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Narrow Band High Resolution Radar Imaging

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A thesis submitted for the degree of Doctor of Philosophy
of the University of London

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July 2007
I, Shirley Lynne Coetzee, confirm that the work presented in this thesis is my own. Where information has been derived from other sources, I confirm that this has been indicated in the thesis.

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Abstract

Most modern radar systems use a monostatic configuration and exploit wide bandwidth to achieve high down range resolution used for target detection and classification. However increasing pressure on occupancy of the radio spectrum and the requirement for ever more accurate target classification poses serious challenges to future radar designs. Techniques based on narrow band radar are thus being investigated as a mean of reducing spectral occupancy. This approach can be coupled with the use of a multiple radar (multistatic) geometry to provide a potentially powerful technique for improving target detection and classification even beyond that of conventional systems.

A multistatic topography allows the application of tomographic techniques for target imaging. Tomography is a process by which a two-dimensional cross-sectional image of an object is obtained via illumination from a variety of differing angles in a variety of differing planes. In radar tomography observations from multiple radar locations enable a three dimensional projection in Fourier space. In this way a three dimensional image of an object can be constructed using techniques such as Backprojection. The use of a narrow band waveform in multistatic radar tomography trades resolution achieved by bandwidth for resolution achieved by spatially diverse angular imaging.

This thesis reports a detailed investigation into a range of narrow band, multistatic geometries and techniques to obtain high resolution imaging of moving targets. Images processed using Synthetic Aperture Radar (SAR) and Inverse SAR (ISAR) configurations, modified to emulate multistatic narrow band configurations, have been investigated for both real and simulated data. The effect on the spatial resolution due to masking, ambiguity and coherency of targets consisting of both sparse and dense scatterers was analysed under a range of conditions.

A cross range resolution of $\lambda/4$ was achieved using simulated data. This analysis was also extended to the case of real data of typical ground targets. In this situation the data is inevitably significantly affected by noise but a resolution of $\lambda/2$ was achieved.
This study concludes with a comparison of modeled narrow band system performance with theoretical predictions leading to a preliminary assessment of the capability of the narrow band radar tomographic imaging technique for potential applications.
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Publications

The following paper was published as a result of this Ph.D research:

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List of Principal Symbols

Nomenclature for all equations is defined in the main body of the document, however a list of the more commonly used symbols is given here.

\[ \begin{align*}
\theta & \quad \text{Angle} \\
B & \quad \text{Bandwidth} \\
\delta_{\text{cr}} & \quad \text{Cross-range resolution} \\
f & \quad \text{Frequency} \\
N & \quad \text{Number, quantity} \\
\phi & \quad \text{Phase} \\
\tau & \quad \text{Pulse width} \\
R, r & \quad \text{Range} \\
\delta_{r} & \quad \text{Range resolution} \\
d & \quad \text{Real antenna length} \\
\omega & \quad \text{Rotation velocity} \\
\xi & \quad \text{Spatial frequency} \\
c & \quad \text{Speed of light} \\
T, t & \quad \text{Time} \\
\nu & \quad \text{Velocity} \\
\lambda & \quad \text{Wavelength}
\end{align*} \]
List of Abbreviations and Acronyms

CAT, CT  Computer-Aided Tomography
CBP  Convolution Back Projection
CDT  Coherent Doppler Tomography
CSA  Chirp Scaling Algorithm
CW  Continuous Wave
DFT  Discrete Fourier Transform
FBP  Filtered Back-Projection
ISAR  Inverse Synthetic Aperture Radar
LFM  Linear Frequency Modulated
MFP  Matched Filter Processing
MTRC  Motion Through Resolution Cells
1D  One Dimensional
PCMFP  Phase Conjugate Matched Filter Processing
PFA  Polar Format Algorithm
PRF  Pulse Repetition Frequency
PRI  Pulse Repetition Interval
RAR  Real Aperture Radar
RCS  Radar Cross Section
RMA  Range Migration Algorithm
SAR  Synthetic Aperture Radar
2D  Two Dimensional
UNB  Ultra Narrow Band
ZDC  Zero Doppler Correction
Chapter 1

Introduction
1.1 Motivation

This thesis describes a research into a novel concept for the production of high resolution imagery using narrow band radar. High resolution, which is normally obtained using wide bandwidths, is achieved instead by using spatially diverse angular imaging. Using this in conjunction with a multistatic topography allows for the application of tomographic techniques and provides a means to produce high resolution images from narrow band radar.

1.1 Motivation

With the continued growth in traditional applications and emergence of new technologies, the electromagnetic spectrum is becoming increasingly congested and new strategies are being adopted to improve spectral efficiency.

For example, in modern radar, increased bandwidth in the form of frequency diversity, modulation or stepped frequencies has been used to determine additional information about objects. This is adding to spectral congestion making the problem even worse. In addition, very wide band radar systems are inherently expensive and more complicated to produce. They tend to require high peak power to give good detection capability and have stringent requirements on waveform, implementation of oscillator stability and phase noise.

Thus alternatives that use less bandwidth while at the same time achieving the same performance become extremely attractive. One approach is to use narrow band waveforms. Operating with inherently Continuous Wave (CW) signals permits a substantial reduction in thermal noise power hence improving overall sensitivity. However, the drawback is that they have little or no effective range resolution and hence seem a poor choice for applications requiring imaging solutions.

To provide image resolution in this system, the frequency diversity in wideband radar is traded for spatial diversity in narrow band radar. This is achieved by making use of tomographic imaging which processes 1D cross-range projections taken from multiple angles around a target so as to produce a 2D image of the target. In order to obtain the
1D projections from narrow band signals, the signals need to be Fourier transformed to form 1D Doppler projections. A tomographic imaging process is then used to form a 2D image of the target.

A geometric diversity of 360 degrees across a large number of sensor sites is well known to offer sub-wavelength resolution (Carrara, 1995), (Soumekh, 1999). However, little research has been reported on the use of narrow band radar in this configuration to create high resolution imagery.

Overall, multistatic narrow band radar appears to be an attractive alternative to conventional high resolution techniques. There are, however, many research issues to be addressed in assessing the feasibility of this technique. This study is thus concerned with investigating the limitations and finding novel solutions to the problems of narrow band imaging.
1.2 Literature Review

In this section, a review of key previously published work on high resolution Synthetic Aperture Radar (SAR) and Inverse Synthetic Aperture Radar (ISAR) has been carried out. The literature relating to tomography, which is of particular relevance to narrow band radar imaging, is then examined. A critique of recent work, specifically on aspects of high resolution narrow band radar tomographic imaging is then presented. This literature review has allowed a number of narrow band imaging concepts to be evaluated and their performance, limitations and applications to be assessed.

In 1967, Brown (Brown1,1967) presented a paper on synthetic aperture radar which characterised the motion, signals and resolution of a stripmap mode SAR, from a mathematical and an optical processing point of view. It was also one of the first papers to describe SAR imaging of a rotating target. However, the mathematics was treated in a stripmap mode manner without realising that the resolution may be also enhanced. This paper does point out though that complex phase errors occurred in both SAR and rotating cases due to motion when the coherent integration time was long enough for point scatterers to move through resolution cells.

In a further paper from Brown et al (Brown2,1969), a partial compensation for Motion Through Resolution Cells (MTRC) was provided by approximating the circular motion as a quadratic. Limitations on resolution were imposed accordingly. The *modus operandi* for processing SAR images continued to be the application of a two-dimensional Discrete Fourier Transform (DFT) to a small length or narrow angle of a stripmap mode geometry to produce images from range-Doppler data given in a cartesian format and thereby avoiding MTRC.

In 1980, Walker (Walker,1980) also characterised the requirements for optically processing coherent radar data collected from targets positioned on a rotating platform. He introduced the concept of the polar format as a layout appropriate for the compensation of MTRC. With MTRC compensated, this gave the possibility of higher resolutions being achieved than is possible using stripmap mode SAR. This concept essentially initiated the development of the spotlight mode SAR and ISAR.

The similarities between spotlight mode SAR / ISAR and Computer-Aided Tomography
1.2 Literature Review

(CAT or CT) were immediately apparent to (Munson,1983), (Mensal,1983), (Mensa2,1984), (Magotra,1991), (McCoy,1991) and others. Since then, many advances have been made in radar using tomographic imaging.

Munson et al (Munson,1983) proposed a tomographic formulation for spotlight-mode SAR. It was shown that a Linear Frequency Modulated (LFM) signal, otherwise known as a chirp signal, taken by a spotlight mode SAR at specific look angles using a narrow beamwidth may be approximated as a tomographic projection (in the form of a one-dimensional Fourier transform) at that angle. By applying a tomographic process known as the Convolution Back Projection (CBP) algorithm, see Chapter 2, a two dimensional image may be produced from the range data taken over many look angles.

However, various radar issues affecting the process were also examined, namely wavefront curvature, residual quadratic phase errors, and the effects of Doppler and time-varying range. These are explained in more detail in Chapter 2.

One of the first studies done was also that of Mensa et al (Mensa1,1983), (Mensa2,1984) who applied this new tomographic imaging process to monochromatic Continuous Wave (CW) radar. Mensa et al 's study of narrow band radar tomography is similar to this research and thus is investigated and used for comparison.

A mathematical function was derived from the tomographic imaging mathematics producing a point spread function which had a central lobe of width of $\lambda/5$ which is an extremely high resolution. Unfortunately though, the first circular sidelobe level was high at only 8 dB below the central peak, severely limiting the dynamic range which could be partially overcome by using bistatic diversity or wideband frequency.

The paper also introduced the Coherent Doppler Tomography (CDT) technique. CDT differs from the most popular tomographic technique, the Filtered Back-Projection (FBP) technique.

In FBP, each projection is Fourier transformed and filtered. Then the transformed projections at each angle are backprojected to position each point at the correct coordinates. In CDT, however, the points of the projections are transformed in angle first by performing a circular convolution on the ring of points occurring at that frequency. Finally the frequencies of all these rings are interpolated to their position.
This leads to the definitions of coherent and non-coherent image processing. In coherent processing, the phase information is preserved throughout the entire imaging process. In non-coherent processing, subapertures are formed using only small pieces of the polar array at a time and the resulting image is formed by combining only the magnitude of each subimage. This is called non-coherent processing because the phase information for each is discarded. This is the method used by MATLAB's iradon process which is applied in this research.

Mensa et al's paper describes the experimentation using a bistatic system and polychromatic CW (over discrete frequencies) radiating a rotating object over 360 degrees. This showed that point scatterers separated by a distance of $\lambda/5$, were successfully resolved provided that the object consisted of sparse arrays of scatterers of similar strength where the scatterers are small compared to a wavelength. However, the imaging of an object consisting of dense arrays of scatterers or of large scatterers was impaired by the high sidelobes of each scatterer.

Although the paper suggested the wide band option to partially reduce the dominating effect of sidelobes, this was not investigated further. The use of weighting functions was not explored in this work.

The mathematics and experiments were designed for two scatterers diametrically opposite each another on the axis of rotation. As experiments in this research shows, the use of symmetrical objects provides the best results when using tomographic imaging. If the object had been unsymmetrical, their results may have been inconclusive as to whether the poor imaging was due to resolution or the size and density of scatterers or due to the characteristics of tomographic processing.

Wicks et al (Wicks1, 2005), (Wicks2, 2005), (Wicks3, 2005), Bonneau et al (Bonneau, 2002) and Bracken et al (Bracken, 2006) collaborated to present a number of papers and presentations on the topic of target detection and interference rejection by using tomographic processing of geometrically diverse, multi-frequency, multistatic, Ultra Narrow Band (UNB) radar data, employing similar ideas to those explored in this thesis.

Their aim was to show that geometrical diversity offers the potential for increased resolution, and is dual to frequency diversity (increased bandwidth) in classical monostatic
The theory proposed in (Bonneau, 2002) and (Wicks1, 2005) is that the expected spatial resolution \( \delta = \lambda_{\text{min}}/3 \), although they do not justify this.

Initial simulations and experiments portrayed a multistatic geometry consisting of two concentric circles centred around an area of interest, one a circle of transmitters and the other, a circle of receivers. Discrete tones of CW frequencies were used and the various transmitter and receiver pairs were treated as bistatic systems.

The objects, consisting of moving targets at various speeds, were spatially resolved based on their Doppler characteristics over specific coherent processing intervals. The simulations and experimentation were shown to agree with the theory.

No tomographic imaging in the sense of adapting medical imaging methods for radar, as discussed previously or as explained in detail in Chapter 3, was performed here. The work throughout the entire collaboration refers to polar format frequency space as tomography.

In (Bonneau, 2002) and (Wicks1, 2005), the spatial resolution was proved using a bank of matched filters to perform correlation to the various targets based on target position and velocities and form radar images at the correct positions based on the matched filter point function. This technique was referred to initially as Matched Filter Processing (MFP) and later as a more advanced version known as Phase Conjugate MFP (PCMFP).

The work was continued in (Wicks2, 2005), (Wicks3, 2005) and (Bracken, 2006) by comparing ISAR imaging to UNB MFP imaging of rotating objects on a turntable. The ISAR used a stepped frequency from 9-10 GHz while the UNB radar operated at a 10 GHz frequency. Background subtraction performance was also examined.

The simulation and experimental results showed that a high resolution (sub-wavelength) image could be achieved using the spatial diversity of UNB signals. It was also shown that wide bistatic measurements are degraded by direct path motion and have a correspondingly degrading impact on the image. Fundamentally, they showed that the target image for narrow band tomography produced a much finer spatial resolution than that of the wideband ISAR. Again the theory employed, matched filter processing and concept of tomography as polar format frequency space remained unchanged in the later papers.
1.2 Literature Review

This review has allowed the baseline for this technology to be evaluated in order to inform the research carried out during this study. Particular issues for further examination became evident from the literature survey including:

- Use of tomographic imaging based techniques using 1D cross range projections taken from multiple perspectives around an object so as to produce a 2D image of the object.

- Resolution performance limits in terms of range, cross range and stationary spatial sampling.

- Examination of the difference between narrow band imaging of sparse and dense objects.

- Practical limitations of this concept.
1.3 Outline of Thesis

This thesis has been organised as follows:

Chapter 2 describes the background theory applicable to this research. In the first section, radar resolution principles are described. Inverse Synthetic Array Radar (ISAR) theory for wide bandwidth, narrow angles is discussed in detail as a comparison against the narrow bandwidth, wide angle ISAR researched here. This section goes on to provide an overview on the synthetic aperture radar imaging techniques used conventionally for narrow angles. The next section of this Chapter gives an explanation on frequency space and how it pertains to radar resolution. The final section gives an explanation of tomographic imaging principles.

The 'Narrow Band High Resolution Imaging' concept is described in Chapter 3. The narrow band radar cross range profiles used in the imaging process as well as tomographic imaging using these profiles is explained. The narrow band spatial frequency theory applied in this work is also discussed.

The software processor that is applied to simulated and real data to investigate the concept is described in detail in Chapter 4. A description is given on how the data is prepared for use then processed to obtain 2D narrow band high resolution images. These images are compared to multilook images using conventional radar imaging techniques.

Chapters 5 and 6 present results and analysis using both simulated and experimental data. Sparse scatterers are examined in Chapter 5 where a comparison is made between results from the simulated and real data. This is followed by comparison and analysis of the results of dense real targets in Chapter 6. The resulting tomographic images are analysed to test the high resolution imaging achieved against the theoretical predictions. The effect on the spatial resolution of masking, ambiguity and coherency as well as the number of target scattering centres is analysed under a range of conditions. In addition, the effect of multipath on imaging of a partially-opaque target type is highlighted and analysed.

In Chapter 7, a theory to describe the limits of achievable resolution possible in radar systems is proposed. The results of a simulation developed to evaluate this theory is also
The conclusions to the study are given in Chapter 8. The development of the theory and processor is discussed and the results and analysis of the simulation and experiments are summarised. The resolution limitations identified from these studies are examined and methods of overcoming them are suggested. Finally, some of the potential applications for this technique are discussed. The recommendations for further work conclude this chapter and the thesis.
Chapter 2

Background Theory
In this chapter, the essential theory of high resolution radar imaging is introduced. This provides the fundamentals from which the narrow band imaging concept can be developed. Firstly, resolution and how it may be achieved in the down range direction is examined. Subsequently synthetic aperture approaches to achieve resolution are considered. Inverse Synthetic Aperture Radar (ISAR) for narrow angles is then discussed as well as the conventional methods used to generate radar images in the narrow angle case. Polar format frequency space and the relationship between frequency space and resolution is also explained. Finally, the process of tomographic imaging is described.

2.1 Radar Resolution

The ability of a radar to distinguish between individual target scatterers is a key measure of its performance. The minimum distance between two point scatterers that may be distinguished from one another is known as the resolution.

In this section, the various methods that have been traditionally used to obtain high resolution are discussed. The combination of these methods to create high resolution radar images is also examined.

2.1.1 Range Resolution from Unmodulated, Pulsed Signals

Consider a radar transmitting unmodulated, pulsed signals to a target, see figure 2-1.
2.1 Radar Resolution

The round-trip propagation time between transmission and reception of each pulse is given by:

\[ T = \frac{2R}{c} \]  

(2-1)

where: \( R \) = range and \( c \) = speed of light

Therefore, the range of targets can be determined from the round-trip propagation time between the transmitted and received signals, as given by:

\[ R = \frac{cT}{2} \]  

(2-2)

The Pulse Repetition Frequency (PRF) is the frequency with which pulses are transmitted. Similarly the Pulse Repetition Interval (PRI) is the time measured between pulses.

In this simple unmodulated, pulsed signal case, the resolution between two targets can only be measured to a minimum of the pulse length separating the two return echoes. i.e. the range resolution is determined by the pulse width \( \tau \) of the signal, as:

\[ \delta_r = R_{tgt1} - R_{tgt2} = \frac{c(T + \tau)}{2} - \frac{cT}{2} = \frac{c\tau}{2} \]  

(2-3)

Correspondingly, in an unmodulated, pulsed signal with bandwidth \( B \approx 1/\tau \), the range resolution is given by:

\[ \delta_r = \frac{c}{2B} \]  

(2-4)

For example, if \( \tau = 1\mu s \) then \( B = 1 \text{ MHz} \) and using either of the above equations gives \( \delta_r = 150 \text{ m} \).

To achieve high resolution, therefore, the pulse length has to be reduced. This reduces transmit energy and ultimately limits sensitivity. To overcome this, pulse modulation can be employed.
2.1.2 Range Resolution from Modulated, Pulsed Signals

As previously discussed, short pulse lengths and high bandwidth are required for high resolution where unmodulated pulses are used. However longer pulses enable higher energy to be directed to the target and improve detection performance. A compromise between these conflicting requirements can be found by using modulation techniques on wider pulses.

By modulating wide pulses, each part of the waveform can be identified, giving enhanced range information. The result of matching a received waveform to that of the transmitted waveform produces a compressed waveform which identifies the delay in the modulation between the two signals and thereby provides a measure of range and range resolution.

Here higher range resolution can be achieved by applying a modulation to the transmitted signal that extends the bandwidth beyond that given by the reciprocal of the pulse length. For example, figure 2-2 shows a signal which has been linearly modulated in frequency over a bandwidth $B$. This modulated signal is known as a chirp signal.

\[
\delta_r = \frac{C}{2B} \tag{2-5}
\]

This technique, known as pulse-compression, produces high range resolution from wide band signals.

For example, if $B = 1$ GHz then $\delta_r = 0.15$ m = 15 cm.
2.1.3 Stepped Modulation Sampling

Stepped modulation sampling is an advance on the above method where a number $N$ pulses, each of modulation bandwidth $B$, collectively serve as a long pulse of bandwidth $= NB$. The term stepped frequency refers to the fact that the frequency of each successive pulse, within each burst of pulses, is increased by a constant frequency step. Stepped frequency systems may use a variety of waveforms. A stepped chirp (linear FM) waveform is shown in figure 2-3. This type of waveform was used throughout this research.

Using stepped frequency techniques, the range resolution can therefore be significantly improved without using very short pulse widths and large bandwidths:

$$\delta_r = \frac{c}{2NB}$$  \hspace{1cm} (2-6)

In the stepped frequency case, the PRI is deemed to be the time between one burst of $N$ stepped frequency pulses to the time of the next burst. The coherent integration time $T$ is the total number of PRIs measured.
2.1.4 Cross-Range Resolution from Beamwidth

Consider a stationary target a distance $R$ from a stationary radar. Without any signal processing, the cross-range resolution $\delta_{cr}$ of a Real Aperture Radar (RAR) is given by the radar footprint $D$, see figure 2-4.

\[
\delta_{cr} = \frac{D}{R}
\]

Figure 2-4: Real aperture radar beam

If the beamwidth $\Delta \theta$ is narrow, the radar footprint $D$ can be approximated by (Nathanson, 1999):

\[
\delta_{cr} = D \approx R \Delta \theta_{4dB}
\] (2-7)

The 2D beamwidth is related to the real antenna length $d$ and wavelength $\lambda$ by (Nathanson, 1999):

\[
\Delta \theta_{4dB} = \frac{\lambda}{d}
\] (2-8)

Therefore, the cross-range resolution is:

\[
\delta_{cr} = R \frac{\lambda}{d}
\] (2-9)

For example if $R = 10$ km, $\lambda = 3$ cm and $d = 1$ m then $\delta_r = 300$ m.

Thus, to obtain a high cross-range resolution, the beamwidth must be very narrow. To maintain a high cross-range resolution as the range is increased, the wavelength must be decreased or antenna cross-range length must be increased. These options are limited in practice.
However, if the respective velocity between the radar and a target of interest changes, the phase shift caused by the change in range may be used to improve the cross-range resolution obtained. This is the basis for the development of synthetic apertures.

2.1.5 Cross-Range Resolution from Stripmap Mode Synthetic Apertures

Consider an airborne radar transmitting pulses from a side-looking antenna towards a strip of ground parallel to the flight path, as shown in figure 2-5. $L_{\text{max}}$ is defined later in equation 2-10.

