Passive seasonally responsive thermal actuators for dynamic building envelopes

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Disclaimer
The concept for the thermal comparator mechanism described in Chapter 4 of this thesis was invented by Professor Stephen Gage at UCL in a disclosure dated 4th October 2007. It is the main claim of novelty in the co-authored patent application number GB 0803947.1 that was filed with the UK’s Intellectual Protection Office (IPO) on 3rd March 2008.

Declaration
I, Christopher Leung, confirm that the work presented in this thesis is my own. Where research work has been carried out in collaboration with other parties, this is clearly credited and this thesis has not been submitted either in the same or different form to this or any other University for the award of a degree. Where information has been derived from other sources, I confirm that this has been indicated in the thesis.

Signature of the Author .............................................................. Dated .........................

Industry sponsor
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Abstract

It is desirable for a glazed façade to have a variable performance to moderate a building’s energy-balance from seasonal variations in solar gains (Davies, 1981). A novel actuator mechanism is developed that self-regulates a façade by physically reconfiguring its movable elements into different environmental control functions. Its practical feasibility is explored with a proprietary thermal actuator that exploits the expansion of wax during melting from its absorption of heat energy. This provides a means to operate a mechanism solely by its response to passive energy exchanges given prevailing weather conditions.

An advance is needed for thermal actuators to respond differently to the seasonal intensity of sunlight. A means to overcome this obstacle is explored through the possibility of linking multiple heat-motors into thermal comparators. A methodology is adopted to iteratively develop virtual models of mechanisms and energy-flows, complemented by practical machines for testing in controlled climate-chambers and observation on uncontrolled sites.

The possibility of differentiating the energy-balance between actuators by amplifying seasonal differences in the daily intensity of sunlight under clear-skies is investigated using hot-boxes. Simulations predict the possibility of a well-differentiated annual repertoire of responses that reconfigures façade elements for daylight access, solar shading and night-time insulation. Practical thermal actuators in hot-boxes arranged with differentiated exposure to sunlight were observed through a cycle of seasons. The results demonstrate the feasibility of a seasonal difference in actuator responses from daily cycles of accumulated and rejected heat-energy.

The development of a passive means to seasonally marshal the state of wax in response to sunlight and use linked thermal actuators to mechanically express it has shifted the conceptual ground away from environmental control that is abstracted. Architects can move towards the design of dynamic building façades that can literally reconfigure in and of itself by virtue of the embodied state of its material components.
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The scheme for the finite element method used to model the diffusion of heat in wax and its phase-change was originally developed by Dr William R. Humphries at NASA’s MFSC and published in his Ph.D on the ‘Performance of finned thermal capacitors’. Data analysis and modelling of solar geometry presented in sections of this thesis have made use of an MicroSoft® Excel module that implements the USA’s NOAA (National Oceanographic and Atmospheric Administration) high-accuracy solar-position algorithm, this is with the kind permission of its author Mr Greg Pelletier at the Department of Ecology, State of Washington, USA.

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<tr>
<td>$A$</td>
<td>Area</td>
<td>$m^2$</td>
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<tr>
<td>$AR$</td>
<td>Aspect Ratio</td>
<td>n/a</td>
</tr>
<tr>
<td>$Bi$</td>
<td>Biot number (for the thermal geometry of an object)</td>
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<tr>
<td>$E$</td>
<td>Energy</td>
<td>joule $(J)$</td>
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<tr>
<td>$F$</td>
<td>Force</td>
<td>newton $(N)$</td>
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<td>Total global instantaneous radiation on a plane under clear-sky conditions</td>
<td>$W \cdot m^{-2}$</td>
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<td>Total global instantaneous radiation on a horizontal plane (as measured)</td>
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<td>$H_f$</td>
<td>Latent heat of fusion</td>
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<td>$I$</td>
<td>Logic input</td>
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<tr>
<td>$IQ$</td>
<td>Isoperimetric Quotient (metric for compactness)</td>
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<tr>
<td>$K$</td>
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<tr>
<td>$MF$</td>
<td>Melt-fraction (ratio of phase-change)</td>
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<tr>
<td>$P$</td>
<td>Pressure</td>
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<td>$Q$</td>
<td>Internal or output logic state</td>
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<tr>
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<tr>
<td>$VF$</td>
<td>View factor in radiant energy exchange</td>
<td>steradian</td>
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<td>$Y_i$</td>
<td>Daily insolation yield (measured vs clear-sky model)</td>
<td>%</td>
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**Lower case**

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<td>Thermal diffusivity</td>
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<td>$c$</td>
<td>Specific heat capacity</td>
<td>$J \cdot kg^{-1} \cdot K^{-1}$</td>
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<tr>
<td>$dn$</td>
<td>Day number</td>
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<tr>
<td>$e$</td>
<td>Mechanical actuation strain</td>
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</tr>
<tr>
<td>$g$</td>
<td>Acceleration of gravity (Earth=9.80616)</td>
<td>$m \cdot s^{-2}$</td>
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<tr>
<td>$h$</td>
<td>Free convection heat-transfer coefficient</td>
<td>$W \cdot m^{-1} \cdot K^{-1}$</td>
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<tr>
<td>$k$</td>
<td>Thermal conductivity</td>
<td>$W \cdot m^{-1} \cdot K^{-1}$</td>
</tr>
<tr>
<td>$l$</td>
<td>Length</td>
<td>metre ($m$)</td>
</tr>
<tr>
<td>$m$</td>
<td>Mass</td>
<td>kg</td>
</tr>
<tr>
<td>$n$</td>
<td>Refractive index of participating medium</td>
<td>unitless</td>
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<tr>
<td>$p$</td>
<td>Perimeter</td>
<td>metre ($m^2$)</td>
</tr>
<tr>
<td>$q''$</td>
<td>Rate of heat-transfer</td>
<td>watt ($W$)</td>
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<tr>
<td>$r$</td>
<td>Reflectivity</td>
<td>coefficient</td>
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<tr>
<td>$s$</td>
<td>Mechanical stress</td>
<td>pascal ($Pa$)</td>
</tr>
<tr>
<td>$t$</td>
<td>Time ((\dagger)) or time interval</td>
<td>second ($s$)</td>
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<tr>
<td>$w$</td>
<td>Wien’s displacement law constant (2.898 × 10(^{-3}))</td>
<td>$m \cdot K$</td>
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**Greek**

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<td>Surface absorption at short wavelength radiation</td>
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<td>$\alpha_{sol}$</td>
<td>Solar elevation (Altitude-Azimuth and terrestrial-centric)</td>
<td>degrees ((\degree))</td>
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<td>$\gamma$</td>
<td>Surface tension</td>
<td>$N \cdot m^{-1}$</td>
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<td>$\varepsilon$</td>
<td>Optical surface emissivity (1=perfect ‘Black body’)</td>
<td>coefficient</td>
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<tr>
<td>$\phi$</td>
<td>Solar azimuth (+clockwise from north)</td>
<td>degrees ((\degree))</td>
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<td>$\lambda$</td>
<td>Wavelength</td>
<td>metre ($m$)</td>
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<tr>
<td>$\theta$</td>
<td>Angle parameter</td>
<td>degrees ((\degree))</td>
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<tr>
<td>$\rho$</td>
<td>Density</td>
<td>$kg \cdot m^{-3}$</td>
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<td>$\sigma$</td>
<td>Stefan-Boltzman constant (5.67 × 10(^{-8}))</td>
<td>$W \cdot m^{-2} \cdot K^{-4}$</td>
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<tr>
<td>$\tau$</td>
<td>Transmittance of light through participating medium</td>
<td>coefficient</td>
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**Subscripts**

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<td>∗a</td>
<td>Air</td>
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<td>amb</td>
<td>Ambient</td>
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<td>Boiling point temperature (vaporization)</td>
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<tr>
<td>c</td>
<td>Heat-motor cylinder</td>
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<td>cp</td>
<td>Congealing point temperature (solidification)</td>
<td>degrees (°C)</td>
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<td>cond</td>
<td>Conduction mode (heat-transfer)</td>
<td>n/a</td>
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<td>conv</td>
<td>Convection mode (heat-transfer)</td>
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<td>Dry bulb temperature</td>
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<td>enc</td>
<td>Enclosure</td>
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<td>ext</td>
<td>External surface of heat-motor</td>
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<td>g</td>
<td>Glass</td>
<td>n/a</td>
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<td>int</td>
<td>Interior of heat-motor cavity</td>
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<td>l</td>
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<td>lat</td>
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<td>Long wavelength infra-red radiation (λp = 10 µm)</td>
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<td>mp</td>
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<td>degrees (°C)</td>
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<td>r</td>
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<td>Solar</td>
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<td>st</td>
<td>Stored (heat-energy)</td>
<td>joules (J)</td>
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<td>sto</td>
<td>Start-to-open temperature</td>
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<td>Short wavelength visible radiation (λp = 0.5 µm)</td>
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<tr>
<td>w</td>
<td>Wax</td>
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**Symbols**

- Duty-cycle (signal waveform) | n/a |
- Rising, and Falling-edge (signal waveforms) | n/a |

* Presentation of scientific and engineering data in accordance with (pp. v-vi in Taylor, 1995) and guidance in (Symbols, Committee, 1975). † Time-based data presented using conventions specified in ISO-8601
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<td>75xxxx</td>
<td>Richard Levine’s ‘Sundow’ insulated shutter</td>
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<td>Feb 2009</td>
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<td>235</td>
<td>090215</td>
<td>Testing a batch of thermal comparators (#303)</td>
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<td>235</td>
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<td>Testing a batch of thermal comparators (#30x)</td>
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Glossary of Terms

During the course of this research, some investigations have been carried out in fields outside the core discipline of architecture. Specifically, these include the field of environmental engineering and building physics, the field of physical chemistry and heat-transfer and the mechanical properties of materials. In respect of these disciplines, the report adopts the relevant terms and conventions that apply to each field.

Consequently some of the terms used may be unfamiliar because they fall outside the common vocabulary of the core discipline of architecture. Where this is the case and the main text does not adequately introduce a term or explain the relevant context of its use, the author apologises in advance. The following glossary tries to cover such terms with a brief description and explanation.

<table>
<thead>
<tr>
<th>Term</th>
<th>Description</th>
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<tbody>
<tr>
<td>Autonomic</td>
<td>The term “autonomic” was coined on (p. 18 in Wigginton and Harris, 2002) as a metaphor for an “intelligent skin” on buildings. It is an analogue of the human nervous system that exhibits involuntary responses to certain types of external stimuli. For example, mechanical forces, impacts or environmental conditions.</td>
</tr>
<tr>
<td>FEM</td>
<td><strong>Finite Element Method</strong> A mathematical method to model processes that occur in a material such as the distribution of temperature from heating or mechanical stress from applied force. A material is divided into discrete elements and the finite differences between the properties of each element is calculated. The technique is helpful to model processes in non-uniform fields such as those found in materials with low thermal conductivity and high latent heat.</td>
</tr>
<tr>
<td>FSM</td>
<td><strong>Finite State Machine</strong></td>
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continued on next page
This is a theoretical description of the state of a machine based on stimuli that determine each of the states it can have and the stimuli that determine its transition from one state to another. Typically, the behaviour of a system is described using a FSM where its component parts have binary states for which state-transitions can be described. FSMs are widely used to describe, model and simulate the operation of electronics and computers because they allow the systematic scrutiny and analysis of a system’s behaviour. For these reasons FSMs are used to describe the operation of the machines developed during this work. In common with real electronic devices a real heat-motor does not instantly transition between binary states and FSM theory does account for this. An analogy can be drawn between variable signal voltage and variable temperatures as the input stimuli to an FSM. FSM theory is described in (chap. 2 in Minsky, 1972) in the context of a theory of machines independently of its digital or analogue embodiment.

**Heat-motor**

The generic term ‘heat-motor’ is used here to refer to a device that converts thermal-to-mechanical energy. The wax-filled mechanical actuator studied in this work is an example of a thermally-activated device. In the literature, the same device is referred to by different names including “wax-actuator”, “paraffin-thermostat”, “heat-switch” and their permutations. The term “heat-motor” is used throughout this work on the basis that it accurately describes the heat energy that this actuating device provides a motor response.

**Insolation**

The energy gains absorbed from solar radiation (in Joules).

**Open-loop**

The term “open-loop” is used here to describe the operation of a system that responds passively by a free exchange of energy caused by a change in the prevailing conditions of its surroundings. The state and responsive behaviour of a “open-loop” system is determined by the inherent properties of its materials and their arrangement and this is without mechanical, electrical or other intervention. This is the behaviour of a system without feedback, unlike classical control theory (Chap. 4 in Weiner, 1948) where a processor (mechanical, electrical or otherwise) determines an error.
vector. This is based on the calculated difference between a desired goal state and the actual state measured by a sensing transducer in the form of feedback. In a “open-loop” system there may be neither a goal state that is declared using symbols nor sensor transducers from which an error vector could be calculated to give feedback to deterministically alter the behaviour of the system against its passive response to prevailing conditions.

**PCMs**  **Phase Change Material**
A generic term for materials that have properties of interest when they transition between states, both from solid-to-liquid and liquid-to-gas states. The term is used in the literature on environmental engineering to refer to materials such as hydrated salts and waxes including vegetable-based and paraffin-based wax. Almost every material will change phase given appropriate conditions, however what qualifies a material to be called a PCM in the context of an application it that operational conditions intentionally coincide with the material’s inherent thresholds of phase transformation.

**Photogrammetry**
Photogrammetry is a surveying technique that measures objects and scenes directly from photographs. It is based on reverse engineering the ‘perspective space’ that is projected through a camera’s lens. Dedicated software is used to process the ‘perspective space’ from two-dimensional photographs to produce three-dimensional vectors. This is the ‘photogrammetric solution’ of a particular object or scene. Photographs are taken using calibrated cameras with known lens properties. Software then calculates a photogrammetric solution from a single or set of suitable images. Photogrammetry is a non-contact measurement method, the advantage of using it for surveying ‘as built’ and operating systems is to produce high accuracy results without interference. Photogrammetric solutions are simple ASCII files that can be imported into computer aided design (CAD) software as three-dimensional vectors. Once the data-set is correctly datum coordinated, orientated and scaled these can verify ‘as designed’ against ‘as built’.
<table>
<thead>
<tr>
<th>Term</th>
<th>Description</th>
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<tbody>
<tr>
<td>Photonastic</td>
<td>A movement in response to light intensity. The term used in the field of plant physiology when describing the behaviour of a plant, typically referring to the movement of leaves during a relatively short interval of a diurnal cycle (After Chap. 4 in Hart, 1990).</td>
</tr>
<tr>
<td>Photoperiod</td>
<td>The interval between sun-rise and sun-set for a given day number ($dn$).</td>
</tr>
<tr>
<td>Phototropic</td>
<td>A movement in response to light direction. Another term from plant physiology used to describe the behaviour of a plant, typically referring to the growth of the plant as a whole over a relatively longer interval of a season (After Chap. 4 in Hart, 1990).</td>
</tr>
<tr>
<td>SEB</td>
<td>Surface Energy Balance</td>
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<tr>
<td></td>
<td>A mathematical model of the energy exchanges that a surface undergoes through the physical heat-transfer processes of conduction, convection and radiation with the surrounding environment.</td>
</tr>
<tr>
<td>$T_{mp}$</td>
<td>The “melting-point” on the cooling-curve of a wax is the only temperature at which the wax can be in both the liquid and crystalline state. This is the temperature that during the solidification process is associated with the isothermal when the latent heat of fusion is released. The “melting-point” is widely used in industry to characterise a particular wax. It is determined using a test method described in three technically equivalent standards; in the UK by BSI (1980) in the US by ASTM 87-74:IP 55/77 and Internationally by ISO 3841:1977.</td>
</tr>
<tr>
<td>VLT factor</td>
<td>Visible Light Transmission</td>
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<tr>
<td></td>
<td>The ratio of incident to transmitted light in the range of Photopic wavelengths where the human-eye is sensitive, between $\lambda = (0.40 \text{ to } 0.68) \mu m$ with a peak at ($\lambda_p = 0.53 \mu m$). The $VLT$ of glazing systems varies widely depending upon glass type, the number of layers and the type of hard or soft coatings applied to the surfaces. A typical value for single-glazing with low iron glass is at the high-end of ($VLT &gt; 0.9$). While for a very high performance triple-glazed system with engineered coatings, it can be as low as $VLT = (0.25 \text{ to } 0.3)$.</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

“The proper study of those who are concerned with the artificial is the way in which that adaptation of means to environments is brought about - and central to that is the process of design itself”.

From p. 132 in “The sciences of the artificial” by Simon (1981)

1.1 Thesis overview

Across the inhabited areas on Earth, our buildings experience the variability of local weather conditions. The amount of variation as well as its seasonal differences characterise the climate of a given location. While buildings and their occupants experience the prevailing weather in the 'here and now' within the time-scale of a day, for many locations including in the warm temperate climates there are differences in the range of diurnal variability across the seasonal time-scale.

The effect on buildings from the seasonal differences in diurnal conditions can be moderated by environmental design strategies, such as measures to adapt the form of a building’s envelope, its orientation and structure in what has been described as a ‘bioclimatic approach’ to architecture (Olgyay, 1963). A range of scientific methods and techniques for analysis have been developed to explore this approach, both empirically through practice as well as systematically by research, while drawing on principles that date back to classical antiquity (Vitruvius, first century BC).

Figure 1.1 illustrates the 'bioclimatic approach' diagrammatically, showing the amplitude of variation in the external climate being progressively dampened by a building’s structure and its envelope. With this approach, the impact of a successful passive environmental design strategy is characterised by dampening the amplitude of curve 2 to the smaller curve 3. The effect of this approach is taken to imply that the quality of a building’s internal environment could be improved as well as reducing the quantity of energy consumed to maintain it.
While a static structure and envelope fabric can be designed to have a beneficial impact, it is usually the case that additional measures are necessary to compensate for large external variations in order to maintain acceptable indoor conditions. Even in the ‘bioclimatic approach’, reconciling the difference between curve 3 and line 4 is achieved by active mechanical servicing.

In an age of resurgent public concern and political debate over what impacts there may be in future from the present rate that finite resources are being depleted to fuel the large proportion of energy consumed operating buildings, it is timely to study how to advance this approach.

The variation in the curves over a diurnal time-scale shows the need for both a positive and negative compensation, while making the present state of a building’s operation implicitly dependent upon the history of its past states. Together, these differences highlight the limitation of a static structure and building envelope to moderate the variability of prevailing weather conditions.

One approach to compensate for this difference is to dynamically adapt the building’s envelope in response to changes in the weather, by pursuing the technical means to integrate materials that have dynamic properties within the façade’s construction. The concept of the “polyvalent wall” by Davies (1981) described how this might be achieved at the micro-scale with glass construction. This concept is relevant for several reasons as it reappraised the propensity for architects in the modern movement to design with glass in buildings by recognising that in many cases its ubiquitous and extensive use had a large negative impact on the building’s environmental performance.

For all the advantages of its material properties, glazing is also arguably the most thermally permeable part of a building’s climate filter. However, the impact of extensive glazing on its energy-balance is climate dependent. Consider the example of the UK and the seasonal variation it experiences in a temperate climate with warm summers. The typical diurnal contribution made to a building’s energy-balance by a south-facing window oscillates seasonally, tending toward a net surplus on hot sunny days in summer, but tending toward a net deficit on overcast days in winter.

**Figure 1.1: The passive dampening effect of a static building structure on external climate**
The “polyvalent wall” acknowledges that a static building element has the limitation of a fixed performance while it is exposed to external conditions that continually change. This contrasts with the performance of a dynamic envelope that could be responsive to diurnal as well as seasonal climatic variations. Despite numerous advances in the technology for manufacturing and processing glass that have been made since the 1980’s when the concept for the “polyvalent wall” was proposed, it has yet to be fully realised. Nonetheless, the motives to pursue a means to respond to varying external conditions through the notion of a ‘wall for all seasons’ remains.

An alternative is to pursue the technical means for a dynamic façade at the macro-scale, by operating movable ventilation louvres, window shutters, blinds and openable windows. The challenges and questions raised by this approach can be illustrated with reference to a built example. Consider the façade of movable elements designed by Jean Prouvé for an apartment building in Paris completed in 1954. It has openable windows that slide vertically on rails in a balanced-hinge arrangement, while an outer panel slides independently and can be tilted outward. Figure 1.2 shows a portion of the south-eastern façade with these elements in a range of configurations.

This façade exemplifies well-integrated design in the sense that a movable element can serve a range of functions by rearrangement alone. The outer panel can be stowed to maximise daylight, or it can be partially deployed for privacy, fully raised and tilted to provide shade as well as adjusted in combination with the window to vary the free area available for ventilation.

However, in this case as with many others, the movable elements depend upon occupants to manually operate them. In addition, occupants need to be present and understand how the movable element is best deployed in response to given conditions. In practice, for many types of buildings, the frequency of occupancy may be low and there may be a lack of understanding about how best to configure the elements in a timely response to prevailing weather conditions.

Figure 1.2: The façade of Immeuble D’habitation square Mozart, Paris 1954

Modeled after measured drawings from pp. 334-5 in “Jean Prouvé Œuvre complète” by Sulzer (2005)
1.1. Thesis overview

In order to operate a movable building element in the absence of occupants or the lack of their timely intervention, some physical actuation force is usually necessary and this can be achieved in many ways such as using electro-mechanical motors. However, energy is expended and some controls are needed together with a connection to a power supply.

Marshalling the **means to respond** as distinct from **responding to different weather conditions with different responses** are usually separate challenges, the following elaborates on the distinction. To **actively** control a system to change the property of a material that affects the environment in a space requires a regenerative mode of intervention as described by Banham (1984). This is true for many materials at the nano, micro and macro-scales as well as for systems that control them.

Some examples include the electrical current needed to switch between the possible states of an electro-chromatic coating on glass or to power electro-mechanical motors and drives. Once the material is in the activated state, it is the case in some examples that a continuous energy expenditure is necessary to maintain its activation, for example electro-chromatic glass.

In order to respond differently to different stimuli, a computer is usually required to process signals from sensing transducers to control an actuation force determined by the operating logic that is programmed in firmware. For an automatic dynamic response to environmental stimuli, the material property that is altered is usually **actively** controlled as a cybernetic system encompassing sensing, processing, output and continuous feedback as described by Weiner (1948).

The range of strategies that are available to control an actuation force in this way using firmware are well-developed as surveyed in (Aström and Murray, 2009). However, an alternative would be to exploit the passive response to the surroundings that is inherent in a material’s properties in order to embody both the operating logic as well as provide the actuation force needed. An advancement in this area would allow a reconsideration of how the difference between curves 3 and 4 in Figure 1.1 might be reconciled *without* resorting to an active regenerative approach.

In order to explore this we turn to the heat-motor, a device that converts the phase transformation of a fusible material into usable mechanical force. The properties of this thermo-hydraulic device are given consideration as a candidate actuator technology.

The relation between temperature and the density of different materials as the basis for novel temperature-activated devices has been the subject of investigations since at least the turn of the last century. An early patent for the invention of a thermostatic valve that uses the principle of the heat-motor with wax was granted in the 1960’s to Mr Sergius Vernet (Vernet, 1961b,a).

However, to exploit this we then need a fusible material with suitable properties. The properties of polymer gels have also attracted attention because of the possibility that they can be synthesized to exhibit stimulus-response behaviour with their surrounding environment as described by Cohen Stuart et al. (2010). In both cases the responsive behaviour is due to changes at the nano-scale of molecules, their responsive behaviour becomes apparent at the macro-scale of a component when given a sufficient bulk of the material.
1.1. Thesis overview

It is at the macro-scale that the dynamic properties of these materials can be usefully converted into the magnitude of actuation force necessary for considering applications in the built environment to operate movable building elements. The material is switched at the phase-change threshold by the exchange of energy with the surroundings. When a wax or polymer gel undergoes phase-transition its useful engineering properties become apparent, hence the literature often refers to them as Phase-Change Materials or PCMs.

Most applications of polymer gels are focused on biotechnological applications at the nano-scale and micro-scale. However, some polymer gels are scalable so larger macro-scale passive devices are possible. State-of-the-Art examples of polymer gel actuators make use of the ‘Maxwell stress’ generated in a dielectric elastomer material when they are electrically activated for applications in robotics as surveyed by Bar-Cohen et al. (1999) and more recently by Ashley (2008).

We find in a historical account that waxes have been used in many applications long before the advent of synthetic polymers because it was relatively easy to extract and refine them from oils and vegetable matter that are widely available, as described by Knaggs (1947). This consideration prompts further curiosity about the suitability of wax as a PCM in this application context.

The property of wax that is altered in response to temperature is its specific volume with a positive coefficient of expansion, it expands in both solid and liquid phases when heated and substantially so during melting. Investigations into wax’s mechanical properties have shown that it can withstand pressures of 1 GPa in a study by Nelson et al. (1960), and the capacity of a wax heat-motor to achieve a blocking force of 50 MPa has been shown by Long and Loveday (2007).

Buildings are expected to continue performing over long time-scales in the order of many decades, this obviously puts a demand on the reliability and longevity of the components that are used. The thermo-hydraulic properties of waxes are competitive with other shape memory materials, they do not appear to suffer from the stress fatigue associated with solid-to-solid (martensitic to austenitic) phase-transitions that occur in metallic Shape Memory Alloys (SMAs) that limits their longevity as shown by Lagoudas et al. (2009).

Evidence for the thermo-physical stability of waxes have been shown by Sharma et al. (1999) from accelerated endurance tests of phase-change, the endurance of heat-motor assemblies themselves have been estimated by Tibbitts (1988) as >50,000 duty-cycles. This suggest that heat-motors would provide a satisfactory endurance given that the MBFR (Mean Between Failure Rate) expected for motors used in buildings is 30,000 cycles as reported in Bordass et al. (2000).

Given these physical properties, waxes appear to be a suitable candidate for the fusible material in heat-motors for operating movable building scale elements and a credible alternative to electro-mechanical or hydraulic actuators over the design life of a building.

The advantages of using wax in heat-motors has been described in previous studies by Tibbitts (1988), however these have been electro-thermally activated under closed-loop control. Heat-motors can also be operated passively, this has the advantage of low-power because the wax
undergoes phase transformation by a free exchange of thermal energy with the ambient surroundings at no cost. Once a wax has changed phase it remains in that state so long as conditions of thermal equilibrium persist, this involves no additional continuous energy expenditure.

The behaviour of a passive heat-motor due to the phase-change of wax can be described using the metaphor of an autonomic response, this is drawn from the somatic and autonomic responses that are exhibited by the human body and its nervous system. This metaphor serves to distinguish between active control that is directed and a passive responsive that is reactive, as described by Wigginton and Harris (2002). In the case of the human body’s nervous system, an example of an autonomic response is the involuntary muscle movement in the lower leg in response to contact stimuli on the centre of the knee.

The output state of a heat-motor can be considered as an autonomic response in the sense that the phase of wax is wholly dependent on its specific energy balance. When wax has a sensible temperature above its melting-point it is in its liquid phase or if it is below the melting-point it is in its solid phase. The implication for the output of a heat-motor in this application context is that there are only two stable states to consider.

The responsiveness of a heat-motor will determine the latency of the rising-edge and falling-edge during transitions between the solid and liquid phases. If these have a non-trivial duration then the latency of the state-transitions themselves may need to be characterised.

There are relatively few cases in which passive changes to a material’s intrinsic properties in response to its surroundings have been used as the basis for a substantial change in the geometry of an element in a building. One example is to exploit the response of layers of wood ply to the amount of moisture that is held in the surrounding air, as investigated by Reyssat and Mahadevan (2009) based on studies of the hygromorphic behaviour in pine cones by Dawson et al. (1997).

An example of this includes an indoor screen constructed of lapped plywood shingles that demonstrated how the screen’s geometry can be “warped” at the macro-scale in response to the ambient level of humidity as shown by Khan et al. (2008). Similar properties have been demonstrated in the “Reactive surface structure” project as described in Schumacher et al. (2010).

We can approach the design of passively actuated elements in different ways, one existing approach is to design a material’s response by manipulating its intrinsic properties during the process of its manufacture. Methods applied to timber, a variety of metals and composites to widen the palette of materials with dynamic properties that can be selectively activated have been suggested by Kolarevic and Klinger (2008). However, few examples using this approach have been shown to be dynamic at the macro-scale with a passive response to the range of variation in environmental conditions that are typical at the outside envelope of buildings.

For the architects and designers that adopt this alternative approach, it represents a recognition that the embodied response of materials to their surroundings is inseparable from the environment in which they will operate.
1.1. Thesis overview

“This approach fundamentally challenges the static nature of these industrialized materials and sensitizes them to the ephemeral and dynamic qualities of the environments in which they are fabricated and eventually deployed.”

Preface on p. 2 in “Reflexive architecture machines” by Khan et al. (2008)

This is a relationship that is evident in the variety of physiological behaviours that biological organisms exhibit in response to the dynamic changes they face in their habitats. An example is the alternation between phototropic and photonastic behaviour exhibited by the leaves of the Oxalis triangularis plant and other species in the Oxalidaceae genus as described on pp. 103-4 by Hart (1990). The viability of marshalling similar behaviours in an artificial system by passive thermal exchanges is encouraged by a premise in the biophysical study of living systems that:

“the flow of energy through a system acts to organise that system”

From p. 2 of Chap. 1 in “Biology and thermal physics” by Morowitz (1968)

All these examples exhibit passive responses, where the adjective use of the word ‘passive’ is qualified as the free exchange of energy between the actuating material and its surrounding environment without resistance or forced intervention. The passive exchange of heat-energy follows the thermodynamic principle that a system will tend toward a thermal equilibrium.

This also applies to the passive exchange of mechanical energy in a mechanism where the actuator’s strain does work against a biasing force. The mechanical energy stored in a mechanism follows the Law of Conservation of Energy and will tend toward a ‘mechanical equilibrium’, while representing the logical ‘state’ of the mechanism at a point in time.

The quote from Khan et al. (2008) describes a ‘reflexive architecture’ in which the materials themselves are the reconfigured component, but it could go further to describe the approach of designing with functional materials that are capable of conditional responses to their surroundings. A key distinction is that while both remain reactive to changes in the environmental stimuli to which they are sensitive, it is the combined reactive properties of two (or more) materials that generates a repertoire of responses.

Recalling the ‘wall for all seasons’ at the centre of the “polyvalent wall” concept, we are reminded that it is desirable to move shades and shutters over windows to different positions depending upon the seasonal variation of outdoor conditions. When occupants are not present or when they are unwilling to operate a shade or shutter into the appropriate position then we rely on an automatic response. In order to marshal the advantages of a passive heat-motor we need to find a means for them to distinguish between seasons with the requisite variety.

If we look at existing applications of passive heat-motors for regulating the temperature in conservatories and greenhouses by operating windows and ventilators, then conceptually the heat-motor emulates the desired state in response to their shared exposure to the prevailing conditions.
Ideally, the heat-motor’s exposure to the prevailing environmental stimuli in the form of air temperature and the heating effect from insolation are representative of how the effects of air temperature and sunlight suit the thermal conditions of a greenhouse.

In the case of the greenhouse, the heat-motor’s output affects conditions inside the greenhouse by admitting air at the ambient external temperature. As such, this also affects the conditions experienced by the heat-motor and embodies a thermal model with negative feedback because the response of the heat-motor does something to change its exposure to the stimuli, in this case to reduce it. Experimentation with miniature greenhouses or ‘hot-boxes’ have an established history as surveyed in Chap. 5 by Butti and Perlin (1981) leading to the concept that they can serve as a model of the Earth’s atmosphere itself, as described by Aarhenius (1908).

The approach to considering the heat-motor placed in a greenhouse as in itself a thermal model of a greenhouse has some similarities with “Behaviour-based robotics”, a branch of robotics surveyed by Arkin (2000). The paradigm put forward by the “Behaviour-based” roboticists is predicated on robust low-level stimulus-response behaviour between a machine and a given environment as proposed by Brooks (1991). In this way, robots can successfully operate in complex and unpredictable environments with an embodied logic instead of relying on a centralised representation using symbols and arbitrated control.

To reconfigure building components using passive heat-motors has similarities with “Behaviour-based” robotics in another way because both use low-level stimulus-response devices that lead to a close-coupling between perception and action. Yet, neither have a centralised representation of the system that marshals an arbitrated form of control. The heat-motor device has something else in common with many robots that have been built by the “Behaviour-based” movement, they are only operational when they are physically located in their application environment, and subject to its forces. Heat-motors are perpetually embodied and have no ‘On/Off’ switch.

The practical challenges faced by “Behaviour-based” roboticists to model and simulate their robots solely in-silica has been outlined as due to the limitations of software tools and the data that is available to construct high-efficacy representations of either the robot’s behaviour or the environment within which they operate, on pp. 274-6 by Pfeifer and Scheier (1999). The close coupling between perception and action at the centre of the “Behaviour-based” paradigm also implies that there is a sensitivity between the two. This is one reason that the practitioners in that field have been motivated to invest resources into building their robots in the real-world where they can rigorously test their behaviour through observation. This is to mitigate against the simplifications that are made by a model’s assumptions, as well as the uncertainties that are inherent in the data used both for generating a simulation environment and its run-time scenarios.

In the field of Architecture and Environmental Engineering, similar challenges are faced when modelling and simulating the performance of buildings as described by Wit (2003). This is expected to be particularly acute in cases that employ passive measures due to uncertainties in material
properties and the availability of meteorological data. It is anticipated that to explore the heat-motor as a thermal model of an enclosure solely in-silica would inherit these limitations.

The efficacy of predicting the behaviour of a heat-motor by harvesting solar radiation will be perturbed by the amount and frequency of cloud turbidity. Working on the assumption that the variability of cloud cover is randomly distributed, which is the common experience for the British Isles then by definition turbidity cannot be predictably characterised. This may result in difficulties predicting the behaviour of a system operated by heat-motors for every type of weather condition. However, one helpful exception is when atmospheric conditions approximate those under a clear-sky and this applies both during daytime and at night.

The electro-chromic glass described earlier is one example of a system with actively directed control, there is no change in the material properties of the coating unless and until the controller’s firmware has determined that the conditions that are necessary for change have been satisfied. Only then will the material properties be changed by inputting the additional energy needed to marshal it. This is fundamentally different from a passive system because at no point does the material properties of the glass substrate or the applied coating change its state in response to the prevailing air temperature or the intensity of solar radiation. The materials themselves have no passive response to their environment that forms a union or intersection with the set of conditions that the controller is using to determine the desired state for the coating.

The key differences between the conventional paradigm of a control system and an autonomic stimulus-response system are compared using the schematics shown in Figure 1.3 (a) and (b) respectively. In each case, the flow of information is shown by the black arrows and the notional flow of energy is shown by the green arrows.

**Figure 1.3: Conventional control compared with an embodied autonomic paradigm**

(a) Conventional control paradigm after p. 74 in Schumacher et al. (2010)

(b) The embodied autonomic 'stimulus-response' paradigm
In a conventional control system the flow of information is linear with the data obtained from sensors symbolically encoded as input, shown flowing from left-to-right. The control system relies on a high-fidelity abstract symbolic representation of the environment with which to compare the input. It then uses strategies to manipulate it in order to arbitrate control over the actuator system. It is connected to the physical environment ‘out there’ only at the start and end through feedback, this leads to a separation between the flow of information and the flow of energy. By contrast, the heat-motor in the autonomic responsive paradigm is both the input sensor and the representation of the output state. The continuous embodiment of the heat-motor in the physical environment as a thermal model of the desired output state means it is situated ‘out there’ at all levels. The flow of information is circular and closely mapped to the flow of energy.

Automatic systems are not generally autonomous because they require a connection back to a power supply as well as some interconnection between components for communications and control. Although this depends upon where the boundary around the system is drawn, in practical terms for this application context some tethering back into the building is needed as well as coordination between structure and services. By contrast, an autonomic system using heat-motors that are ‘always on’ suggests a construction element that could be completely self-contained without the need for external power or coordination with piped or wired building services.

One of the advantages for the designer of a conventional automatic system is the flexibility to reprogram the controller’s firmware as required, with either little or no change to the underlying hardware. By contrast, the challenge for the designer of an autonomic system using the terms proposed here is to manage without making arbitrary separations between functional behaviour and physical embodiment. Since the inherent thermo-physical properties of wax cannot be selectively deactivating independently of prevailing thermal conditions, one implication is that there are restrictions to how an occupant override might be incorporated.

These comparisons highlight the differences in each approach for how a variable that is used to describe a dynamic system’s ‘field’ of possible behaviours can be distinguished from the values used for parameters that constrain them, as defined in Chap. 2 and 6 respectively by Ashby (1960).

The external variables are common to both approaches, as collected by the automatic measuring system and the autonomic device’s exposure to the ambient environment as shown in Figure 1.3(a) and (b) respectively. However, it is the internal variables and parameters that are different because in a conventional automatic system they are decomposed into each of the four boxes in Figure 1.3(a) as abstractions in the controller’s firmware, whereas in the autonomic approach they are embodied in the coupling between the time-dependent response of the material properties and its geometric arrangement to changes in the external variables.

The conventional automatic approach enjoys an advantage from the possibility to design the firmware with the scope to reprogram itself based on an internal assessment of performance feedback that it collects during use. Through a learning-algorithm it could adapt its response both
1.1. Thesis overview

to the uncertainties as well as changes in its operating context that were unforeseeable at design-time. In practice, this may be limited to refining the value of parameters that relate the system’s state-determined behaviour with its response to its environment, such as raising or lowering a temperature threshold. Although the scope to enjoy this flexibility could extend to totally recasting the set of variables and parameters that are used to describe and control a system’s behaviour.

By contrast, in the autonomic approach each stimulus-response device has a mechanical arrangement that is assumed to be fixed at design-time with no or only a limited scope for adjustment and rearrangement once installed. These considerations form the terms of engagement for a designer’s discipline to construct instances of this ‘class of machine’.

The fact remains that a two-state heat-motor has only one threshold, in a greenhouse it can do nothing more than drive open the ventilators to the maximum free-area when the maximum desirable temperature is exceeded. If the wax that powers a heat-motor is treated as a functional material then combinations of heat-motors might provide a passive means to distinguish between different thermal conditions. The heat-motor has received little attention in the literature for its potential to perform evaluations of its thermal environment by varying the exposure of heat-motors to the prevailing weather conditions. There would be an advancement if the repertoire of responses using heat-motors could be varied and shown to match the seasonal variation in outdoor conditions.

We can experiment with the amplification and differentiation effects of sunlight found in the greenhouse environment in a miniaturised form using a hot-box. Historically these were used as an indicator of the energy gained from solar radiation or ‘insolation’, Samuel Langley used one in the 1880’s amongst a suite of instruments during his quest to establish a value for the extraterrestrial solar constant. He observed that the attainable temperature was raised substantially above ambient due to the heating effect of sunlight. He reasoned that an object in a hot-box had a thermal response like the Earth’s surface because of the similarities between the spectrally-selective behaviour of the box’s glass cover and the composition of gases in the atmosphere (Langley, 1884).

The observations he made during an expedition to ascend Mount Whitney showed a proportional relationship of increased heating effect with higher altitude. Attainable temperature could be used as a representation of location in the sense that the effect of insolation on the air trapped inside the hot-box is always dependent on the ambient of a given location.

Based on this well-established finding, the general proposition is that the heating effect in a hot-box could be used to represent the different seasonal effects of insolation at a given location. This proposition could be developed if we can be more specific about exactly what attainable temperature relates the level of insolation with the prevailing ambient for a given site. We could declare a specific temperature on a continuous scale, but in many ways this is arbitrary. An alternative is to define binary ‘states’ either side of a discontinuity on the temperature scale.

It would be feasible to investigate the latter by replacing the modest expansion of mercury over a wide temperature range inside a thermometer with the large expansion of wax over a narrow
temperature range inside a heat-motor, using the motor’s output to indicate the level of insolation. If a passive actuator system can be constructed to successfully exploit this potential, it may be preferable to using an active system to operate the same movable building element.

There is sufficient thermal energy to passively activate a heat-motor by simply exposing it to direct solar radiation under a clear-sky. To match the maximum stress-strain of a heat-motor by using an autonomous electro-mechanical drive system would likely require a high-current supply. The photovoltaic cells needed to meet the instantaneous demand may prove both large and expensive. A robust solution is to store electrical charge in a battery from energy collected by a photovoltaic cell and then power the motor using the battery. Some controls are necessary, in order to switch power from the battery as well as to control its charging cycles. This is inefficient, all these items add cost, add complication and each component has its own longevity and failure rate. Rechargeable batteries are expensive and contain corrosive chemicals and polluting materials, they have limited charge-discharge lives, and if they receive no maintenance their lives are shortened e.g. sulphur crystallisation forming on the electrodes of Pb-acid cells.

In summary, the thesis pursues the premise that buildings can benefit from an envelope with dynamic elements that moderate the exposure of the building to the seasonal variation of external conditions. The thesis topic is to explore the possibility that a novel passive technology may be used as both the means and the method to position movable building elements in a variety of arrangements in response to different prevailing conditions. Having introduced the research topic and its concepts, the problem that the thesis addresses is specified next followed by an outline of the steps that the thesis will take to address them.

1.2 Problem specification

The duty-cycle of a single heat-motor is limited by the single threshold of the wax charge used. Effectively, it has only one threshold of phase transformation. This is a limitation for a heat-motor to operate a movable building element because it is desirable for a façade to respond to the seasonal variations in external conditions by adopting different positions during daytime. It would be an advancement if this limitation were overcome with a means and an operating rationale that provided an appropriate response that is conditional on the seasonal variation of insolation.

1.3 Research question

Can the heat-motor and hot-box arrangement be used to represent the effect of insolation given local conditions as the indicator of ‘state’? This question can be cast in the language of equilibrium physics: To what extent does the seasonal variation in the passive transfers of heat-energy available from insolation perturb the phase-state of heat-motors compared with ambient? The answer to this question characterises how the output of a heat-motor can be used to marshal the desired responses in a building’s façade depending on the prevailing level of solar radiation.
1.4 Aims and objectives

To advance the benefits of a heat-motor, can multiple heat-motors be linked and still inherit the advantages of large load capacity, low power and robustness in order to explore this possibility? Can a mechanism be devised that is operated by heat-motors to represent different ‘states’ that advance the problem specification?

Adopting the autonomic stimulus-response behaviour as the alternative to the conventional control system paradigm, can the output of heat-motors be effectively marshalled by selectively exposing them to differences in the seasonal level of insolation? If mechanisms operated by multiple heat-motors are feasible, can the tendency toward thermal and mechanical equilibrium in such a mechanism be selectively exploited to perform useful work?

1.4 Aims and objectives

The pursuit of answers to the research question is the basis for the aims and objectives of the thesis, they determine the experiments and observational studies that need to be conducted in order to build the knowledge that is necessary to practice the design of mechanisms to address the problem specification.

In order to establish whether it is practical for mechanically linked heat-motors to operate movable building components, practical work will be carried out to construct and test this proposition. To achieve this will raise design and construction issues that fall outside the central thesis question, where relevant these have reported in the Appendices.

The underlying physical phenomenon that determines the duty-cycle behaviour of a passive heat-motor is the thermo-mechanical behaviour of wax during phase-change. This is a non-linear phenomenon and presents problems for exploring the application of heat-motors. Simple linear ‘Steady state’ models of heat-transfer may approximate their behaviour, however there is a lack of models to realistically characterise the non-linear thermo-mechanical behaviour of the actuator.

Given the uncertainties that are inherent in attempting to model and simulate the behaviour of heat-motors solely in-silica, the thesis aims to establish some working values for the basic parameters that determine the physical and thermal behaviour of a heat-motor actuator. In pursuit of this, empirical work is done using a proprietary heat-motor obtained from industry. It will be subjected to testing under laboratory conditions as well as realistic outdoor conditions. The thesis aims to use this data to ground the development of a model and its simulation.

Given that cloud turbidity is stochastic, its effect on the daily accumulation of insolation makes a high-efficacy prediction for the dynamics in an energy-balance uncertain. However, if we restrict our consideration to only days with clear skies then we might be able to negate many of these effects and explore the behaviour of heat-motors in both virtual and physical form.

A case-study building will be constructed to test whether multi heat-motor mechanisms can operate moderately sized movable building elements. This is used to explore the operating properties of a thermally-activated device with a repertoire of responses in this application.
1.5 Outline of the thesis structure

Chapter 2 Literature review
Following the introduction, the Literature Review in Chapter 2 presents a survey of existing heat-motor technology. A detailed review is presented of the relevant properties and characteristic behaviour of waxes suitable for heat-motors. A survey is presented of movable building elements that adopt different positions in response to seasonal variations in external conditions. The Literature review concludes with a summary of the advantages and disadvantages of heat-motors.

Chapter 3 Passive thermal capacitance in heat-motors
An in-depth study of heat-motors using a highly refined wax is conducted. The design of experiments to determine the physical properties of a proprietary heat-motor is presented. The findings of these controlled experiments is reported.

Chapter 4 Development of a thermal comparator
This chapter explores advancing the limitation of a single response from one heat-motor using a plurality of heat-motors. The design of a mechanism to differentially link plural heat-motors to operate a resultant output is presented.

Chapter 5 Comparator response: A seasonal output
A thermal design rationale is developed based on the conditional exposure to seasonal insolation.

Chapter 6 Characterising diurnal duty-cycles
A model is developed to simulate the evolution of heat-diffusion through the hot-box and the heat-motor cylinder to couple the transfer of heat energy by passive gains with the thermo-hydraulic expansion of wax at phase change that drives the heat-motor’s stroke.

Chapter 7 Results from an in-situ observation study
An observation study of a practical system is conducted, the results are presented.

Chapter 8 Analysis and discussion
The thermal design rationale is subjected to axiomatic tests and the efficacy of the simulation is analysed by comparison with the results of the observation study. A discussion considers the limitations of the thermal design rationale in the context of passive energy transfers between the practical embodiment and a realistic test-site.

Chapter 9 Conclusions
The conclusions summarise the findings of this research and the prospects for application to industry. Based on the findings of this work, areas of investigation that could be developed further are suggested and proposals outlined.

Bibliography and Appendixes References are cited throughout the text in the Harvard style in accordance with BS 5605:1990 (BSI, 1990) and BS 1629:1989 (BSI, 1989) with the full details listed alphabetically in the bibliography. The Appendices contain supporting information to the thesis, including detailed tabulations of data. Articles and papers that have been published during the course of the research project are also reproduced.
Chapter 2

Literature Review

Firstly, a brief ‘how it works’ explanation is given for the passive operation of a heat-motor actuator. This establishes the basic terms of reference and these will be referred to throughout the body of text. A detailed review of a selection of existing examples that have been obtained from industry is used to contrast the range of operating parameters and mechanical designs.

The most widely used working fluid in heat-motors are paraffin waxes, the thermo-mechanical properties of this material is reviewed in detail. In particular, the relevant characteristics of wax’s behaviour during the phase transformations between solid and liquid state are described.

A selection of applications are then reviewed where both passive and actively-controlled heat-motors have been used in different ways. These illustrates how both types can be applied to operate thermally regulating systems. These examples characterise the differences between a heat-motor that is a passive device driving an ‘open-loop’ system from a heat-motor that is an actively controlled actuator within a ‘closed-loop’ system.

None of the applications surveyed offer a conditional response to their thermal environment, one that might be afforded by linking two or more heat-motors together to form a mechanism that can perform evaluations of thermal conditions. Selected use-cases for heat-motors to operate building components are presented. These are examples of movable façade elements that serve multiple functions and they are reviewed as candidates for exploring this possibility.

The specific use-case of movable shutters is reviewed in further detail and the advantages of different shutter arrangements are summarised. The literature review concludes with a summary of the advantages and disadvantages of passive heat-motors.
2.1 Heat-motor technology

Having introduced the heat-motor earlier, the following sections draw on examples of proprietary heat-motors from industry for a more detailed review of how they operate and what their characteristics they exhibit. The development of heat-motor technology for commercial applications has been carried out for window openers by Cole (1984), ventilators in greenhouses by Bayliss (2010b), ventilators for air-conditioning by Orbesen (1993), valve control in heating systems by Danfoss (2006) and a variety of actuator mechanisms by Tibbitts (1988). Heat-motor technology has also been investigated for applications in mining and heavy industry by Long and Loveday (2007). It has also been investigated in a miniaturised form by Lee and Lucyszyn (2006) and at the micro-scale by Klintberg (2002) as well as Lehto (2007); Roger (2008) for use in lab-on-a-chip medical devices.

Since a heat-motor converts the expansion of wax into mechanical work, the basic design is determined by how the wax is contained and how its expansion is accommodated. The majority of heat-motors developed by industry are based on versions of a “cylinder and stroke rod” design as shown indicatively in Figure 2.1. These consist of a hollow rigid body that is strong enough to contain the hydrostatic forces that are generated by the expansion of wax, a stroke rod partially contained within the cavity is then proportionally displaced by the change in volume of the wax. Different versions of this design are distinguished by the details of how the change in volume of the wax inside the rigid body is sealed from escaping. Various examples have been disclosed in patent specifications granted for hermetic containment that is either flexible (p. 25 in Vernet, 1946) or an inflexible type (p. 1 in Cole, 1980). When the wax expands the heat-motor performs an actuation duty-cycle and does work against the operating force of a transfer-mechanism. The output of the transfer-mechanism then does work against the working load. The operating force consists of the working load and a biasing force, the latter is needed for the actuator to perform a recovery stroke after an actuation stroke in the absence of a working load.

**Figure 2.1: Section drawings through a “cylinder and stroke-rod” actuator**

Note: General arrangement cross-sections based on the Bayliss Superpower-tube thermal actuator with internal and external fins omitted for clarity.
2.1. Heat-motor technology

It is reported in (p. 214 by Duerig, 1990) that the magnitude of the biasing force in a wax actuator is typically (20 to 30) % of the operating force. It can be provided in a variety of ways, if the dead load of the components has sufficient mass then a gravity balance is the simplest. Otherwise, various compression, extension or nitrogen gas springs have been used.

In all these heat-motors, wax is used as the fusible material because of its large expansion of between (10 to 20) % by volume when it undergoes phase-change between solid and liquid states. A survey carried out by Long and Loveday (2007) compared the performance of a wax actuator with a range of other types of actuators surveyed earlier by Huber et al. (1997). A summary of their findings is shown in Figure 2.2 with the profile of paraffin wax actuators highlighted.

In Figure 2.2 the actuation strain \( e \) is a measure of the displacement length that is achieved during a stroke against a working load when compared with the initial length by \( ((1 + e) \cdot l) \) where \( l \) is the initial length (from p. 2187 in Huber et al., 1997). The actuation stress \( \sigma \) is a measure of the force per unit cross-sectional area of the actuator. Based on these variables, the position of paraffin wax in the upper right-hand corner suggests that it is suitable for applications that require both a large displacement and the capacity to withstand the stress of large loads.

Each actuator has its own stress-strain profile, for a proprietary heat-motor for use in practical tests this imposes a limit on the maximum stress and maximum strain that can be applied. This does not necessarily indicate the maximum limitations of heat-motor technology itself on the basis that a specific device may fail under forces that the wax itself could withstand. The thermo-mechanical properties of wax will be discussed in more detail in a later section.

**Figure 2.2: The strain-stress profile of wax actuators compared with alternatives**

Figure removed due to third-party copyright

Source: From graph on (p. 101 in Long and Loveday, 2007)
2.1. Heat-motor technology

2.1.1 Overview of a passive heat-motor’s operation

A heat-motor operates by absorbing sufficient thermal energy to melt the wax charge. The expansion of wax provides the mechanical work that extends a stroke-rod. Wax phase depends on the balance of the wax’s specific enthalpy at a given point in time. The processes during a heat-motor’s duty-cycle are described based on the transfer of heat energy between the surrounding environment and the heat-motor dominated by passive convection with air.

A duty-cycle is shown in Figure 2.3 as a sequence of cross-sections through a heat-motor, a corresponding temperature-stroke plot is shown in Figure 2.4. Initially the heat-motor is in thermal equilibrium with its surroundings at a temperature below the melting-point of the wax. The wax is solid with a high density and the heat-motor’s output stroke-rod is retracted. The temperature around the heat-motor begins to rise, heat energy is transferred through the heat-motor’s walls to the wax inside. A temperature is reached when heat energy continues to be transferred but the wax temperature is constant, this is associated with the start of latent heat of fusion being transferred to the wax. During the melting process the wax’s density decreases. The expansion of the wax forces the heat-motor’s stroke-rod to extend. At this point, mechanical work will start to be done against any mechanical resistance to the extension of the stroke-rod.

If the temperature surrounding the heat-motor is greater than the wax’s melting-point then heat energy will be transferred through the cylinder to the wax which continues to melt and expand. The expansion of the wax extends the stroke-rod and where resisted more mechanical work will be done. The temperature of the wax during the melting process depends upon the purity of the wax used (Tibbitts, 1988) and the pressure under which it is held (Zoller and Walsh, 1995). High purity wax melts in a narrow temperature range, whereas blends of waxes and low purity wax have been shown by Templin (1956); Lehto (2007); He et al. (2004) to expand over a temperature range. When the latent heat of fusion has been transferred to the wax, expansion drops abruptly. Liquid wax heated above the melt-point will continue to expand, however Grosse and Egloff (1938) showed that the coefficient of linear expansion is much less than the expansion during phase-change.

If the temperature surrounding the heat-motor starts to decrease, heat energy is transferred away from the wax through the heat-motor’s cylinder. A temperature will be reached when heat energy continues to be transferred to the wax but its temperature is isothermal, this is associated with the start of latent heat of fusion being released. The wax starts to crystallise into a structure where wax molecules change from amorphous to a more compact lattice form. The wax starts to solidify and its density begins to increase, this is the melting-point on the cooling curve. At this point a mechanical force is needed to push the stroke-rod back into the cylinder body while the wax is contracting. The force needed to do this can be applied in a variety of ways and typically a mechanical spring or a dead-weight. If no force is applied then in most practical heat-motors the frictional resistance of the seal alone is usually sufficient to hold the stroke-rod at its extended position despite the wax contracting behind it.
Figure 2.3: The sequence of processes during the duty-cycle of a heat-motor

1. Temperature of wax < melting-point
2. Temperature of wax reaches melting-point and expands
3. Wax absorbs Latent heat of fusion
4. Wax at melting-point and starts to release Latent heat of fusion
5. Temperature of wax at melting-point with Latent heat of fusion released

Figure 2.4: The characteristic temperature vs stroke profile of a wax actuator

Figure removed due to third-party copyright

Source: The figure below from (p. 215 in Duerig, 1990)
If the temperature surrounding the heat-motor continues to fall below the melting-point of the wax then it continues to solidify and density increases. A highly refined \( n \)-paraffin wax will solidify at the isothermal temperature. Its exact characteristics are determined by the process of crystallisation, a complex subject area that is beyond the scope of this thesis. A temperature is reached when the rate of density change against temperature slows down abruptly, this is associated with transferring all the latent heat of fusion from the wax. The stroke-rod is then fully retracted.

### 2.1.2 Actively operated heat-motors

If a heat-motor needs to be operated when the surrounding environment is colder than the melting point of the wax, then a heating element can be used to raise the temperature of the wax using a closed-loop control circuit. There are examples of commercial heat-motors that control the heating of the wax in this way. Actively operated heat-motors have been developed for controlling valves in heating systems by Danfoss (2006), actuator assemblies in spacecraft by Tibbitts (1992) and actuators for ventilating buildings for livestock by Brooks et al. (1986). A wide range of AC or DC powered electro-thermal heaters are used to provide the internal heating. A patent specification by Flemming (1978) disclosed the design of a system that heats the wax with duty-cycle controlled pulses and then uses mechanical feedback from the extension length of the stroke rod to control the wax’s expansion. Typically, the heat-motor will consume \((2 \text{ to } 20)\) W of power. However, this depends upon the heat-motor’s design, the thermal characteristics of its operating environment and the type of wax used.

If a heat-motor needs to be deactivated when the surrounding environment is warmer than the melting point of the wax then a heat-pump can be used to lower the temperature of the wax under closed-loop control. A patent specification by Schneider and Mentor (1989) disclosed the invention of a heat-motor controlled in this way using a thermo-electric “Peltier-effect” device that can both heat and cool. However, the searches carried out found no commercial heat-motor that uses this principle. Most practical actively controlled heat-motors use wax with a melting-point above the maximum temperature that is anticipated in its operating environment. In this case the rate of cooling is determined by the power dissipation through the heat-motor’s body just as a passively operated device. Although the temperature of the wax in its melted state in these devices is going to be higher than the temperature of the operating environment, the bandwidth of deactivation depends upon the rate of passive power dissipation to the ambient surroundings.

### 2.1.3 Examples of proprietary heat-motors

In the mechanical engineering industry, heat-motors have been used in thermostatic valves. These can be specified to activate in the temperature range of \((-10 \text{ to } 150)\) °C to provide displacement lengths of \((0.2 \text{ to } 12.7)\) mm when operating against forces in the \((90 \text{ to } 320)\) N range. The heat-motor cylinder is fabricated from a very good thermal conductor to achieve low latency when used in passive applications such as the thermal regulation of pipe-work circuits.
In the space industry, the heat-motors shown in Figure 2.6 are from a range of High Output Paraffin (HOP) actuators. These can be specified to activate at temperatures between \((-20\) to \(120\)\(^\circ\)C\) depending upon the wax formulation (p. 21 in Tibbitts, 1988). A large range of actuation displacement lengths are possible, from precision linear positioning \((1\) to \(10\)\(\mu m\)) to much longer lengths \((100\) to \(200\)\(mm\)) when working against forces up to \(4000\ N\). These types of heat-motors have been used in launch vehicles, satellites and spacecraft since the 1970's because of their simple design, dependability of operation in extreme conditions and their reliability in long-service moderate duty cycle applications. These include hold-down and deployment mechanisms for satellites as well as a range of both active and passively activated positioning devices.

