellite signal in dB-Hz and \( c = 1.1 \times 10^4m^2 \) is a constant [32].

For the case without weighting, \( W_p \) is simply the identity matrix.

![Diagram](attachment:image.png)

Figure 3: Approach proposed in [12].

In this work, as indicated above, we exploit the 3D city model to classify the signals as line-of-sight (LOS) or not. This additional information helps refining further the algorithm proposed in [12], summarised in Figure 3, in several ways:

We are able to define a new weighting matrix, \( W_{p3D} \), for the least-squares solution step in Figure 3, indicated as (e), that incorporates this additional information. This replaces \( W_p \) in equation (1) and is given by

\[
W_{p3D} = \begin{pmatrix}
p_1 \sigma_{p1}^{-2} & 0 & \ldots & 0 \\
0 & p_2 \sigma_{p2}^{-2} & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & p_m \sigma_{pm}^{-2}
\end{pmatrix}
\]  

(9)

where the elements \( p_i, i = 1, \ldots, m \), are the estimated probabilities that each signal is received via a direct LOS path. The derivation of the elements \( p_i \) is as follows: Considering each GNSS satellite, one can compute a shadow map of a city building map (within the region of interest) with respect to a GNSS satellite position by projecting the buildings on the surface region. The direct LOS map of each considered signal is represented as a 2D matrix filled with elements “1” or “0”, each indicating whether or not the receiver has a LOS to the corresponding GNSS satellite at that grid point (we considered a one metre grid spacing). Effectively, the computation of the direct LOS maps makes use of the GNSS satellite’s position and the 3D digital model map of the area (similarly, an NLOS map, which relates NLOS propagation resulting from the GNSS satellites being invisible, could be generated by switching the 1 and 0 values). By calculating a simple average of the “1”s and “0”s across each map, we determine a LOS probability \( p_i \) for each satellite. The overall likelihood weightings are then determined by multiplying the LOS probabilities with signal-strength derived (or \( C/N_0 \) derived) weighting factors and elevation-based weighting factors.

We recall that consistency checking presented in [12] looks at identifying the subset of GNSS measurements that are most consistent with each other on the basis that these are least likely to be contaminated by NLOS reception and severe multipath interference. The subset comparison method works by scoring different subsets of the GNSS measurements according to their consistency and then using the most consistent subset to form the position solution. The basis of this method is the minimal sample set (MSS), a subset consisting only of the minimum number of measurements required to produce an exact solution. Each MSS is used to predict the remaining pseudo-ranges, which are compared with their measured values, both to score the MSS and to identify which of the measurements are consistent with it. Different criteria may be used for this, enabling the method to be adapted to different statistical distributions of the NLOS and multipath errors.

It is not necessary to compute and test every possible MSS. This is because the objective is to obtain the final measurement subset, which may be built up from a number of different MSSs. The algorithm considered is based on a technique known as random sample consensus (RANSAC), which uses random-draw subsets of the measurements and a probability-based stopping criterion for efficiency. The MSS is then assessed, resulting in a consensus set (CS), which is the set of other measurements that are found to be consistent with the MSS (and is determined by comparing the magnitudes of the residuals, \( e_i^j \), of all measurements outside the MSS with a threshold, \( \delta \)), and a common
RANSAC cost function $C^i$ corresponding to the $i$th MSS, which is a measure of the consistency and is defined by (assuming a Gaussian distribution)

$$C^i(e^i) = \sum_{j=1}^{m} k(e^i_j, \delta),$$  \hspace{1cm} (10)

where

$$k(e^i_j, \delta) = \begin{cases} 0 & e^i_j \leq \delta \\
\left(\frac{\delta}{\sigma_{pj}}\right) & e^i_j > \delta 
\end{cases},$$  \hspace{1cm} (11)

where $\sigma_{pj}$ are given by (7) and (8) and $e^i_j$ are the elements of vector $e^i$. This latter is calculated using

$$e^i = z - \hat{z}^i,$$  \hspace{1cm} (12)

where $\hat{z}^i$ is the set of measurements predicted from the $i$th MSS position and time solution, $\hat{x}^i$, given by

$$\hat{x}^i = \hat{x} - H_u^c(e^i) (z^i - \hat{z}^i).$$  \hspace{1cm} (13)

Where $H_u^c$ comprises the rows of the measurement matrix, $H_u^c$, given by (3), which correspond to the $i$th MSS, $\hat{z}^i$ comprises the elements of the predicted measurement vector, $\hat{z}$, given by (2), corresponding to the $i$th MSS and $\hat{x}$ is the predicted state vector, also defined by (2).