![Figure 2-5: Side-looking stripmode SAR beam](image)

This scenario describes that of a stripmap mode synthetic aperture radar (SAR). Common imaging modes in SAR are: stripmap (where the antenna sweeps a strip parallel to the flight-line), spotlight (where the antenna continuously aims on a specific terrain spot) and scan (where the antenna aims on a short strip) (Carrara, 1995), (Stimson, 1998), (Oliver, 1998).

The side-looking stripmode SAR described above receives echoes of the transmitted pulses at intervals during the flight as if received from virtual antenna elements placed at regular positions along the flight path.

By processing the stored echoes as if they were all received at the same time, it is possible to synthesise a very long side-looking linear aperture.

Just like range resolution was improved by making use of frequency bandwidth, so the cross-range resolution can be improved by making use of spatial bandwidth. This is
achieved by noting the change in phase returned in the received signals as the stripmap mode SAR travels past the stationary target.

The phase of the signal is dependent on the round trip time delay \( t_D \) given by equation 2-10.

\[
t_D = \frac{2r}{c} \quad (2-10)
\]

where: \( r \) = range between the radar and the target
\( c \) = speed of light

At time \( t_D \), the phase of the received signal is given by equation 2-11:

\[
\phi = -2\pi f_0 t_D = -2\pi f_0 \frac{2r}{c} = -\frac{4\pi r}{\lambda} \quad (2-11)
\]

where: \( f_0 \) = carrier operating frequency

\( \lambda = c/f_0 \) = carrier wavelength

As a result of the ongoing change in phase, the frequency of the return signal is shifted. This ‘Doppler shift’ in frequency, \( f_D \), is dependent on the radial velocity \( \nu_r \) between the radar and the target, as shown in equation 2-12.

\[
f_D = \frac{1}{2\pi} \frac{d\phi}{dt} = -\frac{1}{2\pi} \frac{d}{dt} \left( \frac{4\pi r}{\lambda} \right) = -\frac{2dr}{\lambda \frac{dt}{dt}} = -\frac{2\nu_r}{\lambda} \quad (2-12)
\]

where the minus sign denotes a receding target, while a plus sign in its place denotes a closing target.
Consider a radar moving past a target in the scenario shown in figure 2-6.

For the above geometry, the point target response has phase shift and Doppler characteristics dependent on cross range similar to those shown in figure 2-7.

Figure 2-6: Stripmap mode SAR geometry

Figure 2-7: Stripmap mode SAR geometry, phase shift and Doppler shift
2.1 Radar Resolution

The Doppler frequency, \( f_D \), is related to the tangential velocity \( v_t \) as

\[
f_D = -\frac{2v_t \sin(\Delta \theta)}{\lambda}
\]  
(2-13)

The corresponding Doppler bandwidth is (Brown1,1967), (Brown3,1969), (Ulander1,1996), (Ulander2,2003)

\[
B_D = f_D(\text{positive}) + f_D(\text{negative}) = \frac{4v_t \sin(\Delta \theta/2)}{\lambda}
\]  
(2-14)

The bandwidth gives a time resolution in the tangential direction of (Carrara,1995)

\[
\delta_t = \frac{1}{B_D}
\]  
(2-15)

The cross-range (or azimuth) resolution expressed for stripmap mode SAR is given by (Ulander1,1996), (Brown1,1967)

\[
\delta_{cr} = v_t \delta_t = \frac{\nu_t}{B_D} = \frac{\lambda}{4\sin(\Delta \theta/2)}
\]  
(2-16)

For a narrow beam, \( \sin(\Delta \theta/2) \approx \Delta \theta/2 \), which approximates \( \delta_{cr} \) to:

\[
\delta_{cr} = \frac{\lambda}{2\Delta \theta}
\]  
(2-17)

For general use, the limit value is calculated based on assumptions of narrow beamwidth, as the following example (Stimson,1998) shows. However, the finest cross-range resolution achievable is limited by a number of factors as Chapter 7 will show.

The beamwidth of a side-looking stripmode SAR is the 2-way version of the real antenna beamwidth given previously in equation 2-8, i.e.

\[
\Delta \theta_{4dB} = \frac{\lambda}{2L}
\]  
(2-18)

where \( L \) is the length of the synthetic aperture.
2.1 Radar Resolution

The cross-range resolution may be approximated by:

\[ \delta_{cr} = R \frac{\lambda}{2L} \]  
(2-19)

However, the beamwidth of the real antenna used in SAR is still governed by equation 2-8, namely:

\[ \Delta \theta_{4dB} = \frac{\lambda}{d} \]  
(2-20)

where \( d \) is the real antenna length.

As both of these equations (2-19 and 2-20) are true, this means that the length of the synthetic aperture \( L \) is limited to a maximum of:

\[ L_{max} = \frac{\lambda}{d} R \]  
(2-21)

and, therefore, the cross-range resolution is limited to a minimum of:

\[ \delta_{cr} = R \frac{\lambda}{2\frac{d}{R}} = \frac{d}{2} \]  
(2-22)

Therefore SAR azimuth resolution is approximately half the real antenna in the azimuth direction, independent of range and wavelength. This is only true for uniformly illuminated synthetic aperture radars such as sideling stripmap mode SAR. However, if the antenna beam is continuously focused on a point of interest such as it is in spotlight mode SAR, circular SAR or Inverse Synthetic Aperture Radar (ISAR) then the cross-range resolution may be improved further.
2.1.6 Cross-Range Resolution from Spotlight Mode Synthetic Apertures

Spotlight mode SAR, shown in figure 2-8, is used to obtain a relatively fine-resolution image of a known location or target of interest.

As the platform passes the target, the beam pointing direction changes to keep the target in view. In that case, the narrow angle approximation given in equation 2-17 can be replaced by the wide angle approximation given by equation 2-16 instead and therefore

$\delta_{\sigma}(\text{spotlight mode}) \leq \delta_{\sigma}(\text{stripmap mode})$ \hspace{1cm} (2-23)

If the spotlight SAR platform follows the arc of a circle with the target directly below the center of the circle, then the concept is similar to circular radar (where the radar is flown in a circle around a specific terrain spot) and to the circular aperture discussed in Chapter 7.

2.1.7 Ambiguities that Impair Resolution

Ambiguities occur that detrimentally affect resolution and are due to the PRF selected for the radar system.

Range ambiguities occur when a return pulse is received simultaneously in the mainlobe along with pulses from other ranges (Stimson,1998). To avoid range ambiguities,

$$PRF < \frac{c}{2W \sin \Delta \theta}$$ \hspace{1cm} (2-24)
where $W$ is the length of the footprint (Curlander, 1991), (Griffiths, 2006), see figure 2-9.

Doppler frequency ambiguities occur when the PRF is not sampling the Doppler frequencies sufficiently. The PRF must be greater than the instantaneous Doppler bandwidth $B_D$ to prevent the detection of two ambiguous targets simultaneously in the antenna lobe (Kingsley, 1992), (Curlander, 1991). The Doppler bandwidth $B_D = \nu_t/\delta_{cr} = \nu_t \cdot 2/d$ where $\nu_t$ is the velocity of a platform in the tangential direction and $d$ is the real aperture length, see figure 2-10. In order to meet Nyquist requirements,

$$PRF > 2\nu_t/d \quad \text{(2-25)}$$

Figure 2-9: Range ambiguities

Figure 2-10: Doppler ambiguities
In terms of antenna theory, this same minimum PRF places the first grating lobe between the first and second sidelobes of the real antenna. A synthetic array's stronger sidelobes may cause false targets to appear on either side of mainlobe targets. The sidelobes may be reduced through amplitude weighting (Kingsley, 1992), (Griffiths, 2006), (Stimson, 1998).

By using wide bandwidth, large spotlight-mode synthetic apertures and avoiding these ambiguities, uncorrupted high resolution may be achieved in radar.
2.2 Inverse Synthetic Aperture Radar

Inverse Synthetic Aperture Radar (ISAR) is similar to uniformly illuminated SAR, such as stripmode SAR, but in this case the radar is stationary and the target is moving.

Two cases of ISAR exist:

- Co-operative ISAR, in which the target motion (usually rotation) is controlled and known. This is used for high-resolution target RCS imaging.
- Non-co-operative ISAR, in which the target motion is not known \textit{a priori}, and needs to be estimated.

Both real and simulated data obtained from turntable ISAR (co-operative) has been used in this research. As is shown in later chapters, the data has been processed and modified to emulate the performance of narrow band radar for testing purposes.

To understand and simulate the experimental process, understanding fundamental ISAR theory is necessary. This section explains basic ISAR theory in the case of wide bandwidth and narrow angles which will be expanded and compared against the case of narrow bandwidth and wide angles in successive chapters.

2.2.1 Inverse Synthetic Aperture Radar Signals

An ISAR radar transmitting a signal of:

\[ T(t) = u(t).\exp(j.2.\pi.f_0.t) \]  
\[ (2-26) \]

where: \( u(t) \) = instantaneous signal

The reflected signals are received from the target are (Mensa2, 1984):

\[ S(t) = \sigma(x,y).\exp.j.\{2.\pi.f_0.t - \frac{4.\pi.f_0}{c}(R_0(\theta) + \nu(t).t - r.\cos(\omega t - \theta))\} \]  
\[ (2-27) \]

where: \( \sigma \) = reflectivity index
\( x \) = parallel cartesian coordinate axes
2.2 Inverse Synthetic Aperture Radar

\( y = \) perpendicular cartesian coordinate axes
\( R_0 = \) range from the radar to a reference point on the target
\( r = \) radial range of the reference point on the target to a specific scatterer
\( \nu = \) translation velocity
\( \omega = \) rotation velocity
\( \theta = \) angle

The ISAR motion parameters are shown in figure 2-11 below.

![Figure 2-11: ISAR motion parameters](image)

After matching the transmitted signal to the received signal, the equation is downconverted to:

\[
S(t) = \sigma(x, y) \cdot \exp\left\{-j \frac{4 \pi f_0}{c} (R_0(\theta) + \nu(t) \cdot t - r \cos(\omega t - \theta))\right\} \tag{2-28}
\]

From the equation above, the ISAR target motion can be separated into three parts respectively (Mensa2, 1984):

a) the orbital motion of the target, dependent on \( R_0(\theta) \)
b) the translational motion of a reference point on the target, dependent on \( \nu(t) \cdot t \)
c) the rotational motion of the target around its centre, dependent on \( r \cos(\omega t - \theta) \)

### 2.2.2 Translational Motion

Translational motion does not occur in turntable ISAR so, although not directly relevant to the work done in this research, it is briefly discussed here to explain the translational motion effects that are similarly named, and may be confused with, those of rotational motion yet these effects occur for different reasons and with different behaviour.

Translational motion is due to a change in translational velocity which occurs in the radial direction between the target and the radar. In the downconverted received ISAR signal, this is the part of the previous equation that is given by equation 2-29.

\[
S(t) = \sigma(x, y).\exp\{-j.\frac{4\pi f_0}{c}(\nu(t).t)\} \tag{2-29}
\]

If the velocity is constant, this results in a motion with a fixed offset i.e. a moving target is observed as having an offset in range from that of a non-moving target at the same range. This effect is known as **range offset**.

However, if the velocity is changing, the motion is due to an accumulative change in range and this effect is known as **range walk**. Figure 2-12 illustrates both effects.

Range offset occurs for frequency-dispersive waveforms such as chirp pulse or stepped frequency waveforms where the change in frequency of the waveform determines both the range and Doppler capabilities. So, for non-dispersive waveforms, such as short pulses, or phase coded waveforms, there is no range offset (Wehner1, 1987), (Wehner2, 1987).

Range walk occurs when sampled positions appear to migrate over cells / profiles (usually in a quadratic response because of the accumulation in range error) (Wehner1, 1987), (Wehner2, 1987). This effect should not be confused with cell migration due to rotational motion which is discussed in the next section. The range walk distortion, both in range and in cross-range profiles, occurs for synthetic ISAR processing, where processing occurs once all the signals have already been sampled (Wehner1, 1987), (Wehner2, 1987). Velocity correction, therefore, is essential to provide profile realignment.
In contrast, in real ISAR processing, as signals are sampled, so each profile is continuously updated and so range walk does not occur. One of the methods used in the real ISAR case for updating the profiles is to track a single prominent scatterer (Wehner1,1987), (Wehner2,1987).

### 2.2.3 Rotational Motion

After compensation for translational motion, or in the case of turntable ISAR, the received signal becomes:

\[
S(t) = \sigma(x,y).exp\{-j.\frac{4.\pi.f_0}{c}(R_0(\theta) - r.\cos(\omega t - \theta))\} \tag{2-30}
\]

The phase associated with the received pulse is:

\[
\psi(t) = -\frac{4.\pi.f_0}{c}.(R_0(\theta) - r.\cos(\omega t - \theta)) \tag{2-31}
\]

The Doppler frequency shift is:
2.2 Inverse Synthetic Aperture Radar

\[ f_D(t) = \frac{1}{2\pi} \frac{d\psi(t)}{dt} = -f_0 \left( \frac{2\omega r}{c} \sin(\omega t - \theta) \right) \]  (2-32)

Rotational motion causes a quadratic phase distortion analogous to range walk. This causes a cell migration effect known as Motion Through Resolution Cells (MTRC). Correction for MTRC is known as focusing.

MTRC occurs when the rotational angle of interest is wider than either one range or one cross-range (Doppler) resolution cell so that scatterers migrate to other resolution cells during the rotation, see figure 2-13.

As the target rotates, the cross-range position remains the longest in the same resolution cell at the beamwidth edges of the turntable. This occurs when the range position is at the centre of the turntable.

The number \( M \) of cross-range resolution cells by which a scatterer will migrate over Doppler resolution cells \( \Delta f_D \) during the rotation time \( T \) is:

\[ M = \text{abs} \left\{ \frac{1}{\Delta f_D} \left( f_D|_{t=0} - f_D|_{t=T} \right) \right\} \]  (2-33)

From 2-31, the Doppler frequency shift is given by:

\[ f_D(t) = \frac{1}{2\pi} \frac{d\psi(t)}{dt} = -f_0 \left( \frac{2\omega r}{c} \sin(\omega t - \theta) \right) \]  (2-34)

Figure 2-13: Motion Through Resolution Cells (MTRC)
2.2 Inverse Synthetic Aperture Radar

Therefore,

$$\bar{M} = \text{abs} \frac{1}{\Delta f_D} \{[f_0 \left( \frac{2 \omega r}{c} \sin(\omega t - \theta) \right)]_{t=0} - [f_0 \left( \frac{2 \omega r}{c} \sin(\omega t - \theta) \right)]_{t=T} \} \quad (2-35)$$

Because $\Delta f_D = 1/T$,

$$\bar{M} = \text{abs} \frac{2 f_0 r \omega T}{c} \sin(\omega t) \quad (2-36)$$

$$\bar{M} \approx \text{abs} \frac{r}{\delta x} \sin(\omega t) \quad (2-37)$$

As the turntable rotates, the range position remains the longest in the same resolution cell at the farthest and closest edges of the turntable. This occurs when the cross-range position is at the centre of the turntable.

The number $M'$ of range resolution cells by which a scatterer will migrate over range resolution cells $\Delta r$ during the rotation time $T$ is:

$$M' = \text{abs} \frac{1}{\Delta r} \{[R_0 - r \cos(\omega t - \frac{\pi}{2})]_{t=0} - [R_0 - r \cos(\omega t - \frac{\pi}{2})]_{t=T} \} \quad (2-38)$$

$$M' = \text{abs} \frac{r \sin \omega T}{\delta r} \quad (2-39)$$

2.2.4 Blur Radius

The blur radius is the radius limit beyond which cell migration occurs.

If we consider narrow angles and a square resolution, where $\delta r = \delta x = \Delta r$, the values for $M'$ and $\bar{M}$ may be considered as

$$M = M' = M' \approx \frac{r \omega T}{\Delta r} \quad (2-40)$$

In this case, the blur radius is given by
2.2.5 Motion Compensation and Polar to Cartesian Conversion

To obtain high cross range resolution from detection over wide angles, motion compensation is required. Techniques have been developed to compensate the range curvature and MTRC such as the Polar Format Algorithm (PFA), Range Migration Algorithm (RMA) and Chirp Scaling Algorithm (CSA) (Curlander, 1991), (Carrara, 1995), (Son, 2001). According to Desai (Desai, 1992), "a polar-to-cartesian interpolation ... is computationally intensive and error prone due to interpolation inaccuracies".

They all, to some extent, apply polar reformatting to modify the polar formatted data into a cartesian format which is explained further in section 2.2.7. Figure 2-14 shows polar reformatting using the PFA algorithm.

\[
r = \frac{2(\Delta r)^2}{\lambda}
\]

(2-41)

Figure 2-14: Polar Format Algorithm (PFA)
Courtesy of (Son, 2001), pg 199, fig. 12.4
2.2.6 Cross-Range Resolution

Over narrow rotation angles, the Doppler frequency shift of equation 2-32 reduces to:

\[ f_D(t) = f_0 \cdot \frac{2 \omega \cdot r_c}{c} \]  

(2-42)

As figure 2-15 shows, if \( r \) is \( r_c \) in this case, then \( \omega r_c \) is the instantaneous scatterer velocity towards the target.

![Figure 2-15: Radial velocity produced by a scatterer on a rotating target](image)

If two scatterers are separated in cross-range by a distance \( r_c \), then the change in the Doppler frequency is

\[ \Delta f_D(t) = f_0 \cdot \frac{2 \omega \cdot \Delta r_c}{c} \]  

(2-43)

This means that, for narrow angles and constant \( \omega \), cross-range is directly proportional to the Doppler frequency and hence cross-range results are scaled versions of the Doppler results.

Doppler resolution is also related to coherent integration time as (Carrara, 1995):

\[ f_D(t) \approx \frac{1}{T} \]  

(2-44)

So further derivations of cross-range resolution are

\[ \delta_{cr} = \frac{c}{2 \omega f_0 T} = \frac{\lambda}{2 \omega T} = \frac{\lambda}{2 \Delta \theta} \]  

(2-45)
This equation agrees with that of narrowbeam SAR in equation 2-17.

These equations show that cross-range is proportional to Doppler and that cross-range resolution is inversely proportional to the rotation angle.

### 2.2.7 Synthetic Aperture Radar Processing

Conventionally, radar images are generated from wide bandwidth and narrow angles. The conventional method of imaging is shown here for later references highlighting the relationship and differences to that of the narrow band tomographic imaging method used in this research.

Conventional radar images are generated in the rectangular format of range versus Doppler. The Doppler axis is proportional to the cross-range axis and, therefore, the radar image takes the format of range versus cross-range. This is achieved as shown in figure 2-16.

![Figure 2-16: Range / Doppler processing](image)

The data is obtained from the received signal in the format shown in the top, left block. The horizontal axis shows the time bins as each burst of pulses is sampled in order to
determine the round-trip propagation time between the radar and the targets. The vertical axis shows the angular intervals at which each burst is sampled as the turntable rotates.

Each row in the block is range compressed or cross-correlated against a data block obtained from the transmitted signal. The range compression of each row creates a compressed pulse which thereby forms a range profile of the targets as measured at each angular interval.

When a Fast Fourier Transform (FFT) is performed on each of the columns in the top, right block, a Doppler profile is generated in the resulting columns as shown in the bottom, right block. The resulting block therefore shows the 2D radar image describing the range versus the Doppler, and thereby the cross-range, characteristics of the targets.

If the radar has been designed to detect elevation as in the case of a interferometer, a third axis would exist describing the elevation characteristics of the targets.

Before the cross-range dimension is processed, motion complexity such as curved trajectory, target rotation, etc, must be compensated and removed. Thereafter, if the exact trajectory is known, the cross-range may be processed as a matched filter also known as a cross-correlation against a given profile. However, if the relative motion is constant, a fast Fourier transform will suffice.

Generally speaking, in the wide bandwidth, narrow angles case, either of the following two methods is used to form 2D radar images.

**2D Fourier Transform**

The radar processing over a narrow beam sector of SAR or ISAR are considered to be in cartesian (rectangular) format frequency space, see figure 2-17.

Cartesian format frequency space can either be described in terms of Fourier space or in terms of K-space or wavenumber space. These concepts are explained further in section 2.3.
2.2 Inverse Synthetic Aperture Radar

The conventional method is to use the 2D Discrete Fourier Transform (DFT) which obtains the reflectivity index as shown in equation 2-46.

\[
\sigma(x, y) = \int \int S(f_x, f_y) \exp\{-j2\pi(x.f_x + y.f_y)\} df_x df_y
\]  

(2-46)

This equation is defined in section 2.3.2. A straightforward 2D FFT is all that is required to form a 2D range vs Doppler image of the target, which can then be scaled to obtain a range vs Doppler or range vs cross-range image of the target.

**Multilook**

The conventional method of imaging often makes use of multilook processing. In this research, multilook images were generated for comparison between the wideband, narrow angle multilook imagery and narrow band, wide angle tomographic imagery. Results in later chapters of this thesis show, analyse and compare between these two imaging methods.

Multilook is a technique used to combine many 2D radar images each generated over narrow angles from different perspectives or at different frequencies. An example of this is shown in figure 2-18 which describes the multilook process for ISAR.
The ISAR grid is divided equally into a number of sections of narrow angles. A 2D range/Doppler image is then processed for each section. The image from each section is rotated back to a selected angle of incidence by a rotation matrix $R_n$ (Vespe, 2005) given by:

$$R_n = \begin{bmatrix} \cos \theta_n & \sin \theta_n \\ -\sin \theta_n & \cos \theta_n \end{bmatrix}$$  \hspace{1cm} (2-47)

The images are then non-coherently summed together to form the final image (Kingsley, 1992). This summing process counteracts masking or occlusion and leads to a reduction in speckle however it also diminishes the image resolution slightly due to its averaging effect (Vespe, 2005), (Kingsley, 1992).

Examples of multilook images may be found in the results shown in Chapters 5 and 6.
2.3 Frequency Space

The relationship between frequency and physical space can be described in terms of frequency space. This is another way that radar signals and their responses may be characterised. For the purposes of this work, it helps visualise resolution and how narrow band, high resolution may be achieved.

Frequency space can be described in terms of polar format frequency space, Fourier space or in terms of K-space (also known as wavenumber space).