Heat-motors have been developed for large stroke and moderate load applications to operate windows and ventilators by a number of manufacturers, these are listed in Appendix J.1. The temperature at which they begin to activate can be selected by the choice of wax formulation, for these heat-motors it is typically between \((9\) to \(60\)\(^\circ\)C\).

**Figure 2.5:** Section drawing through actively controlled “Squeeze-boot and Rod” design

**Figure 2.6:** Actively operated heat-motors with internal heaters

Source: (Top) From (p. 20 in Tibbitts, 1988). (Bottom) © SpaceDev Inc. 2008 (Tibbitts)
2.1. Heat-motor technology

Depending upon the size and volume of the heat-motor’s cylinder, the actuation displacement length is in the range of (30 to 200) mm. Depending upon the cylinder construction, they can do work against applied forces in the range of (150 to 2250) N. These are all passively operated by the prevailing thermal conditions in their application environment. These include greenhouses and conservatories, Figures 2.7 (a to f) shows a selection of proprietary examples.

This section has described the technology of heat-motors and briefly reviewed the range of devices that are available from industry. Since it is the thermo-hydraulic behaviour of wax during phase-transitions that determines the operating characteristics of heat-motors, the following section reviews the relevant thermal and mechanical properties of waxes.

**Figure 2.7: A selection of proprietary passively operated heat-motors**

(a) ‘Gigavent’ heavy-lift type (Manufactured by: Orbesen Teknik A/S)

(b) ‘Superpower’ heavy-lift type (Manufactured by: Bayliss Ltd)

(c) Medium ‘Thermact’ wax hydraulic actuator (Manufactured by: Orbesen Teknik A/S)

(d) Light-duty heat-motor (Manufactured by: Bayliss Ltd)

(e) Light-duty heat-motor (Manufactured by: Thermoforce Ltd)

(f) Small ‘Thermact’ wax hydraulic actuator (Manufactured by: Orbesen Teknik A/S)

*Note: Arranged at approximately the same scale using measuring rule, load capacity is in descending order from top-to-bottom with the stroke-rod. Stroke-rods are shown retracted in the column of images on the left, extended in the corresponding column of images on the right (author’s collection)*
2.2 The thermo-hydraulic properties of wax

2.2.1 Paraffin waxes

Paraffin wax at room temperature is a hard white substance with a slightly rough flaky surface and is odourless. Paraffins are a group of hydrocarbons described by the chemical formula $C_nH_{2n+2}$. In this formula the number ($n$) refers to how many Carbon atoms there are in a molecule of the paraffin. In general, the higher the number of Carbon atoms the higher the melting-point of the paraffin. An analysis of empirical data on even and odd-numbered paraffins by Etessam and Sawyer (1939) derived a logarithmic relationship between molecular weight and melting-point. At room temperature paraffins with ($C < 5$) are a gas, paraffins with ($5 \leq C \leq 16$) are a liquid oil and those with ($C > 16$) are a solid wax.

Paraffins exhibit thermo-plastic behaviour, this means that they can repeatedly undergo phase-change between liquid and solid simply by raising and lowering the temperature around the wax’s melting-point. To review the thermo-physical properties of a paraffin inside a heat-motor it is instructive to understand their chemistry in more detail.

At the molecular level the Carbon atoms can be arranged in different forms, the paraffins that are normal-alkanes or $n$-paraffins have Carbon atoms arranged in a straight-chain. Other arrangements or isomers are possible, these are where the Carbon atoms form one or more branches. For each paraffin with ($C \geq 4$), the number of isomers increases exponentially. The isomers of a wax often exhibit very different thermo-physical properties from their normal-alkane, this includes melting-point, density and many were found by Grosse and Egloff (1938) to be unstable.

In practice, paraffins are supplied in different grades that range from low-cost “technical grade” to laboratory “analytical grade”. Technical grade paraffin consists mainly of straight-chain $n$-paraffin with a percentage of different branched isomers of that paraffin, together with other oils. The “analytical grade” is a fully-refined paraffin consisting of $>99.9\%$ of one straight-chain $n$-paraffin
and only traces of branched isomers. Wax purity is reflected in price, at the time of writing a
0.5 kg slab of un-graded paraffin wax can be obtained for £ (3 to 4) GBP in the UK, while 0.5 kg
of analytical grade \( n \)-pentadecane \((C_{15}H_{32})\) with structure the \( C \rightarrow (C_{13}) \rightarrow C \) costs £ 160 GBP.

Studies by Grosse and Egloff (1938) showed that the change in density of \( n \)-paraffins against
temperature when either in their liquid or solid state are linear. However, a systematic investigation
by Templin (1956) of the volumetric behaviour of \( n \)-paraffins showed they exhibit non-linear be-
haviour during solid and liquid phase-changes. Figure 2.8 is a plot of six different \( n \)-paraffin waxes
showing the change in specific volume against temperature during phase-change. The plot of vol-
ume for wax \( n \)-C\(_{20}\) when \((T \_w < 33 \degree C)\) and after the melting process when \((T \_w > 37 \degree C)\) is linear,
but during the melting process it shows non-linear behaviour. The plot shows that the expansion
of wax tends to occurs over a temperature range where the width of the range depends on purity, a
technical grade wax is somewhat wider (He et al., 2004) than high purity wax (Broadhurst, 1962;
Srivastava et al., 1993). “Analytical grade” is extremely pure (> 99.99 % mol.) consisting of a
single \( n \)-paraffin, this expands within a narrow temperature range (<0.5 \degree C).

The expansion of \( n \)-paraffins during melting is due to the structure of the carbon chains at the
molecular scale changing from compact crystals to an amorphous form. Substances with a ratio of
boiling-to-freezing temperature (> 1.8) are easily supercooled (p. 241 in Turner, 1971), the paraffin
\( n \)-octadecane’s ratio is 1.96 so it can be supercooled when cooled either very slowly or very rapidly,
the ratio for water is 1.36. By definition, supercooling alters the equilibrium temperature that is
normally associated with the solidification processes and is generally undesirable. Wax crystallisation
and nucleation is a complex phenomenon and beyond the scope of this review although wide-ranging
surveys were found in (Avrami, 1939; Uhlmann and Chalmers, 1965; Dirand et al., 2002).

Figure 2.8: Specific volume vs temperature during phase-change

Figure removed due to third-party copyright

Source: From (p. 98 in Freund et al., 1982)
2.2. The thermo-hydraulic properties of wax

Supercooling a liquid paraffin oil leads to a heat-motor remaining activated at temperatures below the melting-point on the cooling curve. Figure 2.9 shows the relationship between specific heat energy and temperature during solid-liquid phase-change.

Time-lapse studies have been used by Henze and Humphrey (1981); Sparrow et al. (1981) to observe the freezing and melting of wax, this is primarily to record the evolution of the boundary front during melting and the formation of dendrites during solidification. Both phenomenon would otherwise be difficult to observe. These techniques are employed again to observe the behaviour of wax during the solidification and melting process. A sequence of stills taken from a time-lapse study of phase-change in a sample of wax under ambient conditions is shown in Figure 2.10, to be read from top-left (a) across and down to bottom-right (f). Note the change in volume from (a to f) against the white graduation at \( V = 60 \text{ ml} \) under \( P = 1 \text{ bar} \) of pressure. The solidification process in \( n \)-paraffin waxes crystallises the chain molecules into lattice structures. Image (b) shows nucleation occurring homogeneously throughout the bulk of the wax as crystals start to form.

The range of dry bulb air temperatures at the external envelope of buildings in the temperate climate of northern Europe is taken to be between \((-10 \text{ to } +30) ^\circ C\). However, there is a heating effect from short-wave (\( \lambda_p = 0.5 \mu m \)) solar radiation on a surface that has a high absorptivity (\( \alpha_{sw} \)). In London (UK) the effect of solar heating on a vertically inclined south-facing surface with an (\( \alpha_{sw} = 0.9 \)) in June on a clear-sky day can exceed temperatures of \( T = 60 ^\circ C \). This extends the range of operating temperatures that a heat-motor device may be exposed to.

A study by Bentilla et al. (1966); He et al. (2004) on the thermo-physical behaviour of “binary mixtures” of \( n \)-paraffins found it was possible to control the freezing-point in increments of \( T_{mp} = 1 ^\circ C \) by varying the proportion of constituents as shown on the plot in Figure 2.11. The work by Lehto (2007) using binary-mixtures of paraffins with higher melting-points exhibited expansion during phase-transition over a wider temperature range.

Figure 2.9: Volume-temperature-enthalpy at phase-change for a high purity wax
2.2. The thermo-hydraulic properties of wax

A review of wax heat-motors as an alternative to electrically operated computer controlled actuators in (pp. 46-9 by Pathan, 2004) identified Orbesen Teknik ApS (2009) as one commercial supplier of waxes that offered a range of operating temperatures. They disclosed the composition of these waxes as “seed oils”, Table 2.1 shows the eleven types of wax that they supply and their operating parameters. The waxes that are assumed to be single formulation are Type D, E, F, G and H, all others are assumed to be mixtures of these based on their labelling. All the mixtures show a widening of the temperature-range over which expansion takes place and a gradual reduction in the amount of expansion compared with a single formulation.

The use of the extended temperature range of expansion property of a wax mixture is that this provides scope for adjustment. The mechanism can be adjusted to clutch the heat-motor’s stroke-rod output to different temperatures without needing to replace the wax charge. Typically, a thumb-turn screw at the end of the mechanism’s outside cylinder is manually rotated.

Figure 2.10: Sequence from time-lapse showing one duty-cycle of wax phase-change

Note: The metal part in the wax is the temperature sensor [Time-lapse of solidification and melting] (WMV, 2 Mb)
It is assumed that all these are all non-paraffinic as claimed, although a review of their operating temperatures of phase-change and the characteristics of their binary-mixtures shows many similarities with those reported in the literature for \( n \)-paraffins. A review of the results from testing binary-mixtures suggests that they suffer some drawbacks when compared to pure \( n \)-paraffins as melting-point can become difficult to determine. Mixing \( n \)-eicosane with \( n \)-docosane showed a degradation of latent heat of fusion from \((H_f = 244)\) to \((H_f = 139 \, J \cdot g^{-1})\) on (p. 17 by Humphries, 1974). Furthermore, the total volumetric expansion appears to be less that the largest expansion from one of the pure constituents. For simplification, the remainder of this study will concentrate on pure \( n \)-paraffins with a narrow temperature range and the largest volume of expansion.

Paraffin in its solid state has a compact crystalline form that is highly incompressible. When hermetically sealed inside the cavity it expands when melted to exert a force equally in all directions. The magnitude of pressure exerted depends upon the area of the cavity walls and the load applied to the stroke-rod. For a force of \((F = 1000 \, N)\) passing into a cavity with an area of \((A = 0.01 \, m^2)\), the pressure exerted by the liquid paraffin oil is in the order of \((P \approx 0.1 \, MPa)\).

Liquid paraffin oil is capable of withstanding elevated pressures, however, isothermal measurements of its density at different pressures by (Zoller and Walsh, 1995) using a GNOMIX PVT machine shows that volumetric expansion is reduced as pressure increases. Specifically, Figure 2.12 plots data from isotherms between ambient room temperature and \(250^\circ C\) at monotonic intervals of \(10^\circ C\) for the paraffin \( n \)-tetracosane. The six lines are cross-plotted at pressures from \((0 \text{ to } 200) \, MPa\), this shows that compressibility is much greater at lower pressures \((0 \text{ to } 40) \, MPa\) than when elevated \((160 \text{ to } 200) \, MPa\).

Figure 2.11: The variation in freezing-point by the composition of wax blend

Figure removed due to third-party copyright

Note: Freezing temperature vs composition for Tridecane-Tetradecane solution, From (p. 16 in Humphries, 1974)
The same applies to non-paraffinic waxes according to one of the manufacturers of heat-motors, who stated that a reduction in actuation displacement is in the order of $\approx 2.5 \% \text{ per } 100 \text{ N of additional load}$ (Orbesen, 2009a). On this basis, if load varies by $\pm 50 \%$ at $60 \degree \text{C}$ the total effect on expansion would be modest in the order of $(2 \text{ to } 3) \%$.

Cavity pressure influences melting-point ($T_{\text{mp}}$) as the crystalline solid and liquid states co-exist in equilibrium 'because the application of a pressure tends to favour the formation of the phase (solid or liquid) which has the smaller specific volume' (p. 2 in Chalmers, 1964).

Paraffins expands during melting ($\rho_s > \rho_l$), therefore an increase in pressure will raise the temperature at which the abrupt volume change associated with phase-transition will occur, as shown in experiments by Nelson et al. (1960). The magnitude of change in ($T_{\text{mp}}$) for a high-purity wax may be estimated using the Claperyon equation (p. 564 in Wark, 1977).

The pressures inside the cylinder cavity in this study are expected to be in the lower of these ranges, the findings of these studies suggests that the output length may exhibit some variation if there is a significant variability in the load, or the possibility that there may be some unpredictability. Ideally, this can be verified inside the specific heat-motor cylinder of interest and with hydrostatic load that is typical in the target application.

<table>
<thead>
<tr>
<th>Wax type</th>
<th>Choice of wax in cylinders</th>
<th>Cylinder and stroke length</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Wax phase-change temperatures ($\degree \text{C}$)</td>
<td>Thermacl</td>
</tr>
<tr>
<td>F32E5D1G</td>
<td>13-34</td>
<td>18-34</td>
</tr>
<tr>
<td>D</td>
<td>6-12</td>
<td>9-12</td>
</tr>
<tr>
<td>E</td>
<td>22-29</td>
<td>24-29</td>
</tr>
<tr>
<td>E5 D</td>
<td>19-27</td>
<td>21-27</td>
</tr>
<tr>
<td>E10 D</td>
<td>14-26</td>
<td>16-26</td>
</tr>
<tr>
<td>E15 D</td>
<td>12-25</td>
<td>15-25</td>
</tr>
<tr>
<td>F</td>
<td>38-45</td>
<td>41-45</td>
</tr>
<tr>
<td>F10 D</td>
<td>24-41</td>
<td>27-41</td>
</tr>
<tr>
<td>F20 D</td>
<td>15-40</td>
<td>20-40</td>
</tr>
</tbody>
</table>

*Source: Orbesen Teknik A/S Accessed 5th December 2009*
2.2. The thermo-hydraulic properties of wax

Ideally, a passive heat-motor will reach thermal equilibrium with its surroundings without any latency. In practice, all materials have a non-zero time-lag that affects latency. An indicator of heat-motor responsiveness is the thermal diffusivity of \( n \)-paraffin waxes used inside them. A substance with a high thermal diffusivity will transfer heat quickly through a given thickness. In the case of a fully-refined paraffin wax such as \( n \)-octadecane, it has a thermal diffusivity of \( (a_l = 1.115 \times 10^{-7} \text{ m}^2 \cdot \text{s}^{-1}) \) in its liquid state. The thermal diffusivity of \( n \)-paraffins is low when compared with most other non-metallic materials, for example the value for water is \( (a_l = 1.45 \times 10^{-7} \text{ m}^2 \cdot \text{s}^{-1}) \). This has been identified as a limitation of using \( n \)-paraffins in the field of thermal storage by Hale and O’Neill (1971); Bailey and Chong-Kwang (1975); Abhat (1983) and for control of thermal excursions by Humphries and Griggs (1977).

Since this is a fixed property of the material, wax’s low thermal diffusivity indicates that large thermal gradients will be created when heating or cooling a given thickness of wax. The very low value for the thermal conductivity \( (k) \) indicates that an increased thickness of wax could result in long time-lags before thermal equilibrium is reached throughout the body of wax.

Methods to reduce the charge and discharge times of wax to overcome this property of wax have been investigated. However, these were mainly for either limiting temperature excursions or improving the charge and discharge times in thermal storage systems, not directly for wax actuated heat-motor technology. They monitored specific heat and melting-point temperature of sample \( n \)-paraffins but did not monitor expansion. The types of fillers that have been tested are itemized in Table 2.2. The study by Bentilla et al. (1966); White et al. (1972) found that Aluminium honeycomb offered high effective thermal diffusivity because the thickness of the sides of the comb could be reduced the most while maintaining contact with the transfer plate.

\[ \text{Figure 2.12: The pressure volume temperature behaviour of the paraffin } n \text{-tetracosane} \]

\[ \text{Figure removed due to third-party copyright} \]

\[ \text{Note: From (p. 27 in Zoller and Walsh, 1995)} \]
It has been demonstrated that aluminium fins can be used internally to reduce the time needed to melt \( \times 2.2 \) and reduce the solidification time by \( \times 4.2 \) by Bugaje (1997). Embedding Lessing rings in the paraffin has been shown to reduce the time taken to change phase still further (Velraj et al., 1999). The use of expanded metal mesh embedded in wax demonstrated a \( \times 4 \) overall improvement in thermal conductivity, and the linear relation shown for net % volume of metal filler suggested that further improvements are possible (Hoogendoorn and Bart, 1992). A selection of the materials that have been tested are shown in Figure 2.13 (a to d).

The analytical studies by Lamberg and Sirën (2002); Henze and Humphrey (1981) of the phase-change process and the numerical modelling of the conduction and convection process in melting and solidification suggests that closer spacing of thinner fins will yield the greatest reduction in both heating and cooling times during phase-change. This is mainly through the promotion of convection processes. Analytical studies have emphasised the importance of the wax container’s internal geometry to maximise the surface area of contact.

A general finding of these studies was that the performance of the filler could be greatly improved by the quality of the bonding with the outer container. Specifically, in a study by Bentilla et al. (1966) the thermal conduction between the honeycomb edges of the metallic foil and the transfer plate were limited by the epoxy resin adhesive used. This suggests heat-motors of monolithic construction would benefit most from these enhancements, this is possible with the advancements in rapid prototyping (RP) manufacturing including Selective Laser Sintering (SLS) and Direct Metal Deposition (DMD) methods (Callicott, 2001) (pp. 190-91 in Noorani, 2006).

The wetting ability of wax has been investigated and shown to have a surface tension coefficient of \( \gamma = 27.5 \times 10^{-3} \, N \cdot m^{-1} \) at room temperature (p. 43 in Hale and ONeill, 1971). This is two and a half times that of water \( \gamma = 73.0 \times 10^{-3} \, N \cdot m^{-1} \) at a similar temperature (p.3 in Gennes et al., 2004). This suggests that \( n \)-paraffin wax would readily mould itself into a cavity with highly irregular geometry to take full advantage of these heat-transfer enhancements.

<table>
<thead>
<tr>
<th>Filler type</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminium finned containers</td>
<td>Humphries, 1974; Bugaje, 1997</td>
</tr>
<tr>
<td>Aluminium foam, wool</td>
<td>Bentilla et al., 1966</td>
</tr>
<tr>
<td>Aluminium honeycomb</td>
<td>White et al., 1972</td>
</tr>
<tr>
<td>Carbon fibre whiskers</td>
<td>pg.17 in Knowles, 1993</td>
</tr>
<tr>
<td>Copper foam</td>
<td>Li et al., 2012</td>
</tr>
<tr>
<td>Expanded metal mesh</td>
<td>Hoogendoorn and Bart, 1992</td>
</tr>
<tr>
<td>Graphite foam</td>
<td>Fukai et al., 2000; Mills et al., 2006; Zhong et al., 2010</td>
</tr>
<tr>
<td>Lessing rings</td>
<td>Velraj et al., 1999</td>
</tr>
</tbody>
</table>
2.2. The thermo-hydraulic properties of wax

The time-lag during a temperature rise above the melt-point was characterised for different thicknesses in a study by (pp. 5-6 by Bentilla et al., 1966). The results shown in Figure 2.14 for each thickness at five different power-dissipations were applied using four different \( n \)-paraffin waxes. The labels (A to D) in Figure 2.14 correspond to the four \( n \)-paraffins \( n=\{14, 16, 18 \text{ and } 20\} \). Given the shallowest depth of wax tested \( (\Delta x = 2.54 \text{ mm}) \) which is the iso-line closest to the plot’s origin, the lowest rate of heat-transfer applied \( (538 W \cdot m^{-2}) \) which is the iso-line closest to the \((X)\)-axis. The time-lag for all four waxes (A to D) is \( (R_t \approx 15 \text{ mins}) \) for a temperature rise above the waxes melt-point \( (\Delta T \approx 10 \text{ } ^\circ \text{C}) \). From this, if the power dissipation is doubled to \( (1080 W \cdot m^{-2}) \) then the time-lag is more than halved to \( (R_t \approx 7 \text{ mins}) \). If the power dissipation is quadrupled to \( (2150 W \cdot m^{-2}) \) then the time-lag is reduced again to \( (R_t \approx 4 \text{ mins}) \).

These studies applied active heating to the wax container using relatively high rates of heat-transfer while the area of the cold-plate used in these experiments was in the order of \( (A = 0.01 \text{ } m^2) \). For applications in the built environment, solar radiation has an intensity at the lower end of this scale. Comparing the rate of heat-energy input from a given temperature difference between ambient and wax melting-point with the heat-flux from solar, a heat-motor would be expected to actuate faster from solar than by convection transfers with the ambient air.

**Figure 2.13: Selection of methods to enhance the heat-transfer in paraffin waxes**

- (a) Aluminium honeycomb (White et al., 1972)
- (b) 3D matrix of copper foam (Li et al., 2012)
- (c) Lessing rings (Velraj et al., 1999)
- (d) Graphite foam (Zhong et al., 2010)
For a heat-motor to be highly responsive to ambient conditions by passive heat-transfers alone, the wax needs to be contained in a cavity with a high surface-area to volume quotient, that has low three-dimensional thickness. It is not in the scope of the thesis to improve the heat-motor device itself, however the proprietary heat-motor that will be used in the practical work will be selected for its high surface area in order to promote heat-transfers.

The rate of heat-transfer by convection between the heat-motor and the ambient is characterized by its thermal geometry. The Biot number ($Bi$) relates the external area and volume of an object with its thermal properties. While no data for Biot numbers is published by the heat-motor manufacturers, it can be estimated by measurement and calculation. Verification will depend upon how uncertain the values are for the thermo-physical properties of the materials. A low Biot number ($Bi < 0.1$) suggests a uniform temperature field and would simplify thermal analysis.

These sections reviewed the properties of paraffin wax as the fusible material to power heat-motors. It also reviewed the thermo-hydraulic operation of a heat-motor actuator. In the following section, attention turns toward some existing uses of heat-motors actuators. The parameters and properties that have been outlined in the previous sections will be referred to where applicable to describe the capabilities of heat-motors in the context of each application.

**Figure 2.14: One-dimensional adiabatic system performance of waxes**

*Figure removed due to third-party copyright*

*Source: From (pp. 5-6 in Bentilla et al., 1966)*
2.3 The application of passive actuators

The following sections review examples of how the heat-motor actuator has been applied. These include a simple passively operated system where the heat-motor is used to control the rate of ventilation in the form of a general-purpose actuated ventilator. Then a more complicated “semi-passive” example is reviewed in which the heat-motor is used to harvest thermal energy that is then converted and stored as mechanical energy.

2.3.1 Window openers for greenhouses and roof-lights

This application exploits the output of a heat-motor to directly operate a window, such as the ventilators in framed greenhouses and skylights in buildings. The operation is straightforward, the heat-motor is connected to a mechanism that links its stroke-rod between the frame of the window aperture and the frame of the window’s opening-light.

A typical arrangement is shown in Figure 2.15, the lower two sections are through an inclined top-hung outward opening skylight fitted with a heat-motor operated ventilator, similar to a proprietary model by Orbesen (1999). Above in Figure 2.15 the skylight is illustrated in its closed position (left) when the heat-motor is inactive and open (right) when actuated. Typically, the heat-motor operated mechanism is at the opposite side of the frame from the hinge-line. This arrangement maximises the motor’s turning moment whilst minimising torsion forces on the mechanism from the window’s dead-load and any imposed live-loads. Wind-loads on an open window can be large and present a risk of damage, and is usually compensated for by damping springs.

This application thermally regulates the free-area available for ventilation through the window. The heat-motor is on the inside, while the window is closed the state of the heat-motor is determined by the energy balance of gains and losses with the surroundings inside. The balance depends on the window’s orientation and the energy exchanges with indoor sources and sinks.

If the gains are in favour of the heat-motor’s temperature reaching the melting-point of the wax, then latent heat of fusion starts to be transferred and the window starts to open. Once the window is open there is a free exchange of air between inside and outside. When the energy balance between the heat-motor and its surroundings cools the heat-motor to the wax’s melting-point on the cooling-curve then it will start to solidify. In the design of these mechanisms, both the window’s dead-load and a biasing spring are used to push the stroke-rod back into the heat-motor to maintain positive pressure between the stroke-rod and the solidifying wax.

Most transfer mechanisms have a manually adjustable connector between the end of the stroke-rod and the window or ventilator opener. In the case of a pure wax, expansion occurs over a narrow temperature and this is used for ‘zero-setting’. In the case of blended waxes that expand across a temperature range, this is used to select the lowest temperature at which the window will start to open in a similar way to adjusting the set-point on a thermostat. The blend provides an approximately proportional relation between the temperature rise and the angle of window opening.
2.3. The application of passive actuator systems

These are self-regulating systems, they *passively* exchange thermal energy with the surroundings in favour of thermal equilibrium. Depending upon the net balance of the wax’s specific heat with respect to its melting-point, the heat-motor will mechanically adjust the free area available for ventilation.

The thermal environment that is desirable for plant growth in greenhouses is between (13 to 24) °C, it is generally held that most plants will suffer damage by sustained temperatures in a greenhouse above 29 °C (pp. 16, 83 in Hessayon, 2006). The following describes the ventilator’s operation in a *warm greenhouse* across a diurnal cycle on a spring day under clear skies. Warm greenhouses use supplementary heating to maintain conditions above 13 °C, they typically have ventilators in the lower walls and either side of the roof’s ridge line as shown in Figure 2.16.

The air temperature inside the greenhouse will tend toward thermal equilibrium with the outside and at night this is likely to fall below the melting-point of the wax. Under a clear-sky the apparent temperature of the sky has been shown to fall as low as \( T_{\text{sky}} = -45 \) °C by Bliss (1961). A temperature difference so large that passive radiant cooling would dominate heat-losses. At sunrise the wax is solidified and the ventilators closed. Insolation will raise the temperature in the greenhouse due to the spectrally selective optical properties of glass to transmit shortwave sunlight but reflect back the longwave ‘Black body’ radiation emitted from inside the greenhouse.

*Figure 2.15: Heat-motor used to operate a roof-light for ventilation*

[Animation of mechanism] (WMV, 1.5 Mb)
2.3. The application of passive actuator systems

When the heat-motor has absorbed sufficient heat energy to raise its sensible temperature to its melting-point, the wax’s expansion opens the ventilator. Buoyant warm air rises and escapes through the ridge vents to drive the ‘stack-effect’, outside cooler air at ground-level is then drawn in. This state will persist so long as gains from insolation balance losses by convection to the cooler outside air in favour of maintaining the heat-motor’s temperature above the wax’s melting-point.

As evening twilight approaches, the gains from insolation continue to fall until the balance of internal heat energy shifts in favour of convective losses to the cooler ambient air. The heat-motor’s temperature starts to fall toward or below the wax’s melting-point. As the latent heat of fusion is released from the wax, it starts to solidify and the ventilator closes.

A further drop in indoor temperature during the night is therefore limited by minimizing the heat-loss due to air changes. In the context of this energy balance the high latent heat of fusion in wax acts as a thermal capacitor, this does have one benefit because it dampens the effects of intermittent sunshine. This serves to maintain the ventilator in a stable open position. Taken together, the heat-motor and greenhouse are an analogue of the ‘Governor’ device invented by James Watt in 1788 because the mechanical arrangement of the component parts embodies the regulating behaviour that it exhibits during operation. It embodies the balance of forces that act to self-regulate as described on pp. 2-3 by Aström and Murray (2009).

**Figure 2.16: Greenhouse with ventilators at the ridge and lower side-panels**

Source: Greenhouse with ventilator courtesy of Mr and Mrs Axmark. [Time-lapse] (WMV, 2.8 Mb)
To improve the match between temperature range and rate of air changes, the manufacturers of heat-motors have blended waxes so that they expand roughly in proportion to temperature over a range that matches that preferred for plant growth (13 to 24)$^\circ$C. As opposed to using a pure wax with a narrow melting-point that would either be fully open or fully closed within (1 to 2)$^\circ$C of the melting-point. However, the limitation of the heat-motor is that it is restricted to one response. It can do no more than maximise the free-area for ventilation when temperatures exceed ($T_{db} = 29^\circ$C) when plants become at risk of being damaged or destroyed. These temperature can easily be attained by heat gains from insolation.

### 2.3.2 Heat-motor in the propulsion system of an ocean glider

The “SLOCUM” is an Autonomous Underwater Vehicle (AUV) developed by the Webb Research Corporation. This is an example of a heat-motor used within a system that is controlled deterministically. Unlike the previous examples, it is not a free-running “open-loop” system that self-regulates. The “SLOCUM” propels itself to descend and ascend in the ocean by manipulating its buoyancy. However, it glides through the ocean by controlling the angle of fins as part of a computer-controlled guidance, navigation and control (GNC) system (p. 25 in Simonetti, 1992).

Figure 2.17 shows a profile of temperature against ocean depth in a tropical climate, this is typically a drop from (24 to 5)$^\circ$C between the ocean surface and a depth of (250 to 500) m called a “thermocline”. The glider’s propulsion system is based on manipulating buoyancy by exploiting the change in density of a working fluid between its liquid and solid state. The working fluid is a $n$-paraffin wax that expands when melted and is less dense than when frozen. The principle of the heat-motor is used by harvesting thermal energy when the glider descends through shallow warm water to colder water at depths below (250 to 500) m. At these colder depths the working fluid in the heat-motor solidifies and contracts, using a transfer system this thermal energy is stored as mechanical energy in an accumulator using ethylene glycol as a transfer fluid.

The transfer fluid is moved between an accumulator, an internal and external bladder under the pressurization of Nitrogen gas, the GNC computer sequences the opening and closing of valves between each of these to achieve the fluid transfer from the internal and external bladders. A detailed account is given by Webb et al. (2001) of the heat-engine’s thermal energy harvest through one descent and ascent dive. In essence, the computer deterministically controls the AUV’s buoyancy depending upon its depth and the mechanical energy that is stored by harvesting the expansion-contraction cycle from the phase-change of the wax.

The integration of the computer makes this an example of selective and controlled discharge of the stored mechanical energy that is harvested by the heat-motor. In this way it is an example of a discrete and discontinuous link that is made between changes in the thermal environment that the heat-motor harvests and the time and place when and where the energy stored by the harvesting is released.
The oceans beyond the continental shelves that surround the Earth’s major landmasses have relatively stable thermal gradient to a depth of \( \approx 500 \text{ m} \). The thermal environment is characterised by diurnal gains from solar radiation at the surface and the thermal inertia of water. The glider exploits this and moves with respect to the stable thermal gradient using a sophisticated auto-pilot, it can be selective as to when or even whether to ascend or descend through the gradient.

The ocean is a very different thermal environment to the Earth’s atmosphere above it, the thermocline leads to a water temperature \((T < 15^\circ \text{C})\) at depths \((< 400 \text{ m})\) everywhere (p. 426 in Bowditch, 2002) while air temperature fluctuates because it has less thermal inertia and the fluctuating effects of sunlight lead to much more varied conditions.

These sections have presented a survey of some existing heat-motor technology, together with the properties of wax that are exploited to power them. In addition, we have reviewed a variety of applications in which heat-motors have been used. We have seen that the heat-motor has a proven ability to generate large hydrostatic forces of the order that could be used to move components of the mass of a construction component. They have a proven capability to duty-cycle within diurnal time-scale. We have now established the credibility of paraffin waxes as an alternative to competing shape memory materials. In the sections that follow, attention now turns to reviewing passively operated movable building elements that can serve multiple purposes.

**Figure 2.17**: The “SLOCUM” ocean glider surfacing and operating profile

*Source: Images (by Douglas, 2008) and diagrams after (p. 17 in Simonetti, 1992)*
2.3.3 Combined roof-light and insulated shutter

The “Skylid” by the Zomeworks Corporation is a movable insulated panel used for reducing the night-time heat-losses through roof-light windows. It is passively actuated by responding to thermal energy from sunlight when available to open while relying on losses to ambient to deactivate. Each panel is balanced about a centre-pivot that rotates open when actuated and then returns to the closed position under gravity. When closed, the panels are designed to provide a continuous line of thermal insulation with the roof construction around the window opening, as shown on the left-side in Figure 2.18. In the open position shown on the right in Figure 2.18, the window admits daylight and solar gains (p. 207 in Langdon, 1980). A patent specification by Baer (1975) discloses the balanced gravity drive method to control opening and closing the panels.

This system does not operate by the thermo-hydraulic force generated in the phase-change of the working fluid, it exploits the displacement of mass between the gas and condensate state during liquid-to-gas phase-changes. When the working fluid is heated and boils from its liquid-to-gas state, the centre of gravity shifts due to the gas state being much less dense than the liquid. The arrangement of canisters that hold the working-fluid matches both the control position as counter-weights as well as the exposure to the system’s input stimuli, the heat energy from sunlight. In essence, it operates by a thermally regulated weight displacement instead.

The manufacturer has disclosed that the working fluid is the refrigerant Freon, if it is Freon-11 (\(\text{FCCl}_3\)) which has a boiling point of \(T_{bp} = 24.1^\circ \text{C}\) then given an indoor temperature of (19 to 21) \(^{\circ} \text{C}\) the lower container would condense the working fluid and close the panels during the night, but during the day even modest solar gains would boil the fluid above 24.1 \(^{\circ} \text{C}\).

When the “Skylid” is closed as shown on the left in Figure 2.18 an enclosure is formed behind the glass by the upper side of the “Skylid” panels, this resembles a solar-heated greenhouse and is the space where the temperature of the upper canister of Freon is determined. During the day, some sunlight would be needed to raise the temperature of the upper canister above the boiling point of the working fluid in order to change the balance of weight that would open the panels. Although driven by mass-displacement rather than thermo-hydraulics, this system relies on the thermal responsiveness when boiling 6.8 kg (15 lb) of Freon to alternate the balance between the two different configurations at the phase-change temperature \((T_{bp})\).

2.3.4 Indoor insulated shutter

An example of a heat-motor operated thermal shutter is the “Sundow”, a movable element invented by Richard S. Levine at the University of Kentucky in 1975 (pp. 128-9 in Shurcliff, 1980). These consist of a pair of thermally insulated shutters that are operated into either an open or closed position by a single-action heat-motor driving a bespoke mechanism. It was designed to provide indoor shutters that maintain a continuous line of thermal insulation with the wall construction during night-time to reduce heat-losses.
The shutters were designed to open in response to sunlight to admit solar gains for internal space heating. At night, the shutters were designed to close when the temperature drops below the melting-point temperature of the heat-motor. The enclosure between the glazing and the pair of shutters when closed is a high absorption solar collector, part of a separate system not discussed here. The thermal environment inside the enclosure determines the state of the heat-motor, if during the early morning there is sufficient solar irradiation to raise the heat-motor temperature above the melting-point then the mechanism opens the shutters.

The mechanism is operated by a single heat-motor that provides a single-action response. Figure 2.19 shows a plan section through the “Sundows” in their closed position. This would be typical under cold night-time conditions, a manual override is provided to close the shutters during excessively hot conditions to prevent overheating. It is not exactly clear from the published drawings and photographs of the project (e.g. Levine, 1983) how the manual override worked.

A pair of “Sundows” were parametrically modelled based on the description given in (Shurcliff, 1980) to clearly illustrate the bespoke mechanism operating the two different configurations of the shutters. The pair of shutters are shown in their closed position on the left-hand side in Figure 2.19 and in their open position on the right-hand side.

Figure 2.18: The “Skylid” by the Zomeworks Corporation

Source: Modelled from the design on p. 3 in (Baer, 1975) [Animation of mechanism] (WMV, 2.6 Mb)
Drawings of the mechanism that translates the heat-motor’s actuation to operate the shutters are shown in Figure 2.19 (lower half) as disclosed on p. 129 by (Shurcliff, 1980). On the left, when the heat-motor is colder than the melting-point temperature of the wax the stroke-rod is fully retracted closing the shutters into the night-time position. On the right, when the enclosure temperature is warmer than the wax melting-point the mechanism drives the shutters open.

This demonstrates how a single-threshold heat-motor could directly operate a building scale component in response to thermal energy. This design was refined into an undisclosed gear mechanism applied to a building; the Raven Run house in Lexington, Kentucky, USA (Levine, 1983, 2010). The architect designed the south-facing façade fitted with ‘Sundows’ to incline at 54°44′ to the horizontal, this strikes a balance with the seasonal difference in solar energy given the site’s latitude at 38°6′N. This is approximately mid-way between the lowest and highest solar elevation angles at the winter ($\alpha_{sol} = 28.5^\circ$) and summer solstice’s ($\alpha_{sol} = 75.3^\circ$) respectively.

No operating data was found in the literature that describes the performance of the heat-motor as a thermally activated positioning actuator for this design of shutter. However, the original construction has been modified to include glazing on the inside that effectively creates a greenhouse enclosure around the heat-motor is reported on p. 316 by (Roaf et al., 2001).

**Figure 2.19:** The “Sundow” indoor thermal shutter, closed (left) and open (right)

Note: Heat-motor highlighted in black, (after p. 129 in Shurcliff, 1980) [Animation of mechanism] (WMV.1.5 Mb)
2.3.5 Seasonal shutters within a 'Solar-wall' construction

This section reviews the repertoire of a movable construction element operating within the façade of a building that embodies the 'bioclimatic approach' in relation to its site. The Wallasey school in the Wirral, UK (Latitude of 53°25' N) completed in 1961 was designed to regulate its energy-balance by the arrangement of its structure, orientation and façade construction given its location. Its environmental design was bold, to maintain habitable indoor conditions solely by internal gains from its occupants, artificial lighting and external gains from solar despite the unfavourable maritime climates of the British Isles, as described on (pp. 279-84 by Banham, 1984).

To achieve this, apart from the highly-insulated roof-slab and walls the building’s main elevation is a 70 m long south-facing full-height glazed ‘Solar-wall’ construction, as shown in Figure 2.20. The construction of the façade consists of two glazed walls 600 mm apart, the outer layer is fully glazed but the inner layer is differentiated depending on room use. The building’s concrete structure and wall surfaces are exposed to the internal gains and provide the thermal-mass to dampens the internal temperature from large diurnal fluctuations. When completed, it was one of the earliest large-scale passive solar heated buildings in the UK.

The environmental design of this building has been extensively analysed by Davies (1986b), briefly the building’s energy-balance depends on passive solar gains to meet a large portion of the heating load. The internal ceiling at first-floor level is sloped outwards at ≈ 13° to match the highest solar-elevation reached at the winter-solstice for this latitude, maximising the penetration of sunlight during winter-time. During summer, the admission of solar gains needs to be selective to avoid the risk of overheating when ambient is also warm. It is also desirable to moderate losses in the energy-balance by reducing night-time heat-flow to a cold ambient. Unusually for a whole building design, a patent for the solar-wall system was granted to its architect Emslie A. Morgan in 1966. In its specification 'shutters of heat insulating material pivoted about vertical axes between the inner

Figure 2.20: The 'Solar wall' in the main block of St. George School in Wallasey (UK)
and outer skins of the solar wall’ are disclosed on p. 4 in Patent GB 1022411 by Morgan (1966). Although shutters were incorporated in the ‘solar wall’ built in 1961 as reported on p. 132 by Davies (1986b), these are not exactly as specified in the patent shown in Figure 2.21. The patent discloses two different strategies for moderating excessive solar gain using the inner wall. It either provides heat insulation by screening and a metal radiator as shown in Figure 2.21 (drawing in top half) or it absorbs heat with thermal mass as shown in Figure 2.21 (model in lower half). In both cases the function of the shutters is to reduce the heat-loss by closing to provide thermal insulation during the cooler conditions at night-time, and when the sun is obscured.

This building is an example of a relatively fine diurnal energy-balance between gains and losses that is regulated to maintain a stable internal temperature, with the ‘solar-wall’ and the movable shutters responding to moderate fluctuations in outdoor conditions. The shutters were designed to be manually operated, however an automated system would be desirable when occupants are not present or unwilling to operate them. The rationale for deploying the shutters in response to accumulated or rejected thermal energy suggests the possibility of emulating the regulation of the building’s energy-balance by the physical arrangement of its movable components. Given a shared exposure to the prevailing conditions this is also a characteristic response of passive heat-motors.

Figure 2.21: Movable thermal shutters within the double wall construction

(a) Vertical section thro ‘solar-wall’
(b) Plan of ‘solar-wall’ with shutters shown open
(c) ‘Solar wall’ shutters closed for thermal insulation
(d) ‘Solar wall’ shutters open to admit solar gains
2.4 Summary

Table 2.3 summarises the advantages of a movable insulated shutter in different positions based on the seasonal variation in external conditions.

Table 2.3: The benefits of movable thermally insulating shutters

<table>
<thead>
<tr>
<th>Advantages</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compensation for the limitations of glazing</td>
<td>In terms of thermal performance, windows are the weak area in the construction of walls and roofs. Typically, they provide less than half the thermal insulation offered by a typical wall construction of the same area. At the time of writing, the UK building regulations for a window are $U$-value $= 2.2 \cdot W \cdot m^{-2} \cdot K^{-1}$ compared to the walls around it at $0.35 \cdot W \cdot m^{-2} \cdot K^{-1}$ (Table 4, p. 7 in UK Approved Document Part-L by ODPM, 2006b). Insulated shutters over windows externally would improve this and reduce heat loss during winter.</td>
</tr>
<tr>
<td>Reduce heat-loss</td>
<td>A survey by Langdon (1980) shows examples of thermally insulating shutters used over windows on residential buildings in the U.S.A. built during the 1970’s and 1980’s. An analysis by Shurcliff (1980) showed that the main benefits of shutters used externally over windows was that when retracted at night during the heating season they substantially reduce heat-losses through the glazing. It has been shown by Quirouette (1980); Nielsen (1987) that a reduction of 20% of night-time losses on an annual basis can be achieved for external thermally insulating shutters that are deployed and retracted over glazing. An empirical study by Galyfianaki (2007) analysing the effects of thermally insulated shutters in a Mediterranean climate on heating loads showed that a residential building on a south-facing elevation external aluminium insulated shutters closed during winter-nights could reduce heat-losses by one-fifth compared to a window without shutters.</td>
</tr>
<tr>
<td>Shutters providing shade from excessive solar gains in summer</td>
<td>Fixed external window shutters have been widely used to provide shading from excessive heat gains due to solar irradiation. An order of magnitude assessment for the impact of insulating thermal shutter on a domestic property in a Mediterranean climate has been carried out by Galyfianaki (2007). It concluded that using a shutter could reduce the cooling load by 39%.</td>
</tr>
</tbody>
</table>

continued on next page
Advantages Description

Property security
Ground floor windows are vulnerable to forced intrusion especially at night-time. A shutter over the window provides protection against forced entry. However, it is problematic to use a passive heat-motor operated mechanism without a form of override to prevent the shutter from automatically opening when there is no occupancy in hot weather.

Based on the findings of this review, the benefits of heat-motors suggest they would be suitable for operating movable building elements by exploiting passive energy exchanges. The following summarises the advantages in Table 2.4 and disadvantages in Table 2.5 of heat-motors powered by paraffin waxes as the fusible material.

### Table 2.4: Advantages of passively operated wax heat-motors

<table>
<thead>
<tr>
<th>Advantages</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Zero CO₂ emissions during operation</strong></td>
<td>The heat-motor is a passively operated device that only needs to harvest thermal energy from its surrounding environment or isolation from the sun. No other primary energy sources are needed, it does not add to electrical energy consumption or contribute to Green House Gas (GHG) emissions</td>
</tr>
<tr>
<td>Availability and low-cost</td>
<td>Paraffin waxes are a by-product of refining crude oil and can be extracted from various other sources such as slack wax, it remains easily available in large quantities from suppliers worldwide. Most standard grade paraffin wax is relatively low-cost at £ (25 to 30) GBP · kg⁻¹. Fully-refined paraffin waxes are more expensive, depending upon the formulation these can cost between £ (40 to 300) GBP · kg⁻¹</td>
</tr>
<tr>
<td>Large volume expansion</td>
<td>They have a large volumetric change of (10 to 20) % between solid and liquid phase, this is necessary to generate large stroke lengths for articulating building scale components</td>
</tr>
<tr>
<td>Selectable melting-point</td>
<td>Waxes are available with a melting-point within the range that is typical at the external envelope of buildings (−10 to 40) °C for the warm temperate climate studied here (Köppen classification ‘Cfb’).</td>
</tr>
</tbody>
</table>

*continued on next page*
## 2.4. Summary

### Advantages

<table>
<thead>
<tr>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Selectable temperature as range of expansion</td>
</tr>
<tr>
<td>High hydrostatic load capacity</td>
</tr>
<tr>
<td>Robustness and reliability</td>
</tr>
<tr>
<td>Thermo-plasticity</td>
</tr>
<tr>
<td>Silent operation</td>
</tr>
<tr>
<td>No known toxicity</td>
</tr>
</tbody>
</table>
Advantages | Description
---|---
Chemical inertness | The word paraffin (para-, -finum) in Latin means “lacking affinity”. Paraffin is chemically inert in the presence of most materials, it is non-corrosive to metals and does not readily dissolve or mix with other substances including water (Lehto, 2007). This also suggests that its behaviour as a material is unlikely to degrade over time from contaminants in the way that other non-paraffin phase-change materials may e.g. Glauber salts (Abhat, 1983)

<table>
<thead>
<tr>
<th>Disadvantages</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exceptionally low thermal conductivity</td>
<td>The thermal conductivity for paraffin waxes is low in both its solid and liquid states ( (k_s \approx 0.14 \text{ W} \cdot \text{m}^{-2} \cdot \text{K}^{-1}) ), this is similar to good insulating materials. This imposes a limitation on the responsiveness of a heat-motor to a given heat flux. The existing work on thermal capacitors suggests improvements in responsiveness are possible with increased surface area for heat-transfers using metal matrices. However, existing heat-motors have not made maximum use of these methods to increase responsiveness</td>
</tr>
<tr>
<td>High latent heat of fusion</td>
<td>The latent heat of fusion of a paraffin wax is relatively high ( (H_f \approx 230 \text{ kJ} \cdot \text{kg}^{-1}) ), compared with water (336 \text{ kJ} \cdot \text{kg}^{-1}), this is the heat energy that must be transferred from the liquid wax phase-change from liquid to solid to freeze. This means that in ambient thermal conditions close to the melting-point there may be a long time-lag while latent heat is transferred</td>
</tr>
<tr>
<td>Moderate fire hazard</td>
<td>Paraffin represents a moderate fire hazard, in practice it does not readily ignite even in the presence of a naked flame. Its flash-point is typically &gt; 275°C</td>
</tr>
<tr>
<td>Hysteresis</td>
<td>While the phase-change of wax is a reversible process, it does exhibit hysteresis due to the transfer of latent heat of fusion. This affects the responsiveness of a heat-motor though a complete duty-cycle. For this reason it is suitable for applications with moderate frequency</td>
</tr>
</tbody>
</table>
2.4. Summary

<table>
<thead>
<tr>
<th>Disadvantages</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>of duty-cycles in the order of (&gt; 10 mins) when operated by passive heat-transfers</td>
<td></td>
</tr>
<tr>
<td>Sensitivity of melting point to cavity pressure</td>
<td>If the heat-motor does work against highly variable loads then it will exhibit a lack of consistent and repeatable stroke-length. This is due to the physical effects of pressure on the density of ( n )-paraffin waxes in their liquid state, increased pressure leads to decreased volumetric expansion. Very high pressures will also raise the equilibrium temperature of melting</td>
</tr>
<tr>
<td>Leveraging actuation displacement length</td>
<td>The maximum actuation displacement length that is available from a stock heat-motor in the survey was 206 mm (Orbesen, 2009a), although specials up to 300 mm have been made by (Bayliss, 2010a). In order to move building-scale components this may require larger distances to be translated. As shown in the greenhouse ventilator mechanisms, this can be achieved by leveraging as part of a translation mechanism. Whilst this may be suitable for a single threshold response, alternative mechanisms may be necessary to achieve this for responses with several thresholds. Further mechanisms may also be necessary to perform conditional responses</td>
</tr>
<tr>
<td>Poor strain resolution in 'open-loop' operation</td>
<td>While there is a distinct volumetric change across the solid-to-liquid phase-change at an ideal isothermal, real heat-motors operating 'open-loop' will experience temperature excursions above and below the paraffin’s melting-point. At these temperatures, approximately linear coefficients of expansion will apply and lead to possible ‘underrun’ by contraction of solid and ‘overrun’ by expansion in heated liquid wax</td>
</tr>
<tr>
<td>Risk of contamination of the stroke-rod</td>
<td>The stroke-rod is the only moving part in most practical heat-motors, during stroke duty-cycling it is exposed to contamination from dust and debris that if allowed to accumulate it may compromise the seal(s). This is acknowledged by two of the leading manufacturers of greenhouse actuators as a weakness in the design of typical heat-motors</td>
</tr>
</tbody>
</table>
Specific applications were reviewed in which the heat-motor operates a movable element in the context of the built environment. Two different applications were reviewed; one with the heat-motor operating in an 'open-loop' and the other as part of a deterministic 'closed-loop' system. The distinction was that 'open-loop' applications had direct and proportional energy conversion between the heat-motor and the final output, whereas the active 'closed-loop' systems convert and store this as mechanical energy to selectively release it on demand. The inclusion of a computer contrasts the difference between the operation of passive 'open-loop' greenhouse application and this active 'closed-loop' system, because it makes the transfer-mechanism selective between the heat-motor’s state and the final output.

Finally, the review turned attention toward the application of the heat-motor’s operating properties to the environmental design of thermally insulated window shutters. The suitability of this application was considered on the basis that each of the shutter’s configurations may be associated with a distinct seasonal condition. The prospect might be that the multi-functional thermal shutter could be operated by a thermally selective actuator that exhibited conditional responses to the seasonal variation of days in a given climate.

Two examples were drawn from the literature to illustrate this, both are cases of internal thermal shutters that are automatically operated by a heat-motor in response to ambient conditions. In both cases the state of the heat-motor was shown to depend upon thermal conditions indoors and gains from solar irradiation through the window. They were also both examples of a 'open-loop' application of the heat-motor and were limited to a single threshold of response.

The main limitation identified for using heat-motors for operating building elements was that they are restricted to a single repertoire of response because only one heat-motor is used. The review also identified that heat-motors exhibit an operational limitation of thermal latency in both the activation and recovery stroke because of the thermal properties of paraffin wax.

The conclusions drawn from the literature review carried out in this chapter suggest that thermally selective actuators that provide conditional responses are useful for repositioning building scale components such as the thermal shutters surveyed in the Literature review. This is not satisfied by a single threshold response, this limitation is evident in the “Skylid” and “Sundows” reviewed earlier. In both cases the shutter leaf will remain open under hot conditions and allow excessive solar gains to be admitted unless the override disclosed on p. 3 in the patent by Baer (1975) for the “Skylid” and the override reported on p. 316 by Roaf et al. (2001) for the “Sundows” are manually operated by an occupant.

Existing applications of heat-motors have not been found to selectively harvest thermal energy by passive transfers conditional on the seasonal variation of insolation. However, examples of opportunities to apply the novel properties of heat-motors were identified, such as building façades with movable elements that are repositioned to serve different functions in response to seasonal variations in climate.
Chapter 3

Passive thermal capacitance in heat-motors

This chapter characterises the duty-cycle behaviour of an actual heat-motor, the proprietary Mk.7 “Superpower Tube” manufactured by Bayliss Precision Components Ltd in the UK. For consistency, this actuator is then used throughout subsequent studies presented in later chapters. This chapter investigates the characteristic responsiveness of this heat-motor and attempts to establish a working approximation for some of its engineering parameters. It would be challenging to conduct this study outdoors in uncontrolled conditions because of the large number of variables involved. Therefore, the controlled environment inside an air-conditioned climate chamber in a thermal studies laboratory is used as an alternative.

In this simplified study, the dominant heat-transfer process is limited to convection between the heat-motor and the air in the climate chambers. By conducting a systematic study, we can ask how quickly the heat-motor reaches thermal equilibrium with its surroundings when conditions change? In this environment, the heat-motor’s responsiveness may be quantified by determining its convective coefficient. An experiment is designed to quantify this for the proprietary heat-motor, a description is given of the experiment methodology.

The melting and solidification of wax occurs at an isothermal while heat-energy continues to be transferred, the implications of this non-linearity on the charge and discharge times of the proprietary heat-motor are described. The results of the experiments will illustrate the relation between the transfer of latent heat of fusion and the evolution of actuation displacement and recovery stroke during each phase-change. The findings of this chapter provide the working values for parameters that are then used in the practical and numerical studies carried out in a later chapters.
3.1 Characterising a proprietary heat-motor

This section moves from the description of a generic heat-motor given in the literature review to establishing the characteristics of a specific proprietary example. If heat-transfers are dominated by passive convection then the rate of heat-transfer is strongly influenced by the surface properties and geometry of the heat-motor. In the absence of data from the manufacturer or the literature, the surface properties of the proprietary heat-motor need to be established empirically. The thermal-geometric characteristics of the heat-motor can be estimated by calculating its ‘Biot number’.

3.1.1 Calculation of the Mk.7 heat-motor’s Biot number

The thermal responsiveness of the heat-motor is determined by the physical properties of the materials and its geometric form. This can be characterised as the relation between surface and volume by the Biot number as expressed in Equation (3.1). The heat-motor is made of an Aluminium alloy with the wax in the cavity, two materials with very different thermal conductivities. The conservative estimate of the heat-motor’s Biot number uses the value for wax as this is the lower of the two. This can be expected to lead to an overestimate of the the actual Biot number.

\[ Bi = \frac{h_c \cdot (V/A_c)}{k} \quad \text{(unit-less coefficient)} \]  

Where:

- \( h_c \) Convective heat-transfer coefficient in air = 10 (est.) \{in W \cdot m^{-2} \cdot K^{-1}\}
- \( k_w \) The thermal conductivity of n-octadecane = 0.146 \{in W \cdot m^{-1} \cdot K^{-1}\}
  
  (p. 6-212 in Lide, 2007)
- \( k_{H_2O} \) The thermal conductivity of water = 0.63 \{in W \cdot m^{-1} \cdot K^{-1}\}
  
  (p. 51 in Goswami and Martin, 2005)
- \( A_c \) Outside area of cylinder = 4.29 \times 10^{-2} \{in m^2\}
- \( V \) Volume of heat-motor = 5.60 \times 10^{-4} \{in m^3\}
  
  (From pg. 219 in Appendix B.1)

\[ Bi = \frac{10 \cdot (5.60 \times 10^{-4}/4.29 \times 10^{2})}{0.146} = 0.89 \quad \text{(2 d.p.) (unit-less coefficient)} \]

In has been shown in (pp. 212-7 by Incropera and DeWitt, 2007) that the simple ‘lumped capacitance method’ can be applied to estimate a coefficient for convection \((h_c)\) provided that the object has a ‘Biot number’ \((Bi < 0.1)\). The ‘as supplied’ Bayliss Mk.7 heat-motor does not meet this criterion, when charged with wax the Biot number is approximately \((Bi = 0.9)\). As the thermal conductivity \((k)\) is the denominator in Equation (3.1) the Biot number is sensitive to the thermal conductivity of the materials used. Paraffinic waxes have an exceptionally low thermal conductivity in the order of \((k_w = 0.14 W \cdot m^{-1} \cdot K^{-1})\), this is over \times 1,700 lower than Aluminium.

In order to simplify this estimate, the wax in the ‘as supplied’ Bayliss Mk.7 heat-motor is replaced with distilled water. Calculating Equation (3.1) with \((k_{H_2O})\) reduces the Biot number to a value much closer to the criterion for estimating \((h_c)\).
3.1. Characterising a proprietary heat-motor

There are further benefits to using distilled water as a substitute, the heat-motors can be heated and cooled around the a wax’s melting-point without the large thermal lag effects on the cooling curve that occur during the phase-change of wax. Secondly, the values for the thermal properties of distilled water are well known and can be used in calculations compared with the uncertainty of thermal data for the 'as supplied' wax.

The following section describes the experiment methodology for estimating \((h_c)\) using the 'lumped heat capacitance method'. We are interested in determining how quickly heat is transferred from the surface of the heat-motor by convection. Since this is a time-dependent study, we need the capability to warm up a heat-motor and then place it in a cooler ambient environment and observe the time-constant during the cooling period. It is also necessary to repeat the heating and cooling cycle in order to gather a distribution of results around the relevant melting-point.

We cannot depend on ambient outdoor conditions for this therefore a climate chamber is used in which temperature and humidity can be programmed and placed under closed-loop control. This also enables us to account for systematic errors in the instrumentation. Using this facility, a batch of heat-motors are heat-soaked in one chamber to a temperature above the melting-point of the wax. When they have reached thermal equilibrium they are transferred to another climate-controlled chamber pre-cooled at a temperature below the melting-point of the wax.

