The Roll Motion of Trimaran Ships

By

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Declaration

I, Thomas James Grafton, confirm that the work presented in this thesis is my own. Where information has been derived from other sources, I confirm that this has been indicated in the thesis.

6th July 2007
Abstract

This thesis reports on research conducted into the roll motion of trimaran ships. After reviewing the relevant literature to determine the state of the art of roll motion prediction for both monohull and multi-hulled ships a hypothesis is set out that:

Accurate trimaran roll motion predictions can be obtained using linear Potential Flow Seakeeping theory with the roll damping term either obtained from a roll decay experiment or augmented with empirically based theoretical roll damping components developed for monohulls.

This hypothesis underpins the work of many of the existing researchers who have investigated the seakeeping performance of trimaran ships, although none have formally proved it to be true.

After conducting theoretical, experimental and combined theoretical and experiment studies using a single trimaran model this hypothesis is subsequently disproved. This leaves a problem: How can accurate trimaran roll motions be determined? The focus of the remainder of the thesis is to understand why the hypothesis is incorrect, investigating in turn each of the assumptions that underpin it. Finally, as a recipe for future researchers, a series of experimental and theoretical investigations has been devised to explore the physics of trimaran roll motion from first principals.

This work has shown that, for a trimaran with significant flare above the waterline on the side hulls, roll decay coefficients cannot be measured from free decay experiments if the motion is characterised by a single degree of freedom roll equation with constant coefficients. Furthermore, it is postulated that heave and roll are strongly coupled for all trimarans. It is shown, using the results of model experiments, that this coupling breaks the assumption of linear theory where an input sinusoidal wave of constant amplitude and frequency leads to output motions which are also sinusoidal with constant amplitude and frequency.
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Equivalent linear roll damping (be) plotted against mean roll angle between two successive peaks (red spots) for trimaran model DVZ without roll damping appendages at a speed equivalent to 25 knots at full scale. The black line is a least squares fit to the data used to obtain linear and cubic damping coefficients (b1 and b3).

Simulated roll decay history (calculated using linear and cubic damping coefficients derived from the least squares fit to the equivalent linear damping values calculated between subsequent peaks) compared with a Savitzky-Golay fit to the measured data for trimaran model DVZ without roll damping appendages at a speed equivalent to 25 knots at full scale.

Variation in the linear non-dimensional linear roll damping coefficient (k1) with speed for a range of appendages tested in the 2002 model experiments on trimaran DVZ.

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Figure 5-4-7  Comparison of measured roll decay histories with initial heel to port for both the 2002 and 2004 model experiments for trimaran DVZ without roll damping appendages at zero speed. The 2004 results were adjusted to remove a bias.

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Figure 5-4-10  Comparison of measured roll decay histories with initial heel to starboard for both the 2004 model experiments for trimaran DVZ without roll damping appendages at zero speed. The presented results were adjusted to remove a bias.
Figure 5-5-1 Variation of equivalent linear roll damping (be) plotted against mean roll angle between two successive peaks (red spots) for a simulation of roll equation without stiffness variations.

Figure 5-5-2 Roll decay, GM variation due to heave, absolute roll angle and equivalent linear roll damping (be) plotted against the mean roll amplitude between successive peaks for a simulation of the roll equation with stiffness variations. The heave frequency is equal to 2 rad/s and there is no heave damping.

Figure 5-5-3 Roll decay, GM variation due to heave, absolute roll angle and equivalent linear roll damping (be) plotted against the mean roll amplitude between successive peaks for a simulation of the roll equation with stiffness variations. The heave frequency is equal to 2 rad/s and bh = 0.08.

Figure 5-5-4 Movement of the trimaran model after removal of the inclining weight.

Figure 5-5-5 Change in GM due to heel about a fixed waterline (note that the displacement changes as haunches are immersed).

Figure 5-5-6 Variation in GZ due to the time varying term GMw. The blue circles are the time values of GZ at each time step in the simulation. The solid black line represents the value of GZ if a constant value of GM is used (142.10 mm).

Figure 5-5-7 Roll decay, GM variation due to heave, absolute roll angle and equivalent linear roll damping (be) plotted against the mean roll amplitude between successive peaks for a simulation of the roll equation with stiffness variations due to rolling around a fixed waterline only.
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<td>Absolute pitch Fast Fourier Transform for signal from 0 – 4.6 seconds.</td>
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Figure 5-6- 8 Absolute pitch Fast Fourier Transform for signal from 4.6 – 13.5 seconds.

Figure 5-8- 1 Roll angle – time history from model experiment in regular waves for trimaran DVZ in beam seas at a speed equivalent to 6 knots (with no roll damping appendages fitted).

Figure 5-8- 2 Wave amplitude – time history from model experiment in regular waves for trimaran DVZ in beam seas at a speed equivalent to 6 knots (with no roll damping appendages fitted).

Figure 5-8- 3 Discrete Fourier Transform of roll angle – time history with wave power scale (y-axis) converted to represent roll amplitude for trimaran DVZ in beam regular waves at a speed equivalent to 6 knots.

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Figure 5-8- 6 Fit of regular wave to recorded heave acceleration – time history using the magnitude of the acceleration of the first peak of the Discrete Fourier Transform for trimaran DVZ without roll damping appendages in beam seas at a speed equivalent to 6 knots.
Recorded heave acceleration – time history compared to fit using two sinusoidal varying terms each with constant amplitude and frequency taking from the first and second peak of the Discrete Fourier Transform for wave frequency 2.80 rad/s for trimaran DVZ in beam seas at a speed equivalent to 6 knots.

Discrete Fourier Transform of heave acceleration – time history for trimaran DVZ in beam seas at a speed equivalent to 12 knots without roll damping appendages.

Pitch angle – time histories for trimaran DVZ in beam seas at a speed equivalent to 6 knots without roll damping appendages.

Discrete Fourier Transform of pitch angle – time history for trimaran DVZ in beam seas at a speed equivalent to 6 knots without roll damping appendages.

Process to further the understanding of trimaran rolling.

Set up for experiment one.

Setup for experiment three, forced heave experiments on simple side hull shapes with and without haunches.

Experimental setup for hull lift calculations.

Classification of section shapes for eddy damping calculations: "U/V" on left and "Full" on right.

Classification of eddy separation conditions: one eddy shed on left and two on right.

Vertical lift force generated during roll motion for a high speed craft with high beam-draught ratio.

Definitions for the bilge keel parameters.

Assumed pressure distribution on hard chine hull shape for eddy damping calculation according to Ikeda et al (145) (146).

Assumed pressure distribution around a skeg for damping calculation according to Ikeda et al (146).
Figure A3-1- 1 Variation of free surface lift loss factor KL with the ratio of appendage depth to chord.

Figure A3-1- 2 Free surface wave function gamma.

Figure A4-1- 1 Comparison of roll RAO calculated from components (theoretically) with model experiment results for trimaran DVZ in stern quartering seas at a ship speed of 6 knots

Figure A4-1- 2 Comparison of roll RAO calculated from components (theoretically) with model experiment results for trimaran DVZ in beam seas at a ship speed of 6 knots

Figure A4-1- 3 Comparison of roll RAO calculated from components (theoretically) with model experiment results for trimaran DVZ in stern quartering seas at a ship speed of 12 knots

Figure A4-1- 4 Comparison of roll RAO calculated from components (theoretically) with model experiment results for trimaran DVZ in beam seas at a ship speed of 12 knots

Figure A4-1- 5 Comparison of roll RAO calculated from components (theoretically) with model experiment results for trimaran DVZ in stern quartering seas at a ship speed of 20 knots

Figure A4-1- 6 Comparison of roll RAO calculated from components (theoretically) with model experiment results for trimaran DVZ in beam seas at a ship speed of 20 knots
Nomenclature

Lower Case Greek Symbols

\( \alpha \) Real part of solution of equation of unforced motion for a spring-mass-damper system
\( \alpha_0 \) Representative flow incidence angle for lift [radians] damping calculations
\( \alpha_1 \) Flow incidence angle at the apparent centre of [radians] lifting pressure on the hull for lift damping calculations
\( \alpha_{bk} \) Coefficient for determining \( C_{bk} \) according to Tanaka
\( \alpha_{ci} \) Coefficient within the effective wave slope coefficient for a catamaran
\( \alpha_{hc} \) Deadrise angle of a two-dimensional cylinder used [radians] in model experiments by Ikeda et al
\( \alpha_{ind} \) Induced angle of attack of an appendage [radians]
\( \alpha_{m1} \) Coefficient within the effective wave slope coefficient for a monohull
\( \alpha_{m2} \) Coefficient within the effective wave slope coefficient for a monohull
\( \alpha_r \) Angle of incidence of the flow for a cross-section of [radians] a SWATH hull (Analogous to the trim angle of a flat plate inclined to the flow)
\( \alpha_s \) The wave slope equal to \( 2\pi \eta_0/\lambda \)
\( \alpha_{se} \) The effective wave slope coefficient
\( \alpha_{vl} \) Flow incidence angle at the apparent centre of [radians] vertical lifting pressure on the hull for lift damping calculations
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tr>
<td>$\beta$</td>
<td>Imaginary part of solution of equation of unforced motion for a spring-mass-damper system</td>
</tr>
<tr>
<td>$\beta_{frc}$</td>
<td>The angle between the y-axis and the lever arm between the centre of gravity and the hull surface element where the friction force is being determined [radians]</td>
</tr>
<tr>
<td>$\beta_{hull}$</td>
<td>The mean hull deadrise angle [radians]</td>
</tr>
<tr>
<td>$\beta_{side}$</td>
<td>The angle between the y-axis and the lever arm between the centre of gravity and the position where eddies are shed on the side hull ($r_{ed-s}$) [radians]</td>
</tr>
<tr>
<td>$\beta_{H-L}$</td>
<td>A factor varying in proportion to the ships forward speed used by Haddara and Leung (124) to determine the Lift Coefficient</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Wave direction relative to the ship ($\pi$ radians is head seas) [radians]</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Damping Factor. Ratio of total roll damping component $B_{44}$ at a given speed to the total roll damping component at zero speed for the same wave frequency and direction</td>
</tr>
<tr>
<td>$\delta_4$</td>
<td>Phase lag of the roll forcing moment from the roll motion [radians]</td>
</tr>
<tr>
<td>$\delta_{bl}$</td>
<td>Boundary layer thickness [m]</td>
</tr>
<tr>
<td>$\delta_i$</td>
<td>Phase lag of motion for unforced spring-mass-damper system [radians]</td>
</tr>
<tr>
<td>$\varepsilon_{;i=0\rightarrow 4}$</td>
<td>Coefficients in the damping model defining the form of the roll damping term $b_{44}$</td>
</tr>
<tr>
<td>$\varepsilon_{bbl}$</td>
<td>Deadrise angle of a particular hull section [radians]</td>
</tr>
</tbody>
</table>
\( \phi \) Phase angle between the force and the motion in the equation of motion for a spring-mass-damper system [radians]

\( \varphi \) Static heel angle [radians]

\( \gamma_r \) Phase lag of roll motion from roll forcing moment [radians]

\( \gamma_{bk} \) Angle associated with the location of the bilge keel with respect to the centre of gravity [radians]

\( \gamma_v \) Ratio of the maximum to the mean flow velocity around a hull section

\( \eta \) Displacement of the sea surface [m]

\( \eta_0 \) Amplitude of the displacement of the sea surface [m]

\( \dot{\eta} \) Velocity of the sea surface [m/s]

\( \dot{\eta}_v \) Vertical velocity of fluid induced by the incoming wave [m/s]

\( \dot{\eta}_h \) Horizontal velocity of fluid induced by the incoming wave [m/s]

\( \dot{\eta}_{hp} \) Horizontal velocity of fluid induced by the incoming wave on the port submerged hull of a SWATH [m/s]

\( \dot{\eta}_{hs} \) Horizontal velocity of fluid induced by the incoming wave on the starboard submerged hull of a SWATH [m/s]

\( \dot{\eta}_{vp} \) Vertical velocity of fluid induced by the incoming wave on the port submerged hull of a SWATH [m/s]

\( \dot{\eta}_{vs} \) Vertical velocity of fluid induced by the incoming wave on the starboard submerged hull of a SWATH [m/s]

\( \ddot{\eta} \) Acceleration of the sea surface [m/s²]

\( \kappa \) Coefficient depending on the hull midships area coefficient
<table>
<thead>
<tr>
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<tr>
<td>( \lambda )</td>
<td>Coefficient for determining ( C_m ) according to Tanaka, or, wavelength of a regular sinusoidal wave</td>
</tr>
<tr>
<td>( \lambda_{1,2} )</td>
<td>Roots of characteristic equation</td>
</tr>
<tr>
<td>( \theta_1 )</td>
<td>Coefficient in exponentially decaying sine plus cosine fit to measured roll decay data</td>
</tr>
<tr>
<td>( \theta_2 )</td>
<td>Coefficient in exponentially decaying sine plus cosine fit to measured roll decay data</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Density of salt water ([\text{kg/m}^3])</td>
</tr>
<tr>
<td>( \sigma_f )</td>
<td>Prandtl's finite span aerofoil factor</td>
</tr>
<tr>
<td>( \sigma_s )</td>
<td>Area coefficient of a ship section</td>
</tr>
<tr>
<td>( \tau_{\text{body}} )</td>
<td>Trim angle of a body of revolution ([\text{radians}])</td>
</tr>
<tr>
<td>( \tau_{\text{ship}} )</td>
<td>Trim angle of the ship or model used for the calculation of the hull lift damping component, ( B_L ) ([\text{radians}])</td>
</tr>
<tr>
<td>( \tau_v )</td>
<td>The virtual trim angle acting on the vertical lift force used in the calculation of the vertical lift damping component, ( B_L ) ([\text{radians}])</td>
</tr>
<tr>
<td>( \nu )</td>
<td>Kinematic viscosity ([\text{m}^2/\text{s}])</td>
</tr>
<tr>
<td>( \omega )</td>
<td>Frequency of motion ([\text{rad/s}])</td>
</tr>
<tr>
<td>( \omega_0 )</td>
<td>Damped natural frequency for forced motion ([\text{rad/s}])</td>
</tr>
<tr>
<td>( \omega_d )</td>
<td>Damped natural frequency for unforced motion ([\text{rad/s}])</td>
</tr>
<tr>
<td>( \omega_e )</td>
<td>Wave encounter frequency ([\text{rad/s}])</td>
</tr>
<tr>
<td>( \omega_h )</td>
<td>Heave natural frequency ([\text{rad/s}])</td>
</tr>
<tr>
<td>( \omega_n )</td>
<td>Natural frequency of motion ([\text{rad/s}])</td>
</tr>
</tbody>
</table>
\( \omega_{nc} \)  
Equivalent natural frequency of roll motion for a catamaran  
\( \omega_{nd} \)  
Non-dimensional frequency parameter  
\( \omega_{nmc} \)  
Equivalent natural frequency of roll motion for a centre hull of a trimaran  
\( \zeta_{bk} \)  
Coefficient for determining \( C_{bk} \) according to Tanaka  
\( \psi_i \)  
Coefficient representing the argument of the Lewis function on the transformed unit circle  
\( \zeta \)  
Non-dimensional damping factor  

**Upper Case Greek Symbols**  
\( \Gamma_{bk} \)  
Angle associated with the location of the bilge keel with respect to the centre of gravity  
\( \Lambda \)  
Frequency tuning factor  
\( \Omega \)  
Free surface wave function used for corrections to lift slope \( C_{La} \) to account for free surface lift losses  

**Other Symbols**  
\( \nabla \)  
Displaced volume  
\( \text{m}^3 \)  

**Capital Letters**  
\( A \)  
\((6 \times 6)\) matrix representing the added mass and added inertia terms for motion in six degrees of freedom  
\( A \)  
Constant in analysis of unforced spring-mass-damper system
\( A_{lk} \) Coefficient depending on Lewis form coefficients of the ship section, used for the calculation of eddy damping by Ikeda et al

\( A_{44} \) Added inertia in roll \([\text{kg m}^2]\)

\( A_{33} \) Added mass in heave \([\text{kg}]\)

\( A_{app} \) Plan area of an appendage \([\text{m}^2]\)

\( A_{bkp} \) Coefficient for determining the bilge keel roll damping component \( B_{bkp} \) according to Ikeda et al

\( A_{rp} \) A representative area \([\text{m}^2]\)

\( A_p \) The projected area of a cross-section of a SWATH hull in the horizontal plane \([\text{m}^2]\)

\( AR \) The aspect ratio of a plate or appendage

\( AR_e \) The effective aspect ratio for lift calculations

\( A_w \) Waterplane areas of ship \([\text{m}^2]\)

\( A_{wc} \) Waterplane area of one hull of a catamaran \([\text{m}^2]\)

\( B \) \((6 \times 6)\) matrix representing the damping terms for motion in six degrees of freedom

\( B \) Constant in analysis of unforced spring-mass-damper system, or the Beam of the ship in metres \([\text{N/(m/s)}]\) or \([\text{m}]\)

\( B \) Coefficient for determining \( C_{tk} \) according to Tanaka

\( B_1 \) Linear roll damping coefficient \([\text{Nm/(rad/s)}]\)

\( B_{1E} \) Coefficient depending on Lewis form coefficients of the ship section, used for the calculation of eddy damping by Ikeda et al

\( B_2 \) Quadratic roll damping coefficient \([\text{Nm/(rad/s)}^2]\)

\( B_3 \) Cubic roll damping coefficient \([\text{Nm/(rad/s)}^3]\)

\( B_{33} \) Heave damping coefficient in the equation of motion for a ship in regular waves \([\text{N/(m/s)}]\)
$B_{33}^{SH}$ Heave damping coefficient in the equation of motion for the side hulls of a trimaran undergoing motion in regular waves

$B_{44}$ Roll damping coefficient in the equation of motion for a ship in regular waves

$B_{b kp}$ Coefficient for determining the bilge keel roll damping component $B_{b kp}$ according to Ikeda et al

$B_c$ Beam of a centre hull [m]

$B_e$ Equivalent linear roll damping coefficient [Nm/(rad/s)]

$B_i$ Waterline beam of the side hull [m]

$B_{sec}$ Beam of ship section [m]

$B_A$ Appendage roll damping component [Nm/(rad/s)]

$B_{AD}$ Roll damping component due to the drag force created by an appendage [Nm/(rad/s)]

$B_{AL}$ Roll damping component due to the lift force created by an appendage [Nm/(rad/s)]

$B_{bk}$ The total bilge keel roll damping component [Nm/(rad/s)]

$B_{bkh}$ See $B_{b kp}$ [Nm/(rad/s)]

$B_{bkl}$ Roll damping component due to lift generated by a pair of bilge keels [Nm/(rad/s)]

$B_{bkn}$ Roll damping component due to the normal force from a pair of bilge keels [Nm/(rad/s)]

$B_{b kp}$ Roll damping component due to the pressure difference on the hull in front and behind the bilge keels [Nm/(rad/s)]

$B_{b kw}$ Roll damping component due to waves radiated from a pair of bilge keels [Nm/(rad/s)]

$B_E$ Eddy shedding roll damping component [Nm/(rad/s)]

$B_{EC}$ Eddy shedding roll damping component for the centre hull of a trimaran [Nm/(rad/s)]

$B_{es}$ Eddy shedding roll damping component for the side hull of a trimaran [Nm/(rad/s)]
Total hull skin friction roll damping component

Hull skin friction roll damping component at zero speed

Hull lift roll damping component

Roll damping component due to a skeg

Roll damping component due to the heave damping of the side hulls

Wave radiation roll damping component

Distance from the vertical centre of buoyancy to the metacentre

\( B_f \)
\( B_{f0} \)
\( B_L \)
\( B_{sk} \)
\( B_{SHH} \)
\( B_w \)
\( BM \)

\( C \)

\( (6 \times 6) \) matrix representing the stiffness terms for motion in six degrees of freedom

\( C \)

Constant in analysis of unforced spring-mass-damper system

\( C_0 \)

Coefficient for determining \( C_{bk} \) according to Tanaka

\( C_{44} \)

Roll stiffness

\( C_{33} \)

Heave stiffness

\( C_a \)

Coefficient depending on Reynolds Number for determining \( C_{bk} \) according to Tanaka, or, local hull roughness allowance for a ship possessing surface roughness

\( C_{bk} \)

Coefficient equivalent to the drag coefficient for a bilge keel

\( C_f \)

Local coefficient of skin friction acting on the hull

\( C_k \)

Coefficient for determining \( C_{bk} \) according to Tanaka

\( C_n \)

Normal force pressure coefficient for a rectangular plate moving with a uniform velocity in the direction perpendicular to its plane

\( \text{[Nm/(rad/s)]} \)

\( \text{[Nm/(rad/s)]} \)

\( \text{[Nm/(rad/s)]} \)

\( \text{[Nm/(rad/s)]} \)

\( \text{[Nm/(rad/s)]} \)

\( \text{[Nm/(rad/s)]} \)

\( \text{[m]} \)

\( \text{[Nm/rad]} \)

\( \text{[N/m]} \)
\( C_x \) Integral around the girth of a hull section
\( C_B \) Block coefficient
\( C_D \) Non-dimensional drag coefficient
\( C_{D0} \) Component of the non-dimensional drag coefficient
\( C_{DF} \) Non-dimensional frictional coefficient
\( C_{IKD} \) Coefficient devised by Ikeda et al for eddy damping calculations
\( C_L \) Lift coefficient
\( C_{La} \) The slope of the lift coefficient \( C_L \) plotted against \( \alpha_{ind} \) at an induced angle of zero degrees
\( C_M \) The inertia coefficient
\( C_p \) Pressure coefficient
\( C_{PF} \) Pressure coefficient for pressure in front of a skeg
\( C_{PR} \) Pressure coefficient for pressure behind a skeg
\( C_{TAN} \) Non-dimensional coefficient based on hull shape developed by Tanaka

\( E \) Energy dissipated by the equivalent linear damping term \( \text{[Nm]} \)
\( E_p \) Factor to modify the lift slope curve \( C_{La} \) to allow for planform effects
\( E_{BL} \) The ratio of the lift developed by the part of the fin in the boundary layer divided by the nominal lift of the fin when no boundary layer is present

\( F^W \) \((6 \times 1)\) vector of the wave forces and moments for motion in six degrees of freedom
\( F \) Force term in the equation of motion for a spring-mass-damper system \( \text{[N]} \)
\( \bar{F} \)  
Coefficient for determining \( C_{\delta k} \) according to Tanaka

\( F_0 \)  
Complex amplitude of force \( F \)

\( F_4 \)  
Roll moment on a ship due to waves in the equation of uncoupled roll motion in regular waves  
[Nm]

\( F_{40} \)  
Complex amplitude of roll forcing moment \( F_4 \)

\( F_n \)  
Froude Number

\( F_T \)  
Total Roll moment acting on a ship in the equation of uncoupled roll motion in regular waves  
[Nm]

\( \overline{GM} \)  
Distance between the centre of gravity and the metacentre  
[m]

\( \overline{GM}_a \)  
Variation in \( \overline{GM} \) due to removal of an inclining weight  
[m]

\( \overline{GM}_E \)  
Value of \( \overline{GM} \) for an equivalent monohull  
[m]

\( \overline{GM}_v \)  
Variation in \( \overline{GM} \) due to heave from haunches on the side hulls after the inclining weight is removed  
[m]

\( \overline{GZ} \)  
Hydrostatic righting lever arm  
[m]

\( H_0 \)  
Half breadth to draught ratio of a ship section

\( H_1 \)  
Coefficient depending on Lewis form coefficients of the ship section, used for the calculation of eddy damping by Ikeda et al

\( I_4 \)  
Roll inertia  
[kg m^2/ rad]

\( I_c' \)  
Roll inertia and virtual inertia of a catamaran calculated assuming the hulls both roll and heave  
[kg m^2/ rad]

\( I_{xx} \)  
Second moment of area of the waterplane about the x-axis  
[m^4]

\( I_{yy} \)  
Second moment of area of the waterplane about the y-axis  
[m^4]
\( I_{zz} \)  Second moment of area of the waterplane about the z-axis \([m^4]\)

\( K \)  Reduced frequency

\( K_{hc-1} \)  Coefficient for calculation of eddy damping component for a hard chine craft

\( K_{hc-2} \)  Coefficient for calculation of eddy damping component for a hard chine craft

\( K_{L} \)  Correction factor to be applied to lift slope \( C_{L,a} \) to account for free surface effects

\( \bar{KB} \)  Distance from the keel to the vertical centre of buoyancy \([m]\)

\( KC \)  The Keulegan-Carpenter Number

\( \bar{KG} \)  Distance from the keel to the vertical centre of gravity \([m]\)

\( \bar{KM} \)  Distance from the keel to the metacentre \([m]\)

\( L \)  Length between perpendiculars of the ship \([m]\)

\( L_{c} \)  Length between perpendiculars of the centre hull \([m]\)

\( L_{rep} \)  A representative length \([m]\)

\( L_{s} \)  Length between perpendiculars of the side hull \([m]\)

\( L_{sec} \)  Length of a ship section \([m]\)

\( L_{sk} \)  Span of a skeg from root to tip \([m]\)

\( \mathbf{M} \)  \((6 \times 6)\) matrix representing the mass and inertia terms for motion in six degrees of freedom

\( M \)  Ship weight \([N]\)

\( M_{i} \)  Coefficient depending on the hull section for eddy damping calculations using Ikeda et al.'s method

\( M_{R} \)  Roll damping moment or roll restoring moment \([Nm]\)

\( M_{w} \)  Roll moment generated by regular waves in beam seas \([Nm]\)
\( \overline{OG} \) Vertical distance from the origin (at the still water position) to the roll axis (centre of gravity) measured positive downwards [m]

\( P_m \) Pressure difference between the port and starboard sides of the hull section when the section rolls to port [N/m²]

\( Q(\nu) \) A non-dimensional energy loss function according to Roberts (90)

\( \text{Re} \) Real part of a number consisting of real and imaginary parts

\( R_n \) Reynolds Number

\( R_{fr} \) Rise of floor [radians]

\( S \) The maximum area cross-plane to the flow. For a bilge keel this is equal to \( l_{bk}b_{bk} \) [m²]

\( S_e \) Plan area of an appendage [m²]

\( S_{bk} \) Plan area of a bilge keel [m²]

\( S_L \) Representative area for lift calculations [m²]

\( S_w \) Hull wetted surface area [m²]

\( S_{WE} \) Wetted surface area of an element on the hull [m²]

\( S_{W\text{sec}} \) Wetted surface area of a hull section [m²]

\( S_{WC\text{sec}} \) Wetted surface area of a section of the centre hull of a trimaran [m²]

\( S_{WS\text{sec}} \) Wetted surface area of a section of the side hull of a trimaran [m²]

\( T \) Draught of ship [m]
\( T_1 \) Coefficient for eddy damping calculations
\( T_2 \) Coefficient for eddy damping calculations
\( T_4 \) Roll period [s]
\( T_{na} \) Natural roll period [s]
\( T_{sec} \) Draught of a ship section [m]

\( U \) Forward speed of the ship [m/s]
\( U_{max} \) The maximum speed in oscillatory motion [m/s]
\( U_p \) Fluid particle velocity [m/s]
\( U_{p-max} \) The maximum particle velocity. [m/s]
\( U_r \) Relative oscillating velocity of a cross-section of a SWATH hull [m/s]
\( \dot{U}_p \) Fluid particle acceleration [m/s^2]

\( V \) Energy loss term. Energy loss per unit roll inertia and added inertia [/s^2]

\( X \) (6 x 1) vector denoting the wave induced motions in six degrees of freedom
\( X_0 \) Complex amplitude of displacement

**Lower Case Letters**

\( a \) Mass term in equation of motion for a spring-mass-damper system [N/(m/s^2)]
\( a_1 \) Coefficient in exponentially decaying sine plus cosine fit to measured roll decay data, or, Lewis form parameter
\( a_2 \) Coefficient in exponentially decaying sine plus cosine fit to measured roll decay data
\( a_3 \) Lewis form parameter
\( a_{sk} \) Distance from the chine of a hull to the start of the negative pressure region on the back face of a skeg [m]

\( b \) Damper term in the equation of motion for a spring-mass-damper system [N/(m/s)]

\( b_1 \) Linear roll damping coefficient per unit inertia \( (I_4 + A_{44}) \) [/s]

\( b_2 \) Quadratic roll damping coefficient per unit inertia \( (I_4 + A_{44}) \) [/s]

\( b_3 \) Cubic roll damping coefficient per unit inertia \( (I_4 + A_{44}) \) [/s]

\( b_4 \) Coefficient associated with the sea surface velocity in the equation of uncoupled roll motion for a ship

\( b_{33} \) Heave damping per unit mass [/s]

\( b_{44} \) Roll damping per unit inertia \( (I_4 + A_{44}) \) [/s]

\( b_{bk} \) Breadth of bilge keel (equivalent to span) [m]

\( b_e \) Equivalent linear roll damping per unit inertia \( (I_4 + A_{44}) \) [/s]

\( b_h \) Heave damping coefficient

\( c \) Stiffness term in the equation of motion for a spring-mass-damper system [N/m]

\( ch \) Chord of an appendage [m]

\( c_4 \) Coefficient associated with the sea surface elevation in the equation of uncoupled roll motion for a ship

\( d_h \) Vertical distance from the horizontal axis to the position on a cross-section of a submerged SWATH hull where the beam is a maximum [m]
\( f(\lambda) \)  
Function of \( \lambda \) for determining \( C_{bh} \) according to Tanaka.

\( f_1 \)  
Factor associated with the number of positions on a hull section that eddy shedding occurs, its equal to one for single point eddy separation and zero for two point separation.

\( f_2 \)  
Correction factor for the pressure coefficient in the Ikeda et al calculation for eddy damping.

\( f_3 \)  
Correction factor for the velocity around the hull section in the Ikeda et al calculation for eddy damping.

\( f_4 \)  
Wave forcing term in the roll equation of motion per unit inertia and added inertia [m²].

\( f_{30} \)  
Steady heave force amplitude per unit mass.

\( f_{hbn} \)  
Correction factor to account for the flow speed increase at the bilge in the vicinity of the bilge keels determined from experiments.

\( f_{he-1}(\alpha_{hc}) \)  
Coefficient for calculation of eddy damping component for a hard chine craft.

\( f_{he-2}(\alpha_{hc}) \)  
Coefficient for calculation of eddy damping component for a hard chine craft.

\( f_D \)  
Drag force acting on a body per unit length: [N/m].

\( f_I \)  
Force per unit length acting on a body in the direction of fluid particle acceleration due to inertia forces [N/m].

\( h \)  
Depth of submergence of a hydrofoil measured from the still waterline to the camber line of the hydrofoil at the fin tip [m].

\( h_a(t) \)  
Time varying heave moment amplitude per unit roll inertia in forced roll experiment [s²].
\[ h_{ctw} \] Distance between the centre lines of the two hulls of a catamaran [m]

\[ h_{sec} \] Offset of a cross-section of the submerged hull of a SWATH from the x-axis [m]

\[ i \] The complex variable, \( \sqrt{-1} \).

\[ k \] Wave number

\[ k_1 \] Non-dimensional linear roll damping coefficient

\[ k_2 \] Non-dimensional quadratic roll damping coefficient

\[ k_a \] Coefficient in exponentially decaying sine plus cosine fit to measured roll decay data

\[ k_b \] Coefficient in exponentially decaying sine plus cosine fit to measured roll decay data

\[ k_{mb} \] Coefficient for determining \( C_{mb} \) according to Tanaka

\[ k_e \] Non-dimensional equivalent linear roll damping term

\[ k_x \] Roll radius of gyration [m]

\[ k_y \] Pitch radius of gyration [m]

\[ k_z \] Yaw radius of gyration [m]

\[ k_L \] Lift Slope coefficient based on trim angle used in the derivation of the hull lift coefficient, \( C_L \)

\[ k_N \] Lift Slope coefficient based on angle of attack \( \alpha_0 \) used in the derivation of the hull lift coefficient, \( C_L \)

\[ l \] Length of an element around the girth of the hull [m]
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_0$</td>
<td>Distance between the roll centre and the point on the hull surface where the flow incidence angle is equal to $\alpha_o$</td>
<td>[m]</td>
</tr>
<tr>
<td>$l_a$</td>
<td>Distance from the roll centre to the axis which the model is free to heel about in experimental setup to measure lift damping component, $B_L$</td>
<td>[m]</td>
</tr>
<tr>
<td>$l_{bk}$</td>
<td>Length of bilge keel</td>
<td>[m]</td>
</tr>
<tr>
<td>$l_{gbk}$</td>
<td>Distance from the root of the bilge keel to the waterline measured around the girth of the hull</td>
<td>[m]</td>
</tr>
<tr>
<td>$l_{hc-1}$</td>
<td>Lever arm for calculation of eddy damping component for a hard chine craft</td>
<td>[m]</td>
</tr>
<tr>
<td>$l_{hc-2}$</td>
<td>Lever arm for calculation of eddy damping component for a hard chine craft</td>
<td>[m]</td>
</tr>
<tr>
<td>$l_{hc-3}$</td>
<td>Lever arm for calculation of eddy damping component for a hard chine craft</td>
<td>[m]</td>
</tr>
<tr>
<td>$l_R$</td>
<td>Lever arm corresponding to the vertical distance from the roll axis to the apparent centre of the horizontal lifting pressure on the hull during roll motion</td>
<td>[m]</td>
</tr>
<tr>
<td>$l_{R-v}$</td>
<td>Lever arm corresponding to the horizontal distance from the roll axis to the apparent centre of the vertical lifting pressure on the hull during roll motion</td>
<td>[m]</td>
</tr>
<tr>
<td>$l_S$</td>
<td>Lever arm corresponding to the distance from the roll axis to the apparent centre of the lifting pressure on the hull in an oblique towing test</td>
<td>[m]</td>
</tr>
<tr>
<td>$l_{sk}$</td>
<td>Lever arm for calculation of skeg damping component</td>
<td>[m]</td>
</tr>
<tr>
<td>$l_{sk-1}$</td>
<td>Lever arm for calculation of skeg damping component</td>
<td>[m]</td>
</tr>
<tr>
<td>$l_{sk-2}$</td>
<td>Lever arm for calculation of skeg damping component</td>
<td>[m]</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Unit</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
<td>-------</td>
</tr>
<tr>
<td>$l_{sk-3}$</td>
<td>Lever arm for calculation of skeg damping component</td>
<td>[m]</td>
</tr>
<tr>
<td>$L_{rep}$</td>
<td>A representative length for calculation of the Keulegan-Carpenter Number</td>
<td>[m]</td>
</tr>
<tr>
<td>$m$</td>
<td>Mass of the ship</td>
<td>[kg]</td>
</tr>
<tr>
<td>$m_1$</td>
<td>Coefficient for determining the bilge keel roll damping component $B_{BKP}$ according to Ikeda et al</td>
<td></td>
</tr>
<tr>
<td>$m_2$</td>
<td>Coefficient for determining the bilge keel roll damping component $B_{BKP}$ according to Ikeda et al</td>
<td></td>
</tr>
<tr>
<td>$m_3$</td>
<td>Coefficient for determining the bilge keel roll damping component $B_{BKP}$ according to Ikeda et al</td>
<td></td>
</tr>
<tr>
<td>$m_4$</td>
<td>Coefficient for determining the bilge keel roll damping component $B_{BKP}$ according to Ikeda et al</td>
<td></td>
</tr>
<tr>
<td>$m_5$</td>
<td>Coefficient for determining the bilge keel roll damping component $B_{BKP}$ according to Ikeda et al</td>
<td></td>
</tr>
<tr>
<td>$m_6$</td>
<td>Coefficient for determining the bilge keel roll damping component $B_{BKP}$ according to Ikeda et al</td>
<td></td>
</tr>
<tr>
<td>$m_7$</td>
<td>Coefficient for determining the bilge keel roll damping component $B_{BKP}$ according to Ikeda et al</td>
<td></td>
</tr>
<tr>
<td>$m_8$</td>
<td>Coefficient for determining the bilge keel roll damping component $B_{BKP}$ according to Ikeda et al</td>
<td></td>
</tr>
<tr>
<td>$m_{a4}$</td>
<td>Moment amplitude per unit roll inertia in forced roll experiment</td>
<td>[s$^2$]</td>
</tr>
<tr>
<td>$p_0$</td>
<td>Coefficient for determining $C_{sk}$ according to Tanaka</td>
<td></td>
</tr>
<tr>
<td>$p_1$</td>
<td>Coefficient for determining $C_{sk}$ according to Tanaka</td>
<td></td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
<td></td>
</tr>
<tr>
<td>$q_{bk}$</td>
<td>Coefficient for determining $C_{bk}$ according to Tanaka</td>
<td></td>
</tr>
<tr>
<td>$\bar{r}$</td>
<td>The average radius of roll for calculation of skin friction damping determined by empirical formula [m]</td>
<td></td>
</tr>
<tr>
<td>$r_o$</td>
<td>Distance from the centre of pressure of an appendage to the roll centre of the ship [m]</td>
<td></td>
</tr>
<tr>
<td>$r_{app}$</td>
<td>Perpendicular distance from the centre of gravity of the ship (roll centre) to the line of action of the lift or drag force generated by an appendage [m]</td>
<td></td>
</tr>
<tr>
<td>$r_{bilge}$</td>
<td>Bilge radius (strictly the radius of the bilge circle linking the flat bottom of an idealised rectangular ship section to the vertical side) [m]</td>
<td></td>
</tr>
<tr>
<td>$r_{bk}$</td>
<td>Lever from the roll centre (centre of gravity) to the mid span of the bilge keel [m]</td>
<td></td>
</tr>
<tr>
<td>$r_{eb}$</td>
<td>Effective bilge radius for eddy damping calculations [m]</td>
<td></td>
</tr>
<tr>
<td>$r_{ed}$</td>
<td>Radius from the roll centre (centre of gravity) to the position where eddies are being generated [m]</td>
<td></td>
</tr>
<tr>
<td>$r_{ed-s}$</td>
<td>Radius from the roll centre (centre of gravity) to the position where eddies are being generated on the side hull of a trimaran [m]</td>
<td></td>
</tr>
<tr>
<td>$r_f$</td>
<td>Distance from element around the girth of the hull to the roll centre (centre of gravity) of the ship for calculation of hull friction damping [m]</td>
<td></td>
</tr>
<tr>
<td>$r_{hc}$</td>
<td>Lever from the point where eddies are shed to the roll centre (centre of gravity) [m]</td>
<td></td>
</tr>
<tr>
<td>$r_{max}$</td>
<td>Maximum distance from the roll axis to the hull surface expressed by an approximate formula [m]</td>
<td></td>
</tr>
</tbody>
</table>
\( r_{sep} \) The distance between the longitudinal plane of symmetry of the centre hull to the longitudinal plane of symmetry of the submerged part of the side hulls. I.e. the separation of one side hull from the centre hull [m]