2.3.1 Polar Format Frequency Space

Polar format frequency space is discussed here as it pertains to any type of synthetic aperture radar affected by range curvature and the same principles apply to both narrow angle ISAR and wide angle ISAR.

As the target rotates in turntable ISAR, samples of target reflectivity versus spatial frequency are received. In section 2.2.1 the following equation was presented:

\[ S(f, \theta) = \iint \sigma(x, y) \exp\left( -j \frac{\omega f_0}{c} R_0(\theta) - r \cos(\theta) \right) dx \, dy \]  

(2-48)

After compensating for \( R_0 \), this is rewritten as

\[ S(f, \theta) = \iint \sigma(x, y) \exp\left( j \frac{\omega f_0}{c} r \cos(\theta) \right) dx \, dy \]  

(2-49)

or writing this in terms of \( x \) and \( y \):

\[ S(f, \theta) = \iint \sigma(x, y) \exp\left( j \frac{\omega f_0}{c} (x \cos(\theta) + y \sin(\theta)) \right) dx \, dy \]  

(2-50)

These are spaced in polar format frequency space \((f, \theta)\) where \( f \) is the frequency and \( \theta \) is the rotation angle, see figure 2-19.
2.3 Frequency Space

2.3.2 Fourier Space and K-space

Fourier space and K-space (also known as wavenumber space) are other methods used to map out the relationship between frequency and physical space. K-space is more commonly used as it simplifies the processing of the cyclic behaviour of the rotational signals.

In Fourier space, the axes are measured in $rad/m$ and defined as:

$$f_x = \frac{2f}{c} \cos \theta, \quad f_y = \frac{2f}{c} \sin \theta$$

In K-space (wavenumber space), the axes are measured in $cycles/m$ and defined as:

$$k_x = \frac{2\omega}{c} \cos \theta = 2k \cos \theta, \quad k_y = \frac{2\omega}{c} \sin \theta = 2k \sin \theta$$

where $k$ is the wavenumber:

$$k = \frac{2\pi}{\lambda} = \frac{2\pi f}{c} = \frac{\omega}{c}$$
The received signal in Fourier space is:

\[ S(f_x, f_y) = \int \int \sigma(x, y).\exp\{j.2.\pi.(x.f_x + y.f_y)\} dx. dy \]  (2-51)

and the received signal in K-space is:

\[ S(k_x, k_y) = \int \int \sigma(x, y).\exp\{j.(x.k_x + y.k_y)\} dx. dy \]  (2-52)

### 2.3.3 Resolution In Frequency Space

Figure 2-20 shows the spatial sampling of a wide band, narrow angle monostatic SAR system. The radius from the centre of the disc, shown by \( \Delta u \), shows the change in signal frequency, i.e. the bandwidth, over the signal. Now the range resolution is inversely proportional to \( \Delta u \). The distance over the arc, shown by \( \Delta v \), shows the change in spatial frequency over the signal. Now the cross-range resolution is inversely proportional to \( \Delta v \).

Figure 2-20: Spatial frequency characteristics of a wide band, narrow angle monostatic SAR system
The range and cross-range resolutions can be calculated from Fourier space as follows:

\[ \Delta u = \frac{2B}{c}, \quad \Delta v = \frac{2f_{\text{max}}}{c} \]

so

\[ \delta r = \frac{1}{\Delta u} = \frac{c}{2B}, \quad \delta x = \frac{1}{\Delta v} = \frac{c}{2f_{\text{max}}} \]

Similarly, the range and cross-range resolutions can be calculated from K-space as follows:

\[ \Delta u = \frac{4\pi B}{c}, \quad \Delta v = \frac{4\pi f_{\text{max}}}{c} \]

so

\[ \delta r = \frac{2\pi}{\Delta u} = \frac{c}{2B}, \quad \delta x = \frac{2\pi}{\Delta v} = \frac{c}{2f_{\text{max}}} \]

### 2.3.4 Related Work

Mensa et al (Mensa1, 1983), (Mensa2, 1984) applied their theory to monochromatic Continuous Wave (CW) radar so their spatial frequency diagram would have been similar. However, they used multi-tonal signals thereby achieving a wider bandwidth, as shown in figure 2-21.
2.3 Frequency Space

Narrow Band, Wide Angle
Monostatic SAR System

Figure 2-21: Spatial frequency characteristics of a multi-tonal, narrow band, wide angle monostatic SAR system

The spatial frequency characteristics in this figure are multi-tonal so have distinct lines occurring at various frequencies. This effect can be described by equation 2-53.

\[ k_x = \sum_{i=1}^{N} 2.k_i \cdot \cos \theta, \quad k_y = \sum_{i=1}^{N} 2.k_i \cdot \sin \theta \]  \hspace{1cm} (2-53)

Wicks et al (Wicks1,2005), (Wicks2,2005), (Wicks3,2005), Bonneau et al (Bonneau,2002) and Bracken et al (Bracken,2006) applied their theory to Ultra Narrow Band (UNB) signals and achieved a wider bandwidth using both bistatic and multi-tonal signals, as shown in figure 2-22 (a). This has been expanded to include multiple perspectives, as shown in figure 2-22 (b), thereby obtaining 360 degrees of spatial observation which further increase the spatial frequencies available.
Neither bistatic nor multi-tonal signals were considered in this work but are recommended for future work with a caveat to remain using narrow band in keeping with the objective to reduce signal congestion.
2.4 Tomographic Imaging

Section 2.2.7 showed that methods, such as polar reformatting, can be used to convert samples in frequency space from polar format to cartesian format. Tomographic processing, however, provides a method which applies the projection slice theorem to reconstruct the target image directly from polar formatted frequency space. Because of the theorem, tomography enables high resolution 2D images to be obtained from high resolution 1D projections.

2.4.1 Background to Tomography

Tomography involves the formation of an image of an object from profiled 1D sections of the object. The word tomography is derived from the term *tomos* which means "cut" in Greek (Liang, 1999). For example, conventional medical Computer-Aided Tomography is performed to obtain cross-sectional images of patients.

The concept of Tomography was first published in 1826 by a Norwegian physicist, Niels Henrik Abel, for an object with axi-symmetrical geometry. In 1917, an Austrian mathematician, Johann Radon, extended Abel's idea for objects with arbitrary shapes. Radon invented the Radon transform which applies the Projection Slice theorem to create 2D or 3D images from 1D profiles of objects, see section 2.4.2. Allan Cormack, a South African physicist at Tufts University developed the initial algorithms in 1964 and Godfrey Hounsfield at EMI Research Laboratories, UK built the first CT scanners in 1972. Godfrey Hounsfield and Allan Cormack were jointly awarded the Nobel prize in 1979 for their pioneering work on X-ray Tomography (Carrara, 1995), (Liang, 1999), (Scudder, 1978).

Ground-breaking work in this field was produced in the sixties and the seventies in the development of both X-ray tomography and Magnetic Resonance Imaging (MRI). Now, a diverse range of tomographic imaging systems are available for both medical and non-medical uses. These systems include X-ray CT (Computer Tomography), MRI, PET (Positron Emission Tomography), SPECT (Single Photon Emission Computed Tomography), MEG (MagnetoEncephaloGraphy), SAR (Synthetic Aperture Radar), and various acoustic imaging systems. These systems use different physical principles for signal gen-
eration and detection such as transmission, as in the case of medical X-ray imaging, or reflection, as in the case of radar tomography. Ultimately though, the underlying signal processing principles for image formation are, to a large extent, the same (Carrara, 1995), (Liang1, 1999).

2.4.2 Projection-Slice Theorem & Radon Transform

Tomographic reconstruction applies the projection slice theorem to compute the image. The projection slice theorem states that the 1D Fourier transform of the 1D projection of a 2D function made at an angle $\theta$ is equal to a slice of the 2D Fourier transform of the function at an angle $\theta$ see figure 2-23 (Mensal, 1983), (Mensa2, 1984), (Son, 2001), i.e.

$$P_{\theta}(f) = \int P_{\theta}(t).e^{-j2\pi ft}dt = S(f, \theta) \quad (2-54)$$

where:

- $P_{\theta}(t)$ = 1D projection of a 2D function made at an angle $\theta$
- $P_{\theta}(f)$ = 1D Fourier transform of the 1D projection of a 2D function made at an angle $\theta$
- $P_{\theta}(t)$ = slice of the 2D Fourier transform of the function at an angle $\theta$

By applying the projection slice theorem where the 1D projection data is mapped from the spatial domain at angle $\theta$ to the frequency domain at angle $\theta$, the combined 2D frequency data formed from a number of projections can be processed and interpolated back to a specific position at angle $\phi$ in the 2D spatial domain.
2.4 Tomographic Imaging

This image reconstruction process is known as the Radon transform given by:

\[
s(r, \phi) = \int_{\frac{\theta}{2}}^{\frac{\theta}{2}} \int_{f_1}^{f_2} P_\theta(f).\exp(-j.2.\pi.f.r.(\cos(\phi - \theta))).|f|.df.d\theta
\]  

(2-55)

where \((r, \phi)\) represents the resulting polar coordinates of a rectangular grid in the spatial domain.

Whereas some algorithms convert the outputs from many radars simultaneously into a reflectivity image using a 2D Fourier transform, the tomographic process generates an image by projecting the 1D Fourier transform of each radar projection individually back onto a 2D grid of image pixels. This operation gives rise to the term Backprojection which is used in techniques such as “Filtered Backprojection (FBP)” or “Convolution Backprojection (CBP)”.

In the case of FBP, the image is formed realised in two distinct steps:

1) Filtering each projection: \(Q_\theta(t) = \int_{f_1}^{f_2} P_\theta(f).\exp(-j.2.\pi.f.t).|f|.df\)

2) Backprojecting: \(\sigma(r, \phi) = \int_{\frac{\theta}{2}}^{\frac{\theta}{2}} Q_\theta(r.\cos(\phi - \theta)).d\theta\)

The above theory and further descriptions of various tomographic techniques are available from (Bhashyam,1999), (Carrara,1995), (Jain,1989), (Kak,1988), (Harris,2001) and others.
2.4 Tomographic Imaging

2.4.3 Sampling Rate of Each Projection

As specified in (Jain, 1989), each 1D projection should be sampled uniformly, at least, at the Nyquist rate of the highest spatial frequency $\xi = 2.f/c$, i.e. the sampling interval in Fourier space $d$ should correspond to:

$$d = \frac{1}{2.\xi} \tag{2-56}$$

This determines the spatial separation of scatterers comprising the target that may be resolved.

2.4.4 Filtering the Projection

The first step of FBP reconstruction is to perform the frequency integration (the inner integration) of equation 2-56. This entails filtering each of the projections using a filter with frequency response $|f|$, see figure 2-24.

![Figure 2-24: Filtering a projection](image)

The filtering operation may be implemented by ascertaining the filter impulse response required then performing convolution or a FFT/IFFT combination to correlate $P_\theta(f)$ against the impulse response.
2.4.5 Angular Intervals

To avoid the signals shifting by more than $\pi$ radians per burst, the data should be sampled at discrete angular intervals given by (Mensa1, 1983), (Mensa2, 1984):

$$\Delta \theta \leq \frac{\lambda}{2D}$$

(2-57)

where $D$ is the maximum dimension of the object.

Equation 2-59 can be recognised as a variation of the cross-range resolution calculation for narrow angles as given previously by equation 2-37, namely:

$$\delta_{\sigma r} = \frac{\lambda}{2. \Delta \theta}$$

(2-58)

In other words, the angular intervals have to be sampled over an angle equal to or greater than that necessary to determine the required cross range resolution.

2.4.6 Number of Projections

According to Jain (Jain, 1989), “if an object in space is limited by a circle of diameter $D$ and if $\xi$ is the largest spatial frequency of interest in the polar coordinates of the Fourier domain, the number of projections required to avoid aliasing effects due to angular sampling in the transform domain are

$$N > \pi D \xi$$

(2-59)

where $\xi = 2.f/c = 2/\lambda$.

This means that the number and location of scatterers comprising the target will determine the number and distribution of apertures required to avoid grating lobes in the tomographic image.

This idea is based on the assumption that the distance $d$ between projections, see figure 2-25, should be a minimum of $d = \lambda/2$. 
2.4 Tomographic Imaging

This is then approximated by \( r \theta = \lambda/2 \) and therefore the number of projections \( N > \frac{2 \pi}{\theta} = \frac{2 \pi r}{\lambda} = \pi D \xi \). However, if we want to sample the angular intervals at \( d = \frac{1}{2 \xi} \), as specified in equation 2-56, then the number of projections would have to be doubled.

2.4.7 Problems

Incoherency occurs because the tomographic process used here (the MATLAB \textit{iradon} function) sums together the scalar magnitudes of each projection to form the image, not the complex values.

Superimposing can lead to the introduction of erroneous point scatterers.

Due to these problems, the density of the scatterers in the object will affect the quality of the image.
2.5 Summary

This chapter has described the existing theory related to narrow band radar, particularly relating to its imaging and resolution capabilities. Realisation of both down range and cross range resolution have been examined including the use of synthetic apertures. Inverse Synthetic Aperture Radar (ISAR) was then discussed and the effects of motion on the ISAR profiles were described. Conventional methods for generation of radar images in the narrow angle case were also discussed. Polar format frequency space and the relationship between frequency space and resolution have also been explained. Finally, the process of tomographic imaging and the resulting ambiguities have been described. The theory described in this chapter has been developed and applied in the next chapter to form the basis of the narrow band imaging concept at the core of this thesis.
Chapter 3

Narrow Band High Resolution Radar Imaging Concept
3.1 Narrow Band Radar Cross Range Profiles

As stated earlier, the electromagnetic spectrum is becoming increasingly congested. That is because conventional radar uses wideband signals to obtain high range resolution features. By decreasing the signal bandwidth, the range resolution becomes increasingly ineffective. This research offers a different approach to obtaining high resolution images while using minimal bandwidth.

The concept underlying this research is to exploit spatial diversity to obtain high cross-range resolution features. By incorporating the use of tomographic imaging, high cross range resolution images may be formed.

To explain the basic concept, consider an aperture forming a 1D cross range profile of an area of interest. The cross-range resolution of the profile depends on the beamwidth in the case of an aperture or the extent in the case of a synthetic aperture.

By positioning another aperture perpendicular to the first, as pictured in figure 3-1, a 2D cross range profile of the area of interest may be produced with 2D resolution.

![Figure 3-1: Two apertures at perpendicular angles forming 1D cross range profiles with 2D resolution. Each aperture gives a cross range resolution perpendicular to the aperture normal.](image-url)
3.2 Tomographic Imaging

By positioning multiple apertures from various perspectives, the focus on the resolved area of interest is honed down to forming a small, approximately circular, shape, see figure 3-2. By combining the apertures from multiple perspectives, the 2D resolution can be improved beyond the 2D resolution achieved in the perpendicular case. Note: this is necessary to remove ambiguities that occur when imaging multiple targets.

Figure 3-2: Apertures at various angles forming 1D cross range profiles with improved 2D resolution

The strategy of this work is to generate high resolution 2D imagery by applying reflection tomography to narrow band profiles produced from multiple perspectives.

3.2 Tomographic Imaging

Tomographic imaging, as discussed in section 2.4, forms a 2D image from 1D projections taken from multiple perspectives around the object.

The term “projection” used in tomographic processing refers to a 1D radar profile that is generated at a specific perspective to an object i.e. from a single aperture orientation.

Figure 3-3 demonstrates the basic principle used in this work to produce 2D high cross range resolution tomographic images.
3.2 Tomographic Imaging

The left-hand side of the figure shows a radar rotating around a rectangular object consisting of four point scatterers. On the left of the image below the geometry at each perspective is also shown the 1D cross range projection formed by the radar at that perspective. Each 1D cross range projection is formed by using either a real or synthetic aperture and has a cross range resolution determined by the aperture angular width. For 2D imaging purposes multiple 1D projections are created at multiple perspectives around the object.

The right-hand side of the figure shows how each 1D cross range projection is given as an input to the tomographic process which interpolates, backprojects and combines the 1D cross range projections together to form a 2D cross range image. Note also that erroneous
scatterers may be formed in the image due to this process.

The resulting 2D cross range resolution of the 2D cross range image is related to a combination of each 1D projection’s cross range resolution.

### 3.3 Narrow Band Spatial Frequency Theory

The concept of this work can be explained in further detail using the spatial frequency domain.

The spatial frequencies that are used in conventional wide band, narrow angle SAR systems were described previously in section 2.3.

However, in a narrow band system, $\Delta u$ is narrow and $\Delta v$ remains, as shown in figure 3-4, thereby worsening the range resolution.

![Figure 3-4: Spatial frequency characteristics of a narrow band, wide angle monostatic SAR system](image)

Instead of using a wide bandwidth to obtain range information with a high range resolution, the approach adopted in this work is to instead make use of spatial diversity and use the narrow band signals to obtain cross-range information with a high cross-range resolution.

Figure 3-5 illustrates the concept of using spatial diversity by positioning narrow band, wide angle signals at multiple perspectives in 360 degrees of space.
3.3 Narrow Band Spatial Frequency Theory

Figure 3-5: Spatial frequency characteristics of narrow band, wide angle synthetic apertures at multiple perspectives

In figure 3-5, each individual 1D projection is essentially cross range in nature. However, this translates to range when viewed from the perpendicular perspective. By making use of spatial bandwidth, each 1D projection therefore appears to have some finite range resolution associated with it when viewed from the perpendicular perspective.

By combining the 1D projections from multiple perspectives, the 2D Fourier space shown in figure 3-6 may be mapped out.

Figure 3-6: Spatial frequency characteristics of combined 1D projections from multiple perspectives

This phenomenon is exploited by tomographic processing as described previously in Chapter 3 in order to combine the spatial frequency spectra. Using this concept, high resolution is achieved utilising spatial frequency instead of bandwidth and through the use of spatial diversity and tomographic imaging, 2D high resolution radar images are formed.
3.3 Narrow Band Spatial Frequency Theory

In chapter 2 the basic theory and practical limitations of narrow band radar imaging were discussed. In this chapter the subsequent development of a new imaging concept to overcome some of these limitations has been developed and described. In the following parts of this thesis, the performance of this novel imaging technique is explored using a specially developed software processor. The design and implementation of this processor is described in detail in the next chapter.
Chapter 4

Narrow Band High Resolution Radar Imaging Processor Design
A software processor was developed and implemented to investigate the narrow band high resolution radar imaging concept.

Figure 4-1 shows the steps used by the processor to form 2D narrow band high cross range resolution radar images that are discussed in this section. These various stages of the high resolution imaging process are subsequently described in more detail below.

The narrow band high resolution radar imaging concept has been tested by applying experimental and simulated data to the various configurations of the processor. The results and analysis of these tests are given in Chapters 5 and 6.

It should be noted that the example figures shown in this chapter are those taken at various stages during the processing of real trihedral data obtained from Thales Sensors.

Details of the experimental data and analysis of the results of this processing are given later in Chapter 5.
A brief summary of the stages of the processing is described below to give an overall picture of the process. Each of these is then explained in detail in the subsequent sections of the chapter.

The **Data Preparation** section shows how the data is prepared to form the basis from which the narrow band high resolution radar imaging concept can be implemented. The data was obtained from an X-band turntable ISAR instrument. Wideband stepped chirp pulses were sampled in high speed bursts as the turntable slowly rotated with respect to the radar. The purpose of the data preparation is to collapse wideband data into a narrow band 1D reflectivity profile of an object or objects on the turntable versus 360 degrees of turntable rotation angle.

The **Narrow Band High Resolution Radar Imaging Process** section gives an overall description of the processes that are used in narrow band high resolution radar imaging.
4.1 Data Preparation

The Multiple Perspectives section describes how synthetic apertures are formed from specific portions of the 1D reflectivity profile versus turntable angle at multiple perspectives around the object. The width of each synthetic aperture angle ($\Delta \theta$) is determined by the cross range resolution specified. To achieve this, each synthetic aperture formed is a copy of the relevant portion of $\Delta \theta$ degrees in length copied from the 1D reflectivity profile versus 360 degrees of rotational angle.

The 1D Cross Range Projections section describes the formation of a 1D cross range projection from each synthetic aperture viewed from multiple perspectives around the object.

The High Cross Range Resolution Motion Correction section details the issues involved and corrective method used in generating high cross range resolution projections.

The Tomographic Imaging section discusses the application of the 1D cross range projections from multiple perspectives to the tomographic imaging process to obtain a 2D cross range image.

In addition, the Multilook Processing section describes the conventional radar imaging process used to produce multilook images for comparison between the two imaging processes.

4.1 Data Preparation

This section explains how the wideband data obtained for this research is transformed into a 1D narrow band reflectivity profile versus turntable angle. Once the data is available in this form, it can be fed into the high range resolution processing software.

The data preparation process is shown in figure 4-2. A description of each part of the process is given below:

- The Data In section represents both the theoretical input derived from the simulated data and the real experimental data used. Use of these data allowed comparison of results based on theory and experiment. The experimental data was obtained from
4.1 Data Preparation

an X-band turntable ISAR. Stepped chirp pulses were sampled in high speed bursts as the turntable slowly rotated with respect to the radar so the data has the format of time bins versus turntable angle. Two sets of data with different bandwidths (A and B) were investigated in the work. The details and parameter values of these data sets are discussed in further detail in this section.

- As explained in the Background Theory chapter, range compression is performed on modulated radar signals to detect range features of the object being radiated. The data obtained from the Data In section is then range compressed by cross-correlating the radar pulses at each turntable angle to obtain a range profile versus turntable angle.

- **Zero Doppler Correction (ZDC)** is then performed to remove the stationary artefacts such as the stationary trihedrals and stationary clutter. The reasons for ZDC are discussed in detail in the next section when the requirement for ZDC becomes evident during cross range processing.

- Finally, the range profile is collapsed into a 1D reflectivity profile versus turntable angle in preparation for the first step in the narrow band high resolution radar imaging concept, i.e. the formation of 1D cross range profiles from multiple perspectives around the object.

Depending on the bandwidth of the data used, different methods were used to form the 1D reflectivity profile, namely:

- if bandwidth A was used, the **multiple range bins** were collapsed into a single range bin.
- if bandwidth B was used, a **single range bin** was selected.