The process is iterated to find a MSS that generates the minimum cost function. This continues until there have been sufficient iterations for the probability of finding a better MSS to fall below a certain threshold [12]. In cases where the MSS with the lowest cost function has less than one measurement in its consensus set (CS), it is not possible to confirm that this measurement subset (or any other) is self-consistent, so consistency checking is deemed to have failed and the all-satellite position solution is used.

In our new approach exploiting the 3D city model, the initialisation step, indicated by (a) in Figure 3, is modified so that the minimal sample sets (MSS) are composed of measurements from GNSS satellites with high LOS probabilities, instead of being selected purely randomly. Furthermore, an additional weighting is applied to the cost function indicated in Figure 3 as (b) and used to derive the consensus set (CS). This results in a modified cost function, $C_{3D}^i$, given by

$$C_{3D}^i(e^i) = \sum_{j=1}^{m} k_{3D}(e^i_j, \delta),$$  \hspace{1cm} (14)

where

$$k_{3D}(e^i_j, \delta) = \begin{cases} 0 & e^i_j \leq \delta \\
(\frac{\delta}{\sigma_{pj}}) & e^i_j > \delta 
\end{cases},$$  \hspace{1cm} (15)

2.3. Terrain Height Aiding

Many conventional maps, dedicated digital terrain models (DTMs) and digital elevation models (DEMs) and all 3D maps provide the terrain height. Land vehicle or pedestrian GNSS user equipment may be assumed to be at a fixed height above the terrain. Therefore, the approximate GNSS horizontal position solution may be used to obtain a height solution from the mapping data or a separate terrain height database. This may then be used as an extra ranging measurement within a GNSS positioning algorithm, a technique known as height aiding [33]. Typically, the height-aiding measurement is treated as a virtual transmitter at the centre of the Earth, the range to which is equal to the (local) Earth radius plus the height (Figure 4).

If the terrain within the search area is not flat, the range may vary over the uncertainty bounds of the approximate GNSS position solution. Height aiding is particularly useful in cases where there are insufficient direct-LOS signals to determine a position solution without using NLOS signals. Under good GNSS reception conditions, height aiding only improves vertical positioning. However preliminary tests using simulated height [12] have shown that in areas such as urban canyons, where the signal geometry is poor, it can also improve horizontal positioning.
and the state vector is as defined in (2). Here we generate height aiding measurements using a terrain height database and the unaided GNSS position solution. Figure 5 summarises the iterative process of computing height aiding comprising three main steps:

1) A position is computed using pseudo-range measurements from all of the satellites tracked as described in equation (1) (using one of the weighting strategies described in Section 2.2).

2) Following the computed position and coordinate transformation from WGS84 to the National Grid Easting and Northing coordinate system, a database containing terrain height information is then queried and the four DTM vertices surrounding the position solution are identified and extracted. These latter are then used in an interpolation process (as described in the next paragraphs) to extract a new height information corresponding to the computed position.

3) Following conversion of the height information to a virtual range measurement, this is then added to the measurement vector and a new position solution is computed. The process is iterated until the difference between the old and new position is smaller than the DTM cell resolution.

The study reported in [34] demonstrates that whether interpolating on mathematical surfaces or DTMs, irrespective of terrain complexity, the higher-order algorithms consistently outperform the simpler linear variant. For this study, two representative high-order interpolation algorithms, bicubic and biquintic, were implemented, as well as the more popular bilinear algorithm, often incorporated in desktop Geographic Information System (GIS) packages.

The most commonly used interpolation method for a regular grid is patchwise polynomial interpolation. The general form of this equation for surface representation is [34]

\[ h(x, y) = \sum_{i=0}^{m} \sum_{j=0}^{n} a_{ij}x^i y^j, \]  

where \( h(x, y) \) is the height of an individual point with rectangular coordinates \( x \) and \( y \), and \( \{a_{ij}, i = 0, \ldots, m, j = 0, \ldots, n\} \) are the coefficients of the polynomial in (18).