\( s \) Span of a hydrofoil or fin [m]

\( s_{\text{average}} \) Average length of the pressure distribution behind the bilge keel [m]

\( s_{hc} \) Length of pressure distribution on one side of the hull of a hard chine craft [m]

\( s_{plate} \) Span of a representative flat plate [m]

\( s_{sk} \) Length of the negative pressure region behind a skeg [m]

\( s_{traps} \) Longest length of a trapezoidal pressure distribution acting on the hull in the vicinity of a bilge keel [m]

\( t_i \) Interval of time [s]

\( u \) Coefficient for eddy damping calculations

\( u_r \) Relative velocity on a submerged hull element for a rolling ship with forward speed [rad/s]

\( x \) Displacement [m]

\( \dot{x} \) Velocity [m/s]

\( \ddot{x} \) Acceleration [m/s²]

\( x_0 \) Amplitude of sinusoidal displacement \( x \) [m]

\( x_3 \) Heave displacement [m]

\( x_4 \) Roll angle [radians]

\( x_{4-01} \) The amplitude of the decaying linear solution to the uncoupled equation of roll motion. [radians]
<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{4,m}$</td>
<td>The mean roll angle (average of the two peaks of the roll decay curve spanning the period examined)</td>
<td>[radians]</td>
</tr>
<tr>
<td>$x_{4,p}$</td>
<td>The amplitude of a specific peak in the roll decay history</td>
<td>[radians]</td>
</tr>
<tr>
<td>$x_{4,p1}$</td>
<td>The amplitude of the first peak in the roll decay history</td>
<td>[radians]</td>
</tr>
<tr>
<td>$x_{30}$</td>
<td>Steady heave amplitude</td>
<td>[m]</td>
</tr>
<tr>
<td>$x_{40}$</td>
<td>Roll amplitude</td>
<td>[radians]</td>
</tr>
<tr>
<td>$\dot{x}_3$</td>
<td>Heave velocity</td>
<td>[m/s]</td>
</tr>
<tr>
<td>$\dot{x}_4$</td>
<td>Roll velocity</td>
<td>[rad/s]</td>
</tr>
<tr>
<td>$\ddot{x}_3$</td>
<td>Heave acceleration</td>
<td>[m/s^2]</td>
</tr>
<tr>
<td>$\ddot{x}_4$</td>
<td>Roll acceleration</td>
<td>[rad/s^2]</td>
</tr>
<tr>
<td>$x_{mid}$</td>
<td>The horizontal distance from a wave crest to the centre of gravity of a catamaran</td>
<td>[m]</td>
</tr>
<tr>
<td>$x_n$</td>
<td>Motion displacement at the $n^{th}$ peak of the free decay of a spring-mass-damper system</td>
<td>[m]</td>
</tr>
<tr>
<td>$x_{sec}$</td>
<td>The perpendicular distance along the x-axis from the origin to the centre of the cross-section of the submerged SWATH hull</td>
<td>[m]</td>
</tr>
<tr>
<td>$x_{FP}$</td>
<td>The distance from the fore perpendicular of a hull to the mid point of an appendage attached to that hull</td>
<td>[m]</td>
</tr>
<tr>
<td>$\dot{y}_h$</td>
<td>Horizontal fluid velocity on hulls of a SWATH induced by the incoming wave</td>
<td>[m/s]</td>
</tr>
<tr>
<td>$\dot{y}_{hp}$</td>
<td>Horizontal fluid velocity on port hull of a SWATH induced by the incoming wave</td>
<td>[m/s]</td>
</tr>
<tr>
<td>$\dot{y}_{hs}$</td>
<td>Horizontal fluid velocity on starboard hull of a SWATH induced by the incoming wave</td>
<td>[m/s]</td>
</tr>
<tr>
<td>$\dot{z}_{sec}$</td>
<td>Vertical velocity of a ship section relative to the local water surface</td>
<td>[m/s]</td>
</tr>
</tbody>
</table>
\[ \dot{z}_v \] Vertical fluid velocity on hulls of a SWATH induced by the incoming wave \[ \text{[m/s]} \]

\[ \dot{z}_{vp} \] Vertical fluid velocity on port hull of a SWATH induced by the incoming wave \[ \text{[m/s]} \]

\[ \dot{z}_{ss} \] Vertical fluid velocity on starboard hull of a SWATH induced by the incoming wave \[ \text{[m/s]} \]

**Names**

- **diff** Difference in the values of peaks between the recorded and fitted or fitted and simulated roll decrements
- **DFT** Discrete Fourier Transform
- **DNV** Det Norske Veritas AS, Norway
- **LCB** Longitudinal Centre of Buoyancy \[ \text{[m]} \]
- **LCF** Longitudinal Centre of Floatation \[ \text{[m]} \]
- **MARIN** Maritime Research Institute of the Netherlands
- **MoD** UK Ministry of Defence
- **NACA** National Advisory Committee for Aeronautics
- **peaks** Number of peaks used for rms error calculation
- **RAO_{Roll}** Motion Response Amplitude Operator, in this case for roll motion
- **rms** Root Mean Square Value
- **SWATH** Small Waterplane Area Twin Hull ship
- **TPC** Tonnes Per Centimetre Immersion \[ \text{[Tonnes]} \]
Dedication

To Junwu Zhang, the father of modern research into trimaran ships, who was taken away from this world too early.

Acknowledgements

I consider the evolution of my post graduate research into this thesis to be a particular personal achievement and were it not for the support, both direct and indirect, of very many people I do not think I would be writing this short note at all! Firstly I would like to thank the two Professors who supervised my research, Professor John van Griethuysen and Professor Simon Rusling. I would also like to thank the other staff in the department for their frequent support and stimulating discussions, in particular Professor David Andrews, David Fellows and Tim McDonald. There is often a time in every challenging task where one begins to loose sight of the light at the end of the tunnel. Sometimes you start to wonder whether you are in fact even in a tunnel at all, rather than at the bottom of a big hole. For pulling me out of that hole and motivating me to find the light at the end of the tunnel I must express particular thanks to Professor Steve Bishop of the Department of Mathematics at UCL. Finally, I would like to thank my work colleagues, friends and family who have all given me their complete support throughout.

Some of the work in this thesis is based on research conducted for the UK MoD Sea Systems Group by UCL and QinetiQ. This included two sets of model experiments undertaken at QinetiQ’s Haslar facilities. I would like to thank all the QinetiQ staff involved in this work, in particular Chris Richardsen and Bob Scrace. Finally I would like to thank both the MoD Sea Systems Group and QinetiQ for allowing me to include some of the results of this research in the thesis.
Preface

This short preface explains the story of the research contained in this thesis. The thesis has been written to take the reader through the work following the most logical path – the path that can only be identified once the work is complete. Thus this logical path does not show any of the inevitable mistakes, problems and blind alleys visited whilst completing the research. Of course, in reality, research is a stop-start process where we try, fail, re-try, fail, think, think some more, try again and in the end we succeed and publish the thesis.

The original aims of the research, determined at the outset, are captured in the points below:-

1. Determine the current state of the art of ship roll motion prediction.
2. Investigate the applicability of current roll prediction methods to trimarans.
3. Either develop new theory or adapt existing theory to predict trimaran rolling.
4. Identify and evaluate suitable appendage types and locations for trimaran roll damping appendages.
5. Develop theories to model the roll damping contribution of the proposed appendages and incorporate with the theory of point number 3.
6. Augment a suitable seakeeping prediction code with the newly developed theory and validate using model tests.

In reading the thesis that follows it will be observed that these aims were not all achieved. Once point 6 was reached it became apparent that the theory did not predict the roll behaviour observed in the recently completed model experiments, regardless of whether or not appendages were fitted. After much investigation the fundamental assumptions started to unravel and the direction of the thesis changed. The remainder of the research focused on understanding why the theory proposed by the author in point 3, for a trimaran without roll damping appendages fitted, did not yield accurate roll motion predictions. At this point, the author was
already three quarters of the way through one thesis and then had no choice but to re-direct the work towards a different thesis. The literature review, documented in Chapters 2 and 3 of this thesis, did not suggest that existing methods, used for monohull roll prediction, would be unsuitable for trimarans.

The original research method is described in the following bullet points:-

- Complete a literature review of monohull and multi-hull roll motion prediction;
- Adapt existing theory to predict the roll motion of a trimaran at zero forward speed and with forward speed;
- Incorporate within the theory the damping contribution of appendages that could be fitted to a trimaran;
- Validate the newly adapted theory using both roll decay experiments and seakeeping experiments in regular waves;

The first two and a half years of the research were spent performing theoretical studies and model experiments to assess the effectiveness of various roll damping devices. These studies are described in Chapter 4 of the thesis. Careful scrutiny of the results showed that the early assumptions about the existing theories applicability to predict rolling (adopted after an extensive literature review) could not be justified.

After this breakdown of the original research direction a revised avenue for the research was devised. After the literature review the following hypothesis was tested:-

*Accurate trimaran roll motion predictions can be obtained using linear Potential Flow Seakeeping theory with the roll damping term either obtained from a roll decay experiment or augmented with empirically based theoretical roll damping components developed for monohulls.*
The research was then re-cast to disprove this hypothesis which was implicit in nearly all previous researchers work on trimaran roll motion predictions. Having disproved the hypothesis, the thesis then goes on to explore why the hypothesis is incorrect using the model experiments that had already taken place (originally aimed at investigating the performance of roll damping devices on a trimaran). This work is described in Chapter 5 of the thesis.

To provide an end point to the work, Chapter 6 of the thesis proposes a series of experiments and theoretical assessments which, in the opinion of the author, will enable the fundamental physics, describing trimaran rolling, to be obtained. The conclusions of the thesis are then set out in Chapter 7.
Chapter (1) Trimaran Design

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1.6 Conclusions ................................................................................................... 88
1.1 Introduction

In this Chapter the trimaran hull configuration will be introduced, followed by a brief review of research and development on large trimaran displacement ships including ships built using the trimaran hull. The focus of the remainder of the Chapter is trimaran design. This is split down to show how the centre hull design is primarily about reducing hull wave making resistance, whereas the side hulls, which provide the majority of the stability, are designed to ensure the ship has sufficient intact and damaged stability, whilst at the same time minimising additional hull resistance. Finally, the impact of hull design decisions on roll motion is discussed. For a trimaran, the declutching of stability from resistance during design, which is not possible with a monohull, gives the designer greater freedom in tuning the roll performance of the ship. This is achieved through selection of appropriately sized hulls and the transverse location of these side hulls. Additionally, the trimaran configuration offers a greater range of positions to locate a roll damping appendage in comparison to a monohull. The final section discusses this issue and shows that many of the possible locations allow a larger roll damping moment to be generated in comparison with a monohull.

Chapters 2 and 3 continue the literature review focusing on roll motion prediction for monohulls and trimarans and highlight current best practice in predicting the roll response of monohulls and trimarans.
1.2 Trimarans – A Brief Review

A trimaran is a triple hulled vessel whose heritage can be traced back to early Indonesian outrigger canoes, with a slender central canoe consisting of a hollowed out tree trunk supported by two widely spaced even more slender side hulls. The concept of a trimaran displacement ship for moderately sized fast ships was introduced in the seminal paper by Pattison and Zhang (1) which outlined the early research undertaken in the Department of Mechanical Engineering at University College London (UCL). This research commenced after Nigel Irens, one of the leading designers of sailing trimarans, approached the department to discuss the powering for a half scale demonstrator of his idea for a fast ferry, which became the iLan Voyager (2).

![Figure 1-2-1: The Trimaran Configuration](image)

The concept of the Trimaran displacement ship is based on a ship with a centre hull with a high slenderness ratio (length divided by displaced volume to the power of one third) supported by two slender side hulls of small displacement designed to provide the necessary stability to keep the ship upright and pass the
various stability criteria. The slender centre hull has low wavemaking resistance, the dominant component of a ship's resistance at high speed, with the two well spaced side hulls easily restoring the stability sacrificed in the centre hull in order to reduce resistance. The three hulls are then joined together by a cross-structure, which may comprise one or more internal decks depending on the size of the vessel. Thus the configuration has considerable upper deck space and, if decks are included in the cross structure, a larger rectangular space near the centre of the vessel for internal arrangement. The arrangement of the trimaran form developed through the research is shown in Figure 1-2-1.

The first trimaran design at UCL, an Advanced Technology Frigate, was produced by students on the MSc Naval Architecture programme (3) and was presented to the 1992 RINA Affordable Warships Symposium (4). Pattison and Zhang point out in the conclusion of their paper (1) that one of the aims of this design was to discover why there are no trimaran ships, with the expectation of finding some basic flaw in the concept. However, this early work showed that not only were trimaran ships possible but they may show real economic advantages for some roles.

The development of the trimaran concept through studies at UCL captured the interest of the UK Ministry of Defence (MoD). This is reported by Summers and Eddison in the discussion following Pattison and Zhang's paper (1), where the then Forward Design Group of the UK MoD was investigating both monohull and trimaran derivatives for a future Anti-Submarine Warfare Frigate. The 5800 tonne, 160 metre trimaran derivative was presented by Summers and Eddison in 1995 (5) and this study confirmed many of the advantages identified by Pattison and Zhang. At the same time, the MoD funded UCL in conjunction with the then Defence Research and Evaluation Agency (DERA) at Haslar (now QinetiQ) to perform a more comprehensive and vigorous test of the concept. A programme of analytical work and ship model tests was undertaken to achieve a greater level of assurance as to the viability of the Trimaran Frigate concept. This work was split into six tasks, see Andrews and Hall (6), namely:-
1. Design of a Trimaran Displacement Ship. A trimaran hullform was developed based on a recent air defence destroyer designed at UCL.

2. Seakeeping Model Tests. Model experiments of the UCL hull form were conducted in the Ocean Basin at Haslar. The model was self propelled and instrumented to obtain responses for a series of wave headings and frequencies for a range of ship speeds.

3. Prediction of Trimaran Motion. UCL adapted an existing in-house seakeeping code to obtain trimaran motion predictions.

4. Comparison of Theoretical and Ship Model Data for Seakeeping.

5. Resistance Model Tests. The hull form was tested with two different pairs of side hulls placed at a number of different transverse positions in the Towing Tank at Haslar.

6. Comparison of Theoretical and Ship Model Test Data for Resistance.

This analysis procedure was identical to that followed when analysing the performance of a new monohull ship. These studies are reported further by Andrews and Zhang (7), Zhang and Andrews in (8) and Zhang and van Griethuysen (9), with a more comprehensive discussion in Zhang’s PhD on the Design and Hydrodynamic Performance of the Trimaran Displacement Ship (10).

During this period further predictions of trimaran motions were carried out by Chan, Incecik, Hall and Bate using a three dimensional linear Potential Flow Theory seakeeping code solving the equation of motion in the frequency domain (11); and a seaworthiness assessment using the same code by Chan, Incecik and Ireland (12). The first study computed motion predictions for the UCL designed hullform and for a similar hull form designed by shipbuilders Vosper Thornycroft (now VT Shipbuilding). The results for the UCL hull were compared with the Haslar ship model experiment results noted previously.

MoD funded research was also undertaken at the Ship Stability Research Centre at the University of Strathclyde to explore the stability and survivability of trimarans, both intact and with flooded compartments, by Vassalos, Helvacioglou and Jasionowski (13). Numerical results were compared to model experiments.
performed on a scale model of a 150 metre trimaran, different from both the UCL-DERA and Vosper Thornycroft hull forms.

During this period the MoD also funded DERA to undertake a programme of research to investigate the structural loading on a trimaran (14) (15) at their facilities in Rosyth. Much of this work has been used to produce the recent rules for the design of steel trimarans by Lloyds Register of Shipping (16). In parallel, signature and propulsion issues were also investigated along with survivability. Using this research the MoD sought to produce a validated trimaran design evaluation toolset which would reduce the likelihood of a trimaran solution being considered of greater risk than the erstwhile monohull one by contractors bidding for future MoD ship projects and thus allow trimaran options to be put forward by them.

As this work gathered pace it became clear that if the trimaran was to be considered for future surface warships then some form of proof of concept, or demonstrator would be required to prove the benefits at large scale. The solution
was to build an ocean going trimaran of sufficient size to allow subsequent trials results to be scaled up to a full size warship. At the end of a period of investigation the research ship RV Triton was designed and launched in 2000, Figure 1-2- 2. The evolution of RV Triton was reported in detail in the 2000 RINA Conference on RV Triton (17), (18), (19), (20), (21) and (22) with initial results from the trials reported in the 2004 RINA Conference on Trimaran Ships (23) and (24).

During this period of time other researchers and shipbuilders became interested in trimarans or trimaran derivatives. Those who had published details by the mid 1990's were Vosper Thornycroft, with a fast trimaran corvette, (25); Basin des Carines (26); and the Finnish Hydrodynamic Research Centre with a 40 knot containership (27).

![Figure 1-2- 3: A BMT Nigel Gee and Associates Pentamaran Ferry Design](image)

In 1995 Nigel Gee and Associates (now BMT Nigel Gee and Associates) developed the five hulled Pentamaran concept (28), see Figure 1-2- 3, which is essentially similar to a trimaran with a long slender centre hull supported by a pair of displacement side hulls located aft, but with large angle stability provided by a further pair of small sponsons located forward above the waterline. The aft side hulls are smaller than those of a trimaran as the forward sponsons provide large angle intact stability and a reserve of buoyancy to ensure sufficient stability in damaged conditions. Thus with less wetted area and smaller displacement side
hulls, even lower resistance at high speed is reported. The concept has been applied to fast ferries, both pure freight and passenger vehicle vessels (29), as well as containerships and warships (30).

In 2003 Austal Ships in Australia, one of the leading designers of high speed catamaran fast ferries, announced that they were to build a diesel powered 126 metre aluminium trimaran fast ferry for operation in the Canary Islands, Figure 1-2-4. The desire to carry larger payloads, without increasing the power requirement to such an extent that Gas Turbines were required, led Austal to start an extensive programme of research, tank testing and other analysis on a new design. Details of the design were reported by Armstrong at the 2004 RINA Conference on Trimaran Ships (31). American subsidiary Austal USA, teamed with the General Dynamics and Bath Iron works consortium, one of two contractors producing prototype designs for the American Littoral Combat Ship programme, are proposing a trimaran hull form based on the fast ferry design (31).

![Figure 1-2-4: The Austal built Trimaran ferry, Benchijigua Express](image)

Other dynamically assisted designs based on the trimaran concept have been considered. North West Bay Ships (32) proposed a trimaran with a pair of very
large active fins attached to the bottom of the centre hull extending to the underside of the two stepped side hulls. At the operational speed, the two side hulls are intended to be clear of the water. The first ship built to this design, the MV Triumphant, was launched in March 2001, just before RV Triton. MV Triumphant is shown in Figure 1-2-5.

Figure 1-2-5: North West Bay Ships Trimaran MV Triumphant

The DAT concept (Dynamically Assisted Trimaran) (33) was originally developed in Norway by TechMan AS and is currently promoted with US partner Island Engineering. The DAT operates as a trimaran at low speed, but rises onto foils fitted to all three of its hulls at higher speeds. The DAT has a long slender central hull with side hulls that are about half the length of the centre hull, with all three hulls having the transom at the same location. The centre hull is reported to have a bulbous bow to maximise hull length. Island Engineering have built a small scale demonstrator of the concept, see Figure 1-2-6.
A more novel trimaran ship, the BGV, has been proposed by Bureau d’etudes Gilles Vaton (www.bgv-international.com). The BGV is a trimaran with a long slender centre hull supported by small aft outriggers using Wing In Ground Effect (WIG) technology on the cross structure between the main and side hulls to provide additional lift. The largest design proposed so far is a 236 metre design intended to carry 176 trucks (6300 tonne deadweight) at speeds up to 35 knots, see Figure 1-2- 7. Smaller designs have been produced with a 155 metre, 1950 tonne car carrying variant operating at speeds of up to 45 knots.
In 2000 a three year programme of research commenced at the Universities of Genoa, Trieste, Napoli, and Ancona investigating the suitability of trimaran hullforms for high speed commercial vessels. Published studies include initial sizing for fast ferries using a multi-attribute procedure (34), trimaran hull design for fast ferry applications (35), the roll motion of trimarans in beam waves (36), (37), seakeeping assessment of trimaran hulls (38), and preliminary structural analysis (39). Other research has been conducted in China, concerning roll decay analysis of a trimaran model (40), and in Korea, on trimaran hydrodynamic design (41).
1.3 Centre Hull Design

As the stability of a trimaran comes in the main from the side hulls, the focus for the centre hull design is thus the reduction of wavemaking resistance and is achieved through a high length to beam ratio. Length to beam ratios of trimarans designed at UCL are generally in the range 14 to 16, (7) and (9). The wetted surface area of a trimaran will be typically about 30% greater than that of a monohull of equivalent displacement. As the advantages of the low wavemaking resistance will only be realised at medium to high speeds, the second consideration is the minimisation of the wetted surface area of the hull. This is especially important if the vessel is to spend a reasonable proportion of its time at low speed where frictional resistance dominates, as is the case for warships. Zhang and van Griethuysen (9) showed the results of studies where wetted areas had been calculated at constant slenderness and block coefficient (0.50) with varying beam to draught ratios. These studies showed that the favourable region for beam to draught ratio for minimising friction resistance was located between 2.0 and 2.5. This was re-enforced by an independent DERA (now QinetiQ) report that showed the minimum wetted surface area occurred at a beam to draught ratio of about 2.4 (42). This can be contrasted with stability considerations for a monohull which typically lead to beam to draught ratios of around 3 to 4.

In design terms, the high length to beam ratio is achieved by inserting spare volume into the hull to drive up the length. Whilst this would appear to be beneficial in giving increased space for internal arrangements, much of this extra volume is located in the slender forward part of the hull which is difficult to utilise practically. The increased slenderness afforded by this extra volume comes at a price: structural weight. Careful weight management is essential for any high speed ship designer. Increased weight means increased machinery and fuel if the desired speed and endurance are to be achieved, which leads to a bigger ship and this larger ship will require even more power and design spirals outwards.

In practice, reducing the beam of the centre hull is limited by the demands of the propulsion plant and these demands become more and more important the smaller
the ship. For a detailed discussion see Greig and Bucknall (43). Bricknell and Carlisle (44) and Skarda and Walker (45) discuss the issue in the context of larger naval trimarans. In the machinery space of a monohull with a twin engine installation, there is normally sufficient space to mount the engines side by side. In the slender trimaran centre hull this may not be possible, requiring the designer to increase the length of the machinery spaces. Typically the percentage of the hull length taken up by machinery is greater in a trimaran when compared with a monohull. Because of the extra volume inserted to achieve the slenderness, this is not usually a problem other than for small high speed trimarans. High speed trimarans are generally propelled by waterjets due to their increased efficiency over propellers at high speeds. Thus, the transom must have sufficient beam to fit in the waterjet outlets and the structure must be strong enough to withstand the thrust whilst accommodating the large holes. Grieg et al discuss these problems in the context of a high speed trimaran Corvette (46).

For small trimarans, Irens proposed a delta hullform that progressively increases the waterline beam right out to the transom (2). Skarda and Walker (45) mention this as a possible solution to the problem of insufficient centre hull beam for waterjets in trimaran frigates, accepting the resistance penalties this gives at lower speeds. They also note that this problem reduces as vessel size increases.

Brizzolara et al (35) compared a fast ferry trimaran with a deep V centre hull to a traditional round bilge hull with V sections forward and U sections aft. The round bilge design had a length to beam ratio of 11.83 compared to 10.86 for the deep V hull. For the deep V hull a wider beam and deeper draught were required to maintain the displacement due to the lower area in the V sections. Models of the two hulls were tested at speeds equivalent to 16.7 to 42.8 knots at full scale. In all configurations tested, the deep V design out performed the round bilge one, with the greatest difference recorded between the two design’s full scale effective power curves of 10%. This follows the trends observed for monohulls (47) and (48). Tests were also performed in head seas in regular waves to determine the added resistance and heave transfer function. The heave motion of the deep V hull was superior, due to the increased heave damping of the V-sections and the wider beam, as well as having generally the lowest added resistance in waves.
One must remember that, for small ships, the intent of a deep V hullform is to generate some dynamic lift thus reducing the amount of ship that has to be propelled through the water. This change in draught should be allowed for when the side hulls are designed to ensure there is sufficient stability.
1.4 Side Hull Design

The design of the side hulls is one of the most important aspects of trimaran design. The purpose of the side hulls is to provide, when at an angle of heel, a righting moment (displacement multiplied by the lever arm $G_{Z}$) that acts to return the ship to the upright condition. This is complicated by the transfer of buoyancy when one side hull comes out of the water. As the ship heels, this manifests itself as a reduction in the slope of the $G_{Z}$ curve until a sufficient amount of the opposite side hull and cross-structure are immersed to provide enough righting moment to recover the original slope of the $G_{Z}$ curve.

For a quasi-static ship, the righting lever ($G_{Z}$) is defined by the relationship:

$$G_{Z} = GM \sin \varphi$$

Where $G$ denotes the position of the centre of gravity and $\varphi$ is the heel angle in radians. $M$ is the metacentre and, for small angles of heel, this is considered fixed and determined from the geometry as shown in Figure 1-4- 1. $GM$ can be determined from the relationship:

$$GM = KB + BM - KG$$

Where K is the nominal keel position or the baseline of the centre hull, B the centre of buoyancy (this moves from B to B’ in the figure as the waterline moves from the blue solid line to the blue dashed line as the ship heels to starboard).
Figure 1-4-1: Quasi-static stability of a trimaran

The importance of the side hulls in providing the righting lever $GZ$ through $GM$ can be shown by considering $BM$. The distance $BM$ is related to the second moment of area of the waterplane, $I_{xx}$, and the ship displaced volume, $\nabla$:–

$$\overline{BM} = \frac{I_{xx}}{\nabla}$$  

For a multihull configuration the second moment of area of the waterplane is calculated about an axis $x-x$ running from forward to aft at a position mid way between the outermost hull on the port and starboard side. For a trimaran this is through the centreline of the waterplane of the centre hull. For a simple rectangular sectioned trimaran, Figure 1-4-2, $BM$ can be calculated from the second moment of area of a rectangular section and the parallel axis theorem:-

$$\overline{BM} = \frac{L_c B_c^3}{12} + 2 \left( \frac{L_s B_s^3}{12} + \left\{L_s B_s \right\} r_{sep}^2 \right)$$  

The subscripts $c$ and $s$ in equation 1-4-4 refer to the centre and side hulls respectively and all other symbols are defined in Figure 1-4-2. For a trimaran
with a displaced volume of 5500 m³ comprising a centre hull with a rectangular waterplane 150 metres by 10 metres and a pair of side hulls with rectangular waterplane 60 metres by 1.60 metres and a separation, $r_{sep}$, of 14.5 metres: $BM$ is 9.62 metres of which the centre hull contributes 24% and the side hulls 76%. Clearly the separation, $r_{sep}$, is the dominant term due to the parallel axis theorem.

For widely spaced side hulls around 80% of $BM$ is provided by the side hulls.

The global aim in trimaran design of reducing resistance leads the designer to minimise the wetted surface area of the side hulls. The dominant parameters for side hull wetted surface area are length and draught and reductions in these dimensions, whilst beneficial from a resistance point of view, also have the effect of reducing the displaced volume of the side hulls which is required for stability, especially when damaged.

The designer's aim so far is clear: Design the side hulls to provide enough $BM$ (generally dominated by the separation of the side hulls from the centre hull) to give adequate $GM$ and hence $GZ$ whilst minimising the wetted surface area by
reducing the length and draught of the side hulls. The chosen $GM$ should be the minimum acceptable to satisfy both the intact and damaged stability standards.

It is the stability standards that have the most profound effect on the size of the side hulls. Andrews and Zhang (49) discuss this in detail for different trimarans designed to the UK Ministry of Defence (MoD) stability standards and the then current (1996) International Maritime Organisation (IMO) proposals for amendment of the Code of Safety for Dynamically Supported Craft which later became the IMO High Speed Craft Code (50). They discussed the important influence the damage stability criteria have on determining the minimum length of the side hulls. If the damaged extent of the ship is calculated based on the relevant stability criteria and denoted the 'Damaged Length' and the total amount of water flooded into the compartments in the hull as the 'Flooded Length' (equal to or greater than the damaged length) then the length of the side hull can be derived from the flooded length. Andrews and Zhang (49) proposed that when a side hull is subjected to damage, about half its length is required to remain intact to provide sufficient waterplane area and buoyancy for the ship to meet the stability criteria. So, the length of the side hull can be initially selected as twice the expected floodable length. For the UK MoD stability requirements (51) this equates to a side hull length of 30% that of the centre hull (based on potential weapon damage) with a minimum value of 15% of the waterline length or 21 metres (based on collision requirements).

The shape of the $GZ$ curve is also important, most crucially when the heel angle is large enough for one side hull to come out of the water. A typical trimaran $GZ$ curve is shown below in Figure 1-4-3. For this trimaran, one side hull comes out of the water around 10 degrees, with the cross structure entering the water at 25 degrees. In this design, extra volume has been placed above the waterline in the form of flare on the inboard face of the side hull (haunches) to avoid an abrupt flattening out of the $GZ$ curve in this region. Without this extra buoyancy the area under the curve below 30 degrees may not be sufficient to pass the stability criteria. The Pentamaran solves this problem by having separate forward sponsons which sit above the still water line in the intact condition and which also
provide the necessary buoyancy to pass the damage stability criteria if the aft side hull is completely flooded. Thus the $GZ$ curve can be split into three distinct zones:

1. $GZ$ increasing linearly with heel angle whilst both side hulls remain in the water (typically 0 to 10 degrees).
2. One side hull comes out of the water, leading to a reduction in the gradient of the $GZ$ curve offset by the immersion of the haunch on the other side hull (typically 10 to 25 degrees).
3. Immersion of the cross-structure deck edge. $GZ$ then increases to a maximum at around 50 – 60 degrees and then decreases (or a point of down flooding is reached at which point the $GZ$ curve is truncated).

![GZ curve for a typical trimaran](image)

If a monohull and trimaran had identical $GM$ values, the loss of waterplane area when a side hull is damaged on the trimaran would result in a larger heel angle and wind heeling angle for the trimaran. Consequently, the value of $GM$ required to meet the stability criteria is generally greater for a trimaran ship than for an equivalent monohull ship. The larger value of $GM$ is provided by either
increasing the beam of the side hull, which will reduce the length of the side hull for constant displacement, or the side hulls are moved further outboard.

Stability analysis should be performed not only in the design condition but also in the light condition (or at least the worst seagoing condition) with sufficient side hull displacement and volume to pass the criteria in both conditions. Pattison and Zhang (1) reported that for their early studies the total displacement of both side hulls together should be around 10% of the total displacement.

In practice, the final chosen value of $\overline{GM}$ may be greater than that required to meet the stability standards because of the relationship between $\overline{GM}$ and roll period:

$$T_{n4} = 2\pi \sqrt{\frac{I_4 + A_{44}}{\rho g \sqrt{\overline{GM}}}}$$

Where $T_{n4}$ is the roll period in seconds; $I_4$ the roll inertia (kg m$^2$); $A_{44}$ the added inertia in roll (kg m$^2$); and $\rho$ the water density (kg/m$^3$). Short roll periods are undesirable as they give rise to high roll accelerations. Pattison and Zhang (1) remind us that the compromise between stability and rolling behaviour is of crucial importance in the design of monohull warships, as the stability criteria are easily met with enough $\overline{GM}$, and only the requirement for long roll periods prevents this. A trimaran design can be tuned to avoid too stiff a roll motion by modifying the location or proportions of the side hulls to achieve a desired $\overline{GM}$. This is discussed by Andrews and Zhang (49) and in more detail in Section 1.5.

The longitudinal location of the side hulls is determined by two factors: the desire to minimise the wave interference between the centre and side hulls; and the arrangement of the upper deck. The extra 'real estate' in the cross-structure should ideally be located where it can be put to the most effective use. The chosen position of the side hulls will also affect the seakeeping performance of the ship in quartering seas, leading to an inevitable compromise between seakeeping, resistance and arrangement.
With the length of the side hulls driven by the damage stability criteria and the beam by the required $GM$ for stability and roll period, then if the wetted area of the side hull is to be minimised the logical conclusion is to reduce the draught. However, if the draught is small, not only will the angle at which the hollow starts to appear in the $GZ$ curve be reduced (this occurred at 11 degrees in Figure 1-4-3) a phenomenon known as parametric resonance may occur.

Parametric resonance in a dynamic system describes motion that results from periodic variation of parameters describing the oscillating system, not motions excited by some external force or moment such as from the sea. In a dynamic system describing a ship, this is most likely to occur if there are cyclic variations in the stiffness (restoring) term. If the ship is pitching, heaving and rolling in a seaway the time varying waterplane will result in a time varying $GZ$ value. If this restoring moment varies sinusoidally about the still water value, and the equation of motion is reduced to uncoupled roll, then the resulting equation is of the form of the Mathieu Equation (see Jordan and Smith (52)). The Mathieu equation is known to exhibit regions of instability where, for small excitation, large roll angles will occur so long as the amplitude of the variation is large enough and the period is appropriate. The restoring moment does not need to be negative for this motion to occur.

Parametric rolling is most likely to occur in monohulls in head or stern seas, which do not normally excite roll motion, for wave lengths roughly equal to the ship length. With the wave crest at amidships, relative to the still waterline, the waterplane area at amidships is similar even when the draught is much greater so long as the midship section coefficient is large (ratio of the midship section area to a rectangle equal in area to the beam multiplied by the draught, both at the design waterline). However if the instantaneous draught at the bow and stern is very small, the variation in the waterplane is large due to the V shaped sections forward and the wide transom and rapid tapering to the shaft housing aft. Consider a merchant ship advancing in head or stern seas, Figure 1-4-4, first with the wave crest at midships (red line) then with a wave crest at each end (green
line). It is clear that the variation in waterplane compared to the still water value (blue line) is greatest when the wave crest is at midships. This variation in waterplane will cause a variation in the restoring moment, due to changes in $BM$ and thus $GZ$ and these will be cyclic and sinusoidal in regular waves.

For a trimaran the condition most likely to initiate parametric rolling is when the wave trough is amidships, or located at the centre of the side hulls. This condition is shown in Figure 1-4-5 which shows that if the side hulls partially emerge from the water (side hull with the solid line in the figure) then the variation in waterplane area in the midships part of the hull will be large, and, if enough of the side hull emerges, the restoring moment could easily become negative. However, if the side hull draught is great enough (side hull with the dotted line in the figure) then this problem is removed.
The theory behind parametric roll motion has been discussed by many authors over the years and a recent overview is given at the beginning of a paper by Ribeiro e Silva, Santos and Guedes Soares (53) as well as by Hashimoto and Umeda (54). Both report references dating back to the 1950's. Until recently the subject has mainly been studied theoretically; in regular (55) (56) and (57) and irregular waves (58); with limited experimentation, (53) (54) (59) (60) (61) (62) (63).

The recent spate of papers and model experiments have been fuelled by a well publicised incident involving a laden post-Panamax containership in 1998, (62) (63). The vessel, the APL China, was overtaken by a storm in the North Pacific Ocean encountering significant wave heights up to 13.4 metres. During this storm the master of the vessel held her bow onto the waves as best he could and roll motions as great as 35 to 40 degrees were reported. The two references listed above report theoretical and model investigations into the incident and conclude that parametric rolling was possible and was the most likely cause of this incident. From the theory, validated by model tests, they state that parametric rolling occurs when the following requirements are satisfied:

- The natural period of roll is approximately twice the wave encounter period
- The wavelength is of the order of the ship length (between 0.8 and 2 times the ship length)
- The wave height exceeds a critical level
- The roll damping is low

From the preceding discussion it can be seen that the design of the side hulls of a trimaran is complex and cannot be finalised until a detailed stability assessment has been carried out. Unlike the monohull ship, the side hull parameters of a trimaran ship cannot be determined along with the centre hull parameters during the initial sizing process. Zhang and van Griethuysen (9) propose a design
procedure for the side hulls split into two stages, building on the work of Andrews and Zhang (49):-

1) The initial sizing of the side hull together with the detailed centre hull design
2) Detailed design of the side hull shape.

The process is best illustrated by a flow chart, Part (1) is given in Figure 1-4-6 and Part (2) in Figure 1-4-7.
SIDE HULL INITIAL DESIGN

DESired GM
• Greater than for a monohull

SIDE HULL LENGTH
• Select as a proportion of centre hull length
• Related to flooded length of a damaged side hull according to the chosen stability criteria

SIDE HULL BEAM AND TRANSVERSE SEPARATION
• To achieve desired GM

LOCATE CROSS STRUCTURE
• Governed by layout considerations

Location acceptable for layout?