Each of these processes are explained in more detail in the following sections.
4.1 Data Preparation

Data in

Burst m = 1

Burst m = M

Range compression

Zero Doppler Correction

Collapse multiple range bins

Select single range bin

Figure 4-2: Data preparation
4.1 Data Preparation

4.1.1 Data In

To examine the narrow band high resolution radar imaging concept, experimental data has been obtained from a real turntable ISAR. A simulation has also been developed to produce simulated turntable ISAR data to compare theoretical performance against that achieved in practice. The data is wideband and will be collapsed later to mimic the narrow band case.

The experimental data has been taken from an X-band stepped chirp ISAR system. The system parameter values that were used in this work are shown in table 4.i.:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial frequency of first pulse in burst</td>
<td>9.5 GHz</td>
</tr>
<tr>
<td>Number of pulses used per burst</td>
<td>1</td>
</tr>
<tr>
<td>Pulse length (T)</td>
<td>341 ns</td>
</tr>
<tr>
<td>Bandwidths (A or B)</td>
<td>500 MHz or 21.429 MHz</td>
</tr>
<tr>
<td>Centre wavelength</td>
<td>0.03 m or 0.0316 m</td>
</tr>
<tr>
<td>Angular intervals</td>
<td>10773 burst intervals per 360 degrees of turntable rotation angle</td>
</tr>
<tr>
<td>Slant range to centre of turntable</td>
<td>80 m</td>
</tr>
<tr>
<td>Turntable diameter</td>
<td>6 m</td>
</tr>
<tr>
<td>Grazing angle</td>
<td>8 degrees</td>
</tr>
</tbody>
</table>

Table 4.i.: X-band stepped chirp ISAR system parameter values used in this work

Three types of experimental data were used, namely that from the detection of a pair of trihedral positioned on opposite sides of an ISAR turntable, a full-sized military tank and a long wheelbase Land Rover.

As described in the Background Theory chapter, section 2.2, a turntable ISAR is a single stationary radar which views objects on a rotating turntable, see figure 4-3.

Figure 4-3: ISAR geometry
4.1 Data Preparation

The ISAR transmitted and received a high speed burst of pulses directed at the turntable. While the signals were being transmitted, the turntable rotated slowly at a rate of 10773 bursts per 360 degrees of turntable rotation. This means that the turntable rotated at an angular rate of approximately 0.0334 degrees per burst or 0.00058 radians per burst which can be approximated as a stop/start situation.

According to the angular interval requirement given in the Background Theory chapter, section 2.4.5, in order to sample the spatial frequency at more then twice the Nyquist frequency, the angle between pulses should be:

\[ \Delta \theta \leq \frac{\lambda}{2D} \] (4-1)

In the case of the largest item, the tank, where \( D=8 \text{ m} \), \( \Delta \theta = 0.001975 \text{ radians} \) or 0.113 degrees for \( \lambda = 0.0316 \text{ m} \). This means that the data's sampling rate of pulse interval per turntable angle meets the spatial sampling requirement.

In order to lower the bandwidth of the bursts used in this narrow band radar research, only a small part of the first pulse per burst was extracted for use in this work so the system can be considered in terms of a single pulses per turntable angle as opposed to a burst per turntable angle. Two sets of data having different pulse lengths and therefore different bandwidths (A and B) were extracted in this way for different requirements in the work.

Bandwidth A is a high bandwidth of 500 MHz which was used initially for generating multiple range bins of 30 cm range resolution each. These multiple range bins were used for generating multilook images for characterising the data and for comparison purposes. The multiple range bins in the range profile were then collapsed into 1D reflectivity profiles for generating low cross range resolution tomographic images.

Bandwidth B is a lower bandwidth of 21.429 MHz which was used in later work, when motion correction became an issue while generating high cross range resolution images. By using a lower bandwidth with a 7 m range resolution, the turntable of 6 m diameter positioned at a slant range of 77 m to 83 m from the radar, fell neatly into a single range bin, the 23rd range bin between 77 m to 84 m, in the range profile. Range cell migration
was avoided by using only the data from the single range bin so that motion correction was only required for cross-range cell migration.

The centre wavelengths shifted slightly as the bandwidths changed, namely from $\lambda = 0.03$ m in the high bandwidth case to $\lambda = 0.0316$ m in the low bandwidth case.

The simulation was developed using MATLAB version 6.5 and version 7 software. Simulated data of various configurations of point scatters were generated. The two types of data shown in this document are of a pair of point scatterers on the opposite sides of the turntable at a radius of either 3 m or a radius of 0.2 m from the centre of the ISAR turntable.

The simulation generated a single transmitted and received chirp pulse at each angular interval of turntable rotation.

The transmitted chirp signal was defined as

$$TX(t) = I(t) + j.Q(t)$$  

where:

$$I(t) = \cos(2\pi f_0 t - \frac{1}{2} K t^2) \ast \text{rect}(t)$$  

$$Q(t) = \sin(2\pi f_0 t - \frac{1}{2} K t^2) \ast \text{rect}(t)$$

The $\text{rect}$ function is a square pulse of length $T$ which has the value of 1 if $-T/2 \leq t \leq T/2$ else 0. The frequency response of the $\text{rect}$ function is a $\text{sinc}$ function of bandwidth $B$ where $B$ is either bandwidth $A$ or bandwidth $B$.

The received signal $RX(t)$ has the same definition as the transmitted signal but the time is shifted by the time delay

$$t = t - t_D$$
4.1 Data Preparation

The receive signals were generated for each for each point target and then summed together to produce the combined receive signal.

Both the experimental and simulated data were produced for range compression in the format shown in figure 4-4.

![Figure 4-4: Data format](image)

The format shows that at each turntable angle, the complex data in each transmitted and received signal was saved in terms of time bins, each time bin of pulse length $T$, per angular interval.

4.1.2 Range Compression

The wideband modulated radar signals detected as the object rotates 360 degrees in angle in the line of sight of a stationary radar were read into the Data In process in the form of a 2D profile of time bins versus turntable angle (measured in angular intervals). This section explains how the profile's time bins are range compressed to detect range features of the object thereby forming a 2D profile of range bins versus turntable angle. This profile is then range collapsed to generate a 1D reflectivity versus turntable angle profile for the initial input to the narrow band high resolution radar imaging process.

This range compression process was described in Chapter 2 of this thesis. At each turntable rotation interval, the range compression process cross-correlates the modulated transmitted signal pulse $TX$ with the modulated received signal pulse $RX$ detected to produce a range profile $S_{out}$ response per turntable rotation interval.
4.1 Data Preparation

i.e. the cross-correlation performed on the range values obtained for each burst is:

\[ S_{\text{out}} = \text{FFT}^{-1}(\text{FFT}(RX) \ast \text{FFT}(TX^*)) \]  \hspace{1cm} (4-6)

By correlating the modulated pulses, a compressed response is formed which details range features of the object at a range resolution \( \delta_r = c/(2B) \), where \( c = 3 \times 10^8 \) m/s = speed of light and \( B \) = pulse bandwidth.

In this way the 2D profile of time bins versus turntable angle is transformed to a 2D profile of range bins (each range bin the length of the range resolution) versus angular intervals. An example of a range profile produced after range compression of the real trihedral data is given in figure 4-5. Analysis of the results of this processing is given later in detail in Chapter 5, section 5.2.1.

![Range profile after range compression](image)

Figure 4-5: Range profile of real data of trihedrals

As illustrated previously in figure 4-3, figure 4-5 shows two trihedrals rotating opposite each other on the turntable in the centre of the image, forming a figure eight response. Due to the polar response of the trihedrals, signals are primarily reflected at specific angles where the trihedrals were facing the radar.
Another two trihedrals were stationary and were positioned on either side of the turntable. They can be seen as straight vertical lines in the range profile as the backscatter from these were independent of the angular intervals from the turntable rotation.

Stationary clutter can also be seen as thinner straight vertical lines.

Variable noise and clutter in the return can also be seen as horizontal and vertical glitches in the range profile.

4.1.3 Zero Doppler Correction

Zero Doppler Correction (ZDC) is performed to remove the stationary artefacts such as the stationary trihedrals and stationary clutter. It will be shown later that where the cross range projections are generated from further processing of this data, the signal energy given by the stationary artefacts combine at zero Doppler in the Doppler profile, or likewise, at the centre of the cross range axis in the cross range projection. This combined signal energy far outweighs, and thereby overwhelms, the signal energy produced from the moving objects which were of fundamental interest in the cross range projections. For this reason, the zero Doppler is removed so that only the non-zero Doppler from moving artefacts remain.

The response of stationary artefacts in the data preparation stage of this work were thus reduced so that the narrow band high resolution radar imaging concept could be investigated without influence from these scatterers. After studying a number of options, the Zero Doppler Correction (ZDC) Estimation and Subtraction algorithm from Showman et al (Showman, 1998), was implemented and adapted for this processor.

The ZDC Estimation and Subtraction algorithm involves averaging together the data in the frequency domain over the 360 degree rotation of the turntable then subtracting this average to remove the constant zero frequency. However, in this research application, instead of using the full 360 degrees of data to form the average, the ZDC process applied a running average over three consecutive PRFs in order to obtain and remove local zero Doppler characteristics to remove specific clutter artefacts.
Figure 4-6 below shows the range profile after ZDC has reduced the response from the stationary artefacts, i.e. from the stationary trihedrals, clutter and stationary noise. The clutter and stationary noise is further reduced by limiting the range of interest to that of only the area of the turntable itself. Thus, the two stationary trihedrals on either side of the turntable were also removed from further processing because they would have shown up as unwanted zero clutter in the Doppler profiles.

Neither clutter nor ZDC has been incorporated into the simulation so ZDC is only applied for the processing of the experimental data. This is because the simulation was used to test the narrow band radar imaging process not to test the corrections to experimental data impediments.

4.1.4 Collapse Multiple Range Bins

When the high bandwidth was used, the range bins in the range profile were 30 cm. In order to obtain a 1D projection for tomographic imaging, all the range bins in the range profile were summed together and collapsed to form a 1D reflectivity profile versus angular intervals. This again is equivalent to generating narrow band data.
The equation used for collapsing the range data was:

\[ S(t) = \sum I(t) + j \sum Q(t) \]  

(4-7)

The reflectivity profile values of \( S(t) \) are then normalised over all angular intervals in order that they can then be compared.

An example of range collapsing is shown in figure 4-7 which shows the one-dimensional data generated from the range collapse of the original range profile shown in figure 4-6.

Collapsing the range at this early stage has the consequence of corrupting the data because the data has not been motion corrected first, see section 2.2.5. This means that all the data at the same angle is summed together even though the individual scatterers may have experienced differing cell migration. However, by collapsing the range profile without motion correction the 1D reflectivity profile represents the profile that would have been received from a CW radar which would also have experienced similar corruption of data.

The data will be motion corrected later in the process, see section 4.2.3.
4.1.5 Select Single Range Bin

When the low bandwidth was used, the range bin size was 7 m and the only range bin of interest was that in which the 6 m diameter turntable was located. In that case, collapsing the range profile was not necessary as the 1D reflectivity profile was taken from the relevant range bin.

Figure 4-8: 1D reflectivity profile of trihedral data after range collapse of single range bin

Analysis of the results of the data preparation processing is given later in detail in Chapter 5.
4.2 Narrow Band High Resolution Radar Imaging Process

This section explains the narrow band high resolution radar imaging process in which 2D tomographic images are formed from 1D cross range projections projected from multiple perspectives around an object.

This procedure is shown in figure 4-9 which shows:

The 1D reflectivity profile versus angular intervals that was prepared previously by the processor is input to the narrow band high resolution radar imaging process.

The Multiple Perspectives section describes how synthetic apertures in the form of 1D reflectivity profile versus synthetic aperture angle are formed at multiple perspectives around the object. The width of each synthetic aperture angle (Δθ) is determined by the cross range resolution specified. To achieve this, each synthetic aperture formed is a copy of the relevant portion of the 1D reflectivity profile versus rotational angle.

The 1D Cross Range Projections section describes the formation of a 1D cross range projection from the synthetic aperture at each perspective by performing an FFT on the synthetic aperture’s profile. The option of applying a Hamming window during the FFT process is also available. The resulting format of each cross range projection is that of a 1D reflectivity profile versus cross range bins.

The High Cross Range Resolution Motion Correction section describes the optional application of a window function to high cross range resolution profiles to correct for rotational motion smearing.

The Tomographic Imaging section discusses the application of the 1D cross range projections as projections to the tomographic imaging process to obtain a 2D cross range image.

Each of these processes are explained in more detail in the following sections.
Figure 4-9: Narrow band high resolution radar imaging process
4.2 Narrow Band High Resolution Radar Imaging Process

4.2.1 Multiple Perspectives

This section describes how portions of the 1D reflectivity profile prepared previously in the processor are allocated to form synthetic apertures at multiple perspectives.

To produce a narrow band tomographic image, 1D cross range projections are required from multiple perspectives around the target. The 1D cross range projections are produced from synthetic apertures as will be explained in section 4.2.2.

Each synthetic aperture is a copy of the relevant portion (subset) of the 1D reflectivity profile versus turntable angle, centred at the relevant perspective angle to the object. The width of each synthetic aperture angle ($\Delta \theta$) varies from $-\Delta \theta/2$ to $+\Delta \theta/2$ and is set according to the cross-range resolution required, where

$$\delta_{cr} = \frac{\lambda}{4\sin(\Delta \theta/2)}$$

(4-8)

where $\lambda$ = wavelength and $\delta_{cr}$ = cross range resolution.

e.g. if a cross range resolution of 0.172 m is required and the wavelength = 0.03 m, then a synthetic aperture angle of 5 degrees is necessary. If 90 perspectives are used, spaced regularly (4 degrees apart) round the object, then the synthetic aperture angle at each perspective would overlap the neighbouring apertures by 0.5 degrees on either side.

In this way, segments of the 1D reflectivity profile are allocated to form synthetic apertures at multiple perspectives.
4.2 Narrow Band High Resolution Radar Imaging Process

4.2.2 1D Cross Range Projections

This section describes how the 1D cross range projections at multiple perspectives are produced from synthetic apertures.

As mentioned above, the synthetic apertures of each perspective are formed from segments of turntable angle copied from the 1D reflectivity profile prepared previously.

A 1D projection is then created from a Fast Fourier Transform (FFT) of the synthetic aperture at that perspective. The aperture effectively has a rect envelope of width $W$ around the phase shifted signal.

As given in equation 4-9,

$$FFT(rect\left(\frac{t}{W}\right)) = W \cdot sinc(fW)$$

when an $FFT$ is performed on the aperture, a Doppler aperture is produced encompassed by a $sinc$ envelope. The envelope has a resolution width of $1/W$ at approximately -4 dB and the closest sidelobes occur at -13 dB.

In this way, a 1D Doppler projection, and therefore proportionally a 1D cross-range projection, can be produced at each perspective.

The $FFT$ of each projection is required to be sampled at a frequency equal to or higher than the Nyquist sampling of the required cross-range resolution. For a very fine cross range resolution and without any additional processing, this is limited because the data is sampled at a maximum rate of one pulse per 0.0334 degrees of turntable rotation.

However, by zero-padding the profile before the $FFT$ process i.e. increasing the number of zero values at the edges of the signal, more data between samples is produced by the $FFT$ process having interpolated values so that finely sampled cross-range profiles are generated.

Ultimately, each 1D cross range projection created from each perspective is collected together with the others to form a 2D profile of cross range versus turntable angle. This 2D profile may then be input into the tomographic process for imaging purposes.
4.2 Narrow Band High Resolution Radar Imaging Process

Zero Doppler Correction (ZDC) & Turntable Only

As illustrated previously in figure 4-3, figure 4-10 shows two trihedrals rotating opposite each other on the turntable in the centre of the image. Again, due to the polar response of the trihedrals, signals are primarily reflected at specific angles where the trihedrals were facing the radar. This figure is showing the cross range profile so is 90 degrees out of phase to the range profile seen in figure 4-5.

In the example shown previously in figure 4-5, any stationary objects such as the stationary trihedrals, clutter and stationary noise are evident as a straight, vertical line at zero Doppler in the centre of the Doppler, and therefore cross range, profile, see in figure 4-10. As the cross range projection reflectivity was overwhelming at zero Doppler, the decision was made to minimise this effect by using Zero Doppler Correction (ZDC).

The ZDC averaging occurred before the range profile was collapsed and its effects are evident by the missing line at zero Doppler in the figures shown from this stage on. The clutter and noise is further reduced by limiting the range of interest to that of only the area of the turntable itself. Thus, the two stationary trihedrals on either side of the turntable were also removed from further processing because they would have shown up as unwanted zero clutter in the Doppler profiles and therefore in the cross range profiles.

Figure 4-10: Doppler profile of real data of trihedrals - before ZDC
Hamming Windows

Hamming windows serve as weighting functions in the FFT process and were only applied to the low cross range resolution profiles. A different window function is applied instead to the high cross range resolution profiles as can be seen later in section 4.2.3.

Figure 4-11 shows that the energy available in figure 4-10 has been increased when applying this method.

Figure 4-11: Doppler profile of real data of trihedrals - after Hamming window

Applying a weighting function to a range or cross range projection affects the bin size. For example, in the case of applying a Hamming window to a profile, the resolution is multiplied by a factor of 1.3. The Hamming window highlights the features of a profile and is applied by the processor in low cross range resolution cases. Unfortunately the Hamming window worsens the resolution so a different weighting window which improves resolution is applied to the high cross resolution cases instead. Detail of the weighting window will be given in the next section.
4.2 High Cross Range Resolution Motion Correction

High cross range resolution imaging was investigated in order to test the theoretical and practical limits of narrow band high resolution radar imaging in the case where the cross range projections are related to the Doppler shifts between the target and the radar.

In order for a synthetic aperture to have high cross range resolution, the aperture angle $\Delta \theta$, and therefore the number of angular intervals used to generate the cross range projection from the synthetic aperture, is wider than the blur angle.

A 2D cross range profile generated from the 1D cross range projections over multiple perspective angles at high cross range resolution is shown in figure 4-12.

![Cross range profile, high cross range resolution of 1.58 cm](image)

Figure 4-12: 60 Degree cross range projection of real data of trihedrals

The figure shows the smearing effect caused by cell migration due to Motion Through Resolution Cells (MTRC). Therefore MTRC correction is required for synthetic aperture angles that are wider than the blur angle which is related to the blur radius in section 2.24.

Most techniques for rotational motion compensation are designed to remove MTRC and produce data that is not only MTRC compensated but also remove the rotational characteristics resulting in a 2D image of the point scatterers in their correct positions. However
the correction adopted here for high cross range resolution profiles is to remove the MTRC characteristics of each cross range projection, leaving only the rotational characteristics as the input for the tomographic processor.

The concept introduced here is that, since the rotational data without MTRC exists for cross range projections generated over angles less than the blur angle, these profiles were used as a matching filter or weighting function for correlation. Because the weighting function used here is dependent on the object being profiled, it can be referred to as an apodisation function which is a function that is non-standard and is dynamically adapted to suit the situation.

An example of the cross range projection of the weighting function in this case is shown in figure 4-13.

Figure 4-13: Weighting function in the form of a low resolution cross range projection

The high cross range resolution profiles were matched against the correlation filter created from the low cross range resolution profiles. Each low cross resolution profile that is used in this case as the correlation weighting function is not the low cross resolution profile formed previously where Hamming windows were applied for imaging purposes. Instead, the low cross resolution profile used in this case to form the weighting function is multiplied by itself to further reduce its resolution. Since each Doppler signal is encompassed by
4.2 Narrow Band High Resolution Radar Imaging Process

a \( \text{sinc} \) envelope, the resolution of the envelope will be reduced by this multiplication process producing a resulting envelope with \( \text{sinc}^2 \) characteristics. This effectively reduces the width from \( 1/W \) to \( 0.72/W \) and lowers the sidelobes from approximately -13 dB to approximately -26 dB.

The resulting high resolution cross range profile produced for the tomographic processor has a similar profile to that of the low cross range resolution profile but the MTRC smearing effects have been reduced and the 1D cross range projection at each perspective has the high resolution intended, see figure 4-14.

![Cross range profile, high cross range resolution of 1.58 cm](image)

**Figure 4-14:** High resolution cross range projection of real data of trihedrals

Using this method, unsmeared high cross range resolution profiles were generated by motion correcting the profiles. By removing the MTRC characteristics of each cross range projection, only the rotational characteristics remained. These high cross range resolution profiles may be input to the tomographic imaging process to form high resolution images.

Note: This option is only applied by the processor in high cross range resolution cases.
4.2 Narrow Band High Resolution Radar Imaging Process

4.2.4 Tomographic Imaging

Tomographic imaging was used in this work to form a 2D image from narrow band signals received from multiple angular perspectives around an object. This section describes how a tomographic image is produced.

Tomographic imaging assumes a constant rotational velocity of the turntable and that the centre of the input data is at the centre of rotation. However, the data of various targets was obtained using slightly different geometries and positions of the scatterers on the turntable. To form a tomographic image, it is necessary that the 2D profile used as input to the process has an equivalent number of cross range bins on either side of the cross range bin at 0 m so that, in effect, the data is centred. In order to centre the object, the projection data was shifted and centred by shifting existing bins, adding or subtracting blank cross range columns to the edges of the profile where necessary (e.g. if there were 3 cross range bins per projection and data only existed in the first cross range bin, the projection was shifted so that the data occurred in the centre bin).

Tomographic imaging suffers from ambiguities which are related to the number of projections used to form the image. In order to prevent tomographic ambiguities, the number of projections required as specified in section 2.4.6, should be

\[ N > \pi D \xi \] \hspace{1cm} (4-10)

where \( D \) is the diameter of the object being imaged, \( \lambda \) is the wavelength and the spatial frequency \( \xi = 2/\lambda \).

Tomographic ambiguities are evident as lines radiating from the centre of the image every \( 360/N \) degrees apart. An example of tomographic ambiguities is shown in figure 4-15 in the case where \( N=171 \) was selected which is insufficient for the diameter of the object and the bandwidth used. Therefore ambiguities occur spaced \( 360/171 = 2.1 \) degrees apart.
4.2 Narrow Band High Resolution Radar Imaging Process

Figure 4-15: Tomographic ambiguities

Despite the tomographic ambiguities, the number of perspectives had to be selected to be 171 as that is the number of files that were obtained per 360 degrees of data.