**Figure 3.1: Schematic of heat-motor test-cell inside one of the climate chambers**
3.1. Characterising a proprietary heat-motor

Figure 3.2: Heat-motor assemblies in climate-chamber during the experiment

Figure 3.3: The interior layout of the climate-controlled chamber

(a) Plan view of chamber interior
(b) Section through A-A Volume ($V = 29.1 \text{ m}^3$)
The heat-motors are then cold-soaked until they reach thermal equilibrium with the temperature of the air in the cold chamber. During the cold-soaking a temperature history is recorded for each heat-motor, the time-series data from alternating cycles of heating and cooling can then be used to calculate the rate of heat-transfer by convection using the 'T-history' method as described by Yinping et al. (1999). This procedure was repeated to obtain a comparative set of data.

A schematic of the experiment set-up is shown in Figure 3.1 and the test apparatus itself in Figure 3.2. The temperature of the chamber air is monitored and recorded, the temperature of the water at the core of the heat-motor’s cavity is measured with a custom temperature probe.

Each climate-controlled chamber is an air-conditioned room measuring $3.8 \text{ m}$ deep by $2.6 \text{ m}$ high as illustrated in Figures 3.3(a and b). Air-conditioning plant supplies air through a fan-driven duct at high-level into each chamber, the probes providing feedback to the plant’s process controller are placed at the exit grille of these supply ducts. Refrigeration coils located below the supply fans provide the cooling. The chamber’s PID-based controller is used to schedule time-temperature profiles at temperatures between ($-10$ to $40)\degree\text{C}$. In these experiments, the two chambers are programmed to maintain a temperature either side of the wax’s melting-point $T_{mp}$, with one chamber maintaining a temperature at $T_{db} = (T_{mp} + 8)\degree\text{C}$ while the other chamber is at $T_{db} = (T_{mp} - 8)\degree\text{C}$.

Heat-transfers are assumed to be dominated by convection on the basis that the surface temperature of the walls and the surface of the heat-motor’s cylinder are within $\pm (1$ to $2)\degree\text{C}$ of each other. Since the chamber’s walls are highly reflective with low emissivity it is assumed that the net exchange of ‘Black body’ radiation at thermal infra-red wavelengths is small enough to be ignored. In addition, the heat-transfers by conduction between the motor components and the apparatus are minimised by using fixings with low thermally conductivity. Six heat-motors were drawn from several different batches to conduct these experiments. These are representative of the heat-motors ‘as supplied’, this is qualified by the results of a mensuration survey of 35 motors tabulated in Appendix B.2 on page 220.

**Figure 3.4: Temperature measurement of wax at the centre of the cavity**

Note: Details of the probe’s design are given in Appendix C.1 on page 224.
We wish to characterise the heat-transfer behaviour of the heat-motor by convection in air, in particular we wish to characterise this during the phase-change of wax. However, the heat-motors are supplied with a proprietary wax with unknown properties. The manufacturer reports that the temperature range of expansion is (16 to 25) °C by Bayliss (2011). An initial pilot study carried out in the thermal laboratory established the critical point at \( T_{mp} \approx 21 \, ^\circ\text{C} \), details of this study are reported in Appendix D.1 on page 226. This is a wide temperature range and suggests that it may be a blend of waxes, this introduces some uncertainties. In order to remedy this, the ‘as supplied’ blend is replaced by a pure wax with a narrow melting-point. This allows us to analyse the results of these experiments using values published in reference tables for the thermo-physical properties of phase-change temperature, heat capacity, latent heat of fusion and density.

Figure 3.5 shows a typical time-temperature plot from these experiments. In this specific case, a cooling curve is shown in which the wax in the heat-motor undergoes solidification. The critical points during this process are annotated in the plot and explained as follows. Immediately after the ‘chamber change-over’ at \( t = 00:00 \, \text{hrs} \) the heat-motor experiences a rapid drop in temperature and the wax exhibits supercooling. The average temperature reached when the wax supercooled was \( T_{sc} = 27.1 \, ^\circ\text{C} \). After a short recovery the wax starts to solidify, during this process it releases its latent heat of fusion for \( H_f \approx 11.8 \, \text{kJ} \). The wax remains isothermal at its melting-point during this transfer of heat-energy. The temperature during the isothermal was stable and very narrow in range, indicating that the wax is of high purity. This experiment was conducted with the cavity under atmospheric pressure only without a working load applied.

**Figure 3.5: Time-lag during cold-soak with solidification phase-change in n-octadecane wax**

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Ambient temperature of air \( (T_{\text{db}}) \) — Temperature at the core of the heat-motor \( (T_w) \)
3.1. Characterising a proprietary heat-motor

After the wax has solidified, it continues to cool until it reaches thermal equilibrium with the ambient ‘cold-soak’ temperature in the chamber. From Figure 3.5 we can see that given a temperature difference of \( \Delta T = -16 \degree C \) the cooling curve has a time-lag of \( R_t = 2.5 \text{ hrs} \). We now compare this result with the indicative time-temperature curves for the same experiment but when the heat-motor is filled with distilled water shown in Figure 3.6 (b).

The difference that wax phase-change has on the time-lag can be seen when comparing the temperature curve (trace —) of water with that of wax during heating in Figures 3.6 (a and c). A first-order comparison between this pair of curves during the heat-soak shows that the effect of wax phase-change during melting in Figure 3.6 (c) adds \( t \approx 1.0 \text{ hrs} \) to the time-lag. During the cold-soak when the water is cooled as a liquid but the wax undergoes solidification, the difference between water and wax is shown in Figures 3.6 (b and d) respectively. The effect of wax solidification in Figure 3.6 (d) adds \( t \approx 1.5 \text{ to 2 hrs} \) to the time-lag compared with water. We can also observe that the time-lag for heating is longer than for cooling.

Based on the results of repeating this experiment on a group of heat-motor cylinders, the following section estimates a value for the coefficient of free convective heat-transfer from the surface of the \( Mk.7 \) heat-motor in air at the melting-point of the wax charge.

**Figure 3.6: Time-lags of heat (left) and cold-soak (right) with water (above) wax (below)**

(a) Time-lag of distilled water during heat-soak  
(b) ‘Newtonian’ cooling curve of water  
(c) Melting phase-change in wax  
(d) Solidification phase-change in wax

---

- Ambient temperature of air \( (T_{db}) \)  
- Temperature at the core of the heat-motor \( (T_w) \)
3.1.2 Calculating the heat-motor’s convective coefficient

Knowledge of the rate at which heat-energy is transferred from the heat-motor’s outside surface by convection with the surrounding air is necessary to model its behaviour. The rate equation for convection is shown in Equation (3.2) from (p. 8 in Incropera and DeWitt, 2007). To determine the rate of heat-transfer in ‘Steady state’ for the proprietary heat-motor depends upon finding a value for the coefficient \( \left( h_c \right) \).

\[
q_{\text{conv}} = h_c \cdot (T_c - T_{db}) \text{ \{in} \ W \cdot m^{-2}\text{\}} \tag{3.2}
\]

Where:

- \( q_{\text{conv}} \) Rate of heat-transfer by convection \text{\{in} \ W \cdot m^{-2}\text{\}}
- \( h_c \) Free convective heat-transfer coefficient in air \{in} \ W \cdot m^{-2} \cdot K^{-1}\text{\}}
- \( T_c \) Temperature of the heat-motor cylinder \text{\{in} \ K}\text{\}}
- \( T_{db} \) Dry-bulb temperature of the ambient air \text{\{in} \ K}\text{\}}

A time-temperature curve of a heat-motor charged with water undergoing cooling is shown in Figure 3.7. The motor was heated to \( (T_0 = 35.7 \, ^\circ C) \) and then placed in a chamber at \( (T_1 = 21.5 \, ^\circ C) \) it follows a typical Newtonian cooling curve. If the Biot number of the heat-motor cylinder can be regarded as \( (Bi < 0.1) \) then the temperature distribution in the heat-motor can be regarded as uniform. The conservative calculation for the heat-motor’s Biot number when charged with water was \( (Bi = 0.19) \). This ignored the good thermal conductivity of the cylinder itself and is taken as a reasonable approximation for this estimate.

**Figure 3.7: Cooling curve of heat-motor charged with distilled water**

---

<table>
<thead>
<tr>
<th>Ambient air ((T_{db}))</th>
<th>Distilled water in heat-motor ((T_w))</th>
<th>Temperature of supercooled wax ((T_{sc}))</th>
</tr>
</thead>
</table>
In this case the lumped capacitance method described in (pp. 212-7 by Incropera and DeWitt, 2007) is used to estimate \( h_c \) given a cooling-curve. Equation (3.3) is the expression for lumped capacitance and is rearranged to solve for the convective coefficient of the cylinder \( h_c \).

\[
h_c = \frac{(m_c \cdot c_{p,c} + m_w \cdot c_{p,w} + m_b \cdot c_{p,b} + m_r \cdot c_{p,r}) \cdot (T_0 - T_{sc})}{A_c \cdot A_1}\ {\text{in } W \cdot m^{-2} \cdot K^{-1}} \quad (3.3)
\]

Where:

- \( h_c \): free convective heat-transfer coefficient of cylinder \{in \( W \cdot m^{-2} \cdot K^{-1} \}\)
- \( m_c \): mass of the heat-motor cylinder \{in kg\} = 0.248
- \( c_{p,c} \): mean specific heat of the cylinder (2024-T6 Alloy) \{in \( J \cdot kg^{-1} \cdot K^{-1} \}\} = 875
- \( m_w \): mass of the charge \{in kg\} = 0.059
- \( c_{p,w} \): mean specific heat of the charge (liquid water) \{in \( J \cdot kg^{-1} \cdot K^{-1} \}\} = 4178
- \( m_b \): mass of the collar \{in kg\} = 0.068
- \( c_{p,b} \): mean specific heat of the collar (brass) \{in \( J \cdot kg^{-1} \cdot K^{-1} \}\} = 380
- \( m_r \): mass of the stroke-rod \{in kg\} = 0.08
- \( c_{p,r} \): mean specific heat of the stroke-rod (stainless steel) \{in \( J \cdot kg^{-1} \cdot K^{-1} \}\} = 510
- \( T_0 \): thermal equilibrium temperature above melting-point \{in K\} = 308.85
- \( T_{sc} \): average temperature at which the wax supercooled \{in K\} = 300.25
- \( A_c \): o/a heat-transfer area of heat-motor surface \{in \( m^{-2} \}\} = 0.047

The value for \( (A_1') \) is evaluated by Equation (3.4) for each experiment as the time-series integration of the water’s temperature \( (T_w) \) in the heat-motor from the end of the heat-soak \( (t_0) \) until the water reaches the temperature of supercooled wax \( (T_{sc}) \) during the cold-soak at \( (t_1) \). This is annotated in Figure 3.7 as the area between the curve and the line at \( (T_1 = 21.5^\circ C) \).

\[
A_1' = \int_{t_0}^{t_1} (T_w - T_{db}) \quad (3.4)
\]

<table>
<thead>
<tr>
<th>Heat-motor</th>
<th>Coefficient ( (h_c) ) ( (W \cdot m^{-2} \cdot K^{-1}) )</th>
<th>St.D ( (n = 5) )</th>
<th>Cooling time-lag when ( \Delta T = -16^\circ C ) ( \text{(minutes)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q</td>
<td>11.2</td>
<td>2.24</td>
<td>37</td>
</tr>
<tr>
<td>R</td>
<td>9.9</td>
<td>1.66</td>
<td>39</td>
</tr>
<tr>
<td>S</td>
<td>11.5</td>
<td>0.25</td>
<td>36</td>
</tr>
<tr>
<td>T</td>
<td>10.3</td>
<td>3.11</td>
<td>38</td>
</tr>
<tr>
<td>V</td>
<td>10.9</td>
<td>1.21</td>
<td>39</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td><strong>10.8</strong> (1 d.p.)</td>
<td>n/a</td>
<td>37.8 (1 d.p.)</td>
</tr>
</tbody>
</table>
The results of experiments are shown in Table 3.1. The estimate of the coefficient of free convection from a proprietary heat-motor surface in air is found to be low, the value was $h_c = (9.9$ to $11.5) \, W \cdot m^{-2} \cdot K^{-1}$. Based on this value, a heat-motor operating solely by passive ‘free convective transfers’ would be slow to undergo phase changes given that the density of heat energy in air is about $(100 \, kJ \cdot m^3)$ at the heat-soak temperature of $(T_{db} = 38 ^\circ C)$ at $(RH = 50 \%)$.

3.2 Relation between actuation displacement and latent heat transfer

The previous sections established the cylinder’s heat-transfer characteristics with air. The following section deals with the relation between the transfer of latent heat and the evolution of actuation in the heat-motor’s stroke-rod. To quantify this, an electronic position transducer is added to the previous experiment to measure the motor’s actuation displacement length. This was calibrated by matching the data from a linear transducer with time-lapse images of metrology markers.

Time-series in Figures 3.8 (a and b) show the actuation displacement (trace —) during melting and contraction during solidification respectively. The first-order observation is the strikingly slow start to expansion during melting compared with the rapid start of contraction during solidification. Secondly, the approximately linear gradient of actuation displacement during the majority of the phase-change under conditions of constant heat-flux.

Close scrutiny of the temperature profile at the wax core (trace —) reveals that it is not isothermal either during actuation or the recovery stroke, unlike the previous experiments where a minimal load was applied to the stroke. This can be explained if we accept that the pressure exerted by the wax inside the cavity varies during a duty-cycle. This explanation is predicated on the assumption that as a liquid oil, pressure acts equally in all directions. Therefore when wax melts and expands it pushes out the stroke rod leading to a reduction in the total surface area of contact between the wax and the cavity’s surfaces as surveyed in Figure B.4 in Appendix B.1.

Figure 3.8: Time-plot of actuation vs wax temperature during heat-soak and cold-soak
3.2. Relation between actuation displacement length and latent heat transfer

The stress applied to the stroke-rod also increased according to the actuation displacement against the load of the biasing force. Therefore, hydrostatic pressure in the cavity actually increases during an actuation displacement. We can evaluate the effect that pressure has on the melting-point as discussed on (p. 2 by Chalmers, 1964) using the Clausius-Clapeyron relation expressed in Equation (3.5). The denominator in Equation (3.5) shows that the relation between the density of a substance in its solid and liquid phase determines whether applying pressure will raise or lower the melting-point. In the case of wax n-octadecane were ($\rho_l < \rho_s$), when the cavity pressure rises we can expect an increase in the equilibrium temperature during phase-change.

Based on this theoretical account of phase-transformation processes, we can explain the variation in the isothermal during an actuation displacement against a working load. Qualitatively, the data shows a rise in the ‘isothermal’ during melting in Figure 3.8(a) as cavity pressure increases during the actuation displacement. Conversely, it is evident as a fall in Figure 3.8(b) during solidification as cavity pressure decreases during the recovery stroke.

We may ask how sensitive the melting-point of a highly refined charge of n-octadecane wax inside the proprietary heat-motor is when the cavity pressure varies? The Clausius-Clapeyron relation for temperature and pressure applies to single constituent substances, as discussed in the theoretical description in (pp. 564-7 by Wark, 1977). We use values from reference data tabulated in Appendix A.1 for the variables in the Clausius-Clapeyron relation in Equation (3.5) as follows:

\[
\text{Clausius-Clapeyron} \quad \Delta P = \left( \frac{H_f}{T_{mp} \cdot \Delta v} \right) \cdot \Delta T \quad \{\text{in } K \cdot Pa^{-1}\} \tag{3.5}
\]

Where:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta P$</td>
<td>Change in the pressure</td>
<td>${\text{in } Pa}$</td>
</tr>
<tr>
<td>$H_f$</td>
<td>Latent heat of fusion</td>
<td>$= 240,900$ ${\text{in } J \cdot kg^{-1}}$</td>
</tr>
<tr>
<td>$T_{mp}$</td>
<td>Absolute melting-point temperature</td>
<td>$= 301.2$ ${\text{in } K}$</td>
</tr>
<tr>
<td>$\Delta v$</td>
<td>Change in volume from solid to liquid</td>
<td>$=(v_l - v_s)$ ${\text{in } m^3 \cdot kg}$</td>
</tr>
<tr>
<td>$v_s$</td>
<td>Specific volume in liquid state</td>
<td>$= 11.6 \times 10^{-4}$ ${\text{in } m^3 \cdot kg}$</td>
</tr>
<tr>
<td>$v_l$</td>
<td>Specific volume in solid state</td>
<td>$= 13.0 \times 10^{-4}$ ${\text{in } m^3 \cdot kg}$</td>
</tr>
<tr>
<td>$\Delta T$</td>
<td>Change in the melting-point temperature</td>
<td>$= \text{Unity}$ ${\text{in } K}$</td>
</tr>
</tbody>
</table>

\[
\Delta P = \left( \frac{240900}{301.2 \times 0.000136} \right) \times 1 = 5.88 \times 10^6 \quad \{\text{in } Pa \cdot K^{-1}\}
\]

The same calculation for water is ($\Delta P = -13.5 \times 10^6 Pa \cdot K^{-1}$), on this basis the melting-point of n-octadecane is over twice as sensitive to a variation in pressure compared with water. Note that the value of ($\Delta P$) for water is negative because solid water ice is less dense than liquid water, this is the opposite of wax.

These sections have characterised the behaviour of the heat-motor itself in some detail, however this is subject to some constraints of the test environment. In the following section, attention turns toward the climate-chamber itself to briefly consider some of its limitations.
3.3 The limitations of the climate-chamber

While the climate-controlled chamber is helpful for emulating changes in ambient air temperature, it does not replicate the effects of an artificial sun irradiating the heat-motor cylinder. It would be possible to use artificial lighting to approximate the characteristics of the sun, such as a quartz lamp with an intensity at \(q_{rad} \approx 1000 \text{W} \cdot \text{m}^{-2}\), peak spectral output at \(\lambda_p = 0.5 \mu\text{m}\) and surface temperature at \(T \approx 5770 \text{ K}\) from p. 81 in Norton (1989). However, these are also the conditions in which the exchange of radiant energy between the cylinder and the atmosphere are significant at long wavelengths \(\lambda_p = 10 \mu\text{m}\). Replicating this in the climate-chamber 'as designed' would have involved modifications outside the scope of this study.

Cloudless skies are characterised by high-intensity solar radiation, however this also opens a window in the atmosphere to outer space through which an apparent sky temperature is colder than ambient, reaching temperatures of \(T_{sky} = -45 \, ^\circ\text{C}\) as reported in (Bliss, 1961). The climate chamber’s walls have low thermal mass and maintain a surface temperature near that of the ambient air in the experiments (16 to 25) \( ^\circ\text{C}\) not the realistic cold temperatures. The walls are also highly reflective in the thermal band as shown in Figure 3.9 (c), this is not representative of real cloudless skies as thermographed in Figure 3.9 (d) with the temperature scale extended to suit.

Figure 3.9: Characteristic difference in the surrounding surfaces under ambient conditions

(a) Standard sky (Granqvist and Eriksson, 1991)
(b) Cooling by emission losses (Bliss, 1961)
(c) Climate chamber walls are high reflective with a temperature close to ambient (IR-band at \(\lambda_p = 10 \mu\text{m}\))
(d) Radiation intensity of the atmosphere under cloudless conditions is very cold (IR-band at \(\lambda_p = 10 \mu\text{m}\))
In addition, the emissivity of stainless steel at \( (T \approx 300 \, K) \) is 0.17 (Incropera and DeWitt, 2007), this is not well matched to the emissivity of a cloudless sky at \( (\varepsilon_{sky} \approx 0.75) \) at long wavelengths of \( (\lambda_p = 10 \, \mu m) \). Therefore, it was not feasible to replicate the net exchange of radiant energy between the outside surface of the cylinder at its temperature around \( (T \approx 300 \, K) \) with counter-radiation from the atmosphere as well as toward outer space (Chap. 4 in Sellers, 1965). The net rate of cooling by radiant heat-transfer has been shown to be as high as \( (-q''_{rad} = 130 \, W \cdot m^{-2}) \) in clear-sky conditions in studies at night-time (Granqvist and Eriksson, 1991).

3.4 Summary

This chapter investigated the duty-cycle behaviour of a proprietary heat-motor. Since the motor’s actuation displacement length and recovery stroke is dependent on the phase-transitions of the wax charge, the thermal behaviour of the motor in relation to its surroundings was studied. In order to restrict the number of dependent variables, an experiment was designed using an environment where the dominant heat-transfer would be limited to convective exchanges between the heat-motor and the surrounding air. These conditions were approximated in two climate-chambers where air temperature could be controlled. This test environment was used to conduct repeated experiments on a sample group of proprietary heat-motors using appropriate instrumentation.

The results from these experiments using a highly-refined wax charge showed a stable isothermal during actuation and recovery strokes. With a temperature difference of \( (\Delta T = -16^\circ C) \) the time-lags exhibited were in the order of \( R_t = (2 \text{ to } 3) \, hrs \) and melting appears to take slightly longer than solidification. This may be accounted for by a small difference in the thermal conductivity of wax between liquid and solid phase, with solid wax having a higher conductivity. However, it is difficult to establish whether or not this observation is due on balance to a difference in the dynamics of convection in the partially liquid melt compared with the crystallisation during cooling.

The data revealed a small sensitivity to changes in cavity pressure above atmospheric. This was attributed to the variation in the working load and the variable geometry of the cavity in the proprietary actuator. The time-plots of cooling-curves clearly show that the transfer of latent heat of fusion accounts for the majority of the interval.

The relatively high Biot number of the motor’s cylinder meant that simple analysis of its convection coefficient was not a good approximation. The experiments were repeated using distilled water as a substitute for the wax charge to obtain a better approximation. The convection coefficient for the cylinder was estimated at \( (h_c = 10 \, W \cdot m^{-2} \cdot K^{-1}) \) a relatively low value.

The overall implication of these findings is that the heat-motor will exhibit relatively large hysteresis since the characteristic cooling-curve differs for the heating-curve by the interval when latent heat of fusion is transferred. The passive transfers by convection with air alone are not very effective and exhibit low responsiveness. These findings provide data to support the studies on the operating behaviour of this actuator in the following chapters.
The previous chapter described the characteristic behaviour exhibited by a proprietary heat-motor during duty-cycles. Earlier, in the Literature review in Chapter 2 the existing application of the heat-motor to operate movable elements in buildings was limited to single motors with one threshold of response. In this chapter, this limitation is advanced by the development of a mechanism operated by a plurality of heat-motors.

The use-case of movable insulated window shutters was reviewed in Chapter 2, we learnt that it had three desirable positions in response to the seasonal variation in insolation. The context of this application is for a building façade with windows in a south-facing elevation. In this chapter, we learn how the operating force of a transfer-mechanism actuated by a plurality of heat-motors can be used to position the shutter in response to three different thermal conditions. The thermal performance of the window and shutter itself is not evaluated here, this falls outside the scope of the research question.

The chapter begins with a section that considers the feasibility of a plural system, it describes in detail the engineering properties in the thermal and mechanical domains of an arrangement with two heat-motors. Using this rationale, a number of practical mechanisms are constructed and then subjected to a testing regime using the climate-chamber environment. The findings of these tests are reported, analysed and their implications discussed.
4.1 A mechanical rationale for linking plural heat-motors

This section considers the feasibility of a mechanism operated by a plurality of heat-motors, it is based on the following assumptions. We learned from Duerig (1990) that the typical work done by a wax actuator against a biasing force is (20 to 30) % of the operating force. We are reminded that a biasing force is necessary because without it a wax actuator will not recover from an actuation displacement ($l \gg 0$) when there is no working load applied to the output.

In this plural arrangement, it will be assumed that two identical heat-motors ($A$) and ($B$) will independently do work against two identical biasing forces ($F_a$) and ($F_b$) respectively. It is also assumed that heat-motors ($A$) and ($B$) are charged with an identical mass of wax and that the phase-change expansion of wax A and B is the same ($\rho_{1,a} = \rho_{1,b}$ and $\rho_{s,a} = \rho_{s,b}$). Finally, the coefficient of expansion in their liquid state is the same ($\Delta \rho_{1,a}/\Delta T = \Delta \rho_{1,b}/\Delta T$).

Consider the arrangement of heat-motors shown in Figure 4.1(a). The actuation of heat-motor ($B$) is connected to a reference point ‘R’. This point coordinates between the actuation of the mechanism’s output ($Q$) and the working load that it operates. The displacement length of heat-motor ($B$) is ($l_b$) and is kept in check ($l_b \rightarrow 0$) by its biasing force ($F_b$), shown here by a compression spring. The body of the second heat-motor ($A$) is connected to the body of heat-motor ($B$) by a rigid link at reference point ‘C’. The displacement length of heat-motor ($A$) is ($l_a$) and is kept in check ($l_a \rightarrow 0$) by an independent biasing force ($F_a$), another compression spring. The actuation of heat-motor ($A$) is linked directly to the mechanism’s output ($Q$).

Given that the output ($Q$) does work against an operating force ($F$) by the actuation displacement of either or both ($l_a$, $l_b$), what are the balance of forces between heat-motor ($A$), ($B$) and output ($Q$) in each of their respective states? To answer this the characteristics of ($F$) needs to be defined as the evolution of the output ($Q$)’s actuation displacement ($l_q$) due to changes in the state of heat-motor ($A$) and ($B$) acts against ($F$) in different ways. We will assume that ($F$) is a dead-load that imposes a force of constant magnitude to resist the actuation displacement ($l_q$) by acting in the opposite direction as indicated by the arrows ($F \searrow$, $\searrow F$) in Figures 4.1(a to c).

The following is an account of how the balance of forces between the two heat-motors and the operating force changes in a progression through the three states in Figures 4.1 from (a to c). Figure 4.1(a) shows heat-motor ($A$) and ($B$) inactive, the biasing forces act independently against the actuation displacement of heat-motor ($A$) and ($B$) to keep the output ($Q$) in check ($l_q \rightarrow 0$).

In Figure 4.1(b) heat-motor ($A$) is active and ($B$) is inactive. Heat-motor ($A$) has a change in actuation displacement and does work against the working load ($F$) and its biasing force ($F_a$) under the stress of ($\sigma_a = (F + F_a)/A_A \{\text{in } N \cdot m^{-2}\}$), where ($A_A$) is the cross-sectional area of heat-motor ($A$). Since the mechanism’s output ($Q$) is directly connected to heat-motor ($A$) the actuation displacement of the output changes to ($l_q = 0 \rightarrow l_a$). While the mechanism’s output ($Q$) is under the stress of the operating force ($F$) it does not overcome the biasing force ($F_b$) applied to heat-motor ($B$) and thus remains stable.
When heat-motor \((A)\) becomes inactive it undergoes a recovery stroke due to the biasing force \((F_a)\) and overcomes the working load \((F)\) until the mechanism reaches \((l_a \rightarrow 0)\). Since heat-motor \((A)\) is directly connected to the mechanism output \((Q)\), the actuation displacement of the output changes to \((l_q = l_a \rightarrow 0)\). The alternative course of events is shown in Figure 4.1 (c) when both heat-motor \((A)\) and \((B)\) exhibit an actuation displacement. In this case heat-motor \((B)\) does work against the biasing force \((F_b)\) and displaces both heat-motor \((A)\) \textit{and} the working load \((F)\).

\textbf{Figure 4.1: Indicative section through comparator operated by plural heat-motors}

(a) In cold ambient conditions, neither heat-motor actuates.

(b) In warm ambient conditions, only the low temperature heat-motor actuates.

(c) In hot ambient conditions, both heat-motors actuate.
Heat-motor \((B)\) is then under stress 

\[
\sigma_b = \frac{(F + F_b + M \cdot l_b)}{A_b}
\]

\(\text{in } N \cdot m^{-2}\), where \((M)\) is the mass of the mechanism and \((A_b)\) is the cross-sectional area of heat-motor \((B)\). The resultant of this actuation displacement is that the mechanism’s output \((Q)\) changes to \((l_q = l_a \rightarrow (l_a - l_b))\) and the balance of mechanical forces remains stable. If heat-motor \((B)\) becomes inactive while heat-motor \((A)\) remains active then the actuation displacement of heat-motor \((B)\) will recover due to the biasing force \((F_b)\) and the output will change from \((l_q = (l_a - l_b) \rightarrow l_a)\).

Using this rationale, if we know the requirement for the operating force \((F)\) then we can specify the magnitude of the biasing forces \((F_a, F_b)\) and the stress and displacement length profiles \((\sigma_a/l_a, \sigma_b/l_b)\) of the two heat-motors \((A)\) and \((B)\) to operate against \((F)\) and maintain \((Q)\) in each of the three different states. Having established how a stable balance of mechanical forces can be achieved for each state, attention now turns to the thermal conditions that distinguish between each of the three stable states.

As we have seen in the previous chapter, it is the temperature of the wax charge that determines whether a heat-motor exhibits an actuation displacement. To demonstrate the operation of the mechanism, the activity of the heat-motors is differentiated by wax melting-point as follows; the melting-point of the wax charge in heat-motor \((A)\) has a low-temperature melting-point \((T_{mp, a})\) and heat-motor \((B)\) has a high-temperature melting-point \((T_{mp, b})\).

Using this, if we adopt the thermal conditions in the climate-chamber as an approximation of ‘Steady state’ conditions of thermal equilibrium as described in the previous chapter, then we can describe the different states of this mechanism in relation to the ambient temperature \((T_{amb})\). We find that the mechanical arrangement shown in Figures 4.1(a to c) behaves as a thermal comparator between ambient \((T_{amb})\) and the two melting-points \((T_{mp, a}, T_{mp, b})\) to provide a high-force mechanical output \((Q)\) with three different states \((a, b\) and \(c)\) as summarised in Table 4.1.

The sequence of transitions between states is constrained because the thermodynamic temperature \((T_{amb})\) cannot transition from \((a)\) to \((c)\) without first passing through the thermal conditions of \((b)\). When we apply this constraint to a diurnal duty-cycle, the mechanism could undergo two sequences of change, specifically either \(\{a \rightarrow b \rightarrow a\}\) or \(\{a \rightarrow b \rightarrow c \rightarrow b \rightarrow a\}\).

### Table 4.1: 'Truth-table' of a plural heat-motor operated temperature comparator

<table>
<thead>
<tr>
<th>Chamber set-point</th>
<th>Heat-motor A</th>
<th>Heat-motor B</th>
<th>Output Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T_{amb}) (\leq T_{mp, a})</td>
<td>Solid 0</td>
<td>Solid 0</td>
<td>0 a</td>
</tr>
<tr>
<td>(T_{mp, a} &lt; T_{amb} &lt; T_{mp, b})</td>
<td>Liquid (l_a)</td>
<td>Solid 0</td>
<td>(l_a) b</td>
</tr>
<tr>
<td>(T_{amb} \gg T_{mp, b})</td>
<td>Liquid (l_a)</td>
<td>Liquid (l_b)</td>
<td>(l_a - l_b) c</td>
</tr>
</tbody>
</table>
4.2 Schematic design of a thermo-mechanical system

Table 4.1 in the previous section enumerated the three stable states of the output \((Q)\) under ‘Steady state’ conditions when the components are in thermal equilibrium. However, this does not account for the state of the output \((Q)\) when the ambient temperature in the climate-chamber is \((T_{amb} = T_{mp,a})\) or when \((T_{amb} = T_{mp,b})\). In order to advance a description of the behaviour of this mechanism under those conditions we need a model that accounts for the time-temperature history of the wax because the state-transitions have a time dependency.

Advancing the mechanical rationale developed here, the following section presents a dynamic model that accounts for the evolution of the output \((Q)\) during phase-change transitions. We are reminded that when a heat-motor operates passively it behaves as an embodied thermo-hydraulic actuator with a phase-state that is inseparable from its thermal state. It is challenging to describe the behaviour of this system in a deliberately simplified way while remaining grounded in the underlying physical processes, this is because the heat-motor exhibits both non-linear behaviour and straddles across the domain of thermal and mechanical phenomenon.

The following section presents a qualitative description of how the plural system behaves using the analogue of an electronic circuit. This is chosen because its components can be representative of the behaviour in both domains and thus can be used to unify them.

4.2.1 Description of the system components

Figure 4.2 shows a model of the plural heat-motor system, it uses an electronic component to represent each part within and across the thermal and mechanical domains. A boundary is drawn around each of the two heat-motors to indicate the components that couple them together. Referring to Figure 4.2 from left-to-right, the following describes each component and what they represent in the actual system.

The input to this system is the set-point of the climate-chamber, it represents the ambient dry-bulb air temperature around the mechanism \((T_{amb})\). The input acts equally on heat-motor \((A)\) and \((B)\), however in both cases the input is coupled to each heat-motor through a resistor and capacitor network. The findings in the previous chapter showed that there is a time-lag between the set-point of the chamber and a heat-motor reaching thermal equilibrium. The resistor represents the resistance offered by the cylinder surface to the transfer of thermal energy from the air in the chamber and is characterised by the coefficient \((h)\). While the capacitance in the network represents the heat-capacity of a motor’s components and is characterised by \((c)\). Technically, \((c)\) represents the specific heat of the motor’s components \((in J)\) and the ground plane for \((c)\) represents absolute zero on the thermodynamic scale.

We consider heat-motor \((A)\) first, the resistor-capacitor network couples the chamber set-point \((T_{amb})\) to the input stage \((in +)\) of comparator \((A)\). The comparator takes the two inputs and compares the levels between them and expresses the positive outcome at its output \((out)\).
This represents the relation between the temperature of heat-motor \((A)\) and the threshold wax phase-change. The junction in the resistor-capacitor network represents the temperature of the wax \((T_a)\). Using this reference point, the temperature is compared against a reference connected to \((-)\) of comparator \((A)\) that represents the phase-change threshold of the wax \((T_{mp.a})\).

Figure 4.2: Thermo-mechanical circuit model of a plural heat-motor system
As we have seen in the previous chapter, the phase-change of wax exhibits hysteresis because the rate of change in the relation between \( T_a \) and \( T_{\text{amb}} \) is different during melting compared with during solidification. Comparator \( (A) \) represents hysteresis at the output stage \( (\text{out}) \) subject to the time-varying value of \( T_a \) at its input \( (\text{in}+) \), for now we will treat the comparator’s behaviour as a ‘black box’.

The output of comparator \( (A) \) crosses the thermal-to-mechanical domain to drive the base junction of a transistor, this then amplifies the signal at its output \( (\text{out}) \). The transistor represents the relation between wax temperature \( T_a \) and its specific density \( \rho_a \). The transistor amplifies the \( (\text{base}) \) at the output \( (\text{out}) \), this represents the change in actuation displacement by \( (\rho_a/l_a) \).

The transistor’s output \( (\text{out}) \) is an ‘open-collector’ type and represents the wax actuator ‘floating’ without a bias to a reference plane. This is representative of the physical behaviour of the actual heat-motor that does not recover from an actuation displacement without a biasing force. A resistor is used to bias the signal at the transistor’s \( (\text{base}) \) to the reference plane ‘C’, it represents the biasing force \( (F_a) \) that is provided by the return-spring in the physical mechanism.

Having described the components of heat-motor \( (A) \), those of heat-motor \( (B) \) are similar except for the following two differences. Firstly, in heat-motor \( (B) \) the reference input \( (\text{in}-) \) to comparator \( (B) \) is the phase-change threshold of the wax \( B \), this is characterised by the melting-point \( (T_{\text{mp}, b}) \) with a band-gap difference from the melting-point of wax \( A \). Secondly, in the mechanical domain the transistor’s output \( (\text{out}) \) characterises the actuation displacement \( (l_b) \) relative to the reference plane ‘\( \text{R} \)’ not ‘\( \text{C} \)’. This represents the difference in reference points of the mechanism shown in Figures 4.1(a to c).

The outputs from heat-motor \( (A) \) and \( (B) \) are connected to the inputs of a final comparator ‘\( \text{C} \)’. The output of heat-motor \( (A) \) is connected to the positive \( (+\text{in}) \) while the output of heat-motor \( (B) \) goes to the negative \( (-\text{in}) \). Comparator ‘\( \text{C} \)’ represents the mechanism linkage shown in Figure 4.1, it performs a differential comparison between heat-motor \( (A) \) and \( (B) \).

The final comparator has neither external feedback nor an internal reference point, therefore the polarity of the output \( (Q) \) on the load \( (F) \) depends upon which reference is used for comparison. In this case, ‘\( \text{R} \)’ is used as the common reference, This represents the linkage in the physical mechanism that compares the difference between the actuation displacement of heat-motor \( (A) \) and \( (B) \) as \( (l_q = l_a - l_b) \) irrespective of a reference point but drives the actuation stress imposed by load \( (F) \) relative to the reference point ‘\( \text{R} \)’.

The output of comparator ‘\( \text{C} \)’ is \( (Q) \) and this represents the actuation displacement \( (l_q) \) that operates the force \( (F) \) to re-position the movable shutter relative to the reference point ‘\( \text{R} \)’. This is shown in the arrangement of actuation displacement in Figure 4.1(c).

This section has established the components in a simplified model of the thermal comparator, and outlined how they are interrelated. The next section considers the time-independent actuation displacement \( (l_q) \) of the output \( (Q) \) for the different state transitions between a, b and c.
4.2.2 The characteristics of the thermal comparator’s output

In the previous section, the detailed workings inside the comparators \((A)\) and \((B)\) were treated as a ‘black box’. However, to characterise the output of the thermal comparator we need a working description of the hysteresis loops for heat-motor \((A)\) and \((B)\) respectively. To this end Figure 4.3(a) illustrates the actuation displacement and recovery strokes when heat-motor \((A)\) and \((B)\) are compounded together. The resultant output \((Q)\) can then be deduced from the operation of the comparator \((C)\) on the linked heat-motors and this is shown below in Figure 4.3(b).

**Figure 4.3: Indicative hysteresis loop by temperature vs actuation in dual wax actuator**

(a) Hysteresis loop for compound of heat-motor \((A)\) and \((B)\), by author after p. 215 in Duerig (1990)

(b) Plot showing the hysteresis loop for the resultant output \((Q)\) and the corresponding state

Note: In the top figure, the dotted line extending the actuation displacement for heat-motor \((A)\) and heat-motor \((B)\) is relative to the fixed link ‘C’ shown in Figure 4.1. The relative actuation displacement of \(A\), \(B\) and \(Q\) are shown indicatively and not to scale. The width of the hysteresis loop (\(X\)-axis for temperature) has been exaggerated for clarity.
4.3 A practical thermal comparator mechanism

Figures 4.3 (a and b) show that while the output \( Q \) follows the profile of heat-motor \( A \) when the temperature is \( T_{\text{amb}} \leq T_{\text{mp,a}} \), wax A continues to expand in its liquid state to form a band-gap in the temperature range \( T_{\text{mp,a}} < T_{\text{amb}} < T_{\text{mp,b}} \). The implication is that the resultant output in state 'b' will not have a stable position if the temperature fluctuates within this range.

The other implication of a band-gap in this arrangement is that the resultant output \( Q \) does not return to zero when wax B has finished melting. The size of the offset depends on the width of the band-gap and is essentially determined by \( (T_{\text{mp,b}} - T_{\text{mp,a}}) \). The larger the difference in melting-points the further offset from zero the resultant output \( Q \) will be in state 'c'.

Once the output \( Q \) has undergone an actuation displacement to reach state 'c', the comparator maintains a stable position due to the equal coefficient of expansion exhibited by wax A and B in their liquid state and since \( (l_q = l_a - l_b) \) they cancel each other out when the temperature exceeds \( T_{\text{amb}} > T_{\text{mp,b}} \).

This section has described the time-independent relationship between the output \( Q \) and ambient temperature, it has defined the parameters for the position of the output \( l_q \) in the three different states. The following section progresses from a theoretical treatment of the mechanical and thermal rationale to the realisation of a practical mechanism that can be tested.

4.3 A practical thermal comparator mechanism

Figure 4.4 shows an assembly of components that embodies the mechanical design rationale for a plural arrangement of heat-motors presented in the previous section. Since Figures 4.1 (a to c) shown earlier is an indicative longitudinal section through this embodiment, the following description of the realised design will be brief.

The assembly is mounted on a background that represents a notional façade panel. The mechanism itself consists of an outer enclosure (part 7) that is open at the front to allow air to circulate and promote convective heat-transfers with the climate chamber’s air-conditioning. An inner enclosure (part 6) slides with one degree-of-freedom against the outer enclosure. The inner enclosure contains the two heat-motors, the heat-motor (part 1) charged with low melting-point temperature wax and heat-motor (part 2) charged with high melting-point temperature wax.

A sliding linkage is used to connect the two heat-motors together with the resultant output (part 4 to 5). The linkage is free to travel in the range \( (0 \text{ to } 0.12) \) m, this easily accommodates the maximum displacement length of either heat-motor \( (l_a, l_b) \).

The inner enclosure also frames the biasing force (part 3) for both heat-motors, here these are provided by a pair coil-type tension springs for each motor. The resulting output of the machine is represented by the position of a mechanical output arm (part 5) with a degree of freedom in rotation about an axis fixed to the outer enclosure. The position of the output arm is connected directly to heat-motor \( A \) charged with the low melting-point temperature heat-motor (part 1) by a drive coupler (part 4).
4.3. A practical thermal comparator mechanism

Sensor instrumentation is incorporated for measuring the linear actuation displacement of the two heat-motors as well as the temperature inside and around the mechanism’s enclosure. Given this description of the practical embodiment, the following section presents an experiment to test the feasibility of the mechanical design rationale using this mechanism.

**Figure 4.4: Mechanism shown assembled (above) and component parts exploded (below)**

(1) Heat-motor (A) charged with low temperature melt-point wax \( (T_{mp,a}) \) highlighted in blue
(2) Heat-motor (B) charged with high temperature melt-point wax \( (T_{mp,b}) \) highlighted in red
(3) Spring assembly as biasing-forces \( (F_a, F_b) \)
(4) Drive coupler as output \( (Q) \)
(5) Transfer-mechanism to shutter pivot as force \( (F) \)
(6) Inner enclosure as common point \( (C) \)
(7) Outer enclosure as reference point \( (R) \)
(8) Mounting frame and brackets (as background building envelope)

Inventor® modelling by Nicholas Browne with development by author [Animation of mechanism](WMV, 2.4 Mb)
4.3. A practical thermal comparator mechanism

4.3.1 Experiments for testing a thermal comparator mechanism

The mechanism can be validated against the mechanical design rationale if it can be demonstrated that each of the two states ‘b’ and ‘c’ that is driven by the heat-motors (A) and (B) can be differentiated between each actuation displacement and recovery stroke. In the first instance, this test is conducted with only the biasing forces applied and without a working load attached. Details of the experiment set-up and methodology are described in Appendix E.1 on page 232.

Following on from the work in the previous chapter, the climate chamber facility is called upon to provide an environment where the ambient temperature (T_{amb}) can be controlled to duty-cycle the two heat-motors. The climate chamber is convenient because the temperature attained can be determined a priori relative to the two different melting-points so the test can also be repeated.

Figures 4.5 (a to c) shows one of the mechanisms during a pilot run of this experiment in the climate chamber. Heat-motor (A) with the low melting-point temperature wax is painted blue and is clearly visible in the centre, heat-motor (B) with the high melting-point temperature wax is behind inside the inner enclosure and hidden in this image.

Each image in Figures 4.5 (a to c) shows the practical mechanism in the state that corresponds to each state described by the mechanical design rationale in Figures 4.1 (a to c). From these, we can see in Figure 4.5 (b) and Figure 4.5 (c) that both heat-motors can successfully overcome their individual biasing forces. Figure 4.5 (c) shows that the mechanism successfully embodies the thermal comparator ‘C’ by subtracting the actuation displacement of heat-motor (B) from (A).

Data from the sensors show that the biasing force is sufficient for both heat-motors to complete a duty-cycle because both undergo a recovery stroke, this is confirmed by reviewing the time-lapse motion capture. This pilot study shows that the practical mechanism validates the mechanical design rationale by successfully differentiating between the three states ‘a’, ‘b’ and ‘c’. Having established this, a batch of thermal comparator mechanisms were constructed and subjected to a battery of duty-cycles in the temperature range of T_{mp} = (10 to 38) °C.

From these tests the size of the band-gap described in Figure 4.3 (b) can be determined, the results are shown in Figure 4.6 and summarised in Table 4.2. The ‘band-gap’ of output (Q) in state ‘b’ has a mean value of (3.4 mm). Scrutiny of the data shows that the actuation displacement (l_q) of the thermal comparator’s output (Q) has a sensitivity of 3.9% of (l_a) to fluctuations in the temperature (T_{amb}) within the range (T_{mp,a} to T_{mp,b}) °C. This result has implications for the stability with which the thermal comparator can position the output (Q) in state ‘b’.

This result also affects the precision with which the mechanism can position the output (Q). Unfortunately the variation in actuation displacement (l_a) between units was found to be the same order of magnitude as the band-gap. This can be explained partly by tolerances in the design and assembly of the prototypes, but the main practical obstacle was the difficulty in establishing a consistent coordination between the start-to-open point of each heat-motor and the zero-point in each mechanism’s linear translation at a defined temperature.
4.3. A practical thermal comparator mechanism

Figure 4.5: The 'as built' thermal comparator during testing in the climate chamber

(a) Verifying the mechanism output when ($T_{\text{amb}} \ll T_{\text{mp},a}$) Cold ambient

(b) Verifying the mechanism output when ($T_{\text{mp},a} < T_{\text{amb}} < T_{\text{mp},b}$) Warm ambient

(c) Verifying the mechanism output when ($T_{\text{amb}} \gg T_{\text{mp},b}$) Hot ambient

Note: Practical mechanism tested without an operating load ($F$). The biasing force is applied by a pair of tension springs and varies from $F_a = (150$ to $500) N$ at a rate of $dF_a/dl_a = 3720 N \cdot m^{-1}$ between the actuation displacement length of $l_a = (0$ to $0.09) m$. While this force increases the cavity pressure from $P_c = (5.2$ to $19) kPa$, the effect on the wax’s equilibrium melting-point is small and will be ignored. [Time-lapse study] (WMV, 4.6 Mb)
Figure 4.6: The temperature-actuation profile of heat-motor A in five† thermal comparators

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Test cell</th>
<th>Output Q 'b'</th>
<th>Output Q 'c'</th>
<th>Heat-motor A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Run</td>
<td>1</td>
<td>Heat-motor A</td>
<td>Heat-motor A</td>
<td>Heat-motor A</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td></td>
<td>max(lₐ) at Tₐmb</td>
<td>Band gap³</td>
</tr>
<tr>
<td>(Date)</td>
<td>(ID)</td>
<td>lₐ at Tₐmb ²</td>
<td>(mm)</td>
<td>(mm) (%)⁴</td>
</tr>
<tr>
<td>2009-03-17</td>
<td>#4</td>
<td>85.5</td>
<td>88.9</td>
<td>93.2</td>
</tr>
<tr>
<td></td>
<td>#5</td>
<td>84.2</td>
<td>88.0</td>
<td>92.6</td>
</tr>
<tr>
<td></td>
<td>#6</td>
<td>84.3</td>
<td>88.0</td>
<td>91.7</td>
</tr>
<tr>
<td></td>
<td>#8</td>
<td>81.9</td>
<td>84.9</td>
<td>89.2</td>
</tr>
<tr>
<td></td>
<td>#9</td>
<td>83.8</td>
<td>87.0</td>
<td>90.5</td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td>83.9</td>
<td>87.4</td>
<td>91.4</td>
</tr>
<tr>
<td>St.D (n=5)</td>
<td></td>
<td>1.30</td>
<td>1.53</td>
<td>1.61</td>
</tr>
</tbody>
</table>

† Sampled from a batch of nine mechanisms. [Time-lapse study] (WMV, 5.3 Mb)

- Chamber temperature (Tₐmb)
- Heat-motor A (Test cell #4)
- Heat-motor A (Test cell #5)
- Heat-motor A (Test cell #6)
- Heat-motor A (Test cell #8)
- Heat-motor A (Test cell #9)

¹ ID of mechanism from batch. ² The ambient air temperature in the climate chamber (Tₐmb).
³ The range within which the temperature fluctuations affect state (b) taken here as (lₐ at 25°C)−(lₐ at 29°C).
⁴ Size of band-gap as a percentage of the actuation displacement at (Tₐmb ≈ Tmp, b).
4.3. A practical thermal comparator mechanism

Figure 4.7: The temperature-actuation profile of output \((Q)\) in four\(^ \dagger \) thermal comparators

![Diagram showing temperature-actuation profile for four thermal comparators.](image)

\(^\dagger\) Sampled from a total batch of nine mechanisms. [Time-lapse study] (WMV, 5.1 Mb)

Figure 4.8: Image showing batch of comparators in climate chamber during testing

![Image showing batch of comparators in climate chamber.](image)

The mechanical design rationale made the simplifying assumption that the mass of wax A is the same as the mass of wax B, this has been verified by a mensuration survey (refer to Table B.2 and B.3 in Appendix B.1 on page 222). The survey also verified that the manufacturing tolerance of the proprietary heat-motors themselves is high \((St.D = 0.001\) in sample size \(n = 23\)).

Accepting this, the data from the duty-cycle tests at temperatures above \((\geq T_{mp,b})\) indicate that the net ratio of expansion during the phase-change of wax B is slightly greater than wax A.
4.3. A practical thermal comparator mechanism

The battery of tests was unable to establish whether this finding can be attributed to limitations in the mechanical design rationale’s two remaining assumptions. It may be explained if the phase-change expansion of wax A and B is the same ($\rho_{l,a} = \rho_{l,b}$ and $\rho_{s,a} = \rho_{s,b}$) but the coefficient of expansion in their liquid state is not $(\Delta \rho_{l,a}/\Delta T) \neq (\Delta \rho_{l,b}/\Delta T)$ or vice versa, or a permutation of both. The attempt to verify and distinguish between these two assumption is hampered because wax A consists of a proprietary blend of waxes ‘as supplied’ while wax B is a single constituent.

Irrespective of this difference, after we have accounted for the small band-gap the implication of this result for the mechanism’s output ($Q$) in state ‘c’ is that it drives the actuation displacement ($l_a$) slightly beyond the zero-point ($l_a \to 0$) as shown by the time-series in Figure 4.7. The other implication of this finding is for the stability of positioning output ($Q$) in state ‘c’ if $(\Delta \rho_{l,a}/\Delta T) \neq (\Delta \rho_{l,b}/\Delta T)$ when the temperature is ($T_{amb} \gg T_{mp,b}$). This arrangement will exhibit a small negative-biased temperature-dependent drift to the net difference in ($l_a = l_a - l_b$).

While this section has established the feasibility of the mechanical design for a plural heat-motor mechanism with a practical embodiment, it has also identified uncertainty in the stability and positioning of the output ($Q$) in states ‘b’ and ‘c’. These have been shown to be partly due to limitations in the assumptions of the mechanical design rationale and partly due to the procedure used to assemble the thermal comparators.

The results of the pilot study show that the main limitation of the thermal design rationale is that it depends on a ‘free running’ balance-of-forces between the heat-motors and the biasing force to maintain the position of output ($Q$) in state ‘b’ and ‘c’. The results show that in fact these are sensitive to fluctuations in temperature ($T_{amb}$) above the melting-points of wax A and B due to the continued expansion of wax in its liquid state. If there is fluctuation in the temperature then there will be some fluctuation in the positioning of the actuation displacement.

We can make a simple modification to the practical thermal comparator mechanism to mitigate against the uncertainty in positioning due to temperature fluctuation. The ‘free running’ actuation of the output ($Q$) can be mechanically constrained by introducing end-stops while maintaining the balance-of-forces between each of the three states. Specifically, we can add an end-stop to the actuation displacement of heat-motor ($A$) to constrain the continued expansion of wax A in its liquid state and maintain a stable position in state ‘b’. If we apply this measure, an equal and opposite mechanical end-stop would be needed to the actuation displacement of heat-motor ($B$) in order to maintain a stable position in state ‘c’ and prevent heat-motor ($B$) overshooting due to the continued expansion of wax B in its liquid state.

Using what we have learnt from the static tests in this section, the following section explores how this can be applied to provide different pre-determined positions for the thermal comparator’s output ($Q$). In order to demonstrate how the comparator differentiates between the states ‘a’, ‘b’ and ‘c’, we need to develop the thermal comparator output ($Q$) to operate the working load of a full-size movable shutter through a transfer mechanism.
4.3.2 Differentiating the output of the thermal comparator

The literature review in Chapter 2 identified desirable positions for an external thermally insulated shutter to serve the functions of night-time insulation and the prevention of overheating during summer-time. These functions could be combined into a movable shutter with a repertoire of three different responses, these are summarised in Table 4.3.

The design of the shutter itself could adopt many different arrangements, as could the degrees-of-freedom in its deployment, these were briefly reviewed earlier in Chapter 2 from reference works such as (Langdon, 1980; Hochberg et al., 2010). The previous section established that the linear actuation displacement available from the ‘as built’ mechanism is in the order of \( l_o \approx (87.4 \text{ to } 91.4) \text{ mm} \), most arrangements of a full-size construction that we attempt to operate would require some mechanical translation. The following is a description of a transfer-mechanism to translate the output of the comparator to position the shutter for each of these three functions.

For this ‘proof of concept’, the actuation displacement of \((l_a)\) is constrained using a track end-stop so that the output \((Q)\) transitions from state ‘a’ to ‘b’ by rotating the shutter leaf from closed-to-open through \(90^\circ\) about the pivot line. The actuation displacement of \((l_b)\) is also constrained using a track end-stop so that the output \((Q)\) transitions from state ‘b’ to ‘c’ by rotating the shutter leaf through \(-45^\circ\) about its pivot. This arrangement is illustrated in Figures 4.9 (a to c), it shows the thermal comparator mechanism operating a shutter leaf into position \((\theta_r = 0^\circ, 90^\circ \text{ and } 45^\circ)\) in the three thermal states ‘a’, ‘b’ and ‘c’ respectively.

**Table 4.3: Shutter position in response to different diurnal conditions across seasons**

<table>
<thead>
<tr>
<th>Season</th>
<th>Description of diurnal conditions</th>
<th>Shutter position</th>
</tr>
</thead>
<tbody>
<tr>
<td>All seasons</td>
<td><strong>During night-time</strong> A window to a heated space will tend towards being a sink of net heat-loss in the building’s heat-balance. There are conductive, convective and radiant losses through the window to ambient.</td>
<td>Insulate the window (Close) See Figure 4.9 (a)</td>
</tr>
<tr>
<td>All seasons</td>
<td><strong>During day-time</strong> A window provides access to daylighting and amenity to view outside</td>
<td>Expose the window (Open) See Figure 4.9 (b)</td>
</tr>
<tr>
<td>Summer</td>
<td><strong>During day-time</strong> The ambient temperature may reach (and exceed) indoor temperatures for prolonged periods, direct insolation may lead to periods of overheating due to excessive gains from radiant energy. While there is still daylight utility.</td>
<td>Shade the window (Part-open) See Figure 4.9 (c)</td>
</tr>
</tbody>
</table>
4.3. A practical thermal comparator mechanism

Figure 4.9: Plan and part-section through thermal comparator operating shutter leaf

(a) Cold ambient temperature, shutter closed

(b) Warm ambient with the low temperature heat-motor actuating, shutter fully opened

(c) Hot ambient with both heat-motors actuating, shutter half-opened

Resultant output \( Q \) translated through CO2 'slider-crank' mechanism. Motion constrained by end-stops
The thermal comparator’s resultant output ($Q$) is translated from linear-to-rotational motion by a ‘slider-crank’, the CO2 that consists of two linked bars and a slider as classified by Reuleaux (1876). The CO2 linkage translates actuation displacement ($l_q$) to rotate the opening angle of a side-hung pivoting shutter leaf. The translation can be described by Euler’s triangles if the actuation displacement ($l_q$) and the desired rotation angle ($\theta_r$) are specified as follows.

Based on the mean value of ($l_q = 0.0874 \text{ m}$) from the previous section to open the shutter by ($\theta_r = 90^\circ$) in order to expose the window to admit daylight, the geometry of the CO2 linkage is solved iteratively using the method described in (pp. 52-3 by Khare, 2007) and plotted in Figure 4.12. From this, to partly close the shutter leaf by rotating it ($\theta_r = -45^\circ$) in order to shade the window, the actuation displacement of heat-motor B needed to achieve this is found by inverse kinematics to be ($l_b = 0.0874 - 0.0596 = 0.0278 \text{ m}$). This determines the position of the mechanical end-stop along the track of heat-motor B’s displacement.

This transfer-mechanism was then constructed with a frame and shutter as shown in Figures 4.11 (a and b) and subjected to a battery of tests in the temperature range (15 to 37) °C. Figure 4.10 is a time-series of the plural heat-motors actuating to drive the thermal comparator through a sequence of states ($a \rightarrow b \rightarrow c \rightarrow b \rightarrow a$) and shows the corresponding deployment angle of the shutter leaf.

**Figure 4.10: Time-series of thermal comparator deploying shutter during test**

![Figure 4.10](image)

*Note: Result sampled from a series of tests # (200 to 209) in April 2008 with thermal comparator Mk.II. The dotted portion of the lower plot is an estimation due to uncertainty in the measurements at positions ($\theta_r > 63^\circ$).*
Figure 4.11: Testing the thermal comparator mechanism to operate a pair of shutter leaves

(a) Thermal shutters driven to the closed position by the mechanism when ambient is cold

(b) Thermal shutters driven to the open shading position by the mechanism when ambient is hot

Note: Images taken inside the climate chamber during a 24-hour diurnal cycle emulating the temperature profile of a hot summer day. Each full-size insulated shutter leaf measures $(1.01 \times 0.63 \times 0.095) \text{ m}$ with a combined mass $m = (17 \text{ to } 18) \text{ kg}$.  

[Time-lapse study] (WMV, 3.6 Mb)
4.3. A practical thermal comparator mechanism

Further bays were constructed and data for the actuation displacement length \( l_q \) and corresponding shutter deployment angle \( \theta_r \) was collected from four thermal comparator and shutter assemblies, and added to the plot in Figure 4.12.