NO  YES

START SECOND STAGE

Figure 1-4-6: Part (1) of side hull design process as proposed by Zhang and van Griethuysen (9)
SIDE HULL DETAILED DESIGN

STABILITY ANALYSIS
• Identify worst damaged cases for the ship

SIDE HULL LENGTH
• Adjust length of side hulls

SIDE HULL SUBDIVISION
• To pass the stability criteria

SIDES HULL SHAPE AND VOLUME
• Refine side hull shape to pass the stability criteria and obtain desired GM
• Focus on getting an acceptable shape for the GZ curve
• Take account changes will have on build cost – especially fine side hull shapes

Required GM achieved?

NO

YES

REFINEMENT OF SIDE HULL LONGITUDINAL POSITION
• Small adjustment of position to minimise resistance
• Minimise effect on stability and layout

FINISH

Figure 1-4-7: Part (2) of side hull design process as proposed by Zhang and van Griethuysen (9)
1.5 Design for Roll Motion

There are two major tasks to be accomplished to ensure roll motions of a trimaran are minimised and attention needs to be focused on both of these when sizing the ship. Firstly, $\overline{GM}$ must be selected not only to pass the stability criteria but also to ensure both synchronous roll motion and stiff roll motions are avoided. Secondly, effective roll damping appendages must be sized and located to reduce roll motions.

1.5.1 Selection of a Suitable GM Value

The roll period of any ship is related to the restoring moment, which for a ship heeled over to a small angle is equal to $\overline{GM}$ as shown in equation 1-4-5 in Section 1.4. The minimum value of $\overline{GM}$ required to meet the stability criteria will be greater for a trimaran than for an equivalent monohull. Andrews and Zhang (49) theorised that for a trimaran, $\overline{GM}$ should be selected to make it retain the same value as an equivalent monohull once it had lost half the waterplane area of one of its side hulls (assuming that the side hull length is set at twice the flooded length after damage). They stated that half the length of the side hull would generally provide around 20% of the total waterplane inertia thus allowing a relationship between the metacentric heights of a trimaran and an equivalent monohull to be formed. Assuming that the trimaran and monohull have equal distances between the centres of buoyancy and gravity and if that distance is approximately half $\overline{KM}$ then they proposed:-

$$\overline{GM} = 1.70\overline{GM}_E$$

Where the subscript $E$ refers to the equivalent monohull. The discussion on side hull design in Section 1.4 has shown there is scope to vary the design value of $\overline{GM}$ above the minimum suggested in equation 1-5-1 by careful transverse placement of the side hulls in order to tune the roll motion away from frequencies
close to resonance. These changes can be made without significant effect on the resistance. This will reduce the likelihood of encountering synchronous roll motion when the excitation period is equal to the natural roll period of the ship. The monohull designer does not have this luxury and increases in $GM$ are provided by increases in beam which will increase resistance.

Andrews and Zhang (49) proposed a procedure to choose $GM$ based on a tuning factor relating the ship's natural roll period to the wave encounter period (i.e. the wave excitation period the ship sees relative to its speed through the water and the direction of the incoming or outgoing wave train). The tuning factor can be plotted against $GM$ for a range of wave directions at a particular speed. The designer can then use these plots with a range of pre-determined critical speed, heading and sea states (based on the operational profile of this ship and likely areas of operation) to ensure that synchronous roll motion does not occur.

Trimarans designed at UCL have tended to have $GM$ values in the range 1.5 to 3.5 metres. For seagoing trimarans this leads to synchronous roll motion between stern quartering and beam seas over the range of likely operational speeds.

Where possible, the trimaran designer has to select $GM$ not only to ensure that synchronous rolling is unlikely but must also avoid too 'stiff' a roll motion which occurs when $GM$ is large, leading to short roll periods and high roll velocities, typical of Catamaran ships.

The shape of the restoring lever arm curve ($GZ$ curve) is also important as outlined in Section 1.4. A jerky roll motion will arise if there is too great a discontinuity in the $GZ$ curve when one side hull comes out of the water (typically at heel angles between 5 and 15 degrees).
1.5.2 Design of Roll Damping Appendages

The configuration of a trimaran ship, with side hulls separated from the centre hull leaving a protected space in-between, offers a much wider range of locations where a roll damping appendage can be placed compared with a conventional monohull. On a monohull, the only practical location is at the turn of bilge. When an appendage is located here it is limited so that the tip does not extend through a vertical line downwards from the edge of the extreme waterline beam if damage is to be avoided when coming alongside, otherwise it must be retracted into the hull which requires some internal space and usually hydraulic power. This advantage of a trimaran should be maximised to ensure comfortable motions.

Zhang and Andrews (8) showed that for a trimaran, appendages could provide the greatest contribution to the total roll damping, due either to the lift they develop at speed or due to drag and eddy shedding during oscillation for appendages such as bilge keels.

The contribution of an appendage to roll damping is discussed in detail in Chapter 2 for monohulls and Chapter 3 for trimarans. The roll damping generated due to the lift force of a fixed appendage, \( B_{AL} \), is proportional to the ship's forward speed, \( U \), the plan area of the appendage, \( S_a \), the lift slope, \( C_{La} \), and the distance from the centre of pressure of the appendage to the roll centre, \( r_a \), squared. This is summed across the total number of appendages:

\[
B_{AL} \propto U \sum C_{La} S_a r_a^2
\]

The lift slope, \( C_{La} \), increases with aspect ratio although there are diminishing returns for large aspect ratios. It is obvious from equation 1-5-2 that large, high aspect ratio appendages located far away from the roll centre will be very effective. For a trimaran this is best achieved by locating them on or near to the side hulls.
The shape and location of the side hulls will place constraints on the size and location of appendages. For this reason it is important to design the roll damping appendages during the detailed side hull design stage outlined in Section 1.4. Both the appendage and side hull design should be revisited after initial seakeeping predictions to maximise the roll damping.
1.6 Conclusions

In this chapter the trimaran configuration has been introduced and it has been shown that research into large powered trimarans has been ongoing since 1994 (1). Since this time a number designs have evolved from the basic trimaran concept and ships have been built using the trimaran hull configuration or derivatives of it.

The review of trimaran design in this chapter has shown that many of the parameters that affect the roll motion of a trimaran can be varied at the early design stage, for example the roll period and radius of gyration depend upon the transverse location of the side hulls. Thus, a fundamental philosophy on how to deal with rolling is required at the design stage to avoid designing in unkindly motion. Because of this, the designer has the ability to tune the roll response of the ship by careful design of the side hulls and variation of the mass distribution in the ship. Such changes have to be balanced against other requirements placed on the ship affecting the layout and the likely desire to obtain high speeds (which determine the hull shape) as well as the need to provide adequate stability (which determines the minimum hull separation).

Furthermore, the importance of roll damping appendages for any ship has been reiterated and, in particular, that for a trimaran, larger more effective appendages can be fitted when compared to a monohull, and these are most likely to be located on, or near to, the side hulls. Such appendages are likely to be much more effective than those traditionally fitted to monohulls as they can be located much further away from the roll centre and can often have much larger plan areas and aspect ratios. Due to this wide range of configurations and locations appendage design should be considered much earlier in the design process so that performance can be maximised.

Having shown that roll motion is important and that the early trimaran analysis procedure essentially followed the approach used for monohulls Chapter 2 will focus on monohull roll motion prediction with Chapter 3 focusing on the roll
motion of multihulls and trimarans in particular. In Chapters 4 and 5 the roll motion of a chosen trimaran design is investigated using the methods identified in Chapters 2 and 3.
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2.1 Introduction

In Chapter 1 trimaran design was reviewed. In Chapters 2 and 3 ship roll motion prediction will be reviewed, first for monohulls and then for multihulls.

When embarking upon analysis of a new ship type it is sensible to follow a similar investigative process to that used for a conventional monohull. The distinct advantage of this is that, at each step in the process, results can be compared to those for a monohull, for which there will be a wide variety of existing information. Areas where there are differences between the new ship type and a monohull will be highlighted.

Whilst reporting on the progress of the MoD research programme on trimarans in 1995, Andrews and Hall (6) noted that “The current message in the exploration of this configuration is that it should be seen not as another hybrid advanced naval vehicle or even a typical monohull but rather a variant of the conventional monohull”. For this reason the early MoD studies into trimaran warships followed traditional monohull analysis processes across a wide range of technical facets, for example resistance predictions and model tests; seakeeping predictions and model tests; structural design and evaluation; and internal layout.

If, as these early studies indicate, a trimaran can be analysed using techniques appropriate for a monohull, then the start of the literature review of roll prediction methods should focus on current best practice for monohulls. Therefore, in this chapter, monohull roll analysis methods are reviewed. Chapter 3 contains a review of catamaran and embryonic trimaran roll motion prediction methods and highlights areas where a different approach has been used when compared to the established monohull best practice.

Essentially, the work in this thesis is starting from a hypothesis that trimaran roll motion can be investigated using the same investigative procedure as is traditionally used for monohulls.
2.2 Linear Seakeeping Theory

The motion of a real ship in an actual seaway is a highly complex problem to assess mathematically, in part due to the somewhat random nature of the sea surface. To solve the problem, engineers and scientists have generally modelled the sea as a superposition of many harmonic waves, which, when the wave amplitude and frequency are constant, have become known as regular waves. In Linear Seakeeping Theory, the problem is further simplified by applying the assumption that the response of the ship to a regular incoming wave propagating towards the ship on a constant bearing is also harmonic and regular. Motions excited in each of the six degrees of freedom by the incoming wave are also assumed to have constant amplitude and frequency.

To allow the linear theory to more properly model an actual seaway, the response of the ship at a number of wave directions either side of the dominant direction are modelled and, by using a spreading function, a seaway can be simulated which includes some waves with incident directions either side of the predominant direction.

The response of a ship to this wave environment can be modelled by considering the ship to be a linear spring-mass-damper system. The mass term is the mass or inertia of the ship, the spring term is the buoyancy or righting moment required to restore the ship to the equilibrium position and friction and turbulence contribute to the damper. Over a given period of time the forcing terms (forces for surge, sway and heave motion and moments for roll, pitch and yaw) will be equal to the sum of the mass or inertia terms multiplied by acceleration, the damper terms multiplied by velocity and the spring terms multiplied by the displacement (in each degree of freedom). Hence to determine the motions of the ship, six coupled linear second order differential equations have to be solved.

The mathematics of linear wave theory will be discussed in the next two sections.
2.2.1 Overview, Ship Motions in Six Degrees of Freedom

In linear seakeeping theory the motion of the ship is assumed to be adequately modelled by a dynamic forced spring-mass-damper system. This classic second order dynamic system is shown below in Figure 2-2-1.

![Classical Spring-Mass-Damper dynamic system](image)

If a time varying force is applied to this system at the free end, the mass, damper and spring each absorb part of the force so that at any particular instant of time the total force is the sum of the mass force, the damping force and the spring force. Making the assumptions that:-

- The spring has no mass and obeys Hooke’s law (spring force directly proportional to displacement)
- The damper has no mass or stiffness and the damping force from the damper varies in direct proportion to the velocity
- The mass contributes only inertia to the system, so the mass force is directly proportional to acceleration

Then, if the mass is \( m \) N/(m/s²), the damper \( b \) N/(m/s) and the spring stiffness \( c \) (N/m), the force, \( F \), to accelerate the mass at 1 m/s² to the right, whilst extending the spring 1 metre, with the damper providing damping at a rate of 1 m/s is described by the equation:-
The dot signifies derivatives with respect to time. This system is Second Order as the highest derivative is the second derivative of $x$, $\ddot{x}$; and linear because each term in the equation varies in proportion to the appropriate derivative of $x$. Real dynamic systems may not be linear, or linear all the time, however non-linear behaviour complicates the overall behaviour of the system and the mathematical treatment of it. It is thus attractive to treat a system as linear where this can be reasonably justified.

This equation can be used to model the motion of a rigid ship, the three horizontal or vertical motions, surge, sway and heave; and the rotational motions, roll, pitch and yaw. Denoting an axis system located at the centre of gravity of a ship when stationary (with zero velocity or acceleration) in still water as the equilibrium axis, these motions, $x_i$, are defined in Figure 2-2- 2 along with the six subscripts, $i$, referring to the six degrees of freedom of the unmoored ship.

Based on centre of gravity in still water (equilibrium position)

<table>
<thead>
<tr>
<th>Subscripts</th>
<th></th>
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<tbody>
<tr>
<td><strong>Translations:</strong></td>
<td></td>
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<tr>
<td>SURGE:</td>
<td>1</td>
</tr>
<tr>
<td>SWAY:</td>
<td>2</td>
</tr>
<tr>
<td>HEAVE:</td>
<td>3</td>
</tr>
<tr>
<td><strong>Rotations:</strong></td>
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<td>Roll:</td>
<td>4</td>
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<tr>
<td>Pitch:</td>
<td>5</td>
</tr>
<tr>
<td>Yaw:</td>
<td>6</td>
</tr>
</tbody>
</table>

**Figure 2-2- 2: Axis convention for a freely floating ship**

If the steady state motion $x$ is described in complex form as:-
\[ x = x_0 e^{i(\omega t + \phi)} \] 2-2-2

Where \( \phi \) is the phase angle between the force and the motion and:

\[ e^{i(\omega t)} = \cos(\omega t) + i \sin(\omega t) \] 2-2-3

For a ship this equates to a regular wave of constant amplitude and frequency. This can be simplified by taking \( X_0 = x_0 e^{i\phi} \) and then equation 2-2-2 becomes:

\[ x = X_0 e^{i(\omega t)} \] 2-2-4

Similarly the force can be described by:

\[ F = F_0 e^{i(\omega t)} \] 2-2-5

Thus, an input sinusoidal wave of fixed amplitude and frequency leads to a sinusoidal response also with fixed amplitude and frequency. Differentiating equation 2-2-4 with respect to time \( t \) and then substituting into 2-2-1 along with equation 2-2-5 and cancelling the \( e^{i(\omega t)} \) terms yields:

\[ \frac{X_0}{F_0} = \frac{1}{(c - a\omega^2) + i\omega b} \] 2-2-6

And:

\[ \phi = \tan^{-1} \left( \frac{b\omega}{c - a\omega^2} \right) \] 2-2-7

If there was no damping, the motion would become infinite at the undamped natural frequency, \( \omega_n \):

\[ \omega_n = \sqrt{\frac{c}{a}} \] 2-2-8
Now, if the non-dimensional frequency tuning factor is defined as:

\[ \frac{\omega}{\omega_n} = \Lambda \]  \hspace{1cm} 2-2-9

And the non-dimensional damping factor as:

\[ \frac{b}{\sqrt{ca}} = 2\zeta \]  \hspace{1cm} 2-2-10

Then equation 2-2-6 can be rewritten as:

\[ \frac{X_0}{F_0} = \frac{1}{c(l - \Lambda^2) + ic(2\Lambda\zeta)} \]  \hspace{1cm} 2-2-11

At zero frequency, \( \Lambda = 0 \) and the response reduces to \( 1/c \). When \( \Lambda = 1 \) the inertia forces exactly balance the stiffness forces and the response reduces to \( 1/i2c\zeta \). For finite damping, the maximum response occurs at the damped natural frequency, \( \omega_0 \), defined by:

\[ \omega_0 = \omega_n\sqrt{1 - 2\zeta^2} \]  \hspace{1cm} 2-2-12

The effect of changing \( \zeta \) from 0.05 (red line) to 0.1 (black line) is shown in Figure 2-2-3, where the modulus of \( X_0/F_0 \), \( |X_0/F_0| \), is plotted against the frequency ratio \( \Lambda \). The mass and stiffness are set to 1 (\( a \) and \( c \)).
Increasing the mass leads to a reduction in the non-dimensional damping factor, equation 2-2-10. Figure 2-2-4 shows the effect of doubling the mass, with stiffness and damping remaining at 1 (c and b). In this example zeta, $\zeta$, changes from 0.5 to 0.3536.
Figure 2-2- 4: Variation of the motion response with mass, red line mass = 1 Kg, black line mass = 2 Kg

Figure 2-2- 5: Variation of the motion response with stiffness, red line c = 1.0, black line c = 0.7 and green line c = 1.3 (b = 0.5 for all three cases)
To highlight the effect of changes in stiffness, $c$, first $b$ is set to 0.5 and then $c$ is varied from 0.7 (black line) through 1.0 (red line) to 1.3 (green line) in Figure 2-2-5. In the figure it can be seen that reducing the stiffness increases the response. However, changing the stiffness modifies the phase difference between the input and the response. Thus, the resonant peak moves to a higher frequency when the stiffness is increased and then to a lower frequency if the stiffness is reduced.

Examination of the free decay of a second order linear dynamic system allows the non-dimensional damping factor, equation 2-2-10, to be determined. So the equation of motion now becomes:

$$a\ddot{x} + b\dot{x} + cx = 0 \quad 2-2-13$$

The corresponding characteristic equation is:

$$\lambda^2 + \frac{b}{a}\lambda + \frac{c}{a} = 0 \quad 2-2-14$$

The roots of this equation are:

$$\lambda_{1,2} = \frac{-b}{2a} \pm \frac{1}{2a} \sqrt{b^2 - 4ac} \quad 2-2-15$$

If $\alpha = \frac{-b}{2a}$ and $\beta = \frac{1}{2a} \sqrt{b^2 - 4ac}$ the two solutions are now: $-\alpha + \beta$ and $-\alpha - \beta$.

When the motion is under damped, $b^2 < 4ac$ and the two roots are complex conjugate so that:

$$\beta = i\omega_d \quad 2-2-16$$

The frequency $\omega_d$ is the frequency of free, unforced decaying motion. So:
\[ \omega_d = \frac{1}{2a} \sqrt{4ac - b^2} = \sqrt{\frac{c - b^2}{a}} \]

\[ \therefore \omega_d = \omega_n \sqrt{1 - \zeta^2} \]

It is important to note the difference between equations 2-2-17, 2-2-12 and 2-2-8; under forced motion the system vibrates at \( \omega_0 \), which is less than the unforced frequency \( \omega_d \), both of which are less than the undamped natural frequency by a margin dependent on the non-dimensional damping factor \( \zeta \).

The roots of the equation of motion are now \(-\alpha + \imath \beta\) and \(-\alpha - \imath \beta\) and the general solution is:

\[ x = e^{-\alpha t} \left( A \cos \omega_d t + B \sin \omega_d t \right) \]

Which can be simplified to:

\[ x = Ce^{-\alpha t} \cos(\omega_d t - \delta) \]

Where:

\[ A^2 + B^2 = C^2 \quad \text{and} \quad \delta = \tan^{-1}\left( \frac{B}{A} \right) \]

Successive peaks in the decay will be bounded by \( Ce^{-\alpha t} \) and troughs by \(-Ce^{-\alpha t}\) and will occur when \( \cos(\omega_d t - \delta) \) is integer multiples of \( \pi \). So every \( n \)th peak will occur when:

\[ x = C \exp\left( - \frac{bn\pi}{2a\omega_d} \right) \]

So, between the \( n \) and \((n+1)\) positive peaks (over one period):-
If $a > d$, this becomes:

$$
\frac{x_n}{x_{n+1}} = \frac{\exp\left(-\frac{bn\pi}{2a\omega_d}\right)}{\exp\left(-\frac{b(n+1)\pi}{2a\omega_d}\right)} = \exp\left(-\frac{b\pi}{2a\omega_d}\right)
$$

$$
\therefore \quad \log_e\left(\frac{x_n}{x_{n+1}}\right) = \left(\frac{b\pi}{2a\omega_d}\right)
$$

$$
\Rightarrow \quad \frac{b}{a} = \frac{2\omega_d}{\pi} \log_e\left(\frac{x_n}{x_{n+1}}\right)
$$

If $\omega_d \approx \omega_n$ this becomes:

$$
\zeta = \frac{1}{\pi} \log_e\left(\frac{x_n}{x_{n+1}}\right)
$$

Using the approach outlined in this section, six equations of motion can be developed accounting for ship motions in surge, sway, heave, roll, pitch and yaw as shown in Figure 2-2-2. To some extent, ship motions in one degree of freedom will be influenced by motions in the other five degrees of freedom, therefore the six equations are coupled together. For a symmetric ship undergoing small amplitude motion the situation can be simplified if the vertical plane motions are not assumed to interact with the lateral plane or "horizontal" motions. In this case sway, roll and yaw can be uncoupled from heave and pitch, and surge can be uncoupled from all the other motions, see Chapter 8 of Lloyd (64).

The first efforts at obtaining the hydrodynamic coefficients of a coupled linear six degree of freedom equation of motion split the ship hull into a number of two dimensional strips (where each strip had a constant cross sectional area). The hydrodynamic coefficients in the equation of motion were then derived analytically using conformal mapping to transform semi-circles to cross sections resembling the actual sections through the ship (known as Lewis Transforms). The conversion of ship sections to circles simplified the mathematics to an extent.
that allowed solution of the coupled system for the first time. This approach is
described in detail by Lloyd (64).

These early strip methods assumed that:-

- The ship is slender, where the length is much greater than either the beam
  or draught
- The hull is rigid so that the ship structure does not flex
- Speed is moderate so that there is no appreciable trim or planing lift
- The ship motions are small
- The ship hull sections are wall sided above the waterline (i.e. only the
  portion of the hull below the waterline is considered in the calculations)
- The water depth is very much greater than the wave length so that deep
  water wave approximations may be applied
- The presence of the hull has no effect on the waves
- The flow is assumed to be irrotational and incompressible and viscosity is
  neglected

This final assumption is important. It follows that the boundary layer acting on
the hull is neglected. If this assumption is to be revoked Reynolds Averaged
Navier Stokes Equation (RANSE) must be used to model the fluid flow.
Modelling the fluid using these equations is thus attractive but significantly
increases the mathematical complexity of the problem. Reynolds Averaged
Navier Stokes Equations form the basis of the methods used in all modern
Computational Fluid Dynamics (CFD) codes. Roll prediction using CFD codes is
still embryonic at the current time.

Today, practical linear seakeeping calculations are performed almost exclusively
using Potential Flow Theory, see Newman (65). Here, if the fluid is modelled
using the velocity potential, the problem is reduced to the solution of just one
linear second order differential equation. Potential Flow Theories can be applied
to any floating body, revoking the slender body assumption of the early strip
methods. Additionally, Potential Flow Theories can account for more complex hull shapes as they split the hull into a large number of small panels.

Methods using Potential Flow Theory in practical use include two dimensional methods, requiring solution of the Green Function over a number of two dimensional panels forming strips representing the submerged hull, see Bertram (66); three dimensional methods, requiring solution of the Green Function for a number of three dimensional panels representing the submerged hull, see Bertram (66); and Rankine Singularity methods, see Bertram (66), which model the entire hull and free surface in the near field and therefore do not assume the hull is wall sided above the still water position. All three methods are amenable to solution in either the frequency or time domains.

A sample of commercially available seakeeping computer prediction codes using these methods are given in Table 2-2-1.

Note that PRECAL is only available to members of the Co-operative Research Ships club and both DNV-WASIM and LAMP can calculate motions using either linear or quasi non-linear methods.
<table>
<thead>
<tr>
<th>Code</th>
<th>Method</th>
<th>Frequency or Time Domain?</th>
<th>Web Reference</th>
<th>Other References</th>
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<td>Frequency</td>
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<td>(68)</td>
</tr>
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<td>Green Function</td>
<td>Frequency</td>
<td>CRS</td>
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<td></td>
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<td><a href="http://www.crships.org">www.crships.org</a></td>
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<tr>
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<td>Rankine</td>
<td>Time</td>
<td>DNV</td>
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<td>Time</td>
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<td></td>
<td><a href="http://www.saic.com">www.saic.com</a></td>
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</tbody>
</table>

Table 2-2-1: Commercially available seakeeping codes

For ship rolling, viscosity is significant as the boundary layer will separate from the hull during roll motion (e.g. at sharp corners such as the edges of bilge keels). Hence Potential Flow seakeeping calculations alone will not yield accurate roll motion predictions. In practice, either roll damping coefficients are measured experimentally, see Section 2.4, or empirical corrections are applied which allow for viscous effects assuming the total roll damping can be split into a number of components – both viscous and non-viscous. The non-viscous components include the wave radiation roll damping which is the only component predicted by Potential Flow Theory, see Section 2.5.
2.2.2 The Linear Uncoupled Equation of Roll Motion in Regular Waves

For a ship using the axis convention in Figure 2-2-2, if the roll motion is uncoupled from the other five degrees of freedom, the equation of motion can be described by balancing the moments acting on the ship:

\[(I_4 + A_{44})\ddot{x}_4 + B_{44}\dot{x}_4 + C_{44}x_4 = F_4\]  

This is a linear second order differential equation as described in Section 2.2.1. The most convenient units for a ship are, roll inertia, \(I_4\), and added inertia, \(A_{44}\), in Nm/(rad/s²); roll damping, \(B_{44}\), in Nm/(rad/s); roll stiffness, \(C_{44}\), in Nm/rad; and external forcing moment, \(F_4\), in Nm. As the ship rolls, the accelerating hull causes changes in fluid velocities adjacent to its surface. The additional moment required to accelerate this water as well is included as the added inertia coefficient, \(A_{44}\). The added inertia of the hull in air is negligible.

Taking the uncoupled equation of roll motion as described in equation 2-2-22, for a given ship at a particular speed and heading with respect to a wave train of regular sinusoidal waves, the moment acting on the ship, \(F_T\), will depend upon the displacement, velocity and acceleration of the sea surface, \(\eta, \dot{\eta}\) and \(\ddot{\eta}\), as well as the roll acceleration, velocity and displacement. Making the assumption that the wave amplitude, \(\eta_0\), is small in comparison with the wave and ship length then the resulting roll motions will be small. This allows a Taylor expansion to be used to obtain a linear approximation for the moment, \(F_T\):

\[F_T = a_4\ddot{x}_4 + b_4\dot{x}_4 + c_4x_4 - \left(A_{44}\ddot{x}_4 + B_{44}\dot{x}_4 + C_{44}x_4\right)\]  

In uncoupled roll motion, this moment will also be equal to the product of roll inertia and roll acceleration:

\[F_T = I_4\ddot{x}_4\]
Combining equations 2-2-23 and 2-2-24 gives equation 2-2-22 where the roll moment due to waves, \( F_4 \), is equal to:

\[
F_4 = a_i \dot{\eta} + b_i \dot{\eta} + c_i \eta
\]  

2-2-25

It is customary in ship motion theory to relate ship motions to a moving origin, \( O \), located on a vertical axis running through the centre of gravity, \( G \), on the mean water level. It is assumed that \( O \) moves with the waves that would be observed at \( O \) if the ship were not present (i.e. *it is assumed that the ship hull does not distort the waves in any way*). So:

\[
\eta = \eta_0 \sin(\omega_e t)
\]  

2-2-26

The quantity \( \omega_e \) is the wave encounter frequency and is described by the relationship:

\[
\omega_e = \omega + kU \cos \chi
\]  

2-2-27

\[
k = \frac{\omega^2}{g}
\]

Where \( k \) is the wave number, \( \omega \) the frequency of the regular waves, \( U \) the forward speed and \( \chi \) the wave direction relative to the ship, where 180 degrees is head seas.

Differentiating equation 2-2-26 twice and substituting into equation 2-2-25 and then using a \( \sin(A + B) \) expansion gives:

\[
F_4 = F_40 \sin(\omega_e t + \gamma_4)
\]  

2-2-28

Where:
\[ F_{40} = \eta_0 \left[ \sqrt{(c_4 - a_4 \omega_e \omega_e^2)^2 + (\omega_e b_4)^2} \right] \]  

\[ \gamma_4 = \tan^{-1} \left( \frac{\omega_e b_4}{c_4 - a_4 \omega_e \omega_e^2} \right) \]  

So, equation 2-2-22 becomes:

\[
(I_4 + A_{44})\ddot{x}_4 + B_{44} \dot{x}_4 + Cx_4 = F_{40} \sin(\omega_e t + \gamma_4)
\]  

The solution to this form of differential equation will be:

\[ x_4 = x_{40} \sin(\omega_e t + \delta_4) \]

Where \( x_{40} \) is the roll amplitude in radians and \( \delta_4 \) is the phase difference between the roll forcing moment and the roll motion.

The solution to this linear theory is that the output motion of a regular sinusoidal input wave will also be sinusoidal. The motion amplitudes will be directly proportional to the excitation amplitudes which will be in turn proportional to the wave amplitude.

The equation of motion describing uncoupled roll motion (equation 2-2-30) has been developed based on linear theory. If sinusoidal motion described by equation 2-2-31 is assumed and this is substituted into equation 2-2-30 the equation can then easily be solved in the frequency domain using a matrix inverse technique, see section 2.2.4.

### 2.2.3 Non-Linear Roll Damping in an Otherwise Linear Equation of Motion

The nature of the roll damping term in the equation of motion, \( B_{44} \), is often investigated by performing free or forced rolling experiments. The procedure is described in detail in Section 2.4.2. Since the work of William Froude (79), it has
been understood that to obtain accurate roll estimates for monohulls both linear
and non-linear roll damping terms are needed. The work of Himeno (80),
Schmitke (81) and Kat (82), shows that linear roll damping is provided by: wave
radiation damping; hull lift damping, caused by the moment generated by the lift
force created by the ship’s hull acting as a low aspect ratio wing during roll
motion at forward speed (see section 2.5.5); lift damping from appendages; and
surface friction. Non-linear roll damping was caused by viscous effects, such as
eddy shedding around the bilge during rolling, bilge keels (not including any lift
forces generated) and non-linear surface friction effects.

A non-linear roll damping term makes the equation of motion non-linear and
linear seakeeping theory is no longer valid. Himeno (80) showed that the
inclusion of these non-linear damping terms is important if accurate roll motion
predictions are to be obtained, see Section 2.5.1.

In the next sub-section the form of the roll damping term in the equation of
motion is considered in detail. In the final sub-section, a process for obtaining an
equivalent linear roll damping term from a damping term with non-linear
components is given so that the equation of motion can once again be solved
using linear seakeeping theory.

2.2.3.1 The Form of the Roll Damping Model

In this section the form of the roll damping term, $B_{44}$, will be discussed. For
convenience the equation of motion in 2-2-22 is recast by dividing through by the
roll inertia and added inertia:-

$$\ddot{x}_4 + \frac{B_{44}}{(I_4 + A_{44})} \dot{x}_4 + \frac{C_{44}}{(I_4 + A_{44})} x_4 = \frac{F_4}{(I_4 + A_{44})}$$

$$\Rightarrow \quad \ddot{x}_4 + b_{44} \dot{x}_4 + \omega_n^2 x_4 = f_4$$

2-2-32
From equations 2-2-8 and 2-2-10, \( b_{uu} \) can be replaced by \( 2\zeta \omega_n \) if desired. As \( \zeta \) is generally used to denote a linear decay coefficient it is sometimes best not to make this substitution because when non-linear coefficients are introduced they are then related to the linear coefficient.

Currently there is no comprehensive theory for the prediction of the roll damping moment of a ship and either empirical or semi-empirical methods have to be used. The formation of a purely empirical method requires a suitable form for the damping model which reflects the physics of how energy is dissipated in rolling. Once a damping model has been chosen, model experiment data is required to determine the magnitude of the coefficients in the damping model.

Bass and Haddara (83) summarise various nonlinear models that can be used to describe roll motion utilising the simple single degree of freedom roll equation. The roll damping coefficient \( b_{uu} \) was broken down into the following components:

\[
b_{uu}(x_4, \dot{x}_4) = [\epsilon_0 + \epsilon_1|x_4| + \epsilon_2x_4^2 + \epsilon_3|\dot{x}_4| + \epsilon_4\dot{x}_4^2]x_4 \tag{2-2-33}
\]

And the different damping models are described in Table 2-2-2. The most commonly used to describe ship roll motion are linear, quadratic and cubic. Many authors refer to these as linear; linear and quadratic; and linear and cubic. The former description shall be used throughout this thesis.

The quadratic model is mainly credited to William Froude (79) and this formulation has been used extensively. Haddara (84) introduced the cubic model to overcome analytical difficulties arising from the use of the quadratic form where the solution to the equation of motion is more complex, see for example Mathisen and Price (85). The other damping models were investigated extensively by Haddara and Bennett (86) and Haddara and Bass (87). They performed analysis on roll decay histories, using either the first one or two cycles or one half of the decay record and checked the consistency of the derived decay parameters by how well they predicted the decay for the remaining cycles.
Haddara and Bennett focused on an R-Class Icebreaker and merchant vessel MV Artic, whereas Haddara and Bass investigated small fishing vessels less than 25 metres in length. All decay analysis was conducted using an energy method developed by Bass and Haddara (83) which is reviewed in Section 2.4.2.

<table>
<thead>
<tr>
<th>Damping Model</th>
<th>Parameter Values ($\varepsilon$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>$\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon_4 = 0$</td>
</tr>
<tr>
<td>Quadratic</td>
<td>$\varepsilon_1 = \varepsilon_2 = \varepsilon_4 = 0$</td>
</tr>
<tr>
<td>Cubic</td>
<td>$\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 0$</td>
</tr>
<tr>
<td>Linear angle dependence</td>
<td>$\varepsilon_2 = \varepsilon_3 = \varepsilon_4 = 0$</td>
</tr>
<tr>
<td>Linear angle dependence plus quadratic</td>
<td>$\varepsilon_2 = \varepsilon_4 = 0$</td>
</tr>
<tr>
<td>Quadratic angle dependence plus quadratic</td>
<td>$\varepsilon_1 = \varepsilon_4 = 0$</td>
</tr>
<tr>
<td>Linear angle dependence plus cubic</td>
<td>$\varepsilon_2 = \varepsilon_3 = 0$</td>
</tr>
<tr>
<td>Quadratic angle dependence plus cubic</td>
<td>$\varepsilon_1 = \varepsilon_3 = 0$</td>
</tr>
<tr>
<td>Full parameter</td>
<td>$\varepsilon_1 \neq 0$</td>
</tr>
</tbody>
</table>

Table 2-2-2: Forms of the roll damping coefficient according to Bass and Haddara (83)

Theoretical decrements predicted using roll damping terms obtained through analysis of the first decay cycle from the models of MV Artic and the R-Class Icebreaker had the best correlation to the remainder of the decay cycles when the cubic model was used. At zero speed a reported error of less than 5% was achieved. However, using roll damping terms obtained through analysis of half the cycles in the decay for the icebreaker, theoretical predictions of the decay showed that either the quadratic or cubic methods provided a good fit to the measured data. For MV Artic, linear angle dependence was shown to be better than quadratic. The reasons for both these results were explained by considering the energy dissipation over time. The largest dissipation of energy was shown to occur during the first roll cycle. After this, the rate of energy dissipation for subsequent cycles over time was shown not to vary considerably. Hence, linear and linear angle dependent coefficients, derived using just the first cycle, will be large and over predict the motion in subsequent cycles unless recourse is taken to
use a more complicated dependence on the angle such as provided by the cubic velocity term. If, as in the case of MV Artic, the rate of energy dissipation per cycle is similar across all the cycles then a linear angle dependent model would be sufficient.

Tests at forward speed on the icebreaker showed an increase in the damping, however this relation was non-linear. The addition of bilge keels reduced the angle dependence of roll damping at low speeds where bilge keels would be expected to be most effective. Without bilge keels, strong angle dependence was noted at low speed, decreasing thereafter.

Three fishing vessel hulls were investigated by Haddara and Bass (87), one with a hard chine form, and two more conventional hull shapes. These studies showed that the best fit to the experimental data was with the quadratic model, when the damping was moderate, and the linear angle dependent model, when the damping was large. Once again, this result was explained by looking at the change in rolling motion energy over time for one roll cycle. The drop of energy in the first quarter of a roll cycle for a heavily damped model is significant whereas thereafter it becomes more gradual. For the lightly damped model the drop in energy level is much more gradual. When using the quadratic model, the fishing vessel with the hard chine form was shown to have a large non-linear damping coefficient, $\varepsilon_3$. Of the two conventional hull shapes, one had a low rise of floor and this also exhibited a high non-linear damping coefficient. This was thought most likely to be due to significant eddy shedding from this hull shape.

The discussion in Section 2.5.1 shows that the linear damping coefficient, $\varepsilon_0$, in equation 2-2-33, is considered to be an indication of the wave dissipation component of the damping moment, augmented by dynamic lift from appendages and the hull itself at forward speed. The work by Haddara and Bass and Haddara and Bennett shows that when the total damping is large, such as with forward speed, this term is dominant, whereas for light damping, such as at zero speed, the non-linear terms dominate. The quadratic and cubic models best describe motions
where the rate of energy dissipation in the first cycle is greater than in subsequent cycles.

The preceding discussion has shown that the choice of a suitable damping model is by no means simple. It can be affected by ship speed, hull shape, level of total roll damping and the portion of the decay history on which the analysis is performed. A search of the literature shows that quadratic was preferred over cubic by Spouge (88); Renyuan (89); Mathisen and Price (85) and Roberts (90), each reporting better correlation with experimental data, although they all used the same experimental results obtained for the Fishery Protection vessel Sulisker. Both quadratic and cubic damping models were investigated by Chan and Huang (91). They showed good results using both the quadratic and cubic models when a non-linear roll restoring moment was used with large initial roll angle. Franscescutto, Nabergoj and Hsiu (92) and Bulian (93) at the University of Trieste prefer a cubic damping model based on their research on large amplitude rolling. Bulian showed using an analytical approach based on the averaging method, that the cubic damping model provided good correlation to an artificial roll decay history obtained from a numerical solution of the equation of motion when the initial roll amplitude was less than 40 degrees. Cotton and Spyrou (94) conducted research into ship capsize and conducted model experiments on a prismatic ship section including free decay. They initiated the decay by releasing the model at the angle of vanishing stability, around 38 degrees. Starting at this large angle they found that a cubic damping model provided the best fit to the experimental data. Looking at their experimental data they noted that a quadratic model would have provided an adequate fit to the decay for roll amplitudes below 20 degrees.