The final low resolution and high resolution cross range tomographic images are shown in figures 4-16 and 4-17 respectively. The achieved resolution in figure 4-17 is 1.58 cm.

Figure 4-16: Low resolution cross range tomographic image
4.2 Narrow Band High Resolution Radar Imaging Process

Image from 171 57.08deg projections, cr-resl=1.58cm

Figure 4-17: High resolution cross range tomographic image
4.3 Multilook Processing

This section describes the conventional radar imaging process, namely multilook processing, for comparison with the narrow band high resolution radar imaging method described above.

Multilook images generated from the simulated and real data are shown and analysed in the next two chapters for comparison with the narrow band radar tomographic images.

As explained previously in section 2.2.7, the multilook imaging process, which is conventionally used in SAR and ISAR processing, involves combining 2D range versus cross range sub-images produced from omni-directional perspectives.

The preparation shown in figure 4-2 shows that to create the multilook image, the range profiles are created as was done for the high bandwidth case above but the range profiles were not collapsed. Instead a $2D \text{FFT}$ is performed to create a 2D cross range profile which gives each projection a sub-image with dimensions of range versus cross range.

The number of range bins and cross-range bins used to create the multilook images were calculated to be proportional in size. This meant that the multilook process was only performed for the high bandwidth (A) case since proportional 2D range versus cross range images could be developed from it as opposed to the 1D images that could be produced from the single range bin in the low bandwidth (B) case.

The sub-images and final image created in the high bandwidth case have a range resolution of:

$$\delta_r = \frac{c}{2B} = \frac{3 \times 10^8}{2.500 \times 10^6} = 30 \text{ cm}$$

(4-11)

The 171 files each had 61 bursts covering an overall angle of 2.1 degrees or 0.0367 radians.

Thus, the cross-range resolution in this narrow angle case is

$$\delta_{cr} = \frac{\lambda}{2 \Delta \theta} = \frac{0.03}{2 \times 0.0367} \approx 41 \text{ cm}$$
4.3 Multilook Processing

To get the range versus cross range bin ratios equivalent, the number of range bins used was

\[ n_r = \frac{41}{30} \times 61 \approx 83 \]

Therefore 83 range bins of 30 cm range resolution each is approximately equivalent to 61 cross range bins of 41 cm cross range resolution each.

However, so that the rotation of the sub-image generated from each of the perspectives overlaps correctly, the range versus cross range bins would have to be square. This was achieved by padding the FFT to 83 when performing the Doppler processing which had the effect of increasing the number of cross range bins and proportionally decreasing the cross range resolution of each cross range bin.

This padding resulted in the sub-images and thereby the resulting image consisting of 83 range bins of 30 cm range resolution each and 83 cross range bins of \( 41 / (83/61) \approx 30 \) cm cross range resolution each.

The final multilook images are shown in figure 4-18.
4.4 Summary

In conclusion, the narrow band high resolution radar imaging concept and processor design show that, by making use of spatial diversity, 2D tomographic images may be formed from 1D projections taken from multiple perspectives around the object.

Instead of using wide bandwidth to achieve resolution, the narrow band radar projections are obtained by making use of spatial frequency to generate cross range profiles of moving objects. However, this means that the apertures used to form the cross range projections have to be sufficiently wide in angle to obtain the required cross resolution.

In the case of high cross range resolution, the aperture angle of each projections is wider than the blur width so motion correction is required to remove the cell migration response.

If an inadequate number of projections are used to form the tomographic image, tomographic ambiguities occur in the form of lines radiating from the centre of the image.

The processing involved to produce both low cross range resolution images and high cross range resolution images are demonstrated in this work, the results of which are analysed in Chapters 5 and 6. The results of conventional multilook processing are also included for comparison.

Chapter 5 deals with narrow band imaging of sparse scatterers using simulated and real trihedral data. Chapter 6 then demonstrates narrow band imaging of dense scatterers using real target data.
Chapter 5

Narrow Band Imaging of Point Scatterers - Results & Analysis
To test the Narrow Band High Resolution Radar Imaging concept, real data was obtained from a wideband, turntable ISAR system. The details of this system were shown previously in table 4.1. Three sets of data were obtained for different objects on the turntable, namely that of a pair of trihedrals, a tank and a Land Rover.

As described in Chapter 3, the concept involved processing narrow band data so as to produce a number of 1D narrow band cross range profiles from multiple perspectives around the object(s), which would then be tomographically imaged to produce a 2D narrow band radar image.

In order to test this concept it was necessary to modify the wide band experimental data into a narrow band form. A simulation involving two pairs of point scatterers were therefore developed to test this process. The results and analysis of the simulated data are discussed in section 5.1.

These are then compared to the real trihedral data, which are discussed in section 5.2. Since the trihedral data is real, it therefore suffers from noise, clutter, and practical limitations so the comparison between the ideal and real images and their respective resolutions are very significant for practical application of this technique.

5.1 Narrow Band Imaging of Simulated Data - Results & Analysis

As mentioned previously, the first stage of this work involved developing a simulation to investigate the principles of narrow band radar tomographic processing. Results using the simulated data are discussed in this section.

Two pairs of point scatterers were simulated. In the first case, a pair of point scatterers were positioned diametrically opposite each other at a radius of 3 m from the centre of the turntable to simulate the experimental data of a pair of trihedrals. In the second case, a pair of point scatterers were positioned diametrically opposite each other at a radius of 0.2 m from the centre of the turntable to investigate how cell migration depends on the blur radius. The geometry of the simulated data scenarios are illustrated in figure 5-1 and
The transmitted ($TX$) and received ($RX$) chirp pulses were simulated at every turntable rotation angle. The range of the scatterer positions was used to calculate the time delays of the received pulses as shown in figure 5-3.
5.1 Narrow Band Imaging of Simulated Data - Results & Analysis

Figure 5-3: Positions and motion of scatterers with (a) 3 m radius and (b) 0.2 m radius

5.1.1 Data Preparation

This section explains how the wideband data simulated for this research is transformed into a 1D narrow band reflectivity profile.

Range Compression

This part of the processor performs range compression to the input data. At each turntable rotation interval, the range compression process cross-correlates the modulated transmitted signal pulse ($TX$) with the modulated received signal pulse ($RX$) detected to produce a range profile per turntable rotation interval.

When a range resolution of 30 cm was used, the range profile resulting from range compression of simulated data is shown in figure 5-4.
5.1 Narrow Band Imaging of Simulated Data - Results & Analysis

In these range profiles, it is clear that the peak of range correlation occurs in the range bins where the scatterers are positioned on the turntable as expected from the illustrations given previously in figures 5-1 and 5-2 respectively.

The range profiles display a sinc response in the range dimension. This is characteristic of the cross-correlation of the transmitted and received chirp pulses.

The intensity of both range profiles also shows an oscillatory response in the turntable angle dimension. This is due to the Doppler shift which is at a maximum when the scatterers are rotating perpendicular to the radar line of sight and at a minimum when the scatterers are rotating parallel to the radar line of sight.

Because a 30 cm range resolution has been used to form the range profiles, the 3 m radius scatterers fit into more than one range bin as shown in figure 5-4 (a). The 3 m radius scatterers, having a 6 m diameter, should fit into approximately 20 range bins. However, it is evident from the figure that these actually cover approximately 22 range bins. This is because of the sidelobes of the sinc response.

In 5-4 (b), the 0.2 m radius scatterers are rotating from 79.8 m to 80.2 m. Because a range resolution of 0.3 m is used, the range bins in the area of interest are positioned at 79.8 m, 80.1 m and 80.3 m. In this case, the dominant range bins are the two in the centre.

When a range resolution of 7 m was used, the range profile resulting from range compression of trihedral data is shown in figure 5-5.
5.1 Narrow Band Imaging of Simulated Data - Results & Analysis

Figure 5-5: Range profile of rotating point scatterers with (a) 3 m radius and (b) 0.2 m radius using a range resolution of 7 m

In these profiles, it is clear that the peak of range correlation occurs in the single 7 m range bin where the scatterers and turntable are positioned.

Again, the profiles have the sinc function in the range domain due to range compression and oscillation of intensity in the turntable angle domain due to the Doppler phase shift.

**Zero Doppler Correction**

It should be noted that noise and clutter was not incorporated into the simulation therefore Zero Doppler Correction (ZDC) was not necessary.

**Collapse Multiple Range Bins**

When a range resolution of 30 cm was used, in order to obtain a 1D projection for tomographic imaging, all the range bins in the range profile were summed together coherently and collapsed to form a 1D reflectivity profile versus angular intervals. This again is equivalent to generating narrow band data as effectively all range resolution has been lost.

The results of range collapsing is shown in figure 5-6 which shows the one-dimensional data generated from the range collapse of the original range profile.
5.1 Narrow Band Imaging of Simulated Data - Results & Analysis

Figure 5-6: 1D reflectivity profile of rotating point scatterers with (a) 3 m radius and (b) 0.2 m radius after range collapse of multiple range bins

The intensities of both reflectivity profiles also show an oscillatory response in the turntable angle dimension due to the Doppler shift which will ultimately be used to form the 1D cross range profiles.

The maximum Radar Cross Section (RCS) values of the profiles have oscillatory effects in figure 5-6 (b) but not so in figure 5-6 (a). This effect is due to the variation in the properties of the coherent phased signals being summed together from the range bins in question. In the 0.2 m radius scatterers case, only two range bins, next to each other and with similar properties were summed together. However, in the 3 m scatterers case, a total of 22 range bins with varying properties, were summed together and therefore interference effects become significant.

Select Single Range Bin

When a range resolution of 7 m was used, only a single range bin was of interest i.e. the range bin that the 6 m diameter turntable lay in. In that case, collapsing the range profile was not necessary as the ID reflectivity profile was taken from the relevant range bin, as shown in figure 5-7.
Figure 5-7: 1D reflectivity profile of rotating point scatterers with (a) 3 m radius and (b) 0.2 m radius after range collapse of single range bin.

Again, the Doppler phase shifts are evident in the profiles.

The noise level is negligible in the cases of the 3 m radius scatterers and 0.2 m scatterers respectively as no noise was simulated and no additional effects due to coherent summing occurred.

In these cases, the profiles remain at the maximum RCS values and no interference has occurred due to summation because these profiles were obtained from single range bins.
5.1 Narrow Band Imaging of Simulated Data - Results & Analysis

5.1.2 Narrow Band High Resolution Radar Imaging Process

In this section the imaging process for the simulated data is explained in detail. Initially multiple perspectives are produced followed by 1D cross range projections at each of these perspectives. This is then followed by generation of a 2D cross range profile. The application of motion correction and weighting is then carried out followed by generation of full tomographic images.

Multiple Perspectives

As described in Chapter 3, tomographic images may be formed from 1D cross range projections obtained from multiple perspectives. The 1D cross range projections are developed from performing an FFT on a corresponding synthetic aperture and the cross range resolution of each projection is determined by the width of the corresponding synthetic aperture angle.

In order to allocate the simulated data over multiple perspectives, the 1D reflectivity profile created in section 5.1.1 was divided up into segments. The 360 degrees of simulated data was sectioned into 171 files, each of which covered 2.11 degrees.

For the 1D projections, in order to obtain the required angular width and thereby the required resolution, 171 synthetic apertures were created from the data file available at each perspective and extended to the data in the neighbouring files.

The blur angle of the simulated data for the scatterers at a radius of 3 m was calculated as 5.90 degrees and the blur angle of the simulated data for the scatterers at a radius of 0.2 m was calculated as 22.78 degrees.

The low cross range resolution profiles for both cases were developed to be a similar angle as the narrower blur angle of the two. However, they extended to an angle of 6.32 degrees which is slightly wider than the blur angle but of negligible consequence. This meant that a low cross range resolution of 14.33 cm was obtained for the consequent apertures, 1D projections and images.

The weighting functions used to weight the high cross range resolution projections were
developed as similar to the relevant blur angle. Again, the angle of the weighting function for the 3 m radius scatterers is 6.32 degrees and a resolution of 14.33 cm. The weighting function for the 0.2 m radius scatterers is over an angle of 31.62 degrees with a resolution obtained in that case of 1.45 cm.

As will be evident later in this chapter, the highest resolution obtained in the high cross range resolution profiles was that of $\lambda/4$ in the simulated data case. This meant that a high cross range resolution of 0.79 cm was obtained over projection angles of 178.95 degrees.

The multiple perspective parameter values used for processing simulated data are shown in table 5.i..

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low cross range resolution</td>
<td>14.33 cm</td>
</tr>
<tr>
<td>Low cross range aperture angle</td>
<td>6.32 deg</td>
</tr>
<tr>
<td>Blur angle for radius of 3 m</td>
<td>5.90 deg</td>
</tr>
<tr>
<td>Blur angle for radius of 0.2 m</td>
<td>22.78 deg</td>
</tr>
<tr>
<td>Weighing function aperture angle for radius of 3 m</td>
<td>6.32 deg</td>
</tr>
<tr>
<td>Weighing function aperture angle for radius of 0.2m</td>
<td>31.62 deg</td>
</tr>
<tr>
<td>Weighing function resolution for radius of 3 m</td>
<td>7.17 cm</td>
</tr>
<tr>
<td>Weighing function resolution for radius of 0.2 m</td>
<td>1.45 cm</td>
</tr>
<tr>
<td>High cross range aperture angle</td>
<td>178.95 deg</td>
</tr>
<tr>
<td>High cross range resolution</td>
<td>0.79 cm $\approx \lambda/4$</td>
</tr>
</tbody>
</table>

Table 5.i.: Processor parameter values used to process simulation data

The 1D cross range projections resulting over each of these synthetic aperture angles is discussed in more detail in the following sections.

### 1D Cross Range Projections

A 1D cross range projection is produced from the FFT of the synthetic aperture at each perspective. A 2D cross range profile of 1D cross range projections versus perspective angles can be built up.

The 1D cross range projections with a low cross range resolution of 14.33 cm obtained from perspective number 1, are shown in the figures below.
5.1 Narrow Band Imaging of Simulated Data - Results & Analysis

Figure 5-8: Low resolution cross range projections for (a) 3 m radius and (b) 0.2 m radius scatterers

The figures 5-8(a) and (b) show the characteristic sinc outline where the sidelobes appear at -13 dBs lower than the main lobe.

Occlusion

When generating the 1D cross range projections, it is evident that occlusion occurs in cross range detection when a scatterer is obscured from radar view by another scatterer. For the scatterers positioned on opposing sides of the turntable, the effects of occlusion are most evident when the scatterers are positioned one behind the other on the radar line of sight.

Figures 5-9 below show how the two scatterers in the 0.2 m radius case mask each other in the cross range projections obtained from perspectives located at 90 degrees, 110 degrees and 113.33 degrees respectively.
5.1 Narrow Band Imaging of Simulated Data - Results & Analysis

Figure 5-9: Occlusion of cross range projection when turntable is at (a) 90 degrees, (b) 110 degrees and (c) 113.33 degrees

It is evident from these occlusion characteristics that the density and positions of the scatterers in an object are of utmost importance when forming cross range projections and cross range images.

Cross range profiles

By collecting all the one dimensional cross-range projections together that were obtained from multiple perspectives, an overall 2D cross range profile may be generated. This shows the cross-range motion of the scatterers as the turntable rotates. This is illustrated in figure 5-10.

Figure 5-10: Low resolution (41 cm) cross range profile for (a) 3 m radius and (b) 0.2 m radius scatterers
5.1 Narrow Band Imaging of Simulated Data - Results & Analysis

As illustrated previously in figure 5-1 and 5-2, figure 5-10(a) and (b) shows two sets of point targets rotating opposite each other on the turntable in the centre of the image. As expected, the cross range profile is orthogonal to the range profile shown in figure 5-4 so that the figure '8' of the range profile has been shifted by 90 degrees as is expected for range versus cross range dimensions.

In the cross range profile, the sinc outlines are evident in the cross range dimension of each projection. These are due to the FFT transforms unlike the range profiles where the sinc outlines were due to range compression. As is characteristic of a sinc function, the sidelobes appear at -13 dBs lower than the main lobe.

It can also be seen that the intensity of the profile is greater where the cross range value of the profile is furthest from the centre as is evident by the brightness of profile (a) at the -3 m and +3 m cross ranges. When compared to the range profiles in figure 5-4, this is where the scatterers are moving perpendicularly across the radar line of sight so that the greatest Doppler shift occurs.

The cross range profile in the 3 m radius case has a gap through the centre of the profile at 0 m cross range. This is not due to ZDC because it was not performed in the simulations. This may be due to supplying asymmetrical data to the FFT process which occurs when the section of the 1D reflectivity profile data used for the synthetic aperture in question is zero-padded unevenly.

When a high cross range resolution of 0.79 cm \( \approx \lambda/4 \) was required, synthetic aperture angles of 178.95 degrees \( \approx 180 \) degrees were used.

Looking at a 1D high cross range resolution projection at 1 angular interval (figure 5-11) it can be seen that the intensity of the 1D projection seems 'averaged out' at a constant level of -10 dBs in the case of the 3 m radius point targets and -5 dBs in the case of the 0.2 m radius point targets. This is because of cell migration which has the effect of smearing the sinc functions so that they appear as rect functions instead.
5.1 Narrow Band Imaging of Simulated Data - Results & Analysis

The combined result of the 1D cross range projections at multiple perspectives gave the high cross range resolution profiles shown in figure 5-12.

When comparing these cross range profiles to the cross range projections that formed these profiles shown in figure 5-11, it can be concluded that the figure ‘8’s are the peak of the profiles and that the dark vertical lines at or near the centre of the profiles are the troughs of the profiles.
Also when comparing this high cross range resolution profile to the low cross range resolution profile in figure 5-10, it can be seen that the noise floor is still present, just smeared so energy is maintained.

High Cross Range Resolution Motion Correction

In order for a synthetic aperture to have high cross range resolution, the aperture angle $\Delta \theta$, and therefore the number of angular intervals used to generate the cross range projection from the synthetic aperture, is wider than the blur angle. However, projections that are wider than the blur angle suffer from smearing effects caused by cell migration due to Motion Through Resolution Cells (MTRC). Therefore MTRC correction is required for synthetic aperture angles that are wider than the blur angle.

The correction adopted here for high cross range resolution profiles is to remove the MTRC characteristics of each cross range projection, leaving only the rotational characteristics as the input for the tomographic processor.

The high cross range resolution profiles were matched against a weighting function created from the low cross range resolution profiles. The resulting cross range projection produced for the tomographic processor have a similar profile to that of the low cross range resolution profiles but the MTRC smearing effects have been greatly reduced and the cross range projection has the high resolution intended as will be shown. Generation of this weighting function is discussed in more detail below.

Weighting function

The weighting used as the matching function is derived from the low cross range resolution profiles. The profile for scatterers at a 3 m radius has a cross-range resolution of 14.33 cm obtained over an angle of 6.32 degrees; and the profile for scatterers at a 0.2 m radius has a cross-range resolution of 2.90 cm obtained over an angle of 31.62 degrees. These angles
are slightly wider than the blur angles of 5.90 degrees and 22.78 degrees respectively but show negligible effects. After multiplying the weighting function to itself, the cross-range resolutions become 7.17 cm and 1.45 cm respectively.

The cross range profiles of the weighting functions is shown in the figure below.

Figure 5-13: Weighting functions for scatterers with (a) 3 m radius and (b) 0.2 m radius

The output shown in the figure below is the 1D projection obtained from perspective number 1. It shows the measure of intensity in that projection over cross-range.

Figure 5-14: Weighting functions in the form of low resolution cross range projections for scatterers with (a) 3 m radius and (b) 0.2 m radius

Because the weighting functions were multiplied by themselves, the resulting sinc function
responses have a sharper intensity with the sidelobes at approximately -26 dBs lower than the main lobes.

When this figure 5-13 (a) is compared to the low cross range resolution profiles shown previously in figure 5-10 (a), the following features can be noted:

The same cross range resolution of 14.33 cm and therefore the same projection aperture width was used in both. This means that the same low cross range resolution profile was used and multiplied twice to form the weighting function.

As expected, due to the multiplication, the weighting function profile is narrower than the low cross range resolution profile.

The heightened intensity of the weighting function at the outer cross range values is more apparent.

When figure 5-13 (b) is compared to the low cross range resolution profiles shown previously in figure 5-10 (b), the following features can be noted:

A cross range resolution of 2.90 cm has been used for the weighting function which is in contrast to the cross range resolution of 14.33 cm used in the low cross range resolution profile. The higher resolution means that the weighting function will be sharper, not only from the multiplication process. That is evident in the figure.

The higher cross range resolution of 2.90 cm, means that projection apertures were each of 31.62 degrees in width. This is above the blur angle of 22.775 degrees for 0.2 m radius scatterers. Cell migration can be seen near the 0 m cross range value of the profile.

However, when comparing the blurred cross range profile of the weighting function to the unblurred cross range profile of the low resolution case, the weighting function is more enhanced especially considering the fact that both profiles are shown at the same scale. In this case, the slight smearing may be disregarded.
High cross-range resolution profile

After applying high cross range resolution motion correction by multiplying the 57.08 degree projection by the weighting function, high cross range resolution projections without MTRC effects are obtained.

A corresponding 1D cross range profile at perspective number 1 is shown in 5-15.

Figure 5-15: High cross range resolution projections for (a) 3 m radius and (b) 0.2 m radius scatterers

The zoomed in version of these projections are shown below.

Figure 5-16: High cross range resolution cross range projections - zoomed in

It is evident in this figure that the noise floor between the scatterers has dropped drastically.
This figure demonstrates that the intensity of the artefacts are standing 70 dBs higher than the noise floor. For this reason, the resulting peaks of the projections are far sharper and of far finer resolution than previously.

After weighting the 180 degree projection, the resulting cross range profile appears as shown here:

Figure 5-17: High cross range resolution (0.79 cm) profile for (a) 3 m radius and (b) 0.2 m radius scatterers

These profiles are more finely defined than either of the two profiles used to form them, namely the 180 degree cross range profile shown in figure 5-11 and weighting function shown in figure 5-13.