The results show that while the four transfer-mechanisms reached (> 96% of \( l_q \)) this only translated to (73.6% of \( \theta_r \)) and therefore did not fully deploy the shutters. This is due to shortcomings during assembly, specifically the difficulty in coordinating the \((\pm 0.015)\ m\) adjustment between the zero-point of part (4) and (5) in Figure 4.4 with heat-motor (A).

The steepness of the gradient in the geometric solution at values near the maximum actuation displacement \((dx/dy of curve \ldots)\) shows that without applying the mechanical constraint the apparently small band-gap has large effect of the shutter positioning. The sensitivity of 3.9% to the temperature fluctuation in the range \( T_{amb} = (T_{mp,a} \text{ to } T_{mp,b}) \) °C when the actuation displacement reaches \( l_q = (0.084 \text{ to } 0.087)\ m\) would have led to a large \( \theta_r = (0 \text{ to } 15)\ °\) uncertainty in the shutter deployment. Even if all the practical comparators had been assembled flawlessly.

If the thermal comparator has this degree of sensitivity to temperature fluctuations in the climate chamber where thermal conditions are be tightly controlled, it would obviously be exacerbated in an environment where temperature will fluctuate unpredictably. Having established the need for mechanical constraints, the following section reviews the move from the laboratory to in realistic environment and the implications for the assumptions made thus far.

Figure 4.12: 'As built' vs model of transfer-mechanism's linear-to-rotational translation
4.4 Regulation with the thermal comparator’s output

The previous sections have progressed from considering the feasibility of a plural heat-motor to verifying the design of a practical embodiment, albeit with limitations that have been identified in validating aspects of its operation. This has been done at the expense of simplifying assumptions that were necessary to conduct practical tests on the mechanism.

While the mechanical design rationale is transferable in the move from an environment with stable pre-determined temperatures to one with dynamic conditions, the thermal design rationale presented in this chapter is confined to the environment of the laboratory. Consequently, the results obtained here represent static conditions and assume a temperature range that can be pre-determined.

In order to develop the thermal design rationale so that it describes the behaviour of the thermal comparator’s output under conditions when the input is dynamic and cannot be pre-determined, the implications of the simplifying assumptions are now revisited.

Firstly, it was declared that the state of the thermal comparator’s output \(Q\) is used to emulate the desired position of the shutter, given that the shutter is a moderator for the exposure of the window to the prevailing external conditions. The behaviour of output \(Q\) can be described qualitatively as a servomechanism because it regulates \(Q\) given the input \(T_{\text{amb}}\). Specifically, the classical definition of a servomechanism as expressed in Equation (4.1) relates the regulating error vector \(e(t)\) with an input \(x(t)\) and output \(Q(t)\) at time \(t\) after p 2 in MacColl (1945).

\[
e(t) = K \cdot x(t) - Q(t) \quad \{\text{unitless}\} \quad (4.1)
\]

We are reminded that the thermo-mechanical hysteresis of the comparator shown earlier in Figures 4.3 (a and b) does not account for the dimension of time, since hysteresis is defined as a rate independent memory effect by Visintin (1994). When we scrutinize the thermo-mechanical model in Figure 4.2 on page 67 against Equation 4.1 in the dimension of \((t)\) we can see that if we cannot determine the temperature of \((T_{\text{amb}})\) then the comparator’s output \(Q(t)\) cannot be dependent on \((T_{\text{amb}})\) to differentiate between wax A and B to regulate the displacement length of \((l_q)\).

Secondly, as we found in Figure 4.7 and 4.6 the phase-change of wax A and B exhibit non-linear discontinuities, therefore the relational value of \((K)\) in Equation 4.1 does not approximate a constant. This would not matter for a system under closed-loop control because it would input extra energy to feedback and compensate for any rate-dependent behaviour.

However, this comparator operates without feedback and solely by passive energy transfers, therefore the thermal design rationale now needs to be reframed to account for the rate-dependent behaviour of \(x(t)\). Specifically, how does the time-dependent evolution of passive energy exchanges with the prevailing conditions affect the behaviour of thermal comparator’s output. In the following section the first implication is addressed in more detail.
4.4. Regulation with the thermal comparator’s output

4.4.1 Differentiating the phase of waxes: Sensible vs Latent heat

This chapter considered how a variation in wax melt-point can be used to evaluate thermal conditions by differentiating between the states of two heat-motors. This assumes a ‘Steady state’ of thermal equilibrium at a temperature that can be determined. Under these conditions the ambient temperature can be compared with the melt-point of the wax charge in each heat-motor.

This thermal design rationale is useful in the laboratory because we can verify the heat-motor’s operation of the thermal comparator’s mechanism. It is feasible to approximate conditions of thermal equilibrium at a temperature that can be determined, and transitions between them can be repeated as needed. However, outside the laboratory where meteorological conditions prevail it is not possible to be deterministic about the attainable temperature. While attainable temperature is obviously un-steady in a diurnal cycle which makes it more difficult to treat analytically, it is also the realistic thermal conditions that exist around the outside of buildings.

Figure 4.13 shows the relationship between displacement, specific heat and temperature for three waxes during phase-change. The difference between considering transient energy flow and ‘steady state’ temperature is highlighted by comparing the displacement vs specific heat (top-left corner) with the displacement vs temperature (top-right corner). The difference between each melting-point is small, but the latent heat of fusion \( H_f \) is large between all of them.

Figure 4.13: Wax expansion at phase-change compared with temperature and specific heat

Note: Comparison based on equal mass of wax \( (m_w = 0.05 \, kg) \), this is approximately the mass of wax in the Bayliss heat-motor. Other physical properties of the three waxes are tabulated in Appendix A.1
4.5 Summary

In this chapter we learnt a mechanical arrangement of plural heat-motors that differentiated between ambient temperatures against the melt-point of two different wax charges. An output was regulated by this mechanism and exhibited three stable states under conditions of thermal equilibrium. Each of these can be defined by differentiating between the melting-points of the fusible material used to charge the heat-motors. The novelty of this arrangement is tested through the UK’s Intellectual Protection Office (IPO) with a provisional patent application for the invention of an “Ambient temperature response window actuator” by (Gage and Leung, 2008).

This mechanism advances the limitation identified in the application of single heat-motors. It had two parts, firstly a rationale for differentiating the state of two or more heat-motors as inputs. Secondly, to construct a linkage and transfer mechanism that evaluates the state of the inputs. Using this mechanical rationale, further comparative evaluations could be developed by devising mechanisms that perform different logical evaluations or by extending the number of heat-motors to differentiate between more states of thermal equilibrium, or a permutation of both.

The three stable states ‘a’, ‘b’ and ‘c’ of the output ‘Q’ that were shown in Figure 4.3 (b) on page 69 are defined by their position outside the range of the control strokes in heat-motors (A) and (B). This contradicts engineering convention because of the portion of the wax actuator’s displacement length that is being used operationally as shown on pp. 214-16 by Duerig (1990). It also highlights the nature of dependence on ‘free’ transfer of thermal energy in passive systems as distinct from ‘forced’ transfer in closed-loop control that was previously considered a limitation of heat-motors as identified in technical reports (p. 8 by Tibbitts, 1992).

We wish to move from the thermal laboratory to realistic outdoor conditions, and take the desirable operating properties of the practical mechanism developed in this chapter with us. However, to achieve this we need to overcome two disparities. One is to account for the limits of the climate chamber to replicate realistic outdoor conditions as detailed earlier in Chapter 3 on page 60. The other is to account for the limits in the rationale to differentiate between the comparator’s inputs using different melting-points in order to reflect the realistic variability in outdoor ambient temperature. This demands that we learn a suitable method to differentiate the state of each heat-motor accordingly.

The results of a pilot-study to observe thermal shutters operated in-situ under external conditions is reported in Appendix H.1 on page 261. This demonstrated the mechanical and thermo-hydraulic viability of physically operating a full-size practical window shutter system.

Rather than develop further mechanisms to compare the inputs in different ways or extend the number of inputs to account for further conditions, we confront the question of how to transition from the laboratory to a realistic environment for the mechanism developed in this chapter. In the next chapter we learn a method to differentiate between the heat-motors under transient conditions and develop the thermal design rationale accordingly.
Chapter 5

Comparator response: A seasonal output

In the previous chapter, a thermal comparator mechanism was developed to demonstrate that a mechanical output could be regulated into three stable states when in thermal equilibrium with the ambient temperature. This chapter considers the implications of moving from the controlled environment of an air-conditioned climate chamber to the uncontrolled environment of synoptic weather conditions. In particular, from where the thermal environment is dominated by convective heat-transfer with air to conditions where the energy-balance is determined by the dynamics of gains from solar radiation versus losses to ambient by convection. This requires a change in how the phase state of the wax is determined, from a 'Steady state' to time-varying transient conditions.

This chapter seeks to develop the thermal design rationale to account for how the comparator mechanism in Chapter 4 could respond to the diurnal differences in the intensity of sunlight under clear-sky conditions. Since realistic conditions are dominated by passive insolation, this chapter investigates the feasibility of selective exposure to sunlight as the means to shift the heat-motor’s energy-balance in favour of undergoing a duty-cycle.

The practical feasibility of this is demonstrated by existing applications using a single heat-motor, as surveyed in Section 2.3.1 during the literature review in Chapter 2. However, this chapter asks whether the thermal design rationale can be developed to exploit the seasonal differences in the diurnal intensity of sunlight. This is in order to selectively differentiate between the duty-cycles of the plural heat-motors as the inputs that operate the thermal comparator.

The context of the thermal comparator’s application is carried forward from the previous chapter, that is for operating external shutters to windows in a south-facing façade. The annual pattern of insolation will be characterised for a south-facing vertically-inclined plane aspect at Latitude 51.5°N, and this chapter considers the climate context experiences in London (UK).
5.1 Thermal design rationale

This chapter begins by reframing the thermal design rationale developed in the previous chapter, this is in order to account for how the heat-motor’s state can be determined by a dynamic energy-balance between passive gains and losses.

Firstly, consider the case of a heat-motor placed inside an enclosure that allows heat-transfers with ambient by convection but shields it from solar radiation. The ‘Stevenson screen’ is an enclosure that approximates these conditions, as described on pp. 116-8 in Appendix V by MetOffice (1913). Data for air temperature has been collected inside ‘Stevenson screens’ by the (MetOffice, 2010) for many decades, this is drawn upon to estimate the annual rates of diurnal duty-cycling. Figure 5.1 shows the first-order results for a range of wax melting-points.

Analysing the results for each duty-cycle category indicates that the fluctuation in the temperature of air across diurnal cycles cannot be depended upon to duty-cycle a heat-motor on a daily basis on its own. Further analysis of time-series for air temperature at hourly intervals from reference meteorological data for London shows that the rate-of-change is \((dT_{db}/dt) < \pm 3\,^\circ\text{C} \cdot \text{h}^{-1}\) for > 95% of hourly-intervals. This observation together with the findings from the thermal laboratory work reported in Chapter 3 suggests that the temperature of the heat-motor will correspond to ambient for most of the time.

If a heat-motor is placed in a ‘Stevenson screen’ then any duty-cycling that it exhibits is solely dependent upon fluctuations in the ambient temperature. To determine the heat-motor’s duty-cycles the wax’s melting-point would need to be related directly to the diurnal evolution of ambient temperature. This is problematic because the air temperature of synoptic weather conditions is both seasonally variable and diurnally unpredictable, on this basis we cannot do any better at determining the duty-cycles. Even if we could, the results from the laboratory work reported in Chapter 3 indicates that any phase-changes that did occur would be slow using the proprietary heat-motor, with response time-lags in the order of hours.

To make a deterministic statement about the annual rates of duty-cycling for a wax with a given melting-point from the passive harvesting of environmental energy, there needs to be an advancement from the unpredictability of the thermal environment inside a ‘Stevenson screen’. Having characterised the thermal behaviour of a proprietary heat-motor of high latent heat of fusion and low convective coefficient, the following explores if this limitation may be remedied by amplifying the ratio of gains from sources against the losses to sinks. To achieve this we turn to capturing insolation as the thermal energy source.

This presents a challenge, because the characteristics of both passive gains and losses involve continuous radiation, convection and conduction processes. In order to simplify the problem, heat-transfers with ambient sinks by conduction will be ignored. The practical implications are that the construction and selection of materials will aim to minimise the effects of thermal bridging between the heat-motor and the ambient surroundings.
This is in contra-distinction to the design, materials and construction of the practical thermal comparator mechanism described in the previous chapter. The difference is that attaining thermal equilibrium between the mechanism and the chamber was treated as largely rate-independent. This distinction further emphasises the shift in the thermal design rationale towards describing the comparator’s state in terms of transient processes that are rate-dependent.

To test the feasibility of this rationale, the ratio of gains from solar radiation is compared with the losses by convection to the ambient temperature given the climate considered in this study. Observations during a pilot-study were encouraging, they showed the proprietary heat-motor inside a glazed tube exposed to direct radiation would duty-cycle despite the ambient temperature remaining below the wax’s critical point, as reported by Leung and Croxford (2007) in Appendix G.1.

There are further grounds for optimism since qualitatively the rate of heat-transfer by radiation is \( \frac{q''_{\text{rad}}}{\Delta T^4} \) after Stefan’s Law compared with the rate of heat-transfer by convection that is only \( \frac{q''_{\text{conv}}}{\Delta T} \) after Fourier’s Law, where \( \Delta T \) is the temperature difference between source and sink. This suggests that it ought to be possible to obtain sufficient gains from solar radiation more predictably than by convection from ambient air.

The implication for the thermal design rationale is that it would rely solely on the effective capture of insolation to melt a wax charge by heating it above the ambient temperature. As a consequence, the rationale would then depend upon ambient as the passive sink to reject accumulated heat gains in order to solidify the heat-motor’s wax.

To summarise, in the previous Chapter 4 the state of heat-motor \( (A) \) was differentiated from \( (B) \) by using a different wax while they shared the same thermal environment. The rationale is now reframed so the state of heat-motor \( (A) \) is differentiated from \( (B) \) by exposing them differently to the dynamics of the prevailing thermal environment while charging them with the same wax. Before exploring how their exposure can be differentiated, a method is needed to determine what wax melting-point \( (T_{\text{mp}}) \) might be suitable in both heat-motor \( (A) \) and \( (B) \).

**Figure 5.1: Histogram of the annual rates-of-change in dry-bulb temperature of air**

![Histogram of annual rates-of-change in dry-bulb temperature of air](image)

*Note: Instances of temperature gradient in each interval bin / % of hrs*

Based on the TRY (Test Reference Year) weather data-set for London UK, from Chap. 2 in (CIBSE, 2007)
5.1. Thermal design rationale

5.1.1 Wax melting-point vs the range of ambient temperature

This section describes how a suitable wax can be found by comparing the melting-point of each fully refined wax in the \( n \)-paraffin series against the minimum temperature that would maintain ambient as a thermal sink by convection. For a given climate, an estimate is made for the percentage of days per annum when a complete melting and solidification cycle would be expected.

Since the melting-point of paraffins can be selected in an approximately stepwise manner by carbon-number \((n)\), a value for \((n)\) is calculated against day numbers \((dn)\) with time-series for air temperatures \((T_{db})\) at intervals \((i)\) across a year as data-set \({(T_{db})}_{1,1} \ldots {(T_{db})}_{366,i}\). This selection is restricted to the temperature range between \(\text{max}(T_{db})\) and \(\text{min}(T_{db})\) from the climate data relevant for London (UK). The result of this numerical study on each refined wax in the \( n \)-paraffin series is shown in Figure 5.2. The first wax in the \( n \)-paraffin series with a melting point that remains above the maximum daily air temperatures on an annual basis is the wax \( n \)-octadecane \((C_{18}H_{38})\) with a melting point of \((T_{mp} = 28.2 ^\circ C)\) for \((>97\%)\) of intervals.

While a higher value of \((n)\) would still satisfy this evaluation, gains are also harvested passively therefore a higher melting-point would demand an even more effective capture of available gains. Therefore, studying this system’s behaviour using the lowest melting-point that still meets the rationale’s criteria ‘hedges one’s bet’ on its most finely-balanced embodiment for the UK’s climate.

Figure 5.2: Melting point temperatures vs annual distribution of ‘duty-cycle’ categories

Note: Evaluation of seven fully refined \( n \)-paraffin waxes and one blended wax with melting-points in the range of \((-10\) to \(40) ^\circ C\) against meteorological data for daily maximum air temperatures in London (UK) from (MetOffice, 2010). See Appendix F.1 for the algorithm used to evaluate \((n)\). The duty-cycles and melting-point of water at \((T_{mp} = 0 ^\circ C)\) is included as a comparative datum.
A complementary empirical study was carried out using a simple physical model with the middle to high-range of melting-points placed in a south-east facing ventilated window box as shown in Figures 5.3 (a to d). Time-lapse observations were made of the diurnal melt-solidification behaviour for this series of waxes, and how these varied on days from late-summer through to mid-winter.

These showed that on mostly sunny days, waxes \( n = 16, 17 \) underwent duty-cycles but this was seasonally dependent on the ambient air temperature as shown in Figures 5.3 (a,b) vs (c,d), time-lags were long. Wax \( n = 18 \) only melted on the warmest days with clear skies. The time-lapse studies support the findings of the numerical study that waxes \( n < 15 \) are mostly liquid when not duty-cycling and waxes \( 15 \leq n < 18 \) are mostly solid when not duty-cycling. The physical model confirmed that wax \( n = 18 \) will rarely duty-cycle by convective exchanges alone. This physical model allowed the free exchange of external ambient air and showed that the convective loss would have some seasonal variation, the following considers the energy balance.

**Figure 5.3: Passive phase-change of seven n-paraffin waxes under ambient conditions**

- (a) Mid-morning in duty-cycle melting \( C_{16} \) at 18.8°C
- (b) Late-evening in duty-cycle solidifying \( C_{16} \) at 15.7°C
  
  Observation on a cool day 24th September 2010 [Time-lapse study] (WMV, 3.5 Mb)

- (c) Mid-morning in duty-cycle melting \( C_{17} \) at 28.1°C
- (d) Late-evening in duty-cycle solidifying \( C_{17} \) at 18.6°C
  
  Observations on a warm day 9th September 2010 [Time-lapse study] (WMV, 1.7 Mb)

*Note:* A physical model of refined waxes in the carbon-number series \( \text{(C}_{13}\text{H}_{28} \text{ to C}_{19}\text{H}_{40}) \) and the Bayliss blend. Arranged by melting-point temperature in ascending order from left-to-right between \( T_{mp} = (5.9 \text{ to } 36.4) °C \). Observations made in London (UK) summer to winter 2010.
5.1.2 Estimating the specific energy to actuate a heat-motor

Having established the melting-point of a wax ($T_{mp}$) that is above the ambient temperature ($T_{db}$) for the majority of the year, the magnitude of convective loss that would have to be overcome by gains to melt the wax is indicated by the deficit of sensible heat below ambient. If we assume that the heat-motor and thermal comparator mechanism does not have a net change in the stored energy across each diurnal interval, then an initial temperature difference ($\Delta T$) for each day number ($dn$) can be calculated using Equation (5.1) from $init(T_{db})$ for the hour before sunrise.

$$\Delta T = T_{mp} - init(T_{db}) \quad \{\text{in } ^\circ \text{C}\}$$

(5.1)

While ($\Delta T$) is dependent on $init(T_{db})$ and therefore may exhibit some seasonal variability, it only accounts for the portion of sensible heat in the heat-motor’s energy budget. However, it also needs to account for the latent heat and any mechanical work done, for that we can apply the Law of Conservation of Energy to determine the energy needed to actuate a heat-motor ($E_{dn}$).

The net amount of energy that needs to be absorbed by the heat-motor through the balance of gains from solar radiation and losses to ambient within the first half of the photo-period in order to duty-cycle the proprietary heat-motor is estimated using Equation (5.2). This indicates the minimum insolation needed each day to melt the wax, from this an estimate can be made of the seasonal variation across ($1 \leq dn \leq 366$).

$$\text{Minimum actuation energy} = (\text{Sensible heat + Latent heat to melt wax}) + \text{Mechanical work}$$

At sunrise ($t_0$) $E_{dn} = \left( \sum_{1}^{n} m_n \cdot c_n \cdot \Delta T + (m_w \cdot H_f) \right) + (F \cdot l_m) \quad \{\text{in } J \cdot d^{-1}\}$

(5.2)

Where:

- $E_{dn}$: Minimum actuation energy
- $t_0$: Time of sunrise of a given ($dn$) from midnight
- $m_n$: Mass of $n$ components
- $c_n$: Specific heat capacity of $n$ components
- $\Delta T$: Temperature deficit of $n$ components
- $T_{mp}$: Melting point of the wax
- $init(T_{db})$: Air temperature at sunrise
- $m_w$: Mass of the wax
- $H_f$: Specific latent heat of fusion
- $F$: Biasing force and working load
- $l_m$: Motor actuation displacement length

Note: (1) Refer to Appendix B.1 on page 218 for values of relevant parameters for proprietary heat-motor. (2) Approximately that found in the proprietary heat-motor when charged with $n$-octadecane wax. (3) See Appendix A.1 for values of the relevant thermo-physical properties.
5.1. Thermal design rationale

In the absence of site-specific data for \( T_{db} \), the mean monthly minimum air temperature taken from climate data is used to find \((\Delta T)\). Using Equation (5.1) the sensible heat portion of the motor’s energy-budget on an annual cycle can be determined. Figure 5.4 shows the annual mean ratio of sensible-to-latent heat is \((1 : 1.7)\) with a variability \((St.D = 0.34, \ n = 366 \ days)\). Therefore, it is the latent heat that is the largest portion of the energy-budget needed to actuate the heat-motor. Figure 5.4 shows that the total budget has some sensitivity to the seasonal variation in the equilibrium temperature at the start of each diurnal cycle \( T_{db} \).

The efficacy of the thermal design rationale to predict heat-motor duty-cycling is predicated on establishing a well-defined seasonal difference in the available insolation compared with \( (E_{dn}, R) \) versus an insensitivity to the seasonal variation at the start of each photoperiod characterised by \( T_{db} \). The magnitude of seasonal insolation can be predicted under clear-skies using models driven by the astrometric relation between the Earth and Sun such as by Brinsfield et al. (1984).

Having defined the minimum actuation energy, the next section considers how “that adaptation of means to environments is brought about” (to quote p. 132 in Simon, 1981) to seasonally vary the thermal comparator’s duty-cycles solely by the passive harvesting of sunlight.

---

**Figure 5.4: Predicted seasonal variation in the ratio of sensible-to-latent heat**

Winter  | Spring  | Summer  | Autumn  
---|---|---|---

Maximum \( E_{dn} = 22.0kJ \) When \((dn = 75)\)

Minimum \( E_{dn} = 17.7kJ \) When \((dn = 225)\)

---

Note: (1) Based on mean monthly minimum air temperatures from climate data for London, UK from (MetOffice, 2010) using properties of the proprietary heat-motor tabulated in Appendix B.1.
5.2 Seasonal differentiation by selective inclination

In order to realise a seasonal response in the thermal comparator’s output \( (Q) \), we need a means to differentiate between the energy accumulated from insolation by heat-motor \( (A) \) from that accumulated by heat-motor \( (B) \). Can this be achieved by exploiting the seasonal difference in the insolation falling on two planes of equal surface area separated only by inclination angle? This question is considered by describing the energy balance of two surfaces of equal area across the time-scale of each diurnal interval.

There are numerous formulation to describe the SEB (Surface Energy Balance) in physical climatology (Chap. 8 in Sellers, 1965) and urban micro-climates (Chap. 2 in Erell et al., 2011). In each case, the starting-point is to assume the Law of conservation of energy holds as stated by:

\[
\text{Net change in energy} = \text{energy gain} - \text{energy loss} + \text{change in stored energy}
\]

If all the gains accumulated during daytime are subsequently lost to ambient within each cycle then it can be assumed that the change in energy stored is zero, the SEB then simplifies to gains equating directly to losses. In this form, the SEB is helpful for use in evaluating whether or not the net change in energy during the daily exchange of gains and losses exceeds the budget of \( (E_{dn} - \Gamma) \) within the intervals that state changes in the motors is desirable.

SEBs can account for many of the physical energy-exchange processes that surfaces close to the Earth may undergo, however, in this study the following simplifying assumptions are made. The latent heat flux from the dynamics of water between the atmosphere and ground-level in its different states will be neglected. This flux is related to the energy absorbed as water vapourises from a surface or is released as it condenses onto a surface from dew in air. Since the thermal design rationale only considers clear-sky conditions when there are no rain-bearing clouds, the flux from the evapotranspiration of water after direct precipitation will be neglected.

Applied to an urban micro-climate, the SEB may account for anthropogenic sources of heat (p. 28 in Erell et al., 2011), however, this term will be assumed to be small enough to be neglected in the context of a prospective test-site that consists of low-density buildings and minimal unheated artificial surfaces. Accepting these assumptions, the SEB simplifies to the expression in Equation 5.3 with energy gains on the left balanced by energy losses on the right.

\[
\text{Solar radiation} + \text{Atmospheric counter-radiation} = \text{Thermal emission} + \text{convection loss}
\]

\[
q''_{\text{rad}, \lambda=0.5 \mu m} + q''_{\text{rad}, \lambda=10 \mu m} = q''_{\text{rad}, \lambda=10 \mu m} + q''_{\text{conv}}
\]  \( (5.3) \)

Both planes experience the same heat-transfer modes, but the magnitude of each varies depending on the surface inclination angle. The horizontal and vertical planes are differentiated by subscripts \( (h, v) \) respectively. Overall sunlight is the dominant source, this is constantly balanced by energy losses to the sinks. These consist of ambient sensible heat mostly to the surrounding air and the net radiative exchange with the atmosphere at long wavelengths.
In the British Isles, the net balance of radiation exchange at long wavelength in the horizontal plane under clear-skies has been estimated at \( (q''_{\text{rad}, \lambda=10\,\mu m}\downarrow - q''_{\text{rad}, \lambda=10\,\mu m}\uparrow) = -8.4\, MJ\cdot m^{-2}\cdot d^{-1} \) by Monteith (1961). This is balanced by shortwave gains, with any difference changing the amount of energy stored. When this rises above the actuation threshold the heat-motor starts a duty-cycle. Figure 5.5 shows the characteristic seasonal variation in the total insolation in a horizontal plane and south-facing vertically-inclined plane (After p. 128 in Anderson and Riordan, 1976). On the horizontal, there is a large variation with the peak and trough coincident with the Summer and Winter solstices respectively. In the vertical plane, the seasonal variation is lower with two peaks at the Spring and Autumn equinoxes and the trough at the Summer solstice.

When the two profiles are compared there is a band between the lowest of both, this is where the vertical plane can provide actuation energy every day while the horizontal only provides enough for a portion of the year. This seasonal ‘energy bandwidth’ is indicated by the grey filled area. The question posed is whether or not the variation in the estimate for \( (E_{dn}) \) developed earlier using the proprietary heat-motor will actual fall within the limits of the ‘energy bandwidth’?

**Figure 5.5: Bandwidth of seasonal differentiation in pre-noon insolation by surface inclination**

*Note:* (1) Insolation on horizontal plane predicted under a clear-sky using empirically derived model by (Haurwitz, 1945) for London (UK) Latitude 51.5°N. (2) Insolation on inclined plane predicted using model by (Oke, 1987). Gains from the insolation curve’s rising-edge between dawn and noon.
If we regard the external area of the heat-motor \((A_c)\) as the surface that is directly exposed to insolation, then the result from Equation (5.2) can be unitised for area by \(\left( E_{dn} \times 1/A_c \right) \) across the annual interval \(1 \leq dn \leq 366\). If we assume that the outside of the heat-motor’s surface approximates a ‘Black body’ absorber of insolation energy and is perfectly insulated against convection losses, then the mean average of daily insolation energy needed to actuate the proprietary heat-motor is \(E_{dn} = 0.87 \text{ MJ} \cdot \text{m}^{-2} \cdot \text{d}^{-1}\).

This is approximately (3 to 6.5) times less than the magnitude of the seasonal ‘energy bandwidth’ shown in Figure 5.5. Based on this estimate, there is more than sufficient energy available to operate the thermal comparator using this rationale. While this estimate suggests a surplus in the available energy, it is based on the simplifying assumptions of an idealised enclosure. The following considers a more realistic but deliberately simplified description that compares the daily-budget with the difference in the available energy for each inclination diurnally and across seasons.

The estimate for \(E_{dn} \) made earlier in Section 5.1.2 applied the Law of Conservation of Energy assuming that solar gains entered a ‘closed system’ and were completely absorbed by the heat-motor. The findings of the pilot-study reported in Appendix G.1 on the effects of glazed enclosures on the behaviour of the proprietary heat-motor are now taken forward, the hot-box is introduced as a real enclosure that approximates some properties of a ‘closed system’.

By placing heat-motor \((A)\) and \((B)\) in separate hot-boxes, the rationale considers if the aperture into each hot-box can selectively differentiate the exposure to insolation of each heat-motor by inclination. This arrangement is shown indicatively in Figure 5.6(a) for the horizontally inclined aperture into a hot-box containing heat-motor \((B)\) while Figure 5.6(b) shows a vertically-inclined south-facing aperture into a hot-box containing heat-motor \((A)\).

In order to scrutinize how well the hot-box approximates a closed system, the following section develops a characterisation of the optical and thermal properties of a real hot-box to passively harvest environmental energy to duty-cycle heat-motor’s \((A)\) and \((B)\).

**Figure 5.6:** Sky view factors of the hot-box apertures with a 90° difference in inclination

![Figure 5.6](image-url)
5.2.1 Characteristics of an ideal hot-box

This section outlines the simplifying assumptions that have been made with reference to an indicative cross-section through the dual hot-box arrangement shown in Figure 5.7. Specifically, the arrangement of each aperture, the construction of the hot-box components and annotations that indicate the mode and direction of each heat-transfer process. This characterisation is made assuming that the hot-boxes are exposed to clear-sky conditions.

- Each heat-motor is insulated from conductive transfers through the walls of the hot-box, this leaves only the glass cover as the surface with direct convective losses to ambient. All of the hot-box’s optically opaque surfaces are assumed to be well-insulated in order to reduce the loss of heat-energy that accumulates inside the enclosure. This also prevents heat-transfers by conduction from local sources as well as losses to sinks around the hot-box. It will be assumed that this can be approximated by lining the hot-box with rigid board insulation with a low $U$-value. This simplification means that the convective loss from the heat-motor to ambient occurs by proxy through the glass cover. Specifically, there are convection losses ($-q'_{conv}$) from the heat-motor to the air in the hot-box cavity, then from the air to the glass and finally from the glass to the outside air. These losses are through the heat-transfer by convection from air-to-glass and finally glass-to-air characterised by the coefficients ($h_{conv, a\rightarrow g}$) and ($h_{conv, g\rightarrow a}$) respectively.

- It is assumed that the depth and arrangement of the heat-motor inside the cavity of each hot-box will be the same for both inclinations. The location and mounting details of the heat-motor assembly within the cavity of each hot-box will be as similar as practical.

- While the two inclination angles are different, it is assumed that the size of each aperture is equal. It is assumed that the coefficient for the rate of convection loss from the glass to the ambient air ($h_{conv, g\rightarrow a}$) will be the same since they are exposed to the same ambient temperature ($T_{db}$). The spectrally-selective properties of glass will be exploited to increase the gain from insolation by reducing the reflected direct insolation component and also reflect back the ‘Black body’ radiation emitted from the heat-motor and the air inside the enclosure. It will be assumed that these properties are approximated by covering the aperture with a single sheet of clear uncoated soda-lime glass.

- The aperture into the hot-box containing heat-motor ($A$) has a view-factor ($VF_v$) of the sky’s irradiance that is restricted to a quadrant looking due south. While the hot-box containing heat-motor ($B$) has a hemispherical view-factor ($VF_h$) of the sky. The characteristics of the incident insolation and the emitted ‘Black body’ radiation within the enclosure walls will be assumed to be grey except for the back of the glass sheet. It is assumed that these properties can be approximated by lining the hot-boxes with a continuous highly-reflective
5.2. Seasonal differentiation by selective inclination

aluminized polyester sheet that has very low emissivity and is spectrally grey. It is assumed that the heat-motor will be uniformly irradiated by incoming insolation from reflection within the enclosure.

- It is assumed that the hot-box cavity is well-sealed with a negligible rate of air-exchange with outside ambient. The temperature of the air in the horizontal hot-box is \((T_{a,h})\) and the vertical hot-box \((T_{a,v})\). It is assumed that the heat-transfer mechanisms are convection from the heat-motor’s cylinder surface to cavity air and radiation to the inside glass surface. These are characterised by coefficients \((h_{\text{conv},c\rightarrow a})\) and \((h_{\text{rad},c\rightarrow g})\) respectively.

- The proprietary heat-motor is treated ‘as supplied’, it is beyond the scope of this study to conduct photometric measurements across the range of relevant wavelengths. These would be needed at \(\lambda = (0.15\text{ to } 4.0)\ \mu m\) to characterise the absorption \((\alpha')\) of solar radiation (p. 19 in Sellers, 1965) and at \(\lambda = (8\text{ to } 15)\ \mu m\) to characterise the emission of ‘Black body’ radiation \((\varepsilon'_{n})\) in the range of cylinder temperatures expected \(T_{c} \approx (260\text{ to } 320)\ K\). Therefore parameter values have to be assumed based on the data in the literature. It will be assumed that the surface is spectrally grey with a value of \((\alpha'_{n} = 0.6)\).

- The dynamics of heat-transfer by ‘Black body’ radiation between surfaces and the sky is complicated to characterise as shown by (Bliss, 1961). When the heat-motor is warmed by insolation it re-radiates this energy to the glass at long thermal wavelengths. Since the glass is opaque at these wavelengths, the amount of energy that is exchanged between them will depend upon their temperature difference. This in turn depends upon the thermal dynamics of the glass surface with the outside ambient environment, since this is subject to the apparent temperature of the sky \((T_{\text{sky}})\) which is sensitive to prevailing synoptic weather. This can only be estimated under clear-sky conditions and for this simple model it will be assumed that \((T_{\text{sky}} = T_{db} - 6)\ K\) after p. 1 in (Atwater and Ball, 1978).

Based on these assumptions, the following sections describe the theoretical thermal behaviour of the horizontal and vertically-inclined hot-boxes with a simple model. The objective of using this model is to simulate the heat-motor’s cylinder temperature with respect to \((T_{mp})\) in each hot-box at intervals during the day for a sample of days across the seasons. Using the simulation results from each hot-box, a first-order prediction can then be made of the state of the ‘thermal comparator’s output at corresponding time intervals.
Figure 5.7: Section showing the heat-transfer mechanisms between components

Note: The dual hot-box system is shown fixed above a window opening that is in a south-facing façade to a single-storey building. It is assumed to have a flat roof without a parapet. The comparator mechanism \( Q \) is represented symbolically; it accepts displacement length \( l_a \) and \( l_b \) as inputs.
5.3 Thermal model of a dual hot-box system

In previous work by (Garg et al., 1983) a model was developed to study the stagnation temperature in solar cookers. This model is adapted for use here on the basis that (Garg et al., 1983) studied the dynamics of heat-transfer in a solar cooker as a closed system that is static, single glazed, optically non-concentrating, coated in moderately reflective surfaces and with a sealed air-filled cavity. Since the proposed hot-box bears a close resemblance to these characteristics, the model by (Garg et al., 1983) and the energy flows shown in Figure 5.7 are treated as comparable.

This model can be represented by a thermal network using the analogy of an electronic circuit, this is illustrated in Figure 5.7 as an overlay onto a section through the components of the two hot-boxes. The heat-transfers are shown as links in the circuit network and the components as nodes with temperatures ($T$). The resistance across each link in the circuit is an analogy of the heat-transfer coefficient for conduction ($h_{\text{cond}}$) and convection ($h_{\text{conv}}$) respectively.

This is a one-dimensional heat-transfer model, is assumes that the components have a strong thermal coupling due to their construction and geometric arrangement. It also assumes that the heat-capacitance of each component can be lumped at the nodes where there are 'Steady state' conditions of thermal-equilibrium during the time-intervals considered. It is assumed that all energy enters and leaves each hot-box through its glass covered aperture within the adiabatic boundary that is drawn to define this. If we apply the Law of Conservation of Energy to this system, the net rate-of-change in internal energy ($q''_{\text{net}}$) of each component is described by Equation (5.4).

$$q''_{\text{net}} = q''_{\text{gain}} - q''_{\text{loss}}$$ (5.4)

On this basis, the rate-of-change ($\frac{dT}{dt}$) in the specific heat ($m \cdot c \cdot \Delta T$) of the principal components in the horizontal hot-box is described in differential form by Equation’s (5.5), (5.6) and (5.7). The rate-of-change in the heat-motor’s cylinder temperature ($T_{c,h}$) based on ($q''_{\text{net}}$) is;

$$m_c \cdot c_c \cdot \left( \frac{dT_{c,h}}{dt} \right) = VF_{h,sky} \cdot G_c \cdot H \cdot \tau_{g,h} \cdot \alpha_c - h_{\text{rad,c}} \cdot VF_{h,c} \cdot \left( T_{c,h} - T_{g,h} \right)$$ (5.5)

$$- h_{\text{conv,c}} \cdot \left( T_{c,h} - T_{a,h} \right)$$

The rate-of-change in the glass cover’s temperature ($T_{g,h}$) based on ($q''_{\text{net}}$) is;

$$m_g \cdot c_g \cdot \left( \frac{dT_{g,h}}{dt} \right) = VF_{h,sky} \cdot G_c \cdot H \cdot \alpha_g + h_{\text{conv,g}} \cdot \left( T_{a,h} - T_{g,h} \right)$$ (5.6)

$$+ h_{\text{rad,c}} \cdot VF_{c} \cdot \left( T_{c,h} - T_{g,h} \right)$$

$$- h_{\text{rad,g}} \cdot VF_{h,g} \cdot \left( T_{g,h} - T_{sky} \right) - h_{\text{conv,g}} \cdot \left( T_{g,h} - T_{db} \right)$$

Finally, the rate-of-change in the cavity’s air temperature ($T_{a,h}$) based on ($q''_{\text{net}}$) is;

$$m_a \cdot c_a \cdot \left( \frac{dT_{a,h}}{dt} \right) = h_{\text{conv,c}} \cdot \left( T_{c,h} - T_{a,h} \right) - h_{\text{conv,g}} \cdot \left( T_{a,h} - T_{g,h} \right)$$ (5.7)
For each time interval \((dt)\) as the hot-box’s components approach a ‘Steady state’ of thermal equilibrium, the differentials;

\[
\left( \frac{dT_{c,h}}{dt} \right), \left( \frac{dT_{g,h}}{dt} \right) \text{ and } \left( \frac{dT_{a,h}}{dt} \right)
\]

will approach 0 as limit \(dx \to 0\).

The mass \((m_c, m_g \text{ and } m_a)\) and specific heat-capacities \((c_c, c_g \text{ and } c_a)\) of the cylinder, glass cover and cavity air respectively are then treated as negligible. Using this qualification and building on the assumption that the hot-box components are thermally coupled, Equations (5.5), (5.6) and (5.7) can be treated as a system of simultaneous linear differential equations.

**Figure 5.8: Network and node model of the heat-transfers in dual hot-box system**

*Note: Arrangement of components is shown indicatively, with the motor cylinder based on the Bayliss type Mk.7 section. The mechanical linkage between stroke-rods \((l_a)\) and \((l_b)\) is shown in dotted lines.*
This is because they contain only first-order derivatives for the heat-transfers by convection
\( q'\text{conv,source} \rightarrow \text{sink} \) and radiation \( q''\text{rad,source} \rightarrow \text{sink} \) as well as having interdependent variables. The
\( q''\text{gains} \) and \( q''\text{losses} \) of the energy-balance are now rearranged by moving the forcing functions
with known variables to the left, while the coefficients and unknown variables are arranged on the
right to give the system of Equations (5.9).

\[
G_c H \cdot \tau_{g, h} \cdot \alpha_c \cdot VF_{h, \text{sky} \rightarrow g} = h_{rad, c \rightarrow g} \cdot VF_{c \rightarrow g} \cdot (T_{c, h} - T_{g, h}) + h_{\text{conv, c} \rightarrow g} \cdot (T_{c, h} - T_{g, h}) \\
-G_c H \cdot \alpha_g \cdot VF_{h, \text{sky} \rightarrow g} = h_{\text{conv, a} \rightarrow g} \cdot (T_{a, h} - T_{g, h}) + h_{rad, c \rightarrow g} \cdot (T_{c, h} - T_{g, h}) \\
- \tau_{rad, g \rightarrow \text{sky}} \cdot VF_{h, g \rightarrow \text{sky}} \cdot (T_{g, h} - T_{\text{sky}}) \\
- h_{\text{conv, g} \rightarrow db} \cdot VF_{c \rightarrow g} \cdot (T_{g, h} - T_{db}) \\
0 = h_{\text{conv, c} \rightarrow a} \cdot (T_{c, h} - T_{a, h}) - h_{\text{conv, a} \rightarrow g} \cdot (T_{a, h} - T_{g, h})
\]

(5.9)

The dominant independent variable in the forcing functions is the time-varying value for insolation
on this inclination \( (G_c H) \). The magnitude of solar radiation is modelled for clear-skies dependent on
the solar elevation angle \( (\alpha_{\text{sol}}) \) using the empirically derived model by (Haurwitz, 1945) as shown
in Equation (5.10). The value for \( (\alpha_{\text{sol}}) \) itself is found at a given time \( (t) \) on a given day \( (dn) \) at
a given geographical location \( (\text{long, lat, ele}) \) using the astronomical algorithms by (Meeus, 1998).
A data-set for \( (G_c H) \) was generated with all the values relevant to the context of this study.

\[
G_c H = 1098 \cdot \cos(\alpha_{\text{sol}}) \cdot e^{-0.057/\cos(\alpha_{\text{sol}})} \quad \{\text{in W} \cdot \text{m}^{-2}\}
\]

(5.10)

The effect of \( (G_c H) \) on the cylinder depends on the transmission of short wavelength rays through
the glass \( (\tau_{g, h}) \). The glass cover specified in the assumptions transmits most short wavelength
radiation, however this assumes a stationary sun that is normally incident. Given that both hot-
boxes are fixed, the sun’s angle of incidence is time-varying at a rate of \( (\approx 0.25^\circ \cdot \text{min}^{-1}) \) due to
Earth’s rotation (Norton, 1989). To account for this, the following characterisation is made for
the annual pattern of the sunlight’s direct beam under clear sky conditions.

Fresnel’s equations of reflection are applied to find the transmission coefficient \( (\tau_{g, h}) \) through
the horizontal glass cover given the incident angle of direct beam insolation \( (\theta_v) \) using Equa-
tion (5.11) and Equation (5.12) respectively (on p. 102 in NASA by Siegel and Howell, 1981).
The direct beam component of insolation is assumed to be un-polarised with an equal mix of
parallel \( (\|) \) and perpendicular \( (\perp) \) polarised rays, this is described for the parallel plane \( (\|) \) by;

\[
\tau_{g, (h, \|)} (\theta_{h, v}) = \left\{ \frac{(n_2/n_1)^2 \cos(\theta_{h, v}) - [(n_2/n_1)^2 - \sin^2(\theta_{h, v})]^{1/2}}{(n_2/n_1)^2 \cos(\theta_{h, v}) + [(n_2/n_1)^2 - \sin^2(\theta_{h, v})]^{1/2}} \right\}^2
\]

(5.11)

Together with perpendicular \( (\perp) \) plane of polarisation by;

\[
\tau_{g, (h, \perp)} (\theta_{h, v}) = \left\{ \frac{[n_2/n_1]^2 - \sin^2(\theta_{h, v})]^{1/2} - \cos(\theta_{h, v})}{[n_2/n_1]^2 - \sin^2(\theta_{h, v})]^{1/2} + \cos(\theta_{h, v})} \right\}^2
\]

(5.12)

The value for the refractive indices of air and glass \( (n_1 \text{ and } n_2) \) respectively is taken from data in
the literature (p. 102 in Siegel and Howell, 1981).
5.3. Thermal model of a dual hot-box system

The insolation arriving at the surface of the heat-motor is characterised by the product of the incident insolation and the average transmission of the two components of polarisation. Scrutiny of Figure 5.7 shows that at any given time the incident angles ($\theta_h$) and ($\theta_v$) into the apertures of each hot-box are different, therefore glass transmission ($\tau_{g,h}$) and ($\tau_{g,v}$) at each ($t$) is calculated for the horizontal and vertical hot-boxes respectively. The annual pattern of transmission predicted by this method is shown in Figure 5.9 plotted in the solar position in azimuth ($\phi_{sol}$) and altitude ($\alpha_{sol}$) relative to the normal of the glass cover for each ($t$) of time across the seasons.

This shows pronounced differences in the pattern of glass transmission ($\tau_{g,h}$) on insolation ($G_cH$) in the horizontal plane compared with ($\tau_{g,v}$) on insolation ($G_cV$) on the vertical. The diurnal variation in ($\tau_{g,h}$) is much larger especially at low angles of incidence. The seasonal variation between inclinations are reversed, in the horizontal plane transmission is lowest during winter when insolation is also lowest. The vertical plane shows less seasonal variation, with the lowest transmission during summer. This shows that the pattern of seasonal variation in the two cover’s optical transmission is biased in favour of the thermal design rationale. Having given a description of the time-varying values for the variables ($\tau_{g,h}$) and ($\tau_{g,v}$) to use in the model, the following considers how to evaluate the remaining coefficients in the equations.

**Figure 5.9: The annual pattern of transmission for direct beam sunlight into each hot-box**

Note: Transmission ($\tau_{g,h}$) of direct sunlight through the glass aperture into the horizontal hot-box is plotted in orthographic projection (left-hand side). The corresponding transmission ($\tau_{g,v}$) through the glass aperture into the vertical hot-box is also plotted in orthographic projection, after the method on p. 239 in Oh et al. (2000) on the (right-hand side). The gray-scale indicates the change in transmission coefficient plotted along the sun’s path at 15 min intervals for every 30th day of the year within each hot-box’s acceptance angle. These projections are used so that they can be overlaid onto spherical reflection images for analysis, after the method shown on p. 78-83 in Olgyay (1963).
To solve the system of Equations (5.9) we need to establish values for the heat-transfer coefficients, we turn to the data in the literature and the results from laboratory work reported in Chapter 3 for working approximations of:

- **Convection transfers**
  
  \[ h_{conv,c\rightarrow g} = 12.5 \cdot W \cdot m^{-2} \cdot K^{-1} \quad \text{(Cylinder to glass cover)} \]
  
  \[ h_{conv,a\rightarrow g} = 12.5 \cdot W \cdot m^{-2} \cdot K^{-1} \quad \text{(Cavity air to glass cover)} \]
  
  \[ h_{conv,g\rightarrow db} = 10.0 \cdot W \cdot m^{-2} \cdot K^{-1} \quad \text{(Glass to outside ambient)} \]
  
  \[ h_{conv,c\rightarrow a} = 12.5 \cdot W \cdot m^{-2} \cdot K^{-1} \quad \text{(Cylinder to cavity air)} \]

- **Radiation transfers**
  
  \[ h_{rad,c\rightarrow g} = 8.0 \cdot W \cdot m^{-2} \cdot K^{-1} \quad \text{(Cylinder to glass cover)} \]
  
  \[ h_{rad,g\rightarrow sky} = 10.0 \cdot W \cdot m^{-2} \cdot K^{-1} \quad \text{(Glass cover to sky)} \]

- **View-factors**
  
  \[ VF_{h} = 0.3 \quad \text{(Sky to glass, } \lambda_{p} = 0.5 \mu m) \]
  
  \[ VF_{c\rightarrow g} = 0.25 \quad \text{(Cylinder to glass, } \lambda_{p} = 10 \mu m) \]
  
  \[ VF_{h,g\rightarrow sky} = \pi / 2 \quad \text{(Glass to sky, } \lambda_{p} = 10 \mu m) \]

- **Optical properties**
  
  \[ \alpha_{c} = 0.6 \quad \text{(Cylinder absorption, } \lambda_{p} = 0.5 \mu m) \]

Having accounted for these coefficients, the system of Equations (5.9) then has three unknown variables \((T_{c,h}), (T_{g,h})\) and \((T_{a,h})\) and the energy-balance expressed in Equations (5.5), (5.6) and (5.7) has been resolved into exactly three expressions. We may be able to solve it for the unknowns using a matrix of coefficients and the determinant matrix in the form:

\[
\begin{bmatrix}
 a_{1} \cdot T_{g,h} + b_{1} \cdot T_{a,h} + c_{1} \cdot T_{c,h} \\
 a_{2} \cdot T_{g,h} + b_{2} \cdot T_{a,h} + c_{2} \cdot T_{c,h} \\
 a_{3} \cdot T_{g,h} + b_{3} \cdot T_{a,h} + c_{3} \cdot T_{c,h}
\end{bmatrix} = k_{1}
\]

(5.13)

Multiplying out the elements in each component’s energy-balance for Equation (5.5) and separating unknowns and coefficients into the form \([a_{1} \cdot T_{g,h} + b_{1} \cdot T_{a,h} + c_{1} \cdot T_{c,h} = k_{1}]\) gives:

\[
(h_{rad,c\rightarrow g} \cdot VF_{c\rightarrow g}) \cdot T_{g,h} + (-h_{conv,c\rightarrow g}) \cdot T_{a,h} + (h_{rad,c\rightarrow g} \cdot VF_{c\rightarrow g} + h_{conv,c\rightarrow g}) \cdot T_{c,h} = G_{c} H \cdot \tau_{g,h} \cdot \alpha_{c} \cdot A_{c} \cdot VF_{h}
\]

(5.14)

Multiplying out the elements in each component’s energy-balance for Equation (5.6) and separating the unknowns and coefficients into the form \([a_{2} \cdot T_{g,h} + b_{2} \cdot T_{a,h} + c_{2} \cdot T_{c,h} = k_{2}]\) gives:

\[
(-h_{conv,a\rightarrow g} - h_{rad,c\rightarrow g} \cdot VF_{c\rightarrow g}) \\
-h_{rad g\rightarrow sky} \cdot VF_{h,g\rightarrow sky} - h_{conv,g\rightarrow a}) \cdot T_{g,h}
\]

(5.15)

Multiplying out the elements in the each component’s energy-balance for Equation (5.7) and separating unknowns and coefficients into the form \([a_{3} \cdot T_{g,h} + b_{3} \cdot T_{a,h} + c_{3} \cdot T_{c,h} = k_{3}]\) gives:

\[
(h_{conv,a\rightarrow g}) \cdot T_{g,h} + (-h_{conv,c\rightarrow a} - h_{conv,a\rightarrow g}) \cdot T_{a,h}
\]

(5.16)

\[
+h_{conv,c\rightarrow a} \cdot T_{c,h} = 0
\]
5.3. Thermal model of a dual hot-box system

This separates out all of the terms in the system of linear simultaneous Equations (5.9). When the terms from Equations (5.14), (5.15) and (5.16) are collected together, they express each coefficient in the matrix (5.13) enumerated as follows;

\[
\begin{align*}
T_{g,h} & = \begin{cases}
   a_1 = -h_{rad,c-g} \cdot V F_{c-g} \\
   a_2 = -h_{conv,a-g} - h_{rad,c-g} \cdot V F_{c-g} \\
   a_3 = + h_{conv,a-g}
\end{cases} & \begin{cases}
   k_1 = G_c H \cdot \tau_{g,h} \cdot \alpha_c \cdot V F_h \\
   k_2 = - G_c H \cdot \alpha_g \cdot V F_h - h_{conv,g-a} \cdot T_{db} \\
   k_3 = 0
\end{cases} \\
T_{a,h} & = \begin{cases}
   b_1 = -h_{conv,c-g} \\
   b_2 = + h_{conv,a-g} \\
   b_3 = - h_{conv,c-a} - h_{conv,a-g}
\end{cases} & \begin{cases}
   c_1 = h_{rad,c-g} \cdot V F_{c-g} + h_{conv,c-g} \\
   c_2 = h_{rad,c-g} \cdot V F_{c-g} \\
   c_3 = h_{conv,c-a}
\end{cases}
\end{align*}
\]

The system of Equations (5.9) can now be solved for the three unknowns \((T_{g,h}), (T_{a,h})\) and \((T_{c,h})\) using Cramer’s rule as shown top-left in Equations (5.18). By substituting the coefficients listed in (5.17) into the solution matrices \((\Delta_{g,h}), (\Delta_{a,h})\) and \((\Delta_{c,h})\) in Equations (5.18) and finding the corresponding determinant matrix \((\Delta_h)\) to give;

\[
\begin{align*}
T_{j,h} & = \frac{\Delta_{j,h}}{\Delta_h} & T_{g,h} & = \frac{\Delta_{g,h}}{\Delta_h} = \frac{\begin{vmatrix}
   k_1 & b_1 & c_1 \\
   k_2 & b_2 & c_2 \\
   0 & b_3 & c_3 \\
\end{vmatrix}}{\begin{vmatrix}
   a_1 & b_1 & c_1 \\
   a_2 & b_2 & c_2 \\
   a_3 & b_3 & c_3 \\
\end{vmatrix}} \{\text{in } ^\circ\text{C}\} \\
T_{a,h} & = \frac{\Delta_{a,h}}{\Delta_h} = \frac{\begin{vmatrix}
   a_1 & k_1 & c_1 \\
   a_2 & k_2 & c_2 \\
   a_3 & 0 & c_3 \\
\end{vmatrix}}{\begin{vmatrix}
   a_1 & b_1 & c_1 \\
   a_2 & b_2 & c_2 \\
   a_3 & b_3 & c_3 \\
\end{vmatrix}} \{\text{in } ^\circ\text{C}\} \\
T_{c,h} & = \frac{\Delta_{c,h}}{\Delta_h} = \frac{\begin{vmatrix}
   a_1 & b_1 & k_1 \\
   a_2 & b_2 & k_2 \\
   a_3 & b_3 & 0 \\
\end{vmatrix}}{\begin{vmatrix}
   a_1 & b_1 & c_1 \\
   a_2 & b_2 & c_2 \\
   a_3 & b_3 & c_3 \\
\end{vmatrix}} \{\text{in } ^\circ\text{C}\} 
\end{align*}
\]

The three solution in Equations (5.18) allow the model to simulate the time-varying temperature of the heat-motor’s cylinder inside the horizontal hot-box. The variables in the forcing functions are the gains are \((G_c H)\) and the losses are \((T_{db})\) and \((T_{sky})\) at each time \((t)\). The values of the other parameters enumerated in (5.17) are calculated or estimated based on the assumptions declared earlier in this section. Having described the model for the horizontal inclination, the next section enumerates the corresponding model for the vertically-inclined hot-box. While the same method is used, the parameters are different as are the dominant forcing functions due to the different inclination angle.
5.3. Thermal model of a dual hot-box system

Given a hot-box of the same construction and using the assumptions as declared for the horizontal hot-box, the Law of Conservation of Energy as expressed in Equation (5.4) is applied to the principal components in the vertical hot-box. The rate-of-change of the specific heat-energy of each component is described in differential form, starting with the temperature of the heat-motor’s cylinder \( T_{c,v} \) based on \( (q'_{\text{net}}) \) as follows:

\[
m_c \cdot c_c \cdot \left( \frac{dT_{c,v}}{dt} \right) = V F_g \cdot G_c V \cdot \tau_{g,v} \cdot \alpha_c - h_{\text{rad}, c \rightarrow g} \cdot V F_c \cdot (T_{c,v} - T_{g,v}) \tag{5.19}
\]

\[
- h_{\text{conv}, c \rightarrow a} \cdot (T_{c,v} - T_{a,v})
\]

The rate-of-change in the temperature of the glass-cover \( (T_{g,v}) \) based on \( (q''_{\text{net}}) \) is:

\[
m_g \cdot c_g \cdot \left( \frac{dT_{g,v}}{dt} \right) = V F_{g,v} \cdot G_c V \cdot \alpha_g + h_{\text{conv}, a \rightarrow g} \cdot (T_{a,v} - T_{g,v}) + h_{\text{rad}, c \rightarrow g} \cdot V F_c \cdot (T_{c,v} - T_{g,v}) - h_{\text{rad}, g \rightarrow \text{sky}} \cdot V F_{g \rightarrow \text{sky}} \cdot (T_{g,v} - T_{\text{sky}}) - h_{\text{conv}, g \rightarrow a} \cdot (T_{g,v} - T_{db}) \tag{5.20}
\]

The rate-of-change in the temperature of the cavity air \( (T_{a,v}) \) based on \( (q''_{\text{net}}) \) is:

\[
m_a \cdot c_a \cdot \left( \frac{dT_{a,v}}{dt} \right) = h_{\text{conv}, c \rightarrow a} \cdot (T_{c,v} - T_{a,v}) - h_{\text{conv}, a \rightarrow g} \cdot (T_{a,v} - T_{g,v}) \tag{5.21}
\]

For each time interval \( (dt) \) as the hot-box components reach a ‘Steady state’ of thermal equilibrium, the differentials;

\[
\left( \frac{dT_{c,v}}{dt} \right), \left( \frac{dT_{g,v}}{dt} \right) \text{ and } \left( \frac{dT_{a,v}}{dt} \right) \text{ will approach } 0 \text{ as limit } dx \to 0 \left( \frac{dy}{dx} \right) \tag{5.22}
\]