What can be concluded from this review is that, at the present, one model does not seem better than another. The quadratic model is the longest established and hence most commonly used and would appear to be better than a cubic model when the roll amplitude is moderate. However, researchers working on large amplitude rolling seem to prefer the cubic model.
2.2.3.2 Converting the Non-Linear Damping Term to an Equivalent Linear Form

To allow for a non-linear damping model in an otherwise linear equation of motion an equivalent linear damping term, \( B_e \), is developed by equating the energy dissipated by this term in the equation of motion to that dissipated by the non-linear effects, usually over one quarter of a roll cycle. This coefficient can then replace \( B_{st} \) in equation 2-2-30 and the equation of motion is once again solvable by linear theory.

Ignoring the phase difference between the motion and the forcing term let the roll motion be described by:

\[
x_4 = x_{40} \sin \omega_d t
\]

Here the roll frequency is \( \omega_d \) which was defined in Section 2.2.2 as the damped natural frequency for unforced motion. This can be measured in a roll decay experiment. The damped natural frequency in forced roll motion, \( \omega_o \), can be measured from a forced roll experiment and this could be used instead if preferred.

In one roll cycle the work done will be equal to the integral of the roll moment multiplied by the angular distance moved. Taking the work done in rolling to be equal to the energy dissipated by the linearised damping term, \( E \):

\[
E = 4 \int_0^{x_{40}} B_e \dot{x}_4 dx_4
\]

\[
E = 4B_e \omega_d \int_0^{x_{40}} x_{40}^2 \cos^2 \omega_d t dt
\]

\[
\Rightarrow E = B_e \omega_d x_{40}^2
\]
Equivalent linear damping coefficients will now be developed for the two most popular damping models, quadratic and cubic, based on the energy dissipated by each of these models over one roll cycle, see also Spouge (95) and Lloyd (64).

An equivalent linear damping term can be developed once an expression for the roll moment is known, $M_R$, either using a damping model from Section 2.2.3.1 or from an expression for the roll damping moment due, for example, bilge keels, appendages, hull friction and hull eddy shedding. The method for obtaining an expression for the energy dissipated in rolling by a known roll moment, which can then be used with equation 2-2-34 to obtain an equivalent linear damping term, is given below.

\[ E = \int_0^{x_m} M_R \, dx \]

\[ E = 4 \int_0^{\frac{\pi}{2a_f}} M_R \dot{x}_4 \, dt \]

\[ E = 4 \int_0^{x_m} M_R \dot{x}_4 \, dx \]

**Quadratic Damping**

Taking the quadratic model for the damping:-

\[ B_e = B_1 + B_2 |\dot{x}_4| \]

Changing the variable of integration to time, the energy dissipated over one roll cycle is:-
Where $T_4$ is the roll period associated with frequency $\omega_d$. Making the substitution for $\dot{x}_4$:

\[
E = 4 \int_0^{T_4} \left( B_1 \dot{x}_4 + B_2 \ddot{x}_4 \right) \dot{x}_4 \, dt
\]

\[
E = 4 \int_0^{T_4} \left( B_1 \omega_d x_{40} \cos \omega_d t + B_2 \left( \omega_d x_{40} \cos \omega_d t \right) \omega_d x_{40} \cos \omega_d t \right) \times \omega_d x_{40} \cos \omega_d t \, dt
\]

\[
E = 4 \omega_d^2 \int_0^{T_4} B_1 x_{40}^2 \cos^2 \omega_d t + B_2 \omega_d x_{40}^2 \cos^3 \omega_d t \, dt
\]

\[
\Rightarrow E = \pi \omega_d B_1 x_{40}^2 + \frac{8}{3} B_2 \omega_d^2 x_{40}^3
\]

Equating this with equation 2-2-34 gives:

\[
B_0 = B_1 + \frac{8}{3\pi} B_2 \omega_d x_{40}
\]

**Cubic Damping**

Taking the cubic model for the damping:

\[
B_0 = B_1 + B_3 \dot{x}_4^2
\]

Changing the variable of integration to time, the energy dissipated over one roll cycle is:

\[
E = 4 \int_0^{T_4} \left( B_1 \dot{x}_4 + B_2 \ddot{x}_4 \right) \dot{x}_4 \, dt
\]
Where $T_4$ is the roll period associated with frequency $\omega_d$. Making the substitution for $\dot{x}_4$:

\[
E = 4 \int_0^{\pi/2} \left[ B_1 \omega_d x_{40} \cos \omega_d t + B_3 (\omega_d x_{40} \cos \omega_d t)^3 \right] \omega_d x_{40} \cos \omega_d t \, dt
\]

\[
E = 4\omega_d^2 \int_0^{\pi/2} B_1 x_{40}^2 \cos^3 \omega_d t + B_3 \omega_d^2 x_{40}^4 \cos^4 \omega_d t \, dt
\]

\[
\Rightarrow E = \pi \omega_d B_1 x_{40}^2 + \frac{12}{16} B_3 \omega_d^2 x_{40}^3
\]

Equating this with equation 2-2-34 gives:

\[
B_4 = B_1 + \frac{3}{4} B_3 \omega_d^2 x_{40}^2
\]

### 2.2.4 Solution of the Equation of Motion in the Frequency Domain

Having converted the non-linear damping term to an equivalent linear form in Section 2.2.3.2, the linear equation of roll motion, equation 2-2-30, can be recast as follows:

\[
(I_4 + A_{44}) \ddot{x}_4 + B_4 \dot{x}_4 + C x_4 = F_{40} \sin(\omega_d t + \gamma_4)
\]

This equation of roll motion can then be coupled with linear equations of motion representing the other five degrees of freedom to create a single equation of motion for the ship. This is best presented in the form of a matrix:

\[
(M + A) \ddot{X} + B \dot{X} + C X = F^w
\]

$X$ is a $(6 \times 1)$ matrix denoting the wave induced motions in the six degrees of freedom and $M, A, B$ and $C$ are $(6 \times 6)$ matrices for the masses and inertias;
added masses and inertias; damping; and hydrostatic stiffness. $F^W$ is a $(6 \times 1)$ vector of the wave forces and moments. If the motions are measured at the centre of mass, in the directions of the principal axis of inertia, $M$, has a diagonal form:

$$
\begin{bmatrix}
  m & 0 & 0 & 0 & 0 & 0 \\
  0 & m & 0 & 0 & 0 & 0 \\
  0 & 0 & m & 0 & 0 & 0 \\
  0 & 0 & 0 & I_{xx} & 0 & 0 \\
  0 & 0 & 0 & 0 & I_{yy} & 0 \\
  0 & 0 & 0 & 0 & 0 & I_{zz}
\end{bmatrix}
$$

Where $m$ is the mass of the ship and $I_{xx}, I_{yy}$ and $I_{zz}$ are the inertias about the three principal axis:

$$
I_{xx} = mk_x^2
$$

$$
I_{yy} = mk_y^2
$$

$$
I_{zz} = mk_z^2
$$

Where $k_x, k_y$ and $k_z$ are the radii of gyration in roll, pitch and yaw respectively.

For a ship with port-starboard symmetry about the x-axis the added mass and inertia matrix is of the form:

$$
\begin{bmatrix}
  A_{11} & 0 & A_{13} & 0 & A_{15} & 0 \\
  0 & A_{22} & 0 & A_{24} & 0 & A_{26} \\
  A_{31} & 0 & A_{33} & 0 & A_{35} & 0 \\
  0 & A_{42} & 0 & A_{44} & 0 & A_{46} \\
  A_{51} & 0 & A_{53} & 0 & A_{55} & 0 \\
  0 & A_{62} & 0 & A_{64} & 0 & A_{66}
\end{bmatrix}
$$

And similarly the damping matrix is:-
The hydrostatic stiffness matrix of a ship symmetrical about the fore-aft axis is:

\[
\begin{bmatrix}
B_{11} & 0 & B_{13} & 0 & B_{15} & 0 \\
0 & B_{22} & 0 & B_{24} & 0 & B_{26} \\
B_{31} & 0 & B_{33} & 0 & B_{35} & 0 \\
0 & B_{42} & 0 & B_{4} & 0 & B_{46} \\
B_{51} & 0 & B_{53} & 0 & B_{55} & 0 \\
0 & B_{62} & 0 & B_{64} & 0 & B_{66}
\end{bmatrix}
\]

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The components of the excitation vector \( F^w \) must all vary harmonically with time at frequency \( \omega_e \). So, if \( \text{Re} \) denotes the real part of a complex number (using \( i \) as the complex variable):

\[
F^w = \text{Re} \left[ \bar{F} e^{im} \right]
\]

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Where \( \bar{F} \) is a complex \((6 \times 1)\) matrix representing the constant amplitude wave forcing terms (forces and inertias) and the phase difference between the input wave and the output forcing terms. Expressing the linear response of the ship in the form of equation 2-2-51 gives:

\[
X = \text{Re} \left[ \bar{X} e^{im} \right]
\]

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Where \( \bar{X} \) is a complex \((6 \times 1)\) matrix representing the constant amplitude output motions and the phase difference between the input wave and the output motions. Hence:-

\[
C_{35} \text{ and } C_{53} \text{ are equal to zero if the ship is symmetrical fore and aft about a transverse axis passing through midships.}
\]
\[
\dot{X} = i\omega_e \text{Re}[\overline{X}e^{i\omega t}]
\]
\[
\ddot{X} = -\omega_e^2 \text{Re}[\overline{X}e^{i\omega t}]
\]

Substituting equations 2-2-51 to 2-2-53 into the equation of motion, equation 2-2-45, and cancelling the \(e^{i\omega t}\) terms yields:

\[-\omega_e^2(M + A)\overline{X} + i\omega_e B\overline{X} + C\overline{X} = \overline{F}\]

Equation 2-2-54 can be solved using a matrix inverse technique if:

\[
[i\omega_e B + C - \omega_e^2(M + A)]\overline{X} = \overline{F}
\]

This equation can only be solved directly when all of the coefficients in each of the matrices are constant. The equivalent linear damping term developed in Section 2.2.3.2 depends on the roll amplitude and so the equation of motion has to be solved in an iterative manner. The iterative process commences with an estimate of the steady roll amplitude, the next iteration uses the output roll amplitude after equation 2-2-54 has been solved, and iterations continue until the initial and output roll amplitude are identical.

The roll and pitch restoring terms, \(C_{44}\) and \(C_{55}\), are equal to the ship displaced volume multiplied by the transverse or longitudinal \(\overline{GZ}\) value obtained at the respective steady roll or pitch amplitude. Once again this would require an iterative solution to the equation of motion. Therefore, \(\overline{GM}\) is substituted for \(\overline{GZ}\) on the assumption that the roll and pitch amplitudes are small.

The remaining terms in the equation of motion are usually determined using Potential Flow Theory; see Newman (65). The most popular approaches are Strip Theory, Green Function methods and Rankine Singularity Methods; see Bertram (66). Bailey, Hudson, Price and Temarel (96) show that, with the exception of the roll damping term, \(B_{44}\), the remaining unknown coefficients in the equation of
motion can be predicted with reasonable accuracy using either Green Function methods or Rankine Singularity methods in the frequency domain. In the paper they compare different researchers' mathematical formulations with available model experiment results for a Series 60 monohull. Further examples are given for a frigate at forward speed in the subsequent discussion.

The heave and pitch response of a deep V hulled fast ferry and a much larger, high speed round bilge ferry, are calculated using Strip Theory, Green Function Methods and Rankine Singularity Methods, all in the frequency domain, by Bruzzone, Gualeni and Sebastiani (69). The results were compared with model experiment results at a range of speeds. The work showed that at forward speed, both Green Function Methods and Rankine Singularity Methods provided a good correlation to experiment data, whereas the Strip Theory only gave good results at low speeds.

Other similar comparisons of monohull ship motions with Potential Flow methods in the frequency domain were undertaken by Hudson, Price and Temarel (97); Maury, Delhommeau, Ba, Boin and Guilbaud (98); and Takaki, Lin, Gu and Mori (99). All of these researchers similarly concluded that either Green Function methods or Rankine Singularity methods were adequate for determining ship motions using Potential Flow Theory with the exception of roll motion.

Thus, it can be concluded that, for a monohull, motions can be predicted with reasonable accuracy in six degrees of freedom in the frequency domain with linear theory using either Green Function methods or Rankine Singularity methods with the exception of roll damping. Roll damping is not accurately predicted because these Potential Flow methods do not account for viscous effects, see the brief discussion at the end of Section 2.2.1.
2.3 Non-linear Seakeeping Theory

Non-linear seakeeping theory allows the ship response to arbitrary wave excitations to be calculated rather than just the response to a regular wave of constant amplitude and frequency with linear theory. One must be careful when using the words “non-linear” in seakeeping theory as it rather implies that the entire equation of motion is considered non-linear; with current theories this is not the case. In fact, what occurs is that parts of the equation of motion are considered non-linear. The most common are the wave excitation force on the right hand side of the equation of motion, see for example the formulation by Ballard, Du, Hudson and Temarel (100), and in addition, some of the coefficients of the equation of motion can be considered non-linear. Examples of non-linear terms used to enhance roll motion prediction are the wave forcing terms, and the roll stiffness term, $C_{44}$. The remainder are calculated using linear Potential Flow Theory (noting that for accurate roll motion predictions the Potential Flow Theory derived roll damping term must be augmented). By making the equation of motion partially non-linear, solution in the frequency domain is impossible and the equation of motion must now be solved in the time domain. The time domain method has the added advantage that the hydrodynamic coefficients of the equation of motion can be calculated at each and every time step based on the instantaneous wave profile. Thus for some hull shapes a regular input wave could give an irregular output response. This is not possible with linear theory.

The mathematics of solving the equation of motion in the frequency domain have been somewhat covered up in the previous sections (the reader is left to peruse the reference papers) and this was due in part to the mathematical complexity involved. With current computing capability, frequency domain calculations can be performed rapidly, however the same cannot be said for time domain calculations. Hence, in general, time domain seakeeping predictions have been the subject of academic research until only very recently. The accuracy of frequency domain Potential Flow Methods (albeit with corrections for roll damping) discussed in Section 2.2.4 has meant that time domain solutions have
only been used for isolated ship types or seaways where the additional computational analysis time is considered worthwhile. Examples include motions in large amplitude waves where the assumption that the ship is wall sided above the waterline in frequency domain methods may lead to inaccuracies, or for ships with hull shapes with considerable changes in yz-plane section shape (i.e. due to flare above the waterline).

Time domain methods are very attractive for roll motion predictions as they offer the possibility of including time varying damping coefficients. No methods currently exist for extracting time varying roll damping coefficients from experiments and, in addition, no theories have been published which derive time varying roll damping coefficients theoretically. Constant damping coefficients are usually measured in roll decay experiments, see Section 2.4. The measured coefficients can be input directly into the equation of motion which is then solved in the time domain. As will be shown in Section 2.5, all of the published theoretical methods for predicting roll damping of monohull ships rely on converting the non-linear damping to an equivalent linear form, with the equation of motion then solved in the frequency domain. Damping coefficients suitable for incorporation in a linear code were favoured as there were no non-linear codes in practical use when these theories were published.

If an equivalent linear damping term is substituted into the equation of motion for the roll damping coefficient, the equivalent linear damping term will depend on the output roll angle and, as with frequency domain solutions, iterations have to be performed until the input and output steady roll amplitude are the same. This creates a problem in the time domain as these iterations would be required at each and every time step and this makes calculation times unfeasibly long. A recent reminder of this is given by Fan and Wilson (101) describing their time domain non-linear strip theory for ship motions.

Two attempts have been made to implement theoretical equivalent linear roll damping theory in a time domain code. Vassalos, Jasionowski and Cichowicz (102) propose an engineering approximation whereby a discrete piece-wise constant treatment of the equivalent linear damping terms is used, taking a roll
amplitude corresponding to the amplitude of the last half-roll cycle. Hence after a
number of time steps the solution will converge to a steady time varying result for
regular waves. Unfortunately, the paper does not give any examples to
demonstrate the accuracy of the method. Pastoor, van’t Veer and Harmsen (75)
discuss implementation of viscous damping in the DNV WASIM computational
seakeeping code, where an approximation of the viscous effects is achieved by
calculating the viscous damping terms using wave amplitudes giving an upper and
lower bound. This second method is discussed in more detail for trimarans in
Chapter 3.

Non-linear time domain analyses have been conducted by: Ballard, Hudson, Price
and Temarel (103), for only heave and pitch motions; Holloway and Davis (104),
validating a method on submerged and floating cylinders; Kara and Vassalos
(105), with results compared to model tests of a Wigley hull in heave and pitch;
Kring, Huang, Sclavounos, Vada and Braathen (73), with results compared to
model experiments of a Series 60 hull in heave and pitch; Lin and Yue (77),
focusing on surge, heave and pitch and comparing results to Series 60 model
experiments; Nakos and Sclavounos (74), using a Rankine Singularity Method;
Westlake, Wilson and Bailey (106), who propose a time domain Strip Theory; and
Salvesen and Lin (78), reporting on the LAMP computational suite.

For conventional monohull ships, the increased accuracy of having a non-linear
wave forcing term and some non-linear coefficients in the equation of motion is
not considered worth the much increased computational effort required in non­
linear time domain methods. Such methods are only considered advantageous
when a novel hull shape is used, motions in large amplitude waves are required, or
a study of ship motions in a specific sea environment is desired. For most ships,
results of linear seakeeping runs, which can then be compared with similar results
from actual ships whose motion characteristics are known and considered
acceptable, is all that is wanted.

If the equation of motion is uncoupled so that analysis is performed on a single
degree of freedom roll equation time domain analysis is simplified and, if the
coefficients are known, the equation of motion can be solved using a Runge Kutta
technique, see (107). Roll motion has been investigated in such a way by Francescutto, Nabergoj and Hsiu (92); Francescutto (108) and Francescutto, Bulian and Lugni, (60), amongst others.
2.4 Roll Motion Prediction using Experimentally Derived Roll Decay Coefficients

2.4.1 Overview

Long before even linear seakeeping theories had been established, ship rolling had been of interest to engineers; see for example Gawn (109), who reports on roll decay experiments performed in 1873. This interest has continued since the development of seakeeping prediction methods, starting with Strip Theory in 1970, Salvesen, Tuck and Faltinsen (110), due to the difficulty in calculating viscous roll damping when Potential Flow Theory is used. This has remained the central difficulty hindering accurate roll motion predictions to the present day. Sections 2.2 and 2.3 show that all theoretical methods still rely on Potential Flow Theory as the basis of roll motion prediction, often with empirically based enhancements which will be described in Section 2.5. The usual route to determine the roll response of a ship is to perform some form of model experiment and results from these experiments are often used to replace or augment the Potential Flow Theory roll damping coefficient. The experiments generally used are explained in the next section and semi-empirical theories developed using a range of experimental results are discussed in Section 2.5.

2.4.2 Prediction by Model Experiment

Roll decay coefficients can be determined from either forced or free roll decrement experiments. In the former, a pure sinusoidal roll moment is applied to the model and the resultant steady roll amplitude and phase difference are recorded along with the forcing moment amplitude and frequency. The test is repeated over a range of forcing moment amplitudes so that results are obtained for different roll amplitudes. In a roll decrement experiment the model is given an initial heel angle in calm water and then allowed to roll freely, the resulting decaying roll motion is measured. The model's roll motion will decay at the
damped natural frequency $\omega_d$. Using these model results and making the assumption that, in both cases, pure roll motion had been recorded (that is roll motion is not coupled with surge, sway, heave, pitch or yaw) the linear and non-linear roll damping coefficients can be measured. These, along with the other coefficients in the uncoupled equation of roll motion, are normally considered to be constant.

Spouge (95) gives a review of methods of obtaining the linear and non-linear damping coefficients from either free or forced rolling experiments for an uncoupled linear equation of roll motion. All the methods reviewed in Spouge's paper use only the peaks of the roll angle response time history rather than the complete cycle. Parameter Identification Techniques can also be used to determine the constant coefficients of the equation of motion. In these techniques a form of the equation is assumed, the unknown terms in this equation are estimated and then the equation is simulated using numerical methods. The simulation is compared with the actual data and a mathematical routine is used which varies the coefficients in the equation until the solution converges with the measured result. Parameter Identification Techniques for determining coefficients in the uncoupled equation of roll motion have been discussed by Roberts, Kountzeris and Gawthrop (111), Contento, Francescutto and Piciullo (112) and Francescutto, Contento, Biot and Schiffrer (113) amongst others.

In the next two sections, the methods available to analyse free decay results are reviewed. For further information on analysis of forced roll experiments see Spouge (95). The preference for free roll decay here is due to the fact that a special rig is required to force roll a model and this has to be fitted to a carriage in a towing tank to conduct experiments at forward speed. Not all model testing facilities have the capability to perform forced roll experiments on large models at forward speed.
2.4.2.1 Methods using the Peaks of Roll Decrement Time Histories

One of the most popular methods for analysing free decay data is the quasi-linear method. This is simply a linear analysis of the decay of the peaks of the roll decrement time history used to determine damping at various points in the decrement. The damping values are determined at specific positions and are effectively equivalent linear values (see Section 2.2.3.2). Non-linear damping coefficients are determined from their variation with roll amplitude.

In the quasi linear method, the damping is measured over one half of a roll cycle. Spouge (95) shows that, assuming a linear roll restoring moment, this equivalent linear damping term can be expressed as:

\[
 b_e = \frac{2\omega_d}{\pi} \ln \left[ \frac{x_{4p}}{x_{4p+1}} \right] \tag{2.4-1}
\]

Where \( b_e \) is the equivalent linear roll damping term per unit roll inertia and added inertia and \( x_{4p} \) and \( x_{4p+1} \) are the amplitude of successive roll peaks describing half a roll cycle. This is equivalent to the equation used to obtain \( \zeta \) in Section 2.2.1. The equivalent linear damping terms developed in Section 2.2.3.2 when divided through by roll inertia and added inertia can be expressed as:

\[
 b_e = b_1 + \frac{8}{3\pi} b_2 \omega_d x_{4m} \tag{2.4-2}
\]

\[
 b_e = b_1 + \frac{3}{4} b_3 \omega_d^2 x_{4m}^2 \tag{2.4-3}
\]

Where \( b_1 \), \( b_2 \) and \( b_3 \) are the linear, quadratic and cubic damping coefficients per roll inertia and added inertia respectively and a suitable form for \( x_{4m} \), the mean roll angle between successive peaks in the decay history, is:
The damping coefficients are found by plotting \( b_e \) values against \( x_{4m} \), both determined between successive roll peaks defining half a roll cycle. This plot is very useful as it shows the spread of the roll damping throughout the complete decay. A least squares fit to the \( b_e - x_{4m} \) points using either equation 2-4-2 or 2-4-3 can be performed to determine the damping coefficients. The spread of points around this fitted straight line (equation 2-4-2) or curve (equation 2-4-3) gives a measure of the adequacy of the chosen damping model.

The classic method developed by Froude for analysing free decay data equates the energy lost to damping in each half cycle to the work done by the restoring lever in reducing the roll amplitude; see for example Spouge (95). This produces an expression for the slope of the decrement of the peaks in the roll-time history, \( \frac{dx_{4p}}{dt} \). For linear restoring, expressing the decrement over one half a roll cycle this becomes:

\[
\frac{dx_{4p}}{dt} = \frac{\pi x_{4m} b_e}{\omega_d} \tag{2-4-5}
\]

A linear least squares fit of equation 2-4-5 with \( b_e \) expressed using equations 2-4-2 or 2-4-3 to the peak decay will yield the linear and non-linear damping coefficients.

Roberts (90) treated the roll decrement as an energy loss function, i.e. damping is considered as energy lost from the system. Roberts considered an uncoupled single degree of freedom roll equation with either a linear or non-linear form for both the roll damping and the roll restoring moments. In the case of the roll restoring moment a linear or linear and cubic form were considered. Spouge (95) expresses Roberts non-dimensional energy loss function \( Q(V) \) assuming linear restoring as:-
\[
Q(v) = \frac{1}{2\omega_d} b_v
\]  
2-4-6

This energy loss function can also be determined by measuring the gradient of a plot of the energy loss \( V \) at each peak over time:-

\[
Q(v) = -\frac{dV/dt}{2\omega_d V}
\]  
2-4-7

Where the energy loss is determined at each peak (with linear restoring) as:-

\[
V = \frac{1}{2} \omega_d^2 x_{4p}^2
\]  
2-4-8

So, the energy loss \( V \) is determined for each peak of the decay and plotted against time (taking the absolute value of the decrement so that both positive and negative peaks are included). From this plot the gradient of the energy decay \( dV/dt \) is calculated between successive peaks and \( Q(v) \) is determined using equation 2-4-6. The change in \( Q(v) \) over time for the decay is plotted and a least squares method is used to fit equation 2-4-6 to this assuming either quadratic, equation 2-4-2, or cubic, equation 2-4-3, forms of the roll damping term.

Spouge (95) also describes an averaging method attributed to Krylov and Bogoliuboff. This assumes that the motion during the roll decrement is sinusoidal with slowly varying amplitude and phase, and that for any cycle the rates of change of amplitude and phase are constant at their average values during the cycle. Spouge states that this assumption is reasonable for lightly damped models, and in general for the phase angles, but is questionable for heavily damped models whose amplitude changes rapidly during the early part of the roll decrement. The method obtains a value for the roll amplitude of successive peaks \( x_{4p} \) by integrating the mean rate of change of amplitude and phase in one cycle. With a quadratic damping model this is expressed as:-
Where $x_{4p}$ is the amplitude of the initial roll peak. Using the known initial peak amplitude the linear and quadratic damping coefficients can be found by fitting equation 2-4-9 to the experimental decrement using a non-linear least squares fit.

The final method available to obtain the roll damping coefficients from the decrement of the roll peaks is the perturbation method. This makes the assumption that the non-linear terms in the equation of motion are small in comparison to the linear terms. The solution of the equation of motion is then equal to the solution of the linear equation plus a series of smaller perturbations representing small disturbances to the simple linear solution. Mathisen and Price (85) developed a method using both a quadratic and cubic model for the roll damping term in the equation of motion with a linear roll restoring moment. Using this method the roll amplitude of successive peaks $x_{4p}$ is determined using the following expression, in this case for a quadratic damping model:

$$x_{4p} = \left[ 1 + \left( \frac{8b_2 \omega_d}{3nb_1} \right) x_{4p1} \right] \exp \left( \frac{b_1 n \pi}{2 \omega_d} - \frac{8b_2 \omega_d}{3nb_1} \right)^{-1}$$

$$n = 1,2, \ldots, n$$

$$2-4-9$$
2.4.2.2 Methods using the Complete Roll Decrement Time History

Bass and Haddara (83) proposed a parameter identification technique to determine constant coefficients of an uncoupled non-linear equation of roll motion. The roll damping term $b_{44}$ could be expressed in a number of forms, see equation 2-4-11 and Table 2-4-12, and the restoring form could be linear or linear and cubic. The advantage of parameter identification techniques is that the complete roll decrement time history is used rather than just the peaks of the decrement. This permits the coefficients in the equation of motion to be determined when only a short roll decrement time history is available. This is the case with heavily damped systems such as ship fitted with highly effective roll stabilisers.

The method uses the equivalence of the rate of energy dissipated in damping to identify parameters in the roll damping model chosen. Taking the example of a quadratic damping model and linear roll restoring moment in the roll decay test from time $t_1$ to $t_2$, the loss of ship energy (assumed to be through only roll motion) equals the energy dissipated in the damping moment:

$$ V(t_2) - V(t_1) = -\int_{t_1}^{t_2} (b_1 \dot{x}_4 + b_2 \ddot{x}_4) \ddot{x}_4 dt \quad \text{(2-4-11)} $$

Where the total energy of the ship per unit moment of inertia and added inertia at time $t$ is:

$$ V(t) = \frac{1}{2} \dot{x}_4^2 + \int \omega_y^2 \dot{x}_4 dt \quad \text{(2-4-12)} $$

Here the restoring moment per unit roll inertia and added inertia is taken to be approximately equal to $\omega_y^2$ during roll decay. Thus using equations 2-4-11 and 2-4-12 the energy dissipated in damping can be equated to the energy loss per roll cycle and the damping coefficients $b_1$ and $b_2$ are determined using a least squares method. This requires the roll velocity to be determined from the roll decrement.
If the recorded roll decrement is noisy, this will be amplified when the signal is differentiated and some curve fitting to the recorded data is generally required to achieve stable damping coefficients.

The method of Bass and Haddara forms the basis of the Parameter Identification Method used by Contento, Francescutto and Piciullo (112) to investigate the roll motion of a model of a ro-ro ferry in beam regular waves. Francescutto, Contento, Biot and Schiffrer (113) extended this work using it to identify parameters in the uncoupled equation of roll motion in beam regular waves for frigate, destroyer and fishing vessel models as well as the ro-ro ferry used in the previous paper.
2.5 Roll Motion Prediction using Semi-empirical Roll Damping Components

During the early stages of a design when undertaking concept studies, or when the hull shape is being refined, the need to perform model rolling experiments to obtain roll motion predictions is not very attractive as a large number of models would be necessary. In fact, more likely, the number of permutations of the hull design would be limited by the funds available for model testing. Therefore, researchers have tried to come up with theoretical methods for predicting the roll damping term in the equation of motion, $B_{44}$, which can be easily incorporated in Potential Flow seakeeping computer codes. The theories that have been developed are described briefly in this section (and expanded further in Appendix 1).

2.5.1 Overview

The most common approach used to assess the roll damping including viscous effects is to assume the total roll moment (generally calculated for transverse sections of the ship to allow easy incorporation into a Strip Theory seakeeping code) can be split into a combination of separate components. A large amount of work on the subject was completed in Japan and this is documented by Himeno (80) covering the work of Ikeda (114), (115), (116), (117), (118), and for a recent summary see Ikeda (119). Schmitke (81) also developed a series of component damping formulae around the same time in Canada. Each component represents a roll damping moment $M_R$ and non-linear components are linearised using the procedure outlined in Section 2.2.3.2. This approach was also supported by Kat (82). The components generally considered are given in Table 2-5-1.
<table>
<thead>
<tr>
<th><strong>Component</strong></th>
<th><strong>Conventional Symbol</strong></th>
<th><strong>Linear or Non-linear</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Skin Friction on the hull</td>
<td>$B_F$</td>
<td>Both</td>
</tr>
<tr>
<td>Eddy Shedding from the corners of the hull sections</td>
<td>$B_E$</td>
<td>Non-linear</td>
</tr>
<tr>
<td>Radiated free surface waves</td>
<td>$B_W$</td>
<td>Linear</td>
</tr>
<tr>
<td>“Lift Damping” due to the lift force created because of the angle of attack</td>
<td>$B_L$</td>
<td>Linear</td>
</tr>
<tr>
<td>between the rolling hull and the inflow of water at forward speed</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bilge Keel damping</td>
<td>$B_{BK}$</td>
<td>Mainly Non-linear</td>
</tr>
<tr>
<td>Damping from appendages other than bilge keels</td>
<td>$B_A$</td>
<td>Linear</td>
</tr>
</tbody>
</table>

Table 2-5-1: Component damping terms

Most of the formulae proposed are semi-empirically derived from a series of model experiments. These experiments generally utilised two-dimensional ship sections, cylinders or flat plates. Each roll damping component was isolated in the experiments as far as practical and so interaction among them is not considered. This subdivision of roll damping into additive components is not justifiable since hydrodynamic interaction between some of them is unavoidable, however this subdivision is convenient and has been shown to provide adequate predictions of ship roll damping. This was pointed out by Chakrabarti (120) who gives a good overview of the different formulae.

As can be seen from the table, determining whether a component is viscous or non-viscous is open to some debate. For bilge keel damping, Himeno (80) subdivides this term into three components, one for the contribution due to the change in pressure around the hull caused by fitting the bilge keels; a second due to the normal force on the bilge keels during rolling, both considered non-linear; and a final linear component due to the waves propagated by the bilge keels during rolling. Furthermore Kat (82) states that the frictional damping is linear, whereas both Brook (121) and Himeno (80) state that it is non-linear. Perhaps
more accurately, Chakrabarti (120) states that the part of the component due to laminar flow past the ship is linear whereas the modification for turbulent flow is non-linear. These differences of opinion come about, in part, because the subdivision of the components is not based on hydrodynamics, rather on a practical basis for convenience in predicting roll damping and carrying out experiments to determine the coefficients thus selected.

Those semi-empirical components that rely on model experiments of representative ship sections are derived from a small number of experiments on ship and barge hull shapes. Their application to unusual hull shapes is not recommended without thorough inspection of the original papers which describe the work in detail. As a word of caution, some researchers report poor correlation to ship or model experiments, for example Yücel Odabasi points this out in a discussion to a paper by Kat and Paulling (122). Kat himself makes the comment "with some adjustments the formula given...provided estimates of roll damping that were correct to at least the same order of magnitude as the values obtained by experiment" (82).

2.5.2 Magnitude of the Equivalent Linear Terms

In this section the magnitude of each of the equivalent linear damping components identified in Table 2-5- 1 is considered based on the published results of Himeno (80) and Schmitke (81).

2.5.2.1 Wave Radiation Damping

This component is the only one predicted by Potential Flow Theory. Both Schmitke (81) and Himeno, using the theory of Ikeda et al, (80), showed that this component is insignificant at low frequencies away from resonance and only becomes significant at high frequencies and high Froude Numbers, see Figure 2-5- 1, Figure 2-5- 2 and Figure 2-5- 3. The term 'WAVE-MAKING' in Figure 2-5- 1 refers to the wave radiation damping component, $B_w$. At constant frequency,
there is a peak in the wave making damping when $U\omega/g$ is equal to $\frac{1}{4}$ (80), this can clearly be seen in Figure 2-5-2.

Note that in Figure 2-5-2 and Figure 2-5-3 the sum of the terms $B_{BKN}$, $B_{BKH}$ and $B_{BKW}$ is equivalent to $B_{BK}$ in Table 2-5-1. The terms $B_{BKN}$ and $B_{BKH}$ are described in greater detail in Section 2.5.6 where $B_{BKP}$ is used instead of $B_{BKH}$.

Figure 2-5-1: Monohull destroyer roll damping components at 20 knots, taken from Schmitke (81).

Figure 2-5-2: Change in roll damping components with increasing advance speed using the method of Ikeda et al, taken from Himeno (80).
Ikeda et al (123) showed how a half-breadth to draught ratio, $H_0$, effected the magnitude of the wave radiation damping, $B_w$, for Lewis form cylinders (see Lloyd (64) Chapter 11) of unit length and cross-sectional area at fixed roll frequency. The trend is reproduced in Figure 2-5- 4 which shows that wave damping is a minimum when $H_0$ is between 1.2 and 1.5. Most monohull ships have beam to half-draught ratios in the range 1 to 2 which means that wave radiation damping will be small.

The roll damping predicted by Potential Flow Theory comprises only the Wave radiation damping component, $B_w$. However, as this component is in general only a small part of the overall roll damping, accurate monohull roll motion predictions using Potential Flow Theory requires the total roll damping to be calculated by summation of this component with other semi-empirical components ($B_E$, $B_F$, $B_L$, $B_{BK}$, and $B_A$) or by using model experiment results.
2.5.2.2 Hull Friction Damping

For the conventional monohull merchant ships hulls considered by Ikeda et al, see Himeno (80), the hull friction damping component, $B_h$, provided only a small contribution to the total roll damping, see Figure 2-5-2 and Figure 2-5-3.

Himeno (80) points out that the scale effects between hull friction damping measured at model scale and full scale can be quite large; the value for an actual ship will be 20 to 30 times less than that of a model.

2.5.2.3 Eddy Damping

The eddy damping component, $B_e$, was shown to increase with increasing roll frequency, see Figure 2-5-3, and was at its maximum value at zero speed. The component then decreases with increasing forward speed, see Figure 2-5-2.
2.5.2.4 Lift Damping

Once a ship has forward speed the vast majority of the roll damping will be accounted for by the Lift Damping, $B_L$, and the contribution of any appendages. Lift damping is zero when the ship has no forward speed and it was suggested by Ikeda et al (114) and (115) to increase linearly with ship speed thereafter. However, Haddara and Leung (124) showed that lift damping varied in a more complex way than Ikeda et al proposed and showed that the variation with speed and roll amplitude was non-linear. Thus, at high forward speed this component provides a large portion of the total roll damping.

2.5.2.5 Bilge Keel Damping

The bilge keel damping components ($B_{bkh}$ and $B_{bkn}$ in Figure 2-5-2 and Figure 2-5-3) provide a significant portion of the total roll damping for any given speed and frequency. Their contribution is shown to increase with roll frequency and remain constant with forward speed. If, in addition, the lift force generated by the low aspect ratio bilge keel fin is considered, the effectiveness of the bilge keel will increase slightly with forward speed. Schmitke (81) developed theory to account for this.

2.5.2.6 Other Appendages

Lloyd (64) showed that the lift forces generated by appendages increase linearly in proportion with forward speed. In addition, their contribution to roll damping increases in proportion with the square of the distance of the appendage from the point at which the ship rolls about. Thus, it is apparent that lift damping from appendages will be very significant at forward speed.