The peaks and troughs of both contributing profiles have both enhanced and diminished parts of the resulting profiles so that the intensity obtained at specific turntable angles is distorted. The intensity at the inner cross range values appears to be segmented and the intensity at the outer cross range values is not only dominant, but instead of following the shifted figure '8' curves, these areas in the profiles appear linear. These linear characteristics will have an effect on the resulting tomographic images created because, instead of following the circular response expected of the tomographic process, the scatterers will appear as having shifted slightly from their constant positions on the turntable as the turntable rotates.
Tomographic Imaging

The final stage of the imaging process was to build a 2D tomographic image from the multiple perspective data. This section shows the tomographic images produced from a 2D low resolution cross range profile and 2D high cross range resolution profile. These 2D profiles each consisted of 171 1D projections.

The 2D low resolution cross range tomographic images of the scatterers with (a) 3 m radius and (b) 0.2 m radius are shown in figure 5-18.

![Image from 171 6.32deg projections, cr-resl=14.33cm](image1.png)

![Image from 171 31.62deg projections, cr-resl=2.90cm](image2.png)

Figure 5-18: Low resolution cross range tomographic images of (a) 3 m radius and (b) 0.2 m radius scatterers

The foremost observation is that a true image of the scatterers in 2D space has been generated using this narrow band tomographic processing technique.

These images have been scaled by the expected cross range resolution. This verifies the validity of this approach. The scales in these images have been shown as cross range X versus cross range Y. This is because a square image has been built up from a set of projections from 171 viewing angles spaced over 360 degrees around the turntable. So any dimension in the resulting image is that of cross range.

The scatterers in each image are at a slight angle from horizontal due to an offset in the starting angle position input into the tomographic imaging process. Outer circles occur around the central image of each point scatterer. This is due to the cross range projection sidelobes also being tomographically imaged.
In order to prevent tomographic ambiguities, the number of projections required as specified in section 2.4.6, should be

\[ N > \pi D \xi \] (5-1)

where \( D \) is the diameter of the object being imaged, \( \lambda \) is the wavelength and the spatial frequency \( \xi = 2/\lambda \). As the value of \( N \) was fixed at 171, in order to avoid tomographic ambiguities, the dimension \( D \) of the objects being imaged had to be smaller than 81.6 cm. The point objects being imaged here are smaller than this so no tomographic ambiguities appear.

The resulting tomographic images of the compensated high cross range resolution profiles shows the scatterers as sharp points at high resolution, see figure 5-19.

Figure 5-19: High resolution cross range tomographic images of 3 m and 0.2 m radius scatterers

Figure 5-20 shows the same images as before but from a zoomed in perspective.
5.1 Narrow Band Imaging of Simulated Data - Results & Analysis

The resulting cross range images show the scatterers as being positioned the correct distance apart from each other.

The scatterers appear smeared. The smearing effect may also be partially due to the cell migration from using wide angle apertures to obtain high cross range resolution. However, when comparing figure 5-17 (a) to figure 5-17 (b), the cell migration effect for the high cross range profiles used to generate the tomographic images are far less in the 3m radius case than in the 0.2 m radius case, which contrasts with the smearing that resulted in the tomographic images.

The smearing effect could not be due to misaligning the high cross range profiles input to the tomographic processor because the same definitions to form the images were used to form the low cross range resolution images shown previously in figure 5-18 which demonstrate no smearing effects.

Again, the diameter of the point objects being imaged here are smaller than the 81.6 cm requirement for 171 perspectives so no tomographic ambiguities have occurred.

Testing of these two cases showed that the finest cross range resolution that could be produced for projection apertures wider than 167 degrees up to 180 degrees was of 0.79 cm. This confirms the theory that the finest cross range resolution achievable would equal \( \lambda/4 \), i.e. 0.79 cm when \( \lambda = 0.0316m \).
5.1.3 Multilook Processing

Conventional multilook processing was also performed to give a comparison with the narrow band high resolution radar imaging process. 171 projections are used, each having a 2D image with 30 cm range resolution versus 41 cm cross range resolution.

The final multilook images are shown in figure 5-21.

![Image from 171 projections, cr-resl=41.00cm](image1)

![Image from 171 projections, cr-resl=41.00cm](image2)

Figure 5-21: Multilook images of 3 m and 0.2 m radius scatterers

The figure shows the two sets of point targets for the 3 m radius scatterers and the 0.2 m scatterers respectively. The point targets appear circular due to the circular process of multilook imaging. They also appear clear and uncorrupted.

In the 0.2 m radius case, the scatterers fall into 3 cross range bins, particularly the centre bin which varies from -15 cm to +15 cm. This is due to the $41/(83/61) \approx 30$ cm cross range resolution of each cross range bin overlapping with 20 cm radius scatterer positions. The images above show proportionality in range and cross range has been achieved by using the above method.

However, the point target images in the 3 m radius case, which were used to compare to the trihedral data, are positioned correctly and provide a good simulation of the trihedral data.
5.1.4 Summary Of Simulated Data Findings

The simulation involving point scatterers proved that good quality 2D imagery can be generated from Narrow Band Radar data using the processes developed in this study.

Using this process, a 2D image with a high resolution of $\lambda/4$ was achieved. This is a far higher resolution than would have been achieved by conventional high bandwidth methods.

The results also verified the validity of the methods applied to convert wide band data to narrow band data and to create high resolution cross range profiles by making use of a weighting function to minimise the effects of MTRC.

Using cross range to profile and image the scatterers has shown to be subject to occlusion. This means that only reflections from the unobstructed scatterers can be detected. This also means that multiple perspectives are required, not only to prevent ambiguities in tomographic imaging, but also to detect each scatterer at a sufficient spatial resolution and at every angle possible. The effect of occlusion on dense scatterers in objects such as in the tank and the Land Rover is investigated in Chapter 6.

Overall these results show great promise for using these methods to produce high quality 2D tomographic images using narrow band radar. These results were however obtained using simulated data. In order to address the practical application of this technique, it is necessary to also examine this process applied to real data. Therefore, this process has also been carried out on real data and the results are discussed in the next section.
5.2 Narrow Band Imaging of Real Trihedral Data - Results & Analysis

A similar process to that used in the last section was now used to process the real ISAR data described in Chapter 4. The experimental trihedral data was used in this analysis and the steps in the process were effectively identical to those described in the last section. These are explained in more detail below.

In the trihedral case, the objects detected by the ISAR were four trihedrals, or corner reflectors as they are otherwise known, such as that shown in figure 5-22.

Figure 5-22: Trihedral

Two of the trihedrals were stationary and positioned at slant ranges 69 m and 88 m from the radar respectively, on either side of the turntable which was centred at a slant range of 80 m from the radar. As only moving targets are of interest, these will be removed during processing as explained previously in section 4.1.

The other two trihedrals were positioned opposite each other on the turntable at a 3 m radius from the turntable centre. However, they were not diametrically opposite each other but slightly offset from each other so their rotation past the radar can not give a true figure of eight response. This also means that the data is not symmetrical at the 0 degrees aspect angle. The geometry of interest of the real trihedral scenario is illustrated in figure 5-23. The direction of the radar is shown because the trihedral RCS response is related to the direction of the trihedrals with respect to the radar as explained below.
5.2 Narrow Band Imaging of Real Trihedral Data - Results & Analysis

Trihedrals are designed to have the characteristics of point targets and are often used for radar calibration. The figure below shows the theoretical Radar Cross Section (RCS) distribution obtained from a trihedral at the 8 degrees grazing angle used here.

Note that the RCS is at a maximum when the radar beam reflects from the centre of the trihedral and can be expressed (Knott, 2004) as

\[ \sigma = \frac{12\pi l^4}{\lambda^2} \]  

(5-2)
where each trihedral side is of length $l$.

The RCS then has a gradual roll off as the beam is reflected at an angle. The RCS then sharply increases at aspects angles of $+/-$ 45 degrees when the radar beam is reflected off the sides of the trihedral. These peaks at the edges of the response are sometimes referred to as the 'ears'. The RCS values then remain low at all other aspect angles not shown in the figure.

The width of the RCS response depends on the grazing angle of the radar beam and is characteristic of the shape of the trihedral.

This RCS response is evident in the profiles and images formed from trihedrals.

### 5.2.1 Data Preparation

Data preparation was similar to that explained previously for the simulated data.

**Data In**

The modulated transmitted signal ($TX$) data and the modulated received signal ($RX$) data input into the processor was in the form of 2D complex profiles of time bins versus angular intervals.

**Range Compression**

This part of the processor performs range compression to the input data. At each turntable rotation interval, the range compression process cross-correlates the modulated transmitted signal pulse ($TX$) with the modulated received signal pulse ($RX$) detected to produce a range profile per turntable rotation interval.

When a range resolution of 30 cm was used, the range profile resulting from range compression of trihedral data is shown in figure 5-25.
Figure 5-25: Range profile from real trihedral data using a range resolution of 30 cm

The presence of the four trihedrals are clearly visible. The geometry of the two trihedrals on the rotating turntable was shown previously in figure 5-23 and their trajectories are evident in the centre of the image, forming a figure of eight response. Due to the RCS response of the trihedrals, signals are primarily reflected at specific angles where the trihedrals were facing the radar. Another two trihedrals were stationary and were positioned on either side of the turntable when this data was detected. The stationary trihedrals can be seen as straight vertical lines in the range profile as the backscatter from these were independent of the turntable rotation.

Since this is real data, clutter and noise were expected to be present in this data. Clutter can be seen as thinner straight vertical lines and is relatively constant in magnitude. Noise in the return can be seen as horizontal and vertical glitches in the range profile.

When a range resolution of 7 m was used, the range profile resulting from range compression of trihedral data is shown in figure 5-26.
5.2 Narrow Band Imaging of Real Trihedral Data - Results & Analysis

Figure 5-26: Range profile of rotating trihedrals using a range resolution of 7 m

In the profile, it is clear that the peak of range correlation occurs in the single 7 m range bin where the scatterers and turntable are positioned.

The profiles have an oscillation of intensity in the turntable angle domain due to the Doppler phase shift and they show sinc characteristics in the range domain due to range compression.

The trihedral RCS response highlighting the trihedral end points or 'ears' are also evident.

Zero Doppler Correction

Zero Doppler Correction (ZDC) is performed to remove the stationary artefacts such as the stationary trihedrals and stationary clutter. The clutter and stationary noise is further reduced by limiting the range of interest to that of only the area of the turntable itself. This is obviously particularly important in this experimental data. Thus, the two stationary trihedrals on either side of the turntable were removed from further processing because they would have shown up as unwanted zero clutter in the Doppler profiles.
When a range resolution of 30 cm was used, the results following ZDC were as shown in figure 5-27.

![Range profile after ZDC](image)

Figure 5-27: Range profile of rotating trihedrals after ZDC using a range resolution of 30 cm

The signal energy response of the two trihedrals rotating opposite each other on the turntable is still evident in the centre of the image, forming a figure of eight response and the level of signal energy is unchanged.

The signal energy response of the two stationary trihedrals were shown previously as straight vertical lines on either side of the range profile. By processing the turntable only, these lines have been removed.

The red area on the top and bottom of the figure are due to sidelobes of the high intensity areas (shown by the white areas) formed by the RCS response of the two rotating trihedrals.

When a range resolution of 7 m was used, the results of ZDC were as shown in figure 5-28.
5.2 Narrow Band Imaging of Real Trihedral Data - Results & Analysis

Figure 5-28: Range profile of rotating trihedrals after ZDC using a range resolution of 7 m.

After ZDC has been applied, the signal energy response of the two stationary trihedrals, shown previously as straight vertical lines on either side of the range profile, have been removed and only the 7 m range bin where the rotating scatterers on the turntable are positioned remains.

Collapse Multiple Range Bins

When a range resolution of 30 cm was used, in order to obtain a 1D projection for tomographic imaging, all the range bins in the range profile were summed together and collapsed to form a 1D reflectivity profile versus angular intervals. This again is equivalent to generating narrow band data.

An example of range collapsing is shown in figure 5-29 which shows the one-dimensional data generated from the range collapse of the original range profile.
5.2 Narrow Band Imaging of Real Trihedral Data - Results & Analysis

Figure 5.29: 1D reflectivity profile of trihedral data after range collapse of multiple range bins

This figure shows the one dimensional amplitude of the sum of all the range bins given in the range profile after ZDC processing shown previously in figure 5.27.

Again, the trihedral RCS response highlighting the maximum RCS at specific trihedral positions and the end points or 'ears' are clearly evident. The response is also very noisy and lower than the hypothetical RCS response shown in figure 5-24. It is also shifted in turntable rotation angle from the ideal response.

The noise level is found to be at approximately -18 dB below the maximum RCS value. This noise level is high due to the combination of the noise measured in each range bin.

The 1D reflectivity profile shown above in figure 5.29 looks very different to its simulated version seen previously in figure 5-6(a). This can be explained to some extent by comparing the range profiles used to form the reflectivity profiles. By comparing the trihedral range profile in figure 5-25 to the simulated range profile shown in figure 5-4(a), it can be seen that the figure '8' profile is very similar. However, the intensity of the simulated version is constant whereas the intensity of the trihedral profile is weighted due to the trihedral profile as illustrated in figure 5-24. Thus, when the intensities of each range bin in the range profile are summed together to form the 1D reflectivity profile, the intensities are weighted accordingly. The other factor that can be seen on comparison of the two reflectivity profiles, is the phase modulation that occurs over the turntable angle. This
5.2 Narrow Band Imaging of Real Trihedral Data - Results & Analysis

Phase modulation is due to the rotational motion of the turntable and is used in the following 1D Cross Range Projections section to produce the Doppler aperture and thereby the cross-range aperture. The phase modulation is not evident in the reflectivity profile of the trihedral data due to noise which is present in the real data. However, this will become apparent in the following sections.

Select Single Range Bin

When a range resolution of 7 m was used, only a single range bin was of interest i.e. the range bin that the 6 m diameter turntable lay in. In that case, collapsing the range profile was not necessary as the ID reflectivity profile was taken from the relevant range bin, as shown in figure 5-30.

![1D reflectivity profile of trihedral data after range collapse of single range bin](image)

Figure 5-30: 1D reflectivity profile of trihedral data after range collapse of single range bin

This figure essentially shows the one dimensional amplitude of the range bin in question given in the range profile after ZDC processing shown previously in figure 5-28.

Again, the trihedral RCS response highlighting the maximum RCS at specific trihedral positions and the end points or ‘ears’ are clearly evident and are similar to a hypothetical RCS response shown in figure 5-24. However, it is shifted in turntable rotation angle according to the maximum trihedral RCS positions.
The noise level is found to be at approximately -28 dB below the maximum RCS value. This noise level is lower than that measured for multiple range bin collapsing because the noise of only one range bin is detected.

For the same reasons given previously in the section on multiple range bins above, the 1D reflectivity profile shown in figure 5-30 looks very different to its simulated version seen previously in figure 5-7(a).

5.2.2 Narrow Band High Resolution Radar Imaging Process

Multiple Perspectives

The model described in Chapter 3 that was used to obtain 1D cross range projections from multiple perspectives for the simulated data was now applied to the real trihedral data.

For ease of use, the low cross range resolution projections and therefore the corresponding images were developed using 171 perspectives, each extending over an angle of 2.11 degrees. The development of synthetic apertures and thereby the 1D projections over these narrow angles gave the projections a low cross range resolution of 41 cm. However, after a Hamming window was applied to each of the projections, a resolution of 53 cm resulted.

Unlike the simulation, the highest resolution obtainable in which the high cross range resolution profiles and images were discernible in the trihedral data case, was that of \( \lambda/2 \). This meant that a high cross range resolution of 1.58 cm was obtained over project angles of 57.08 degrees. This will be evident later in this chapter.

To summarise, the multiple perspective parameter values used for processing trihedral data are shown in table 5.ii.

The 1D cross range projections resulting over each of these synthetic aperture angles will be discussed in more detail in the following sections.
5.2 Narrow Band Imaging of Real Trihedral Data - Results & Analysis

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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</thead>
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</tr>
<tr>
<td>Low cross range aperture angle</td>
<td>2.11 degrees</td>
</tr>
<tr>
<td>Blur angle</td>
<td>5.90 degrees</td>
</tr>
<tr>
<td>Weighting function aperture angle</td>
<td>6.34 degrees</td>
</tr>
<tr>
<td>Weighting function resolution</td>
<td>7.13 cm</td>
</tr>
<tr>
<td>High cross range aperture angle</td>
<td>57.08 degrees</td>
</tr>
<tr>
<td>High cross range resolution</td>
<td>1.58 cm ( \approx \lambda/2 )</td>
</tr>
</tbody>
</table>

Table 5.ii.: Processor parameter values used to process trihedral data

1D Cross Range Projections

A low cross range resolution profile with Hamming window is shown in figure 5-31 with a cross range resolution of approximately 41 cm \( \times \) 1.3 \( \approx \) 53 cm.

![Cross range profile after Hamming window](image)

Figure 5-31: Cross range profile of rotating trihedrals using a cross range resolution of 53 cm.

As illustrated previously in figure 5-23, figure 5-31 shows two trihedrals rotating opposite each other on the turntable in the centre of the image. As expected, the cross range profile is orthogonal to the range profile shown in figure 5-27 so that the figure ‘8’ of the range profile has been shifted by 90 degrees as is expected for range versus cross range dimensions.

The cross range profile is also affected by the trihedral RCS characteristics.
A single 1D cross range projection at perspective number 1 is shown in figure 5-32. It can be seen that the sidelobe level is approximately at -13 dBs. The response from the two trihedrals is clearly evident. The minimum signal level between the two peaks is approximately -28 dBs.

When a high cross range resolution of 1.58 cm was used, the combined result of the 1D cross range projections at multiple perspectives is the cross range profile shown in figure 5-33.

The shifted figure '8' of the two trihedrals is still evident.
Cell migration is clearly evident by the smeared appearance of the profile. This is due to the expected cross range cell migration that occurs once the projection angle is wider than the blur angle.

When comparing this cross range profile to the low cross range resolution profile in figure 5-31, it can be seen that the trihedral end points or 'ears' have shifted in turntable angle, i.e. from 330 to 70 degrees instead of from 300 to 50 degrees. This is due to flipping the array of data from left to right during processing so the final figures may be mirror images of each other.

Also when comparing this cross range profile to the low cross range resolution profile in figure 5-31, it can be seen that the noise floor is still present, just smeared because the energy is maintained.

Looking at a single 1D cross range profile at perspective number 1, see figure 5-34, it can be seen that the intensity of the 1D projection seems 'averaged out' at a constant level of -25 dBs. That is because the trihedrals were positioned at the edges of the turntable so the MTRC from the centre of the turntable to the edges produced what is essentially a noise floor.

![Cross range, persp 1, cr-resl=1.58cm](image)

*Figure 5-34: High resolution cross range projection*
High Cross Range Resolution Motion Correction

The high resolution motion correction involving a weighting function to remove MTRC used here is similar to that explained previously for the simulated data.

Weighting function

The weighting function used as the matching function uses a cross range profile with a resolution of 14.26 cm generated over an aperture angle of 6.34 degrees. This is slightly wider than the blur angle but has negligible effects. After multiplying the weighting function to itself, the cross-range resolution becomes 7.13 cm.

The cross range profile of the weighting function is shown in the figure below.

![Cross range profile, low cross range resolution of 7.13 cm](image)

---

Figure 5-35: Weighting function in the form of a low resolution cross range profile

The output shown in figure 5-36 is the 1D projection obtained from perspective number 1. It shows the measure of intensity in that projection over cross-range.
5.2 Narrow Band Imaging of Real Trihedral Data - Results & Analysis

When comparing this cross-range profile to that measured in the low resolution case in figure 5-32, one can see that the profile and example 1D projection are sharpened in intensity due to a higher cross-range resolution multiplied by itself to increase the weighting function even further. The noise floor is thereby lowered to approximately -50 dB.

When the profile in figure 5-35 is compared to the low cross range resolution profile shown previously in figure 5-31, the following features can be noted: The cross range resolution has improved from 53 cm to 7.13 cm so the weighting function profile is finer. No Hamming window was applied to the weighting function. The noise level has been lowered so it is not significant in the weighting function profile. The sidelobes are still observable in the weighting function profile. No smearing is evident even though the weighting function aperture angle is wider than the blur angle.

High cross-range resolution profile

After applying high cross range resolution motion correction by multiplying the 57.08 degree projection by the weighting function, high cross range resolution projections without MTRC effects are obtained.

A corresponding 1D cross range profile from perspective number 1 is shown in 5-37.
5.2 Narrow Band Imaging of Real Trihedral Data - Results & Analysis

This figure demonstrates that the intensity of the artefacts are standing 70 dBs higher than the noise floor. This results in an extremely high resolution point target.

The high cross-range resolution profile is shown here:

Figure 5-37: High cross range resolution projection after correction

This profile is more finely defined than either of the two profiles used to form it, namely the the 57.08 degree cross range profile and the weighting function.

The peaks and troughs of both contributing profiles have both enhanced and diminished
parts of the resulting profiles so that the intensity obtained at specific turntable angles is distorted. The intensity at the inner cross range values appears to be segmented and the intensity at the outer cross range values is not only dominant, but instead of following the shifted figure '8' curves, these areas in the profiles appear linear. These linear characteristics will have an effect on the resulting tomographic images created because, instead of following the circular response expected of the tomographic process, the scatterers will appear as having shifted slightly from their constant positions on the turntable as the turntable rotates.

**Tomographic Imaging**

As mentioned in the previous section, tomographic imaging has been used to form 2D images from narrow band signals received from multiple angular perspectives around an object. This section shows and analyses the resulting tomographic images for the experimental data.

Once again, tomographic images were produced from low resolution and high resolution 2D cross range profiles, each consisting of 171 1D projections.

The final low cross range resolution tomographic image is shown in figure 5-39.

![Image from 171 2.1 deg projections](image.png)

**Figure 5-39: Low resolution cross range tomographic image**
It is found, as in the case of the simulated data, that a true image of the scatterers in space has been generated using this narrow band tomographic processing technique. This is believed to be the first time that this has been demonstrated using real wide band data.