Given the same construction and materials as the horizontal hot-box, the mass \( (m_c, m_g \text{ and } m_a) \) and specific heat \( (c_c, c_g \text{ and } c_a) \) of the cylinder, glass-cover and cavity air respectively are treated as negligible. The energy-balances expressed in Equations (5.19), (5.20) and (5.21) can then be arranged as a system of simultaneous linear differential equations in the form:

\[
G_c V \cdot \tau_{g,v} \cdot \alpha_c \cdot V F_v = h_{\text{rad}, c \rightarrow g} \cdot V F_{c \rightarrow g} \cdot (T_{c,v} - T_{g,v}) + h_{\text{conv}, c \rightarrow g} \cdot (T_{c,v} - T_{a,v})
\]

\[
-G_c V \cdot \alpha_g \cdot V F_v = h_{\text{conv}, a \rightarrow g} \cdot (T_{a,v} - T_{g,v}) + h_{\text{rad}, c \rightarrow g} \cdot V F_{c \rightarrow g} \cdot (T_{c,v} - T_{g,v}) - h_{\text{rad}, g \rightarrow \text{sky}} \cdot V F_{g \rightarrow \text{sky}} \cdot (T_{g,v} - T_{\text{sky}}) - h_{\text{conv}, g \rightarrow a} \cdot (T_{g,v} - T_{db})
\]

\[
0 = h_{\text{conv}, c \rightarrow a} \cdot (T_{c,v} - T_{a,v}) - h_{\text{conv}, a \rightarrow g} \cdot (T_{a,v} - T_{g,v}) \tag{5.23}
\]

Upon scrutiny, the key differences between the system of Equations (5.9) for the horizontal hot-box and Equations (5.23) for the vertically-inclined hot-box are the variables for radiation exchange. The insolation on vertically-inclined south-facing hot-box aspect is \((G_c V)\) rather than \((G_c H)\). While the sun remains ‘in view’, the angle subtended by the sun onto the glass cover is different from the other inclination, therefore the light transmission through the glass \( (\tau_{g,v}) \) is calculated for each corresponding \((t)\) using Equations (5.11 to 5.12) described earlier.
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The view-factor for ‘Black body’ emission from the heat-motor’s cylinder to the glass cover ($V F_{\nu,h,c \rightarrow g}$) is the same for both hot-boxes (‘Configuration 4’ on p. 340 in Appendix A by Sparrow and Cess, 1978). However, the $90^\circ$ separation in inclination angle gives different view-factors for short-wave radiation absorption and ‘Black body’ emission from the glass to the sky, summarised as follows:

View-factors

$$V F_{\nu} = \pi/4 \quad \text{(Vertical quadrant, } \lambda_p = 0.5 \mu m)$$

$$V F_{\nu,g \rightarrow sky} = \pi/4 \quad \text{(Glass to sky, } \lambda_p = 10 \mu m)$$

$$V F_{\nu,c \rightarrow g} = 0.25 \quad \text{(Cylinder to glass, } \lambda_p = 10 \mu m)$$

Optical properties

$$\alpha_c = 0.6 \quad \text{(Cylinder absorption, } \lambda_p = 0.5 \mu m)$$

The system of Equations (5.23) can be solved in the same way as Equations (5.9). Firstly, the terms in the system of Equations (5.23) are multiplied out and then separated into the unknowns ($T_{g,\nu}$), ($T_{a,\nu}$) and ($T_{c,\nu}$) and coefficients in the matrix form:

$$\begin{align*}
\begin{bmatrix}
a_1 \cdot T_{g,\nu} & + & b_1 \cdot T_{a,\nu} & + & c_1 \cdot T_{c,\nu} \\
a_2 \cdot T_{g,\nu} & + & b_2 \cdot T_{a,\nu} & + & c_2 \cdot T_{c,\nu} \\
a_3 \cdot T_{g,\nu} & + & b_3 \cdot T_{a,\nu} & + & c_3 \cdot T_{c,\nu}
\end{bmatrix} & = k_1
\end{align*}$$

(5.24)

Multiplying out the elements in each component’s energy-balance for Equation (5.19) and separating the unknowns and coefficients into the form $[a_1 \cdot T_{g,\nu} + b_1 \cdot T_{a,\nu} + c_1 \cdot T_{c,\nu} = k_1]$ gives:

$$(-h_{rad,c \rightarrow g} \cdot V F_{c \rightarrow g}) \cdot T_{g,\nu}$$

$$+ (-h_{conv,c \rightarrow g}) \cdot T_{a,\nu}$$

$$+ (h_{rad,c \rightarrow g} \cdot V F_{c \rightarrow g} + h_{conv,c \rightarrow g}) \cdot T_{c,\nu} = G_v \cdot \tau_{g,\nu} \cdot \alpha_c \cdot V F_{\nu}$$

(5.25)

Multiplying out the elements in each component’s energy-balance for Equation (5.20) and separating the unknowns and coefficients into the form $[a_2 \cdot T_{g,\nu} + b_2 \cdot T_{a,\nu} + c_2 \cdot T_{c,\nu} = k_2]$ gives:

$$(-h_{conv,a \rightarrow g} - h_{rad,c \rightarrow g} \cdot V F_{c \rightarrow g})$$

$$-h_{rad,g \rightarrow sky} \cdot V F_{g \rightarrow sky} - h_{conv,g \rightarrow a}) \cdot T_{g,\nu}$$

$$+ (h_{conv,a \rightarrow g}) \cdot T_{a,\nu}$$

$$+ (h_{rad,c \rightarrow g} \cdot V F_{c \rightarrow g}) \cdot T_{c,\nu} = -G_v \cdot \alpha_g \cdot V F_{\nu}$$

(5.26)

Multiplying out the elements in each component’s energy-balance for Equation (5.21) and separating the unknowns and coefficients into the form $[a_3 \cdot T_{g,\nu} + b_3 \cdot T_{a,\nu} + c_3 \cdot T_{c,\nu} = k_3]$ gives:

$$(h_{conv,a \rightarrow g}) \cdot T_{g,\nu}$$

$$+ (-h_{conv,c \rightarrow a} - h_{conv,a \rightarrow g} \cdot T_{a,\nu}$$

$$+ (h_{conv,c \rightarrow a}) \cdot T_{c,\nu} = 0$$

(5.27)

This separates out all the terms in system of linear simultaneous Equations in (5.23).
When the terms from Equations (5.25), (5.26) and (5.27) are collected together, they express each coefficient in the matrix (5.24) and are enumerated as follows:

\[
\begin{align*}
T_{g,v} &= \begin{pmatrix}
    a_1 &= -h_{rad,c\rightarrow g} \cdot V F_{c\rightarrow g} \\
    a_2 &= -h_{conv,a\rightarrow g} - h_{rad,c\rightarrow g} \cdot V F_{c\rightarrow g} \\
    a_3 &= +h_{conv,a\rightarrow g}
\end{pmatrix} \\
T_{a,v} &= \begin{pmatrix}
    b_1 &= -h_{conv,c\rightarrow g} \\
    b_2 &= +h_{conv,a\rightarrow g} \\
    b_3 &= -h_{conv,c\rightarrow a} - h_{conv,a\rightarrow g}
\end{pmatrix} \\
T_{c,v} &= \begin{pmatrix}
    c_1 &= h_{rad,c\rightarrow g} \cdot V F_{c\rightarrow g} + h_{conv,c\rightarrow g} \\
    c_2 &= h_{rad,c\rightarrow g} \cdot V F_{c\rightarrow g} \\
    c_3 &= h_{conv,c\rightarrow a}
\end{pmatrix}
\]

(5.28)

The system of Equations (5.23) can now be solved for the three unknowns \((T_{g,v}), (T_{a,v})\) and \((T_{c,v})\) using Cramer’s rule shown in the top-left of Equations (5.29) by substituting the coefficients listed in (5.28) into the solution matrices \((\Delta_{g,v}), (\Delta_{a,v})\) and \((\Delta_{c,v})\) in Equations (5.29) and finding the corresponding determinant matrix \((\Delta_v)\) to give:

\[
\begin{align*}
\text{Cramer’s rule} & \quad T_{g,v} = \frac{\Delta_{g,v}}{\Delta_v} = \begin{vmatrix}
    k_1 & b_1 & c_1 \\
    k_2 & b_2 & c_2 \\
    0 & b_3 & c_3
\end{vmatrix} \quad \{\text{in } ^\circ C\} \\
T_{a,v} = \frac{\Delta_{a,v}}{\Delta_v} = \begin{vmatrix}
    a_1 & k_1 & c_1 \\
    a_2 & k_2 & c_2 \\
    a_3 & 0 & c_3
\end{vmatrix} \quad \{\text{in } ^\circ C\} \\
T_{c,v} = \frac{\Delta_{c,v}}{\Delta_v} = \begin{vmatrix}
    a_1 & b_1 & k_1 \\
    a_2 & b_2 & k_2 \\
    a_3 & b_3 & 0
\end{vmatrix} \quad \{\text{in } ^\circ C\}
\end{align*}
\]

(5.29)

Using the three solutions in Equations (5.29) the temperature of the heat-motor’s cylinder in the vertically-inclined hot-box can be simulated by inputting values for the time-varying variables for gains \((G_c V)\) and losses \((T_{db})\) and \((T_{sky})\).

At this point the value for the cylinder temperature \((T_{c,h})\) in the horizontal hot-box, the corresponding temperature in the vertical hot-box \((T_{c,v})\) can be predicted and compared with their common wax melting-point temperature \((T_{mp})\) at time \((t)\) in each diurnal cycle sampled at day numbers \((dn)\) across seasons during the year. The next section reports the results of simulating this first-order model against a database of climate data and diurnal profiles of solar radiation under clear-sky conditions from the models (Haurwitz, 1945; Oke, 1987).
5.3.1 Seasonal pattern in the thermal comparator’s output

The results of the simulation is visualised in Figure 5.10 with the diurnal pattern of duty-cycles of both heat-motor (A) and (B) plotted in pairs along the Y-axis by time (t). The seasonal pattern of variation in the duty-cycling of each pair is plotted along the X-axis by day number (dn).

The simulation predicts that the balance of energy in the vertical hot-box will lead to heat-motor (A) overcoming losses to ambient and duty-cycle every day. The higher insolation during winter balances the larger losses to the colder ambient conditions. While during summer, lower insolation on the vertical plane is balanced by smaller losses to the warmer ambient conditions. The balance of energy in the horizontal hot-box is predicted to lead heat-motor (B) to duty-cycle only when insolation is highest and losses are lowest between \((155 \leq dn \leq 255)\), otherwise the losses to ambient are not overcome. Recalling the thermal comparator mechanism developed in the previous Chapter 4, with heat-motor (A) and (B) operating the resultant output \((Q)\) this simulation can also be used to predict the annual pattern of the window shutter’s position. The comparator’s output \((Q)\) moves the window shutter into either state ‘A’ to close it \((\theta_r = 0^\circ)\), state ‘B’ to open it \((\theta_r = 90^\circ)\) or state ‘C’ for shading \((\theta_r = 45^\circ)\) at the times annotated in Figure 5.10.

Figure 5.10: Pattern of seasonal duty-cycles predicted for dual hot-box thermal comparator

Note: The outputs of heat-motor (A) and (B) are plotted on the ordinate axis, then in pairs at intervals of \((dn = 10 \text{ days})\) apart on the X-axis. Neither heat-motor (A) nor (B) is actuated at night, therefore the comparator’s output \((Q)\) is expected to close the shutter during night-time as the thermal design rationale intended. For clarity, output state is only plotted during daylight-hours.
5.4 Summary

In this chapter the thermal design rationale was developed to exploit insolation as the dominant thermal source while maintaining ambient as the thermal sink. This allowed a deterministic statement to be made about the duty-cycling of a heat-motor based on its melting-point relative to the climate in a given location.

By using this method, the paraffin wax \(n\)-octadecane was selected as the candidate for both heat-motors (A) and (B). This hedges one’s bet on testing the thermal design rationale with a finely-balanced embodiment by attempting to dominate the energy budget as much as possible with only the latent heat needed to express logical ‘state’. While waxes with a higher melting-point would still satisfy the rationale’s criterion, they also weaken the differentiation between each logical ‘state’ by adding a larger proportion of sensible heat to the energy balance needed for actuation.

The thermal design rationale was taken forward to propose that the duty-cycles of heat-motor (A) and (B) could be differentiated by exploiting the seasonal variation in the daily accumulation of gains from insolation by exposing them to different inclination angles. This is based on the seasonal change in gains and how they are balanced against the losses by convection to ambient. A numerical estimate showed that the wax’s latent heat of fusion represents a large proportion of the energy budget, this suggests that it would play the defining role in differentiating between the state of each heat-motor.

The feasibility of the rationale was tested by implementing a first-order model to simulate the approximate annual duty-cycle pattern of an idealised dual hot-box system against simplified meteorological data. The results of its simulation showed that the duty-cycles of the two heat-motors can be differentiated both diurnally and seasonally by selective exposure, in spite of uncertain variation in ambient temperature.

The simulation predicts that the comparator would operate the shutter open to provide daylight access every day. There would be a seasonal cut-off from the horizontal hot-box to operate the window shutter to provide shading from mid-summer to early-autumn under clear-sky conditions. The central locus in the seasonal pattern of the shutter’s shading response falls approximately (4 to 6) weeks after the summer solstice. While this may seem later than we might expect, it is consistent with the Earth’s summer-time thermal ‘flywheel effect’ that is typical in temperate and cool climates (p.16 in Henderson and Roscoe, 2010).

While this has provided a first-order validation of the thermal design rationale, it is predicated on the model’s simplifying assumptions. Of the limitations that have been outlined, it is the omission of a description for phase-change that is expected to lead to uncertainties in the predicted compared with a realistic system. To remedy this, what is needed is a description of how heat-energy diffuses through the heat-motor’s cavity coupled to the hydrostatic forces generated by the expansion of wax. To pursue this advancement, the following chapter applies a two-dimensional description of the heat-equation and phase transition of wax in the heat-motor’s cavity.
Chapter 6

Characterising diurnal duty-cycles

In the previous chapter, the thermal design rationale for differentiating the duty-cycling of the dual heat-motors in order to operate the thermal comparator mechanism was developed further. It accounted for the different pattern of insolation that is available at each inclination on a seasonal basis. A numerical estimate was presented to show how this could be exploited to differentiate the duty-cycles between each heat-motor.

The numerical estimate was limited to accounting for the total energy balance in each diurnal interval, this model alone does not account for the diurnal evolution of the energy exchange. In this chapter, the dimension of time is reframed from spanning across seasons to that of an individual diurnal interval within each season. Accordingly, the account of heat-energy transfer is reframed from the gross total specific energy to actuate the heat-motor \( E_{dn,A} \) to the net evolution of duty-cycles in heat-motor \( (A) \) and \( (B) \) across each day number \( (dn) \).

The linear model implemented in the previous chapter did not address the transfer of latent heat of fusion between the heat-motor’s wax and ambient during phase-change because of two main obstacles. Firstly, how are the discontinuities of sensible temperature in the motor’s cavity due to the low thermal conductivity of wax accounted for? Secondly, how are the thermal discontinuities due to the phase-change of wax accounted for? Here, these two limitations are addressed by describing the diffusion of heat-energy through the wax in the heat-motor’s cavity using the Finite Element Method (FEM) and implementing an existing solution to Stefan’s phase-change problem.

In order to account for the thermal comparator’s output over each diurnal interval, a coupling is needed between the thermal and the mechanical processes that determine the state of each heat-motor’s actuation displacement. This chapter seeks to develop a description that accounts for the phase-transition between liquid and solid wax that can be coupled to the expansion of wax during an actuation displacement and contraction during a recovery stroke.
This chapter begins with a detailed review of the diurnal pattern of duty-cycles that is expected for the resultant output of the thermal comparator \((Q)\) when operated by the state of heat-motors \((A)\) and \((B)\). Figure 6.1 shows an indicative diurnal interval in two contrasting seasons. The column on the left in Figure 6.1(a) shows time-series on a day in early-spring, while the column on the right in Figure 6.1(b) shows time-series on a day in late-summer. The top row of plots show the evolution of insolation that is incident on each inclination. The middle two rows show the indicative evolution of the logical state \((I_{a,b})\) of each heat-motor, the bottom row shows the output of the thermal comparator \((Q)\) and the corresponding position of the window’s shutter.

At this point we can reflect on the conclusion drawn at the end of Chapter 4, where accounting for the attainable temperature of the air in the climate-chamber was in itself insufficient to determine the phase state of the heat-motor at a point in time. Likewise in this chapter, accounting for the level of instantaneous incident radiation on the horizontal \((GcH)\) and vertical \((GcV)\) south-facing plane is also in itself insufficient to determine the heat-motor’s phase state.

As shown in Figure (5.4) of the previous chapter, the role played by the transfer of latent heat of fusion is pivotal to the energy balance. However, the extent of the discontinuities it introduces is uncertain. The absorption and release of latent heat of fusion is highlighted indicatively as intervals of time for each heat-motor in the middle two rows in Figure 6.1. The following section takes the first step to clarify their relation with a detailed description for the diffusion of heat in the wax.

**Figure 6.1: Indicative pattern of diurnal duty-cycles for dual hot-box thermal comparator**

(a) Duty-cycle in heat-motor \((A)\) only  
(b) Duty-cycle in heat-motors \((A)\) and \((B)\)  
*Note: The level of insolation in each inclination is shown indicatively for a clear-sky. The mechanism output \((Q)\) conforms to the mechanical constraints specified on page 78 in Chapter 4*
6.1 Modelling the heat-motor cavity

Our experience of a candle burning gives an indication of how large a temperature gradients can develop within a given thickness of wax, the flame of a paraffin candle can reach (>130 °C) yet within mm’s and two states of matter from its combustion the candle’s outer body is barely above room temperature, as thermographed in Figure 6.2(a). In this study, non-concentrated solar radiation is used to heat the cylinder’s surface but even this low energy-density source will heat the surface enough to lead to a non-uniform temperature field exhibiting time-lags before reaching ‘Steady state’ conditions of thermal equilibrium with the surrounding ambient.

The first question is, how large are the thermal gradients that develop between the modest temperature differences $\Delta T = (5$ to $40) °C$ that are expected across the ($\Delta x = 14 mm$) thickness of wax in the heat-motor’s cavity? To address this question, a model is proposed for the spatial distribution of the temperature field through a cross-section of the wax inside the motor’s cavity.

This two-dimensional model could then be used to investigate the dynamics of the spatial distribution of thermal energy when the temperature of a discrete element reaches the melting-point of the wax, given the parameters selected in the previous chapter. When heat-energy continues to be diffused through the spatial distribution to a discrete element in this state, it does not lead to a rise in temperature but to the transfer of its latent heat of fusion. This is an example of the one-phase ‘Stefan problem’ because during melting an isothermal ‘moving boundary’ develops.

The central concern of this study is to exploit the useful mechanical work converted by the heat-motor during an actuation displacement. This prompts the question of how the thermal processes can be coupled to the mechanical actuation to describe the expansion of the wax? Research on heat-transfer in wax tends to focus on its heat storage properties and generally ignores the volume change during phase-transitions, since this further complicates the problem by introducing another non-linearity. The useful work performed by a heat-motor from the expansion of wax is central to this study and cannot be ignored, in response the following is proposed.

Figure 6.2: Thermographing the modelling problem: Paraffin wax’s low thermal diffusivity

(a) Paraffin tea-light burning (IR-band at $\lambda_p = 10 \mu m$)  (b) Comparative scale of heat-motor (Visible-band)
6.1. Modelling the heat-motor cavity

6.1.1 A description of the heat-transfer model

In the previous Chapter, the dominant heat-transfer processes were represented in a cross-section through both hot-boxes and heat-motors by Figure 5.8 on page 99. This is now developed further with a grid for each model overlaid on the cylinder cavities as shown in Figure 6.3.

In the previous chapter, the heat-transfer by conduction could be ignored because of the very low mass and the high thermal conductivity of the components considered. Here the focus is on the thermal behaviour of the wax which cannot be approximated in this way. Heat-transfer in the wax will be treated as a pure conduction problem between discrete points.

Figure 6.3 shows a model in which the heat-transfer by conduction \( q''_{\text{cond}} \) from the node at the cylinder’s surface with temperature \( (T_{c,h}) \) through the wall to the wax node at the centre of the cavity with temperature \( (T_{w,h}) \). These are the nodes in the horizontal hot-box. In the vertical hot-box they correspond to the conduction \( q''_{\text{cond}} \) between nodes at temperatures \( (T_{c,v}) \) and \( (T_{w,v}) \) respectively.

An adiabatic boundary is defined by a sector with radii along the lines of axial symmetry from the centre of the heat-motor’s cavity out through the centrelines of the radial fins separated by 60°. This is indicated by a dashed-line boundary over each heat-motor in Figure 6.3. The heat-motor’s lines of axial symmetry are exploited on the assumption that the thermal behaviour of wax during melting is agnostic to orientation in the \((X-Y)\) plane in a \((1g)\) environment. This has the advantage of simplifying the problem and reducing the computation time.

Given the arrangement shown in Figure 5.7 both heat-motors would be mounted in the horizontal position lengthwise, it will be assumed that the depth is shallow enough that the mass-transport effects of convection can be ignored. An overall adiabatic boundary is defined around each heat-motor as indicated by the dashed-line up to the glass aperture covers, this separates the motor’s into each hot-box.

A two-dimensional grid is applied over the 60° sector in the \((X-Y)\) plane with a spatial resolution of \((l = 2.5 \times 10^{-4} m)\) squared. When polar arrayed this provides a total of \((n = 3640)\) discrete elements of wax across the cavity in each heat-motor. This resolution balances model accuracy with the demands on computational time. This is a simplified approach using a fixed grid with each element an equal size in the \((X-Y)\) plane. In the perpendicular \(Z\)-axis, the sector is then taken mid-way along the cylinder’s length. The ratio of the sector’s radius that corresponds to the wax cavity depth compared with the cylinder’s length is \((> 1 : 20)\). On this basis, the remainder of the cylinder’s cavity is treated as a semi-infinite slab and conduction in the \(Z\)-axis is ignored.

In this arrangement, the specific heat \( (c_w) \) and the latent heat of fusion \( (H_f) \) will be spatially distributed across all elements rather than lumped at node \( (T_{w,h}) \) and \( (T_{w,v}) \). Grid ‘H’ is applied to the horizontal hot-box and another grid ‘V’ to the vertical hot-box. It is the diffusion of heat-energy through the spatial distribution of the grids applied to the heat-motor in each hot-box that is the advancement over the linear model.
Since the spatial distribution will account for the portion of latent heat in each discrete element, by implication it also accounts for the element’s melt-fraction. When the temperature of an element is below the melting-point, the portion of latent heat is zero and in this state the melt-fraction is assumed to be zero as well.

If heat-energy then diffuses through the field and raises the element’s temperature to equal the melting-point as well as starting to transfer some latent heat, then the melt-fraction increases. Specifically, it will be assumed that the melt-fraction is the proportion of the latent heat of fusion transferred compared to the total required to change the phase for each discrete element in the field. In this way, each element’s density can be coupled to its melt-fraction in an inversely proportional relation. Such that when an element’s melt-fraction is zero its density is assumed to be solid. As its melt-fraction increases then its density decreases proportional to the difference between its density in its solid and liquid state. When all the latent heat of fusion in an element has been transferred its melt-fraction is one and its density that of liquid at the melt-point temperature.

To model the expansion of wax in the cavity as a whole, each element’s density is accounted for as follows. For orientation, please refer to Figure 6.3 showing a detailed cross-section through the hot-boxes, each heat-motor and its cavity. The expansion of the wax in the cavity is constrained within the boundary of the cylinder’s wall in the (X-Y) plane, therefore an element’s expansion can only be in the Z-axis. Using this arrangement, the inversely proportional relation between melt-fraction and density will be represented by the variable length of each element in the Z-axis. This is colinear with the axis in which the stroke-rod moves, therefore the sum of all element lengths is taken to be representative of the motor’s overall actuation length. The overall actuation displacement length from each heat-motor \( l_a \) and \( l_b \) is shown indicatively mechanically passing between the hot-boxes through thermally-isolated linkages as inputs to the thermal comparator transfer-mechanism with output \( Q \).

While using the melt-fraction variable to describe the major change in density of an element during its phase-transitions, this model may be improved upon to also account for the expansion of the wax in its liquid state at temperatures above the melting-point. At these temperatures an element’s melt-fraction remains equal to one however, a coefficient of linear expansion still applies because it is proportional to the sensible temperature of each element.

Given the low thermal diffusivity of wax through the depth of the cavity, if the variability in the distribution of temperatures above the melting-point within the grid is high, this will have implications for the overall actuation length. This was highlighted by overruns and variability in actuation length from the analysis of the climate-chamber studies in Chapter 4.

Having outlined the modelling problem and the questions that it’s simulation seeks to address, the following declares the governing heat-transfer equations and implements this model with an existing Finite Element Method (FEM) developed for studying the ‘Performance of finned thermal capacitors’ by Humphries (1974).
Figure 6.3: Network and node model of the heat-transfers through dual hot-box system

Note: The FEM grids 'V' and 'H' are shown at \((dX, dY = 1 \text{ mm}^{-2})\) respectively for clarity but are implemented by the model at \(\times 4\) finer resolution.
6.1.2 Heat-transfer equations and the finite element method

The previous section gave an overview of the model at the macro-scale, this section considers the individual elements of the grid at the micro-scale. The analogy of the electronic circuit is applied again to each element. This is shown in Figure 6.4 with the network links representing the heat-transfers between an element and its four nearest neighbours. On the left in Figure 6.4 is the description of an element’s sensible temperature $T_{(x,y)}$ and the accumulated specific heat $c_w(x,y)$. On the right in Figure 6.4 is the description of the same element’s melt-fraction $MF_{(x,y)}$ and the accumulated latent heat of fusion $H_{f(x,y)}$.

The diffusion of heat-energy is characterised by the resistance to heat-conduction $(RV, RH)$ across the network links for an element’s $h_{(x,y)}$ width $(dX)$ and height $(dY)$ respectively at time $(dt)$. From this, the heat-transfer between the aluminium cavity wall and the wax charge is treated as a pure conduction problem applied between each of the $(n = 3640)$ grid elements in the FEM using Fourier’s Law expressed in Equation (6.1) from (p. 4 in Incropera and DeWitt, 2007).

$$\text{Fourier’s Law} \quad \frac{dE}{dt} = -k_w \cdot \frac{dT}{dx} \quad \{\text{in } W \cdot m^{-2}\}$$  \quad (6.1)

Where:

- $\frac{dE}{dt}$ Rate of heat energy transfer \{in $W \cdot m^{-2}$\}
- $k_w$ Thermal conductivity of the wax \{in $W \cdot m^{-2} \cdot K^{-1}$\}
- $dT$ Temperature difference between points \{in $K$\}
- $dx$ Thickness of the wax \{in $m$\}

When the temperature of an element $n_{(x,y)}$ reaches $T_{(x,y)} = T_{mp}$ the melting process of phase-change begins, an element in this state is at the front of the ‘moving boundary’. This is Stefan’s phase-change problem, it can be solved in one-dimension for a ‘moving boundary’ along a plane using his solution as shown in Equation (6.2) from (p. 3 in Groulx and Ogoh, 2009).

Figure 6.4: Element representation in network for the finite element method (FEM)
When this is applied to the geometry inside the cylinder, there is an extended surface formed by the internal fins. The melting front is unlikely to develop as a simple plane around these in two-dimensions. Therefore, this relation is approximated by the FEM by applying the one-dimensional relation across each of the network links between the elements in the \((X)\) and \((Y)\)-axis.

In summary, the FEM presented in this section will be applied as a numerical method to describe the diffusion of heat-energy through a two-dimensional section of each motor’s cavity. The FEM as originally developed by Humphries (1974) used a ‘feed forward’ scheme. Specifically, the diffusion of heat-energy through the grid at time \((t)\) is calculated based on its difference with that of node \(n_{(x,y)}\) at temperature \(T_{(x,y)}\) at time \((t + \Delta t)\).

Then the obstacle is where does the source for the diurnal evolution of temperature \(T_{(x,y)}\) at node \(n_{(x,y)}\) come from? There is no readily available analytical solution that describes the whole two-dimensional network of heat-transfers described earlier by Figure 5.7 in Chapter 5. Therefore, the alternative that is adopted here is to use the synthetic time-series for temperatures that were reported in the previous Chapter 5.

Of all the heat-motor’s components, the cylinder’s surface is the most exposed to the thermal exchanges between the hot-box and ambient because it is directly exposed to the dominant gains from sunlight. On this basis, it is selected as the location for the nodes that drive each simulation the of FEM model. Specifically, the surface temperature \(T_{c,h}\) and \(T_{c,v}\) of each cylinder as represented at their corresponding nodes \(n_{(x,y)}\) in the FEM grids ‘H’ and ‘V’ respectively.

Having established the terms of reference for the diffusion of thermal energy through the heat-motors, the next section develops the model further by specifying how the simulation accounts for the wax’s change in density during phase-change.

\[
\text{Phase-change heat-transfer } \quad k_{w,s} \cdot \nabla T_s - k_{w,l} \cdot \nabla T_l = \rho \cdot H_f \cdot \left( \frac{dx}{dt} \right) \quad (6.2)
\]

Where:

- \(k_{w,s}\) Thermal conductivity of the wax (solid) \(0.146^{(1)}\) \{in \(W \cdot m^{-2} \cdot K^{-1}\}\)
- \(T_s\) Temperature of the wax (solid) \{in \(K\}\)
- \(k_{w,l}\) Thermal conductivity of the wax (liquid) \(0.146^{(2)}\) \{in \(W \cdot m^{-2} \cdot K^{-1}\}\)
- \(T_l\) Temperature of the wax (liquid) \{in \(K\}\)
- \(\rho\) Density of wax \((770 \text{ to } 860)^{(1)}\) \{in \(kg \cdot m^{-3}\}\)
- \(H_f\) Latent Heat of Fusion \(240,900^{(1)}\) \{in \(J \cdot kg^{-1}\}\)
- \(dx\) Position of melt-front in x-axis \{in \(m\}\)
- \(dt\) Change in time \{in \(s\}\)

\(^{(1)}\) See Appendix A.1 for values of the relevant thermo-physical properties for the paraffin \(n\)-octadecane used in this study. \(^{(2)}\) It is difficult to obtain reliable data in the literature for this value so it is assumed to be the same as for solid wax.
6.2 Coupling the ‘moving boundary’ with actuation displacement length

A method is needed to account for the non-linearity during melting and solidification when an element is at the front of the moving boundary and isothermal. Specifically, the element’s volume needs to be determined dynamically depending on the fraction of latent heat of fusion \( H_f \) that it has absorbed during melting. The variable melt-fraction \( MF(x,y) \) of each element will be used to quantify this and relate total specific heat-energy to its variable density.

To determine the displacement length during phase-change, \( MF(x,y) \) is integrated for all \( n \) elements in the grid at each time \( t \). This is to account for the thermal gradient that is generated and the non-linear behaviour of wax during the phase-change. The actuation displacement \( l \) can then be found by applying the linear function \( f(x) \) to the volume displacement of all \( n \) elements.

When all of the elements in the cavity have been melted \( MF(x,y) = 1 \) and \( T(x,y) > T_{mp} \), there is still a temperature dependent volume change. This is assumed to be linear with a volume expansion coefficient that can be applied proportionally to the temperature of an element \( n \) above the melting-point. Finally, all of these variables and parameters are brought together in Equation (6.3) to couple the FEM solution with an aggregate displacement length \( l \).

\[
l = f(x) \cdot \int_{n_{tot}}^{1} \left\{ \begin{array}{ll}
V_s(n) & ; MF(x,y) = 0 , T(x,y) < T_{mp}, \text{solid} \\
V_{pc}(n) & ; 0 < MF(x,y) < 1 , T(x,y) = T_{mp}, \text{melt} \\
(T(x,y) - T_{mp}) \cdot \frac{\Delta \rho_l}{\Delta T} & ; MF(x,y) = 1 , T(x,y) > T_{mp}, \text{liquid}
\end{array} \right\} \{\text{in } m}\]

Where terms are defined as follows:

- \( l \) Actuation displacement length of heat-motor at \( t \) \{\text{in } m}\)
- \( f(x) \) Displacement volume to actuation function \(^{(1)}\) \( n/a \)
- \( n \) Element in the FEM grid at coordinate \( (x,y) \) \( n/a \)
- \( n_{tot} \) Total number of elements in cavity \( n/a \)
- \( V_s(n) \) Volume of a solid element \( \{\text{in } m^3\} \)
- \( MF(x,y) \) Melt-fraction of element \( n \) at coordinate \( (x,y) \) \(^{(2)}\) \( \text{ratio} \)
- \( T(x,y) \) Temperature of element \( n \) at coordinate \( (x,y) \) \{\text{in } °C\} \)
- \( T_{mp} \) Melting-point of wax \{\text{in } °C\} \)
- \( V_{pc}(n) \) Volume of element undergoing phase transition \( \{\text{in } m^3\} \)
- \( V_l(n) \) Volume of a liquid element \( \{\text{in } m^3\} \)
- \( \frac{\Delta \rho_l}{\Delta T} \) Expansion coefficient of liquid wax \(^{(3)}\) \{\text{in } kg \cdot m^{-3} \cdot K^{-1}\} \)

Note: \(^{(1)}\) The function \( f(x) \) was obtained from a mensuration survey of a sample of cylinders and stroke rods by taking the means, details are reported in Appendix B.1 on page 218. \(^{(2)}\) When an element \( n \) at grid location \( (x,y) \) has a melt-fraction \( MF = 0 \) then it is solidified wax, when the melt-fraction \( MF = 1 \) it is liquid paraffin oil. \(^{(3)}\) See Table 4, p. 10 in Grosse and Egloff (1938).
6.2. Coupling the 'moving boundary' with actuation displacement length

6.2.1 Evolution of the moving boundary-front during phase-change

A visualisation has been developed to animate the evolution of each FEM simulation over a diurnal interval. This enables the detailed study of each motor’s duty-cycle, as well as developing the FEM code itself. Figures 6.5 is drawn from \((dn = 177)\) and 6.6 from \((dn = 288)\) that show sequences of phase-transition during melting and solidification respectively. Each of the images represents a 2D cross-section through heat-motor \((A)\) midway along its length and extends up to the adiabatic boundary relevant for analysing the vertically-inclined south-facing hot-box.

The left-hand side column (a to d) in both Figure 6.5 and 6.6 shows the diffusion of heat-energy through the motor’s cylinder wall and wax in its cavity. It is rendered by the temperature of each node \(T_{(x, y)}\) solved using the FEM described in the previous sections. The sequence of images from the top running down Figure 6.5 shows the evolution of the isothermal during melting. It clearly shows the ‘moving boundary’ highlighted in white, these are the nodes at the melting-point temperature \((T_{mp})\).

The ‘moving boundary’ partitions the fraction of solid wax from the liquid, with the liquid slowly heating up in its wake. Both sequences illustrate that the diffusion of heat-energy evolves gradually, especially during the transfer of latent heat of fusion. This also highlights that the process has a dependency on the depth of the cavity as much as the geometry of the cavity walls. In particular, it also shows that the internal fins play a role in promoting the transfer of heat along the sides further into the cavity than would otherwise have been the case.

The right-hand side column (e to h) in both Figure 6.5 and 6.6 shows the phase of the wax. Each row matches the temperature with melt-fraction at the same time-step in the simulation (a with e) to (d with h). The ratio of melt-fraction is rendered using the relation developed in Equation 6.3. Derived from this is a numerical value for each heat-motor’s actuation displacement length \((l)\) as previously described.

Acknowledging that a ‘T-history’ applies to the diffusion of thermal energy through the hot-box here just as much as it did to the climate-chamber experiments reported earlier in Chapter 3, the conditions for the start and end of each simulation need to be defined. In the climate-chamber we were afforded the luxury of determining the initial ‘Steady state’ conditions by programming the air-handling’s PID controller. However, under realistic conditions we have to make do with the assumption that conditions of thermal equilibrium prevail an hour before sunrise each day.

A simulation run is stopped at the end of the solidification process when the heat-motor reaches \((< T_{mp})\) or sunset occurs, whichever is the later. This means that the elapsed real-time of each simulation is variable because the start condition \((t = 0)\) varies as a function of \((dn)\) and the stop condition depends on \((dn)\) and the seasonal differences in mean ambient temperature.

This section has established a coupling between the thermal and phase-state domains in the FEM simulation, this can now be used to address the questions posed about the implications of the simplified first-order linear model that were identified at the start of this chapter.
6.2. Coupling the 'moving boundary' with actuation displacement length

Figure 6.5: FEM simulation showing evolution of 'moving boundary' during melting

(a) Pre-dawn thermal equilibrium \( T_{x,y} < T_{mp} \)

(b) Heat-flux from insolation \( T_{x,y} \geq T_{mp} \)

(c) Heat-flux from insolation \( T_{x,y} \geq T_{mp} \)

(d) Wax completely melted \( T_{x,y} > T_{mp} \)

(e) Wax is solid throughout cavity \( MF_{x,y} = 0 \)

(f) Formation of melt boundary-front \( 0 \leq MF_{x,y} \leq 1 \)

(g) End of melt boundary's evolution \( 0 \leq MF_{x,y} \leq 1 \)

(h) Latent heat of fusion absorbed \( MF_{x,y} = 1 \)
6.2. Coupling the 'moving boundary' with actuation displacement length

Figure 6.6: FEM simulation showing phase-change during solidification

(a) Cooling down to melting-point ($T_{x,y} > T_{mp}$)

(b) Cooling down to ambient ($T_{x,y} \geq T_{mp}$)

(c) Nearing end of insolation ($T_{x,y} \geq T_{mp}$)

(d) Wax completely solidified ($T_{x,y} < T_{mp}$)

(e) Wax is liquid throughout cavity ($M_{F,xy} = 1$)

(f) Start of solidification process ($0 \leq M_{F,xy} < 1$)

(g) Nearing end of solidification ($0 \leq M_{F,xy} < 1$)

(h) Latent heat of fusion released ($M_{F,xy} = 0$)
6.3 Simulating actuation in the dual passive hot-boxes

Earlier in this chapter, the time-plots in Figure 6.1 (a and b) showed the diurnal duty-cycle pattern in spring and summer. It contrasted the seasonal difference of the thermal comparator’s response to the changing pattern of insolation. On a day in spring the comparator undergoes a single duty-cycle while on a day in summer the comparator undergoes dual duty-cycling. It was an indicative diagram and based on the predictions made from simulating the first-order model reported in Chapter 5.

This chapter has developed a more detailed thermal model for simulating the heat-diffusion and corresponding displacement length in both heat-motor’s (A) and (B) in each hot-box. The implications of this for characterising the differences between diurnal duty-cycles in winter and summer is considered as follows.

Figure 6.7 plots the temperature and displacement lengths for each heat-motor from the simulation of a winter day using the climate and insolation profile for the 27th January. Figure 6.8 illustrates the state of both heat-motors in the respective hot-boxes simulated at \( t = 08:46 \text{ hrs} \) rendered in the thermal domain.

**Figure 6.7: Time-series from the FEM simulation of a winter day**

(a) The diurnal evolution of actuation displacement length

(b) The diurnal evolution of cavity temperature

Note: Simulation run assuming clear-sky insolation for 27th January. In (b) the temperature \( T \) is plotted for the element \( n(x,y) \) that is closest to the centre of the cavity in FEM grids ‘H’ and ‘V’.
Figure 6.8: Frame from FEM simulation showing heat-motor duty-cycling on winter day

[Animation of diurnal FEM simulation study] (AVI, 17.7 Mb)
In Figure 6.7, the column of plots on the left-hand side show heat-motor (A) in the vertically-inclined hot-box, while the column of plots on the right-hand side show heat-motor (B) in the horizontal hot-box. In Figure 6.9 (a), the top row of plots show the actuation displacement length \(l\) for each motor, while in Figure 6.7 (b) the bottom row of plots show the temperature \(T_w\) at the core of the cavity inside each motor.

The heat-motor’s actuation displacement is derived from the FEM model using the phase-ratio variable as described earlier, this is then plotted in the time domain. The duration of each trace reflects the assumptions that were made about each simulation start and end condition, as defined earlier in this chapter. The time of sunrise and sunset has been added to each plot to relate the duration of any duty-cycle predicted with the hours of daylight access.

The findings from testing the practical thermal comparator in Chapter 4 showed that real actuators have a displacement length that is variable when the temperature is above the critical point \(T_{mp}\). In the FEM developed here, the coupling between thermal and melt-fraction in the wax accounts for this. Reading the time-plot of temperature and displacement on the left in Figure 6.7 (a and b) shows that the actuator’s behaviour is consistent with this expectation.

Using the same mechanical design rationale for operating the comparator’s output \(Q\) here that was developed earlier in Chapter 4, the actuation length \(l\) of either heat-motor is not proportional except during the melting and solidification processes. This implies that if the wax is solid in either heat-motor, then the mechanism position is ‘Off’ regardless of how much the cavity temperature \(T_w\) might fall below the critical-point \(T_{mp}\) during an interval.

In the opposite case, once the wax in either heat-motor has melted then the mechanism position is ‘On’ regardless of how much the cavity temperature is above \(T_{mp}\). A specific displacement length needs to be defined for the ‘On’ position in a full duty-cycle and \((l_a, b = 60\ mm)\) is used here to evaluate the simulation results as annotated on each of the actuation plots.

As a reminder to the reader, the practical thermal comparator mechanism designed and tested earlier constrained the motor’s displacement within the ‘On’ and ‘Off’ positions by using mechanical stops (See page 78 in Chapter 4). The results from testing a batch of Bayliss type motors found \((l = 89\ mm)\) under ‘no load’ conditions (See page 222 in Appendix B.1). In a practical mechanism using the proprietary heat-motor studied here, this would need to be adjusted down and therefore this definition is probably realistic.

Having set-up these parameters, the headline finding from the simulation of a winter’s day is that only heat-motor (A) inside vertically-inclined hot-box would undergo a duty-cycle. Heat-motor (B) barely reaches a temperature above ambient during the whole simulation interval. This is highlighted in Figure 6.8 with a frame from the FEM in that clearly shows the start of the single duty-cycle in heat-motor ‘A’, the slowing up of the diffusion of heat energy is captured in the animation of the FEM simulation. Close scrutiny of the trace for temperature in Figure 6.7 (b) at the time of melting and solidification shows evidence of time-lags in the order of (0.3 to 0.5) hrs.
6.3. Simulating actuation in the dual passive hot-boxes

Considering a day in a contrasting season, Figure 6.9 shows the simulation results for a summer’s day. For comparison, the time-series are presented in the same arrangement used for the winter’s day. The temperature in the cavity of heat-motor (B) rises above ambient and is sufficient to exceed the critical-point ($T_{mp}$), this reflects both warmer ambient temperatures and a more intense insolation profile onto the horizontal aperture during summer. The temperature and actuation displacement length reached by heat-motor (A) is similar to a winter’s day.

The main findings of this simulation is that both heat-motor (A) and (B) undergo duty-cycles. Figure 6.9 (b) shows that time-lags are exhibited and have slightly longer duration than in winter. The duration of ‘On’ time in (B) falls within and is less that the duration of ‘On’ time in (A), consistent with a response to daylight access and the risk of overheating within that time.

Figure 6.10 shows a frame from a FEM simulation of 19th August at ($t = 10:16$ hrs) rendered in the thermal domain. It captures the wax melting in heat-motor (B) and clearly shows the large temperature gradients that are generated in the cavity. At this point, heat-motor (A) has already melted and is at a similar temperature to the other components in the separate vertical hot-box.

**Figure 6.9: Time-series from the FEM simulation of a summer day**

![Time-series from the FEM simulation of a summer day](image)

**Note:** Simulation run with clear-sky insolation profile for 19th August. In (b) the temperature ($T$) is plotted for the element $n(x,y)$ that is closest to the centre of the cavity in FEM grids ‘H’ and ‘V’.
Figure 6.10: Frame from FEM simulation showing heat-motors duty-cycling on summer day
6.3. Simulating actuation in the dual passive hot-boxes

Having presented the FEM simulations for two days in contrasting seasons and remarked on the differences between the pattern of diurnal duty-cycles, the following draws out some generalisations from the results shown in the time-plots of the main variables.

In both cases, the detailed FEM simulation of the heat-diffusion process has underlined that phase-transitions in the proprietary heat-motor occur gradually as the ‘moving boundary’ evolves through the wax. Since this FEM model has directly coupled the thermal energy exchanges to the mechanical density change of wax through the depth of the cavity, it has shown that the response time of actuation is complicated to describe because of its non-linear characteristics.

One advancement that has been made by the detailed FEM simulation compared with the first-order model in Figure 5.10 of Chapter 5 is to account for time-lags. The simple model showed every duty-cycle as symmetrical about noon, the subtle effect of the time-lags exhibited in the FEM simulations will shift the phase-transitions to the right in each time-plot.

When a duty-cycle does occur, it is in all cases bounded between sun-rise and sun-set. This implies that the heat-energy absorbed from insolation is always rejected back to ambient before the sun sets. This is consistent with the thermal design rationale that assumes insolation energy is dominant during the morning charging period, while ‘Black body’ radiation is dominant during the evening discharge period when the sun is obscured from either aperture’s view-factor.

In both Figures 6.7 (b) and 6.9 (b), the simulation predicts that the cavity temperatures attained may be well above the critical point \(T_{mp}\). Given all the uncertainties outlined thus far, it is challenging to assess whether we should regard this as simply a characteristic property to be expected in the energy balance of this basic version of a passive system, or if this qualifies as an undesirable ‘thermal excursion’. At this point, a conservative engineering approach is taken and the statement made by one of the manufacturers (Netra, 2012) that the heat-motor’s maximum operating temperature is \(T_{max} = 60^\circ C\) is taken prima facie. On this basis, all predictions for temperatures attained above \(T_{max}\) are classified as ‘thermal excursions’.

Inextricably connected to the variation in temperature above \(T_{mp}\) is the variation in actuation displacement. If far more thermal energy is absorbed than is needed for phase-transition, then the simulation also predicts that displacement lengths will tend to extend well beyond the mechanical ‘On’ position. Again, it is challenging to assess whether this should be treated as a property inherent in using heat-motors in passive mechanisms, or if it qualifies as an undesirable ‘mechanical overrun’. At this point, a measure to mitigate against the possible ill effects of a ‘mechanical overrun’ would be to keep displacement length in check with the travel stops described earlier in the mechanical design rationale for the practical thermal comparator.

In summary, this section has delved into the detail of the FEM simulation on two days that show the contrast between winter and summer responses to selectively inclined apertures for differentiating the duty-cycling of the two heat-motors. The following section takes this forward by simulating the characteristic diurnal cycles across a whole year.
6.4 Seasonal variation in the evolution of diurnal actuation

The spatial, temporal, thermal and mechanical domains that have been brought together in this chapter are now visualised in a set of simulations at approximately 15-day intervals across a year, a sample frame is shown in Figure 6.11. One final variable is added to the visualisation, the output \( Q \) of the thermal comparator based on the mechanical design rationale for linking the actuation displacements of heat-motors (A) and (B) developed in Chapter 4. The output displacement length is annotated by the comparator symbol in the lower right-hand side corner in Figure 6.11.

The remainder of this section draws on the results of these simulations to account for the predicted variation in seasonal response between the two hot-boxes, these are summarised in Table 6.1. Reading the columns left-to-right, the type of duty-cycle predicted on each day is listed in Output \( Q \). An entry of 'Single' means that the thermal comparator would open the insulated window shutter fully to provide access to daylight, and entry of 'Double' means that it would both open to provide access to daylight and provide shading protection for a portion of daylight hours. The seasonal variation in this response is that access to daylight is provided for a portion of the day all year, whereas shading is provided for part of each day between day numbers \( dn = (165 \text{ to } 244) \).

The response latency during phase-transitions are tabulated for each hot-box inclination in column order with the prefix \( (\uparrow) \) for melting and \( (\downarrow) \) for solidification when applicable. The vertically-inclined hot-box is annotated by ‘V’ and the horizontally-inclined by ‘H’ respectively.

Figure 6.11: Frame from the animation of coupled thermal and melt-fraction simulation

Note: The complete set of animations is listed in the furthest left-hand side column of Table 6.1.
### Table 6.1: FEM simulations showing the seasonal difference in diurnal duty-cycles

<table>
<thead>
<tr>
<th>Date</th>
<th>Output</th>
<th>Response latency</th>
<th>Excursions</th>
<th>Animation of FEM: heat-diffusion/melt-fraction</th>
</tr>
</thead>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[hyperlink](format, size)</td>
</tr>
<tr>
<td><strong>Winter</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6\textsuperscript{th} Jan</td>
<td>Single</td>
<td>0.25</td>
<td>0.25</td>
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</tr>
<tr>
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<td>Single</td>
<td>0.25</td>
<td>0.25</td>
<td>N/A</td>
</tr>
<tr>
<td>11\textsuperscript{th} Feb</td>
<td>Single</td>
<td>0.30</td>
<td>0.25</td>
<td>N/A</td>
</tr>
<tr>
<td>26\textsuperscript{th} Feb</td>
<td>Single</td>
<td>0.32</td>
<td>0.31</td>
<td>N/A</td>
</tr>
<tr>
<td>5\textsuperscript{th} Mar</td>
<td>Single</td>
<td>0.33</td>
<td>0.32</td>
<td>N/A</td>
</tr>
<tr>
<td>15\textsuperscript{th} Mar</td>
<td>Single</td>
<td>0.35</td>
<td>0.32</td>
<td>N/A</td>
</tr>
<tr>
<td><strong>Spring</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6\textsuperscript{th} Apr</td>
<td>Single</td>
<td>0.36</td>
<td>0.34</td>
<td>N/A</td>
</tr>
<tr>
<td>16\textsuperscript{th} Apr</td>
<td>Single</td>
<td>0.36</td>
<td>0.34</td>
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</tr>
<tr>
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<td>0.36</td>
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</tr>
<tr>
<td>12\textsuperscript{th} May</td>
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<tr>
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<td>0.38</td>
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</tr>
<tr>
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<td>Double</td>
<td>0.41</td>
<td>0.38</td>
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</tr>
<tr>
<td>25\textsuperscript{th} Jun</td>
<td>Double</td>
<td>0.39</td>
<td>0.36</td>
<td>0.88</td>
</tr>
<tr>
<td><strong>Summer</strong></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>1\textsuperscript{st} Jul</td>
<td>Double</td>
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</tr>
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<td>0.34</td>
<td>0.78</td>
</tr>
<tr>
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<td>0.33</td>
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<td><strong>Autumn</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>1\textsuperscript{st} Sep</td>
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<td>0.33</td>
<td>1.00</td>
</tr>
<tr>
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<td>0.33</td>
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</tr>
<tr>
<td>30\textsuperscript{th} Sep</td>
<td>Single</td>
<td>0.32</td>
<td>0.32</td>
<td>N/A</td>
</tr>
<tr>
<td>15\textsuperscript{th} Oct</td>
<td>Single</td>
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<td>0.28</td>
<td>N/A</td>
</tr>
<tr>
<td>22\textsuperscript{nd} Oct</td>
<td>Single</td>
<td>0.28</td>
<td>0.26</td>
<td>N/A</td>
</tr>
<tr>
<td>13\textsuperscript{th} Nov</td>
<td>Single</td>
<td>0.23</td>
<td>0.23</td>
<td>N/A</td>
</tr>
<tr>
<td>15\textsuperscript{th} Nov</td>
<td>Single</td>
<td>0.25</td>
<td>0.22</td>
<td>N/A</td>
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</table>

continued on next page
6.4. Seasonal variation in the evolution of diurnal actuation

continued from previous page

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<tr>
<th>Date</th>
<th>Thermal comparator</th>
<th>Output</th>
<th>Response latency</th>
<th>Excursions</th>
<th>Animation of FEM: heat-diffusion/melt-fraction</th>
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<td>25th Nov</td>
<td>Single</td>
<td>0.23</td>
<td>0.20</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>dn 329 (AVI, 13.7 Mb)</td>
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</table>

| Winter (1) |                      |        |                  |            |                                               |
| 10th Dec   | Single              | 0.23   | 0.20             | N/A        | +8.1                                          |
|           |                     |        |                  |            | dn 344 (AVI, 13.2 Mb)                         |
| 22nd Dec   | Single              | 0.23   | 0.21             | N/A        | +6.8                                          |
|           |                     |        |                  |            | dn 356 (AVI, 13.0 Mb)                         |

Note: Each simulation is run for both the horizontal and vertically-inclined hot-box for heat-motor (B) and (A) respectively. The duration of each simulation depends upon the portion of the day when duty-cycling is expected and varies on a seasonal basis. (1) British seasons after (Manley, 1974).

All of the simulated phase-transitions exhibited some response latency, these varied from \( R_t = (0.25 \text{ to } 1.0) \text{ hrs} \). A qualitative comparison between the two hot-boxes suggests that the shallower the daily maximum elevation angle, the longer the response latency. The difference between charge (melt) and discharge (solidification) response latency is small for the vertical, but the horizontally inclined hot-box has larger differences.

If we compare the differences in the daily energy-balance with the differences in charge and discharge response latency between the hot-boxes, the data suggests that the closer the daily energy-balance approaches the threshold for phase-change the less symmetrical the latencies become. At this point, it is difficult to assess whether this predicted sensitivity on days when the passive energy-budget is finely balanced is an artefact of the FEM simulation alone, or something that we should expect in observe in a realised machine.

In the previous section, ‘thermal excursions’ were identified as a risk based on the definition of \( (T_{\text{max}}) \). When analysing the simulation time-plots, each instance and magnitude of an excursion has been accounted for. These are reported in Table 6.1 under column ‘Excursions’ for the horizontal and vertically-inclined hot-box annotated ‘h’ and ‘v’ respectively. A qualitative evaluation is that the horizontal hot-box would not attain temperatures that exceed the limit at any time during the year. However, the vertically-inclined hot-box would exceed the limit during late-autumn \( (dn = 232) \) through winter until \( (dn = 75) \) in spring when the insolation flux is highest for that inclination despite the colder ambient temperature.

In the last column on the left are listed the animation of each simulation, these visualise the micro-scale diffusion of heat-energy coupled to melt-fraction in the wax cavity of both heat-motors in their hot-box enclosures. The evolution of these changes are visualised at the time-scale of seconds, minutes, hours and then days across a year.
6.4.1 Regulation with the dual hot-box thermal comparator

In this and the previous chapter, the focus has been on addressing the thermal design rationale in the move away from the climate-chamber toward realistic outdoor conditions. The dual hot-box system has moved on from Chapter 4 in other ways, it marks a departure in the cybernetic relation between the flow of energy and the comparator’s output state.

In the mechanism described in Chapter 4, the heat-motors were in the same enclosure and shared the same exposure to variations in ambient conditions. Here, the heat-motors are in separate enclosures and the whole rationale turns on them being exposed differently to the variations in gains from the surroundings.

The mechanical design rationale in Chapter 4 demonstrated an approach to regulate the thermal comparator without feedback, however the logic of this is embodied in the arrangement of the linkage itself. The difference with the dual hot-boxes is that stable states are based on individual energy budgets that are met by the energy-balance in each respective hot-box not a shared equilibrium temperature. A contrast shown by comparing Figure 4.3 in Chapter 4 with Figure 6.12 that shows a state-machine describing the heat-motors in the hot-boxes.

The numerical annotation used for each state (1 to 5) in Figure 6.12 maps to the phase-changes described on page 19 in the Literature review. Only state 6 is added to indicate the end of latent heat transfer during freezing. The subscripts \((x_a)\) and \((x_b)\) refer to heat-motors \((A)\) and \((B)\) respectively. Overall, the logical state of the output \((Q)\) is mapped to stable states other than when the latent heat of fusion is being transferred. State 7 represents the connector between the two state-machine loops, it has two sets of state-transition arrows for the following reasons. Firstly, it recognises that the linkage mechanism is reversible. Secondly, at this point we do not know if this arrangement will perform as the thermal design rationale predicts. The state-machine in Figure 6.12 can be used to link the FEM simulation’s output to a dynamic geometric model.

**Figure 6.12:** State-machine diagram for the 'dual' hot-box thermal comparator mechanism
6.5 Summary

In this chapter, the Finite Element Method (FEM) was applied to simulate the passive thermal behaviour of the proprietary heat-motors inside each of the idealised hot-boxes. The FEM was used to model the effects due to wax’s unusual combination of physical properties, its low thermal conductivity and high latent heat of fusion. It was shown that the low thermal conductivity generated large thermal gradients, even with modest insolation fluxes. However, it was modelling the high latent heat that was crucial to characterising phase-transitions.

While the FEM scheme used here was adopted from earlier work by Humphries (1974), it has been developed further by coupling the heat-diffusion process with the melt-fraction of wax during phase-transitions. This advancement was necessary in order to simulate the evolution of actuation displacement in the heat-motors that provides the useful mechanical work central to this study.

In the previous chapter, the feasibility of whether the response of heat-motors can be differentiated by the passive flow of energy was investigated with an outline thermal design rationale. The simulations reported in this chapter provide an engineering basis for testing its feasibility. This is by allowing the investigation of the duty-cycles based on the energy-balance at the diurnal time-scale. This has been achieved by modelling the underlying multi-physical processes that determine the behaviour of the heat-motor actuators in the proposed hot-boxes.

The results support the proposition that the design for an idealised hot-box could differentiate the duty-cycles between two otherwise identical heat-motors using apertures of equal area by selectively admitting insolation from a vertically-inclined south-facing orientation from a horizontal one. The FEM predicts subtle effects will be exhibited in the timing of the duty-cycle on both diurnal and seasonal time-scales due to the time-lags in the heat-diffusion process. The FEM also predicts that excessive heating in the energy-balance may present a risk to a practical mechanism.