In summary, at high forward speed the roll damping would be expected to comprise of the lift damping component and the appendage components with all
other terms negligible in comparison. At low speed all the components identified in Table 2-5-1, with the exception of appendages other than bilge keels and lift damping will contribute to the overall roll damping. A review of the mathematics behind each of the damping components now follows. Alternatively, the reader can safely skip to section 2.6 without losing the flow of the thesis.

### 2.5.3 Skin Friction Damping

Both Schmitke (81) and Ikeda (114) developed equivalent linearised damping formulae to represent the friction damping based on the work of Kato (125):

\[ B_F = \frac{4}{3\pi} \rho \alpha x \omega_0 C_{DF} S_w \bar{r}^3 \]  

2-5-1

Where \( S_w \) is the hull wetted surface area, \( C_{DF} \) is the non-dimensional frictional coefficient, \( \omega \) is the wave frequency and \( \bar{r} \) is defined as the average radius of roll. Ikeda proposed that the hull wetted surface area and the roll radius could be determined using empirical formula based on beam, draught, length and block coefficient, see Appendix 1. These formulae were developed for merchant ships with very high midship section coefficients and high block coefficients.

Schmitke preferred to allow the friction to be calculated element by element and presented equation 2-5-1 as:

\[ B_F = \frac{4}{3\pi} \rho \alpha x \omega_0 C_{DF} \int \int_{C_x} r_f (y \frac{dy}{dl} + z \frac{dz}{dl})^2 dl \]  

2-5-2

Schmitke took an element measured around the girth of the hull, \( dx \), of length \( l \) positioned at \( (x, y, z) \) a distance \( r_f = \sqrt{y^2 + z^2} \) away from the centre of gravity (assumed to be the roll centre), \( L \) and \( C_x \) on the integrals are over length and around the girth respectively, see Figure 2-5-5. This allows the hull wetted surface shape to modelled more accurately than with Ikeda's method.
In the case when forward speed is non-zero Schmitke proposed that the Schoenherr line based on smooth turbulent flow is used to calculate \( C_{DF} \). It was pointed out in the discussion following Schmitke’s paper (81) that the steady flow Schoenherr skin friction drag coefficient could not really be justified to describe an oscillating phenomenon at low to moderate forward speed and Schmitke conceded that other methods would probably give a better approximation.

If forward speed is zero then Schmitke suggested that a method developed by Kato (125) should be used:

\[
C_{DF} = 1.328R_n^{-0.5} + 0.014R_n^{-0.114} \]

The first part of equation 2-5-3 is due to laminar flow (linear damping) whereas the second part is due to turbulent flow (non-linear damping). Where \( R_n \) is the Reynolds number.

Ikeda used Kato’s method (equation 2-5-3) and modified the whole formula (equation 2-5-1) by the following factor to account for speed effects:
The empirical coefficient of 4.1 was proposed by Tamiya et al (126) based on theoretical modelling and experimental results for rolling ellipsoid models. This was shown to be merely adequate at Froude numbers less than 0.2 by investigations carried out by Ikeda, Himeno and Tanaka (118) who proposed a more complicated theoretical method. Using Tamiya et al’s theory, with their own model tests on ellipsoid models, they found that a coefficient of 5.8 provided a better correlation for Froude numbers above 0.2, however they conclude that it may not be necessary to change the original value. This is most likely to be due to the focus of their work on relatively slow merchant vessels and because the ratio of the frictional damping to total roll damping decreases with increasing advance speed (see Section 2.5.2).

Himeno (80) quoted equation 2-5-1 using Kato’s formula for the friction coefficient, equation 2-5-3, in equivalent linear form as:-

\[
B_{F_0} = 0.787 \rho_S \bar{r} \sqrt{\omega u} \left\{ 1 + 0.00814 \left( \frac{\bar{r}^2 x_{40}^2 \omega_d}{\nu} \right)^{0.386} \right\}
\]

2.5.4 Eddy Damping

The first real effort to examine the roll damping of a ship due to the shedding of eddies around the hull during roll motion was made by Tanaka (127). The original work is published in Japanese but has been summarised by Schmitke (81), Chakrabarti (120) and Ikeda, Himeno and Tanaka (117) in English. Tanaka assumed a non-linear damping model (quadratic) and attributed the wave making damping and frictional damping to the linear term for either a ship, representative ship shape, or two dimensional ship section at zero speed. The remainder of the roll damping was taken to be due entirely to eddy shedding damping and this was considered non-linear.
Consider a two dimensional ship section as shown in Figure 2-5-6 rolling with velocity $\dot{x}_4$. At the point where eddies are shed, the local flow velocity in the plane of the hull section will be $\dot{x}_4 r_{ed}$ m/s. Therefore the moment due to eddy making drag will equal:—

$$\text{Moment} = \left[ \frac{1}{2} \rho (\dot{x}_4 r_{ed})^2 S_{sec} C_{TAN} \right] r_{ed}$$  \hspace{1cm} 2-5-6

Equivalent linearization of equation 2-5-6 leads to a term for the eddy damping:—

$$B_E = \frac{4}{3\pi} \omega \rho x_{40} r_{ed}^3 S_{sec} C_{TAN}$$  \hspace{1cm} 2-5-7

Where $B_E$ is the equivalent linear eddy damping, $S_{sec}$ is the wetted surface area of the hull section under consideration, $C_{TAN}$ is a non-dimensional coefficient based entirely on hull shape and $r_{ed}$ is the radius from the roll centre (centre of gravity) to the position where eddies are being generated, see Figure 2-5-6.

Tanaka's approach was to measure the non-linear damping term from forced roll experiments and use this to determine suitable values of $C_{TAN}$. The formulae developed to represent $C_{TAN}$ is given in Appendix 1 which is shown to depend on: the draught and beam of the transverse section of the ship, the steady roll amplitude, $x_{40}$, the radius from the roll centre (centre of gravity) to the position where eddies are being generated, $r_{ed}$, the location of the centre of gravity and the angle $\alpha$, in Figure 2-5-6.
Ikeda, Himeno and Tanaka (114) (117) proposed a different formulation with the drag force opposing roll motion due to eddy shedding. They developed an equivalent linear eddy damping term of the following form:-

\[ B_E = \frac{4}{3\pi} \omega \rho A_{40} T_{sec}^4 L_{sec} C_{IKD} \]  \(2-5-8\)

Where \(T_{sec}\) and \(L_{sec}\) are the draught and length of the ship section for which the drag force is being calculated and \(C_{IKD}\) is a coefficient devised by Ikeda et al. This equation is effectively the same as Tanaka’s (equation 2-5-7) except Ikeda et al make the assumption that the representative area term in the drag force calculation (equation 2-5-6) can be taken as the product of the sectional length multiplied by the sectional draught, \(L_{sec} T_{sec}\), see Ikeda, Himeno and Tanaka (115). This makes Ikeda, Himeno and Tanaka’s formula, equation 2-5-8, very sensitive to changes in draught due to the presence of a term to the fourth power of the section draught. The coefficient \(C_{IKD}\) is developed in Appendix 1. This expression is somewhat more complicated to derive than \(C_{TAN}\) with many more empirical correction factors.

In both methods, to obtain the total eddy making damping for the ship hull, the coefficients \(C_{TAN}\) and \(C_{IKD}\) are calculated for a number of two-dimensional transverse ship sections representing the ship hull. Equation 2-5-7 or equation 2-
5-8 are then used to determine the eddy damping contribution of that particular section and calculations are repeated for the other hull sections and summed together to get the total contribution of the hull.

The eddy damping term thus calculated was for zero forward speed. Ikeda et al proposed a correction factor to take account of the reduction in eddy damping they observed with increasing forward speed:--

\[ B_e = B_e \frac{(0.04K)^2}{1 + (0.04K)^2} \quad 2-5-9 \]

Where the reduced frequency, \( K \), equals:-

\[ K = \frac{\omega L}{U} \quad 2-5-10 \]

Examining the differences between the two methods presented, Brook (121) points out that Ikeda et al’s coefficient \( C_{IKD} \) does not depend on the roll amplitude and makes no allowance explicitly for the angle at the edge of the hull shedding eddies, \( \alpha_e \), unlike Tanaka’s \( C_{TAN} \). Furthermore, Ikeda et al’s formulation for the equivalent linear eddy damping coefficient depends on the fourth power of the sectional draught. This becomes the dominant part of the formula and small changes in draught will lead to large changes in the damping moment. For this reason, it is especially important to understand the bounds of the model tests that Ikeda et al’s work is based on. They used data from either flat plates, two-dimensional sections, or Series 60 model hull shapes. Details of the models are given in Table 2-5-2.
<table>
<thead>
<tr>
<th>Model Type</th>
<th>$L_{sec}$ (m)</th>
<th>$B_{sec}$ (m)</th>
<th>$T_{sec}$ (m)</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-D Rectangular Cylinders</td>
<td>0.8</td>
<td>0.28</td>
<td>0.112</td>
<td>Bilge Radius from 0 to 3 cm</td>
</tr>
<tr>
<td>Flat Plates</td>
<td>0.8</td>
<td>0.024 − 0.064</td>
<td>0.07 − 0.15</td>
<td>Different roll axis positions tested</td>
</tr>
<tr>
<td>2-D Ship Sections</td>
<td>0.8 or 1.0</td>
<td>0.151 − 0.388</td>
<td>0.096 − 0.193</td>
<td>Displaced volumes from 0.00933 − 0.0549 m³</td>
</tr>
<tr>
<td>3-D Ship Models</td>
<td>1.75 or 1.8</td>
<td>0.2365 − 0.2769</td>
<td>0.095 − 0.1108</td>
<td>Series 60 or Single Screw Merchant Vessels</td>
</tr>
</tbody>
</table>

Table 2-5-2: Details of models used by Ikeda et al (117) to develop their formula for eddy damping

Models with beam to draught ratios greater than 2.5 are not catered for and this method should be used with extreme caution for values greater than this. This deficiency led to further research mainly focused at barge-like offshore structures with high beam to draught ratios (128) (129) (130) (131). Ikeda then proposed a simplified formula for barges, see (129):

$$B_E = \frac{2}{\pi} \rho \omega_{40} L_{sec} T_{sec}^4 \left( H_0 + 1 - \frac{\overline{OG}}{T_{sec}} \right) + \left( H_0^2 + \left[ 1 - \frac{\overline{OG}}{T_{sec}} \right]^2 \right)$$

Where $H_0$ is the half-breadth to draught ratio and $\overline{OG}$ is the vertical distance from the origin (located at the still waterline) to the roll axis (centre of gravity) measured positive downwards.

Ikeda pointed out (119) that both formulae (equation 2-5-8 and equation 2-5-11) are only applicable for small roll angles, less than about 5 degrees. Ikeda proposed a further additional component due to the eddy shedding of a bulbous
bow in 1994 (132) based on the bulbous bows drag coefficient. Once again this formula was not accurate for large roll angles, greater than 15 degrees in this case. Ikeda hypothesised that the poor predictions at larger roll angles could be because the interaction between the water surface and shedding vortices cannot be ignored at larger roll angles (119).

Chakrabarti (120) demonstrated that disproportionately high values of the coefficient $C_{IKD}$ occurred for sections at the bow and stern of a three-dimensional barge hull. The empirical formulas for eddy damping using Ikeda et al’s method were derived from experiments with two-dimensional ship sections with end plates provided to avoid end effects. At the ends of the ship the ship sectional area coefficient, $\sigma_s$, and half-beam to draught ratio, $H_0$, used to determine $C_{IKD}$ (see Appendix 1), fall outside of the range tested by Ikeda et al’s experiments and so the formula breaks down. Chakrabarti (120) recommends a reduction in the area coefficient, $\sigma_s$, of 30% at the extreme bow and stern sections to take account of the inaccuracies. This correction is strictly empirical in nature in order to reduce the end effect.

So, in summary, the total eddy damping is made up of the sum of a number of prismatic ship sections representing the hull shape rolling at zero forward speed. Forward speed is accounted for using an empirical correction. The formulae developed are only applicable for small roll angles less than 5 degrees. The empirical formulae are based on model experiments on two-dimensional rectangular cylinders (midship section coefficient equal to 1), flat plates, two-dimensional prismatic ship sections and three-dimensional ship models. The formulae published by Schmitke (81) are applicable to rectangular, triangular and U-Shaped ship sections whereas those published by Ikeda, Himeno and Tanaka (114) (117) are based on model experiments on models with beam to draught ratios less than 2.5 and for wall sided ship sections with flat bottoms and vertical side strakes linked by a circular sectioned bilge of small radius.
2.5.5 *Lift Damping*

For a ship hull moving with forward speed undergoing roll motion, the hull itself will act as a low aspect ratio wing and generate a horizontal lift force and the resulting moment will counteract the roll angle. The area of the hull generating lift will be equal to the length multiplied by the draught with an aspect ratio equal to the draught divided by the length. For a ship, this area will be very large and so the lift generated will be significant at high speed.

Ikeda, Himeno and Tanaka (114) (115) investigated this roll damping moment experimentally and proposed that it could be represented by:

\[
\text{Moment} = \frac{1}{2} \rho U^2 S_L \frac{l_R}{C_L}
\]

Where the representative area \( S_L \) is taken as \( L_{sec} T_{sec} \), \( C_L \) is a lift coefficient and \( l_R \) is a lever arm corresponding to the distance from the roll axis to the apparent centre of the lifting pressure on the hull, see Figure 2-5-7. The difficulty here is determining the lift coefficient and the lever arm with any certainty.

![Diagram of a ship hull generating lift force](image)

*Figure 2-5-7: The generation of a horizontal lift force on the hull due to an angle of attack between the flow and the hull created during roll motion with forward speed*
Ikeda et al then took the assumption that the lift coefficient was proportional to a certain representative flow incidence angle \( \alpha_0 \):

\[
C_L = k_N \alpha_0
\]

\[
k_N = \frac{dC_L}{d\alpha_0}
\]

\[
\alpha_0 = \frac{l_0 \cdot \dot{x}_4}{U}
\]

Where the new lever \( l_0 \) is the distance between the roll centre and the point on the hull surface where the flow incidence angle is equal to \( \alpha_0 \). Yumuro and Mizutani (133) developed assumptions for these terms which were shown by Ikeda et al (114) (115) to be adequate for Series 60 hullforms with block coefficients of 0.70 and 0.80, a cargo ship with a block coefficient of 0.71 as well as for both a tanker and containership model. Yumuro’s assumptions are as follows:

\[
k_N = 2\pi \frac{T}{L} + \kappa \left( 4.1 \frac{B}{L} - 0.045 \right)
\]

\[
\kappa = \begin{cases} 
0 & C_M \leq 0.92 \\
0.1 & 0.92 < C_M \leq 0.97 \\
0.3 & 0.97 < C_M \leq 0.99 
\end{cases}
\]

\[
l_0 = -\overline{OG} + 0.3T \quad , l_R = -\overline{OG} + 0.5T
\]

Where \( C_M \) is the midship section coefficient, equal to the sectional area below the waterline at midships divided by the product of the beam and draught. Hence the (linear) lift damping is determined from the formula in equation 2-5-17, where the term in the brackets is an empirically derived correction applicable when the roll centre is not located on the still waterline. The correction was based on the model experimental work of Ikeda et al (115) using Series 60 hullforms with block coefficients of 0.70 and 0.80, a cargo ship model and a tanker model. The two
levers $l_0$ and $l_R$ are given in equation 2-5-16 as approximations measured from a roll axis.

$$B_L = \frac{1}{2} \rho U L T k_N l_0 l_R \left[ 1 - 1.4 \frac{\bar{O}G}{l_R} + 0.7 \frac{\bar{O}G^2}{l_R l_0} \right]$$

2-5-17

Ikeda et al (115) showed that the product $k_N l_0$ could be determined experimentally from an oblique towing test in which the model is free to heel. Results published in the same paper using values established in this way correlated no better to model test data than the semi-empirical formulas presented in equations 2-5-14 to 2-5-16. The procedure for determining these values from model experiments is given in Appendix 1.

The component developed above in equation 2-5-17 is linear (varies in proportion with the roll velocity, $\dot{x}_r$). In a recent résumé of his work Ikeda showed some results from model experiments with a slender frigate hull with a big bulbous bow, representative of a sonar dome (119). The results from these experiments showed that the lift coefficient of the hull, $C_L$, becomes non-linear when the angle of attack, $\alpha_0$, equation 2-5-13, is greater than 0.1 radians. Further experiments with high speed fishing boats were shown and the lift damping component, $B_L$, was shown to be non-linear at Froude Numbers above 0.4 due to trim effects.

Haddara and Leung (124) conducted oblique towing tests of two fishing boat models, one with a hard chine and one with a round bilge, to investigate the (horizontal) lift damping component proposed by Ikeda et al (115). For both models, graphs of the side force against Froude Number were given and these showed that the force and hence moment is a non-linear function of Froude Number. This goes against Ikeda’s formula in equation 2-5-17 where the roll damping is proportional to forward speed.
Haddara and Leung noted that in Ikeda et al's formulation the lift coefficient is independent of both forward speed and the roll angle. However, results from their model experiments showed that the lift coefficient varied with both the angle of attack of the lift force due to roll motion and with forward speed. As a result of their experimental work they proposed the following formula for the roll moment:

$$\text{Moment} = \frac{1}{2} \rho U^{(2-n)} L T (1+n) \beta_{H-L} \dot{\theta}^n$$

$$n = 1, \ldots, n$$

2-5-18

The roll damping coefficient ($B_L$) is simply equation 2-5-18 divided by the roll velocity. When $n > 1$ this equation becomes non-linear and it must be made equivalent linear if it is to be used in a linear Potential Flow seakeeping computer program. $L$ and the coefficient $\beta_{H-L}$ have to be measured experimentally. Haddara and Leung showed that the coefficient $\beta_{H-L}$ could be modelled by a quadratic polynomial.

Ikeda et al’s formula, equation 2-5-17, was shown to significantly under predict the roll damping at high Froude Number when compared with Haddara and Leung’s experimental findings. Thus, Ikeda et al’s formulation is only valid for certain hull shapes and at low speeds.

Both Blok and Aalbers (134), reporting on research at the Maritime Institute of the Netherlands (MARIN), and Ikeda and Katayama (135), published similar methods applicable to fast displacement ships and planing craft respectively. Both methods make the assumption that the horizontal lift force, depicted in Figure 2-5-7, is small in comparison to the asymmetric vertical lift force, acting on the bottom of the hull during roll motion, for hulls with high beam to draught ratios typical for these types of ship. When the ship has forward speed the vertical velocity of the transverse section of the hull undergoing roll motion induces an angle of attack to the incoming flow and hence a vertical lift force is
developed. The distribution of this force along the length of the ship creates a trim angle.

Both methods are described in greater detail in Appendix 1. The lift damping formula developed by Blok and Aalbers (134) was:

\[
B_L = (k_L) \frac{\rho S_w U B^2}{L/B + 1}
\]

\[
k_L = \text{const} \times (1 - \sin \beta_{\text{null}})
\]

Where \( \beta_{\text{null}} \) is the mean hull deadrise angle in radians, \( k_L \) is the lift slope coefficient based on trim angle and the constant, \( \text{const} \), is dependent on frequency and is determined from a MARIN experimental data base. No results for either of these values were published.

The formula developed by Ikeda and Katayama (135) was:

\[
B_L = \frac{1}{24} \rho B^4 U k_L
\]

No empirical formula was offered for the lift coefficient \( k_L \) and this would have to be determined experimentally.

\textbf{2.5.6 Bilge Keel Damping}

Schmitke (81) reported on a method developed in Japan by Kato (136) for the prediction of an equivalent linear roll damping component due to a pair of bilge keels. Once again the bilge keel force contributing to the roll damping moment was expressed in the form:

\[
\text{Force} = \frac{1}{2} \rho U^2 S_{\text{bk}} C_{\text{bk}}
\]
Where \( S_{bk} \) and \( C_{bk} \) are the bilge keel plan area and a coefficient equivalent to the bilge keel drag coefficient respectively. Taking \( S_{bk} \) to be the product of the bilge keel length and breadth, \( l_{bk} b_{bk} \), and assuming the force on the bilge keel acts at the mid span, a lever from the roll centre (centre of gravity) to this position is defined as \( r_{bk} \). Hence the equivalent linear damping coefficient is obtained by integrating the energy dissipated by the damping moment:-

\[
B_{BK} = \frac{1}{\pi^3} \rho l_{bk} b_{bk} r_{bk}^3 \alpha x_{40} C_{bk}
\]

The coefficient \( C_{bk} \) can be broken down into a number of components depending upon the ship form and Reynolds Number. Expressions for obtaining this term are given in Appendix 1. The calculations to determine \( B_{BK} \) are intended to be performed on a number of hull transverse cross-sections (in the region where the bilge keel is located) summing up across the number of sections used to define the bilge keel. In this case, the bilge keel length, \( l_{bk} \), refers to the length of bilge keel in that hull section, not the overall length.

Both Schmitke (81) and Ikeda and Katayama (119) (132) developed formula to represent the lift force generated by the bilge keel by considering it to represent a low aspect ratio wing. This linear coefficient can be expressed for a pair of bilge keels as (81) (119):

\[
B_{BKL} = \rho \pi r_{bk}^2 b_{bk}^2 U
\]

An alternative approach was put forward by Ikeda, Himeno and Tanaka (116) (118) who started to investigate the effect of bilge keels by conducting experiments on three-dimensional ship models of simplified geometry. They performed experiments on a model with circular cross section and an ellipsoidal waterplane area with the still waterline passing through the centre of the circular section. This model was fitted with pairs of bilge keels with differing breadths and free roll decay tests were carried out, both with and without the bilge keels, to determine the roll damping moment. The components relating to the bilge keels...
alone were separated by subtracting the frictional damping (using the method of Section 2.5.3). Eddy damping is zero for a circular sections and as the model experiments were conducted at zero speed the lift damping was also zero.

The bilge keel roll damping component was considered to comprise of two parts: one due to the normal force developed by the bilge keel, \( B_{BKN} \), and a second due to the modification of the pressure acting on the hull in the region of the bilge keel, \( B_{BKP} \).

The roll damping due to the normal force created by the oscillating bilge keel was:

\[
B_{BKN} = \frac{8}{3\pi} \rho r_{bk} \beta b_{bk} l_{bk}sx_4 f_{bn} 2 C_D \quad 2-5-24
\]

Where, \( C_D \) is the drag coefficient of the bilge keel and \( f_{bn} \) a factor to account for the flow speed increase at the bilge in the vicinity of the bilge keels. Both terms were determined from experiments and the formulae developed are given in Appendix 1.

The bilge keel roll damping due to the pressure distribution around the bilge keels was determined as:

\[
B_{BKP} = \frac{4}{3\pi} \rho r_{bk} \omega x_{40} f_{bn} 2 \int_G \int C_p \text{levers} dG \quad 2-5-25
\]

Where \( \text{levers} \) is the lever arm from the centre of gravity to the point of action of the force due to the pressure difference across the faces of the bilge keels and \( G \) indicates that the integration is around the girth (around the outside edge of the hull section). \( C_P \) is a pressure coefficient and the integral containing this at the end of the formula was obtained from an empirical formula the details of which are given in Appendix 1.
The empirical coefficients derived by Ikeda et al in both equation 2-5-24 and 2-5-25 were determined from model experiments utilising models used in their other research, see Table 2-5-2, as well as some additional models shown in Table 2-5-3 below.

<table>
<thead>
<tr>
<th>Model Type</th>
<th>$l_{sec}$ (m)</th>
<th>$b_{sec}$ (m)</th>
<th>$t_{sec}$ (m)</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-D Rectangular Cylinders</td>
<td>0.5 – 0.8</td>
<td>0.25 - 0.28</td>
<td>0.1 – 0.122</td>
<td>Bilge Radius from 1 to 3 cm</td>
</tr>
<tr>
<td>Ellipsoidal Model</td>
<td>1.5</td>
<td>0.30</td>
<td>0.15</td>
<td>Used for the normal force bilge keel experiments as well</td>
</tr>
<tr>
<td>2-D Ship Sections</td>
<td>0.80</td>
<td>0.237 – 0.398</td>
<td>0.096 – 0.193</td>
<td>Displaced volumes from 0.01775 – 0.0549 m^3</td>
</tr>
</tbody>
</table>

Table 2-5-3: Details of additional models used by Ikeda et al (116) to develop their formula for bilge keel damping

The two components representing the roll damping of the bilge keels, $B_{BKN} + B_{BKP}$ were compared to experimental results from forced roll experiments. The friction damping was removed using equation 2-5-5 (the model experiments were performed at zero speed) and the remainder was assumed to be due only to the bilge keels. As already discussed, eddy damping and wave damping could be neglected due to the shape of the model and the location of the roll axis respectively. Good agreement was demonstrated between the theory and experiment. Model experiments were then conducted with forward speed. It was demonstrated that the bilge keel component to the roll damping moment remained almost constant in the Froude Number range 0 – 0.20. Later work, to incorporate forward speed effects due to lift generated by the bilge keel as though it were a low aspect ratio wing, culminated in equation 2-5-23 as already discussed.
2.5.7 Other Appendages

Adrian Lloyd (64) in his book on seakeeping presents a method for determining the roll damping contribution for any generalised appendage based on the lift and drag forces produced. Consider a ship undergoing oscillatory roll motion as shown in Figure 2-5-8.

Figure 2-5-8: Lift and Drag Forces acting on an appendage for Lloyd's Method (64)

The drag force acting on the appendage will depend on the drag coefficient, \( C_D \), and the plan area of the appendage, \( A_{app} \), equal to the span multiplied by the chord for a rectangular appendage. Using the method outlined in Section 2.2.3.2 the moment due to the drag force is then used to obtain an equivalent linear damping coefficient:

\[
B_{Ad} = \frac{4C_D^2}{3\pi} \rho x_{40} \omega \sum A_{app} r_{app}^3
\]

Where the summation is across the total number of appendages. In this case, Lloyd recommends a value of 1.17 for the drag coefficient.

Moving on to the lift force, the total force on the appendage will be equal to:-
\[ \text{Force} = (\text{Lift}) \cos \alpha_{\text{ind}} + (\text{Drag}) \sin \alpha_{\text{ind}} \quad 2-5-27 \]

If the induced angle of attack on the foil is small, \( \alpha_{\text{ind}} \), then equation 2-5-27 reduces to just the lift component \((\cos \alpha_{\text{ind}} \approx 1)\). If the forward speed is non-zero an angle of attack is induced on the appendage. This angle can be approximated by:

\[
\alpha_{\text{ind}} = \tan^{-1}\left( \frac{r_{\text{app}} \dot{x}_4}{U} \right) \quad 2-5-28
\]

\[
\approx \left( \frac{r_{\text{app}} \dot{x}_4}{U} \right) \quad r_{\text{app}} \dot{x}_4 \ll U
\]

The (lift) force on the appendage becomes:

\[
\text{Force} = \frac{1}{2} \rho U^2 \sum A_{\text{app}} C_L
\]

\[
= \frac{1}{2} \rho U \sum A_{\text{app}} r_{\text{app}} C_{L\alpha} 
\]

Where \( C_{L\alpha} \) is the slope of the lift coefficient \( C_L \) plotted against the induced angle of attack \( \alpha_{\text{ind}} \) at an induced angle of zero degrees. This force is proportional to the roll velocity and so no equivalent linearization techniques are required. Hence, the damping moment per radian per second is:

\[
B_{\text{Al}} = \frac{1}{2} \rho U \sum A_{\text{app}} r_{\text{app}}^2 C_{L\alpha} 
\]

Lloyd suggests that the lift slope curve can be calculated from the formula developed by Whicker and Fehlner (137) converted to SI units, where \( AR_e \) is the effective aspect ratio of the appendage:

\[
C_{L\alpha} = \frac{1.8\pi AR_e}{1.8 + \sqrt{AR_e^2 + 4}} 
\]

2-5-31
This theory was developed based on wind tunnel experiments on fins attached to a backboard with aspect ratios similar to those used for roll stabilisers on a monohull frigate. The presence of the large backboard causes the effective aspect ratio for calculations to be double the geometric aspect ratio. The applicability of this theory for ratios of effective to geometric aspect ratio other than two is not known.

This relationship is plotted in Figure 2-5-9, where it can be seen that the lift slope increases with increasing effective aspect ratio, but that there are diminishing returns for larger effective aspect ratios.

![Relation Between Aspect Ratio and Lift Slope](image)

Figure 2-5-9: Relationship between lift slope and effective aspect ratio according to Whicker and Fehlner (137)

2.5.8 Other Components

Klaka et al (138) (139) (140) (141) (142) (143) conducted investigations into the roll motion of yachts at zero speed; in particular he focused on the damping moment of the keel. Large keels are fitted to yachts to counter the heel angle developed when sailing as a result of the transverse component of lift developed
by the sail rig. Two series of model experiments were conducted, the first were free decay of a semi-circular cross section model with a keel attached (142) (143). A further series of forced rolling experiments was conducted with four three-dimensional flat plates to determine the roll moment and the plate motion (140). This second series of experiments were deemed necessary due to inconsistencies in the analysis of the free decay results, the values obtained were highly sensitive to the portion of the data used (142). A further series of full scale trials were conducted using a 10 metre yacht, where both free decay in calm water and motions in irregular waves were recorded (138) (141).

Ikeda, Tanaka and Himeno (123) (144) investigated the effect of hull shape on roll damping by considering hull shapes typical of small Japanese fishing vessels. The vessels had beam to draught ratios in the range 3.5 – 5.0 with rise of floor and a hard chine joining the bottom of the hull to the side. Model experiments were conducted and they showed that these hulls have comparatively large wave damping (in comparison with the merchant ships Ikeda et al had tested previously described) which increases with increasing rise of floor. They showed that a vessel with a hard chine has about twice the roll damping of a similar vessel with a round bilge. The eddy damping created by the hard chine increases roll damping significantly but decreases with increasing rise of floor. On the last point Ikeda et al pointed out that this was partly due to the reduction in the moment lever to the point where eddies were generated (this point moves vertically upwards and slightly inwards with increasing rise of floor) as well as the change in local flow velocity at the chine.

For this type of hull shape an improved formula for the eddy damping was proposed (145) (146). This formula is presented in Appendix 1.

They also considered the effect of an overhung deck which is wider than the waterline beam of the ship. This serves to increase the buoyancy and hence righting moment of the vessel at large roll angles. Forced roll experiments were conducted on a variety of models and the results showed that the overhung deck did not have a significant effect on the roll amplitude at resonance. The damping effect was reduced if there was shear in the overhung deck as less of it would then
be submerged for a given roll angle. These results are not surprising because even if the overhung deck were immersed for the first few roll cycles of a free decay, thereafter, as roll amplitude decreases, it would not be immersed and have no effect on the roll damping.

Another investigation was to consider the effect of a skeg, large examples of which were commonly fitted to Japanese fishing vessels. The damping contribution of two types of skags was measured in forced roll experiments on three two-dimensional ship shaped sections representative of fishing vessels. From this series of experiments an empirical formula was developed based on the pressure distribution around the skeg. The formula developed is given in Appendix 1.

Ikeda and Katayama (132) conducted experimental investigations on models of high speed slender vessels to measure the roll damping. Forced roll experiments were performed on two models, both representative of high speed hulls fitted with a skeg, twin propellers and twin rudders, one of which had a bulbous bow. Results from the model experiments were compared with roll damping predictions using Ikeda et al's existing theory for damping components due to hull friction, lift, eddy shedding, bilge keels and a skeg (described in Sections 2.5.3 to 2.5.6) with the wavemaking damping calculated using Potential Flow Theory. This showed that the roll damping calculated for the model with the bulbous bow was less than that measured by experiment. Ikeda and Katayama then proposed an empirical damping component for the bulbous bow based on the model experiment results for that particular model only. A further series of model experiments would be required to make a more general formula.
2.6 Assumptions in Roll Motion Prediction of Monohulls

Current methods for predicting monohull roll motion assume that:-

1. The motions of the ship in any single degree of freedom are adequately modelled by a forced spring-mass-damper system. Thus, motions in each degree of freedom are represented by a second order differential equation. The complete motion of the ship is determined by coupling the six equations representing surge, sway, heave, roll, pitch and yaw in a suitable manner.

2. The unknown terms in the equation of motion can be determined using Potential Flow Theory which assumes that the flow is incompressible, irrotational and neglects viscosity.

Assumption 1 is usually further simplified by the use of linear theory which assumes in addition that:-

a. Motions in each degree of freedom are represented by a second order linear differential equation with constant coefficients.

b. A coupled linear equation of motion assumes that input sinusoidal waves of constant amplitude and frequency give rise to motions in each degree of freedom which are also sinusoidal with constant amplitude and frequency. There will, however, be a phase difference between the forcing term and output motions.

c. When a wave is encountered by a ship the ship is assumed not to distort the wave in anyway as it passes.

d. For assumptions b and c to hold true, the wave amplitude is considered to be small when compared to the ship length and wavelength. This implies that the amplitude of the output motions will also be small.

e. The ship is assumed to be wall sided above the waterline and so no account is taken of any changes in hull shape above the waterline.
Returning to the first two assumptions, the second assumption causes problems with the prediction of roll motion as viscous effects are important. To get around this problem one of the following two approaches is used:

A. The roll damping term in the equation of motion is determined from roll decay or forced rolling experiments. The form of the roll damping term is generally taken to be non-linear. There are then two further options:

i. The non-linear term is made equivalent linear and the final term is used in a Linear Potential Flow Seakeeping Code to predict the roll response. The equation of motion is solved using an iterative approach to obtain the output steady roll amplitude.

ii. The linear and non-linear roll damping terms measured in the roll experiments are used with a non-linear formulation of the equation of motion which is then solved in the time domain.

B. The roll damping term in the equation of motion is determined by summation of a number of components which are assumed to represent the total roll damping. These components can be either linear or non-linear and where they are non-linear they are generally made equivalent linear (and they will then vary as a function of the output steady roll amplitude). The equation of motion can then be solved using an iterative approach to obtain the output steady roll amplitude. These components are usually developed from model experiment results on ship models or representative ship sections.

The roll experiments described in Option A have the following assumptions:

- In a free or forced rolling experiment the roll motion can be modelled by an uncoupled equation of roll motion.
- The output roll motion in the experiment is pure roll, unaffected by motions in the other degrees of freedom.
- Unless a Parameter Identification Technique is used, see Section 2.4.2.2, it is generally assumed that the inertia and added inertia term, \( I_a + A_{44} \), and
the roll stiffness term, $C_{44}$, are constant. This implies that over the range of roll angles tested the $GZ$ curve is linear.

- The roll damping coefficients derived from the model experiments are constant.
- The chosen roll damping model, e.g., quadratic with linear and quadratic damping coefficients, adequately models the recorded roll motion.

A note of caution about option A(ii): As discussed in Section 2.3, non-linear seakeeping theories are still in their infancy and in general, it would be more correct to refer to them as linear theory with particular non-linear extensions. The literature review has revealed no methods which can take linear and non-linear roll damping coefficients (which vary in proportion to the roll velocity) and utilise Potential Flow Theory to solve the equation of motion for all six degrees of freedom. The closest approach to this which has been achieved so far uses a non-linear Potential Flow seakeeping code with equivalent linear roll damping terms. Thus, roll is still treated as a linear term in the equation of motion. Currently, the only methods which allow the use of both linear and non-linear damping coefficients are those which investigate a single or twin degree of freedom equation of motion which is solved using an analytical technique. In this case, either the coefficients of the equation of motion, with the exception of roll, are known, or they are determined using a Parameter Identification Technique.

Option B, which utilising a range of semi-empirical components has the following assumptions:

- The components chosen adequately model the roll damping of the hullform in question, including all appendages.
- The assumptions used to develop each component are appropriate for the hullform in question.
- The range of model experiments which underpin each component are for hull shapes or hull sections with similar properties to the hull in question (e.g. half-length to draught ratio, block coefficient, midships section coefficient, etc.).
When using Option B it would be sensible to ensure that similar hull shapes to the one in question have been investigated previously using the component damping theories and, in addition, results from these analyses have been shown to give good predictions of roll motion when compared to experimental results.
2.7 Monohull Roll Prediction Best Practice

In the previous sections of this chapter the theories available for predicting monohull roll motions have been set out. It remains to demonstrate that these tools are used by researchers and ship designers and to identify which are considered to represent best practice.

Section 2.2.4 showed that, with the exception of rolling, motions in the other five degrees of freedom, surge, sway, heave, pitch and yaw, could be predicted with adequate accuracy, when comparing computational results to model experiments, using Linear Potential Flow Theory solving the equation of motion in either the frequency or time domain, see Bertram (66); Bailey, Hudson, Price and Temarel (96); Hudson, Price and Temarel (97); Bruzone, Gualeni and Sebastiani (69); Maury, Delhommeau, Ba, Boin and Guilbaud (98); and Takaki, Lin, Gu and Mori (99).

Chan (147), Ikeda (119) and Schmitke (81) show that Linear Potential Flow Seakeeping Codes augmented by semi-empirical theoretical formula, such as those described in Section 2.5, give adequate predictions of roll motions when compared with model test results. Dallinga (148) uses such methods to scope a range of model experiments to investigate the hydro-mechanic aspects of fin stabilisers.