These images have been scaled by the expected cross range resolution. Using this scaling, the fact that the points end up the correct distance apart (at \(-3\) m and \(+3\) m) within cross-range resolution limits verifies the validity of this approach.

The scales in these images have been shown as cross range X versus cross range Y. This is because a square image has been built up from a set of projections from 171 viewing angles spaced over 360 degrees around the turntable. So any dimension in the resulting image is that of cross range.

The scatterers in each image are at a slight angle from horizontal due to an offset in the starting angle position input into the tomographic imaging process.

Outer circles occur around the central image of each point scatterer. This is due to the cross range projection sidelobes also being tomographically imaged.

The trihedral objects being imaged here are smaller than that required to avoid tomographic ambiguities so no tomographic ambiguities appear.

The final high cross range resolution tomographic images of the trihedrals are shown in figures 5-40 and 5-41.

![Image from 171 57.08deg projections, cr-resi=1.58cm](image)

Figure 5-40: High resolution cross range tomographic image
The high cross range resolution image zoomed in and centred to show each cross-range bin shows:

![Image from 171 57.08deg projections, cr-resl=1.58cm](image)

Figure 5-41: High resolution cross range tomographic image

The resulting cross range images show the scatterers as being positioned the correct distance apart from each other.

The scatterers appear smeared. This is most likely to be due to the linear qualities of the motion corrected profile shown previously in figure 5-38.

The smearing effect may also be due to the cell migration from using wide angle apertures to obtain high cross range resolution. However, when comparing figure 5-40 to figure 5-39, the cell migration effect for the high cross range profiles used to generate the tomographic images is negligible.

The smearing effect could not be due to misaligning the high cross range profiles input to the tomographic processor because the same definitions to form the images were used to form the low cross range resolution images shown previously in figure 5-39 which demonstrate no smearing effects.

The results shows that the tomographic images can be considered equivalent to the multilook images in this case. Due to the practical limitations mentioned previously, the best cross range resolution achieved using the trihedral data is \( \delta x = \frac{\lambda}{2} = 1.58 \text{ cm} \), where \( \Delta \theta = 57.08 \text{ degrees} \).
5.2.3 Multilook Processing

Multilook processing, was used for comparison with the narrow band high resolution radar imaging process. Again, 171 projections were used, each having a 2D image with 30 cm range resolution versus 41 cm cross range resolution.

The final multilook images of the trihedrals are shown in figure 5-42.

![Multilook image of trihedrals](image.png)

Figure 5-42: Multilook image of trihedrals

The figure shows the two trihedrals clearly as if they were point targets. The trihedrals appear circular due to the circular process of multilook imaging. They also appear uncorrupted by smearing or ambiguities.

However, note that these trihedrals are not proportional in range versus cross range. This is due to the trihedral RCS response which is wider over cross range.
5.2 Narrow Band Imaging of Real Trihedral Data - Results & Analysis

5.2.4 Summary Of Trihedral Data Findings

In this chapter, it has been shown for the first time that 2D tomographic images can be reconstructed accurately from both simulated and experimental narrow band cross range data. This proves the validity of the processing methods developed here and proves this technique is viable even with clutter and noise present in the data.

Two 2D images with low cross range resolution of 53 cm and high cross range resolution of \( \delta x = \lambda/2 = 1.58 \) cm were generated. The high resolution image has much better resolution than the conventionally multilook image with 30 cm range resolution by 41 cm cross range resolution being achieved.

The results showed that the method used to minimise noise and stationary clutter was effective.

During the course of this part of the study, a number of novel techniques have had to be developed. These include:

- A technique to convert wideband data to narrow band data.
- Ways of generating high resolution profiles using a weighting function to minimise the effects of MTRC.
- Forming 2D cross range profiles from 1D cross range projections generated using synthetic apertures.
- Making use of 2D cross range profiles to form tomographic images.

Taken together, the above represents a significant advance in tomographic processing with considerable potential for practical applications. However, the cases considered so far only deal with the slightly unrealistic case of sparse scatterers. Most real targets can be considered as a matrix of densely packed objects. In the next chapter, the case of dense scatterers is considered and the method is applied to more realistic target scenarios.
Chapter 6

Narrow Band Imaging of Dense Scatterers - Results & Analysis
In the previous chapter, the simulation of point scatterers proved that the concept used was valid and the processing of real trihedral data showed that concept worked well when applied to two, widely spaced scatterers. However, these results also showed that the use of cross range to profile and image scatterers is affected by occlusion.

Thus, only reflections from the unobstructed scatterers can be detected and therefore fully contribute to the imaging process. This implies that multiple perspectives are required, not only to prevent ambiguities in tomographic imaging, but also to detect each scatterer at a sufficient spatial resolution. The effect of occlusion on dense scatterers is investigated in this chapter by examining real targets, namely a tracked vehicle (tank) and a wheeled vehicle (Land Rover). In this chapter real ISAR data was used using the same radar parameters as described in Chapter 4.

The first target detected by the ISAR was a tank similar to that shown in figure 6-1 on the turntable. A true picture of the tank cannot be shown because it is classified. On either side of the turntable were two stationary trihedrals which were also detected by the ISAR.

The dimensions of the tank are chassis length of approximately 6 m, width of approximately 3.5 m and height of approximately 2.5 m. The geometry of interest of the real tank scenario is illustrated in figure 6-2.
Again, two of the trihedrals were stationary and positioned at slant ranges 69 m and 88 m from the radar respectively, on either side of the turntable which was centred at a slant range of 80 m from the radar. As only moving targets are of interest, these will be removed during processing as explained previously in section 4.1.

The second target detected by the ISAR was a Land Rover. The experimental set up was identical to that described above with the tank being replaced by the Land Rover on the turntable. The Land Rover was similar to that shown in figure 6-3 although the Land Rover used had a tarpaulin roof over the load space instead. A true picture of the Land Rover cannot be shown because it is classified.
The majority of the radar signal reflections in this case were associated with the metal superstructure of the vehicle, which is similar to that shown in figure 6-4. There were also reflections expected from glass and metal surfaces inside the vehicle which penetrated the glass windows. This resulted in significant multipath effects.

The dimensions of the Land Rover are chassis length of approximately 4.5 m, width of approximately 1.7 m and height of approximately 2.0 m. The geometry of interest of the real Land Rover scenario is illustrated in figure 6-5.
Again, two of the trihedrals were stationary and positioned at slant ranges 69 m and 88 m from the radar respectively, on either side of the turntable which was centred at a slant range of 80 m from the radar. As only moving targets are of interest, these will be removed during processing as explained previously in section 4.1.

6.1 Data Preparation

Data preparation was similar to that explained previously for the simulated data shown previously in section 5.1.1.

6.1.1 Range Compression

The 30 cm resolution range profile range compression of tank and Land Rover data is shown in figure 6-6 (a) and (b) respectively.
Figure 6-6: Range profile of rotating (a) tank, (b) Land Rover at 30 cm range resolution

Again, the stationary trihedrals and stationary clutter appear as straight vertical lines in the range profile.

It is instructive to pick out some of the features of this data before collapsing to the narrow band situation. The tank data in figure 6-6 (a) shows some interesting features. This profile is related to and can be compared against figure 6-2. The central section of the figure appears to show the tank facing the radar where the front of the tank is shown on the left of the figure and the rear of the tank is shown on the right of the figure. The opposite is true in the other two sections. The fact that these sections are almost symmetrical implies that the body of the tank is symmetrical. However, the two empty areas A and B which are shown at the rear and front of the tank are not shown when the tank is rotated 180 degrees around which implies that the front and the rear of the tank have solid ends as also evident by the bright spot C in the front of the tank and the bright spot D at the rear of the tank. The two yellow lines E and F on either side of C are reflections from the front corners of the tank.

When viewing the tank from the front, i.e. the middle section of the profile, the tracks are more distinguishable in the rear of the tank than they are when viewing the tank from behind, i.e. the top and bottom sections of the profile. This is because the rear tracks are
6.1 Data Preparation

masked by the RCS of the solid body.

The gun barrel is not visible in the range profile. This is due to masking by the stronger reflections from the body of the tank. The method of processing narrow band images in this work makes use of the range profile to extract the relevant data. Information about the gun barrel was not present in the range profile in this case thus this does not appear in the cross range profile.

Again, the Land Rover range profile shown in figure 6-6 (b) can be compared to figure 6-5. It can be seen that the red curves along the left of the profile forming a shifted ω shape will be used in the tomographic imaging to form the straight sides of the Land Rover. The length and width of the vehicle can be seen to correspond to the widest and narrowest parts of the response respectively.

Most significantly, the top half of the figure has resulted from the Land Rover facing the radar and the bottom half of the figure has resulted from the Land Rover facing away from the radar.

The two bright points A and B in the top half of the figure point to the front of the vehicle and the back of the cabin respectively while the two bright points C and D in the bottom half of the figure point to the rear of the vehicle and the back of the cabin respectively. The bright spots E at the top of the figure and F in the centre of the figure are from the sides of the Land Rover that are closest to the radar.

The fact that the intensity is greater on the left of the figure than the right of the figure shows that the beam was reflected primarily by the metal structure of the Land Rover and very little by the glass windows, the cabin and the load space.

The top half of the figure has two dominant lines. These were the reflections of the radar beam from the back of the load space at the rear end of the Land Rover. The faint lines behind them on the right of the figure are due to multipath as the beam went through the load space and was reflected within. The lines diminishing in the centre of the top half show where the beam was not reflected back.

The bottom half of the figure has a central dominant line. This is attributed to the
6.1 Data Preparation

reflection of the radar beam from the bonnet at the front end of the Land Rover. The faint lines behind it on the right of the figure are due to multipath from the interior of the cabin. This multipath delay is noticeably shorter in this case compared with that in the load space due to the smaller dimensions of the cabin. The lines diminishing in the sides of the bottom half show where the beam was not reflected at an angle to the windscreen.

When a range resolution of 7 m was used to mimic the narrow band case, the range profiles resulting from range compression of tank and Land Rover data is shown in figure 6-7 (a) and (b) respectively.

![Range profile after range compression](image)

Figure 6-7: Range profile of rotating (a) tank, (b) Land Rover at 7 m range resolution

In the profile, it is clear that the peak of range correlation occurs in the single 7 m range bin where the scatterers and turntable are positioned. The response in the turntable range bin shows several peak RCS reflections with coincide with those of the low range resolution figure 6-6.

6.1.2 Zero Doppler Correction

When a range resolution of 30 cm was used, the results of Zero Doppler Correction (ZDC) were as shown in figure 6-8.
Figure 6-8: Range profile of rotating (a) tank, (b) Land Rover after ZDC at 30 cm range resolution

The RCS responses of the tank and the Land Rover are unchanged from the respective range compressed versions but the two stationary trihedrals, stationary clutter and noise have been removed as expected.

When ZDC was applied in the case of 7 m range resolution, the results shown in figure 6-9 were obtained.

Figure 6-9: Range profile of rotating (a) tank, (b) Land Rover after ZDC at 7 m range resolution
6.1 Data Preparation

Again, the only 7 m range bin remaining after ZDC is that in which the turntable was located.

6.1.3 Collapse Multiple Range Bins

When a range resolution of 30 cm was used, in order to obtain a 1D projection for tomographic imaging, all the range bins in the range profile were summed together and collapsed to form a 1D reflectivity profile versus angular intervals. This again is equivalent to generating narrow band data.

Examples of range collapsing are shown in figure 6-10 which shows the one-dimensional data generated from the range collapse of the original range profile.

![Figure 6-10: 1D reflectivity profiles of (a) tank, (b) Land Rover data after range collapse of multiple range bins](image_url)

This figure shows the one dimensional amplitude of the sum of all the range bins given in the range profile after ZDC processing shown previously in figure 6-8.

In the profile, it is clear that the peak of range correlation occurs where the scatterers and turntable are positioned. The response in the turntable range bin shows several peak RCS reflections with coincide with those of the low range resolution figure 6-6.

The noise level is found to be at approximately -12 dB below the maximum RCS value for the tank data and -18 dB below the maximum RCS value for the Land Rover data. These noise levels are high due to the additive effect of the noise measured in each range bin.
6.1.4 Select Single Range Bin

When a range resolution of 7 m was used, only the range bin in which the 6 m turntable was present was of interest. In that case, collapsing the range profile was not necessary as the ID reflectivity profile was taken from the relevant range bin, as shown in figure 6-11.

![1D reflectivity profile](image)

Figure 6-11: 1D reflectivity profile of (a) tank data, (b) Land Rover data after range collapse of single range bin

This figure essentially shows the one dimensional amplitude of the range bin in question given in the range profile after ZDC processing shown previously in figure 6-8.

Again, it is clear that the peak of range correlation occurs where the scatterers and turntable are positioned. The response in the turntable range bin shows several peak RCS reflections with coincide with those of the low range resolution figure 6-6.

The noise level is found to be at approximately -18 dB below the maximum RCS value in the case of the tank and -22 dB below the maximum RCS value in the case of the Land Rover. These noise levels are lower than those measured for the multiple range bin collapsing case because the noise from only one range bin is detected.
6.2 Narrow Band High Resolution Radar Imaging Process

6.2.1 Multiple Perspectives

The same model that was applied to obtain 1D cross range projections from multiple perspectives from the real trihedral data in section 5.2.2 was now applied to the real tank and real Land Rover data.

The multiple perspective parameter values used for processing the data are shown in table 6.i.

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<thead>
<tr>
<th>Parameter</th>
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<td>Low cross range resolution</td>
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</tr>
<tr>
<td>High cross range aperture angle</td>
<td>57.08 degrees</td>
</tr>
<tr>
<td>High cross range resolution</td>
<td>1.58 cm ≈ λ/2</td>
</tr>
</tbody>
</table>

Table 6.i.: Processor parameter values used to process tank and Land Rover data

The 1D cross range projections resulting over each of these synthetic aperture angles is discussed in more detail in the following sections.

6.2.2 1D Cross Range Projections

A 1D cross range projection was then produced from the FFT of the synthetic aperture at each perspective. A 2D cross range profile of 1D cross range projections versus perspective angles can then be built up.

Low cross range resolution profiles with Hamming windows of the tank and Land Rover are shown in figure 6-12 with a cross range resolution of approximately 41 cm * 1.3 ≈ 53 cm.
6.2 Narrow Band High Resolution Radar Imaging Process

The ZDC processing has removed the centre of the profile which occurs at zero Doppler.

Figure 6-12(a) and (b) are related to the geometries shown in figure 6-2 and 6-5 respectively. As expected, the cross range profiles are orthogonal to the range profiles shown in figure 6-6(a) and (b).

In figure 6-12 (a), it can be seen that the curves of the profile forming a figure of eight shape will be used in the tomographic imaging to form the straight sides of the tank. The length and breadth profiles of the tank can clearly be distinguished by their marked difference in width. The vertical bright lines A and B in the narrower parts of the profile correspond to the ends of the tank.

Because the tomographic imaging process forms images by backprojecting each 1D projection formed at multiple perspectives, this implies that the straight lines of the tank image will be enhanced from curved lines in the profile.

The specks above the left section of the profile and below the right section of the profile, hint at the full length of the tank with the vertical lines C and D corresponding to the reflection from the front, centre and rear, centre of the tank, respectively.
6.2 Narrow Band High Resolution Radar Imaging Process

The rounded lines $E$ and $F$ in the profile form straight lines in the image which have been attributed to the turret.

The gun barrel is not visible in the cross range profile because, as described earlier, it was not visible in the range profile used to form this profile.

In figure 6-12 (b), it can be seen that the red curves along the left of the profile forming a shifted $\omega$ shape will be used in the tomographic imaging to form the straight sides of the Land Rover. Therefore the widest response in range is showing the length of the Land Rover and the narrowest response is showing the breadth of the Land Rover. The vertical bright lines $A$ and $B$ in the narrower parts of the profile will form the ends of the Land Rover.

Because the tomographic imaging process forms images by backprojecting each 1D projection formed at multiple perspectives, it might be expected that the straight lines of the Land Rover image will be enhanced from curved lines in the profile.

The rounded lines $C$ and $D$ in the profile form straight lines in the image. The reasons for these may be understood better with the formation of the final image.

The ZDC processing has removed the centre of the profile which occurs at zero Doppler.

Tank and Land Rover 1D cross range projections of aperture width 2.11 degrees, located at perspective number 1 are shown in figure 6-13. In this case, a Hamming window was used and the cross range resolution is approximately 53 cm. It can be seen that the sidelobe level is approximately at -5 dBs. The noise floor level has been reduced by the Hamming window envelope.
6.2 Narrow Band High Resolution Radar Imaging Process

A high cross range resolution of 1.58 cm was made for multiple synthetic apertures each of 57.08 degrees of turntable rotation. Examination of the 1D high cross range projections at perspective number 1 in figure 6-14 shows that the intensity of the 1D projection appears to be ‘averaged out’ at a constant level of -5 dBs. That is because in each case, either the tank or the Land Rover covered the entire turntable so the MTRC from the centre of the turntable to the edges produced what is essentially a noise floor.

![Cross range profile](image)

Figure 6-13: (a) Tank and (b) Land Rover low resolution cross range projections

The combined results of the 1D cross range projections at multiple perspectives are the cross range profiles shown in figure 6-15.

![Cross range profile](image)

Figure 6-14: (a) Tank and (b) Land Rover high resolution cross range projections

The combined results of the 1D cross range projections at multiple perspectives are the cross range profiles shown in figure 6-15.
The familiar tank profile is still evident because, by comparing this cross-range profile to that measured in the low resolution case in figure 6-12, one can see that the same artefacts are present although emphasised by smearing of the profile.

The smeared appearance of the profile is due to the cell migration that occurs when the projection angle is wider than the blur angle.

### 6.2.3 High Cross Range Resolution Motion Correction

High resolution motion correction was applied at this stage. A weighting function was again used to remove MTRC as previously described for the simulated data.

#### Weighting function

The weighting function used as a matching function was derived from the cross range profile obtained for low resolution projections over an angle of 6.34 degrees. The weighting function has a cross-range resolution of 7.13 cm.

Combining the projections from 171 perspectives result in the cross range profiles of the weighting functions shown in figure 6-16 below.
6.2 Narrow Band High Resolution Radar Imaging Process

Figure 6-16: Weighting functions in the form of a low resolution cross range profile for the (a) tank, (b) Land Rover

When comparing these cross-range profiles to those measured in the low resolution case in figure 6-12, one can see that the same artefacts are present. The profiles and example 1D projections are sharpened in intensity due to a higher cross-range resolution which is multiplied by itself. This has the effect of increasing the weighting function even further. The noise floor is thereby lowered to approximately -50 dB in the case of the tank and -25 dB in the case of the Land Rover.

When this figure 6-16 is compared to the low cross range resolution profiles shown previously in figure 6-12, the following features can be noted:

The same cross range resolution of 14.33 cm and therefore the same projection aperture width was used in both. The weighting function was formed by multiplying the low cross range resolution profile twice thereby decreasing the cross range resolution to 7.17 cm. So as expected, due to the multiplication, the weighting function profile is narrower than the low cross range resolution profile.

The heightened intensity of the weighting function at the outer cross range values is more apparent.

The smearing due to wider apertures is most evident for the vertical lines as the aperture angle widens.
High cross-range resolution profile

After applying high cross range resolution motion correction by multiplying the 57.08 degree projection by the weighting function, high cross range resolution projections without MTRC effects are obtained.

The high cross-range resolution profiles is shown here:

![Cross range profile, cr-resl=1.58cm](image1.png)

![Cross range profile, cr-resl=1.58cm](image2.png)

Figure 6-17: (a) Tank, (b) Land Rover high resolution cross range profiles after correction

This profile is more finely defined than either of the two profiles used to form it, namely the the 57.08 degrees cross range profile and the weighting function.

These artefacts are at an even higher resolution than the weighing function. This means that the features of the tank that provided the highest return due to larger RCS, such as the tank outline, and the greatest changes in Doppler have been given priority.

6.2.4 Tomographic Imaging

As in the case of the simulated data, tomographic imaging has been used to form low and high resolution 2D images from narrow band signals adapted from the wideband tank and Land Rover data. Once again, tomographic images were produced from low resolution and high resolution 2D cross range profiles, each consisting of 171 1D projections.

The low cross range resolution tank and Land Rover images are shown in figure 6-18 (a) and (b) respectively.
The ZDC processing is manifested after tomographic imaging as a hole in the centre of the images.

Note that the rear of the tank, seen at the bottom of image (a), appears different to that expected from two parallel tracks. This is because, the sides or corners that moved the fastest were shown more intensely so the image describes that the rear corners of the tank dominated both by RCS and Doppler speed. These proportionally got less towards the rear centre of the tank. The front of the tank does not seem to be affected in the same way and hints of a solid front end. This may be due to other obscurations.

As suspected from the range profile, the rear tracks are more distinguishable than the front tracks. The front appears to be solid although this may be due to the processing and summation techniques used in narrow band tomographic imaging.

As expected, the prominent points in the tank which reflect the radar signal are the sides and the corner reflectors. The turret is indistinguishable from the tank body although this may be resolved by using a shallower grazing angle than the 8 degrees used here. The gun barrel is again not visible in the image because it was not visible in the range profile and thereby was not incorporated in the cross range profile used to form the image above.
6.2 Narrow Band High Resolution Radar Imaging Process

The Land Rover image prominently shows the scatterers that have had the biggest change in Doppler, namely the corners of the Land Rover and the cross-mark.

The cross-mark visible in the lower half of the figure cannot be due to any physical object in the Land Rover. However, it is deduced that this is due to multipath within the load space of the Land Rover since significant multipath effects were evident in the range profile in figure 6-6 and in the low cross range resolution image in figure 6-18. The multipath is a property of the frequencies used in this work so it is possible that this effect could be removed by using another carrier frequency. This effect may also be because there are pronounced right angles inside the Land Rover, so it is behaving as multiple trihedral corner reflectors.