While the choice of variables and the value of parameters used to model these processes and simulate their evolution has been substantiated from data in the literature, this chapter concludes with two lingering doubts. Firstly, was it justified to use the synthetic time-series for \((T_{c,h})\) and \((T_{c,v})\) obtained from the linear model to drive the ‘feed forward’ scheme that simulates the diffusion of heat-energy? Secondly, what are the implications for the efficacy of the simulation to predict duty-cycles when relying on climatic rather than synoptic weather data?

The FEM scheme used here is a numerical method not an analytical solution for the temperature \(T_{(x,y)}\) in the time-domain. Therefore, in order to verify the FEM scheme a time-series of \(T_{(x,y)}\) would need to be collected from actual heat-motors. Addressing the second doubt is more complicated, it requires that the simulation results are validated by scrutinising how well the effects of diurnal insolation hold compared with actual insolation on the same inclined planes.

Addressing these uncertainties points in one direction, the same one encountered toward the end of Chapter 4 that led to the construction and testing of a practical mechanism. The next chapter follows this through with an empirical validation by testing in-situ.
Chapter 7

Results from an in-situ observation study

This research project has acknowledged that there is no readily available analytical solution that will fully describe the behaviour of the heat-motors in the dual hot-box arrangement proposed here. The efficacy of the findings from the numerical FEM simulation presented in the previous chapter is dependent upon the soundness of the simplifying assumptions that have been made.

In order to test the efficacy of the rationale, there would ideally be practical hot-boxes that embody a good approximation of the thermal design rationale’s simplifying assumptions. This is on the basis that a comparison with the data obtained is meaningful only if it shares the same terms of reference that are used by the model and its simulation.

However, there are obstacles to carrying out a validation due to the lack of numerical data on the behaviour of the proprietary heat-motor operated in this arrangement. Neither the manufacturer of the proprietary heat-motor that has been used in this study had carried out a systematic study, nor was suitable data found to be published anywhere in the literature.

This chapter aims to make a contribution by collecting an empirical source of data that is comparable with the synthetic source generated by simulating the first-order thermal model. The chapter begins with an overview of the empirical study for obtaining the database of results to use for comparison. This outlines the methodology adopted and the data structure of the findings.
7.1 A practical dual hot-box system

In the absence of a source of numerical data, the same empirical approach adopted earlier in Chapter 4 is pursued again here. By the design and construction of practical hot-boxes that are suitable for extended testing. A campaign to monitor the response of the hot-boxes is then conducted. This is intended to implicitly structure a set of results for the variables in the record of ‘as measured’ to be comparable with the ‘as predicted’ results generated by the simulations.

As far as practical, two hot-boxes were constructed to the specifications given in the thermal design rationale in Section 5.2 in Chapter 5. The boxes are built with the same internal dimensions as the thermal comparator mechanism that was developed earlier in Chapter 4. The main differences are that the aperture is covered with glass and that all the conduction paths are replaced with thermally insulating and isolating construction.

Figure 7.1 (a) shows the completed system prepared for observations, one hot-box is mounted on a test-stand with its single-glazed aperture in the horizontal inclination and the other vertically-inclined and south-facing. Figure 7.1 (b) provides an annotated key to the main components. A detailed account of the methodology used for the monitoring campaign itself and the design of the instrumentation package is reported in Appendix I.1.

Figure 7.1: General arrangement of dual hot-boxes for the observation study

(a) Hot-box and heat-motor system on test-stand
(b) Annotation of system components listed below

| (1) | Pyranometer: Total global horizontal solar radiation $G_i = (0 \text{ to } 1500)$ in $W \cdot m^{-2}$ |
| (2) | Horizontal hot-box containing heat-motor ($B$) |
| (3) | Vertically-inclined south-facing hot-box containing heat-motor ($A$) |
| (4) | Both apertures covered with single 4 $mm$ thick clear soda lime glass sheet |
| (5) | Both hot-boxes lined with 100 $mm$ thick rigid insulation board |
| (6) | Outer layer of box lined with single layer of aluminium coated polyester sheet for high reflectivity |
| (7) | Inside of box lined with layers of aluminium coated Mylar sheet for high reflectivity and low emissivity |
| (8) | Thermally isolating couplers of Delrin® ($k = 0.31 W \cdot m^{-1} \cdot K^{-1}$) from (p. 1 in Ensinger, 2008) |

**Note:** The instrumentation mast for insolation measurement is omitted for clarity.
7.1. A practical dual hot-box system

Ideally, there would be an exact correspondence between the modal, spatial and temporal variables used by the virtual model to simulate the system’s behaviour and the instrumentation that measures the behaviour of an actual system. Striving for this match shaped the design of the instrumentation system for the hot-box and the conduct of the monitoring campaign.

Figure 7.3 shows the thermal model in its network form, this is then superimposed over the cross-section. The highlighted nodes represent the variables at the connection between each network link. The sensor for the corresponding modal type is annotated in bold. Specifically, thermocouples and thermoresistors for temperature \( T_{x,y} \), pyranometers for insolation \( G_{cH}, G_{cV} \) and linear motion transducers for actuation displacement length \( l_x \).

The thermal design rationale depends upon successfully differentiating between the specific energy that is absorbed by heat-motor \( A \) compared with \( B \). We wish to establish whether the differences operate the thermal comparator into one of its three possible states. The first aim of the observation study is to establish whether or not a practical hot-box exhibits the seasonal differentiation that was predicted by the simulation.

If the results show that it does, the subsequent questions are when during the year are the thresholds crossed and with what magnitude of insolation for each Day Number \( d_n \). To achieve this objective, the duration of the observational study spanned across a complete cycle of seasons from September 2011 through to September 2012. In preparation for the campaign, a pilot-study commenced in June 2011 to carry out testing and instrument calibration while the hot-boxes were built. In addition, monitoring was extended until March 2013 to monitor the trends in the level of daily insolation and maximum ambient temperature.

Ideally, within each seasonal interval there would be enough days with approximately clear-sky conditions to obtain a well-distributed sample for a year. In total, the campaign succeeded in making over one-hundred observations on days with qualifying conditions. Overall these were fairly evenly distributed. However, the main exception was the month of June in 2012 that experienced particularly bleak weather with very few days enjoying clear-sky conditions.

The outcome of the monitoring campaign is a set of time-series for the cylinder surface temperature of each heat-motor \( T_{c,h} \) and \( T_{c,v} \) for each diurnal cycle sampled across all of the seasons. In order to capture the operating context, both the conditions inside each hot-box and the prevailing outdoor ambient environment were also recorded. Taken together, these are all the variables that match those used in the simulation of the model as illustrated in Figure 7.3.

Figure 7.2 shows the two hot-boxes assembled in-situ in the centre lower right-hand side. To give an indicative arrangement of the component parts inside each hot-box, the parts are shown disassembled above and to the left-hand side. Each hot-box contains the same heat-motor and return-spring arrangement that was reported earlier in Chapter 3 for the experiments in the thermal laboratory. The heat-motor cylinders themselves are positioned to be fully exposed to sunlight but recessed approximately (50 to 60) \( mm \) behind the aperture cover.
With the exception of the aperture, the environment created around the heat-motors is an optically reflective and thermally insulating shell. In common with the earlier climate-chamber experiments, the in-situ study has an aperture of the same area through which the dominant energy exchanges occur. The difference is that instead of free convection with air, this is replaced by insolation from solar radiation and convective exchanges with ambient.

**Figure 7.2: Vertical and horizontal hot-box component parts shown disassembled**

Note: The vertical and horizontal hot-boxes are shown assembled on the test-frame. The disassembled components of sheet material are shown partially un-folded for clarity. The drawing is indicative only.
Having outlined the database of information that the observation study sets out to collect, the following sections report the findings. These are presented at two different time-scales, from the seasonal to diurnal intervals. Broadly, the seasonal time-scale corresponds to the thermal design rationale as developed in Chapter 5 and will test the definition of a seasonal ‘energy bandwidth’. While the diurnal time-scale corresponds to the FEM simulation of the first-order model developed in Chapter 6 and tests the characterisation that it made. Taking each in turn, the following reports what effect the aperture’s inclination angle had on the annual pattern of duty-cycles.

Figure 7.3: Section showing the correspondence between model-nodes and sensors locations

Notes: The section A-A is taken through Figure 7.1 (b) as cross-referenced. See page 283 in Appendix I.1 for a detailed specifications of each sensor used to measure the variables highlighted.


7.2 Study findings: The extent of seasonal duty-cycles

Figure 7.4 shows the actuation energy that was observed for each inclination. Specifically, the amount of energy that was accumulated from solar radiation or insolation up to the time of actuation. This was calculated by the integration of the instantaneous radiation onto each inclination angle ‘H’ and ‘V’ respectively. The integration interval is between the start of exposure at \((t_{x,0})\) and the time of actuation at \((t_{x,1})\) as expressed for each inclination by the two Equations in (7.1). The insolation on each day number \((dn)\) in the horizontal plane \((E_{dn,h})\) and vertically-inclined south-facing plane \((E_{dn,v})\) is plotted in the ordinate \((Y\text{-axis})\). Each data-point represents an instance of when a heat-motor underwent a duty-cycle.

In Chapter 5, the feasibility for the rationale to seasonally differentiate duty-cycles rested on whether or not the insolation into each hot-box falls within the seasonal ‘energy bandwidth’ that separates the two inclinations. To test this, the extent of the seasonal ‘energy bandwidth’ is shown in Figure 7.4 with the grey horizontal band.

On the horizontal plane, there was a seasonal cut-off at \((E_{dn,h} \approx 2.6 \text{ MJ} \cdot \text{m}^{-2})\) below which no duty-cycles occurred, this low value corresponds to when ambient was warmer. In the months between the seasonal cut-offs, the maximum daily insolation on the horizontal plane for the hot-box to duty-cycle is \((E_{dn,h} \approx 5.3 \text{ MJ} \cdot \text{m}^{-2})\), this high value corresponds to when ambient was colder. The horizontal hot-box duty-cycled within the limits of the seasonal ‘energy bandwidth’, there were only four outliers to this result from \((n = 108 \text{ days})\) of observations.

\[
\begin{align*}
\text{(Horizontal plane)} \quad E_{dn,h} &= \int_{t_{h,0}}^{t_{h,1}} G_i H \cdot dt \quad \{\text{in } J \cdot m^{-2}\} \\
\text{(Vertically-inclined south-facing plane)} \quad E_{dn,v} &= \int_{t_{v,0}}^{t_{v,1}} G_i V \cdot dt \quad \{\text{in } J \cdot m^{-2}\}
\end{align*}
\]

Where:

- \(G_i H\) Total global instantaneous solar radiation horizontal \{in \text{W} \cdot \text{m}^{-2}\}
- \(G_i V\) Total global instantaneous solar radiation vertical \{in \text{W} \cdot \text{m}^{-2}\}
- \(dt\) The sample time interval See note (1) \{in \text{s}\}
- \(t_{h,0}, t_{v,0}\) Start of exposure to direct beam radiation See note (2) \{in \text{s}\}
- \(t_{h,1}, t_{v,1}\) Time when heat-motor (A) and (B) reach melting-point temperature \{in \text{s}\}

Note: (1) The value of \((dt)\) is a balance between neither under-sampling nor over-sampling the fastest changing variable. Examining the time-series obtained during the pilot studies showed that a large variation in fluctuations between the parameters, under a clear sky the yield is typically \((Y \geq 67\%)\) temperature changes steadily but actuation displacement typically extends/retracts at a rate of \((\delta l/\delta t \approx 1/30 \text{ mm} \cdot \text{s}^{-1})\). (2) Based on solar geometry for each day number \((dn)\) for this geographic location and assumes an un-obstructed view of the horizon.
7.2. Study findings: The extent of seasonal duty-cycles

Turning attention to the vertically-inclined and south-facing aperture. The results for actuation energy \( (E_{dn, v}) \) during winter, spring and autumn fall within the seasonal 'energy bandwidth'. However, during late-spring and summer the amount of energy \( (E_{dn, v}) \) accumulating when the hot-box exhibited duty-cycles started to fall below the seasonal 'energy bandwidth'. This is partly due to warmer ambient conditions and partly due to a fault that developed in heat-motor \((A)\) that led to a small amount of wax leaking from the cylinder.

**Figure 7.4: Seasonal energy threshold for duty-cycle activity for each hot-box inclination**

The band of daily accumulated insolation to seasonally differentiate heat-motor actuation

1. Predicted total insolation received on the horizontal plane pre-noon \( (1) \)
2. Observed energy \( (E_{dn, h}) \) received \( (2) \) by heat-motor \((B)\) up until the time of actuation \( (3) \)
3. Predicted total insolation received on vertical south plane pre-noon \( (1) \)
4. Observed energy \( (E_{dn, v}) \) received \( (2) \) by heat-motor \((A)\) up until the time of actuation \( (3) \)

**Note:** Results of observations made in London (UK) at geo-coordinates Latitude 51°29′11″N by Longitude 0°23′W at Altitude 13 m a.m.s.l. to the WGS 84 datum. Campaign duration (Sep 2011 to Dec 2012). \( (1) \) Assumes clear-sky conditions based on empirically-derived models by Haurwitz (1945) and Oke (1987). \( (2) \) The measurements for each data-point of a heat-motor's duty-cycle is filtered by the daily insolation yield at \( (Y > 67\%) \) measured in the horizontal plane. \( (3) \) An actuation is qualified as the rising-edge \( (\mathcal{I}) \) in duty-cycles when \( (l_m = 0 \rightarrow 60 mm) \) within a diurnal cycle. The number of sample points \( (n = 108 \text{ days}) \).
During the monitoring campaign from Sept (2011 to 2012), the mechanical seal in heat-motor (A) began to leak wax during duty-cycles in months (5 to 7) of 2012. The implication of a reduced mass of wax ($m_w$) for the energy-balance in Equation (5.2) is that the amount of insolation that needed to be captured for actuating the heat-motor gradually declined. This accounts for the data-points that are below the ‘energy bandwidth’ between the months (5.5 to 9) in Figure 7.4.

These results give us an engineering rule-of-thumb for the seasonal threshold needed to actuate the proprietary heat-motor in this arrangement. Exposing the aperture to ($E_{dn, h} \approx 5 \, MJ \cdot m^{-2}$) of insolation overcomes ambient losses to change the actuator’s energy-balance in favour of actuation. Less than this threshold either due to inclination or orientation, then the heat-motor is highly unlikely to actuate. Even if ambient temperatures were to reach the climate norms for summer and minimise ($\Delta T$). This suggests that the rationale is robust because while ($\Delta T$) in Equation 5.1 has a sensitivity of ($-26 \, to \, +18 \, \%$) to the variation in mean air temperature, while ($E_{dn, v-c}$) has an insensitivity of ($+70 \, to \, +150 \, \%$) to the variation in the available insolation compared with the actuation threshold.

In summary, the results of the observation study show that heat-motor (B) in the horizontal hot-box only duty-cycled for part of the year. The season began at day number ($dn = 55$) and continued until day number ($dn = 300$). Heat-motor (A) in the vertically-inclined hot-box was found to duty-cycle on almost every ($>98 \, \%$) occasion throughout the year. This demonstrates in principle that the seasonal threshold falls within the seasonal ‘energy bandwidth’ used by the thermal design rationale as the basis for differentiating between inclinations.

### 7.3 Study findings: The characteristics of diurnal duty-cycles

While Figure 7.4 showed the seasonal threshold for the accumulation of insolation before the thermal-to-mechanical energy conversion process drives the actuators open, once actuated it gives no sense of how long the logical ‘On’ state actually lasts. Neither does it visualise a comparison between the behaviour that is predicted by the simulations and that observed in the practical embodiment. To remedy this, the diurnal duty-cycling for each heat-motor is plotted on an annual basis in Figures 7.5 and 7.6.

To aid the comparisons, the duty-cycle activity of heat-motor (A) is separated from (B) with the ‘as predicted’ shown above and the ‘as measured’ below in each of Figures 7.5 and 7.6 respectively. The portion of the plots that are shown in grey indicate the logical ‘Off’ state during the photo-period. This shows the extent of duty-cycles compared with when insolation is available assuming clear-sky conditions.

The headline result for the horizontal hot-box shows it is active across more of the year than predicted and active for longer within each diurnal interval. The vertical hot-box is active across the whole year as predicted, but it is active for less within each diurnal interval. The observed time-lags in the melting-process are longer than predicted for both inclinations across the year.
7.3. Study findings: The characteristics of diurnal duty-cycles

Figure 7.5: Horizontally inclined hot-box - heat-motor (B)

Figure 7.6: Vertically-inclined south-facing hot-box - heat-motor (A)

Note: Measurements of actuation displacement in heat-motors (A) and (B) from duty-cycling in hot-boxes during observations (2011 to 2012) in London (UK) under ($Y \geq 67\%$) clear-sky conditions.
7.4 Implications for operating the thermal comparator

A set of the diurnal interval are plotted as cross-sections through the (Y-axes) of Figures 7.5 and 7.6 in Section I.1.2 of Appendix I.1 on pages (292 to 317) with accompanying notes. The plots are complemented by a summary of the observed Yields (Y_i) and minimum actuation energy values (E_{dn,Γ}) in Table I.2 on page 318.

This section has summarised the results for heat-motor (A) and (B) on an individual basis against the predictions made from simulating the linear model, the following section considers these results when the duty-cycles of (A) and (B) are linked together in the context of operating the thermal comparator mechanism that was developed earlier in Chapter 4.

7.4 Implications for operating the thermal comparator

This section draws on the results of the duty-cycling in heat-motors (A) and (B) to consider the resultant output (Q) of the mechanism developed earlier in Chapter 4. This is visualised in Figure 7.7 with the dimension of time on two axes, diurnal intervals on the ordinate axis and seasonal intervals on the abscissa. The results of the FEM simulations reported earlier in Chapter 6 are carried forward as shown in the top-half of Figure 7.7. In order to allow a like-for-like comparison, the 'as measured' actuation data for each inclination is plotted in lower-half.

For clarity, the actuation displacements of both heat-motors are interlaced side-by-side in pairs for each observation day. The logical state of a duty-cycle is shown during daylight hours only. The exception is when an actuation displacement overruns into dusk. In all other cases, when the heat-motors were closed during daylight hours the intervals are plotted in grey.

A first-order comparison between the observed and predicted annual pattern of duty-cycles for the annual duty-cycles of heat-motors (A) and (B) is that the horizontal hot-box duty-cycling was observed across a wider extent of the year than predicted. The season started (100-days) earlier and continued for (45-days) longer.

In the vertical hot-box, while the annual extent of duty-cycling matches the predicted there are disparities between the durations of actuation in each diurnal interval across seasons. In the winter months it was shorter than predicted, but during the summer months it was longer. The annual pattern does appear to match the slight shortening during summer compared with the widening around either of the equinoxes compared with winter, although the differentiation is weak.

The 'as measured' results, between dn≈(100 to 250) show that the rising-edge of the duty-cycle in the horizontal hot-box began earlier than in the vertical hot-box. This would have been a mechanically undesirable sequence of actuation for the embodiment of the thermal comparator developed in Chapter 4 because the shutter would have been rotated inward.

The most striking overall difference between the observations and the predicted pattern is a shift in the duration of actuation to later in the day. The time-lag is in the order of one or more hours and is a characteristic common to both hot-boxes. Qualitatively, this would have led to the shutter failing to provide access to daylight for a portion of the morning.
7.4. Implications for operating the thermal comparator

Figure 7.7: The 'as measured' and 'as predicted' duty-cycles in heat-motors (A) and (B)

Note: The plot (above) shows the simulation results. [Animation of simulation] (AVI, 11.0 Mb).
The plot (below) shows the observation results. [Animation of measurements] (AVI, 14.0 Mb)
7.5 Summary

In Chapter 5 and 6, a thermal design rationale for operating the comparator mechanism was tested by a simulation study of a first-order mathematical model. Apart from the laboratory work reported in Chapter 3 and the qualitative findings of the in-situ pilot-studies reported in the Appendices, that exercise was conducted solely in-silica. A simplified outdoor environment was assumed and it did not directly draw upon an empirical source of data for component temperatures.

This chapter presented the results of an empirical data collection campaign, this was carried out under realistic in-situ conditions using a practical dual heat-motor hot-box over an annual cycle of clear-sky days. The objective was to provide an empirical source of data to compare with the synthetic results from simulating the thermal model.

The results of the observation study support the thermal design rationale for differentiating between the duty-cycles of the heat-motors on a seasonal basis. However, the comparison between the empirical and synthetic time-series has also highlighted discrepancies. Specifically, the predicted seasonal and diurnal extent of horizontal duty-cycles was under-estimated and the diurnal extent of vertical duty-cycles was over-estimated. The time-lags associated with melting phase-change was considerably under-estimated for both inclinations.

These findings have implications for operating the thermal comparator mechanism. The discrepancies between the diurnal differentiation of heat-motor (A) and (B) across seasons would lead to an undesirable sequence of mechanical operations made by the comparator’s output (Q) on the thermal shutter leaf. The over-estimate of predicted diurnal duty-cycling of heat-motor (A) would most likely come into conflict with an occupant(s) preference for access to daylight early in the morning. This may lead to demands for an override to the comparator’s output state.

No attempt was made to optimise the design of the practical dual hot-box to exactly match the simulation results, since the variables are modelled idealistically in one-dimension and the parameter values are estimated. These all represent sources of uncertainty. The contribution made by studying the practical dual hot-box has been to demonstrate the viability of seasonally differentiating the duty-cycling between heat-motors (A) and (B) by the inclination of aperture alone. The following chapter analyses the discrepancies in the findings reported here and considers the scope for improvement to the simulation and synthetic model in order to narrow the gap between them.
Chapter 8

Analysis and discussion

In Chapter 5 and 6, the simulation of a first-order model was developed to predict the annual pattern of the comparator mechanism’s duty-cycles. The results reported in Chapter 7 provided an empirical ‘as measured’ set of duty-cycles from observing actual heat-motors in-situ over a corresponding interval under approximately similar meteorological conditions. At the end of the last chapter, disparities were identified between the ‘as predicted’ and the ‘as measured’ data-sets.

In order to carry out a comparative analysis, a critical evaluation is made of the thermal design rationale itself. Axiomatic tests are applied using the ‘as measured’ data to scrutinise the robustness of the simplifying assumptions that were made during its development in Chapter 5. By testing ‘that adaptation of means to environments’ (Simon, 1981), the practical hot-boxes tested in this research are found to represent a finely-balanced embodiment of the thermal design rationale. This characteristic becomes the focus of considerations in an analysis of the discrepancies between predicted and measured.

This chapter carries out an analysis of the disparities in an effort to establish the efficacy of the model and its simulation. Drawing on the benefit of the ‘as measured’ data from Chapter 7, the FEM applied in Chapter 6 to simulate heat-diffusion and couple it to actuation displacement will be tested for its internal consistency. The comparative analysis of predicted and measured informs a discussion about the sensitivities of the rationale to the variation of seasonal and diurnal conditions and how each heat-motor’s state might be further differentiated by the selective exposure to available insolation in the context of ambient prevailing meteorological conditions.

Accepting that the duty-cycle profile of the thermal comparator when operated solely by passive transfers will not always match the preferences of an occupant when they are present, the scope for overriding the autonomic response is discussed.
8.1 Verifying the criteria for the selection of wax melting-point

The thermal design rationale is based on maintaining ambient \(T_{db}\) as the passive 'cold sink' as close to \(T_{mp}\) as a site’s climate data suggests. This is needed to maintain a \((-\Delta T)\) for as much of the year as synoptic weather conditions allow, as defined in Equation (5.1) on page 90. Did the selection of the wax melting-point \(T_{mp}\) actually provide a continuous thermal sink given the synoptic meteorological conditions that prevailed in-situ during the observation study?

This section tests this assumption by drawing on the contextual data collected during the monitoring campaign. Figure 8.1 plots the maximum air temperature that was recorded each day. Observations made under conditions classified as approximating clear-skies are highlighted (●), while those data-points marked (○) are for all other conditions of atmospheric turbidity.

Figure 8.1: Observed daily maximum air temperatures vs climatic mean and melt-point

- Maximum daily outdoor dry-bulb air temperature on days with (≥67%) clear-skies
- Maximum daily outdoor dry-bulb air temperature (All other observations)
- Monthly mean maximum outdoor air temperature
- Nominal melting-point temperature \(T_{mp}\) of heat-motor wax

Note: The maximum daily outdoor air temperature was less than the wax melt temperature in (96%) of diurnal cycles, sample size \(n = 835\) observation days. ¹ Data collected in London (UK) Latitude 51.5°N during monitoring campaign (part-2008, 2009, 2011 and 2012). ² Climate data shows the mean monthly maximum temperatures for London (1948 to 2010), (Source: UK MetOffice, 2010). ³ Melting-point of paraffin wax \(n\)-octadecane \(T_{mp} = 28.2^\circ\text{C}\) at standard pressure 1020 Pa (Source: p. 126 in Dirand et al., 2002). Seasons in the British Isles defined after (Manley, 1971).
The climatic monthly mean temperature that was used as the input to the first-order linear model is plotted in Figure 8.1 for comparison. Given the prevailing climate for the in-situ observation study, the rationale’s criteria selected the wax melting-point \( T_{mp} = 28.2°C \). The assumption for this selection would be sound if it can be shown that the observed temperatures for mean daily \( \text{max}(T_{db}) \) actually did remain below the melting-point.

When the distribution of ‘as measured’ data-points is compared against the climatic mean, the autumn and winter seasons show a reasonably good fit, but the ‘as measured’ for spring and summer was higher than average, as shown in Figure 8.1 in the interval \((60 < \text{dn} < 280)\). Overall, this is still a fairly good fit given the comparatively short time-span of the monitoring campaign compared with the (30 to 40) y data-base used to calculate the climate means. The results under worst-case conditions show for a few excursions that the daily \( \text{max}(T_{db}) \) recorded were below \( T_{mp} \), validating the selection of \( n \)-octadecane wax for both heat-motors \( (A) \) and \( (B) \).

The wider implications of this finding suggests that if the rationale used here can select the melting-point of a wax against a local climate database, then it may also be viable for other geographic locations with its own specific climates. The range of extremes in dry-bulb temperatures found on the surface of Earth is large, in the order of \((-80 < T_{db} < +60)\) in (Artemis, 2009), although within the range of wax melting-points found in the review in Chapter 2.

It is a matter of speculation as to the efficacy of this rationale over a building’s operating life in the context of a changing climate. A rise in the near-surface temperature within the UK in the order of \( (T_{mean} = +3.5°C) \) by 2100 is likely to have an impact on the rationale’s efficacy. However, an evaluation is very difficult because it is not clear what seasonal variation the predicted rise might have. This particular value for \( (T_{mean}) \) represents a relatively small source of uncertainty, but this is also a conservative prediction based on a medium emissions trend (MetOffice, 2009).

8.2 Validating the start conditions for the FEM simulations

Do the practical hot-boxes reject all of the internal heat-gains that accumulate during daytime, so that by dawn on the following day the conditions approximate an equilibrium with ambient? This question scrutinises the assumption about the simulation’s start condition, since the rationale depends on a ‘thermal reset’ occurring before each dawn.

If it turns out that the thermal inertia of the hot-box was much greater than the simulation results suggest, then this has implications for the assumed start-conditions. It would suggest that in reality they are both un-steady and higher than the assumed ambient temperature at dawn.

This section draws on the results from the observation data to analyse the annual distribution of hot-box temperatures by comparing the difference in temperature \( (\Delta T) \) between the cylinder in each hot-box and ambient at the time of sun-rise. This aims to establish whether it was justified to assume thermal equilibrium at the start of each FEM simulation by initialising all the elements in the model grid to \( (T_{dp}) \).
8.2. Validating the start conditions for the FEM simulations

Figure 8.2 plots the annual distribution of \(\Delta T\) with the sun-rise time annotated for comparative reference. The plot shows that \(\text{init}(T_{db})\) typically lies within \((\Delta T = \pm 1 ^\circ C)\) of \((T_{db})\) for both inclinations at the time of sunrise when the ‘thermal reset’ would ideally have occurred. This verifies the start-condition assumption made in the thermal design rationale.

A qualitative comparison between the two inclinations shows that the vertically-inclined south-facing hot-box takes longer than the horizontal one to reject the heat gains accumulated during daytime. This may be explained by the effect of aperture orientation on the rate of heat-loss by passive radiant cooling. While both apertures cool by radiant cooling to the same sky with apparent temperature \((T_{sky})\), the rate of radiant cooling by emission at long wavelengths \((\lambda_p = 10 \mu m)\) depends strongly on the view-factor that each respective aperture has with the sky vault.

In this observation study, the hot-boxes were separated by \((\theta = 90 ^\circ)\) with the horizontal aperture subtending a solid angle of \((VF_h = \pi/2)\) steradians with the sky while the vertical aperture subtends only \((VF_v = \pi/4)\) steradians. The model made the simplifying assumption that losses were to an apparent sky temperature \((T_{sky})\) that was uniform. Thermographs taken of the observation study site in Figure 8.3 show the actual sky is somewhat anisotropic. Particularly at zenith angles of \(> 70 ^\circ\) where temperatures start to approach ambient.

Figure 8.2: Annual pattern of temperature difference between cylinder and ambient

![Figure 8.2: Annual pattern of temperature difference between cylinder and ambient](image)

Note: The top half plots \((T_{c,h})\) in the horizontal hot-box, the lower half plots \((T_{c,v})\) in the vertical hot-box against ambient \((T_{db})\) for each Day Number \((dn)\) under clear-sky conditions.
8.2. Validating the start conditions for the FEM simulations

The implication is that \((VF_v)\) is biased toward an slightly warmer average temperature than \((VF_h)\) that ‘sees’ mostly the coldest portions of the sky. Both apertures also cool by convection losses to the same ambient air temperature \((T_{db})\) but assuming each has identical construction it is likely they have the same coefficient of heat-transfer. This is even if the rates of heat-loss are slightly different due to differences in absolute temperature.

Overall, the data shows thermal equilibrium between the outside surface of both apertures and the effective sky temperature \((T_{sky})\) reinforcing the validity of the assumed start-condition. Having established the validity of the simulation starting-point, the following section considers the assumption about the uniformity of the thermal sink over each diurnal interval across seasons.

**Figure 8.3: View-factors to atmospheric emission seen by the hot-box apertures**

(Above) Thermograph of the atmospheric emission at long-wavelengths \((\lambda_p = 10 \mu m)\) from a spherical reflection. The horizontal hemisphere is shown (Left) with the zenith point \((Z)\) in the centre. The vertically-inclined south-facing quadrant is shown (Right). Reflections of the corresponding sky view-factors (Below) in the visible-band. Zenith angle highlighted at \((75\%)\) threshold in net losses.
8.3 The implications of climate average as a simulation input

The simple linear thermal model reported in Chapter 5 was simulated assuming that the ambient thermal sink could be characterised by ‘worst case’ climatic data. Specifically, by using the average minimum air temperature drawn from (30 to 40) °C of monthly means. It was also assumed that the ambient thermal sink would be flat across each diurnal interval.

The reasons for this were that a priori to running the first-order simulations, there was uncertainty about how different the diurnal profile of ambient temperature would be on a day with clear skies compared with one that was heavily overcast. Whilst this was useful for an approximate characterisation, our common experience is that under clear-sky conditions the ambient air temperature at dusk tends to be somewhat higher than at dawn.

What are the implications of assuming that the thermal sink temperature was constant across each diurnal cycle? In addition, what if any differences are there across seasons from the effects of the thermal fly-wheel on the atmosphere. These are questions about the efficacy of the input that was used to drive the simulation. It was expected that this would lead to a conservative prediction that under-estimated duty-cycle activity.

Figure 8.4: The observed difference in ambient diurnal thermal sink temperature

\[ \Delta T = (T_{mp} - T_{db}) \text{ @ dusk} - (T_{mp} - T_{db}) \text{ @ dawn} \]

Note: Sample size \( n = 835 \) observation days. The data was collected in London (UK) Latitude 51.5 °N during a monitoring campaign from (part-2008, 2009, 2011 and 2012).
8.3. The implications of climate average as a simulation input

On the basis of the effects of solar heating and the thermal inertia of the near-surface air mass together with re-radiated heat-energy from surrounding surfaces, this section discusses the implications that this assumption holds for the findings made by the first-order simulation.

Figure 8.4 plots the difference in temperature ($\Delta T$) between the ambient sink and the wax melting-point for dawn compared with dusk during the monitoring campaign. Figure 8.4 plots an annual distribution with those observations made under clear-skies highlighted with a (●). A statistical analysis of the results shows that on average the days with clear-skies had a sink temperature difference across the day of ($\Delta T = 5.4^\circ C$). This is twice that for all other conditions of atmospheric turbidity. The variability of ($\Delta T$) on clear-sky days is only slightly greater than for others, ($St.D = 2.86$) over ($n = 153$) observations on days with clear-skies compared with ($St.D = 2.50$) over ($n = 578$) observations on days with lower insolation yields ($Y_i < 67\%$).

The implication of not accounting for this variability in the simulation of the first-order model can be described with respect to Figure 5.10 in Chapter 5 on page 107. This shows an actuation’s duty-cycle to be symmetrical about ($t = 12:00 \text{hrs}$), whereas this variability means the recovery stroke after the start of the falling-edge in a duty-cycle will always occur later in the afternoon than the prediction in Figure 5.10 suggests.

This can be explained by the results showing dusk to be almost always warmer than dawn, ($\Delta T$) is lower compared with the melting-point ($T_{mp}$) and therefore ambient becomes a less effective sink for convective thermal heat-loss. The simulation of the model is conservative in this regard because it over-estimates the rate of heat-rejection by convection to ambient in the afternoon and early evening proportional to $\text{init}(T_{db}) + 5.4^\circ C$ for a given ($dn$) with a ($St.D = 2.86$).

This result informs the analytical question of how sensitive the thermal design rationale is to the variation. Since section 8.1 established that ($T_{db} < T_{mp}$) for nearly all intervals, the proportion of the sensible heat that this variation represents in the initial energy-budget can be evaluated. Based on the evidence of results under approximately clear-skies, it is (33 to 85) % of the sensible heat budget. This is a large spread, but still less than half the overall energy budget for actuating the proprietary heat-motor.

The impact on the robustness of the thermal design rationale is that prevailing meteorological conditions will make predicting the profile of the recovery stroke in a duty-cycle have a larger margin of error than the actuation portion. These findings can be applied by using the parameters values above when generating synthetic weather data-sets (e.g. using the method reported on pp. 64-71 in Chap. 3 by Wit, 2003). As part of further work, this could be used as input to run the first-order simulation of the model for producing a prediction for the spread of variability in the pattern of actuation duty-cycles.

Having tested the thermal design rationale’s assumption and established areas of robustness and limitations, the following section returns attention to the characteristics of the duty-cycles ‘as measured’. In particular, the high response latency observed in the practical hot-boxes.
8.4 Characterising response latency

Returning to the disparities in the findings when comparing the simulated with the measured, the most striking difference in the pattern of duty-cycles exhibited by the practical device is that they are all shifted to later in the day. The time-lag or 'response latency' is in the order of one or more hours and shared by both hot-boxes. This section attempts to explain the large discrepancies in the simulation’s under-estimation of the time-lags.

In order to characterise the response latency reported in Chapter 7, the data from the observations are analysed for their differentiation across diurnal and then seasons intervals. This was done by categorising the duration of every melt in Figure 8.5 (a) and solidification in Figure 8.5 (b) separated by hot-box inclination in planes with \((Z-X)\) and \((Y-X)\) for each day number.

The results show that seasonal differences are more pronounced between the inclinations for melting than they are for solidification. For actuation, the inclinations can be compared in Figure 8.5 (a) on the left. In the \((X-Z)\) plane is a subtle trend showing the latency shortening slightly during the summer months compared with winter. Since direct beam radiation enters the vertical hot-box every day, this can be explained qualitatively by the subtle effect on latency of a seasonal oscillation in the energy-balance between decreasing insolation and a smaller \((-\Delta T)\) sink during summer versus a gradually increasing insolation but a larger \((-\Delta T)\) sink during winter.

In the \((X-Y)\) plane the trend is much more distinct because while during the summer it resembles the latency in \((X-Z)\) plane, it rises dramatically in spring and late-autumn just as the direct beam radiation incident on the aperture reaches the cut-off angles.

These two differences in the melting latency can be contrasted with the trends for solidification as shown in Figure 8.5 (b) on the right. The value of the \((\Delta T)\) thermal sink is lower during summer leading to an increase in latency compared with winter, a trend shared by both inclinations.

**Figure 8.5: Latency of actuation during wax melting (left) with solidification (right)**

(a) Latency during actuation \((l_m = 0 \rightarrow 60 \text{ mm})\)

(b) Latency of recovery stroke \((l_m = 60 \rightarrow 0 \text{ mm})\)

*Note: Data from observation days with \((Y \geq 67\%)\) clear-skies in London (UK) Latitude 51.5°N during (2011 to 2012). Latency evaluated while actuation displacement is within \((l_m = 0 \rightarrow 60 \text{ mm})\). Maximum daily displacement length \((l_m = 0 \rightarrow l_{\text{max}})\) may exceed \((l > 60 \text{ mm})\) by a variable amount.*
8.4. Characterising response latency

This plot shows no dramatic differences in the latency in the (X-Y) plane at the spring and late-autumn cut-offs, this underlines the consistency of the radiant cooling and convective loss processes to ambient that dominate the solidification part of the duty-cycle across all seasons.

Given the characteristics of response latency in the practical system, the simplest explanation for the discrepancies with the simulations arise from simplifying the two-dimensional optical arrangement of the aperture with a one-dimensional thermal model. The motion of the sun does not irradiate the whole length of the motor’s cylinder immediately when the azimuth and altitude angles between sun and aperture intersect. It gradually exposes the cylinder surface during actuation and then gradually recedes during solidification. In addition, the cut-off angles are further constrained as the heat-motors are recessed (50 to 60) mm inside the hot-boxes. In hindsight, it was an over-simplification to assume that the highly reflective linings would uniformly irradiate the cylinders with incoming radiation subject only to the variation in glass transmission.

Discrepancies in the solidification part of the duty-cycle can be explained further through characterising the radiant energy transfers, the description of the dominant heat-transfer processes in the dual hot-box system during daytime for the actuation part of the duty-cycle are now revisited.

Figure 8.6: Seasonal variation of the temperature differential from aperture covers and sky

Note: Passive cooling by radiant heat-transfer by the difference in surface temperatures ($\Delta T = T_{g,h} - T_{sky}$). Temperature time-series of aperture covers from observations on days with ($Y \geq 67\%$) clear-skies in London (UK) at Latitude 51.5° N during (2011 to 2012). Data for the effective sky temperature ($T_{sky}$) for the corresponding interval from the University of Reading (Reading, 2012)
Figure 8.7: Nocturnal heat-transfer processes between the hot-boxes and ambient

Note: Cross-section through the dual hot-box enclosures, heat-motor system and shuttered window. The general arrangement of components is shown indicatively and not in proportion.
Specifically, by referring to Figure 8.7 that illustrates the thermal sources, sinks and transfers processes that characterise the system for the solidification part of the duty-cycle. This is after sunset or when the sun falls outside either aperture’s view. Using the monitoring data, the annual pattern of temperature difference attained between the aperture and sky is visualised in Figure 8.6. If we recall that the difference between the transfer rate of radiant and convective processes is $(\Delta T^4)$ compared with $(\Delta T)$, then this plot underlines the dominance of the radiant cooling toward the end of each day when the daily insolation gains are cut-off at dusk.

Focussing attention on the $(-q''_{rad})$ processes that represents the heat-rejection by ‘Black body’ radiation from the surfaces of the heat-motor to the ambient surroundings through the hot-box aperture. The geometric view-factor between the cylinder and the back of the glass is the same inside both hot-boxes, so assuming the same temperature difference they have the same rate of heat-exchange. Therefore, differentiation in the rate of exchange for the two inclinations depends on differences between the view-factors of the outside glass surface with the sky and surroundings. As discussed earlier, the theoretical difference is $(V_F g \rightarrow \text{sky} = \pi/2)$ given the sink $(T_{\text{sky}})$ and the horizontal aperture would reject heat at a faster rate. The data during the summer interval does support this but the differentiation is weak. It maybe fair to state that the variability of apparent sky temperature and the influence of the ground-reflected component and surrounding building surfaces makes this assessment very difficult to verify.

Turning attention to the $(-q''_{cond})$ processes that represents the heat-absorption and rejection by conduction, the one-dimensional model inferred each heat-motor’s actuation state solely from the cylinder temperatures $(T_{c, h}, T_{c, v})$. While this has established the feasibility of the rationale, it also simplified the thermal behaviour of the wax in the cylinder’s cavity. Chapter 3 showed that treating the proprietary heat-motor’s wax as a ‘lumped capacitor’ is a poor approximation because its Biot number is $(Bi \gg 0.1)$. Based on the profiles of time-temperature-actuation displacement reported on page 58 in Chapter 3 from the thermal laboratory work, the likely effect of wax’s thermal properties on the time-varying behaviour of $(T_c)$ simulated by this model is to introduce discontinuities. The reason why these effects were not described by the first-order simulation was that the one-dimensional linear model of the heat-balance was solved by negating the capacitance of the components and assuming a uniform temperature field, the step in Equations (5.8 and 5.22). This assumption is defended on the basis that the mass of the air inside the hot-box is small and the motor’s aluminium cylinder has exceptionally high thermal conductivity.

Turning attention to the realities of the test site, to simplify the first-order simulations it was assumed that insolation exposure came from unobstructed views of the horizon. However, a later survey of the in-situ observation site measured the elevation angle of obstructions as shown in Figure 8.8. This found obstructions at most angles of azimuth but with relatively low angles of elevation. The main implication of obstructions to the east was to slightly delay the melting portion of the duty-cycle until later in the morning across all seasons. This is consistent with the
discrepancies shown in response latency found on the melting portion of the duty-cycle due to its dependence on the direct beam component of solar radiation.

Lastly, the complicated process of solidification is only quasi-modelled by the FEM and this partly explains why the simulated duty-cycles are symmetrical given an ambient thermal sink at constant temperature. The temperature and actuation data from the observation study allows the simulations to be re-run to compare them. By testing the evolution of the measured actuation displacement against the measured temperatures, the robustness of the FEM itself can be tested as a description of the conduction process, this is pursued in the following section.

Figure 8.8: Sky view-factor of each hot-box showing site-specific obstructions

Note: Spherical reflection images (below) of the sky vault with altitude and azimuth grid in orthogonal projection superimposed. Annual sun-path shown (above) at intervals of 30 days, for each day at 15 min intervals. See page 288 in Appendix I.1 for a description of the methodology. Observational study site, London (UK) at Latitude 51.5°N in September 2012 [Time-lapse study] (WMV, 6.4 Mb)
8.5 Evaluating the robustness of the FEM implementation

This section conducts a test on the FEM schema to evaluate how well it describes the behaviour of the thermo-hydraulic actuation. Specifically, it numerically analyses how well simulating the model couples the evolution in the diffusion of heat-energy to changes in actuation displacement length.

Verifying the FEM element displacement-to-temperature coupling

The original implementation of the FEM relied on a temperature time-series as the simulation input to drive the ‘feed forward’ schema (pp. 166-77 by Humphries, 1974). Recall that the simulation results reported in Chapter 6 were driven by a synthetic time-series that carried simplifying assumptions. We can now benefit from the empirical data reported in Chapter 7’s observation study to use ‘as measured’ cylinder surface temperatures to drive the FEM schema instead.

A sample of time-series from observations days across the seasons are used to re-run the FEM simulations, the results are then compared for the diurnal evolution of the resulting displacement length between the ‘as simulated’ and ‘as measured’. Figure 8.9 plots three pairs of time-series for comparison on days across contrasting seasons. These show that the FEM solution is a closer fit during melting than solidification and that the FEM solution tends to underestimate the volume expansion of the wax above the critical-point temperature.

Analysing the monitoring campaign data as a whole, a residual is calculated based on the mean difference between the time-series from simulating the model and measurements made for each day. This is expressed in a general form by Equation (8.1) (After p. 67 in Chatfield, 2004). Residuals that are close to zero with a random distribution would indicate that the simulation is a ‘good’ description of the observed behaviour.

\[
\text{Residual (variable) = Observation (t) - Fitted value (t-1)}
\]  

Each simulation is then categorised by day number on the (X-axis), these are sampled from days when observations are available for the performance of the hot-box in the database compiled in Chapter 5. The simulation results for each day are then categorised by the orientation of each heat-motor’s hot-box, horizontal plotted in the (X-Y) plane and vertical plotted in the (X-Z) plane. The evolution of heat-diffusion in each motor is categorised into the interval of melting and solidification. This is because the FEM schema handles melting and solidification processes differently. Melting is modelled as a pure-conduction phenomenon whereas the mass transport phenomenon associated with convection in the wax melt is only approximately modelled, the crystallisation processes associated with solidification is not accounted for in the model.

The first residual is evaluated for temperature by comparing points in the temperature field. This is restricted to points in the thermal model grid that can be spatially and temporally coordinated with the location if corresponding sensor used in the monitoring campaign. One of these is fixed because the FEM uses a ‘feed forward’ schema that takes the observed temperature \(T_c\) at the cylinder surface \((x_1, y_1)\) at time \(t\) to seed the next time interval in the simulation.
A hard test for the efficacy of the heat-diffusion description would be to find the point in the temperature field *furthest* away from the cylinder’s surface at the centre of the cylinder. Conveniently, this is where the cavity temperature is measured so we can compare this with the history of temperature \( T_w \) for element at coordinates \((x, y)\) nearest to the cavity core \((x_2, y_2)\) at the previous time step \((t - 1)\) during a simulation run.

With this hierarchy of categories for the evaluation of the simulation’s description of heat-diffusion, a second residual is evaluated for the evolution of actuation displacement and recovery stroke. This tests the description for the coupling of specific gravity and specific enthalpy for all the elements in the FEM by its total displacement volume during the evolution of actuations.

**Figure 8.9: Validation of coupling between hydraulic and thermal diffusion in FEM simulation**

(a) Observation in winter (25th November 2011) [Animation] (AVI, 10.6 Mb)

(b) Observations in a late-autumn (22nd October 2011) [Animation] (AVI, 12.3 Mb)

(c) Observations in autumn (30th September 2011) [Animation] (AVI, 13.7 Mb)

Results of using temperature time-series data from monitoring campaign to seed the FEM simulation for comparing the predicted actuation displacement from the simulations against the observed.
8.5. Evaluating the robustness of the FEM implementation

From the general residual expressed in Equation (8.1), the residuals for evaluating the evolution of actuation displacement derived from the heat-diffusion can be found for each case in each category of the simulation and is expressed in Equations (8.2).

\[
\text{Residual } (T) = \{T_t, T_{(t+1)}, \ldots, T_{(t+n)}\}_{\text{observed}} - \{T_{(t-1)}, T_{(t)}, \ldots, T_{(t+n-1)}\}_{\text{simulation}} \text{ in } ^\circ\text{C} \\
\text{Residual } (l) = \{l_t, l_{(t+1)}, \ldots, l_{(t+n)}\}_{\text{observed}} - \{l_{(t-1)}, l_{(t)}, \ldots, l_{(t+n-1)}\}_{\text{simulation}} \text{ in mm}
\] (8.2)

The results of all the residuals are plotted in Figures 8.10 (a and b) to show the efficacy of the FEM for simulating heat-diffusion and Figures 8.11 (a and b) to show the evolution of actuation displacement. The simulation residuals for heat-diffusion are low across and in-between all categories indicating that the efficacy of the model is a good fit with the observed. The residuals for the evolution of displacement length are not as good. The evolution of actuation displacement is underestimated during melting and overestimated during solidification. This result is consistent between the two hot-box orientations and also appears independent of a seasonal variation.

**Figure 8.10:** The residual (T) from simulating the heat-diffusion through wax in the cavity

(a) Melting during actuation \((l_m = 0 \rightarrow l_{\text{max}})\)  
(b) Solidification in recovery stroke \((l_m = l_{\text{max}} \rightarrow 0)\)

**Figure 8.11:** The residual (l) from simulating the evolution of actuation displacement

(a) Melting during actuation \((l_m = 0 \rightarrow l_{\text{max}})\)  
(b) Solidification in recovery stroke \((l_m = l_{\text{max}} \rightarrow 0)\)

Note: Plotted from sample of \((n=24)\) simulated days compared with data from observation studies collected under \((Y > 67\%)\) clear-skies in London (UK) during (2011 to 2012).
Allowing for calibration errors and instrumentation tolerances this still suggests the underlying trend. This exercise has tested the description in Section 6.2 “Coupling the moving boundary with actuation length” of Chapter 6. The residuals validated the FEM’s simulation of the diurnal profile of actuation displacement length based on a time-series for the cylinder’s surface temperature using the ‘feed forward’ scheme.

The residuals also identified short-comings in the efficacy of FEM using the synthetic time-series because of differences with the actual profiles measured in-situ. This had implications for the overall simulation results. Further work could refine the accuracy of the model to account for the factors that limit a more realistic profile for the cylinder’s temperature in each hot-box. While it is reasonable to expect that further effort could improve the efficacy of the simulation’s prediction of duty-cycle patterns, the sensitivity of the FEM to variations in the actual temperatures attained in the practical embodiment indicates the relatively fine balance of the rationale’s energy budget.

The sensitivity of the energy balance in the context of passive heat-transfers has been a recurring attribute of this study. In this regard, the following section discusses the consequences stemming from the selection of a wax melting-point so close to the range of ambient temperatures.

8.6 Robustness in a repertoire of passively-driven ‘states’

The central research question asked whether a heat-motor can be used to represent the effects of insolation by a well-defined logical ‘state’. In addition, whether compounds of linked heat-motors can be arranged to provide a repertoire of responses to the different seasonal effects of insolation. By restricting itself to an investigation of the characteristic expression of ‘state’ by thermo-mechanical means solely from the evolution of passive heat-transfers, this study had to define a thermal design rationale for how ‘state’ would be differentiated in a given climate.

At this point, the reader is reminded that the ideal means for determining the motor’s logical ‘state’ is by the absorption and rejection of the wax’s Latent heat of fusion alone. In which the phase-transitions occur close to the isothermal, with a fractional increase in the wax’s temperature above the melting-point for a logical ‘On’ and a fractional decrease below it for a logical ‘Off’. The two logical ‘states’ would correspond to a stable thermal as well as mechanical equilibrium, each with its own well-defined specific energy. The portion of sensible heat between these two states would be vanishingly small.

In order to operate the thermal comparator mechanism’s output \( Q \) into each of its desirable states, the hot-boxes had to oscillate the heat-motor’s energy-balance between the equilibrium state that represents logical ‘Off’ and logical ‘On’. Ideally, the difference in the arrangement of each hot-box serves to differentiate when heat-motor \((A)\) and \((B)\) are maintained in one or other of the equilibrium states within each diurnal interval and across seasons.

Operating the thermal comparator using heat-motors in this way went against engineering convention (Duerig, 1990) because the output state is not the gradual change in actuation displace-
ment length where the sensible heat portion mattered, but whether or not the wax is completely melted or solidified in which the Latent heat portion of the budget matters most.

To reiterate, the energy source that drives the transition from logical ‘Off’ to ‘On’ comes from harvesting insolation. While this has a well-defined diurnal profile under clear-sky conditions, the total insolation under each profile varies seasonally. The energy sink that drives the transition from logical ‘On’ to ‘Off’ comes from maintaining a deficit with the ambient surroundings. This varies in proportion to \((-\Delta T)\) and is both unpredictable and variable on an annual basis.

The design challenge for the hot-boxes was to successfully differentiate between the duty-cycles of each heat-motor by selectively harvesting the available insolation from a pattern of exposure that changes seasonally. Arguably, the robustness of this rationale depends on maintaining each heat-motor’s energy-balance close to one or other of the well-defined energy-budgets that corresponds to each respective logical ‘state’. It should constrain it from deviating excessively due to the large variability in the source with respect to the energy-budget threshold, and to a lesser extent the sink. The larger these deviation are, the greater the uncertainty in predicting behaviour because the temperature difference increases the portion of sensible heat in the energy budget.

To investigate the practical application of this actuator operating in-situ, a threshold had to be defined for the difference between states relative to the site’s climate. To carry out a practical study within these terms of reference, the thermal design rationale could not ignore the implications on the dynamics of the transition between each logical ‘state’ due to the low thermal diffusivity and conductivity of wax. It had to accept that uncertainties would arise from its non-linear behaviour during phase-changes between liquid and solid.

In this sense, selecting a wax melting-point so close to the climate norm at \((T_{mp}) \geq \max(T_{db})\) ‘hedged one’s bet’ on an embodiment that strikes a fine-balance. It had to admit enough insolation to generate a temperature gradient that transferred Latent heat at a sufficiently high rate to give an acceptable time lag while also avoiding the temperature deviating excessively.

We can reflect on this dynamic in the practical embodiment by drawing on the data from the observation study to discuss the limits of the thermal design rationale for differentiating between logical ‘states’. To help visualise this, the data-reduction that was applied to plot the results in Figures 7.5 and 7.6 on page 140 is rolled back by one dimension, with temperature data substituting actuation displacement as a proxy for logical ‘state’ to discuss the dynamics of the phase-transitions.

To orient the reader, each plot shows the temperature of the heat-motor cylinder’s in the \((Z)\)-axis, the diurnal profiles are in the \((Y-Z)\)-plane with the annual profile projected by linear interpolation along the seasonal intervals in the \((X)\)-axis. The temperature threshold between logical ‘states’ at the melting-point of the wax is through the \((X-Y)\) plane at \((T_{mp} = 28.2^\circ C)\), with the portion rendered white representing phase-transitions. The portion of the surface above is rendered in warmer colours and represents logical ‘On’, while the surface below that is rendered in cooler colours represents logical ‘Off’.
Figure 8.12: Comparison of surface temperature on cylinder of heat-motor (B)

(a) Evolution of $(T_{c,b})$ ‘as predicted’ by the FEM simulation of the H-grid model ($T_{\text{mean}}$ in 1949 to 2010)

(b) Evolution of $(T_{c,b})$ ‘as measured’ in horizontal hot-box during observation study (2011 to 2012)
Horizontally-inclined hot-box

Referring to Figure 8.12 that shows the annual patterns of temperature for heat-motor (B), with those predicted in the upper plot 8.12 (a) against the temperature measured on the motor inside the horizontal hot-box in the lower plot 8.12 (b).

The main discrepancy is that the maximum temperatures attained in the practical embodiment were considerably higher than predicted. To explain this, the assumption that the hot-boxes had comparable apertures by virtue of equal-area alone comes under scrutiny. This may be true when comparing surfaces with aspect-ratio of (AR = 1) on a square-metre basis, however, the aperture covers reused from the pilot-study in Appendix H.1 were rectangular with (AR = 0.64). It was also assumed that the view-factor of (VFh = π/2) between the aperture and sky would be orientation agnostic. Specifically, that net radiant energy exchanges would be independent of azimuth angle or yaw rotation about the (Z)-axis as shown in Figure 5.7 on page 97.

For practical reasons, the longest length was set-up oriented north-south rather than east-west, compared with the vertically-inclined aperture which had the shortest length aligned north-south. While the view-factor of (VFh = π/2) is true geometrically, the radiation-exchange under a real cloud-free sky is not uniform but an-isotropic (Muneer, 2004). This is complicated to characterise especially for its interactions through glass and was not incorporated into the first-order model.

The implication of this simplification on the amount of direct beam radiation admitted by the horizontal aperture was an under-estimation by the model. Especially during the summer season when the difference in aspect-ratios reached a maximum of (1 : 1.56). This led to an under-estimation of the attainable temperatures by the simulation which accounts for the discrepancy between the predicted and measured pattern of duty-cycles.

It is unlikely that this discrepancy is explained by an under-estimation in the prediction of the available insolation. A comparative analysis carried out between the measured and modelled levels of solar radiation incident on the horizontal aperture (GiH) vs (GcH) found a good fit with observations across the seasons. For further details, see Appendix I.1 for a seasonal spread of diurnal profiles plotting measured vs modelled.

Having accounted for the main discrepancies in the predicted pattern of duty-cycles for the horizontal hot-box, the following considers those found in the vertically-inclined hot-box.

Vertically-inclined south-facing hot-box

Referring to Figure 8.13 that shows the annual pattern of temperature for heat-motor (A), with the predicted in the upper plot 8.13 (a) against the temperature measured on the motor inside the vertically-inclined south-facing hot-box in the lower plot 8.13 (b).

While the annual profile of the measured data has a similar overall shape to the predicted, the main discrepancy is that the magnitude of the peaks predicted in months (1 to 2) and (11 to 12) are not as pronounced in the measured data. To explain this, a comparative analysis between the measured and predicted levels of solar radiation was carried out.
8.6. Robustness in a repertoire of passively-driven ‘states’

**Figure 8.13: Comparison of surface temperature on cylinder of heat-motor (A)**

(a) Evolution of \((T_c, v)\) ‘as predicted’ by the FEM simulation of the V-grid model (\(T_{mean}\) in 1949 to 2010)

(b) Evolution of \((T_c, v)\) ‘as measured’ vertical hot-box during the observation study (2011 to 2012)
This showed a discrepancy in the vertically-inclined plane in which the measured \( (G, V) \) deviated by approximately \( (20\%) \) less than the clear-sky model \( (G_c, H) \) during the winter months. In the vertical plane, the accumulation of insolation is understandably more sensitive to any site-specific obstructions due to the dominance of lower angles of solar elevation relative to the horizontal. This would explain the discrepancies in the magnitude of the two profiles because the model overestimated the level of solar radiation which led to higher temperatures being predicted.