Lloyd and Crossland (149) showed how roll motion predictions could be obtained using the results from roll decay experiments on a model and a Strip Theory Linear Potential Seakeeping Code. The results were then compared with seakeeping experiments with a model in regular waves showing adequate correlation over a range of wave headings. Jenson, Mansour and Olsen (67), obtained roll motion predictions using estimates of the non-dimensional damping factor, $\zeta$, to get correlate theoretical roll motion predictions, obtained using a Linear Potential Flow Seakeeping Code, with model experiment results in regular waves for a range of ship types.
Non-linear Potential Flow Seakeeping Computer Codes have gained popularity in recent years due to the increase in available computational processing speed. Until very recently solution of the equation of motion using linear or non-linear theory was impractical due to the computational effort required. The key advantage of such methods is that in contrast to the linear approach, in which the body boundary condition is satisfied on the portion of the hull under the mean water surface, non-linear methods satisfy the body boundary condition exactly on the portion of the instantaneous body surface below the incident wave. In addition, linear free surface boundary conditions are satisfied on the incident wave surface rather than on the undisturbed mean water surface as with linear theory. Using this approach, both the body motions and incident waves can be large.

Salvesen and Lin (78) utilised a Non-linear Potential Flow Seakeeping Computer Code (LAMP) to predict motions of a CG47 AEGIS Cruiser in storm conditions. Even in this extreme sea, when the LAMP code was run in the linear mode acceptable motions were predicted in heave and pitch. Roll motions were accounted for using semi-empirical theory described in Section 2.5. However, the non-linear theory was required to get adequate predictions of the vertical hull bending moment.

Thus, best practise in monohull roll prediction is to determine the roll motion using a Linear Potential Flow Seakeeping code augmented by either roll damping coefficients measured in rolling experiments or by using a collection of empirically derived theoretical components. Where the experimental results or theoretical components are non-linear they are made equivalent linear to allow the equation of motion to be solved. The preference is for computer codes which solve the linear equation of motion in the frequency domain as calculations are an order of magnitude quicker than when the equation of motion is solved in the time domain. Non-linear Potential Flow Seakeeping codes are only used for large amplitude motions and where the hull section shape varies considerably above the mean water line. Computer codes available which utilise a non-linear formulation for the equation of motion still treat the roll damping term, $B_{44}$, as linear and
either experimentally derived or empirical derived theoretical damping components are required to obtain accurate roll motion predictions.
2.8 Conclusions

Adequate roll motion predictions can only be obtained using Linear Potential Flow Theory if the roll damping term in the equation of motion, $B_{uu}$, is either replaced by experimentally derived roll decay coefficients or augmented by a collection of empirically derived theoretical roll damping components.

Potential Flow Theory only accounts for contribution of wave radiation to roll damping. Generally speaking, wave radiation damping has been shown to provide less than 30% of the total roll damping for a monohull ship. Roll damping due to appendages, lift generated by the hull during rolling at forward speed and viscous effects such as roll damping due to eddy shedding from the hull need to be included if accurate roll motion predictions are to be achieved.

Potential Flow Theories can be used to solve the equation of motion when formulated using linear theory or with some non-linear terms. The linear equation of roll motion can be solved rapidly in the frequency domain using Potential Flow Theory. Solution of this equation in the time domain takes significantly longer. The non-linear methods require solution in the time domain but have the advantage that the body boundary condition is satisfied exactly on the portion of the instantaneous hull surface below the incident wave. With the linear formulation, the body boundary condition is satisfied only on the portion of the hull under the mean water surface and so the submerged hull shape above the mean water surface is not considered. Thus, non-linear Potential Flow methods are better suited to large amplitude waves or for hull shapes with considerable variation in the hull shape above the mean water position.

All existing formulations of the equation of motion using non-linear Potential Flow methods assume that the relationship between the roll damping term, $B_{uu}$, and the wave forcing is linear. Therefore non-linear damping coefficients measured from model experiments or non-linear empirically derived theoretical roll damping components can only be included if they are made equivalent linear.
This increases the computational time required by time domain non-linear Potential Flow Seakeeping codes by an order of magnitude because, after equivalent linearization, the non-linear damping terms vary in proportion with the output roll amplitude and this then has to be determined by iteration at each and every time step.

Roll motion results using the more complex non-linear theory have not been shown to be vastly superior to results obtained using linear theory, except when the hull shape varies considerably above the still water position or when motions in large amplitude waves are required. Therefore, considering non-linear calculations take a very significant length of time using current computational processing abilities, linear methods are still used in preference to non-linear methods.

Hence, best practise for determining monohull roll motions theoretically is as follows:-

- Either measure the roll damping in a model experiment or determine it using an appropriate range of empirically derived roll damping components.
- Replace or augment the roll damping term in the equation of motion with that derived above and then solve the equation of motion using a Linear Potential Flow method. This requires any non-linear terms representing the roll damping to be made equivalent linear.

Having explored in detail the methods used to predict monohull roll motion in this chapter, establishing current best practice, a literature review of existing research into multihull roll motion prediction is required to see how it differs, if at all, from the monohull approach. This is the focus of the next chapter.
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3.1 Introduction

In Chapter 2 monohull roll motion prediction methods were reviewed and best practice established. This showed that accurate roll motion predictions could be obtained using linear Potential Flow Theory so long as the roll damping term in the equation of motion, $B_{44}$, was either replaced by damping coefficients measured in a roll decay test (converting any non-linear coefficients to an equivalent linear form) or augmented with a series of empirically based theoretical damping components. Quasi-non-linear methods were only required if large amplitude motions were considered or if the hull shape varied significantly above the mean water surface.

In this chapter multi-hull roll prediction methods will be reviewed, focusing on the differences when compared with monohull methods, so that best practice for trimarans can be established. This review may conclude that specific analysis methods are required for accurate determination of trimaran roll motion. Alternatively, it may identify a number of pre-existing monohull approaches that can either be applied directly to trimarans, or that can be readily adapted to make them more suitable for trimarans.
3.2 Overview

There are many configurations of multi-hulled ships which have been proposed for various purposes over the years. This chapter focuses only on multi-hull configurations where there has been published research on the roll response or theoretical approaches for predicting the roll response. A review of the available literature shows that this confines the study to Catamaran's, Small Water-plane Area Twin Hull Ships (SWATHS) and or course Trimarans. The intent is to contrast these methods with those identified for a monohull in Chapter 2. The reader is once again reminded of the comments of Andrews and Hall (6) when reporting on the progress of the MoD research programme on trimarans in 1995:-

"The current message in the exploration of this configuration is that it should be seen not as another hybrid advanced naval vehicle or even a typical monohull but rather a variant of the conventional monohull". If this assertion is true, then trimaran analysis roll techniques should expected to be broadly similar to those suited to a monohull.
3.3 Catamarans and SWATHS

The roll response of a catamaran or SWATH can be investigated in a similar fashion to a monohull using a Potential Flow Seakeeping computer code. Calculations in either the frequency or time domain allow the roll damping due to wave radiation to be determined. As discussed in Chapter 2, viscous effects are not accounted for in the Potential Flow formulation and these viscous effects form an important contribution to the total roll damping. The time domain approach has the added advantage that as the hull moves relative to the sea surface Potential Flow methods can be applied that adjust the immersed hull shape as the solution proceeds and thus incorporate non-linear effects due to the local variations of the draught of the immersed part of the ship and local changes in beam through time.

Catamaran and SWATH motions have been computed in the frequency domain using Strip Theory by Lee and Curphey (150) and Centeno, Fonseca and Guedes Soares (151) (152). Green Function methods have been utilised by Fang, Chan and Incecik (153) using a two dimensional pulsating source panel method, as well as by Chan (154) (155) and by Schellin and Rathje (156) (157) using three dimensional panel methods. Hermundstad et al (158) performed a linear hydroelastic analysis using High Speed Strip Theory; see Faltinsen (159). Catamaran and SWATH motions have been computed in the time domain by Holloway and Davies (160) (161) using a two dimensional time stepping Strip Theory and similarly by Fang, Chan and Incecik (162) and Fang and Lin (163) (164).

Of the researchers mentioned in the previous paragraph who discuss viscous motion damping, all bar one, Holloway and Davis (160) (161), use the same empirical approach to determine the viscous damping effects in heave, pitch and roll. This is known as the Viscous Lift and Cross-flow Drag Method. The method is described in the next section.
3.3.1 The Viscous Lift and Cross-flow Drag method of Determining Viscous Damping in Heave, Pitch and Roll

The Viscous Lift and Cross-flow Drag Method was first used for SWATHS by Lee and Curphey (150). The foundation of the method is theory based on experimental results of side forces generated on slender bodies of revolution with moderate trim angle reported by Thwaites (165). This theory would appear to be reasonable for the submerged hulls of a SWATH. However, catamaran hulls are generally not bodies of revolution and they are not fully immersed in the fluid. Hence the theoretical basis of this method is not really suited to catamarans.

Lee and Curphey (150) postulated that the semi-submersible hulls of the SWATH configuration do not generate surface waves when oscillating in the vertical plane. This means that the wavemaking damping component for motion in the vertical modes (heave, pitch and surge) will be small. As roll motion of a twin hull ship could also be described as motion caused by a moment due to differential heaving of the two hulls the same could be said to be true in roll motion too. Thus, the viscous damping will form the majority of the total damping for a twin hull ship and the percentage of the total damping due to viscous effects will be greater than for a monohull. Therefore, accurate roll motion prediction will depend on the accuracy of the method used for predicting viscous damping.

Lee and Curphey (150) attempted to predict the motions using damping terms (in heave and pitch) determined from forced oscillation of a model. They found that motion predictions using damping terms derived from these model tests were unrealistically large when compared with motions measured in model experiments in regular waves. They were unable to offer any reason for the poor correlation and stated:-
"The foregoing description [replicated above] is merely intended to emphasize the complexity of the problem we are attempting to solve and thus demonstrates that a simplified theory could hardly be expected to provide more than qualitative agreement. In the present analysis an empirical approach will be adopted to determine the supplemental damping required to reasonably predict the motion."

This is re-emphasized to remind the reader that the Viscous Lift and Cross-flow Drag method at best provides predictions of the damping of the correct order of magnitude by modelling a complex situation in a simplified manner.

The basic idea is, for a harmonically oscillating slender body of revolution advancing with forward speed in regular waves with moderate trim angle, that the relative fluid motion on the body acts across the flow, hence the term cross-flow drag. In addition, the angle of attack between the relative fluid velocity and the body gives rise to a viscous lift force.

Thwaites (165) showed that the side force acting on the body can be expressed in the form:-

\[ Force = \frac{1}{2} \rho U^2 \sum A_p \sin \tau_{body} \sin \alpha_r \left[ \left( C_L \cot \alpha_{body} \right) + C_D \right] \]  

Where the first term in the brackets is due to the viscous lift and second term due to the cross-flow drag. The summation is along the length of the submerged SWATH hull for a number of sections. \( A_p \) is the projected area of a cross-section of the submerged SWATH hull in the horizontal plane and \( \tau_{body} \) is the trim angle of the body of revolution. The drag coefficient, \( C_D \), is known as the cross-flow drag coefficient and the lift coefficient, \( C_L \), as the viscous lift coefficient. For a harmonically oscillating body in the vertical plane in regular waves with constant forward speed, \( U \), equation 3-3-1 was assumed by Lee and Curphey (150) to take the following form:-
Force = $\frac{1}{2} \rho \sum A_p \left( U^2 C_L \alpha_r + C_D |U_r| U_r \right)$  \hspace{1cm} 3-3-2

Where $\alpha_r$ is the angle of incidence of the flow for a cross-section of the SWATH hull (analogous to $\tau_{body}$) and $U_r$ is the relative oscillating velocity of the cross-section of the SWATH hull. In the vertical plane, $U_r$ is a function of roll angle, pitch angle, heave displacement, maximum breadth of the hull, transverse separation of the hulls and the vertical velocity of the fluid induced by the incoming wave. The $C_D |U_r| U_r$ term in equation 3-3-2 results in the cross flow drag term varying as a non-linear function of motions influenced by the relative oscillating velocity $U_r$, i.e. roll, pitch and heave. The viscous lift term is linear with respect to roll, heave or pitch motion and increases in proportion with the square of the oscillating velocity.

In roll motion, the linear viscous lift, the first term in the brackets in equation 3-3-2, is analogous to Ikeda et al.’s (115) monohull roll damping moment due to hull lift, see Chapter 2, Section 2.5.5. The second cross-flow drag term captures all other forms of viscous damping, for example eddy shedding.

A damping moment can be derived by multiplying the force acting on a section of the submerged SWATH hull by a lever arm measured from the roll centre (taken as the centre of gravity). For roll motion, this lever arm is the perpendicular distance from the centre of gravity to the middle of the hull section. A roll damping term can be derived by dividing this moment by the roll velocity, $\dot{x}_r$.

As already discussed, the cross-flow drag part of this term will vary as a non-linear function of roll velocity and so the newly developed roll damping term has to be made equivalent linear before it can be used in a Potential Flow seakeeping code.

The lift and drag coefficients in equation 3-3-2 are assumed to be constant, remaining the same for each hull section. \textit{Values for these two coefficients must be determined experimentally.} Thwaites (165) gave values based on experiments
on airships with circular or polygonal sections (the polygonal sections had between 15 and 19 sides). He postulated (on page 416 of the first edition) a good value for $C_L$ to be 0.07 and values of $C_D$ between 0.4 and 0.7. Schellin and Rathje (156) compared theoretical motion predictions obtained using the viscous lift and cross-flow drag approach in heave and pitch to model experiment results for two single strut SWATH ships. They used values of the lift coefficients of 0.027 and 0.07 (I believe these to be misquoted in the paper as 0.0027 and 0.007) and corresponding values of the cross-flow drag coefficients of 0.40 and 0.60. The chosen values gave good correlation between the theory and model experiments.

Chan (155) compared computations from his three dimensional Potential Theory, calculating the viscous lift and cross-flow drag using Lee and Curphey's method (150), with model experiment results for a catamaran and a SWATH vessel in all six degrees of freedom. As already mentioned, the application of this theory to catamaran hulls is somewhat questionable due to the body cutting the free surface and because the hulls are not bodies of revolution. Chan used values of the viscous lift coefficient and cross-flow drag coefficient of 0.07 and 0.4 respectively. For the SWATH ship theoretical roll motion predictions were compared with model experiment results at zero speed in beam and bow quartering seas. With constant viscous damping coefficients the roll damping was predicted correctly in bow quartering seas but over predicted in beam seas. Thus, using constant viscous damping coefficients may not yield accurate roll motion predictions.

3.3.2 Davis and Holloway's method of Determining Viscous Damping in Heave, Pitch and Roll

Davis and Holloway (160) developed a new two-dimensional Potential Flow Strip Theory based time domain seakeeping prediction method. In this approach the strips representing parts of the hull included both the hull surface and transient motion of the surrounding water sections. This theory accounted for the radiated
wave damping of the hull and viscous friction effects were included by the addition of a vertical damping force on each hull section.

Assuming a slender hull, small motions and high speed Davis and Holloway (160) showed that the vertical force contributing to motion damping is:

\[
\text{Force} = \frac{1}{2} \rho U C_D \sum B_{\text{sec}} L_{\text{sec}} \dot{z}_{\text{sec}}
\]

Where \( \dot{z}_{\text{sec}} \) is the vertical velocity of a ship section relative to the local water surface and the summation is over the number of sections defining the hull. Assuming this force to apply only during upward motion gives an average (heave) damping coefficient over each cycle:

\[
B_{33} = \frac{\text{Force}}{2 \dot{z}_{\text{sec}}} = \frac{1}{4} \rho U C_D \sum B_{\text{sec}} L_{\text{sec}}
\]

Davis and Holloway (160) point out that application of this simple theory to unsteady flow with a substantial cross-flow element is questionable and stated that they accept the approximation for the purpose of obtaining an order of magnitude estimate of the viscous damping. The drag coefficient, \( C_D \), was assumed to be the same for each of the strips representing the hull.

For a catamaran with wide transverse hull spacing, roll motion is better described as differential heave of the two hulls and so increasing the heave damping will reduce roll motion. Davis and Holloway (160) used this additional damping and predicted motions for three catamarans in heave, pitch and roll and compared them with published model experiment results. In addition, Holloway and Davis (161) conducted model experiments for two Semi-SWATH catamaran models and compared motions in heave and pitch with theoretical predictions using this approach. In both cases the authors selected drag coefficients that gave the best correlation between heave and pitch motion predictions and model experiment results at resonance. Values for the drag coefficient in equation 3-3-4 for catamarans according to Davis and Holloway (160) ranged from 0.090 and 0.105
for Series 64 models depending on the hull spacing, with the larger number being at the lower hull spacing; 0.023 for a NPL hull for each of two values of hull spacing; and 0.024 for a Delft University of Technology hull. The later two hulls had well-rounded section shapes and hence lower viscous drag. For the SWATH models of Holloway and Davis (161) values ranged from 0.185, for a Froude number of 0.2, to 0.0 at a Froude number of 0.7 for one hull shape and from 0.100 at a Froude number of 0.2 to 0.026 for a Froude number of 0.7 for another. In the catamaran studies the drag coefficient was not varied with forward speed.

Comparisons of the experimental and theoretical roll RAO for the NPL hull with wide spacing and the Series 64 hull were good. However, whilst the magnitude of the maximum roll RAO was well predicted for the NPL hull with narrow hull spacing, the encounter frequency at which the peak RAO occurred was too low. The theoretical encounter frequency of the maximum roll RAO was also too low for the Delft hull which had narrow hull spacing similar to the NPL catamaran. The peak roll RAO for both the Delft and NPL hulls was not well defined in the model experiment results because the resonant peak occurred close to the lower end of the range of frequencies used in the experiments and so it is difficult to comment further on the theoretical roll motion predictions.

3.3.3 Investigations with a Single Degree of Freedom Equation of Motion

Francescutto and Cardo (37) explored the roll motion of multihulls in beam seas in regular waves to investigate the most appropriate form of the uncoupled single degree of freedom roll equation of motion. For a catamaran they came up with two different linear versions of the equation of motion in beam regular waves. The first treated the catamaran in the same way as a monohull and the second considered the roll stiffness to be accounted for as a function of the heave stiffness of the twin hulls and the wave forcing moment to be influenced by the heave of the side hulls.
The form of the equation of motion Francescutto and Cardo used for the monohull was based on the work of Contento, Francescutto and Piciullo (112). It is assumed that the wavelength is much greater than the beam of the monohull:

\[ \ddot{x}_4 + b_{44} \dot{x}_4 + \omega_n^2 x_4 = \alpha_s \alpha_{se} \omega_n^2 \cos \omega t \]

\[ \alpha_s = \frac{2\pi \eta_0}{\lambda} \]

Where \( \alpha_s \) is the wave slope and \( \alpha_{se} \) is known as the effective wave slope coefficient. For a monohull, Francescutto, Contento, Biot and Schirrfrer (113) defined this effective wave slope coefficient as:

\[ \alpha_{se} = \alpha_{m1} - \alpha_{m2} \left( \frac{\omega_s}{\omega_n} \right)^2 \]

Where \( \alpha_{m1} \) and \( \alpha_{m2} \) are two constant coefficients. Francescutto, Contento, Biot and Schirrfrer showed how a parameter identification technique could be used to determine values for the unknown coefficients in equations 3-3-5 and 3-3-6 for a monohull by fitting these equations to model experiment results in beam seas. This exercise was carried out on five different models of ro-ro ferries. The form of the damping term \( b_{44} \) in equation 3-3-5 was taken as cubic (with linear and cubic damping terms).

In the catamaran based model the assumption was made that the beam of a single catamaran hull was much smaller than the horizontal separation of the hulls. Making this assumption the equation of motion was recast in the following form for a catamaran undergoing forced roll motion in regular beam waves (37):

\[ I_c \ddot{x}_4 + B_{44} \dot{x}_4 + M_R(x_4) = M_W(t) \]

Where \( M_W(t) \) is the time varying wave disturbing moment and \( M_R(x_4) \) is the roll restoring moment which will be a function of the instantaneous roll angle \( x_4 \).
The roll inertia and added roll inertia for the catamaran is $I'_c$, which is the sum of the roll inertia and added inertia of the two hulls and the moment caused by the heave mass and added mass:

$$I'_c = 2(I_4 + A_{44})_{cat} + 2(M + A_{33})_{cat} \left( \frac{h_{cat}}{2} \right)^2$$  
3-3-8

Where the subscript $cat$ signifies that the quantities in the brackets are for a single catamaran hull and $h_{cat}$ is the horizontal distance between the centre lines of the two hulls of the catamaran. The $h_{cat}^2$ term will make the second term in equation 3-3-8 much larger than the first and hence this first term can be ignored:

$$I'_c \approx 2(M + A_{33})_{cat} \left( \frac{h_{cat}}{2} \right)^2$$  
3-3-9

Rather than assuming the roll restoring moment $M_R(x_4)$ to be due to the hydrostatic righting moment (i.e. the lever $\overrightarrow{GZ}$), the assumption is made that the extra heave force generated when the ship rolls due to the increase in draught of one catamaran hulls causes the majority of the righting moment. For small roll angles, $\tan x_4 \approx x_4$, and assuming the catamaran hulls are wall sided above the waterline this roll restoring moment can be expressed as:

$$M_R(x_4) \approx (C_{33}) \left( \frac{h_{cat}}{2} \right)^2 x_4$$

$$= \rho g A_w \left( \frac{h_{cat}}{2} \right)^2 x_4$$  
3-3-10

$$= \frac{1}{2} \rho g A_{wc} h_{cat}^2 x_4$$

Where $C_{33}$ is the heave stiffness and $A_w$ and $A_{wc}$ are the waterplane area of the ship and of one catamaran hull respectively. If this roll restoring moment is
divided through by the roll inertia term $I'_c$ an equivalent “roll natural frequency”
can be expressed for this equation of motion. Making the assumption that the
heave added mass $A_{33}$ is equal to the ship mass $M$ this equivalent natural
frequency, $\omega_{nc}$, can be expressed as:-

$$\omega_{nc}^2 = \frac{A_{wc} g}{2\mathcal{V}}$$ 3-3- 11

Finally Francescutto and Cardo (37) gave the wave induced rolling moment $M_w$
as:-

$$M_w = \alpha_{cl} \rho g A_{wc} 2\eta_0 h_{cat} \sin\left(\frac{kh_{cat}}{2}\right) \sin(kx_{mid} - \omega_s t)$$ 3-3- 12

$$\frac{M_w}{I'_c} = \alpha_{cl} \frac{2\omega_{bc}^2 \eta_0}{h_{cat}} \sin\left(\frac{kh_{cat}}{2}\right) \sin(kx_{mid} - \omega_s t)$$

Where $\alpha_{cl}$ is a coefficient to scale the wave forcing moment. The position of the
wave peak relative the centre of gravity of the catamaran is $x_{mid}$. This wave
excitation moment is proportional to the wave amplitude, $\eta_0$, (linear seakeeping
theory) and becomes oscillatory when the term $\sin(kh_{cat}/2)$ tends towards one.
Once again a parameter identification technique was used to determine all the
unknown coefficients in this equation of motion.

Having identified the unknown parameters in both formulations of the equation of
motion using a parameter identification technique, Francescutto and Cardo (37)
re-simulated the equation of motion using these parameters. The steady roll
amplitude produced from these simulations was compared with results from
model experiments for a range of wave excitation frequencies. Two different
catamaran hull shapes were tested, one with a round bilge hull and the other with
a hard chine hull. The two hull shapes were each tested with a range of different
hull separations.
Francescutto and Cardo concluded that the monohull formulation of the equation of motion accurately modelled the behaviour of the catamaran at low values of hull separation \( h_{cat} \) whereas the catamaran formulation of the equation of motion accurately modelled the behaviour of the catamaran at high values of hull separation \( h_{cat} \). Both formulations adequately modelled the behaviour for intermediate hull separations.

### 3.3.4 Process for Determining Roll Motion

The roll motion of a catamaran or SWATH is determined using the same approach as outlined for monohulls in Chapter 2 (using the same assumptions) with the exception that the additional roll damping not predicted by Potential Theory is predicted using one of two theories, each requiring coefficients which have to be estimated or measured in model experiments. Fully theoretical methods for predicting catamaran and SWATH roll motion are not available. The process is as follows:-

1. Determine motions in all six degrees of freedom using a Potential Flow seakeeping code. Formulations in the frequency domain using linear theory have been shown to be adequate.

2. Determine the additional viscous damping in heave, pitch, roll and yaw using the Viscous Lift and Cross-flow Drag method of Lee and Curphey (150) or the vertical drag force method of Davis and Holloway (160). The Cross-flow Drag term in Lee and Curphey’s formulation varies as a nonlinear function of roll, heave and pitch and so has to be made equivalent linear before the equation of motion can be solved.

3. The coefficients necessary for the viscous roll damping prediction have to be estimated or measured in a suitable model experiment.
3.4 Trimarans

Trimaran motions have been calculated theoretically using Potential Flow methods by a number of authors using linear theory in both the frequency and time domain as well as using quasi non-linear theory in the time domain.

Frequency domain Strip Theory was used by Doctors and Scrace (166), who obtained motion predictions for RV Triton both ignoring and allowing for the hydrodynamic interactions between the hulls; Floden, Kim and Ottoisson (167), who obtained motion predictions for a small high speed trimaran containership; and Kang, Lee, Kim and Cho (41), for a trimaran frigate. Begovic, Bertorello and Boccadamo (38) predicted motions of a trimaran fast ferry using High Speed Strip Theory (see Faltensen (159)).

Green function approaches in the frequency domain were used by Bingham, Hampshire, Miao and Temarel (14); Chan, Incecik and Ireland (12); Chan, Incecik, Hall and Bate (11); Hudson, Price and Temarel (97); and Zhang and Andrews (8). Chan, Incecik and Ireland (12) and Chan, Incecik, Hall and Bate (11) predicted motions of a trimaran frigate using a three-dimensional Green Function method that they had previously used for predicting catamaran motions (154) (155). Bingham, Hampshire, Miao and Temarel (14) predicted motions of a trimaran frigate in order to determine the wave loadings on the vessel. They considered the ship as a rigid body (the traditional approach used in Potential Flow Theory) and as a flexible body for a hydro-elastic analysis. For the rigid body analysis both a three-dimensional pulsating source method and a three-dimensional translating pulsating source method were used in the frequency domain to obtain heave and pitch motions whereas for the flexible ship only the pulsating source method was used. Hudson, Price and Temarel (97) compare motions predicted using a three-dimensional Green Function method in head seas for a monohull fitted with outriggers with model experiment results. Finally Zhang and Andrews (8) predicted motions for a trimaran frigate and focused their studies on rolling.
As was demonstrated in Section 3.3, the theories of Chan et al (154) (155) used the Viscous Lift and Cross Flow Drag method to predict the viscous roll damping for a catamaran. Additionally, Chan (147) had used the theory of Himeno (80), discussed in Section 2.5 of Chapter 2, to predict the roll damping when applying the same Potential Flow theory to a monohull. How Chan et al determined the roll damping of a trimaran was not discussed in the above mentioned papers (11) (12).

Both linear and non-linear Potential Flow methods in the time domain, using a Rankine Singularity Method (DNV-WASIM), as well as in the frequency domain using a Green Function method (PRECAL), were adopted by Pastoor, van't Veer and Harmsen (75) to compare theoretical motions with model experiment results for a trimaran frigate.

Kim and Weems (76) reported on the heave and pitch motion of a trimaran using a non-linear Potential Flow method in the time domain. They used the LAMP computer suite which also uses the Rankine Singularity Method.

All of the research documented above showed, as was the case with monohulls reported in Chapter 2, that heave and pitch motions were adequately predicted by Potential Flow theories (few focused on motions other than heave, roll and pitch). However, roll motions were not adequately predicted by Potential Theory and viscous effects and lifting contributions of the hull and any appendages had to be determined separately.

Of the papers reviewed in the previous paragraphs few give roll motion results (8) (11) (12) (75) (166) and of these only Doctors and Scrace (166); Pastoor, van’t Veer and Harmsen (75); and Zhang and Andrews (8) mention explicitly how the additional roll damping to account for viscous effects and appendages were dealt with.

As the discussion in Section 2.2.1 of Chapter 2 showed, the roll response of a ship, when approximated as a linear spring-mass-damper system with six degrees
of freedom, is dominated by the roll damping term, $B_{\alpha}$, in the region of roll resonance where roll motions are greatest. Therefore, the next few sections will focus on those researchers that attempted to determine the roll damping term or investigated the effects of varying this term for a trimaran.

### 3.4.1 Trimaran Roll Damping According to Doctors and Scrace

Doctors and Scrace allowed for the extra roll damping not considered by the potential theory to comprise of two components, skin friction damping and appendage lift roll damping. In addition, they modified the added inertia in roll to include the inertia of the appendage (Schmitke did this as well (81)). Where Schmitke (81) and Ikeda et al (114) considered the friction damping at zero forward speed due to the velocity induced on the hull surface during rolling, Doctors and Scrace considered the longitudinal friction force (i.e. acting fore-aft on the submerged hull surface) and noted that the force on a single hull element could be determined by:-

$$Force = \frac{1}{2} \rho U^2 \left( C_f + C_a \right) S_{WE}$$  \hspace{1cm} 3-4-1

In equation 3-4-1 $C_f$ is the coefficient of skin friction acting on the appropriate hull and can be defined using the International Towing Tank Convention 1957 formula:-

$$C_f = \frac{0.075}{\left[ \log(Re) - 2 \right]^2}$$  \hspace{1cm} 3-4-2

$C_a$ is a local hull roughness correlation factor usually set to 0.004 and $S_{WE}$ is the wetted surface area of an element on the hull surface. When moving with forward speed the rolling ship will have a local velocity on the hull surface acting across the flow. The relative velocity on the hull element, $u_r$, is the difference between...
this and the roll velocity. Hence the roll moment is the sum of the forces acting on each hull element:

\[
\text{Moment} = \sum \frac{u_r}{U} (\text{Force}) r_f \cos \beta_{fric} \quad 3-4-3
\]

Where \(\beta_{fric}\) is the angle between the y-axis and the lever arm between the centre of gravity and the hull surface element where the friction force is being determined. Dividing this moment by the roll velocity (\(u_r\) is a function of this) gives the roll damping term due to skin friction according to Doctors and Scrace.

Appendage lift was accounted for using the same approach as Lloyd (64); see Section 2.5.7 in Chapter 2. However rather than using the formula for the lift slope developed by Whicker and Fehlner (137) Doctors and Scrace use one developed by Germain (168):

\[
C_{la} = \frac{2\pi AS}{AR_e^2 + 2AR_e + 16\pi^{-2} \ln[1 + \pi \exp(-7/8)AR_e]} \quad 3-4-4
\]

The difference between this formula and that due to Whicker and Fehlner is shown in Figure 3-4-1 from which it can be seen that the two formulae give very similar results for effective aspect ratios less than 4.

Comparison of this theory with model experiment results for RV Triton showed that the frictional damping component was very small and could almost be ignored. Hence all of the additional damping (not computed by potential theory) was attributed to the appendages. It is unsurprising, considering the importance of eddy shedding damping at low speed (pointed out by Himeno (80) amongst others), that the correlation between the model and theoretical results is poor at low speed and better at higher speeds.
3.4.2 Trimaran Roll Damping According to Zhang and Andrews

To date, the most comprehensive study of trimaran roll motion has been by Zhang and Andrews (8). Following the well established monohull investigative approach, they predicted the motions in all six degrees of freedom using a frequency domain Potential Flow method using the Green Function approach and either augmented the roll damping term with a suitable range of existing empirically based theoretical formulae, or they used roll damping coefficients measured in roll decay experiments.
3.4.2.1 Roll Motion Predictions using Roll Decay Experiment

Results

In the roll decay experiments they assumed a single degree of freedom uncoupled roll equation with constant coefficients and used a quadratic roll damping model comprising linear and quadratic terms to represent the roll damping term. The linear and quadratic roll damping coefficients were determined using the theory of Bass and Hadarra (83). To incorporate the non-linear damping term in the linear equation of motion (in the Potential Flow seakeeping code) it was converted to an equivalent linear form using the approach set out in Section 2.2.3.2 of Chapter 2.

The roll damping predicted in the model experiment is the total roll damping. The Potential Theory seakeeping computer code predicts only the wave radiation damping so this must be subtracted from the total damping obtained in the roll decay experiment to give the sum of viscous and appendage roll damping. Zhang and Andrews make the assumption that the damping measured in the roll decay experiment is equivalent to the roll damping at resonance in regular sinusoidal waves. The theoretical wave radiation damping at resonance is subtracted from this to give the sum of the viscous and appendage damping at resonance. This value is then used to represent the sum of viscous and appendage damping for all other frequencies. This is not an unreasonable assumption since, as shown in Section 2.2.1 of Chapter 2, the magnitude of the roll damping only affects the motion response in the vicinity of resonance.

Roll decay experiments were conducted over a range of speeds and the results were used with the Potential Flow Theory seakeeping code to obtain roll motion predictions. These predictions were then compared with the results from model tests in regular waves. Using this approach, the correlation between the theoretical roll motion prediction and the model experiment results in the form of a roll RAO were acceptable at high forward speed where the linear roll damping coefficient was dominant and the non-linear coefficient small. However, at zero speed, where the non-linear damping coefficient was large, the correlation was poor.
3.4.2.2 Roll Motion Predictions using Component Damping Theory

The second method Zhang and Andrews used to predict the roll motion was to apply a range of component damping formulae (as described in Section 2.5 of Chapter 2) to augment the wave radiation damping predicted by the Potential Flow seakeeping code. The approach followed was to adapt the theory of Schmitke (81) to obtain components for eddy damping and friction damping and to use the theory of Lloyd (64) for the appendages. Formulas for the friction damping and eddy shedding damping were developed for both the centre and the side hulls.

Zhang and Andrews calculated the friction damping for the trimaran by applying Schmitke's method, equation 2-5-2 in Section 2.5.3, to each panel on the hull surface used by the Potential Theory seakeeping code and summed the results across the total number of panels to obtain the friction damping for the trimaran hull.

Zhang and Andrews then applied the formula proposed by Schmitke for the eddy damping of a monohull to the centre hull, see Section 2.5.4 of Chapter 2, repeated here:

\[ B_{E_{C}} = \frac{4}{3\pi} \omega \rho x_{0} \sigma_{ed}^{3} S_{W_{C_{sec}}} C_{TAN} \]  

Where \( B_{E_{C}} \) refers to the eddy damping component of the centre hull and \( S_{W_{C_{sec}}} \) is the wetted area of a hull section of the centre hull. Zhang and Andrews followed the same approach to arrive at the following formula for the eddy damping component of one side hull:

\[ B_{E_{S}} = \frac{4}{3\pi} \omega \rho x_{0} \sigma_{ed-s}^{3} \left( \sin^{2} \beta_{side} \right) S_{W_{S_{sec}}} C_{TAN} \]
Where $B_{ES}$ refers to the eddy damping component of the side hull and $S_{WS \text{sec}}$ is the wetted area of a hull section of the side hull. The other terms in equation 3-4-6 are defined in Figure 3-4-2.

Finally the total eddy damping component for the trimaran is obtained by summing over the various hull sections along the length of the ship:

$$B_E = \sum_{L_s} B_{EC} + 2 \sum_{L_s} B_{ES}$$

Andrews and Zhang recommend that values of the coefficient $C_{TAN}$ are selected for the centre hull and side hull using Tanaka’s method reported on in Section 2.5.4 of Chapter 2. Whilst this is plausible for trimaran centre hulls it is not applicable to trimaran side hulls. Values for $C_{TAN}$ are based on model test results on a limited range of monohull ship sections rolling about their fore-aft axis of symmetry. A trimaran side hull does not roll about its own fore-aft axis, it rotates about the centre of gravity of the ship which will be in the vicinity of the centre hull fore-aft axis of symmetry. Furthermore, the geometries of trimaran side hulls are very different to the geometries of ship sections on which Tanaka’s work was based.
Theoretical predictions using this method were compared with model experiment results in regular waves at a speed equivalent to a ship speed of 18 knots. At this speed the theoretical results correlated reasonably well with the model experiment results.

The trimaran frigate that Zhang and Andrews used was representative of a frigate and was fitted with twin rudders and bilge keels on the inboard side of the side hulls. Both the rudders and the bilge keels were accounted for using the appendage damping method of Lloyd (64). Examination of the roll damping components determined using this theoretical method at frequencies close to resonance, showed that at both zero speed and at ship speed of 18 knots the majority of roll damping was due to the appendages. Below resonance all other components were negligible. At frequencies greater than resonance, the wave radiation damping increased and accounted for nearly all of the roll damping at zero speed and up to half the roll damping at 18 knots.

3.4.3 Roll Investigations by Pastoor, van’t Veer and Harmsen

Pastoor, van’t Veer and Harmsen’s (75) compared theoretical roll motion predictions with results from a series of model experiments, using a configuration of hulls representative of a trimaran frigate. The experiments were conducted at the Maritime Institute of the Netherlands (MARIN) in 1996. Two different Potential Flow seakeeping codes were used to obtain roll motion predictions: The DNV-WASIM code, which uses the Rankine Singularity method in the time domain and allows for some non-linear terms in the equation of motion, and the linear frequency domain Green Function code PRECAL, which could be run both with and without hydrodynamic interactions between the three hulls.

Roll damping in PRECAL was determined using the Potential Flow result for the wave radiation damping and the theory of Ikeda et al, reported by Himeno (80), to obtain roll damping components due to eddy damping, friction damping and lift damping. How these formula were applied to the side hulls, if at all, is not
discussed. The appendages were accounted for using a more complex theory than that of Lloyd (64). This theory was reported to calculate the lift slope of the appendage, based on three-dimensional theory and empirical data from an extensive range of model experiments, accounting for both the proximity of the appendage to the free surface and effective aspect ratio modifications due to the proximity of the appendage to the hull. No further (mathematical) details of the method were given.

Roll damping using DNV-WASIM was calculated in a more simplistic fashion. A linear and non-linear roll damping coefficient could be input and the non-linear term would be converted to an equivalent linear format before the equation of motion was solved. Roll decay experimental results for the trimaran frigate model were not available and so an upper and lower bound were determined using the component damping theory of Zhang and Andrews (8). Appendages could be accounted for separately using the same theory as implemented in PRECAL but with calculations in the time domain.