Bilinear, Bicubic and biquintic interpolations make use of the 4-term, 16-term and 36-term functions, respectively, and can be represented by (18) with \( m = n = 1 \), \( m = n = 3 \), and \( m = n = 5 \), respectively. Since the coordinates of each grid vertex are known, the values of \( \{a_{ij}, i = 0, \ldots, m, j = 0, \ldots, n\} \) can be determined from a set of simultaneous equations based on (18), one for each known point, or its derivative. Having determined the coefficients, \( a_{ij} \), the height for a location with known horizontal coordinates can be determined using (18).

For the bicubic interpolation, the 16 values used to derive the coefficients are the heights at the four vertices of the grid cell, together with three derivatives. The first derivatives with respect to \( x \) and \( y \), \( \partial_x h(x, y) \) and \( \partial_y h(x, y) \), express the slope of the surface in the \( x \) and \( y \) directions, respectively, whilst the second-order cross derivative, \( \partial_x h(x, y) \), represents the slope in both \( x \) and \( y \). For the bicubic interpolation it is necessary to estimate the derivatives or slopes at the DEM vertices. Slope values will influence the shape of the interpolating surface function in a more valuable and accurate way than just using additional DEM vertices [35]. To estimate these slopes from the grid heights, we used finite difference approximations [36] where the different slopes are calculated as follows:
\[
\partial_x h(x, y) = \frac{[f(x + 1, y) - f(x - 1, y)]}{2}
\]
\[
\partial_y h(x, y) = \frac{[f(x, y + 1) - f(x, y - 1)]}{2}
\]
\[
\partial_{xy} h(x, y) = \frac{[f(x + 1, y + 1) - f(x - 1, y) - f(x - 1, y + 1) + f(x + 1, y)]}{4}
\]  \hspace{1cm} (19)

2.4. Initialization Process

Relying on the conventional GNSS solution as a basis for the selection of the search area is not a robust approach especially in a deep urban environment where the conventional GNSS solution might be tens of metres away from the true position. There are a number of different approaches to initialization. Here, we iterate the 3D-model-aided positioning algorithm, initializing the search area from the previous iteration. Figure 6 illustrates this.

![Diagram](Image © 2013 Bluesky © Google)

Figure 6: Initialization process for 3D-model aided positioning.

The idea is to take the conventional GNSS receiver position solution and consider a large search area (in our work we have selected an initial search radius of 100m with a grid spacing of 15m). We then apply the height aiding and 3D-city-model aided signal selection and weighting, considering the height averaged across the search area. The resulting position solution is then considered as the centre of a new search area with a reduced search radius (in our work we reduced the search radius with decrements of 20m) with the new height still averaged across the search area and a reduced grid spacing (in this work we reduced the grid spacing by decrements of 3 metres). The process is iterated until we reach a search area of 40m. Following the exit of the initialization process, a full application of the height aiding (considering the actual height extracted from the 2D city model) and 3D-city-model aided signal selection and weighting is performed considering the last position with a search area of 40m radius and 1m grid spacing.

3. EXPERIMENTAL RESULTS

The combined height aiding and 3D model-based NLOS prediction was tested on old and new GPS and GLONASS data collected using a Leica Viva GS15 survey-grade mult constellation GNSS receiver in Central London (test sets 1 and 2) and using u-blox evaluation kit (EVK-M8T), which is a multiconstellation GNSS receiver (GPS/GLONASS and Galileo ready) [37] (Test Set 3). The first set of test data was collected near Moorgate underground station on 8th April 2011. There are three sites within the test data set, each occupied for about 38 minutes. Figure 7 shows an overview of the test sites. The truth was established using traditional surveying methods and is accurate at the cm-level. The second test data set was collected near Fenchurch Street station on 23rd July 2012. Overall 22 sites were occupied to cover a variety of environments. Each site was occupied for two periods of about 10 minutes approximately 3 hours apart. Figure 8 depicts an overview of the test sites. The truth was established to decimetre-level accuracy using a 3D city model with tape measurements from landmarks.

![Figure 7](Image © 2013 Bluesky © Google)

Figure 7: Locations of the test set 1 sites (Background Image © 2013 Bluesky © Google).