The combined simplifications of ignoring site-specific obstructions, an-isotropic solar radiation from the sky, differences in the relative aspect-ratios of the apertures and low-levels of cloud turbidity that were stochastically distributed are taken to account for the main discrepancies between the predicted and measured pattern of duty-cycles. Within the scope of this study which was to investigate the feasibility of differentiating between heat-motors, these factors have not been compensated for retrospectively by further development of the first-order model and its simulation. Despite these limitations, using only the different energy sources \( (G_{cV}, G_{cH}) \) and the view-factors with the energy sink \( (VF_v, VF_h) \) in the V-grid and H-grid models to distinguish between each hot-box, the simulation successfully described the overall pattern of seasonal differentiation in the diurnal duty-cycling of the thermal comparator mechanism.

Returning to a discussion about the robustness of the design rationale from the measured data, specifically in terms of the equilibrium states that were common to both hot-boxes. As passive open-loop systems, neither of the hot-boxes imposed a limit on how much the accumulation of insolation in summer or the cold ambient in winter drives the energy balance to deviate from the phase-change threshold. As the data in Figures 8.12(b) and 8.13(b) shows, the consequence of this is that the deviations above the threshold are much higher than below it. Apart from suggesting a bias in favour of a higher predictive efficacy for transitions from logical ‘Off’ to ‘On’ compared with vice versa, this means that a stagnation temperature when the optical efficiency falls to zero will tend to occur between \( (12:30 \text{ to } 15:30) \text{ hrs} \) slightly after the peak intensity of solar radiation.

Given these terms of reference for robustness, the ideal responses for operating the thermally-insulated window shutters reported in Appendix H.1 might be met by a convergence between the pattern of duty-cycling measured in Figures 8.12(b) and 8.13(b) with the simulated in Figures 8.12(a) and 8.13(a). This could be achieved by improvements to the thermal design rationale that manipulates when the optical efficiency of the aperture to reach a ‘Steady state’ of stagnation at a lower temperature and earlier in the vertically-inclined hot-box in order to bring forward access to daylight closer to the start of the photo-period, and delay ‘Steady state’ conditions until later as well as shorten its duration in the horizontal hot-box to limit the shading response around mid-day when the risk of excessive gains are greatest.

Finally, this leaves us with the possibility that refinements to the responsive behaviour of the thermal comparator mechanism could be achieved by relatively simple iterative adjustments made to the parameters of the aperture and the orientation of the hot-boxes.
8.7 Occupant preference

Thus far, the premise has been that the comparator mechanism operates the window shutter in order to moderate indoor conditions when occupants are not present to operate them manually. Specifically, to passively adjust the shutter’s deployment over areas of glazing to regulate against excessive gains from insolation as well as to reduce thermal losses in a contribution toward a building’s overall environmental design strategy. At this point, we might ask for qualification as to what extent a building’s operation is in fact left to its own devices in this way?

In terms of quantifying occupancy, convention in the literature has defined space utilisation as a function of occupancy rate together with frequency rate. In which occupancy rate is how much a space is occupied in relation to its designed capacity and frequency rate is how often a space is used in relation to its availability, as defined in Section 2.25 on p. 22 by Bourn (1996).

The results of studies on the levels of occupancy in a range of buildings with different types of accommodation has shown that space utilisation is surprisingly low. Specifically, a survey during 2003-4 of space utilisation across a range of accommodation in UK Higher Education Institutions within normal working hours found the median utilisation rate to be only (27 %), as reported on p. 8 in (SMG, 2006). Other public sector buildings such as schools also suffer particularly low utilisation frequencies in spaces such as assembly halls due to institutional schedules for space usage. In the commercial sector, studies of desk utilisation in offices have shown that within normal hours of business over a year of working days the pattern of single-occupancy desk usage ranges from a maximum (75 to 85) % for a sedentary worker to only (10 to 20) % for workers who are often absent, as reported on p. 13-5 in (Eley and Marmot, 1995).

These findings support the basic premise that automated changes to the shutter’s position for the benefit of the building’s operation is desirable given the extent that spaces are left unoccupied. Especially since significant changes in outdoor conditions occur when occupants are also likely to be absent. This applies to a range of accommodation types, consider the case of commercial offices during summer-time, both dawn and dusk occur outside normal hours of business. Acknowledging this evidence, attention turns to the operation of the comparator when occupants are present.

Ideally, the comparator’s passive response to prevailing external conditions would moderate the indoor environment within the tolerance of an occupant’s satisfaction. There are reasons to think that for some building types particularly the healthcare sector, the idealised annual pattern of shutter duty-cycles operating passively could provide a range of benefits to occupants without the need for intervention. Research into human neurophysiology has suggested some health benefits from a selective exposure to daylight as well as sunlight (Foster and Kreitzman, 2009). A response to these findings by the architect Carlo Volf is the concept of a ‘circadian light clock’ for controlling a building’s diurnal admission of sunlight (Volf, 2013). This would support human bio-rythms by providing a timely exposure to the external cues from sunlight that synchronise human chronocycles for the normal regulation of levels of serotonin and melatonin as well as blood-pressure.
In practice, the literature on studies that observed buildings in-use found fully automatic systems may not be the complete answer, because occupant satisfaction is related not only to psychometric variables but also on their perception of the freedom to choose (Bordass et al., 1994, 1997). The premise of design guidance in the literature is that occupants 'are not interested in technology, only the results' (p. 4 in Bordass et al., 2007), we should not expect that occupants will make a distinction between an autonomic and automatic response at the point of use.

Therefore, an ideal form of control would be flexible enough to accommodate that an occupant’s satisfaction is conditional upon their individual preferences. In doing so, it acknowledges that satisfaction is affected by the visual, thermal and other aspects of their environment as well as contextual factors of building accommodation type and their particular activity within it at a point in time. This section discusses the potential benefits of an override for the occasions when the shutters operate in a way that does not suit an occupant for these reasons.

A range of scenarios are discussed assuming that prevailing meteorological conditions approximates clear-skies. Under these conditions, shutters would follow the pattern of duty-cycles idealised in the thermal design rationale developed earlier in this study. A discussion about the benefits of an override given the different seasonal responses of the comparator is complicated by the shutter’s multi-function design. In order to ground the scenarios, two accommodation types in different building uses with contrasting space utilizations are considered.

Firstly, a classroom in a school building is considered. The following assumptions are made based on the design guidance given in the Building Bulletins (BB) series from the UK’s Department of Education and Skills (DfES) and time-tables from an independently run secondary school in the London Borough of Southwark. Schools have three academic terms per year, totally 190 days of attendance with the school building unoccupied during the summer break from the end of July to the start of September. During term-time, pupil attendance is five-days a week with a daily time-table that is typically divided into eight periods between 08:55 hrs to 16:00 hrs. The longest break is typically 1h20min over lunch between 12:30 hrs to 13:50 hrs local time (GMT/BST).

Based on the typical area scheduled for a general teaching space in a UK secondary-school class room of \((A = 60\, m^2)\) from (Patel, 2004), overall dimensions of \((7.2\, m \times 8.5\, m)\) are assumed. One wall is assumed to be external and south-facing \((l = 7.2\, m)\) long in structural bays of \((l = 3.6\, m)\) wide. Within which an array of shutters in bays \((l = 1.2\, m)\) wide are installed over windows covering \((A = 40\% )\) of the façade area, the maximum proportion from p. 6 in (DfES, 2003). It will be assumed that a thermal comparator’s output may be used to operate a group of shutters but that the position of each shutter can be individually overridden.

Secondly, an administration office in a commercial building is considered. The typical pattern of occupation is assumed to follow a working year of 228 days with a five-day week between normal office hours from 09:00 hrs to 18:00 hrs. The office-plan has the same dimensions as generic classroom together with external wall orientation and shutters in the same arrangement.
8.7.1 The benefits of an occupant override

Since the thermal comparator only reacts passively to the level of external radiant energy from sunlight, it does not make compensations for the state of indoor services. This includes whether or not artificial lighting is on or the level to which heating and cooling plant may be running. The following scenarios consider the possible cues of discomfort that may arise from conflicts between the shutter’s position under clear-sky conditions outdoors and the potential imbalances that may be experienced by an occupant indoors.

Based on the evidence from observational studies in a range of building types including schools, an occupant is prompted to exercise an override based on their ‘needs of the moment’ (Haigh, 1982). Faced with different potential imbalances, the benefits of an occupant override are discussed through the triggers for taking action to alleviate sources of discomfort.

During the winter season on clear-sky days

For a portion of mornings, the comparator’s response latency will keep the shutters closed when the first occupant arrives to a darkened space. The benefits of an override to open the shutters would be to allow amenity views to the outside and access to daylight. This may reduce electrical consumption from artificial lighting, however, this would be balanced against the possibility of increased heating load from thermal losses to the cold ambient conditions outdoors.

It is likely that there will be differences in the impact of this benefit in the context of the two accommodation types. In a classroom the occupant driving change is more likely to be the teacher managing the needs of the class or responding to complaints, rather than the pupils themselves. The teacher is arguably also the person most likely to physically operate the override control, they may even be the only person authorised to do so. In exercising this, they may override all the shutters the length of the classroom. By contrast, in the context of an office the occupant driving change is more likely to be an individual acting locally. They may only override the shutter located closest to their workstation, leaving the remainder to operate passively later in the morning.

When the shutter is fully open there is the possibility of thermal discomfort and visual glare from direct sunlight at low angles of elevation. Arguably, this would be best mitigated by adjusting an indoor screen such as a roller-blind. Alternatively, an override would allow an occupant to either adjust the shutter to the shading position, or to close it. While either choice would benefit the occupant by reducing their discomfort, the latter is arguably more likely. Operating the override to close the shutters would mitigate against a different source of thermal discomfort if the shutters are moved to the shading position. Shaded from direct sunlight, a large asymmetry between sources of radiant heat may form between the cold glazing surface and the rest of the interior, an effect that would be felt most acutely on the exposed skin by those closest to the windows.

In terms of the room’s energy-balance, closing the shutter would increase the demand on artificial lighting but reduce heat-losses to a cold outside ambient. This would be balanced against fore-shortening the opportunity to capture passive solar gains that offset the heating load.
During the spring and autumn seasons on clear-sky days

For the majority of normal hours of occupation, the shutters will be fully open. There is the risk of thermal discomfort and visual glare from direct sunlight in the hours either side of midday. Again, this could be mitigated by adjusting indoor screens over individual windows. Alternatively, an override would allow a shutter to either be adjusted to the shading position or closed. Again, while the benefit of either choice is to reduce occupant discomfort, in this case the former is arguably more likely. Operating a shutter to give shade allows an occupant to continue enjoying access to daylight and views out. There is a lower the risk of discomfort for an occupant close to the window from large asymmetries between radiant sources because ambient conditions are warmer.

In terms of the room’s energy-balance, it would benefit from access to daylighting to offset the demand for artificial lighting. However, this would be balanced against reducing the duration that beneficial passive gains could be captured. In practice, the net difference may be small because ambient conditions will tend to be warmer than in winter.

These alternatives may unfold differently during the course of a day depending upon the accommodation type. In the classroom, the teacher could be expected to apply an override equally to all pupils and therefore act globally across all six shutters. By contrast in an office, it may be more likely that an individual decides on which of the two options and acts only on one shutter, located closest to them. The overall benefit is the flexibility to cater for individual preferences.

During summer-time on clear-sky days

Since the thermal comparator operates on ‘local solar time’ while space utilisation is scheduled on British Summer Time, the shutters will be open or in the shaded position by the start of occupied hours on most days. While the shaded position is intended to mitigate against the risk of excessive gains when it is exceptionally hot outside, the generous \( A = 40\% \) glazing area combined with large internal gains may lead to indoor overheating. Once reached, it is unlikely that this can be compensated for by operating an override to adjust the shutter’s position alone.

This is complicated to address as it depends upon a room’s overall servicing strategy. If it is naturally-ventilated with openable windows in the \( A = 40\% \) glazing area, then it would be desirable to keep the shutters in the shading position. This would allow the free-area available for ventilation to be adjusted in order to regulate the rate of air-changes to suit. In a mixed-mode strategy, cooling demand could be supplemented by chilled beams or radiant panels operating at surface temperatures below ambient. In the case where the room is sealed and cooling demand is serviced by air-handling plant, it may instead be desirable to override the comparator and close the shutters in order to minimise thermal exchanges through the building envelope.

In either case, while an override brings benefits, an occupant would also need to understand that responding to cues of discomfort from overheating cannot be alleviated by operating the shutter alone. In this scenario, the benefit of an override would be to allow preventative action to take place by anticipating the comparator’s passive response earlier in the day.
Benefits that are independent of seasonal responses

The previous scenarios focused on aspects of an occupant’s thermal discomfort in relation to imbalances in the room’s physical environmental influenced by the shutter’s passive responses. In addition to these, there are occasions when it would be necessary or desirable to override the shutter’s position irrespective of the season or time of day. The following briefly outlines some of these scenarios, and in each case considers the different benefits of an occupant override.

- **Fire-safety**
  In cases where it is unavoidable that a room’s windows are designated to provide occupants with a means of escape in the event of an emergency, the shutter cannot obstruct the route to egress. In such cases, an override would be needed to open the shutter behind an opening window when it is closed or shading in order to satisfy statutory fire safety regulations.

- **Specialised activities**
  Some activities place a specific demand on a room’s lighting control. In the case of a school classroom, one example is the adjustments that are necessary for Information Communication Technology (ICT) such as data projectors and white-boards. The display wall or projection surfaces need to be sufficiently shaded to provide adequate contrast. While arguably this need could be met by adjusting black-out blinds, the shutters could be used as a substitute given the light-tightness provided by its seals. The benefit of an override would be to close them regardless of the comparator’s sequence of passive regulation.

- **Property security**
  Windows at ground floor level can represent a security risk where there is a vulnerability from forced entry by intruders. In cases where school classrooms are left unattended during the five school holidays with an average annual frequency rate of less than (48%) including forty-two consecutive days during the summer break, there could be a security risk while the shutters duty-cycle open. This pattern of space utilisation could benefit from a specific override that completely suppresses the comparator’s output and closes each shutter with a secure ‘holiday lock’.

In each of these scenarios the general benefit of an override is to provide the occupant with a timely means to resolve their discomfort, albeit in different ways. Apart from the immediate effect of a manually override, it would also be readable by providing feedback that the expected result of the action took place. Specifically, each individual shutter would either open, shade or close behind the window local to an occupant and in response to their particular ‘crisis of discomfort’. The findings of Post-occupancy Evaluations (PoEs) suggest that these qualities, in particularly a rapid response would be significant factors in satisfying an occupant by supporting the resolution of their ‘needs of the moment’ (Leaman, 1993).
8.7.2 Hypothetical modifications to the comparator mechanism

In spite of these benefits, we should heed the 'Law of unintended consequences' as a caution when there is the freedom to override the shutter’s position carte blanche to alleviate discomfort in the 'here and now'. Operating overrides in an ad-hoc fashion will leave an aftermath that unfolds over the course of a day, the effects of which are different for occupants than they are for the effective regulation of conditions in the room.

The evidence from PoEs suggests that once an occupant takes decisive override control to satisfy their immediate needs, there is a tendency to either do nothing remedial should circumstances change or to take only piecemeal action (p. 54 in Haigh, 1982). In practice, the conditions that lead to a crisis of discomfort for one occupant who operates an override early in the day may change, but the effects of the override may then not transfer to other occupants who arrive later. This may be particularly acute in the context of diurnal space utilisations with high turnovers in occupation, such as school classrooms which have up to eight per day.

An unintended consequence on a room’s energy-balance is the possibility that the accumulated benefits from the comparator’s otherwise passive sequences of transitions would be discounted. Opportunities may be missed to prevent heat-loss, capture advantageous gains and protect against overheating. By incorporating an override at all, we would have to accept that the resulting thermal comparator system implicitly adopts the standpoint that energy use in buildings is to support occupant activities first and foremost, rather than buildings themselves.

The combination of occupant overrides and the thermal comparator’s duty-cycles could be complementary, with the comparator providing a back-stop to ad-hoc overrides by restoring the shutter’s link to its passive duty-cycling after the last occupant has left. In the manner of Reginald Jeeves, the fictional valet who dutifully restores order at the end of the day, regardless of the preceding chaos (Wodehouse, 1925). The following outlines how the comparator could be adapted with modifications to the transfer mechanism to perform this function.

The interface to an occupant override could be a mechanical lever located internally and connected through to the comparator above the window. The position of the lever would correspond to the shutter’s position outside the window, providing clear and immediate feedback through an unambiguous link between the occupant’s action and the opening mechanism’s response.

When the occupant operates the lever, any existing connection between the comparator’s output and the shutter’s linkage mechanism is decoupled and the shutter is moved in the direction of the occupant’s choosing. Giving the occupant freedom to change from the opened, closed or shaded positions to either of the other two. When the occupant releases the lever, the shutter’s linkage is latched to a stay at that position. Either a hold-open, hold-shading or hold-stowed latched to the window frame. While the thermal comparator passively responds to prevailing conditions and next passes through the stowed position in the duty-cycle, it un-latches the shutter’s linkage mechanism from the hold to the window frame and reconnects it to the comparator’s output.
8.8 Summary

Three sections reported axiomatic tests on the thermal design rationale to evaluate its robustness. Each drew on the evidence collected from the monitoring campaign reported in Chapter 7. Firstly, the selection of the melting-point temperature was validated by comparative evaluation. Secondly, the limitations of the ‘thermal reset’ before every sunrise as a start condition for simulations was established. Thirdly, the extent of the simplification of using the climate data averages as an approximate seasonal value the ambient sink at ($-\Delta T$) was established.

The ‘as measured’ diurnal profiles were found to be both asymmetrical and offset into the afternoon compared with the ‘as predicted’. A differential analysis of the response latency in duty-cycles showed that seasonal patterns for the rising-edge are well-defined between the hot-box apertures, but that there are uncertainties in the falling-edge during solidification for both apertures. Response latency took longer than predicted in all cases.

The internal consistency of the FEM simulation and its coupling to actuation displacement reported in Chapter 6 was tested using the observation data. The simulation of heat-diffusion through the motor cavity was validated and found relatively low residuals for a range of observation days. The coupling of thermal-to-mechanical processes was tested using the actuation displacement data and found moderate residuals. This shows that the largest source of uncertainty is in the dynamics of heat-transfers during the non-linear phase-transitions. This is where simplifications in the first-order model have imposed limits on the efficacy of predicting the pattern of duty-cycles.

The implications of testing the thermal design rationale with relatively finely-balanced practical hot-boxes was analysed. The large differences between the measured and predicted seasonal extent and diurnal duration of duty-cycling in the horizontal hot-box was analysed on a comparative basis with the vertical hot-box. Limitations were identified in the underlying model for solar radiation, site-specific factors and the relative aspect-ratios of each aperture to account for the discrepancies.

Accounting for these discrepancies, the rationale for differentiating the output of the two heat-motors to operate the thermal comparator mechanism developed in Chapter 4 in-situ by a difference in inclination angle alone was shown to be feasible and operated reliably in a bracket of daily insolation yields $Y_i = (67$ to $100)$ % of clear-sky conditions.

Finally, accepting that there will be occasions when the output of the thermal comparator could come into conflict with an occupant’s preferences, the possible benefits of an occupant operated override were discussed. This elaborated on several scenarios when the shutter’s passively operated arrangement would need to be adjusted to reconcile prevailing indoor conditions with an occupant’s preference, even if this is occasionally at the expense of the building’s net energy-balance.
Chapter 9

Conclusions

The prospect of a building envelope that could physically reconfigure itself with a variety of appropriate responses to seasonal changes in its surrounding environment for improved envelope performance has inspired architects to investigate façade designs with movable elements. When occupants are not present to manually move the elements, the conventional approach to coordinating each desired configuration with each different prevailing meteorological condition has been dominated by automatic controls, with firmware using actuators that consume power.

This has the characteristic of separating the ‘means used to respond’ from ‘responding to different conditions with different responses’. Specifically, where the firmware embodies the rationale for the automatic control abstractly from the actuator, which then consequently needs to consume power to compensate for the difference between its passive state and that desired at any point in time. While this thesis has not problematised the separation per se, it has proposed an alternative approach that attempted to reconcile them in the context of one specific application for window shutters that serve multiple functions.

This thesis has sought to show that a novel means of actuation could also provide the requisite variety of responses and that its ‘autonomic’ properties bring with it advantages, including an entirely passive operation without the need for an additional power source. The candidate for the ‘means used to respond’ needed robust properties to match the requirements of moving building elements. It also needed to represent an embodiment of logical ‘state’ in mechanical terms in response to the prevailing conditions. The wax-filled heat-motor that converts thermal-to-mechanical energy possesses these qualities, for which a range of proprietary examples are available.

Part of the research effort has been an investigation into how the properties of wax’s phase-change could be exploited as a functional material in its own right. Moreover, it is marshalling the state of the wax inside the heat-motor into either its liquid or solid phase on a conditional basis that is central to the feasibility of this alternative approach. This brings with it the prospect that it can provide a repertoire of responses to a variety of prevailing conditions. It has shifted the conceptual ground away from abstracted forms of control toward a building’s envelope that literally reconfigures in and of itself by virtue of the embodied state of its material components.
These considerations motivated the central research question to ask whether a heat-motor and hot-box arrangement could be used to represent the seasonal difference in the intensity of sunlight. Specifically, as an indicator of different ‘states’ by directly using the heat-motor’s mechanical output. This forms the connection between the arrangement of the actuators and the building’s façade through their shared exposure to the same seasonal differences in the prevailing environment. The former embodying different logical ‘states’ and the latter having different physical configurations driven by the mechanical actuation of the former. This shifts the conceptual ground toward the actuator itself being the control rationale by virtue of its embodied arrangement.

There were obstacles to investigating this approach as few studies have reported the properties and operating dynamics of heat-motors. The design problem that had to be overcome was how to advance from the limitations of the binary response from a single actuator to mechanical arrangements with a repertoire of responses using multi-input comparators and logical operands.

From the outset, this research adopted a practical approach to these problems with an applied outlook, hence an engagement was sought with the properties and behaviour of an actual heat-motor rather than assuming a set of idealised characteristics. A proprietary heat-motor from industry provided the basis for the investigations throughout this research. While this characterises the findings as specific to one version of this actuator type, it is likely that some or all of its properties are shared to some extent by the range of alternative types reviewed in Chapter 2. This is on the basis that the experimental work in Chapter 3 revealed as much about the thermo-mechanical properties of wax as it did about the motor itself, and it is the same wax that would be used in the alternatives. Differences between them could be established with further work through a comparative study, drawing on the methodologies developed in Chapter 3 and 7.

The climate-chambers provided a controlled environment where the effects of passive energy-transfers on different mechanical arrangements of the heat-motors could be repeatedly tested. This point ought to be stressed as over one-hundred tests were carried out during the development of the mechanism presented in Chapter 4 because of its highly iterative prototyping. Had this research been dependent solely on un-controlled conditions outdoors, this would have impeded progress. The overall outcome of the laboratory-based tests was to establish working values for the actuator’s properties during thermal-to-mechanical energy conversion. It also identified and quantified some of the engineering constraints to the motor’s mechanical actuation displacement.

These insights informed the development of a thermal comparator made from a dual heat-motor arrangement linked by a novel transfer-mechanism. Testing a practical mechanism showed that the advantages of the underlying thermal-to-mechanical actuator technology were inherited while extending the repertoire of its responses to conditionally evaluate the thermal environment. Having achieved the embodiment of a comparator logic through Chapter 4, a pilot-study set out to demonstrate the application of this mechanism to physically reconfigure an array of movable thermally-insulated window shutters across the façade of a pavilion building.
An in-situ observation study over four months verified that a different pattern of responses to prevailing conditions was achievable. This was characterised on mild and hot days by the amount of insolation under clear-skies. It showed in principle that the dual heat-motor mechanism could mechanically operate the shutters into the desired positions solely by passive gains from insolation. The terms of reference were set for a rationale that needed to marshal the passive transfers of thermal energy to transition the thermal comparator into its different states on a seasonal basis.

Chapter 5 pursued how to make the transition from the climate-chamber laboratory to in-situ conditions by using a low temperature phase-change material as the working fluid in the heat-motor. It needed to overcome how the net energy-balance could be differentially biased to completely melt and re-solidified a wax charge within diurnal cycles across seasonal variations, while still maintaining the advantages of the thermal comparator’s mechanical actuation.

By recasting this problem as a thermal design rationale to oscillate between equilibrium states, Chapter 5 proposed to 'hedge one’s bet' on the lowest melting-point wax against the highest mean air temperature in order to maintain ambient as the thermal sink. The implication of this rationale was that insolation would be harvested as the sole energy source. The rationale proposed that the states of the heat-motors would be differentiated by differences in their daily and seasonal exposure to insolation as each was filled with a wax having a melting-point that continually tended towards solidification to the cooler ambient temperature.

No work on modelling hot-boxes at differential inclination angles to duty-cycle heat-motors in this way was found in the literature. To test the feasibility of the thermal design rationale, a desk-exercise, a first-order numerical model was developed to predict the annual pattern of diurnal duty-cycles that would be generated by a dual heat-motor operated thermal comparator in hot-boxes separated by ninety-degrees from horizontal to vertically-inclined and south-facing. Simulations of the first-order model suggested that an annual pattern of diurnal duty-cycles that resembled the desirable properties for operating the thermally-insulated window shutters to provide insulation, access to daylight and shading from excessive solar gains was in principle possible.

The simplifying assumptions in the first-order model and its simulation introduced uncertainties that were difficult to account for, affecting the confidence in its efficacy under in-situ conditions. Firstly, because of the lack of an analytical solution for the phase-change processes inside the heat-motor. Secondly, the lack of accurate characterisations of the simulated environment in the hot-boxes used to drive the different dynamic thermal exchanges on samples of days across seasons.

This recalls the point made in the introduction about the obstacles to verifying the efficacy of simulating models of systems with embodied state, because they by definition exhibit sensitivities to the prevailing environment. In the event, it proved infeasible to attempt these simulations using standard hourly-interval weather data alone, when the evolution of the heating gains from insolation changes at rates of \((\approx 0.25^\circ \cdot \text{min}^{-1})\) and the results from the different pilot-studies clearly showing the subtle exchanges of energy within the hot-box evolving at sub-1 min intervals.
Learning from the data relating temperature and actuation displacement obtained during the laboratory work in Chapter 3, it is the non-linear behaviour during phase-transitions that pose the greatest difficulty for validating the efficacy of the thermal design rationale’s proposed embodiment. Chapter 6 sought to address the latency of response with a FEM model through the heat-motors, but this still relied on a synthetic time-series for cylinder temperature. Only Chapter 7 tested the efficacy of the thermal design rationale directly by constructing practical hot-boxes and carrying out a year-long observation study to monitor the resultant behaviour of the heat-motors.

The results of the observation study showed the feasibility of the thermal design rationale to passively drive a seasonally different diurnal pattern of duty-cycling in the dual heat-motor system by separating the angle of inclination between two hot-boxes. The consistency of the results for duty-cycles under clear-skies were encouraging because it demonstrated robustness in spite of uncertainty in the model predictions for the intensity of solar radiation and the seasonal variability of ambient as the thermal sink.

A comparative analysis between the results in Chapter 7 and those in simulations and FEM studies in Chapters 5 and 6 broadly agreed but also showed discrepancies in extent and magnitude of duty-cycles. Explanations for these discrepancies were given based on an analysis of the data and a post-study survey of the observation site itself.

It is probably reasonable to think that with further work to iteratively improve the hot-box design and its construction together with improvements to the thermal models and their simulation to account for the discrepancies found in this study, that these two approaches would converge toward a description with progressively lower residuals. The area in which a greater insight has been gained arises from exploring the possibility of an environmentally-driven conditional logic from the multiple heat-motor operated comparator mechanism itself.

This research demonstrated the feasibility of linking multiple heat-motors together into mechanisms that perform a useful comparative logic response to different prevailing environmental conditions. The output has the advantages of robustness and high actuation force suitable for physically reconfiguring building elements. A thermal design rationale demonstrated how the seasonal variation in diurnal insolation patterns could be used to drive the thermal energy balance of these novel mechanisms to provide a repertoire of stable states that respond to the prevailing conditions solely by passively exchanging thermal energy with its surrounding environment.

The thermal design rationale developed during this research tested a relatively finely-balanced version of the energy-budget needed to actuate heat-motors, as discussed in Chapter 8. While further work could contribute toward engineering improvements to its robustness as well as the predictability of its pattern of duty-cycles, its potential applicability to physically reconfigure movable elements in the built environment in response to prevailing conditions may be wider than the limited climate context of the UK studied here. The following section considers how the thermal design rationale could migrate to respond to other climates as well as diversify its range of responses.
9.1 Scope for adaptability and diversification

This research has been carried out in the south-eastern UK, for which the findings from testing the thermal design rationale in Chapter 7 are site-specific. Although the degree to which it is transferable has yet to be tested either numerically or practically, the underlying dynamics of the energy-balance developed in Chapter 5 may be applicable to other locations in the UK. Its success depends upon the factors analysed in Chapter 8 that centres on the uncertainty of how robust the different passive thermal exchange processes are when applied elsewhere.

Within the UK and other locations that experience a similar climate, mitigating regional and local differences to transfer the rationale of seasonal responses may prove to be possible with only modest adaptations. At the meso-scale, there are implications for the dynamics of the energy-balance in Chapter 5 from the impact of the ‘urban heat-island’ effect as described on (pp. 218-21 in Bridgman and Oliver, 2006) due to the differences between buildings in urban locations compared to those in rural areas. Specifically, in urban environments the effects from a higher density of buildings, lower average surface albedo as well as anthropogenic heat-sources all contribute to increase the average ambient air temperature. Recalling from Chapter 5 that the basic daily actuation energy \( (E_{dn,J}) \) of the proprietary heat-motor is finely-balanced against local ambient, adjustments within the meso-scale may be made to the wax melting-point \( (T_{mp}) \) by applying the methodology in Appendix F.1 using data for mean maximum \( (T_{db}) \) specific to the site.

Depending upon the scale of regional and local variations it is perhaps adaptable to similar warm temperate climates within a band of similar latitudes, on the basis of a relatively small variability in the energy sources and sinks for the transfer processes in the energy-balance developed in Chapter 5. There is some climatological evidence for this as well as an approximately symmetrical magnitude at latitudes north and south of the equator (Table 1 in Sellers, 1965). This may be achieved by compensating for different solar elevation cut-off angles by adjustments to the geometric arrangement of the hot-boxes for locations within the temperate zone \( (23.26^\circ < \text{latitude} < 66.30^\circ) \) north and south of the equator.

The UK experiences a warm temperate climate \( (C) \) that is fully-humid \( (f) \) with warm summers \( (b) \) as classified under the Köppen-Gieger scheme (After Kottek et al., 2006). The areas that experience a similar climate as well as its sub-categories is large, including across the continental USA \( (Cfb) \), the east-coast of the USA that experiences hot summers \( (Cfb) \). Parts of south-eastern China \( (Cfa) \), southern Japan \( (Cfa) \), eastern South-America \( (Cfb/Cfa) \) and the east-coast of Australia \( (Cba/Cfb) \) that together account for \( (5.87\%) \) of Earth’s land area.

It may be possible for the approach developed in this work to migrate further afield, where the climate is also characterised by well-defined seasons that are linked to the pattern of seasonal insolation. The following section considers some possibilities, and how the rationale might be diversified to exploit other arrangements of passive energy sources and sinks that organise the heat-motor mechanical state through its energy-balance.
9.1.1 Thermal design rationale: Adaptability to other climates

Migrating further afield than temperate climates implies a more significant shift as seasonal variations change significantly. Toward the torrid zone between the tropic of Cancer and Capricorn at parallels (23°26′N≥latitudes≤23°26′S) respectively, the seasonal differentiation in daily sunlight is lower. The noon sun-angles are higher leading to a higher solar intensity. Toward the frigid zones above and below the Arctic and Antarctic circles at parallel latitudes (66°30′N) and (66°30′S) respectively, the seasonal difference is more extreme oscillating between periods of darkness and 24-hour daylight. The noon sun-angles are lower leading to lower solar intensities. The overriding point of these well-known characteristics is that any thermal design rationale developed using this approach would migrate with the building it serves, for which the appropriate repertoire of responses would be different from the multi-function shutter explored in this study.

Adaptations to the thermal design rationale could be made to the optical design of the aperture and surface properties of the actuator in order to account for the abundance of solar energy and hot ambient air temperatures that would drive a surplus in the former. While increasing the effectiveness of the hot-boxes and optical properties of the actuator to account for the deficits of solar energy and very cold ambient conditions in the latter. Further analysis would be needed to evaluate the different ratios of variation in solar availability and the range of ambient temperatures with respect to the energy-budget to actuate the heat-motor for different climates.

It is more important in the author’s view to address the prospect and design opportunity for furthering the variety of dynamic building elements using the principles explored in this study, both within the facade as well as deeper within the construction of buildings. Many possibilities are apparent, for example a repertoire of responses to an extreme diurnal range of ambient conditions as experienced in hot arid environments. The possibility of a building that could benefit from dampening internal temperature fluctuations using wall and floor void with variable geometry, either for depth or the extent of surface exposure to act as selective plenums for night-time cooling and daytime air redistribution. Heat-motor’s could be embedded into wall and floor construction as well as within the voids and control the arrangement of elements by adapting to fabric temperature relative to ambient temperature or the duration of sunlight in the preceding diurnal photo-period.

The repertoire of configurations that is desirable for a building’s envelope in response to these seasonal differences would need to be reconsidered, spurring further design challenges. For which the thermal actuators and variations on a hot-box arrangement and linkages that migrate along with the building they serve would need to adapt to suit.

Having shown the feasibility of one case of seasonal responsiveness, further effort could see the diversification of functions for a comparator and mechanisms that embodies other comparators and conditional logic that could be beneficially applied to other climates. The author’s aspiration is that this possibility will serve to spur further invention. As a prompt, the following section suggests the development of a complementary means for passively transition the heat-motor’s phase.
9.1.2 Thermal design rationale: Functional diversification

Reflecting on the results of Chapter 7, while it established how the energy balance of the actuators can be selectively melted by gains from insolation on a south-facing façade, it raises the intriguing possibility of its inversion for applications to a north-facing façade. This section briefly considers its potential as a diversification of the means to passively drive state-transitions.

The rationale in Chapter 5 was ‘actuation positive’, in the sense that the thermal actuator’s duty-cycle is driven by a diurnal change in the energy balance that is net positive ($G - H$). It is dependent upon gains ($G_iH$) to overcome ambient conditions that always tend to solidify the wax. Since the thermal actuator’s mechanical output no directional bias, it is arbitrary whether its mechanical output ‘state’ is driven by the wax in its liquid or solid phase. It is when the phase-change occurs that the thermal-to-mechanical energy conversion is harvested.

In principle, the design rationale could be ‘recovery-stroke negative’ by driving the thermal actuator’s duty-cycle with an energy balance that is net negative ($\Delta G$) within a diurnal interval. The following considers how the thermal design rationale could be diversified to exploit losses by passive cooling to overcome ambient conditions to drive duty-cycles.

The constituent gases in the Earth’s atmosphere have spectrally-selective optically properties which have been exploited as a thermal sink for passive radiant cooling in previous studies carried out in numerous previous studies (Granqvist, 1981; Kreider and Kreith, 1982; Granqvist and Eriksson, 1991). The underlying physics relates to cloud-free conditions opening an ‘atmospheric window’ to space in the wavelength band $\lambda = (8$ to $14) \mu m$, as thermographed in Figure 9.1.

**Figure 9.1: Thermal emission from the atmosphere under clear-skies: A free cold-sink**

Notes: Hemispherical reflection of the sky in the visible-band (Left) with corresponding thermograph (Right) showing atmospheric thermal emission at ($\lambda_p = 10 \mu m$) under clear-sky conditions. Effective sky temperature toward the zenith is ($T_{sky} < -40^\circ C$). The annual sun-path is shown at intervals of 30 days for latitude $51.5^\circ N$, with the sun ($\odot$) at $30 \min$ intervals diurnally, indicating the portion of the sky view-factor that would need to be screened out. London UK January 2014. The nomogram of radiation intensities ($\bullet$) is overlaid onto the hemisphere, adapted from p. 38 in (Olgyay, 1963).
9.1. Thermal design rationale: Scope for adaptability and diversification

9.1.2.1 North-light: A single-action thermally-insulated shutter

In this alternative, the physics of the energy-balance is also dominated by radiant and convective energy transfer processes. Unlike those modelled earlier in Chapter 5, a north-facing façade receives less direct gains. Instead, the dominant exchange of radiant energy is at long wavelengths between the ground and sky as surface thermal emissions to the sky are balanced by atmospheric counter-radiation back to the ground. The tendency toward a net loss to space from this exchange is largest under clear-sky conditions (Sellers, 1965) as Figure 9.1 indicates.

The impact of radiant energy loss on windows and roof-lights in north-facing building façades is a tendency toward a higher rate of heat-loss under clear skies, compared with overcast skies. Especially given that plain glazing has a relatively high surface emissivity. The typical value of \( \varepsilon = 0.873 \) represents a closer approximation to a ‘Black body’ emitter than a plain metal cladding panel from (p. 6 in BSI, 2001). This effect is exacerbated at northern temperate latitudes for windows with deep reveals as the diurnal energy-balance is not compensated for by gains from direct sunlight.

The heat-losses could be mitigated by adjusting a movable thermally insulated shutter to close over the window in response to clear-skies, this would reduce passive cooling by radiant exchange. This presents an avenue of further work that could diversify the thermal design rationale to harvest passive exchanges for the selective conversion of thermal-to-mechanical energy by a thermal actuator to perform this function. An advance is needed in the means to achieve this, as it requires an inversion of the hot-box developed in this study. Rather than amplifying gains, the thermal actuator’s shared exposure with the window to prevailing cloud-cover conditions would amplify cooling processes instead.

This may be feasible with a ‘coolth-box’ that has ideal properties for exploiting the cold sky to selectively reject heat-energy under clear-sky conditions to solidify the wax. It would have to achieve this despite ambient temperatures that are warmer than its melting-point. Further work could investigate aperture arrangements with a sky view-factor that selectively exposes the thermal actuator to the thermal sink while shielding it from direct insolation and ambient.

In principle, the operation of such a system would adjust the shutter’s position to respond to the synoptic state of the sky vault. As cloud-cover shifts between overcast and clear conditions, the coolth-box amplifies the change in radiant emission thereby duty-cycling the thermal actuator. The thermo-mechanical energy conversion operates the shutter to alternate in synchrony with the shared exposure of the window and coolth-box aperture to the prevailing apparent sky temperature.

Under clear-skies when the apparent temperature is coldest, the thermal actuator would solidify and close the shutter to reduce the angle factor and limit heat-loss. While under overcast conditions when the apparent temperature of the sky is closer to ambient, it is warmer than the wax’s melting-point. The thermal actuator would then melt and fully open the shutter to maximise access to diffuse daylight through north-facing windows.
The combination of a window and movable shutter can match the thermal performance of a standard wall, even using plain window glazing. There are architectural reasons in favour of simpler glass assemblies rather than very high performance glazing systems, because a generous daylighting factor can be achieved with plain windows of modest proportions due to its high VLT. Its broad spectrum transmission of natural daylight also improves the colour rendering of interiors.

Figure 9.2: Section through coolth-box showing the dominant heat-transfer processes

Coolth-box shown above a window in a north-facing façade operating a side-hung vertically-pivoted shutter leaf. The construction of the components is shown indicatively and not drawn in proportion.
A section through a north-facing façade with a window and vertically-inclined coolth-box is shown in Figure 9.2 to indicate the general arrangement of components. The following briefly describes the relation between the components in spatial and thermal terms. The coolth-box contains a single thermal actuator (A) with its mechanical actuation displacement (l_a) linked to the shutter’s opening mechanism (Q). A one-dimensional model of the thermal processes is overlaid onto the section showing a simplified network of the dominant thermal exchanges between the thermal actuator, coolth-box aperture and sky quadrant.

The dominant radiation processes are solar and atmospheric, the box provides insulation and shielding against gains by radiant transfers from direct-beam solar radiation (G_{dir}) by its north-facing orientation and the 'east-west sunhood' shown in Figure 9.2. The box isolates the actuator from conductive transfers with the surrounding sources and sinks through thermal insulation, low-thermal conductivity connectors and high-emissivity linings.

The aperture into the box is covered with a screen (s) to limit the gains from outside ambient by convection transfers. The screen consists of two layers of HDPE (High Density Polyethylene), with the outer layer at temperature (T_s) approximating ambient air temperature (T_{db}) through convective exchanges (+q'_{conv}) with coefficient (h_s) to minimise variable wind-driven losses. The inner layer further isolates the actuator to the convective transfers (+q'_{conv}) between the screen, actuator and the air inside the cooth-box at temperature (T_b) with coefficient (h_a).

The screens provide a transparency of (\tau_s = 0.8) at long wavelength for radiation exchange between the actuator surface with an emissivity (\varepsilon_a) and the apparent temperature of the sky (T_{sky}). The vertically-inclined north-facing orientation of the aperture subtends a solid angle of approximately (V_F = \pi/4) steradians with the sky vault. The longwave optical properties of the screen itself approximates a 'Black body' with \varepsilon_{\lambda=10\mu m} = (0.84 to 0.93) typical of a plastic, from data on p. 146 in (Systems AB, 2006) and Table 1 in (IT, 2012).

The north-facing window provides daylight for the majority of the year, only during the summer months there will be some direct sunlight. The shutter is shown here in a vertically-pivoted side-hung arrangement for readability with the drawings presented earlier in Chapter 5. It could be operated in a variety of other configurations as discussed in Appendix H.1. Specifically, the top-hung open outward arrangement would be more effective for reducing the view-factor to the sky while providing clear horizontal amenity views to the outside.

The inversion of the thermal design rationale presented in Chapter 5 depends upon the feasibility of solidifying a wax at a temperature below the coldest average ambient conditions. Otherwise, effectively the actuators’s wax charge would remain liquid for most of the year if it were only exposed to ambient conditions in a 'Stevenson type' enclosure described in Section 5.1.1 of Chapter 5. By inverting the criterion used in Appendix F.1 a first-order estimate gives a candidate wax (n = 14) with melting-point (T_{mp} = 279 K). The following section briefly considers the ideal properties of the coolth-box aperture’s optical design.
9.1.2.2 Balancing spectral-selectivity: The optical properties of a coolth-box

For the coolth-box to reject sensible and latent heat of fusion to a temperature below ambient, the aperture cover would need to be qualitatively different from the hot-box investigated earlier in this work. Further work could test the feasibility of a screen that is optically transparent at long wavelengths by testing if it is effective enough at promoting passive cooling from inside to exchange with atmospheric counter-radiation outside while providing an effective convective barrier to buffer against gains from the surrounding ambient air.

The optical properties of the coolth-box screen would ideally align the spectral emission of the actuator with that of the thermal sink. Figure 9.3 shows the potential for this and a comparison with the hot-box approach. A practical coolth-box enclosure is constructed and thermographed to illustrate the optical spectral properties of a glass cover (left) vs the HDPE screen (right). In the top row of images taken in the visible-band, the glass is transparent (left) while the HDPE is opaque (right). In the row of images below taken in the thermal-band, a refrigerated actuator cylinder is placed inside the box at a temperature of \( T_c \approx 285 \) K. The Black body emission peaks around \( \lambda_p = 10 \mu m \) to which the glass is opaque (left) while the HDPE is transparent (right).

The spectra involved in the dominant radiant energy processes are shown in the lower half of Figure 9.3, they correspond from top-to-bottom to outer-space, the earth’s atmosphere, the coolth-box and the actuators surface, plotted against wavelength in (\( \mu m \)) on the x-axis respectively. Specifically, spectra (1) is short wavelengths solar radiation \( (G_{tot}) \) under clear-skies. Given the sun-path for London (UK) at Latitude \( (51.5^\circ N) \) direct beam radiation \( (G_{dir}) \) to a north-facing façade could be screened by shading between azimuth angles \( \phi = (60 \text{ to } 300)^\circ \). Exposure to diffuse radiation \( (G_{diff}) \) from sunlight scattered by the atmosphere would be partly reflected by the low transmissivity of the HDPE screen at shortwave or otherwise unavoidable.

Spectra (2) plots the absorption \( (\alpha) \) in the longwave band in the lower \( (0 \text{ to } 3000) \) m of the Earth’s atmosphere, showing the ‘window’ that opens up to the cold thermal sink of outer space in the wavelength interval \( \lambda = (8.5 \text{ to } 11.0) \mu m \). This is typical under cloudless conditions when water vapour pressure is low enough to limit atmospheric counter-radiation (p. 41 in Sellers, 1965). The aperture’s sky view-factor determines the effective exposure to outgoing radiation exchange, this varies with zenith angle from a maximum at the zenith to \( (\approx 75 \%) \) of this value at \( (\alpha_{sol} = 30^\circ) \) from (p. 62 in Sellers, 1965). The effects of screening the ground-reflected component as well as surrounding anthropogenically heated surfaces could also be investigated.

Ideally, the screen would be transparent \( (\tau_{\lambda=10\mu m} = 1) \) while opaque to solar radiation \( (\tau_{\lambda=0.5\mu m} = 0) \). However, a search found no single material with these ideal properties. Evaluations of materials in the literature suggest candidates such as HDPE and Tedlar (Granqvist, 1981). Spectra (3) plots the transmission through a proprietary filter engineered by (KubeAG) for peak transmissivity at \( \lambda = (7 \text{ to } 14) \mu m \) allowing ‘Black body’ emission from the actuator’s surface to radiate thermal energy through the aperture to the cold sky vault above.
Figure 9.3: A comparison of the different spectrally selective optical properties

Glass hot-box (Active-high $\downarrow\uparrow$)

(a) Transparent to shortwave ($\tau_{0.5\mu m} = 0.9$)

(b) Opaque at longwave ($\tau_{10\mu m} = 0.1$)

HDPE coolth-box (Active-low $\downarrow\uparrow$)

(c) Opaque at shortwave ($\tau_{0.5\mu m} = 0.2$)

(d) Transparent to longwave ($\tau_{10\mu m} = 0.8$)

Notes: Thermal actuator cylinder treated with black-dye anodisation for higher emissivity ($\varepsilon \approx 0.8$) then refrigerated to ($T_c = 13^\circ C$). Spectra 1: Solar radiation transmitted by Earth’s atmosphere under clear-skies from (Boer, 1977). Spectra 2: Absorption by atmospheric under clear skies from ground to 3000 m from (Evan, 2010). Spectra 3: Transmission spectra through coolth-box aperture’s using a HDPE (High-Density Polyethylene) filter, data courtesy of Kube AG (Switzerland) 2013. Spectra 4: Black body emission from the actuator at the phase-change temperature of wax ($n = 14$).
Spectra (4) shows the thermal energy ($E$) emitted from the actuator at ($T_{mp} = 279 K$), the temperature that the 'recovery stroke negative' rationale must cool to reach the wax’s melting-point. It assumes a ‘Black body’ with an emissivity ($\varepsilon_c = 1$) with a peak around ($\lambda_p = 10 \mu m$). Where spectra (4) overlaps (3) and (2) is the alignment between the surface emission, HDPE filter transmission and the atmospheric ‘window’ clear-skies. The intensity is overestimated for ‘as supplied’ actuator, however, it may be approximated with coatings such as dye anodisation as thermographed in Figure 9.3 or optical blacks ($\varepsilon > 0.95$) such as (Acktar, 2013). Having described the components and the spectral alignment of the heat-transfers between source and sink, a first-order energy-balance of the the coolth-box rationale is expressed in Equation (9.1).

$$Atmospheric\ counter-radiation + ambient\ convection = Thermal\ emission\ of\ motor$$

$$q_{rad, \lambda=10 \mu m} \downarrow + q_{conv} = q_{rad, \lambda=10 \mu m} \uparrow$$

(9.1)

An approximate assessment is that the net rate of cooling by passive radiant loss under a clear sky is a modest $q_{rad, \lambda=10 \mu m} = (-70 \text{ to } -130) W \cdot m^{-2}$. If most gains from shortwave radiant energy during the photo-period can be excluded by effective shielding, given an operating temperature of $\Delta T = T_{db} - (5 \text{ to } 10)^\circ C$ the actuator’s rate of convective gain from ambient is $q_{conv} = (50 \text{ to } 100) W \cdot m^{-2}$. It may be feasible for the ($\approx -30 W \cdot m^{-2}$) deficit from passive cooling to overcome ambient. During the nocturnal period, the balance is further in favour of duty-cycling since shortwave gains ($q_{rad, \lambda=0.5 \mu m} = 0$) and ambient is colder. It is encouraging to be reminded that the rate of radiant cooling is ($q_{rad} \propto \Delta T^4$) after Stefan’s law compared with the rate of convective gains from ambient at ($q_{conv} \propto \Delta T$) after Fourier’s Law.

Further investigation could test the practical feasibility of exploiting the free passive ‘coolth’ sink provided by the Earth’s atmosphere to duty-cycle the proprietary actuator and operate shutters over north-facing windows. A starting point could adapt the in-situ observation system described in Chapter 7 using the suite of instruments detailed on page 136. An extension of the set-up would be to add a net pyradiometer to measure the incoming and outgoing thermal emissions in the longwave band central to the dynamics of this rationale’s energy-balance.

This section outlined the thermal design rationale for a coolth-responsive thermal actuator system, as a possible avenue that diversifies the possibility for passively reconfiguring a building’s envelope. Recalling the generation of ’requisite variety’ from the possible states of the system’s variables (Ashby, 1960). The coolth-box extends the possibilities of this approach as another variable describing a system’s state. Further investigation and feasibility testing may lead to the development of a hybrid comparator mechanism, exploiting both positive and negative inputs to output duty-cycles. This could differentiate between south-facing and north-facing apertures, extending the variety of possible responses with the such as a conditional output to the degree of cloud-cover in opposing north and south sky quadrants. Having considered a possible diversification of the means used to drive actuation, the next section turns attention to the actuator itself.
9.2 Further work to develop thermal-actuator technology

The following sections suggest areas of development, broadly divided between improvements to the underlying actuator performance and variations of the mechanical linkage between actuators.

Reflecting on the findings of this work, a proprietary thermal actuator was investigated ‘as supplied’ and this study found that some of its intrinsic properties did constrain particular characteristics of the ‘as measured’ pattern of duty-cycles. This suggests specific properties and areas that may be improved with further work.

9.2.1 Actuator cylinder: Selective absorption and emission of radiation

This study has shown that the effectiveness of the hot-boxes for driving the thermal actuators by insolation is determined by a relatively finely balanced energy-budget. This was clearly shown by the result for the seasonal threshold, the range of \( \langle dn \rangle \) when the daily energy-budget required to actuate the motor in the horizontal hot-box was a surplus.

The optical property that characterised the variability of the radiation components in net energy budget is the absorption and emission coefficients of the cylinder’s surface. The proprietary thermal actuator was tested ‘as supplied’ with a mill-finished Aluminium surface assumed to be spectrally grey. Specifically, a modest-to-low absorption of short-wave radiation \( (\alpha_{sol}) \) and a modest-to-low emissivity \( (\varepsilon) \) in the long-wave thermal band. This assumption was based on data found in the literature rather than specific radio-metric and photo-metric measurements.

The implication for the actuation threshold in the seasonal energy-budget of a spectral ratio of \( (\alpha_{sol}/\varepsilon \approx 1) \) was an equal sensitivity to the magnitude of gains and losses. It may be desirable in further work to reduce the sensitivity to losses while increasing the sensitivity to gains with spectrally-selective treatment to the actuator’s surface, this would further amplify the optical effectiveness of the hot-boxes for the same range of ambient temperatures.

![Figure 9.4: Modification of the proprietary actuator cylinder for selective absorption](image)

Notes: The difference in the spectral response to shortwave (visible) and longwave (thermal) bands between a mill finish and black anodized surface is shown on the left (From p. 37 in Sparrow and Cess, 1978). Image on the right shows the proprietary actuator components supplied in standard finish (top) and a black-dye surface anodisation treatment (below). Courtesy of Bayliss Ltd (UK)
9.2. Further work to develop thermal-actuator technology

In quantitative terms, this would be modifying the cylinder surface absorption at short-wavelengths ($\lambda_p \approx 0.5 \mu m$) from $\alpha_{sol} = (0.1 \text{ to } 0.2)$ to $\alpha_{sol} = (0.8 \text{ to } 0.9)$ while reducing the emissive cooling by ‘Black body’ radiation at long wavelengths ($\lambda_p \approx 10 \mu m$) even lower ($\varepsilon < 0.15$) achieving a ratio of ($\alpha_{sol}/\varepsilon \approx 5$). By altering the dynamics in this way, more insolation is absorbed while also less is emitted in the thermal band.

All other parameters being equal, this would lead to a greater differentiation in the overall energy-balance as reported by Andersson et al. (1980). This could contribute towards advancing ways to reduce the sensitivity of the daily energy budget to fluctuations in solar radiation intensity from moderate levels of cloud turbidity. Including reducing the daily yield for passively operating the thermal comparator below the ($Y_i \geq 67\%$) found in this study.

At its simplest, the effects of selective absorption and emission could be investigated with a range of surface anodizations. The difference in the effective thermal emission from the cylinder’s surface from anodisation treatment is indicated by the comparative thermograph on the right-hand side in Figure 9.5. A starting-point could be to compare the mill finished surface of the ‘as supplied’ with cylinders treated using dye anodisation such as the sample shown in Figure 9.4.

An observation study similar to the methodology reported in Chapter 7 could be carried out with a pair-wise comparison between two sets of hot-boxes, each with the different surface treatments. The anticipated net effect for operating the thermal comparator would be that in each diurnal interval the rising-edge of actuation would be brought forward while the falling-edge during the recovery-stroke would be prolonged.

Figure 9.5: Comparative thermograph showing the difference in emissivity of motor surface

Notes: (Left) Image taken in the visible-band showing (top-half) the ‘as supplied’ cylinder finish and (lower-half) showing the cylinder with black-dye anodisation corresponding to (Right) image showing thermal emission in the infra-red ($\lambda_p = 10 \mu m$). Both cylinders brought to a steady-state temperature of ($T_c = 50^\circ C$) in a temperature controlled oven and imaged while cooling in ambient air at approximately ($T_{db} = 20^\circ C$). Opaque adhesive tape with high-emissivity ($\varepsilon = 0.92$) was bonded to the right-hand end of each cylinder, these are where the spot temperature measurements were taken, using the method on (pp. 39-40 Systems AB, 2006). FLIR Thermal imaging equipment courtesy of the Department of Construction and Property at the University of Westminster.
9.2.2 Improvements to the heat-motor responsiveness

The adoption of a readily available proprietary heat-motor from Bayliss Ltd enabled the practical studies to be carried out without resorting to designing a dedicated actuator from scratch. However, in reviewing the findings of this study it would have been desirable to improve the slow response found in the ‘as supplied’ actuator. This would reduce system latency, both to maximise the access to daylight as well as timely response to the risk of overheating from direct sunlight.

The FEM simulations in Chapter 6 showed that the heat-diffusion through wax is dependent on the characteristic thermal geometry of the cylinder’s cavity. While latency can be partly attributed to the low thermal conductivity and high latent heat transferred during phase-transitions, it is also affected by cavity depth.

Compared with a plain cylinder, the proprietary Bayliss cylinder is an improvement by reducing compactness from an isoperimetric quotient of ($IQ = 1$) down to (0.24) from the extended area of the six fins in the cavity. An avenue for further work is to re-engineer the extrusion with a smaller fin separation distance, leading to a more compact cavity geometry.

An initial suggestion is shown in Figure 9.6(b) on the right, with a proposed extrusion section ‘Mk.8’. The cylinder’s section shares the same area, mass and cavity containment area as the proprietary cylinder, that is shown alongside in Figure 9.6(a) on the left for comparison. Simply by reducing the fin separation distance, this achieves a $\times 20$ improvement in compactness from ($IQ = 0.238$) down to ($IQ \approx 0.012$).

Figure 9.6: Comparative cross-sections through proprietary cylinder vs a low-IQ alternative

(a) The Bayliss cylinder used in this study

(b) Extrusion Mk.8 of equal area with reduced IQ

<table>
<thead>
<tr>
<th>Property</th>
<th>Bayliss: Mk.8</th>
<th>Area of section through heat-motor cylinder</th>
<th>Perimeter of conduction interface ($p_{int}$)</th>
<th>Area of wax charge in cavity</th>
<th>Extent of stroke-rod shown behind</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{int}$</td>
<td>227 mm$^2$</td>
<td>Area of section through heat-motor cylinder</td>
<td>Perimeter of conduction interface ($p_{int}$)</td>
<td>Area of wax charge in cavity</td>
<td>Extent of stroke-rod shown behind</td>
</tr>
<tr>
<td>$p_{ext}$</td>
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<td>480 mm</td>
<td>223 mm</td>
<td>0.012 ratio</td>
<td></td>
</tr>
<tr>
<td>$A_{int}$</td>
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<td>227 mm$^2$</td>
<td>227 mm$^2$</td>
<td>227 mm$^2$</td>
<td></td>
</tr>
<tr>
<td>$\Delta x$</td>
<td>3.7 to 6.6 mm</td>
<td>$\Delta x = 0.7$ mm</td>
<td>$\Delta x = 0.7$ mm</td>
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<td>$p_{int}$</td>
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<td>$IQ$</td>
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</tbody>
</table>
9.2. Further work to develop thermal-actuator technology

Based on the minimum practical wall-thickness using the aluminium extrusion process of (0.7 mm), a rough estimate would be an improvement in the order of $\times (35$ to $40)$ for the rate of conduction ($q''_{\text{cond}}$). It is difficult to assess what impact this would have on the rate of convection losses, however, based solely on recalculating the change in surface area the effect would be small relative to the improvement in conduction area.