Theoretical roll RAO's are compared with model experiment results for both PRECAL and DNV-WASIM run in linear mode. Theoretically these are equivalent methods the only difference being that with DNV-WASIM in linear mode the calculation is in the time rather than the frequency domain. Comparisons were given at a speed equivalent to 22 knots with a wave incidence of 65 degrees (where 0 degrees is stern seas). In this condition the only appendages on the model were twin rudders and shaft brackets. The model RAO curve had two distinct peaks which are not replicated by either of the theoretical methods and so the correlation between the theoretical and experimental results is not particularly good. A pair of fins was then fitted at the base of the side hulls around amidships, extending horizontally towards the centre hull, which could be actively controlled if required. In both active and passive mode the model experiment RAO curve had a single peak and both codes predicted roll RAO's which were quite similar to the model experiment results.

DNV-WASIM was then run in the non-linear mode. Roll RAO's based on the principal harmonic of a Fourier analysis of the time domain outputs were
compared with the experimentally derived roll RAO's at 22 knots with a wave incidence of 65 degrees. The time domain non-linear DNV-WASIM code was able to replicate the double peak in the roll RAO curve observed in the model experiments. The variation of the roll response when the non-linear damping term was varied between the specified limits was reported as being as much as 20%, however the experimental results generally fell within these bands. When the passive and active fins were fitted the correlation between model experiment and theory was once again good with DNV-WASIM generally predicting more damping from the appendages than was actually achieved.

For the model trimaran with appendages, good correlation between theory and experimental roll at high speed is reliant on the accurate prediction of the contribution of the appendages as they are likely to provide the vast majority of the roll damping. This was shown by Zhang and Andrews (8) in Section 3.4.2.

The only research focused at a more fundamental investigation of the form of the equation of roll motion for a trimaran was conducted by Francescutto (36) and Francescutto and Cardo (37). Their work is discussed in the next section.

### 3.4.4 Research into the form of the Equation of Roll Motion in Beam Waves

Francescutto (36) and Francescutto and Cardo (37) expanded their work on the roll motion of catamarans and monohulls in beam seas, discussed in Section 3.3.3, and looked at an appropriate form of the equation of motion for a trimaran rolling in regular beam seas. The form of the equation of motion was based on that of the monohull, equation 3-3-5, but the wave forcing term was modified to include the forcing from the heaving side hulls, equation 3-3-12. The final equation that Francescutto and Cardo arrived at was:

\[ \ddot{x}_4 + b_{44} \dot{x}_4 + \omega_n^2 x_4 = \left( \alpha_s \alpha_{m_e} \omega_{mc}^2 - \alpha_c \rho g A_{mc} 2 \eta_0 r_{sep} \sin(kr_{sep}) \right) \sin \omega_s t \] 3-4-8
Where $\omega_{nc}$ is the natural frequency attributed to the centre hull of the trimaran alone. This term could not be determined directly because it is in a product with the unknown parameter $\alpha_{se}$. Note that the second term in the curly brackets on the right hand side of this equation is the contribution from the heaving side hulls. A parameter identification technique can be used to find the product of these two terms.

Francescutto (36) performed model experiments on a trimaran comprising of three Wigley hulls. The side hulls were scaled from the centre hull having half the length, beam and draught respectively. Model experiments were conducted at zero speed in beam regular waves. The side hull separation from the centre hull ($r_{sp}$) and their longitudinal position were varied and the roll response was measured. As with the catamaran model experiments conducted by Francescutto and Cardo (37) the steady roll amplitude was recorded for a number of wave frequencies and simulations of the equation of motion using parameters identified from equation 3-4-8 were compared with the model experiment results. In addition, Francescutto also tried out the monohull based and catamaran formulations of the equation of motion detailed in Section 3.3.3. For all values of separation and longitudinal location of the side hulls the combined trimaran formulation of the equation of motion was preferred.

Francescutto and Cardo conclude that the trimaran behaves like a highly damped monohull and the form of the equation of motion that best models the roll motion of a trimaran is based on that of a monohull with modifications to the wave forcing term due to the heaving of the side hulls.

### 3.4.5 RV Triton

The seakeeping performance of the trimaran demonstrator RV Triton, built after a frenetic period of UK MoD inspired research into the trimaran hull configuration, was initially assessed by a series of model experiments, see Scrace (20). This was because, at the time of her design (around 1998), no theoretical prediction
methods had been proven – RV Triton was the first large trimaran displacement ship. The experimental research mimicked the approach followed with monohulls, and in a later paper reporting on the correlation between sea trials and the initial model experiments, Renilson, Scrace, Johnson and Richardsen (23) concluded that:-

"These trials have shown that, in general, the techniques give good prediction of the full scale performance of RV Triton, and as such can be used with confidence in the design of a future trimaran warship."

This series of experiments included free roll decay experiments to measure the roll damping.

One conclusion that can be drawn from this successful series of experimental research is that, by following well established monohull investigative techniques, trimaran motions can be determined with sufficient accuracy.

3.4.6 Process for Determining the Roll Motion of Trimarans

The literature review in the previous sections shows that, in general, the process followed to obtain monohull roll predictions have been used with reasonable success on trimarans. The only author to consider anything different was Francescutto (36) who modified the wave forcing term in a single degree of freedom roll equation to account for forcing due to the heaving side hulls. However, the remainder of the equation of motion was identical to the equation generally used for monohulls derived from a spring-mass-damper system.

As with a monohull, the roll damping was shown to be poorly predicted if only the wave radiation damping, calculated using Potential Flow Theory, was considered. Allowance had to be made for viscous effects and any lifting forces generated by the hull(s) or appendages. These effects were determined by one of two methods: 1) By free decay model experiment and 2) Using modifications of the component damping theory of Schmitke (81) with separate calculations for the
appendages. The contribution of the appendages was shown by Zhang and Andrews (8) and Pastoor, van’t Veer and Harmsen (75) to be particularly important. The adapted theory of Schmitke for the eddy making roll damping contribution was considered not to be applicable to trimaran side hulls. Using either of these theories, generally speaking, the roll predictions at low speed were worse than those at high speeds.

Potential Flow methods were shown to be adequate for predicting motions in the other six degrees of freedom with authors generally preferring linear frequency domain methods. Solutions in the time domain with non-linear parts to the equation of motion were not shown to give hugely improved predictions of roll motion.
3.5 Thesis Hypothesis

The research into trimaran roll motion prediction documented in this chapter has shown that generally speaking, the monohull best practice approach highlighted at the end of chapter 2 is applicable to trimarans. As well as this, theoretical improvements to the monohull procedures have been proposed by various authors to further improve the roll motion prediction of a trimaran. Therefore, a hypothesis can now be stated which can be proved or disproved in the subsequent chapters of the thesis:

Trimaran roll motion can be accurately predicted using monohull best practice. More specifically, accurate roll motion predictions can be obtained using linear Potential Flow Seakeeping theory, with the roll damping term either obtained from a roll decay experiment or augmented with empirically based theoretical roll damping components developed for monohulls.
3.6 Conclusions

The literature review contained in this chapter has shown that the monohull best practice approach for predicting roll motion outlined in Chapter 2 has been used to predict trimaran roll motion in regular waves to an acceptable level. The most accurate roll motion predictions were obtained when roll damping coefficients were measured from free decay experiments.

The only other method developed for predicting roll motion formulated a new uncoupled equation of roll motion based on existing work on catamarans. To predict coupled ship motions in six degrees of freedom the equation of motion of the ship should be developed from scratch using this equation to describe the roll motion. This method still required linear and non-linear roll damping coefficients which would need to be estimated or measured from roll decay experiments.

Thus, based on this literature review, it would seem sensible to conduct research into trimaran roll motions using the process employed successfully over many years to predict the motions of monohulls. Therefore, the focus of the next chapter will be to prove (or disprove) the hypothesis developed in section 3.5.
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4.1 Introduction

The purpose of this chapter is to test the hypothesis proposed at the end of chapter 3. This was that accurate trimaran roll motion predictions can be obtained using linear Potential Flow Seakeeping theory with the roll damping term either obtained from a roll decay experiment or augmented with empirically based theoretical roll damping components developed for monohulls. Many of the existing researchers whose work was documented in the literature review in chapter 3 have assumed implicitly that this hypothesis is true; (8) (11) (12) (14) (41) (97) (166) and (167). However, in the published literature, this hypothesis has never been properly tested. Therefore any serious attempt to research in detail the roll motion of trimarans should start by addressing this point.

To test the hypothesis, a series of free decay model experiments were performed on one trimaran model of a 160 metre frigate design at a range of speeds from 0 to 25 knots full scale. The level of roll damping was changed by adding a range of appendages. The same model was then tested in regular waves to obtain the roll response at different wave frequencies for a number of incident wave directions. Firstly, the roll damping coefficients measured from the experimental roll decay results were used with a linear frequency domain Potential Flow Seakeeping Code to obtain roll motion predictions and these were then compared with the results from the seakeeping experiments. Secondly, theoretical roll motion predictions were obtained using the same seakeeping code as before. However, this time the roll damping was determined using both the Potential Flow results and a range of empirically based theoretical roll damping components first developed for monohulls. Roll motion predictions using this approach were then compared with the results from the seakeeping model experiments in regular waves. This package of work is documented in the subsequent sections of this chapter.
4.2 Roll Motion Prediction using Experimentally Derived Roll Decay Coefficients

In this section, roll damping predictions are obtained for a trimaran design using a linear Potential Flow Seakeeping Program which solves the equation of motion in the frequency domain, with the roll damping determined using the results of free roll decay experiments. Before this research could commence a suitable seakeeping code needed to be selected.

4.2.1 Selection of a Suitable Seakeeping Code

When this research commenced there were very few commercially available seakeeping codes capable of modelling a trimaran hull. Of these, two were only available to members of the Co-operative Research Ships club (led by MARIN in the Netherlands, www.crships.org), the Strip Theory code TRIMO and boundary element Green Function code PRECAL; the only other commercial code was DNV's WASIM, which could be run in linear or quasi non-linear modes in the time domain. The only other types of code available were those developed by universities. One such code was TRISKP, this was used by Zhang and Andrews for their trimaran roll analysis (8) and was developed at UCL, hence the code listing was also available to the author.

For the research in this chapter, two methods were to be used to determine the roll damping; the first using results from roll decay experiments and the second using a range of empirically based theoretical damping components. The seakeeping code selected had to have the flexibility to incorporate both methods prior to the solution of the equation of motion. Of the available codes, only TRISKP allowed this and therefore this code was selected. The trimaran roll damping investigations published by Zhang and Andrews (8) were by far the most comprehensive when the work in this thesis commenced. Therefore, selection of
the same seakeeping code used by Zhang and Andrews (TRISKP) had the added benefit of allowing direct comparison of any new work with their results.

TRISKP is a boundary element linear Potential Flow seakeeping code which uses the Green Function method. The theory behind it can be found in two papers by Wu (169) and Wu and Eatock-Taylor (170).

4.2.2 Organisation of the Experimental Research

To support the research in this chapter two sets of experiments were planned:-

1. Free roll decay tests in calm water over a range of speeds both with and without a selection of different appendages to modify the damping.
2. Seakeeping experiments in regular waves at a range of wave directions, speeds and wave frequencies. These were conducted both with and without roll damping appendages.

Both of these series of model experiments were conducted in the Ocean Basin at QinetiQ’s Haslar facilities in Gosport in the United Kingdom (UK) using a trimaran model known as DVZ with side hull option B. The model is referred to as DVZ throughout this thesis. The roll decay experiments took place in 2002 and the seakeeping experiments in regular waves in 2004.

QinetiQ trimaran DVZ had been designed to represent a 160 metre 5000 tonne displacement frigate with a similar operational role to that of a UK Type 23 Frigate. Trimaran DVZ had been used in an extensive package of research starting in 1997, see Richardsen et al (171), including resistance, seakeeping and manoeuvring experiments with six different sets of side hulls (A-E). Side hull B was thought to give the best compromise and was selected for this work. The side hull spacing and $KG$ had been chosen to ensure adequate stability, both intact and damaged, and an acceptable roll period and roll motions. A picture of DVZ with side hull B is given in Figure 4-2-1 with the dimensions and model condition
used for both sets of experiments in support of this work given in Table 4-2-1, Table 4-2-2 and Table 4-2-3.

![QinetiQ Trimaran Model DVZ](image)

**Figure 4-2-1: QinetiQ Trimaran Model DVZ**

<table>
<thead>
<tr>
<th>CENTRE HULL</th>
<th>Model Scale</th>
<th>Full Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>Waterline Length (m)</td>
<td>7.152</td>
<td>160.20</td>
</tr>
<tr>
<td>Maximum Waterline Beam (m)</td>
<td>0.482</td>
<td>10.80</td>
</tr>
<tr>
<td>Design Draught (m)</td>
<td>0.232</td>
<td>5.20</td>
</tr>
<tr>
<td>Design Displacement (kg or Tonne)</td>
<td>396.10</td>
<td>4571.78</td>
</tr>
</tbody>
</table>

**Table 4-2-1: Particulars of the centre hull of QinetiQ model DVZ**

<table>
<thead>
<tr>
<th>SIDE HULL</th>
<th>Model Scale</th>
<th>Full Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>Waterline Length (m)</td>
<td>2.884</td>
<td>64.60</td>
</tr>
<tr>
<td>Maximum Waterline Beam (m)</td>
<td>0.071</td>
<td>1.59</td>
</tr>
<tr>
<td>Design Draught (m)</td>
<td>0.128</td>
<td>2.87</td>
</tr>
<tr>
<td>Design Displacement (kg or Tonne)</td>
<td>13.95</td>
<td>161.03</td>
</tr>
</tbody>
</table>

**Table 4-2-2: Particulars of QinetiQ side hull B fitted to model DVZ**
### Table 4-2- 3: Particulars of QinetiQ trimaran model DVZ

<table>
<thead>
<tr>
<th></th>
<th>Model Scale</th>
<th>Full Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>Waterline Length (m)</td>
<td>7.152</td>
<td>160.20</td>
</tr>
<tr>
<td>Maximum Waterline Beam (m)</td>
<td>1.357</td>
<td>30.40</td>
</tr>
<tr>
<td>Design Draught (m)</td>
<td>0.232</td>
<td>5.20</td>
</tr>
<tr>
<td>Design Displacement (kg or Tonne)</td>
<td>424.0</td>
<td>4893.84</td>
</tr>
<tr>
<td>KG (m)</td>
<td>0.345</td>
<td>7.73</td>
</tr>
<tr>
<td>Model Test GM (m)</td>
<td>0.140</td>
<td>3.14</td>
</tr>
<tr>
<td>Model Test Roll Period (s)</td>
<td>2.25</td>
<td>10.65</td>
</tr>
<tr>
<td>Pitch Radius of Gyration (m)</td>
<td>1.75</td>
<td>39.16</td>
</tr>
<tr>
<td>Planned Roll Radius of Gyration (m)</td>
<td>0.372</td>
<td>8.33</td>
</tr>
<tr>
<td>Model Test Roll Radius of Gyration (m)</td>
<td>0.420</td>
<td>9.40</td>
</tr>
<tr>
<td>LCB (m, abaft the centre hull ordinate 1)</td>
<td>0.230</td>
<td>5.15</td>
</tr>
<tr>
<td>VCB (m, above baseline)</td>
<td>0.143</td>
<td>3.20</td>
</tr>
</tbody>
</table>

#### 4.2.3 Variations in the Level of Roll Damping

The author conducted roll decay experiments using trimaran DVZ fitted only with a skeg and rudders and compared the roll damping performance of this bare hull to that with a range of appendages fitted. The appendages were either bilge keels, fins fitted to the inboard face of the side hulls, fins linking the centre to the side hulls or inverted T-Foils below the side hulls. Details of the results of these experiments were published in a paper (172) which is reproduced in Appendix 2. The appendages increased the level of roll damping.

To show the effect of increasing the damping in this thesis the results from two of the appendages will be used and compared to the bare hull case. The two appendages considered are a pair of link-fins, linking the centre to the side hulls, and a pair of T-Foils below the side hulls. The two pairs of appendages each have an identical plan area (for the T-Foil this is the plan area of the strut and base fin). The dimensions of these appendages are given in Table 4-2- 4, the T-foil is shown...
in Figure 4-2-2 and the link-fin in Figure 4-2-3. Both appendages were effective in reducing the roll motion especially at higher speeds.

<table>
<thead>
<tr>
<th>Name and Descriptor</th>
<th>Plan Area per Fin (Full Scale) ($m^2$)</th>
<th>Dimensions (Model Scale) (m)</th>
<th>Dimensions (Full Scale) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Link-fin</td>
<td>20</td>
<td>0.399 x 0.100</td>
<td>8.982 x 2.227</td>
</tr>
<tr>
<td>T-Foil</td>
<td>20</td>
<td>STRUT: 0.104 x 0.0832</td>
<td>STRUT: 2.330 x 1.864</td>
</tr>
<tr>
<td></td>
<td></td>
<td>BASE: 0.1875 x 0.1664</td>
<td>BASE: 4.200 x 3.728</td>
</tr>
</tbody>
</table>

Table 4-2-4: Details of appendages carried forward for model experiments on QinetiQ trimaran DVZ

![Figure 4-2-2: The T-Foil fitted under the side hull of trimaran model DVZ](image-url)
4.2.4 Roll Decay Experiments

A comprehensive range of roll decay experiments were planned with trimaran model DVZ. For these experiments the model was instrumented so that it was free running, using an autopilot to maintain heading. These experiments aimed to capture the effects of:

- Forward speed
- Level of roll damping (through the addition of appendages)
- Relative contribution to the total roll damping of the three hulls compared to the appendages

The model was tested at speeds representing ship speeds of 0, 6, 12, 20 and 25 knots. The maximum speed, 25 knots, was the greatest speed the model could achieve with the electric motors fitted. The link-fins and T-Foils were fitted to the side hull at its midships position as this was where the transverse cross-section was thickest (making attachment of the fins easier). This position is aft of
midships position of the complete model, the midships position of the side hull is located 15.81 metres aft that of the complete ship. For the runs with forward speed the model was set to run at a chosen heading and an autopilot was used to keep the ship's actual course on the required heading. A weight was added to the model to heel it to 8 degrees. At this angle all of the appendages would be immersed, although the end of the link-fins on the side hull would be close to the waterline. These experiments were far more comprehensive than those reported by Zhang and Andrews (8) which were also conducted at QinetiQ's Haslar facilities.

4.2.4.1 Assumptions

The first assumption that is made in a roll decay experiment is that “pure” roll motion is measured, that is there are no surge, sway, heave, pitch or yaw motions, hence the recorded roll decay is not influenced by these other motions. The second major assumption is that there is no forcing from waves. This assumes the water remains calm at all times and there is no reflection of waves generated by the hulls during roll decay from the edges of the tank or reflection of waves between the three hulls.

In the model experiments, the roll decay was initiated by removing a weight from the side of the model. Heave motion induced by removing this weight is assumed to decay quickly and hence does not influence the roll results. Crossland, Wilson and Bradburn (173) showed, in experiments with a monohull free to move in the vertical plane, that heave motion decays quickly. Therefore, this is considered a reasonable assumption.

Having made these assumptions, the equation of motion for the ship reduces to the equation of unforced roll motion. A suitable form for the roll damping is then selected, see Section 2.2.3.1 of Chapter 2, usually quadratic or cubic (with linear and either quadratic or cubic damping terms). The next assumption is that the coefficients in the equation of motion remain constant throughout the roll decay and than the techniques discussed in Section 2.4.2 of Chapter 2 can be used to
obtain the roll damping terms. The assumption of constant coefficients implies that, over the range of angles measured in the decay, the roll stiffness term, $C_{44}$, varies in direct proportion to the roll angle, i.e. a linear $GZ$ curve over the range of angles considered.

The above approach has been used over a great number of years to investigate the roll damping of monohull ships, see Spouge (95) for a detailed discussion.

One of the problems encountered when analysing roll decay experiment results is noise in the measured decrement. Zhang and Andrews (8) performed roll decay experiments on trimarans and they fitted a mathematical expression to the recorded decay and performed all subsequent analysis on this fitted decrement. The reduction of noise is important for any analysis method where the signal has to be integrated to obtain the roll velocity, see for example energy methods in Section 2.4.2 of Chapter 2, as integration will greatly amplify any noise. The damped natural frequency of the model can then be obtained from the fitted decrement. As this is a mathematical fit the damped natural frequency should be constant if measured between subsequent peaks. This frequency is then used to obtain the roll stiffness.

The decay was to be initiated by removing a weight from the side of the model. This was achieved at forward speed by attaching a rope to the weight and pulling it clear of the model to start the decay. At zero speed the decay was initiated by pushing down on one side of the model and releasing it. This may induce a small amount of heave motion and possibly pitch and sway motion. These motions are assumed to decay away rapidly, however, as pointed out by Spouge (95), this may lead to some uncertainty in the roll decay and so Spouge suggested removing the first peak from the decay history before analysing the decrement. This approach is adopted here for the subsequent analysis of the roll decrement.
4.2.4.2 Analysis

The roll decrement data was analysed using an energy method developed by Bass and Haddara (83) described in Section 2.4.2.2 of Chapter 2 using a quadratic damping model (with linear and quadratic damping coefficients). This model had been used successfully by Zhang and Andrews (8) in their roll decay analysis of trimarans. The literature review in Chapter 2 indicated that the quadratic model was preferred by many authors researching roll decay at smaller roll angles. The principal behind the method is that the rate of change of total energy in the system is equal to the energy dissipated in damping.

Once the damping coefficients have been obtained the equation of motion can be simulated using the measured coefficients by a Runge Kutta numerical technique and the simulated data can be compared to the fitted decrement to get a measure of the accuracy of the determined damping coefficients. Examples of the closeness of the measured data and fitted data (using a mathematical expression to model the recorded data) to the simulated data (using the Runge Kutta numerical technique) are given in Figure 4-2- 4 and Figure 4-2- 5.
Figure 4-2- 4: Comparison between the recorded and simulated data for the Link-fin with the first peak removed and the data truncated at a suitable value for peak analysis.

Figure 4-2- 5: Comparison between the fitted and simulated data for the Link-fin with the first peak removed and the data truncated at a suitable value for peak analysis.
To compare the simulated decrement with the mathematical fit of the decrement to the recorded data it is desirable to compare the complete decrements. However, as pointed out by Spouge (95), small errors in the frequency accumulate over a long decay history until the fitted and simulated data histories become out of phase which would produce an enormous root mean square (rms) difference. So, an rms error term was calculated by comparing the peaks of the fitted and simulated decrements. The rms error in degrees is expressed as follows, where \( \text{peaks} \) refers to the number of peaks used for the calculation and \( \text{diff} \) is the difference between the magnitude of the peaks:

\[
\text{rms(\text{deg})} = \sqrt{\frac{1}{\text{peaks}} \sum \text{diff}(x_i)^2}
\]

Perusal of the rms error term for the model both with and without the two pairs of appendages, Table 4-2-5, shows that roll damping coefficients determined using the energy method give a simulated decrement which is very close to the fitted decrement. The largest error, taken across the model speeds was 0.1650 degrees.

<table>
<thead>
<tr>
<th>Appendix:</th>
<th>RMS Error, Fitted to Simulated Curve (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
</tr>
<tr>
<td>Bare Hull</td>
<td>0.0707</td>
</tr>
<tr>
<td>Link-fin</td>
<td>0.0719</td>
</tr>
<tr>
<td>T- Foil</td>
<td>0.0729</td>
</tr>
</tbody>
</table>

Table 4-2-5: RMS Error term (degrees) averaged across the model speeds

Combinations of the linear and quadratic damping coefficients for a given decrement can be compared by considering the equivalent linear damping term, \( B_e \), developed in Section 2.2.3 of Chapter 2. This term can be made non-dimensional by dividing through by the sum of the roll inertia and added inertia and by the natural roll frequency, see Spouge (95):

\[
k_e = \frac{B_e}{2\omega_n(I_4 + A_{44})}
\]
For a quadratic damping model this can be re-written in the following form:

\[ k_e = k_1 + \frac{8}{3\pi} k_2 \omega_d x_{40} \]  \hspace{1cm} 4-2-3

Where \( k_1 \) and \( k_2 \) are the linear and quadratic non-dimensional roll damping coefficients.

When the non-dimensional equivalent linear roll damping term is plotted against roll angle, if the damping is predominantly linear then the gradient of the curve will be small. The equivalent linear damping at the natural roll frequency (derived separately from each roll decrement) is plotted against roll angle in degrees both with and without the two pairs of appendages at the different speeds given at full (ship) scale in Figure 4-2-6 to Figure 4-2-8. At zero speed in all the model conditions there is significant non-linear damping whereas at the highest two speeds, 20 and 25 knots, the damping is dominated by the linear term. As was shown in Table 2-5-1 in Section 2.5 of Chapter 2, hull lift damping and appendage damping are linear damping components and these were shown to provide the most significant contribution to the overall damping at forward speed. Hence, once the total roll damping is dominated by the appendages and hull lift damping then the gradient of the equivalent linear damping term plotted against roll amplitude should be small.
Figure 4-2-6: Non-dimensional Equivalent Linear Damping ($k_e$) against roll angle for trimaran model DVZ without roll damping appendages. Speed given at ship rather than model scale.

Figure 4-2-7: Non-dimensional Equivalent Linear Damping ($k_e$) against roll angle for trimaran model DVZ fitted with a pair of link-fins. Speed given at ship rather than model scale.
The different appendages can be compared at increasing ship speed by consideration of the non-dimensional equivalent linear damping at a given roll angle. This is plotted at roll angles of 3 degrees; 7 degrees, at this angle the outboard end of the link-fins is just touching the water surface; and 10 degrees, when the side hull opposite to the direction of roll is just about to come out of the water. See Figure 4-2-9, Figure 4-2-10 and Figure 4-2-11.

The results show that the two sets of appendages increase the level of damping even at zero speed. As speed increases the performance of the appendages improves, as would be expected as the lift they generate increases. At 6 knots the results are a little confused by the predominance of quadratic damping (which is angle dependent). This gives a pronounced hump in the results at the highest roll angle, see Figure 4-2-11.

At 12 knots the T-Foils are seen to be very effective dampers providing more damping than the link-fins. At 20 knots the T-Foils are still outperforming the link-fins, however at 25 knots the damping of the T-Foil appears to drop off. According to the theory it should increase in proportion to forward speed. Closer inspection of the decrements showed that they were quite noisy at the highest speed and because of the decision to remove the first peak from the decrement, for
the link-fins and T-Foils in particular there may only be one complete cycle to analyse using the energy method. Hence the results are least accurate at the highest speed.

**Figure 4-2-9:** Plot of the equivalent linear damping, $k_e$, against speed for a roll angle of 3 degrees. Speed given at ship rather than model scale.

**Figure 4-2-10:** Plot of the equivalent linear damping, $k_e$, against speed for a roll angle of 7 degrees. Speed given at ship rather than model scale.
During the model experiments a number of interesting observations were noted:

- The roll motion never completely decayed away. The model would continue to roll at small amplitudes (typically less than 1 degree) for a very long period; the data logger was generally switched off after one to two minutes. For analysis, the recorded decrement was usually truncated once the roll angle was less than 0.2 to 0.5 degrees as peaks and troughs in the decay were difficult to identify at angles lower than this.

- During the beginning of the roll decrement one side hull of the trimaran was only just in the water. In this condition the side hull fin and link-fins were very close to the surface and this could lead to a reduction in the lift over this period of the decay. For a discussion on free surface effects on appendage lift see Du Cane (174). This effect was counteracted by some extent by the removal of the first decay cycle prior to analysis.

- It is possible that the autopilot controlled rudder could have induced yaw motions at high speed thus breaking the uncoupled roll assumption. Furthermore, as the trimaran rolls the wetted surface of one side hull is greater than the other and this could lead to a increase in drag giving rise to yaw motions. Yaw motion of the model was not measured in these
experiments and so it is not possible to quantify whether either of these concerns are well founded.

The analysis of the roll decrements has highlighted one important issue:

- Removal of the first peak from the decrement obtained at high speed leaves only a very short decay record to analyse, often with only one complete roll cycle. This makes the fitting of a mathematical expression to the decay more difficult and increased the rms error between the recorded and fitted decrements. The energy method used to analyse the fitted decrement requires at least one complete cycle, see Bass and Haddara (83), and so this increases the possibility of further errors.

4.2.4.3 Using the Roll Decay Results to Obtain Roll Motion Predictions

To obtain roll motion predictions the roll decay results were used with the TRISKP linear Potential Flow Theory seakeeping computer code. The approach adopted was to use TRISKP to obtain the wave radiation damping over a range of frequencies and to assume the other roll damping components at each frequency are equal to the total roll damping measured in the decay experiment minus the wave radiation damping at the natural frequency. Thus, it is assumed that the roll damping in the free decay test is equal to the damping at resonance in forced motion and that, with the exception of the wave radiation damping, roll damping does not change with frequency. It was shown in Section 2.2.1 of Chapter 2 that the roll damping coefficient only affects the motion response over a narrow band of frequencies around resonance hence this approximation is considered reasonable.

In the analysis the following assumptions are made:

- The wave radiation damping for a trimaran in regular sinusoidal waves is predicted by TRISKP
• In roll decay experiments, the model rolls at the damped natural frequency which is approximately equal to the natural frequency. This assumption implies that the damped natural frequency for all the decay tests must be the same. In the analysis of the roll decrements described in Section 4.2.4.2, the damped natural frequency was measured for each run and this frequency was used in the subsequent calculations for determining the damping coefficients. The assumption is made for practical reasons. If the existing roll decay analysis were to be used the subtraction of the wave damping from the damping measured in the decay test would need to be done for every appendage and speed combination. Whereas, if one frequency value is assumed it is then only necessary to perform this calculation once for each ship-appendage combination. This is outlined in more detail below.

• The roll damping (with the exception of the wave radiation damping) determined at the natural frequency is identical to the roll damping at all other frequencies.

• The only term in the equation of motion modified using the roll decay experiment results is the roll damping term \( B_{44} \). Cross coupling of terms between roll and other degrees of freedom are calculated by TRISKP.

• The Potential Theory Seakeeping code TRISKP uses the linear and quadratic damping coefficients to calculate the equivalent linear damping term \( b_e \). This term depends on the roll amplitude hence the code has to perform iterations until the input and output roll amplitudes are the same. This increases the run time of the code, hence the need to minimise the amount of times this calculation is performed.

As mentioned above, the roll decay results were re-analysed before being used with TRISKP to predict roll motion amplitudes. The only difference was that rather than using the measured damped natural frequency from each decrement, giving a different frequency for each appendage at each speed, one damped natural frequency was used and this was then considered to approximate the natural frequency. The reason for this is that if the roll natural frequency was different for each speed and appendage combination then the TRISKP code would
need to be run at each of these different natural frequencies to determine the wavemaking damping. This was undesirable due to the long runtime of the TRISKP code.

The approach taken to arrive at this single natural frequency was, for each model-appendage combination, to obtain the damped natural frequency at zero speed. At this speed there will be the largest number of cycles in the measured decay. The average of the damped natural frequencies obtained at zero speed for each model-appendage combination was then taken, giving a natural roll period of 2.232 seconds. This period is different from the one given in Table 4-2-3 (2.25 seconds) obtained by measuring the roll period with a stop watch. Measuring the roll period from the decay history will always give a more accurate result.
4.3 Seakeeping Experiments in Regular Waves

A second series of model experiments were conducted with trimaran DVZ in the Ocean Basin at QinetiQ’s Haslar facilities in 2004. In these experiments the model was configured to run with a desired heading using an autopilot in regular sinusoidal waves of constant steepness (constant wave slope). The model was run in beam and stem quartering seas for a range of wave frequencies with motion in all six degrees of freedom recorded. The model was set up to be in a condition identical to that of the 2002 roll decay experiments. The motion Response Amplitude Operators (RAO’s), see Lloyd (64), in any degree of freedom were calculated from the relevant steady motion time history recorded by the onboard data logger. For roll motion, the roll RAO is the steady roll amplitude, $x_{40}$, divided by the wave slope which is the multiple of the steady wave amplitude, $\eta_0$, and the wave number, $k$, and is a non-dimensional quantity:-

$$ RAO_{roll} = \frac{x_{40}}{k\eta_0} $$

4.3.1 Assumptions

In seakeeping model experiments the following assumptions are made when deducing RAO’s:-

- Input sinusoidal regular waves cause sinusoidal output motion in six degrees of freedom.
- The model autopilot is capable of maintaining the desired heading to the waves.
- The recorded motion time history settles to sinusoidal motion of constant amplitude and frequency.
- “Actual” ship motion is recorded, that is coupling effects with other degrees of freedom and rudder induced motions are included.
4.3.2 Overview

In this second set of model experiments the model was tested at three speeds equivalent to 6, 12 and 20 knots and six wave frequencies. The wave slope, \(2\pi \eta_0/\lambda\), was maintained at 1/50. The model was run in beam and stern quartering seas (90 and 60 degrees respectively) at the speeds and wave frequencies given in Table 4-3- 1. For these experiments only two appendages were fitted, the link-fin and T-Foil which had identical plan areas (this is the sum of the plan area of the base fin and profile area of the strut for the T-Foil). These were the pairs of T-Foils and Link-fins discussed in Section 4.2. In addition, the model was tested without either of these pairs of appendages fitted. A picture of the model during these experiments is given in Figure 4-3- 1.

<table>
<thead>
<tr>
<th>Model Speed (m/s)</th>
<th>Ship Speed (Knots)</th>
<th>Froude Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.652</td>
<td>6</td>
<td>0.078</td>
</tr>
<tr>
<td>1.304</td>
<td>12</td>
<td>0.156</td>
</tr>
<tr>
<td>2.174</td>
<td>20</td>
<td>0.260</td>
</tr>
</tbody>
</table>

Wave Frequency (Model Scale) (rad/s) | 3.789 | 3.098 | 2.802 | 2.576 | 2.400 | 2.187

Wave Frequency (Ship Scale) (rad/s) | 0.801 | 0.654 | 0.592 | 0.544 | 0.507 | 0.462

Table 4-3- 1: Speed, Froude number and wave frequency at both model and ship scale
4.3.3 Results and Comparison with Roll Motion Predictions using Experimentally Derived Roll Decay Coefficients

The correlation between motion predictions using TRISKP, with roll damping determined using the results from the roll decay experiments, and the seakeeping model experiments for the model without roll damping appendages is at best reasonable. In beam seas the peak of the TRISKP-Decay Test roll RAO curve is generally greater in magnitude than the model experiment results would suggest and is located at a lower frequency, see Figure 4-3-3. This indicates that the damped natural frequency used to represent the natural frequency in the TRISKP-Decay Test method was a little too low. In stem quartering seas the TRISKP-Decay Test method predicts a peak in the RAO curve at around 0.60 rad/s after which the RAO reduces. In the model experiments, except at 6 knots, the RAO values continue to increase after this frequency, see for example Figure 4-3-4 for the ship in stem quartering seas at 12 knots. At 6 knots they decrease but not to the extent predicted by the TRISKP-Decay Test method.
Figure 4-3-2: Comparison between model experiment and TRISKP-Decay Test roll RAO's for the model without roll damping appendages in stern quartering seas at a speed equivalent to 6 knots at ship scale.

Figure 4-3-3: Comparison between model experiment and TRISKP-Decay Test roll RAO's for the model without roll damping appendages in beam seas at a speed equivalent to 6 knots at ship scale.
Figure 4-3-4: Comparison between model experiment and TRISKP-Decay Test roll RAO's for the model without roll damping appendages in stern quartering seas at a speed equivalent to 12 knots at ship scale.

Figure 4-3-5: Comparison between model experiment and TRISKP-Decay Test roll RAO's for the model without roll damping appendages in beam seas at a speed equivalent to 12 knots at ship scale.
Figure 4-3- 6: Comparison between model experiment and TRISKP-Decay Test roll RAO's for trimaran model DVZ without roll damping appendages in stern quartering seas at a speed equivalent to 20 knots at ship scale

Figure 4-3- 7: Comparison between model experiment and TRISKP-Decay Test roll RAO's for trimaran model DVZ without roll damping appendages in beam seas at a speed equivalent to 20 knots at ship scale
At 20 knots the TRISKP-Decay Test RAO's are all lower than the model test results in stern quartering seas. In beam seas the peak in the TRISKP-Decay Test RAO curve is not replicated in the model experiment results where there is no noticeable peak. At higher frequencies the experimental RAO's are much greater than the TRISKP-Decay Test predictions, see Figure 4-3-7.

These results show that the TRISKP-Decay Test method has not yielded particularly good predictions of the roll motion for a trimaran, even though the damping was measured directly from experiments with the same model. The deviation of the experimental RAO's from the theoretical prediction at higher frequencies indicates that the roll inertia, added inertia and stiffness may vary and this effect is not picked up by the linear theory. This highlights that some of the assumptions of the linear frequency domain method given in Section 2.2.1 may not be appropriate. The shape of the TRISKP-Decay Test RAO's does not always match the shape of the experimental RAO's, this is especially the case in stern quartering seas.

Results with the appendages fitted are better. Once again, in beam seas the peak of the TRISKP-Decay Test RAO occurs at a slightly lower frequency than the experimental results suggest, see for example Figure 4-3-11 and Figure 4-3-13. Experimental results at higher frequencies are still much greater than the theory suggests proving that this effect is not directly associated with damping, see for example Figure 4-3-14 and Figure 4-3-16. The correlation between the model experiment results and TRISKP-Decay Test predications is generally best in beam seas.
Figure 4-3-8: Comparison between model experiment and TRISKP-Decay Test roll RAO’s for the model fitted with the link-fins in stern quartering seas at a speed equivalent to 6 knots at ship scale.