Height aiding and consistency checking without 3D model-based NLOS prediction was tested using a number of different algorithm configurations. These results are given in Table 1 summarising the RMS horizontal and vertical position error with conventional GNSS positioning and terrain height aiding for both C/N₀ and elevation based weighting and using OS DTM 5 and DTM 50 [38].

![Figure 8](Image © 2013 Bluesky © Google)

Figure 8: Locations of the test set 2 sites (Background Image © 2013 Bluesky © Google).
<table>
<thead>
<tr>
<th>Positioning Algorithm</th>
<th>Average RMS Positioning Error (m)</th>
</tr>
</thead>
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<td></td>
<td>Horizontal</td>
</tr>
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<td>Conventional GNSS</td>
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<tr>
<td>(using C/N0 weighting)</td>
<td></td>
</tr>
<tr>
<td>Terrain Height Aiding and 3D model-based NLOS Prediction</td>
<td>20.7</td>
</tr>
<tr>
<td>Accuracy Improvement</td>
<td>55%</td>
</tr>
</tbody>
</table>

Table 2: Position errors obtained using terrain height aiding and 3D model-based NLOS prediction – Results averaged for test sets 1 and 2.

A third dataset was collected near Fenchurch Street station on the 15th of May 2015 using u-blox EVK-M8T GNSS receiver and where the selected sites are illustrated on Figure 11 and Figure 12. The truth was established to decimetre-level accuracy using a 3D city model with tape measurements from landmarks.

We have evaluated the combined 3D aided signal selection and height aiding approach using this data. Figure 13 and Figure 14 illustrate the positioning error using conventional GNSS positioning and the new approach, respectively. Table 3 summarises the improvement in position accuracy for each location. We can conclude that the positioning performance is degraded further going from S1, S2, S3 and S4 and that the proposed approach improves the horizontal accuracy by 61% and the vertical accuracy by 75%.
Figure 11: Locations of the Test Set 3 sites (Background Image © 2015 Bluesky © Google).

Figure 12: Locations and skylines for Test Set 3.

Figure 13: Position error using conventional GNSS positioning – u-blox data collected at test site 3. The figures in the legend are RMS errors in metres.
Figure 14: Position error using 3D aided signal selection and height aiding approach – u-blox data collected at test site 3. The figures in the legend are RMS errors in metres.

Table 3: Position accuracy improvement using terrain height aiding and 3D model-based NLOS prediction (compared to conventional GNSS positioning with consistency checking) – Locations S1, S2, S3, and S4 on Figure 12.

<table>
<thead>
<tr>
<th>Location</th>
<th>Improvement in Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Horizontal</td>
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<tr>
<td>S1</td>
<td>70%</td>
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<tr>
<td>S2</td>
<td>52%</td>
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<tr>
<td>S3</td>
<td>51%</td>
</tr>
<tr>
<td>S4</td>
<td>72%</td>
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</tbody>
</table>

4. CONCLUSIONS

The ability of height aiding to improve GNSS positioning in dense urban areas using an automated iterative process has been assessed using data collected at multiple sites. Using a height aiding measurement from a 3D city model or separate terrain height database significantly improves single-epoch positioning accuracy, horizontally as well as vertically, due to the improved solution geometry. Considering the Leica Viva receiver measurements and comparing to the conventional GNSS positioning using $C/N_0$ weighting, horizontal accuracy is improved by 43% where vertical accuracy improvement was of 78%.

Augmenting the height aiding with the 3D city model aided signal selection and applying the approach on the same sets of data resulted in further improving accuracy by 20% horizontally and 12.8% vertically. The combined improvement was 55% horizontally and 81% vertically. The u-blox EVK-M8T data was also subjected to the augmented height aiding using the 3D aided signal selection and the overall accuracy improvement across the four locations was of 61.2% horizontally and 75.2% vertically. The overall improvement, considering the data collected using the Leica Viva and u-blox EVK-M8T receivers, was 58% horizontally and 78% vertically.

In future work we plan to integrate the techniques presented here with GNSS shadow matching [26], a concept known as intelligent urban positioning [10]. Applications that could benefit from this include vehicle lane detection for intelligent transportation systems (ITS), location-based advertising, augmented-reality, and step-by-step guidance for the visually impaired and for tourists.

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