The cross-mark has been generated using the two very different methods of multilook processing and tomographic processing and yet the cross-mark itself is very sharply and clearly imaged despite the non-coherent summing of these two different methods. This means that the attributes were not generated by erroneous processing but can most likely be attributed to the multipath properties of the return signal for this target vehicle.

The high cross range resolution tomographic images are shown in figure 6-19.

![Figure 6-19: High resolution cross range tomographic image](image.png)
The above high cross range resolution image magnified are shown in figure 6-20.

When the image of the tank shown in figure 6-19 (a) is compared to the low resolution image obtain in figure 6-18, the details are similar although the tank outline is clearer and better defined in the high resolution case. A lower resolution would have improved the image but the highest resolution possible was being investigated in this case.

The well-defined outline of the high resolution tank image shows that the tomographic imaging process which backprojects and sums the 1D projections from multiple perspectives, works very well to coordinate the thin lines together.

The fact that a sharp outline with such high resolution was achieved shows the very positive effect of the weighting function which highlighted the prominent points in each 1D projection and minimised the MTRC effects.

In the case of the Land Rover image shown in figure 6-19 (b), the weighting function has again helped to highlight the prominent points in each 1D projection and minimised the MTRC effects.

The image definition in this case is poor. This is attributed to the significant multipath present when observing this target resulting in multiple displaced image formation.

When this image is compared to the low resolution image obtained in figure 6-18 (b) and
to the multilook image in figure 6-21 (b), only the outline of the vehicle and parts of the cross mark are similar. Parts of the cross-mark have been reduced in the centre of the Land Rover by the weighting function but the outer parts of it have been enhanced due to the tomographic ambiguities.

In both images, tomographic ambiguities are evident as lines radiating from the centre at every 2.1 degrees. In order to have prevented tomographic ambiguities, the number of projections used as specified in section 2.4.6, should have been \( N > 1257 \) in the case of the 6 m diameter tank and \( N > 943 \) in case of the 4.5 m diameter Land Rover.

Due to the practical limitations mentioned previously, the best cross range resolution achieved using experimental data is \( \delta x = \lambda/2 = 1.58 \text{ cm} \), where \( \Delta \theta = 57.08 \text{ degrees} \).

### 6.3 Multilook Processing

Multilook processing of the tank and Land Rover data was again performed for comparison with the narrow band high resolution radar images.

Figure 6-21 (a) and (b) shows the multilook images of the tank and Land Rover respectively, each having 30 cm range resolution versus 41 cm cross range resolution.
Figure 6-21: Multilook images of (a) tank and (b) Land Rover

The image in figure 6-21 (a) shows the top view of the tank. The length and width of the tank are clearly visible. The tank is facing the top of the figure, the turret can be seen as the disk in the centre of the tank and the gun barrel is pointing towards the lower, left hand corner. The tank is in the form of a rectangle from the front to the back of the turret with the two tracks protruding past each back corner, leaving a gap in the rear centre.

When compared to the low cross range resolution image in figure 6-18 (a), the front of the multilook image also appears to be solid although there is an empty space behind it and again this may be due to the processing and summation techniques used in multilook imaging.

As suspected, the rear tracks are more distinguishable than the front tracks. The front appears to be solid although this may be due to the processing and summation techniques used.

The multilook image shown in figure 6-21 (b) are of the top view of the Land Rover. The Land Rover is facing left with the bonnet on the left hand side and boot on the right hand side of the image.

The two bright lines shown on either side at the centre of the image are reflections from
the sides of the windscreen. The two bright spots right of the figure are due to the rear corners of the Land Rover. These lines and spots are prominent points due to the corners having greater Doppler shifts than the front and sides of the vehicle.

The four light specks on the sides of the image were also created by multipath as the signals bounced inside the load space of the Land Rover, therefore, these specks are positioned at multiples of the range between the width of the vehicle.

The Land Rover appears to be of fairly uniform width, however the front of the vehicle is not clearly defined and is only visible as two lines down the centre of the vehicle's bonnet. This implies that the centre of the bonnet was more apparent to the radar at that grazing angle than the sides of the bonnet. This seems to be borne out by examination of the physical structure of the vehicle as shown in figure 6-3.

A cross-mark similar to that seen in the tomographic processing can also be seen in the right half of the figure. This tends to confirm that this effect is multipath related rather than a processing artefact.
6.4 Summary Of Findings

This chapter has examined the imaging of targets which can be considered as a dense collection of scatterers. Real data has been used to construct profiles of a wheeled and a tracked vehicle and tomographic imaging has been compared with conventional multilook processing for these targets.

It has been shown that a high resolution image of a real target can be obtained by using a tomographic imaging process which backprojects and sums the 1D cross range profiles from multiple perspectives. A more advanced method of motion compensation and the use of multiple perspectives from two dimensional view points has been shown to improve the image quality further.

The tank image was enhanced by the opaque nature of the target. In the case of the Land Rover, significant multipath from the interior of the vehicle was found to seriously affect the image. Tomographic ambiguities also occurred in the images which can be compensated by using more perspectives during processing.

The highly resolved outline achieved for the opaque target is attributed to the weighting function which highlighted the prominent points in each 1D projection and minimised the MTRC effects.

Due to the practical limitations mentioned previously, the best cross range resolution achieved using experimental data was found to be $\delta x = \lambda/2 = 1.58$ cm, where $\Delta \theta = 57.08$ degrees.

In summary, this chapter has demonstrated that the tomographic methods developed in this work can be successfully applied to real target data. Imaging limitations due to target and radar characteristics such as masking, ambiguity and coherency have been identified and some possible solutions suggested.

In the final part of this study the resolution limits for this method are investigated and compared with that obtainable using a circular array of stationary sensors observing a stationary target.
Chapter 7

Narrow Band Radar Resolution Limits
The use of narrow band radar trades resolution achieved by bandwidth for resolution achieved by spatially diverse angular imaging. This method requires the use of Doppler of moving targets to achieve cross range. Another possible method of achieving cross range information is by using the phase return from a circular aperture of sensors detecting a stationary target in the aperture centre. In this chapter, the high resolution limits achievable for both these cases are investigated and compared.

7.1 Narrow Band Resolution Limits

In standard radar designs, the maximum possible sampling frequency is determined by the Nyquist rate. Thus the limits on resolution are effectively set by the sampling rate which has a maximum value of twice the highest frequency present (i.e. the Nyquist rate) and of course the highest frequency is the illumination frequency. In this part of the study, we have investigated whether a similar principle can be applied in the angular sampling domain.

Using bandwidth in the time domain provides a measure of range and range resolution. Range resolution is given by:

\[ \delta_r = \frac{c}{2B} \]  \hspace{1cm} (7-1)

where: \( \delta_r \) = range resolution

\( B \) = bandwidth

\( c \) = speed of light

In the extreme case, \( B = 0 - 2f_0 \), therefore, range resolution reaches a minimum of:

\[ \delta_r = \frac{c}{2.2f_0} = \frac{\lambda}{4} \]  \hspace{1cm} (7-2)
7.1 Narrow Band Resolution Limits

where: \( f_0 = \) carrier frequency
\( \lambda = \) wavelength

As discussed previously in section 2.1.5, Brown (Brown1,1967), (Brown3,1969) and Ulander (Ulander1,1996), (Ulander2,2003) described the wide angle cross range resolution as given by:

\[
\delta_{cr} = \frac{\lambda}{4\sin(\Delta \theta/2)} \quad (7-3)
\]

It can be shown that as \( \Delta \theta \to 180 \text{ deg} \), the cross range resolution reaches a minimum of:

\[
\delta_{cr} \to \frac{\lambda}{4} \quad (7-4)
\]

In the case where the wide angle calculation given by equation 2-4 is collated from 360 degrees of narrow angles instead then the cross range resolution is given by:

\[
\delta_{cr} = \frac{\lambda}{2.\Delta \theta} \quad (7-5)
\]

So the cross range resolution limit expected is:

\[
\delta_{cr} = \frac{\lambda}{2.2.\pi} \approx \frac{\lambda}{12} \quad (7-6)
\]

Realistically, for both wide angle and narrow angle cases, a practical minimum value of

\[
\delta_{cr} \approx \frac{\lambda}{2} \quad (7-7)
\]

may be expected due to real system physical limitations, range ambiguity, range curvature, SNR limitations, etc. This practical limit occurs at an angle of approximately 60 degrees.

The minimum cross range resolution values given by \( \lambda/2, \lambda/4 \) and \( \lambda/(4.\pi) \) are shown in figure 7-1.
7.2 Circular Aperture Resolution

The above analysis has established the cross range resolution limits for cases which exploit the Doppler information from a moving target. In this section, we now compare and contrast the resolution limits obtainable in the case of a stationary target in the centre of a circular aperture of multiple stationary sensors. In this case we have investigated a geometry consisting of a semi-circular array of sensors around the target.

Although, this concept is similar to circular radar (where the radar is flown in a circle around a specific terrain spot), and therefore similar to spotlight mode SAR, it does not take Doppler into account. Instead the sensors and the target are stationary and spatial comparisons are used.

The approach provides further valuable insight into the fundamental resolution limits in microwave imaging.
7.3 Circular Aperture Concept

Cross range resolution achieved in synthetic aperture radars is determined by the relative motion between the radar and the object. However, if cross range measurements are taken using only spatial sampling from a sequence of stationary measurement locations, the link between Doppler generated bandwidth (and therefore viewing angle) is broken. Thus, the limitation due to relative motion is avoided and improved spatial resolution may potentially be obtained. It may be expected that the resolution in this case will be largely determined by the number of spatial samples, the spacing of the spatial samples and number of image projections employed.

For example, consider an antenna forming a circular aperture with a single point object at its centre as shown in figure 7-2. The element positions are shown as the series of dots mapping out a semi-circle. These elements can be as closely spaced as practical considerations allow. In this way a viewing angle of 180 degrees can be achieved.

![Figure 7-2: A stationary circular aperture consisting of many elements detecting a stationary point target](image)

Each sensor can determine the cross range resolution of the object as \( \delta_{cr} = \frac{R}{d} \) as discussed initially in section 2.1.4. However, there are a number of methods of improving upon this figure and these are discussed below.

By collecting together all the return signals from the sensors that make up the circular aperture, they effectively form the individual samples in a synthetic aperture. Because the
object is stationary and equidistant from each sensor, the phase of each received signal is fixed and identical in value for each element of the aperture.

The response from the synthetic aperture thereby forms a *rect* function.

\[
s(t) = \sum_{n=1}^{N} A_n^\theta(t).\exp[j\theta_n^\theta(t)]
\]  

(7-8)

where \( N \) is the number of sensors that make up the synthetic aperture.

The sampling rate is related to position and is not limited by time. Because there is no phase shift in the combined signals of the aperture, performing an *FFT* on the combined response would not result in or be limited by Doppler characteristics. Again, this method makes use of an *FFT* of the synthetic aperture as shown in the equation below:

\[
S(f) = FFT(s(t))
\]  

(7-9)

and a 1D projection of a *sinc* function is produced.

Hypothetically, in the case of a single (infinitesimally small) point object at the centre of the aperture consisting of an infinite number of elements, the *sinc* function becomes a *delta* function in the frequency domain.

In synthetic aperture theory, cross range is derived from the frequency shift due to the relative velocity between target and radar. To derive the cross range in the stationary circular aperture case, synthetic aperture theory is applied under the assumption that the relative velocity, in this case, is infinitesimally small. So the cross range may be derived from the infinitesimally small frequency shift indicated by the *delta* function. In this way, an infinitely high cross range resolution may be obtained theoretically.

By rotating the aperture, multiple synthetic apertures and thereby multiple projections may be formed from which a second dimension of resolution may be obtained. By putting these multiple 1D projections from multiple perspectives into a tomographic imaging process, a 2D image of the point object may be produced and an image with infinitely high cross range resolution may result.

In practice, resolution will be limited by effects such as tomographic ambiguities which
may arise a result of the number of projections. Ambiguities may also arise as a result of the sampling rate since these are sampled apertures.

The practical limitations to this system are also influenced by effects such as oscillator instability, inaccuracy of positioning, computation time, the non-infinitesimally, non-point like nature of real targets, time to acquire, scatterer self-occlusion, etc.

7.4 Circular Aperture Simulation

The case of a circular aperture, as shown in figure 7-2, was simulated where 10 000 elements were positioned a distance of 31.42 cm apart to form a semi-circle at a radius of 1 km around a point object. Each node sequentially transmitted a 10 GHz truncated narrow band waveform at the object and measured the phase of the received signal.

The aperture width is determined by the number of sensors independent of their spacing.

By performing an $FFT$ on the aperture signal, a $sinc$ function is produced having a resolution width of $1/W$. In this case $W = 10000$ so $1/W = 0.0001$, which means that the $sinc$ function has become a $delta$ function effectively.

At low cross range resolutions, the cross range and the Doppler shift that it is derived from are of the same order. Using this rough approximation, a 1D cross range projection was produced from the frequency projection, giving the cross range resolution obtained here as $\delta_{cr} \approx \Delta f$ i.e. $\delta_{cr} \approx 1/W$

By repeating this 1D projection process for multiple perspectives around the target, a 2D cross range profile was produced. In this simulation, the process was then repeated for 100 1D projections spanning the full 360 degrees to be used to form a tomographic image. Figure 7-3 shows the resulting tomographic image formed from simulated data for the circular aperture.
The circular aperture in this simulation produced a cross-range resolution of \( \delta_{cr} \approx 0.1 \text{ mm} \), which is equivalent to \( \delta_{cr} \approx \lambda/300 \). This is a better resolution than can be achieved using conventional synthetic aperture processing and illustrates the improvement possible using this approach. It is also possible that changing the parameters used in the simulation could improve this resolution further.

Thus by avoiding frequency sampling which is limited either by the bandwidth in range or Doppler sampling in cross-range and introducing stationary spatial sampling, infinitely small resolution can theoretically be obtained. Clearly, in a practical application this would not be possible since an infinite number of elements could not be deployed. Nevertheless it is clear that the use of a circular aperture has the potential to allow the formation of high resolution tomographic images of a stationary target.
7.5 Summary

In this chapter, the theoretical limits on resolution using spatial sampling of the aperture rather than frequency sampling as in the case of SAR and ISAR have been investigated. It has been found that this method, in principle, enables very high resolution to be achieved. In fact, there is no theoretical limit of the resolution of the sensor in the case of an infinite number of sensor elements.

There are of course a number of practical limitations on the application of this method. The main practical factor would be the limitation on the number of sensor elements and the effect of varying the radar frequency. Greater numbers of elements will clearly give a finer resolution. It has also been found that the effect of geometry dependent ambiguities needs to be taken into account when evaluating the performance of this type of sensor.
Chapter 8

Summary of Findings, Conclusions and Future Work
The objective of this study was to examine the potential for the use of narrow band radar in performing target imaging. The main findings of this work are summarised in the following section.

8.1 Summary of Findings

The literature review in Chapter 1 has allowed the baseline for this technology to be evaluated in order to inform the research carried out during this study. Particular issues for further examination became evident from the literature survey including:

• Use of tomographic imaging based techniques using 1D cross range projections taken from multiple perspectives around an object so as to produce a 2D image of the object.

• Resolution performance limits in terms of range, cross range and stationary spatial sampling.

• Examination of the difference between narrow band imaging of sparse and dense objects.

• Practical limitations of this concept.

Chapter 2 is concerned with the existing theory related to narrow band radar, its associated cross range resolution and imaging. Its contributions include:

• A review of how high resolution can be achieved in the range direction and the cross range direction and by combining these techniques with SAR methods.

• Inverse Synthetic Aperture Radar (ISAR) was then discussed and the effects of motion on the ISAR profiles were described. The conventional methods used to generate radar images in the narrow angle case were also discussed.

• Polar format frequency space and the relationship between frequency space and resolution were then described and explained.

• Finally, the process of tomographic imaging and ambiguities related to the process were described.

Chapter 3 is concerned with the fundamental concepts developed in Chapter 2 can be
8.1 Summary of Findings

applied to narrow band radar to achieve high resolution tomographic imaging. Its contributions include:

• The development of these concepts through the adaption and application of narrow band theory, spatial frequency theory and tomographic theory.

• The exploitation of spatial diversity to obtain high cross range resolution features.

• The construction of high cross range resolution tomographic images using the above techniques.

Chapter 4 describes the development of the prototype processor system to process either simulated and real experimental data. Its main contributions are:

• It has been show that spatial frequency can be used instead of bandwidth to achieve high resolution.

• Methods have been developed using the above principle to enable the generation of cross range images if moving objects.

• The limitations on the apertures used to form the cross range projections have been investigated and defined.

• It has been shown that in the case of high cross range resolution, the aperture angle of each projections is wider than the blur width so motion correction is required to remove the cell migration response.

• It has been further demonstrated that if an inadequate number of projections are used to form the tomographic image, tomographic ambiguities occur in the form of lines radiating from the centre of the image.

• Finally in this chapter, the processing required to produce both low and high cross range resolution images has been developed and described. Results of conventional multilook processing have also been analysed for comparison.

Chapter 5 deals with narrow band imaging of sparse scatterers using simulated and real trihedral data. The main contributions are:

• It has been shown for the first time that 2D tomographic images can be reconstructed to an accuracy within cross-range resolution limits from both simulated and experimental
narrow band cross range data. This proves the validity of the processing methods developed here. It has also been shown that this technique is viable even with clutter and noise present in the data.

- Two 2D images with low cross range resolution and high cross range resolution have been generated using the methods developed in Chapter 3. It was shown that higher resolution images of sparse scatterers can be obtained than that obtained by conventional narrow band multilook processing with a resolution of $\lambda/4$ being achieved for the simulated data.

- Using real data, 2D images with low cross range resolution of 53 cm and high cross range resolution of $\lambda/4 = 1.58$ cm were generated. The high resolution image has much better resolution than the conventional narrow band multilook image with 30 cm range and 41 cm cross range resolution being achieved.

The high resolution image has much better resolution than the conventionally multilook image.

- The results showed that the method used to minimise noise and stationary clutter was effective.

- During the course of this part of the study, a number of novel techniques have had to be developed. These include:
  - A technique to convert wideband data to narrow band data.
  - Ways of generating high resolution profiles using a weighting function to minimise the effects of MTRC.

Methods to form 2D cross range profiles from 1D cross range projections generated using synthetic apertures.

- The use of 2D cross range profiles to form tomographic images.

Chapter 6 then demonstrates narrow band imaging of dense scatterers using real target data. Its contributions include:

- Real data has been used to construct tomographic images of a wheeled vehicle in the form of a Land Rover and a tracked vehicle in the form of a tank. These have been compared with conventional multilook processing for these targets.
8.2 Conclusions

• It has been found that the image of the tank was enhanced by the opaque nature of the target. In the case of the Land Rover, significant multipath from the interior of the vehicle was found to seriously affect the image. Tomographic ambiguities also occurred in the images. It was shown that this can be compensated for by using more perspectives during processing.

• A clearly resolved image outline for the opaque target (tank) has been achieved. This is attributed to the weighting function which highlighted the prominent points in each 1D projection and minimised the MTRC effects.

In Chapter 7, the theoretical limitations on resolution are determined by spatial sampling of the aperture rather than frequency sampling as in the case of SAR and ISAR. In this case, use of the phase return from a circular aperture of sensors detecting a stationary target has been investigated. This has been compared to the moving target cases analysed in Chapters 5 and 6. Its contributions include:

• It has been shown to give much higher resolution than the single sensor methods previously described. In fact, theoretically infinite cross range resolution could be obtained.

Conclusions may be drawn from this summary and these are discussed in the following section.

8.2 Conclusions

The key achievement of this work is to demonstrate that high resolution images of real targets can be obtained using narrow band radar. This has required development of a number of novel processing techniques to compensate for the loss of bandwidth by using spatial sampling.

The main methods developed include the use of a weighting function to minimise cell migration, techniques to generate 2D cross range profiles from 1D cross range projections, and the subsequent tomographic imaging process using these profiles. Using these methods, images of rotating targets consisting of both sparse and dense scatterers have been generated using both simulated and real data. Issues with imaging opaque and partially transparent targets have been identified.
A cross range resolution of $\lambda/4$ has been achieved using simulated data. This analysis has also been extended to the case of real data of typical ground targets. In this situation the data is inevitably significantly affected by noise but a resolution of $\lambda/2$ has still been achieved.

A technique using an array of stationary sensors to form a circular aperture has also been investigated for imaging stationary targets. This method has been shown to have potentially very high (theoretically infinite) resolution bounded only by real world practical limitations of the system.

This work was done for a set of fixed radar parameters so the effect of changing parameters on the imaging process would need further investigation. This work should act as a good basis for further development of practical applications.

8.3 Future Work

It is evident from this study that there are a number of limitations to these narrow band techniques which require further investigation. Some recommendations for future work to address some of these issues are given below followed by a discussion of a few practical implementations of this technique.

Future work recommended is:

- Advanced motion compensation seems likely to be required for MTRC and could be investigated further. Using one of the many motion compensation processes discussed in section 2.2.5, has the potential to improve the profiles and images obtained during this work and optimise the high resolution concepts developed here.

- Bandwidth interpolation/extrapolation should be investigated as a means to fill in areas of poor resolution. This could be achieved by incorporating multi-tonal signals and using bistatic/multistatic geometries.

- In a similar way that a clutter map is used to subtract the unwanted clutter from an image, so the unwanted multipath signals, once characterised, may be removed from the signals. This should improve image quality significantly but will only be applicable to targets with easily characterisable and fixed multipath.
8.3 Future Work

- The occlusion characteristics found on narrow band signals may be compensated to some degree by tracking prominent scatterers.

A number of practical implementations of this technique have been briefly considered as the conclusion to this work. A summary of these are as follows:

- Airport security: Monitoring of people could be carried out by channeling them through an arched doorway which is surrounded by narrow band radar sensors. By using K, Ka, V or W band radar sensors, the sensors may see through the person's clothing to detect hidden objects such as weapons.

- Ground penetrating radar: A moving CW, narrow band radar or phased array radar could be used to image objects underground.

- Multistatic battlefield radar: In this case it may be possible to image the outline of air or sea targets to enable rapid target identification.

Therefore, high resolution narrow band imaging may be extended to these applications and many more in future use.
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