Figure 4-3-9: Comparison between model experiment and TRISKP-Decay Test roll RAO’s for the model fitted with the link-fins in beam seas at a speed equivalent to 6 knots at ship scale.
Figure 4-3- 10: Comparison between model experiment and TRISKP-Decay Test roll RAO’s for the model fitted with the link-fins in stern quartering seas at a speed equivalent to 12 knots at ship scale

Figure 4-3- 11: Comparison between model experiment and TRISKP-Decay Test roll RAO’s for the model fitted with the link-fins in beam seas at a speed equivalent to 12 knots at ship scale
Figure 4-3-12: Comparison between model experiment and TRISKP-Decay Test roll RAO's for the model fitted with the link-fins in stern quartering seas at a speed equivalent to 20 knots at ship scale.

Figure 4-3-13: Comparison between model experiment and TRISKP-Decay Test roll RAO's for the model fitted with the link-fins in beam seas at a speed equivalent to 20 knots at ship scale.
Figure 4-3-14: Comparison between model experiment and TRISKP-Decay Test roll RAO's for the model fitted with the T-Foils in stern quartering seas at a speed equivalent to 6 knots at ship scale.

Figure 4-3-15: Comparison between model experiment and TRISKP-Decay Test roll RAO's for the model fitted with the T-Foils in beam seas at a speed equivalent to 6 knots at ship scale.
Figure 4-3-16: Comparison between model experiment and TRISKP-Decay Test roll RAO's for the model fitted with the T-Foils in stern quartering seas at a speed equivalent to 12 knots at ship scale.

Figure 4-3-17: Comparison between model experiment and TRISKP-Decay Test roll RAO's for the model fitted with the T-Foils in beam seas at a speed equivalent to 12 knots at ship scale.
Figure 4-3-18: Comparison between model experiment and TRISKP-Decay Test roll RAO’s for the model fitted with the T-Foils in stern quartering seas at a speed equivalent to 20 knots at ship scale.

Figure 4-3-19: Comparison between model experiment and TRISKP-Decay Test roll RAO’s for the model fitted with the T-Foils in beam seas at a speed equivalent to 20 knots at ship scale.
4.4 Roll Motion Prediction using Semi-empirical Roll Damping Components

In this section the empirically based theoretical methods for predicting roll damping components outlined in Section 2.5 of Chapter 2 will be used with TRISKP to predict roll motion RAOs.

So far roll damping components have been developed for:-

- Wave Radiation Damping
- Eddy shedding
- Hull friction due to rolling
- Horizontal or vertical lift developed by the hull due to the angle of attack generated between the flow and hull at forward speed
- Appendages

In Chapter 3 the work of Zhang and Andrews (8) was discussed which adopted variations of all of these, except for the hull lift force derived roll damping component, to predict trimaran roll motion.

Zhang and Andrews (8) used the TRISKP Potential Theory code to obtain the wave radiation roll damping component and applied the viscous damping theory of Schmitke (81) to both the centre and side hulls of the trimaran as well as using the appendage damping theory presented by Lloyd (64), as discussed in Chapter 3. Using this approach they reported reasonably good agreement between the theoretical roll RAO predictions and model experiment results in beam and stern quartering seas at a ship speed of 18 knots. The reader may recall that the author did not consider their formulation for eddy damping to be applicable to trimaran side hulls. Zhang and Andrews method was based the work of Schmitke (81) which used Tanaka’s (127) model experiment results for a limited number of monohull ship sections rolling about their own fore – aft axis of symmetry. Trimaran side hulls roll about the centre of gravity of the ship located in the
vicinity of the centre hull fore – aft axis of symmetry. Thus this theory is not applicable to trimaran side hulls.

Ikeda et al's work (114) (115) showed that the second largest roll damping component after appendage damping was lift damping due to the lift developed by the hull in roll motion. This was not included in Zhang and Andrews (8) formulation. Thus, given the inappropriateness of applying Schmike's eddy damping component to trimaran side hulls and the omission of the lift damping component by Zhang and Andrews there is clearly some scope to develop a more appropriate component damping model for a trimaran.

In the subsequent sections a theoretical roll damping prediction method will be developed based on the component damping approach of existing published theories (8) (81) (114) and (115). This theory will be based on the summation of a number of component damping terms applicable to both the centre and side hulls of a trimaran. Before proposing any of the components developed in the existing theories for inclusion in the new prediction method the applicability of using each theory on a trimaran will be discussed. Components will only be applied if considered theoretically robust for application to either the centre and side hulls of a trimaran or the centre or side hulls in isolation. New components will be developed where it can be shown the existing methods are deficient.

4.4.1 Trimaran Roll Damping Components

All of the existing theoretical methods for determining components of the total roll damping of a monohull were presented in Section 2.4 of Chapter 2. Existing research published by Ikeda et al (114) (115) (117) (118) and Schmitke (81) had proposed damping components due to hull friction, eddy shedding, hull lift, wave radiation as well as damping effects of bilge keels and other appendages. Theories for appendages will be discussed in Section 4.4.4. With the exception of wave radiation damping, which can be calculated by Potential Theory, these theoretical formulations were all based on model experiments on a small range of representative ship shapes or ship sections. Before putting forward any of these
existing theories for determining roll damping components of a trimaran centre hull, the experimental data set underpinning them must be examined to ensure that typical trimaran hull geometries are covered.

**Friction Damping**

Both Schmitke (81) and Ikeda et al (114) developed a component damping formula for friction damping based on the work of Kato (125), which required estimates of the hull wetted surface area and the non-dimensional frictional coefficient and a term called the average radius of roll. Ikeda determined the hull wetted area and average radius of roll using an empirical formula developed for conventional hull shapes of cargo ships with block coefficients from 0.56 to 0.85 and Froude numbers up to 0.25, see Ikeda (119). Trimaran designs presented in the literature, for example (6) (7) and (8), have centre hull block coefficients between 0.4 and 0.55 and operate at Froude numbers (calculated using centre hull length) in excess of 0.4. Thus Ikeda's formula for the average roll radius and hull wetted area are not appropriate for trimaran centre hulls. Schmitke (81) preferred to allow the friction to be calculated on an element by element basis for a faceted representation of the hull.

Ikeda and Schmitke also differed in how they dealt with forward speed. Schmitke suggested that the Schoenherr line based on smooth turbulent flow could be used to calculate the non-dimensional frictional coefficient, whereas Ikeda applied a purely empirical correction using the work of Tamiya et al (126) which is based on model experiments with rolling ellipsoids. This method was shown to be adequate for Froude Numbers less than 0.2. However, Schmitke's approach using the Schoenherr line is not well suited to oscillating phenomena at low to moderate forward speed; this was pointed out in the discussion after the paper.

It was shown in Section 2.5.2 of Chapter 2 that frictional damping provides a very small part of the total roll damping for monohulls based on results from published work. Considering this and the likely poor prediction of friction damping at high Froude Numbers of both methods, Schmitke's approach is chosen over Ikeda's due to the more accurate modelling of the wetted hull surface.
The formula presented by Schmitke for friction damping can be applied to the entire trimaran hull below the still water line. To facilitate this approach the hull must be modelled by a large number of flat panels. TRISKP is a panel method seakeeping code and thus such an approach can be easily incorporated.

**Eddy Damping**

Ikeda, Himeno and Tanaka (117) and Schmitke (81) published formulae for the prediction of the eddy damping for monohulls. The formulae published by Schmitke are applicable to rectangular, triangular and U-Shaped ship sections whereas those published by Ikeda, Himeno and Tanaka are based on model experiment results for models with flat bottoms and vertical side strakes linked by a circular sectioned bilge of small radius. This hull section geometry is typical of merchant ship hulls, not the high speed slender hull shapes used for trimaran centre hulls. Thus, Schmitke’s formulae are preferred as they are better suited to the slender hull geometries typically used for trimaran centre hulls.

**Lift Damping**

Of the available theories for predicting the hull lift damping component described in Section 2.5.2 of Chapter 2, the method developed by Ikeda, Himeno and Tanaka (114) is most suited to trimaran centre hulls. This method is based on the horizontal lift force developed by the hull during roll motion using theory derived from the lift of a plate aligned perpendicular to the still waterline undergoing roll motion. This theory is suited to displacement hull forms with low beam to draught ratios. Work was conducted at MARIN in the Netherlands, reported by Blok and Aalbers (134), to develop an alternative approach suited to high speed hull forms with large beam to draught ratios. Their theory was based on the vertical lift force developed on a hull during roll motion using theory derived from the lift on a flat plate aligned parallel to the still water surface with a trim angle. A similar formulation was also developed by Ikeda and Katayama (135).
The horizontal lift damping component of Ikeda, Himeno and Tanaka is better suited to the centre hull of a trimaran as the beam to draught ratios used for this method (all less than 2.5) are similar to those of typical trimarans. Zhang and van Griethuysen (9) showed that centre hull beam to draught ratios should be in the range of 2.0 to 2.5 to minimise hull frictional resistance. None of the published methods are applicable to trimaran side hulls which have very low beam to draught ratios.

Ikeda Himeno and Tanaka’s method requires knowledge of the magnitude of a hull lift coefficient, $C_L$. They showed that this coefficient could be obtained using data from on oblique towing test, where the model is towed down a tank with a fixed yaw angle with the model free to heel. In the absence of this data, an empirically based formula was given which was based on model experiments on Series 60 hullforms with block coefficients of 0.70 and 0.80, as well as a cargo ship and a tanker model. These hulls are not typical of trimaran centre hull geometries.

Based on the above discussion the following components are considered applicable to trimaran hulls:-

<table>
<thead>
<tr>
<th>Component</th>
<th>Best Applicable Theory</th>
<th>Hull</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wave Radiation</td>
<td>$B_w$</td>
<td>Potential Flow Theory</td>
<td>Centre and Side Hulls</td>
</tr>
<tr>
<td>Hull Friction</td>
<td>$B_f$</td>
<td>Schmitke, based on Kato</td>
<td>Centre and Side Hulls</td>
</tr>
<tr>
<td>Eddy Shedding</td>
<td>$B_E$</td>
<td>Schmitke, based on Tanaka</td>
<td>Centre Hull Only</td>
</tr>
<tr>
<td>Hull Lift</td>
<td>$B_L$</td>
<td>Ikeda, Himeno and Tanaka</td>
<td>Centre Hull Only</td>
</tr>
</tbody>
</table>

Table 4-4-1: Roll damping theories to be applied to a trimaran hulls

The application of these theories and other new theoretical methods are discussed in the next two sub sections.
4.4.1.1 Components for the Centre Hull

Based on the discussion above, the roll damping of the centre hull of trimaran can be determined using the following roll damping components:

- Wave radiation damping
- Eddy Shedding
- Skin Friction
- Horizontal Hull Lift

The wave radiation damping was calculated using the TRISKP code. The friction damping is calculated for each element on the mesh of the underwater part of the centre hull used by the TRISKP code using Schmitke's theory (81). This is the wetted surface of the hull below the still waterline including both the centre and side hulls. The eddy shedding damping will be calculated for the centre hull only using the theory given by Schmitke (81). The horizontal hull lift damping component will be determined using the theory of Ikeda, Himeno and Tanaka (114). No oblique towing experiments had been performed on trimaran DVZ which would allow this component to be determined accurately, instead the empirically based formula given by Ikeda, Himeno and Tanaka must be used. This formula is not particularly suited to the centre hull geometry of a trimaran. Any appendages fitted to the model were also modelled, for further details of appendage calculations see Section 4.4.3.

4.4.1.2 Components for the Side Hulls

The wave radiation damping for the side hulls is calculated by TRISKP. The friction damping is calculated for each element on the mesh of the underwater part of the side hulls used by the TRISKP code using Schmitke's method (81). This method calculates the friction damping of the submerged part of the hull below the still waterline. During roll motion the draught of each side hull will vary
during one roll cycle and may at times be zero. Thus this method can only be expected to give order of magnitude estimates of the side hull friction damping.

As discussed above, the existing formulations for determining the eddy damping and the lift damping are not appropriate for trimaran side hulls.

When a trimaran rolls the side hulls move around the circumference of a circle with a radius equal to $r_{sep}$, see Figure 4-4-1 (a). If this radius is large, the effect will be similar to the side hull heaving up and down perpendicular to the still waterline, see Figure 4-4-1 (b). Assuming that the centre hull is not heaving up and down, and thus it is only the side hulls that are moving in the vertical plane, then the vertical motion of the side hulls will be damped by the heave damping of the side hulls. Damping the side hull vertical motion will also damp the roll motion and so this effect should be accounted for in the theoretical prediction of roll damping.

The roll damping moment induced by side hull heave damping is:

$$
Moment = B_{33}^{SH} (r_{sep} \dot{x}_4) \times r_{sep}
$$

Where $(r_{sep} \dot{x}_4)$ is the angular roll velocity and $B_{33}^{SH}$ is the side hull heave damping in units of kN/(m/s). A roll damping term due to side hull heave damping, $B_{SHH}$, can be obtained by dividing equation 4-4-1 by the roll velocity:

$$
B_{SHH} = B_{33}^{SH} \left( \frac{r_{sep}}{r_{sep}} \right)^2
$$
Figure 4-4-1: Roll damping due to side hull heave. (a) Actual case (b) Roll motion approximated by heaving side hulls only

This term varies in proportion with the roll velocity and so is a linear term. Unless the side hulls are wall sided (with no flare) the term $B_{33}^{SH}$ will vary with roll angle which would make the roll damping induced by side hull heave damping, $B_{SHH}$, non-linear. Note that the side hull heave damping, $B_{33}^{SH}$, can be calculated in either the frequency or the time domain. If there are haunches on the side hulls or significant flare then computations in the time domain will be more accurate as the instantaneous waterline on the side hulls during roll motion will be used at each time step.

The roll damping induced by side hull heave is likely to be significant as it varies in proportion to the separation of the side hull from the centre hull, $r_{sep}$, squared. For trimaran DVZ this separation is 14.40 metres at ship scale.
The side hull heave damping was obtained over a range of wave excitation frequencies at different speeds for trimaran DVZ at ship scale (with the side hull draught equal to the still water value of 2.87 metres). The variation of side hull heave damping of both side hulls with wave excitation frequency is given in Figure 4-4-2 for a ship speed of 6 knots.

![Diagram](image)

**Figure 4-4-2: Variation of Side Hull Heave Damping with wave frequency for trimaran DVZ at full scale in beam seas at 6 knots**

This analysis assumes the heave damping of the side hulls is constant and does not vary when the roll angle changes. For trimaran DVZ this is unlikely to be the case, as this trimaran has moderate flare on the outboard side of the side hulls and a haunch on the inboard side. When the trimaran rolls, the draught of one side hull will increase whilst that of the other will decrease. If there are haunches or flare on the side hull, the heave damping of the deeper side hull will increase whilst that of the shallower side hull will reduce. These effects can only be properly accounted for in the time domain.

### 4.4.1.3 Magnitude of the Different Components

The TRISKP code was adapted so that the roll damping was determined by augmenting the Potential Flow theory derived wave radiation damping with a selected of theoretical roll damping components. These components, just
described, account for centre and side hull friction damping, centre hull eddy damping, centre hull horizontal lift damping and side hull roll damping induced by heave. The non-linear terms, eddy damping and friction damping, were converted to an equivalent linear form (see Section 2.2.3.2 of Chapter 2) and vary with roll amplitude. TRISKP iterates on the roll angle to find the steady solution to the equation of motion when these terms are included in the roll damping term in the equation of motion.

This new version of TRISKP was used to predict the roll damping of trimaran DVZ. An additional damping component was included to account for the lift and drag damping of the twin rudders fitted to the model using the theory of Lloyd (64) with various corrections, see Section 4.4.3. Results were calculated for ship speeds between 0 and 25 knots in beam and stern quartering seas, 90 and 60 degrees wave incidence respectively. Graphical plots of the results from TRISKP were obtained using MATLAB (www.mathworks.com) and these are shown in Figure 4-4-3 to Figure 4-4-5. Note that the roll resonant frequency occurred at an excitation frequency between 0.55 and 0.60 rad/s depending on the encounter frequency (which changes with speed and wave direction). The key to the legend text in these figures is given in Table 4-4-2.

Perusal of these figures shows that at zero speed the majority of the theoretical roll damping is from damping induced by side hull heave. As speed increases the damping due to horizontal lift on the centre hull and due to the twin rudders become increasingly important. The breakdown of the damping in beam seas with forward speed, at the damped natural frequency, is given in Figure 4-4-6. The damped natural frequency was taken as 0.595 rad/s. This value was determined from the zero speed roll decay experiments reported on in Section 4.3. This shows the dominance of the roll damping induced by side hull heave at low speeds and that the centre hull lift damping and the damping from the twin rudders become more and more significant as speed increases.
**Figure Legend Text**

<table>
<thead>
<tr>
<th>Damping Component:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Be</td>
<td>$B_e$ Total equivalent linear damping (kN m/rad/s) comprising components due to wave radiation, friction, eddy shedding, centre hull lift, side hull heave and appendage damping</td>
</tr>
<tr>
<td>Bwave</td>
<td>$B_w$ Wave radiation damping calculated using TRISKP</td>
</tr>
<tr>
<td>Bfriction</td>
<td>$B_F$ Friction Damping</td>
</tr>
<tr>
<td>Beddy</td>
<td>$B_E$ Eddy Damping (Centre Hull Only)</td>
</tr>
<tr>
<td>Blift</td>
<td>$B_L$ Lift Damping (Centre Hull Only)</td>
</tr>
<tr>
<td>Bshhve</td>
<td>$B_{SHH}$ Roll Damping Induced by Side Hull Heave</td>
</tr>
<tr>
<td>Bapp</td>
<td>$B_A$ Appendage Damping</td>
</tr>
</tbody>
</table>

**Table 4-4-2: Key to legend text in figures showing the breakdown of the damping components calculated theoretically**

**Figure 4-4-3: Breakdown of damping components from the theoretical method for trimaran DVZ at ship scale at a speed of 0 knots in stern quartering seas without roll damping appendages**
Figure 4-4-4: Breakdown of damping components from the theoretical method for trimaran DVZ at ship scale at a speed of 12 knots in beam seas without roll damping appendages.

Figure 4-4-5: Breakdown of damping components from the theoretical method for trimaran DVZ at ship scale at a speed of 25 knots in stern quartering seas without roll damping appendages.
4.4.2 Comparison with Model Experiment Results in Regular Waves

Roll motion predictions calculated using the newly adapted TRISKP code are now compared with the 2004 seakeeping experiments on trimaran DVZ. Comparisons between the theoretical and experimental RAO’s at ship speeds of 6, 12 and 20 knots are given in a range of graphs from Figure 4-4- 7 to Figure 4-4- 12. The theoretical RAO’s are not particularly accurate, giving a resonant peak much greater than the model experiments results would seem to suggest. The predictions improve with increasing forward speed. Thus, it is most likely that the roll damping at low speed has been severely under estimated. In addition the damped natural frequency predicted by the theory is lower than that indicated by the peak of the model experiment RAO curve. The predictions are less accurate than when the roll damping was determined using roll decay experiment results in
Section 4.3.3. Once again the shape of the theoretical curve does not match the experimental points at high excitation frequencies especially in beam seas.

![Graph showing roll RAO comparison](image)

**Figure 4-4-7**: Comparison of roll RAO calculated from components (theoretically) with model experiments results for trimaran DVZ in stern quartering seas at a ship speed of 6 knots

The effect that the different damping components developed in Section 4.4.1 have on the roll RAO is discussed in Appendix 4. The work presented in Appendix 4 uses the trimaran without roll damping appendages fitted as an example. This model was operated by remote control and therefore was fitted with a pair of rudders to ensure it could maintain a given course. The roll damping contribution of these appendages is incorporated using the theory of Du Cane (174). This theory is presented in Appendix 3 and discussed in further detail in Section 4.4.3.1.
Figure 4-4- 8: Comparison of roll RAO calculated from components (theoretically) with model experiments results for trimaran DVZ in beam seas at a ship speed of 6 knots

Figure 4-4- 9: Comparison of roll RAO calculated from components (theoretically) with model experiments results for trimaran DVZ in stern quartering seas at a ship speed of 12 knots
Figure 4-4-10: Comparison of roll RAO calculated from components (theoretically) with model experiments results for trimaran DVZ in beam seas at a ship speed of 12 knots.

Figure 4-4-11: Comparison of roll RAO calculated from components (theoretically) with model experiments results for trimaran DVZ in stern quartering seas at a ship speed of 20 knots.
4.4.3 **Components for the Appendages**

The literature review in Chapter 2 examined all the theoretical methods for determining the contribution of roll damping appendages that fit into the framework of predicting roll damping using either linear or non-linear components. These methods could account for skegs and bilge keels, as well as conventional fin appendages attached to the hull of a monohull. These theories consider the appendages to be deeply submerged so that the free surface does not influence the lift calculations and assume that lift is not influenced by the boundary layer around the hull.

For a trimaran, a wider variety of appendages can be fitted and if these are located on or near to the side hulls they are likely to be much closer to the free surface than is the case with monohull appendages. Thus free surface lift losses need to be included in appendage lift calculations.
One approach that can be used to include these is to model the appendages (or both the appendage and hull) in the time domain and calculate the time varying angle of attack and lift force during rolling. If, in addition, the free surface is modelled free surface lift losses can be accounted for as well. This theoretical approach has been followed by authors such as Fontaine, Huberson and Montagne (175); Luit et al (176); and Fang and Lin (163), who did not model the free surface. These approaches are theoretically complex and cannot be included in frequency domain seakeeping codes such as TRISKP. Any detailed analysis of the appendage lift and drag characteristics would need to follow this approach. The approach taken here instead is to obtain order of magnitude estimates by selecting theoretical methods that can be incorporated into the TRISKP code.

4.4.3.1 Lift Force Estimates for Appendages

A simple approach to determining the lift characteristics of the appendages was sought that would capture effects such as free surface lift losses and allow determination of lift slope curves for the wide variety of appendages that could be fitted to a trimaran. After a review of relevant literature, theory based on hydrofoil craft was selected from a book on High Speed Small Craft by Peter Du Cane (174). This theory is presented in Appendix 3 and allows for free surface effects, finite span appendages and Planform Effects for non-elliptical fin loading. The method can be easily incorporated in a frequency domain linear seakeeping code.

The two appendages that were tested in the 2004 seakeeping experiments, the pair of link-fins and the pair of T-Foils were modelled in TRISKP. This would allow damping predictions from TRISKP including the appendage theory of Du Cane to be compared with model experiment results.

To use the theory given by Du Cane the effective aspect ratio of the appendage must be known. For the base fin of a T-Foil and a link-fin this is difficult to quantify without conducting model experiments in a wind tunnel or CFD modelling. As a first somewhat conservative estimate, the effective aspect ratio
was set to be equal to the geometric aspect ratio. This assumes, for the base fin of the T-Foil, that the presence of the strut does not modify the lift. For the strut of the T-Foil and the link-fin this assumes that having an end of the appendage attached to a hull of the trimaran also does not modify the lift generated.

The effect of lift losses on the two appendages tested in the 2004 model experiments, calculated using the theory of Du Cane explained in Appendix 3, is given in Table 4-4- 3. From this it can be seen that for the link-fin, which is located closer to the free surface than the base fin of the T-Foil, lift losses of up to 25% are possible. For the T-Foil the maximum lift loss is a little less at 19%. The effect of forward speed on lift loss is most noticeable.

<table>
<thead>
<tr>
<th>Speed (Knots)</th>
<th>Link-fins</th>
<th>T-Foils, Base Fin Only</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Lift Slope $C_{La}$</td>
<td>Lift Slope $C_{La}$</td>
</tr>
<tr>
<td></td>
<td>Without Free Surface Lift Losses</td>
<td>With Free Surface Lift Losses</td>
</tr>
<tr>
<td>0</td>
<td>3.53</td>
<td>2.49</td>
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<tr>
<td>6</td>
<td>3.37</td>
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<tr>
<td>25</td>
<td>2.96</td>
<td>18.23</td>
</tr>
</tbody>
</table>

Table 4-4- 3: Calculated reduction in lift slope for the link-fin and T-Foil due to free surface effects

**4.4.3.2 Magnitude of the Appendage Damping Component**

The total roll damping for trimaran DVZ, fitted with either the pair of link-fins or the pair of T-Foils, was calculated using the adapted TRISKP code using the appendage lift theory described in sections 4.4.3.1 and Appendix 3. The variation of the different damping components with wave excitation frequency when
trimaran DVZ is fitted with the pair of link-fins is shown in Figure 4-4-13 to Figure 4-4-15 over a range of ship speeds. The key to the legend text in these figures is once again given in Table 4-4-2 in section 4.4.1.3. The damping component breakdown with increasing forward speed, calculated at the damped natural roll frequency, is given in Figure 4-4-16. Similar graphs are given in Figure 4-4-17 to Figure 4-4-20 for the case when the pair of T-Foils are fitted.

These two sets of figures show that, once the forward speed is greater than around 5 knots, the appendages provide more damping than the rest of the damping components added together and so provide the majority of damping at high speed. Note that across the speed range, the roll damping induced by side hull heaving is significant, even at high speeds where appendage lift damping is large. When the pair of link-fins is fitted, the damping induced by side hull heave still provides 16% of the total damping at 25 knots at the damped natural frequency.
Figure 4-4-14: Breakdown of damping components from the theoretical method for trimaran DVZ at ship scale at a speed of 12 knots in beam seas fitted with the link-fins.

Figure 4-4-15: Breakdown of damping components from the theoretical method for trimaran DVZ at ship scale at a speed of 25 knots in stern quartering seas fitted with the link-fins.
Figure 4-4-16: Damping breakdown at the damped natural frequency (0.595 rad/s) with forward speed in beam seas for trimaran DVZ fitted with the link-fins.

Figure 4-4-17: Breakdown of damping components from the theoretical method for trimaran DVZ at ship scale at a speed of 0 knots in stern quartering seas fitted with the T-Foils.
Figure 4-4-18: Breakdown of damping components from the theoretical method for trimaran DVZ at ship scale at a speed of 12 knots in beam seas fitted with the T-Foils.

Figure 4-4-19: Breakdown of damping components from the theoretical method for trimaran DVZ at ship scale at a speed of 25 knots in stern quartering seas fitted with the T-Foils.
Figure 4-4-20: Damping breakdown at the damped natural frequency (0.595 rad/s) with forward speed in beam seas for trimaran DVZ fitted with the T-Foils

4.4.4 Comparison with Model Experiment Results with Appendages in Regular Waves

4.4.4.1 The Link-fins

Comparisons between the roll RAO’s calculated theoretically are compared with the experimental results from the 2004 model tests for trimaran DVZ fitted with the pair of link-fins in Figure 4-4-21 to Figure 4-4-26. Theoretical RAO’s are shown both with (red line) and without (green line) free surface lift losses assuming the effective aspect ratio is equal to the geometric aspect ratio. Due to the uncertainty in the selection of the effective aspect ratio for this type of appendage, theoretical RAO’s are also shown for an effective aspect ratio twice the geometric aspect ratio (blue line) with free surface lift losses.

In Section 4.4.2 it was shown that without roll damping appendages fitted the theoretical method did not provide a good fit to the experimental data at the lowest speed (6 knots). So, even if the appendage damping has been predicted
accurately, a good fit to the experimental data at 6 knots should not be expected. At this speed the fit is better in beam seas compared to stern quartering seas. Moreover, the fit is generally better in beam seas at each of the three speeds considered.

At the highest two speeds, 12 and 20 knots, doubling the effective aspect ratio from 1 to 2, or not accounting for the free surface lift losses, improves correlation of the theoretical predictions to the experimental points at some combinations of speed and wave direction and makes the correlation worse at others. Thus, all that can be concluded from this analysis is that order of magnitude predictions of roll motion may be achieved using the new theoretical method at higher speeds.

Figure 4-4-21: Roll RAO for trimaran DVZ at ship scale with changing ratios of effective to geometric aspect ratio and without free surface lift losses for the link-fins at 6 knots in stern quartering seas
Figure 4-4-22: Roll RAO for trimaran DVZ at ship scale with changing ratios of effective to geometric aspect ratio and without free surface lift losses for the link-fins at 6 knots in beam seas.

Figure 4-4-23: Roll RAO for trimaran DVZ at ship scale with changing ratios of effective to geometric aspect ratio and without free surface lift losses for the link-fins at 12 knots in stern quartering seas.
Figure 4-4-24: Roll RAO for trimaran DVZ at ship scale with changing ratios of effective to geometric aspect ratio and without free surface lift losses for the link-fins at 12 knots in beam seas.

Figure 4-4-25: Roll RAO for trimaran DVZ at ship scale with changing ratios of effective to geometric aspect ratio and without free surface lift losses for the link-fins at 20 knots in stern quartering seas.
4.4.4.2 The T-Foils

Comparisons between the roll RAO’s calculated theoretically are compared with the experimental results from the 2004 model tests for trimaran DVZ fitted with the pair of T-Foils in Figure 4-4-27 to Figure 4-4-32. Once again results are also shown without free surface lift losses (green line) and with the effective aspect ratio used for lift calculations doubled from 1 to 2 (blue line).

For the T-Foil, doubling the effective aspect ratio reduces the RAO by a greater margin than with the link-fin discussed in the previous sub section. This is because, as shown in Section 2.5.7 of Chapter 2, the lift slope, $C_{La}$, increases with effective aspect ratio, however there are diminishing returns once the aspect ratio is greater than 3. The geometric aspect ratio of the link-fin was 4 whereas the base fin of the T-Foil has a geometric aspect ratio of 1.13. This type of
appendage is less affected by free surface lift losses (as it is deeply submerged) and removal of the free surface lift correction from the theoretical calculation has minimal effect on the calculated RAO’s.

Once again the fit of the theoretical data to the experimental results is poor at the lowest speed (6 knots) and provides only an order of magnitude estimate at higher speeds. The fit is best at high speeds when the effective aspect ratio of the base fin of the T-Foil is considered to be equal to the geometric aspect ratio.

![Figure 4-4-27: Roll RAO for trimaran DVZ at ship scale with changing ratios of effective to geometric aspect ratio and without free surface lift losses for the T-Foils at 6 knots in stern quartering seas](image-url)
Figure 4-4- 28: Roll RAO for trimaran DVZ at ship scale with changing ratios of effective to geometric aspect ratio and without free surface lift losses for the T-Foils at 6 knots in beam seas

Figure 4-4- 29: Roll RAO for trimaran DVZ at ship scale with changing ratios of effective to geometric aspect ratio and without free surface lift losses for the T-Foils at 12 knots in stern quartering seas
12 Knots 90 Degrees Wave Incidence

Figure 4-4-30: Roll RAO for trimaran DVZ at ship scale with changing ratios of effective to geometric aspect ratio and without free surface lift losses for the T-Foils at 12 knots in beam seas.

20 Knots 60 Degrees Wave Incidence

Figure 4-4-31: Roll RAO for trimaran DVZ at ship scale with changing ratios of effective to geometric aspect ratio and without free surface lift losses for the T-Foils at 20 knots in stern quartering seas.
Figure 4-4-32: Roll RAO for trimaran DVZ at ship scale with changing ratios of effective to geometric aspect ratio and without free surface lift losses for the T-Foils at 20 knots in beam seas.
4.5 General Comments

The analysis in this chapter is very interesting. One would expect accurate roll motion predictions in the region of roll resonance if the roll damping had been measured by a suitable experiment. What is surprising is that following the traditional approach of obtaining roll damping measurements from free decay experiments does not appear to give accurate roll motion predictions. This questions the validity of the results of the free decay experiments and thus the assumptions underpinning them.

The theoretical method for predicting roll damping using semi-empirically derived damping components indicated that, at low speed, it is likely to be the side hulls that provide nearly all of the roll damping. However, at high speed accurate roll damping predictions were most likely to rely upon accurate predictions of the contribution of any appendages and the lift generated by the trimaran hulls during rolling at forward speed. Roll damping predictions obtained using this method led to roll motion predictions in the region of roll resonance which were worse that those obtained using the roll damping measured in roll decay tests.

In the seakeeping experiments a number of interesting observations were made:-

- At wave encounter frequencies close to the roll resonant frequency of the trimaran and at low speed, one side hull often emerged from the water, exposing the strut of the T-Foil and much of the link-fin when they were fitted. This must reduce the effectiveness of these damping devices.
- If one side hull is not in the water, the instantaneous roll stiffness, inertia and added inertia will have changed. In addition, the location of the roll centre will have moved.
- In stern quartering seas, the autopilot on the model had difficulty maintaining the required heading with respect to the incoming wave direction. Roll and pitch motions couple together because the midships position of the side hulls is located aft of the midships position of the
centre hull and the centre of flotation. In some runs the pitch motions were large enough to expose part of the twin rudders above the water which reduced the model's ability to turn. In some cases the model was yawed around until it was beam on to the waves.

- The appendages were not as effective in damping roll motion in stern quartering seas as they were in beam seas. This could be because the motion was being driven by coupling of roll with pitch and yaw.

Neither of the methods utilised to predict roll motions of the trimaran model were particularly accurate both at excitation frequencies in the region of roll resonance, dominated by accurate roll damping predictions, and at excitation frequencies away from resonance, dominated by the accuracy of the seakeeping code.

Thus, the hypothesis set out at the beginning of this chapter has been disproved.
4.6 Conclusions

At the beginning of this chapter a hypothesis was proposed which was to be proved or disproved by a series of theoretical and experimental investigations. The hypothesis was:

*That accurate trimaran roll motion predictions can be obtained using linear Potential Flow Seakeeping theory with the roll damping term either obtained from a roll decay experiment or augmented with empirically based theoretical roll damping components developed for monohulls.*

Four important conclusions can be drawn from this research that categorically disproves this hypothesis. Firstly, recall from Chapter 2 that for a linear spring – mass – damper system the damping term in the equation of motion only affects the output motion in the region of resonance, and that this is the region where the peak of the output motion generally occurs. Therefore, when theoretical roll motion predictions (using linear theory) are compared with model experiment results, if the damping is accurate the theoretical results would be expected to closely match the experimental points in the region of resonance where the peak roll RAO is located. When the linear Potential Flow TRISKP code was used with roll damping measured from roll decay experiments for the trimaran without roll damping appendages the peak predicted RAO did not match the experimental results. There was no observable trend – the predictions were not consistently greater or less than the experimental results. Results with the appendages fitted, which should increase the roll damping, were sometimes better and other times no better than the correlation between the experiments and theoretical predictions without the roll damping appendages fitted. Thus it can be concluded that either the roll damping has not been accurately measured in the roll decay experiments or that the linear Potential Flow method coupled with the experimental roll decay data is not suitable for predicting peak roll motions in the resonant region.

Secondly, if we now consider the correlation between the peak theoretical roll motions predicted using the semi-empirically derived roll damping components
and the model experiment results (occurring in the region of roll resonance) for the trimaran without roll damping appendages fitted, it was shown that the theoretical results were, in general, significantly under-damped. The correlation between the peak in the theoretical roll motions and the experiment results improves with forward speed. This highlights the importance of the centre hull lift damping term which increases with forward speed. Once the appendages were included, using a simplistic theory, the correlation between experimental and theoretical results in the region of resonance improved, most notably at forward speed when the lift generated by the appendages would be greatest. However, this improvement was very dependant on assumptions made about how the lift characteristics were modified due to the proximity of the appendage to the free surface and the connection between the appendage and the ship. Interestingly, the appendage roll damping component provided over half of the total roll damping at only 6 knots. Thus, roll motion predictions in the region of resonance can only be obtained for an appended trimaran if the contribution of the appendages is determined with high accuracy. This all leads to the conclusion that roll motions in the region of resonance cannot be accurately predicted using the chosen semi-empirical components (based on logical extensions of published methods) and linear Potential Flow Theory when calculations are performed in the frequency domain.

The first two points have focused on the inability to predict the roll damping accurately. Attention is now focused on roll motion at excitation frequencies away from resonance. Regardless of how the roll damping was predicted, the correlation between theoretical roll motion predictions and experimental results at the highest excitation frequencies measured were always poor. The experimental roll RAO was always greater than the linear Potential Flow theory derived result. Recalling the work on the linear spring – mass – damper system in Chapter 2, this indicates that any or all of the following have not been predicted accurately: - the roll inertia, roll added inertia and roll stiffness. From this, the obvious conclusion is that the linear frequency domain method does not yield accurate trimaran roll motion predictions.
The fourth and final point can only really be inferred from the experimental observations and to some extent the work on the theoretical prediction of the appendage roll damping component. During the seakeeping experiments, when rolling heavily, one of the trimaran side hulls could come completely out of the water. Any appendages attached to the side hull would then either partially emerge or be very close to the water surface. In the development of a theoretical method to predict the damping contribution of the appendages it was shown that free surface lift losses were significant. Thus, the lift of any appendage attached to a side hull which is not deeply immersed will vary throughout the roll cycle. Furthermore, if a side hull is coming out of the water the roll stiffness will vary during one roll cycle along with the total roll inertia and added inertia of the three hulls. This variation will be further affected by the haunches and any flare on the side hulls. Therefore the final conclusion is that, due to these observations, linear Potential Flow theory when solved in the frequency domain is not adequate for trimaran roll motion predictions. Further investigations are required to prove this postulation and should focus on time domain methods which accurately model the instantaneous position of the three hulls and any appendages.

The thesis now continues by conducting further research to try to understand why this hypothesis is incorrect, and in particular, to identify which parts of the analysis process followed in this chapter were not suitable for a trimaran hull configuration.