Another alternative is to explore three-dimensional geometries using Rapid Prototyping (RP) processes to significantly increase the cavity area and improve its compactness. We can look to the biological world for many examples of effective geometries based on variations of hexagonal-walls (Thompson, 1961; Pearce, 1978; Ball, 2009). An initial suggestion is modelled in Figure 9.7 after the ear of the domesticated tall grass *Zea mays* (corn maize). This could be further improved by inserting metallic foams and graphite fibres as reviewed on Page 2.2 of Chapter 2.

The implications of these geometries is unlikely to be restricted to improving latency alone. The results characterising the temperature variation during phase-transition due to cavity pressure in Section 3.2 of Chapter 3 may also be affected. The crystallisation dynamics at solidification, as well as super-cooling may be affected by the higher conduction area-to-volume ratio. These sections lower the Biot number for more effective convection to ambient, this may affect the overall energy-balance. These properties could be tested using the methodology reported in Chapter 3 to establish the convection coefficient and Chapter 7 to characterise duty-cycling.

Having considered measures to improve latency in the cylinder itself, the following section considers how the hot-boxes could be varied for improved harvesting of solar gains.

**Figure 9.7: Improving latency with the development of large surface area cavities**

Three 25 mm long cylinder sections modelled with a geometry of radially tapered hexagonal-walled cells of increasing density from left-to-right. The cavity surface area in contact with the wax is highlighted in red. The diameter of the void at the cylinder’s centre has been exaggerated for clarity.
9.2. Further work to develop thermal-actuator technology

9.2.3 Engineering the optical design of the hot-box apertures

Given all the unknowns at the start of this study, there was no attempt to optimise the angle separating the hot-boxes or each of the inclination angles with respect to the insolation that would be available on a unit area basis. Given the fundamental driver of seasons on Earth is its angle of obliquity to the solar system’s ecliptic, at \( (23^\circ26') \) the seasonal variation in daily insolation for London at Latitude \( (51.5^\circ) \) is visualised in Figure 9.8. The observation study reported in Chapter 7 tested the seasonal response of the hot-boxes \( (\theta = 90^\circ) \) apart, however, further work to study the impact of angle adjustments within this range might refine the rationale and reduce uncertainty.

Specifically, reviewing the range of daily insolation levels predicted across a \((0 \text{ to } 90)^\circ\) sweep of inclinations in Figure 9.8, the curve with the highest average lies at \( (\theta = 50^\circ) \). Suggesting that an adjustment of \( (\theta = +40^\circ) \) from the horizon to the aperture irradiating heat-motor \( (A) \) increases both the average daily insolation and evens out the differences from the peaks at the equinoxes and the trough in summertime. The resultant benefit would be to increase the duration of daylight response. Further parametric studies of these variables could refine each aperture for a closer match between available seasonal insolation and the actuator’s energy budget.

Figure 9.8: The seasonal variation in pre-noon insolation for different inclination angles

Notes: ¹ Solar radiation model under clear-sky conditions on horizontal plane after (Haurwitz, 1945), other inclinations ² after (Oke, 1987). Seasons in the British Isles after (p. 402 by Manley, 1974)
9.2.4 **Boosters and masks: Trimming the diurnal energy-balance**

The previous section considered further work to investigate how adjustments could be made to the inclination angle to account for variations in the geographic latitude of a site, at the scale of relocation within the temperate zone. This section considers a complementary investigation, into how fine adjustments to the aperture’s solar exposure could be made to account for variations at the scale of a site. These could be both positive and negative, either by using reflectors to boost insolation intensity or shading patterns to selective block excessive exposure to direct sunlight.

**Figure 9.9: Trimming the solar exposure curve with aperture masks**

Notes: Thermographs of the sky under clear-sky conditions during the photo-period in the infra-red band at ($\lambda_p = 10 \, \mu m$) (Top) showing the horizontal hemispherical sky view-factor (left) and south-facing vertically-inclined (right). The corresponding sky view-factors in the visible-band (Below). Shading masks are shown indicatively, applied differentially to each hot-box apertures. An outline pattern of shading areas could be applied by baking opaque ceramic fritting onto the outside glass surface of the aperture covers. FLIR Thermal imaging camera and imaging processing software courtesy of the Department of Construction and Property at the University of Westminster.
9.2. Further work to develop thermal-actuator technology

Recalling the results of the observation study reported earlier in Section 7.4 of Chapter 7 on page 138. While it was encouraging to demonstrate how in practice there was ample insolation available to duty-cycle actuators compared to the minimum budget, the analysis also showed that sensible temperatures were much higher than predicted. The large surplus of gains that occurred, especially during summer would need to be limited in order to protect the actuator against damage to its seals. Conversely, there were also cases of an undesirable deficit in the actuator's energy-budget on those occasions when the measured insolation fell short of the predicted.

The design of the aperture and its optical performance matters as much to the thermal actuator as the design of the window and its cill, head, jambs and reveal do to the building’s internal space. The performance of the basic geometric arrangement and materials used in the test-cells could be trimmed using masks and external reflector surfaces or boosters. Taken together, these passive measures could be applied to both suppress surplus exposure as well as compensate for site-specific obstructions to the sun-path.

It may also be beneficial to apply masks that are selectively reflective in the thermal band in order to screen against undesirable gains from anthropogenic thermal emission. In an urban context of high-density buildings, surfaces within the aperture's field of view may present a large proportion of the view factor with high intensity sources of thermal emissions, for example from uninsulated walls to plant rooms, boiler flues and fan-coil extraction units.

An indicative shading mask arrangement is shown in Figure 9.9, in this example the late-evening sunlight during the warmer seasons is selectively screened, rendered in gray-tiles at azimuth angles $\phi = (210$ to $300)$°. This could be applied specifically to reduce the latency observed in the results from the site study in Chapter 7 attributed to excessive gains during late afternoon. By a fore-shortening of the exposure to direct sunlight, the afternoon and early evening cooling process is brought forward. This and other modifications to the basic hot-box design and inclination angle arrangement using boosters and masks is one approach to achieving a convergence between the discrepancies found in the idealised predictions and the results observed in this study.

Reflecting on the imaging and visualisation techniques that have been applied throughout this study, taken together these provide a suite of methods to survey prospective installation sites. Whether these are existing buildings that will undergo refurbishment or vacant sites in the case of new-build. By providing an effective coordination between the thermal, visual and spatial relationships of the component parts, an integrated modelling and simulation tool could be developed for evaluate the actuator's performance and making finer adjustments to the aperture and inclination angle on a site-specific basis. The set of still images in Figure 9.9 shows one example of a site with unobstructed views to the horizon.

Finally, the following section briefly considers one example of linking thermal actuator into a simple logic gate, and discusses the possibility of its permutations.
Computing without computers: Logical evaluators operated by heat-motors

In chapter 4, a linkage mechanism was developed to duty-cycle two heat-motors in an open-loop against ambient temperature in an air-permeable enclosure. This linkage was carried forward in the subsequent chapters albeit with an air-sealed enclosure that was driven by gains from direct insolation. The possibility that was not pursued at the end of Chapter 4 was to develop further thermal comparators operated by linking together multiple heat-motors in other permutations.

Instead, the attempt was made to advance the underlying thermal design rationale that operated the heat-motors rather than the mechanisms themselves because it would only have been speculative and restricted to the environment of the climate-chamber. The result of this decision has been to establish useful engineering values and parameters that contribute toward a stronger engineering basis for describing the heat-motor’s dynamics. Accepting the limitations of the dialectic between ‘as predicted’ and ‘as built’ behaviour outlined in Chapter 7, some possibilities for further mechanisms can now be considered.

Linkage mechanisms that embody George Boole’s logical operations are very desirable because they have universal application, as enumerated in (Chap. 2 by Boole, 1997). In the restricted case of a logic operand with only two inputs, the feasibility of different arrangement could be tested using the state-machine Figure 6.12 in Chapter 6. An embodiment of the Boolean operand ‘XOR’ (exclusive disjunction) is shown in Figure 9.10 with all \(2^2\) input patterns. With further work, the dynamics of heat energy absorption and rejection in each of the two heat-motors could be simulated to describe the annual pattern of inputs.

**Figure 9.10: Truth table for Boolean XOR embodied in mechanical form**

(a) Input: \(0 \oplus 0 = 0\)

(b) Input: \(0 \oplus 1 = 1\)

(c) Input: \(1 \oplus 0 = 1\)

(d) Input: \(1 \oplus 1 = 0\)

*Note: An indicative arrangement of a rack and pinion gear mechanism that operates an external shutter on a centre-pivot. Heat-motor (A) is coloured blue and mechanically XOR’s with the heat-motor (B) coloured red. Enclosing hot-boxes omitted. [Animation of mechanism](WMV, 4.7 Mb)*
9.3 Concluding remarks

Returning to the thesis starting point and Herbet Simon’s statement about the endeavour of the designer to be ‘concerned with the artificial is the way in which that adaptation of means to environments is brought about’ (Simon, 1996). Researching this thesis topic has drawn together the time-varying properties of a ‘functional material’ with the possibilities of a building fabric that is dynamic on the physical environment’s terms of engagement. Completely irreverent to conventional boundaries between academic disciplines, this has been a constant challenge to cultivating ideas about how to beneficially exploit environmental energy sources (and sinks) that act on buildings as also the means to alter their physical arrangement.

This also cuts across the design responsibilities that are cast by professional affiliations in a construction team when drawn along conventional contractual lines. From an architect’s perspective coming from conventional architectural practice, the pursuit of this research topic demanded a holistic approach. Transferring this knowledge back into practice would need a continued collaboration with the engineering disciplines, as well as engagement with the manufacturing processes and methods of testing. The author often reflects that the reality of the practical work has shaped many of the key questions that the virtual models and their simulation only serve as approximations of, a well-integrated iterative approach with physical prototyping still has to be commended.

The recurring characteristic of this research was how the passive behaviour of component parts could not be disaggregated from the building’s design as a whole. This consideration pushed the practical mechanism developed in Chapter 4 out of the controlled environment of the thermal laboratory and toward field tests. The benefit of this move was to gain a realistic quantification for the different variables that describe the state of thermal actuators.

Accepting that the state of a passive thermal actuator is inseparable from its embodiment in the history of prevailing conditions, this gives a designer the constraints that are needed to define a dynamic system’s ‘field’ (Ashby, 1960). The findings in this work could contribute toward diversifying the application of thermal actuators, the design challenge ahead is to recast the thermal design rationale to be appropriate for other functions in different climate contexts.

Revisiting the quote from (Khan et al., 2008) in the thesis introduction, adopting the approach developed in this work could sensitise a building’s envelope with a repertoire of responses to different prevailing conditions in a given climate. While this work developed a seasonal repertoire of responses, this could be diversified for functions in other climates by striving for a convergence between the repertoire of desirable changes to the arrangement of a building’s envelope and the passive response of actuators to the changes in prevailing conditions.

On a final note, the author recognises that this research been been an in-depth study of the technical means for achieving passive responses, somewhat disaggregated from the design of a building as a whole. The aspiration is for future research efforts to continue to be complementary and lead toward further demonstration buildings.


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Appendixes
Appendix A

A.1 Material properties

A.1.1 Standard tests for the critical properties of wax

Conducting research on this topic is sensitive to the values of the thermo-physical properties of the waxes used. The thermo-physical data for waxes supplied by manufactures has been taken *prima facie* based on the disclosures made in their own test results, these are reproduced in Figure A.1.

The methods used to obtain these results are harmonised with the methods described in the ISO international standards listed in Table A.1. However, reviewing the actual methods described in each respective BS, ISO and ASTM specification reveals the slight differences in the observed phenomenon of wax behaviour that is used as the basis for defining its melting-point. The implications of this uncertainty are clarified to some extent by the experimental work done using a specific wax in a proprietary heat-motor presented in the body of the thesis.

### Table A.1: International test methods for the thermo-physical properties of \( n \)-paraffin waxes

<table>
<thead>
<tr>
<th>Organization or Company</th>
<th>Drop-point (Standard)</th>
<th>Terminal (Standard)</th>
<th>Melting (Standard)</th>
<th>Congealing (Standard)</th>
<th>Expansion (Standard)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ISO</td>
<td>6244:1982</td>
<td>n/a</td>
<td>3841:1977</td>
<td>2207:1980</td>
<td>n/a</td>
</tr>
<tr>
<td>ASTM</td>
<td>n/a</td>
<td>n/a</td>
<td>D87-74</td>
<td>D938</td>
<td>n/a</td>
</tr>
<tr>
<td>BS EN</td>
<td>2000-133</td>
<td>n/a</td>
<td>4695:1980</td>
<td>2000-76</td>
<td>n/a</td>
</tr>
<tr>
<td>Astor†</td>
<td>DST-007</td>
<td>DST-007</td>
<td>DST-007</td>
<td>n/a</td>
<td>DST-007</td>
</tr>
</tbody>
</table>

† Data and description of methodology provided by IGI International

A.1.2 Thermo-physical properties for a selection of waxes

For reference, a set of values for the physical properties of three \( n \)-paraffin waxes in their liquid and solid state including the one used in these studies is provided in Table A.2. These are either from the literature (where cited) or calculated from values given in the literature.
A.1. Summary of the material properties of \( n \)-paraffin waxes

Table A.2: A summary of the properties of three even-numbered \( n \)-paraffin waxes

<table>
<thead>
<tr>
<th>Properties</th>
<th>Symbol</th>
<th>Fully-refined normal-alkane paraffin waxes</th>
<th>Units</th>
<th>[Source]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chemical formula</td>
<td></td>
<td>( n )-hexadecane ( n )-octadecane ( n )-eicosane</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CAS number</td>
<td></td>
<td>544-76-3 593-45-3 112-95-8</td>
<td></td>
<td>[NIST]</td>
</tr>
<tr>
<td>Latent heat of fusion ( (H_f) )</td>
<td></td>
<td>235.5 240.9 246.8</td>
<td>( kJ \cdot kg^{-1} )</td>
<td>(p. 126 in Dirand et al., 2002)</td>
</tr>
<tr>
<td>Melting point on cooling curve ( (T_{mp}) )</td>
<td></td>
<td>291.2 (18.1) 301.2 (28.1) 309.5 (36.3)</td>
<td>( ^\circ C )</td>
<td></td>
</tr>
<tr>
<td>Expansion (at 1 bar pressure) ( (\Delta \rho) )</td>
<td></td>
<td>16.5 18.3 18.7</td>
<td>% Volume</td>
<td>(p. 1 IGI, 2007a)</td>
</tr>
<tr>
<td>Ratio of ( (T_{bp} : T_{mp}) )</td>
<td></td>
<td>n/a 1.92 1.96 1.99</td>
<td>n/a</td>
<td></td>
</tr>
</tbody>
</table>

Gas phase

| Boiling point \( (T_{bp}) \) (At 1 bar pressure) |        | 561 (288) 590 (317) 616 (343)     | \( K \)                | (p. 52 in Grosse and Egloff, 1938) |

Solid phase (paraffin oil)

| Density \( (\rho_l) \) At temperature |        | 761.2 769.9 776.9                | \( kg \cdot m^{-3} \)  | (p. 15 in Bailey and Chong-Kwang, 1975) |
| Specific heat capacity \( (c_l) \) At temperature |        | 1700 1700 1940                       | \( J \cdot kg^{-1} \cdot K^{-1} \) | (p. 23 in Humphries and Griggs, 1977) |
| Expansion coefficient \( (\Delta \rho / \Delta T) \) (20 to 100) \( ^\circ C \) |        | \(-6.71 \times 10^{-4} \) \(-6.76 \times 10^{-4} \) \(-6.65 \times 10^{-4} \) | \( kg \cdot m^{-3} \cdot K^{-1} \) | (p. 10 in Grosse and Egloff, 1938) |
| Thermal conductivity \( (k_l) \) At temperature |        | 0.140 0.146 0.1505                   | \( W \cdot m^{-1} \cdot K^{-1} \) | (p. 6-212 in Lide, 2007) |
| Thermal diffusivity \( (a_l) \)                  |        | \( 8.7 \times 10^{-8} \) \( 1.12 \times 10^{-7} \) \( 9.99 \times 10^{-8} \) | \( m^2 \cdot s^{-1} \) | (pp. 5-9 in Hale and O’Neill, 1971) |
| Dynamic viscosity \( (\eta) \)                   |        | 3.34 3.08 4.14                       | \( mPa \cdot s \)      | (p. 156 in Humphries and Griggs, 1977) |

\( n \)-hexadecane \( n \)-octadecane \( n \)-eicosane

continued on next page
### A.1. Summary of the material properties of \( n \)-paraffin waxes

*continued from previous page*

<table>
<thead>
<tr>
<th>Properties</th>
<th>Symbol</th>
<th>Fully-refined normal-alkane paraffin waxes</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( n )-hexadecane</td>
<td>( n )-octadecane</td>
</tr>
<tr>
<td>Surface tension</td>
<td>(( \gamma ))</td>
<td>27.6</td>
<td>26.6(^*)</td>
</tr>
<tr>
<td>Liquid velocity</td>
<td>(( \gamma/\eta ))</td>
<td>8</td>
<td>8.6</td>
</tr>
</tbody>
</table>

**Solid phase (paraffin wax)**

<table>
<thead>
<tr>
<th>Density</th>
<th>At temperature</th>
<th>Specific heat capacity</th>
<th>At temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>(( \rho_s ))</td>
<td>840</td>
<td>860</td>
<td>860</td>
</tr>
<tr>
<td></td>
<td>283 (10)</td>
<td>298 (25)</td>
<td>288 (15)</td>
</tr>
<tr>
<td></td>
<td>1450</td>
<td>1450</td>
<td>1440</td>
</tr>
<tr>
<td></td>
<td>291.1 (-18)</td>
<td>291.1 (-18)</td>
<td>291.1 (-18)</td>
</tr>
</tbody>
</table>


**Figure A.1:** Technical data sheet for thermostatic wax 'HA-18' from wax refinery IGI

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Note: The refined wax used throughout the practical studies, Copyright © 2008 IGI (IGI, 2007b)
Appendix B

B.1 Survey of a proprietary heat-motor

The proprietary heat-motor that was used throughout the studies reported here was the “Mk.7 Superpower” supplied by the Bayliss Precision Components Ltd. Table B.1 presents a list of values for parameters used during the studies carried out in the thesis. Where possible measurements were taken from a selection of disassembled heat-motors, for other parameters values had to be assumed and their source(s) in the reference literature is cited where applicable.

A detailed dimensional survey was made of the proprietary heat-motor. Figure B.2 shows the simplicity of the proprietary heat-motor, it consists of only eleven components with one moving part; the stroke-rod (2). The wax is prevented from pushing the stroke-rod completely out of the cylinder by an alignment clip peg (11). The wax itself is contained within the cylinder cavity by a three-part seal (8:7:5) around the stroke-rod at the cylinder collar (3) end, with a rubberised rings (5) and (6) at either end of the cylinder body. For clarity, cross-refer to Figure B.1 showing components assembled and disassembled.

Figure B.1: Simple construction of the proprietary heat-motor: parts shown disassembled

Note: Components courtesy of Bayliss precision components Ltd (UK)
### Table B.1: A summary of the properties of the “Mk.7 Superpower” heat-motor

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of heat-motor (as supplied)</td>
<td>(m)</td>
<td>0.4722</td>
<td>kg</td>
</tr>
<tr>
<td>Mass of heat-motor (without wax charge)</td>
<td>(m_{hm})</td>
<td>0.423</td>
<td>kg</td>
</tr>
<tr>
<td>Mass of heat-motor cylinder</td>
<td>(m_c)</td>
<td>0.248</td>
<td>kg</td>
</tr>
<tr>
<td>Surface area of heat-motor (outside)</td>
<td>(A_c)</td>
<td>0.0429</td>
<td>m(^2)</td>
</tr>
<tr>
<td>Volume of heat-motor cylinder</td>
<td>(V_c)</td>
<td>8.5e-5</td>
<td>m(^3)</td>
</tr>
<tr>
<td>Thermal conductivity of cylinder (as supplied)</td>
<td>(k_c)</td>
<td>177 est.</td>
<td>W (\cdot m^{-1} \cdot K^{-1})</td>
</tr>
<tr>
<td>Density of cylinder (as supplied)</td>
<td>(\rho_c)</td>
<td>2770 est.</td>
<td>kg (\cdot m^{-3})</td>
</tr>
<tr>
<td>Heat capacity of cylinder</td>
<td>(c_c)</td>
<td>875 est.</td>
<td>J (\cdot kg^{-1} \cdot K^{-1})</td>
</tr>
<tr>
<td>Surface absorptivity of cylinder (oxidized) (\lambda_p = 0.5 \mu m)</td>
<td>(\alpha_s)</td>
<td>0.60 est.</td>
<td>coefficient</td>
</tr>
<tr>
<td>Surface absorptivity (Black nickel oxide) (\lambda_p = 0.5 \mu m)</td>
<td>(\alpha_s)</td>
<td>0.85 est.</td>
<td>coefficient</td>
</tr>
<tr>
<td>Surface emissivity of cylinder (oxidized) (\lambda_p = 10 \mu m)</td>
<td>(\varepsilon_c)</td>
<td>0.30 est.</td>
<td>coefficient</td>
</tr>
<tr>
<td>Surface emissivity of cylinder (anodized) (\lambda_p = 10 \mu m)</td>
<td>(\varepsilon_c)</td>
<td>0.06 est.</td>
<td>coefficient</td>
</tr>
<tr>
<td>Volume of wax containment (T_w &lt; T_{mp})</td>
<td>(V_c)</td>
<td>5.337e+4 mm(^3)</td>
<td></td>
</tr>
<tr>
<td>Mass of wax</td>
<td>(m_w)</td>
<td>0.049</td>
<td>kg</td>
</tr>
<tr>
<td>External surface area of cylinder (wax solidified)</td>
<td>(A_e)</td>
<td>0.0442</td>
<td>m(^2)</td>
</tr>
<tr>
<td>Internal surface area of cylinder (wax solidified)</td>
<td>(A_{int})</td>
<td>0.0307</td>
<td>m(^2)</td>
</tr>
<tr>
<td>Typical actuation displacement length</td>
<td>(l)</td>
<td>0.09</td>
<td>m</td>
</tr>
<tr>
<td>Typical actuation strain (\lambda_p = 0.5 \mu m)</td>
<td>(e)</td>
<td>0.24</td>
<td>n/a</td>
</tr>
<tr>
<td>Typical actuation stress (\lambda_p = 0.5 \mu m)</td>
<td>(s)</td>
<td>0.51e10(^6)</td>
<td>Pa</td>
</tr>
</tbody>
</table>

\(\dagger\) Standard ‘as supplied’ mill finish with a layer of atmospheric oxidization. \(\dagger\dagger\) A special surface finish not used in the observation studies, from data in (p. 39 in Goswami and Martin, 2005). \(\dagger\dagger\dagger\) After definition in Table 2 on p. 2187 by Huber et al. (1997). \(\ddagger\) Source data from (p. 21 in NASA, 1988)
B.1. Mensuration survey of the proprietary Bayliss Mk.7 heat-motor

Figure B.2: Anatomy of a proprietary heat-motor: Part longitudinal section

Note: Not drawn to a standard scale. Designed by Bayliss precision components Ltd (UK)
B.1. Mensuration survey of the proprietary Bayliss Mk.7 heat-motor

The area of the heat-motor’s internal cavity varies in proportion to the actuation displacement of the stroke rod. Equation B.1 describes the relation between area and length and this is plotted in Figure B.4 for the valid range of displacement lengths.

\[ A_{\text{int}} = 30739 - 25.133 \times l_s \quad (\text{for the interval } 0 \leq l_s \leq 100 \text{ mm}) \quad \{\text{in } mm^2\} \quad (B.1) \]

Where:
- \( A_{\text{int}} \) is the total surface area of containment of wax \( \{\text{in } mm^2\} \)
- \( l_s \) is the displacement length of the stroke-rod \( \{\text{in } mm\} \)

Figure B.4 shows that cavity area is inversely proportional to actuation displacement. When the stroke-rod is fully extended there is a 10% reduction in the total surface area. The implication for the operation of this design of heat-motor is that 10% more area is in contact with the wax for heat-transfer during melting than is available during freezing.

Figure B.3: Cross-sections through proprietary heat-motor body showing fins inside and out

Figure B.4: The total area of wax containment as a function of actuation displacement
B.1. Mensuration survey of the proprietary Bayliss Mk.7 heat-motor

B.1.1 Variation in the proprietary “Superpower tube” heat-motor

Between 2005 and 2008 approximately thirty-five heat-motors of the same model were obtained, from these a random selection have been surveyed, results are shown in Table B.2.

<table>
<thead>
<tr>
<th>Heat-motor</th>
<th>Total mass</th>
<th>Wax A</th>
<th>Actuation displacement length</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1) m</td>
<td>(2) m&lt;sub&gt;W&lt;/sub&gt;</td>
<td>(3) l&lt;sub&gt;1&lt;/sub&gt;</td>
</tr>
<tr>
<td>M V5 -</td>
<td>0.474</td>
<td>0.049</td>
<td>-</td>
</tr>
<tr>
<td>A • P8 MkIII</td>
<td>0.470</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>B • P8 MkIII</td>
<td>0.471</td>
<td>0.048</td>
<td>-</td>
</tr>
<tr>
<td>C • P8 #2</td>
<td>0.474</td>
<td>-</td>
<td>59.0</td>
</tr>
<tr>
<td>D • P8</td>
<td>0.470</td>
<td>0.049</td>
<td>60.5</td>
</tr>
<tr>
<td>E • P8</td>
<td>0.472</td>
<td>-</td>
<td>59.0</td>
</tr>
<tr>
<td>F • P8</td>
<td>0.472</td>
<td>0.049</td>
<td>57.5</td>
</tr>
<tr>
<td>G • P8</td>
<td>0.474</td>
<td>-</td>
<td>58.0</td>
</tr>
<tr>
<td>H • P8</td>
<td>0.471</td>
<td>0.049</td>
<td>56.0</td>
</tr>
<tr>
<td>J • P8 R(5)</td>
<td>0.473</td>
<td>-</td>
<td>57.0</td>
</tr>
<tr>
<td>K • P8 #3</td>
<td>0.470</td>
<td>0.049</td>
<td>56.5</td>
</tr>
<tr>
<td>L • P8</td>
<td>0.472</td>
<td>-</td>
<td>61.5</td>
</tr>
<tr>
<td>M • P8</td>
<td>0.473</td>
<td>0.049</td>
<td>58.0</td>
</tr>
<tr>
<td>N • P8 #3</td>
<td>0.472</td>
<td>-</td>
<td>60.0</td>
</tr>
<tr>
<td>P • P8 #1</td>
<td>0.472</td>
<td>0.049</td>
<td>59.5</td>
</tr>
<tr>
<td>Q • P8 R(1)</td>
<td>0.472</td>
<td>-</td>
<td>60.0</td>
</tr>
<tr>
<td>R • X7 #6</td>
<td>0.473</td>
<td>0.049</td>
<td>57.5</td>
</tr>
<tr>
<td>S • P8 #6</td>
<td>0.473</td>
<td>-</td>
<td>57.5</td>
</tr>
<tr>
<td>T • P8 #2</td>
<td>0.471</td>
<td>0.049</td>
<td>57.0</td>
</tr>
<tr>
<td>U • X7</td>
<td>0.473</td>
<td>-</td>
<td>59.0</td>
</tr>
<tr>
<td>Y • X7</td>
<td>0.473</td>
<td>0.049</td>
<td>61.0</td>
</tr>
<tr>
<td>X • P8 #4</td>
<td>0.474</td>
<td>-</td>
<td>55.5</td>
</tr>
<tr>
<td>W • P8 #1</td>
<td>0.473</td>
<td>-</td>
<td>59.5</td>
</tr>
</tbody>
</table>

Mean 0.472 0.049 58.5 147.2 88.7

St.D (Sample size) 0.001 (23) 0.003 (11) 1.68 (20) 1.79 (20) 1.59 (20)

(1) Charged with the ‘as supplied’ wax. (2) Measured without a biasing force. (3) At T<sub>amb</sub> = 10.9°C when wax has solidified. (4) At T<sub>amb</sub> = 23.5°C when wax has melted. (5) Used to operate shutter to stack ventilators.
In order to conduct the trial of a batch of plural heat-motors with a equal mass of wax using the HA-18 wax charge, a sample of the standard heat-motors were modified to accommodate a small increase in wax volume. The modified cylinders were weighed before and after being charged with HA-18 wax, the results are shown in Table B.3.

<table>
<thead>
<tr>
<th>Heat-motor</th>
<th>Total mass (1)</th>
<th>Wax B (2)</th>
<th>Actuation displacement length (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit</td>
<td>ID</td>
<td>Cell</td>
<td>m</td>
</tr>
<tr>
<td>(n/a)</td>
<td>(n/a)</td>
<td>(n/a)</td>
<td>(kg)</td>
</tr>
<tr>
<td>B • P8 MkIII</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>D • P8</td>
<td>-</td>
<td>0.458</td>
<td>0.048</td>
</tr>
<tr>
<td>F • P8</td>
<td>-</td>
<td>0.460</td>
<td>0.049</td>
</tr>
<tr>
<td>H • P8 #3</td>
<td>0.460</td>
<td>0.048</td>
<td>-</td>
</tr>
<tr>
<td>K • P8 #1</td>
<td>0.461</td>
<td>0.049</td>
<td>-</td>
</tr>
<tr>
<td>M • P8 #2</td>
<td>0.461</td>
<td>0.049</td>
<td>-</td>
</tr>
<tr>
<td>P • P8</td>
<td>0.461</td>
<td>0.049</td>
<td>-</td>
</tr>
<tr>
<td>R • X7</td>
<td>0.462</td>
<td>0.049</td>
<td>-</td>
</tr>
<tr>
<td>T • X7</td>
<td>0.462</td>
<td>0.049</td>
<td>-</td>
</tr>
<tr>
<td>Y • X7</td>
<td>0.462</td>
<td>0.049</td>
<td>-</td>
</tr>
<tr>
<td>Mean</td>
<td>0.461</td>
<td>0.049</td>
<td>-</td>
</tr>
<tr>
<td>St.D (Samples)</td>
<td>0.001 (9)</td>
<td>0.005 (10)</td>
<td>-</td>
</tr>
</tbody>
</table>

(1) Modified cylinder charged with type ‘HA-18’ wax. (2) Type ‘HA-18’ wax from IGI batch no. #U0860090.
Appendix C

C.1 Measurement of wax temperature

This appendix describes the methodology for measuring the time-dependent thermo-mechanical processes during the phase-change of wax when the heat-motor undergoes duty-cycles. To investigating this requires the measurement of the wax temperature and the heat-motor’s actuation displacement. To achieve the former, the temperature of the wax is measured at the core inside the heat-motor’s cavity where the sensor is thermally insulated from the conductive Aluminium cylinder walls, as well as the ambient surroundings.

A rigid stainless-steel thermoresistor probe with a fast-response was used within the heat-motor cavity and kept well-insulated from the walls with a purpose-made adapter. The adapter was machined from an engineering thermo-plastic (PET) with a low thermal conductivity ($k_s = 0.3\, W\cdot m^{-2}\cdot K^{-1}$) (Ensinger, 2005) to hold the temperature sensor at the centre of the cavity and avoid contact with the cylinder’s internal fins as shown in Figure C.3. The temperature sensor was a platinum thermoresistive (type Pt100) to BS EN IEC 60751 Class B defined in (BSI, 2008) giving a tolerance of ($T = \pm 0.3^\circ C$) (p. 55 in TC-Ltd, 2007). The shop drawing for the part is reproduced in Figure C.2. Each temperature sensor and data-logger channel was calibrated at ($T = 0^\circ C$) in a crushed ice-bath.

Time-lapse observations of passive duty-cycle tests carried out at room temperature on a south-facing window cill showed that air was being trapped in the voids amongst the dendrites that form during solidification. This occurs either at nucleation or the point of critical opalescence, a sample is shown on the right in Figure C.1. It was expected that a higher-density solid wax would sink in a melt solution, however they often remained buoyant until the majority of the bubbles are released, as a sample shows on the left in Figure C.1.

De-gasing is an important process step because air dissolved into liquid paraffin as it solidifies under atmospheric pressure is significantly more compressible, especially under large loads which leads to uncertainty in behaviour (p.194 in Klintberg et al., 2002). To swap out the ‘as supplied’ with the narrow melting-point wax, a procedure was required to de-gas the dissolved air. An adapter was machined to fit a vacuum pump hose to the heat-motor and then repeated melt-freezing cycles under a moderate vacuum before sealing the end-stop.
C.1. Design of a sensor to measure temperature in the wax core

Figure C.1: Air bubbles trapped in the dendrites during solidification at 1 atmos

Figure C.2: Assembly drawing of sensor for monitoring wax temperature

Figure C.3: Thermoresistive sensor to monitor the wax temperature inside heat-motor

Note: Instrumentation design, modelling, fabrication, assembly and images by author
Appendix D

D.1 Pilot study of heat-motors in a climate-chamber

The “Superpower” Mk.7 heat-motors from Bayliss Ltd are supplied with a wax blend, this expands across a temperature range between approximately $T = (16$ to $25)\, {^\circ}C$. An initial pilot study was carried out in a thermal study laboratory’s climate-chamber to develop the experimental hardware and practical techniques that would be used in a later thorough study of the motor’s behaviour.

These results characterise the response of the heat-motor’s cylinder to heat-transfers by free convection between its surface and the chamber’s ambient air. Heat-transfers by the exchange of radiant energy are ignored on the basis that the temperature of the climate chamber’s walls are the same as ambient air. This is justified on the basis that the wall surface is polished stainless steel sheet constructed with a substantial layer of glass wool insulation behind. The net exchange of radiant heat energy between the cylinder and the wall will be very low because the surface of stainless steel has low emissivity in the long wave band ($\varepsilon_{\lambda=10\, \mu m} = 0.17$) from (Incropera and DeWitt, 2007). The heat-motor’s Aluminium surface also has a moderately low optical emissivity that is typical for a metal without any surface treatment.

This Appendix reports two sets of data, the first is for the Mk.7 motor filled with the supplied wax blend. The second set for the Mk.7 motor filled with the highly refined $\text{n-octadecane}$ wax that expands over a narrow temperature range. These two sets of experiments were carried out several years apart, as a consequence the technical procedures, instrumentation in the second set had the benefit of time to make numerous improvements.

In the first set of data for blended wax, the discharge times of the recovery stroke for a range cooling rates are tabulated in Table D.1. The response curves for actuating the heat-motor for a range of heating rates are shown in Figures D.1(a to f). Finally, the thermal hysteresis loop for each rate is plotted from cross-plotting the heating and cooling gradients together.

In the second set of data for the refined wax, the results of heat-soak followed by cold-soak experiments are reported. This focused on approximating the conditions for finding the gradient in the heat-motor’s characteristic Newton’s cooling curve.
D.1. Pilot study of heat-motors in a climate-chamber

D.1.1 Time-lags for recovery stroke: Wax blend

The following results are reported for the experiments in the climate chamber when the ambient air temperature in the environment around the heat-motors was gradually cooled at a range of rates using the air conditioning unit’s PID controlled cooling-coils.

The characteristic temperature range over which the solidification processes occurred in the blended wax can be seen in column four (from left) in Table D.1, it averages (3°C) and noticeably less distinct at rapid rates of cooling.

The series of plots shown in Figure D.1(a to f) clearly show the gradual separation that occurs between the temperature of ambient air and the wax as the rate-of-change increases. The temperature time-series starts to diverge when the rate-of-change > (+2.0°C·h⁻¹) indicating the conditions when we can start to expect time-lags between ambient and the actuator. This can be explained as likely due to a low coefficient of heat-transfer and the thermal inertia due to phase-transition. The data shown in Figure D.2 also indicate the gradual opening up of the hysteresis loop as the rate-of-change in cooling increases.

<table>
<thead>
<tr>
<th>Cooling rate</th>
<th>Band-gap</th>
<th>Recovery time-lag</th>
</tr>
</thead>
<tbody>
<tr>
<td>∆T/∆t (°C·h⁻¹)</td>
<td>Tmp (°C)</td>
<td>Tcp (°C)</td>
</tr>
<tr>
<td>-0.5 (slow)</td>
<td>21.4</td>
<td>18.0</td>
</tr>
<tr>
<td>-1</td>
<td>20.8</td>
<td>17.7</td>
</tr>
<tr>
<td>-2</td>
<td>20.8</td>
<td>18.0</td>
</tr>
<tr>
<td>-4</td>
<td>20.5</td>
<td>18.0</td>
</tr>
<tr>
<td>-8</td>
<td>20.8</td>
<td>17.4</td>
</tr>
<tr>
<td>-16</td>
<td>20.5</td>
<td>18.0</td>
</tr>
<tr>
<td>-32 (fast)</td>
<td>20.5</td>
<td>17.9</td>
</tr>
<tr>
<td>Mean average</td>
<td>20.9</td>
<td>17.9</td>
</tr>
<tr>
<td>St.D</td>
<td>0.47</td>
<td>0.28</td>
</tr>
</tbody>
</table>

Note: * As controlled by the A/C units. † The congealing point in the wax.
Figure D.1: Actuating the heat-motor by heating in air at increasing rates

(a) Rate of heating $+0.5 \, ^\circ\text{C} \cdot \text{h}^{-1}$

(b) Rate of heating $+1.0 \, ^\circ\text{C} \cdot \text{h}^{-1}$

(c) Rate of heating $+2.0 \, ^\circ\text{C} \cdot \text{h}^{-1}$

(d) Rate of heating $+4.0 \, ^\circ\text{C} \cdot \text{h}^{-1}$

(e) Rate of heating $+8.0 \, ^\circ\text{C} \cdot \text{h}^{-1}$

(f) Rate of heating $+16.0 \, ^\circ\text{C} \cdot \text{h}^{-1}$
Figure D.2: Hysteresis loop during phase-change when heated/cooled in air

(a) Rate of heating/cooling $\pm 0.5 ^\circ C \cdot h^{-1}$

(b) Rate of heating/cooling $\pm 1.0 ^\circ C \cdot h^{-1}$

(c) Rate of heating/cooling $\pm 2.0 ^\circ C \cdot h^{-1}$

(d) Rate of heating/cooling $\pm 4.0 ^\circ C \cdot h^{-1}$

(e) Rate of heating/cooling $\pm 8.0 ^\circ C \cdot h^{-1}$

(f) Rate of heating/cooling $\pm 16.0 ^\circ C \cdot h^{-1}$
D.1.2 Time-lags for recovery stroke: Fully-refined wax

This section reports the characteristic cooling-curve of the Bayliss Mk.7 actuator. Further to Figure D.3 in Chapter 3, the impact of phase-transition on time-lag is compared between cylinders filled with water and wax. The results of repeated pair-wise experiments is shown in Table D.2. wax solidification adds (2 hrs) to the time-lag compared with water-filled cylinders.

<table>
<thead>
<tr>
<th>Material</th>
<th>Repeat</th>
<th>Treatment</th>
<th>Motor</th>
<th>Super-cooling</th>
<th>Time-lag of recovery stroke</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-)</td>
<td>Run</td>
<td>ΔT</td>
<td>(°C)</td>
<td>(°C)</td>
<td>Unit</td>
</tr>
<tr>
<td>water</td>
<td>1</td>
<td>-15.6</td>
<td>V</td>
<td>n/a</td>
<td>00:31</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>S</td>
<td>n/a</td>
<td>00:35</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>-15.6</td>
<td>V</td>
<td>n/a</td>
<td>00:46</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>S</td>
<td>n/a</td>
<td>00:34</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>-15.8</td>
<td>V</td>
<td>n/a</td>
<td>00:43</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>S</td>
<td>n/a</td>
<td>00:35</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>-16.1</td>
<td>V</td>
<td>n/a</td>
<td>00:34</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>S</td>
<td>n/a</td>
<td>00:34</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>-16.1</td>
<td>V</td>
<td>n/a</td>
<td>00:39</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>S</td>
<td>n/a</td>
<td>00:23</td>
</tr>
<tr>
<td>wax</td>
<td>1</td>
<td>-16.0</td>
<td>D</td>
<td>27.4</td>
<td>02:28</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>M</td>
<td>26.8</td>
<td>02:34</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>-16.0</td>
<td>D</td>
<td>27.7</td>
<td>02:18</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>M</td>
<td>27.2</td>
<td>02:42</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>-16.0</td>
<td>D</td>
<td>27.3</td>
<td>02:16</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>M</td>
<td>26.8</td>
<td>02:37</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>-16.1</td>
<td>D</td>
<td>27.8</td>
<td>02:27</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>M</td>
<td>27.5</td>
<td>02:48</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>-10.0</td>
<td>D</td>
<td>28.3</td>
<td>02:33</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>M</td>
<td>27.7</td>
<td>03:39</td>
</tr>
</tbody>
</table>
Figure D.3: Characteristic cooling-curve response of Mk.7 heat-motor

Chamber B
Air temp at 36°C
to heat motor
8°C above wax $T_{mp}$

Chamber A
Air temp at 20°C
to cool motor
8°C below wax $T_{mp}$

---

---

Ambient temperature of air ($T_{db}$)  Temperature at the core of the heat-motor ($T_w$)
---

Wax melt-point temperature ($T_{mp}$)  Actuation displacement length ($l$)
Appendix E

E.1 Verifying the thermal comparator’s mechanical design

This appendix describes the experimental methodology adopted to test the operation of the practical thermal comparator mechanisms developed in Chapter 4. These were constructed and prepared for use in a climate-controlled chamber. A schematic of the experiment is shown in Figure E.2.

To monitor the behaviour of the mechanism in its operating context, a suite of measuring instruments were used. Including position sensors to measure the linear motion between the sliding enclosure and the reference frame. Temperature sensors positioned inside and outside the mechanism’s enclosure to monitor the thermal gradient with the chamber’s programmed temperature set-point. Sensor data was recorded at regular intervals by a data acquisition system, supplemented with a visual record of the experiments from time-lapse controlled cameras as sampled in Figure E.1. By synchronising the recording of images and sensor measurements, the movies of these experiments provided a complementary source of data for analysis.

The images proved useful because they allow the corroboration of critical events recorded by the sensor data by compressing into a human-readable time-scale the long duration of these experiments, that in some cases extended into several diurnal cycles. The visual record is also used directly as image-based metrology with the metric rulers placed in the camera’s field of view to monitor the translation of the moving parts. This assists calibration of the linear motion sensors as well as providing an ‘eyeball’ guide to the progress of each experiment.

![Figure E.1: Pair-wise duty-cycling test of thermal comparators in climate-chamber](image-url)

(a) Mechanism closed, output $Q = 0$  
(b) Mechanism open, output $Q = 1$
E.1. Experiment methodology to verify mechanical design of thermal comparator

These aim of these tests were to establish if the mechanism can apply the desired conditional logic given the controlled ambient temperature inside the climate-chamber. The tests also quantify the responsiveness of the comparator by obtaining the time-temperature profile of the chamber’s air with the displacement of the mechanism’s output. The results of repeated tests for single and dual duty-cycles on groups of 2, 3 and 5 comparator mechanisms are summarised in Table E.1.

Figure E.2: Schematic of experiment to test the prototype thermal comparator
## Table E.1: Variation in the actuation displacement in batch of comparators

<table>
<thead>
<tr>
<th>Experiment Date</th>
<th>Test Cell</th>
<th>Motor A Output Q 'b'</th>
<th>Motor B Output Q 'c'</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$l_a$ at $T_{amb}$ (25 to 27$^\circ$C)</td>
<td>Overall band gap (2)</td>
</tr>
<tr>
<td>(YYYYMMDD) (ID)</td>
<td></td>
<td>(mm)</td>
<td>(%)</td>
</tr>
<tr>
<td>2009-03-14 #4</td>
<td></td>
<td>83.7</td>
<td>-</td>
</tr>
<tr>
<td>[Time-lapse] (WMV, 9.3 Mb) #5</td>
<td></td>
<td>81.5</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td>82.5</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td>79.9</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td>78.7</td>
<td>-</td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td>80.4</td>
<td>-</td>
</tr>
<tr>
<td><strong>St. D (n=5)</strong></td>
<td></td>
<td><strong>2.79</strong></td>
<td>-</td>
</tr>
<tr>
<td>2009-02-02 #4</td>
<td></td>
<td>83.4</td>
<td>-</td>
</tr>
<tr>
<td>[Time-lapse]</td>
<td></td>
<td>85.3</td>
<td>-</td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td>84.3</td>
<td>-</td>
</tr>
<tr>
<td>2009-02-03 #4</td>
<td></td>
<td>83.0</td>
<td>-</td>
</tr>
<tr>
<td>[Time-lapse]</td>
<td></td>
<td>84.9</td>
<td>-</td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td>83.9</td>
<td>-</td>
</tr>
<tr>
<td>2009-02-04 #4</td>
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<td>82.7</td>
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<td>[Time-lapse]</td>
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<td>84.6</td>
<td>-</td>
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<tr>
<td>Mean</td>
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</tr>
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</tr>
<tr>
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<td>83.3</td>
<td>-</td>
</tr>
<tr>
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### Experiment methodology to verify mechanical design of thermal comparator

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### E.1. Experiment methodology to verify mechanical design of thermal comparator

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(1) ID of mechanism. (2) Range of actuation displacement between \(l_a \) at 37°C \(\) \(-\(l_a \) at 27°C\). (3) The proportion of overall band-gap to the maximum actuation displacement at 37°C.
Appendix F

F.1 Methodology to select wax melting-point

In Section 5.1.1 of Chapter 5 a thermal design rationale was presented based on selecting a suitable equilibrium temperature for phase-change ($T_{mp}$) with a wax ($n$) from a range of $n$-paraffins. Ideally, the temperature ($T_{mp}$) represents a balance-point between the effects of heating from radiant transfers and the cooling from convective losses to ambient in a given climate. The pseudo-code listed in Algorithm 1 shows the method that was used to determine the melting-point.

The algorithm requires the following inputs; a time-series of diurnal external dry-bulb air temperatures for day numbers ($dn$) across an annual cycle for the given site, a range of wax melting-points and climate-data averages for minimum and maximum temperature. Algorithm 1 categorises the duty-cycle activity that would be expected to occur in a heat-motor charged with each wax against the time-series of temperature. It assumes that heat-transfers are only by convection between the wax container and ambient. The results are classified into four categories of duty-cycle activity and divided into bins of (% of days in a year) for each wax, defined as follows:

(1) No duty-cycle, the wax remains solidified

(2) A full duty-cycle with both melting and solidification occurring

(3) No duty-cycle, the wax remains liquid

(4) Unclassified by (1), (2) or (3)

Algorithm 1 selects a wax by melting-point from the list of candidates that represents the least sensitivity to duty-cycling with ambient and requires the least energy from radiant gains to overcome passive losses to ambient given climate data for a site location.

Since most climate database have time-series of dry-bulb air temperature at hourly (or sub-hourly) intervals, this method could be use for feasibility studies. There are numerous sources of this data for locations worldwide (e.g. Remund and Kunz, 2004). Some limited data is available for near-by planetary bodies in the solar system with (and without) atmospheres, specifically at lander locations around equatorial latitudes on the near-side of the moon and some locations on mars.
Algorithm 1: Procedural algorithm to select melting-point ($T_{mp}$) of wax by ($n$)

**Input:** Meteorological time-series data for dry-bulb air temperature

$\{T_{db,i}\}_{i=1}^{366}$. List of the melting-points for $n$-paraffin series of waxes (Carbon number) \( (C_nH_{2n+2}) = \{T_{mp,1}, ..., T_{mp,n}\} \). Climate data $\max(T_{db})$, $\min(T_{db})$

**Result:** The Carbon number ($n$) of a $n$-paraffin wax with melting-point ($T_{mp}$)

```
for each wax $n = (T_{mp,n} > \min(T_{db}))$ to $(T_{mp,n} < \max(T_{db}))$ do
  for each day $dn = 1$ to 366 do
    for each $T$ in diurnal cycle = 1 to $\text{size}((T_{db})_{dn,i})$ do
      Classify the phase-state(s) of wax ($n$) during this diurnal-cycle;
      if $T_{mp,n} < T = 1$ : $\text{size}((T_{db})_{dn,i})$ then
        Wax ($n,dn$) = 1; % remains frozen all day;
      else
        if $T_{mp,n} < T_{db} = 1$ AND $T_{mp,n} > T_{db} = 2$ :
          $\text{size}((T_{db})_{dn,i}) - 1$ AND $T_{mp,n} < \text{size}((T_{db})_{dn})$ then
            Wax ($n,dn$) = 2; % undergoes a full duty-cycle;
          else
            if $T_{mp,n} > T_{db} = 1$ : $\text{size}((T_{db})_{dn,i})$ then
              Wax ($n,dn$) = 3; % remains liquid, does not duty-cycle;
            else
              Wax ($n,dn$) = 0; % duty-cycle behaviour unclassified;
            end
          end
        end
      end
    end
  end
end
```

Now parse ($n,dn$) to find the first value of ($n$) than satisfies criteria:

```
for each wax $n = (T_{mp,n} > \min(T_{db}))$ to $(T_{mp,n} < \max(T_{db}))$ do
  if 97 percentile of ($n,:$) = 1 then
    if $T_{mp,n} > \max((T_{db})_{dn,i})$ then
      break, end;
    end
  end
end
```
Appendix G

G.1 Pilot study of heat-motor in glazed enclosure

This appendix reports the results of a pilot study that was conducted to test whether a heat-motor actuator could be used to passively operate a weather responsive sculpture. This study was carried out in collaboration with the Sixteen*(makers) group and the design of the sculpture itself was led by Dr Phil Ayres at the School of Architecture, Århus, Denmark. The author’s contribution was to integrate the heat-motor with the sculpture’s deployment mechanism, together with designing and carrying out the observation study.

The close collaboration between the project’s protagonists has made it difficult to completely disaggregate the design of the sculpture from its in-situ context and the task of engineering a working mechanism to animate it. Given that the design of the sculpture was a project in its own right, a brief account will be given in order to place the pilot study into context.

The study was carried out in the Kielder forest, this is located in the North of the United Kingdom at Latitude $55^\circ$ N at the invitation of the region’s curator of artworks. The study’s development has been strongly influenced by the climate and local characteristics of the forest. The forest’s woodland and the large reservoir located at its centre attracts many visitors, who apart from enjoying the outdoor recreational amenities contribute to the economy of the region. The responsive behaviour of a sculpture that is animated by the prevailing synoptic weather conditions adds a distinctive feature of interest amongst the forest’s other attractions. To further advance this, it would be desirable to passively operate the sculpture in this way on as many days as possible.

In order to achieve this given the site’s climate and the remoteness of the location, a design is proposed for a proprietary heat-motor actuator enclosed in a glazed tube with its displacement length linked to a deployment mechanism. This arrangement is intended to duty-cycle the heat-motor by exploiting the ‘greenhouse effect’ that develops in the glazed tube due to gains from solar radiation.

This appendix gives a brief account of the sculpture’s design and describes the methodology adopted to carry out an observation study capable of recording the sculpture’s range of movement. Then the results of the observation study are reported to show the sculpture ‘as built’ and operated in the context of one day with a moderate level of sunshine.
G.1.1 The design of a weather responsive sculpture

This was a collaborative project, as a group we started with a shared aspiration that any intervention in the territory of Kielder should be related to and hold in common with the processes that we found evidence for on the site. To this end, we made repeated visits during (2003 to 2006) to photograph, develop an understanding of and a familiarity with the landscape and its history.

We came to accept the perspective that Kielder forest represents an artificial landscape, this is simply because the majority of its trees are in fact planted as a commercial crop. Kielder presents the visitor with a paradox because while it has retained aspects of picturesque landscape beauty from before the introduction of large-scale silva-culture, it is now an intensively managed and mechanised farm that extends over thousands of hectares.

Despite this paradox, amongst the heavily managed growing and harvesting cycle of the dominant tree crop, Kielder is increasingly a destination for outdoor recreation. A series of artworks and small buildings have been and continue to be developed around the large water reservoir located in the middle of Kielder forest. In this context, the specific interest that we have taken is the changing relation that occurs between living plants and the surrounding environment during the tree’s growth and harvesting cycle.

We propose the design of a sculpture to be placed within this artificial landscape, and to devise ways to animate its arrangement in response to changes in its ambient environment. In particular, we propose to embody the characteristics of change over time that are either forced by a harvesting cycle or inherent in changes to a site’s meso and micro-climate that results from it. This has driven the sculpture’s design and shaped the strategy for investigating its relation to its site specific context as described by Ayres (2005).

With this background, to approach Kielder with a proposal for an art installation was to acknowledge that changes over different time-spans that were perceptible to us during our visits and historical research we hoped could be shared with the perceptions of Kielder’s many visitors. Specifically, we sought embodiments of change that are apparent at different time-scales from a person’s lifetime, the changing seasons across each year and the hourly changes in temperature that occur across a diurnal cycle.

Taking the long-term view, we speculated that these changes would be most evident at the intersections between adjacent areas or ‘coups’ of trees planted at different stages in the tree’s growth and harvest cycle. Specifically, a coup planted with young saplings opposite a mature coup reaching harvest time together with a juvenile coup midway between the two and a coup left fallow. This was the rationale for selecting an installation site for the sculpture.

Given access to the forestry commission’s management database, we found the planting year of each coup and together with the topological map of Kielder (OS, 2002) we could then interpret the available aerial photographs to identify candidate sites. After several reconnaissance visits driving through the forest’s myriad of logging tracks, a site matching this description was selected.
The concept for the sculpture itself can be traced to a number of antecedents, in particular the ‘Tarim machine’ shown in Figure G.1. This sculpture by the Dutch artist Gerrit van Bakel was proposed to move across the dry basin of the Tarim desert (PRC) in a linear path. The Tarim’s wheels are connected by gears to chambers filled with oil (pp. 25-7 in Bakel, 1987). The wheels are driven by harvesting the daily fluctuation in the ambient temperature to alternately expand and congeal the oil due to the exchange of thermal energy with the surroundings.

This means of propulsion is passive since it is due solely to the environmental energy transferred in each warming and cooling cycle. The pace at which it proceeds is determined by the magnitude of the diurnal fluctuations on a synoptic and seasonal basis. It is predicted that the transit across the Tarim desert’s 1100 km long basin would span climatic or even geological time in the order of 17,000 y progressing at the stately pace of ($\Delta l \approx 6.57 \text{ m} \cdot \text{y}^{-1}$) as reported in (Wentick, 1992).

One can speculate that the traces left on the desert basin by the Tarim’s wheels are the record of the fluctuations that propelled it. Its examination would reveal both the synoptic daily fluctuations as well as the climatic fluctuation over the centuries that it will take to cross the desert. The variations in the accumulated trace is a record that is analogous to the variations in width of growth rings in a tree as a dendrochronologist would interpreted them.

While the ‘Tarim machine’ may appear without a function, it is a working technical object with wheels that do turn to leave a real record in the landscape that it moves through. The function it serves the artist with is to use ‘The machine as a metaphor for thinking’ (Veenstra, 1985). We shared this point-of-view in the design of a sculpture for Kielder, it is a technical object placed in an artificial landscape that is exposed to silva-culture’s processes as it changes the micro-climate of the installation site. It allows us to think about a place and its environment through the record of the machine’s in-situ experience of it. Its response to synoptic conditions signals to us as much as passing visitor the state of the micro-climate at any given time.

**Figure G.1: Antecedent powered by passively harvesting environmental energy**

Figure removed due to third-party copyright

The ‘Tarim machine’ by artist Gerrit van Bakel (Photographs by Tom Haartsen Copyright © 1982)
G.1.2 Ambient energy and the balance of states

Defined in engineering terms, the sculpture will be animated in response to synoptic fluctuations in the site’s micro-climate when this matches the thermal energy required to duty-cycle the heat-motor. The thermal climate is summarised in Table G.1 and irradiation levels in Figure G.2. Kielder is typical of a northern temperate climate with average air temperatures between $T_{db} = (6.2 \text{ to } 19.5) ^\circ C$, the availability of solar energy is a moderate $(900 \text{ kWh} \cdot m^{-2} \cdot y^{-1})$.

The proprietary actuator used in this pilot has a known actuation temperature with a critical point of phase-change at $(T_{mp} = 20.9 ^\circ C)$. However, the ‘as supplied’ wax charge is a blend that expands across a temperature range spanning from $T = (16 \text{ to } 25) ^\circ C$. The actuator’s responsiveness when absorbing heat-gains and rejecting heat-losses is uncertain, a generous assumption will be made that actuation state is comparable to temperature with no time-lags.

Based on the climate data in Table G.1, the sculpture will not respond to fluctuations in the prevailing conditions for a portion of the year because the maximum daily temperatures does not reach the critical-point $(T_{mp})$. In order to extend the range of fluctuations that the sculpture responds to, the heat-motor will be placed inside a single glazed cylinder enclosure. This is to exploit the ‘greenhouse effect’ and shift the energy-balance in favour of actuating the heat-motor on days when the ambient temperature is below the critical-point.

**Figure G.2: Global irradiation on the horizontal plane for the UK**

| Table G.1: The daily minimum and maximum outdoor temperatures in Kielder, UK |
|-----------------|---|---|---|---|---|---|---|---|---|---|---|---|
| $T / ^\circ C$ | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
| $T_{max}$ | 6.8 | 7.0 | 9.1 | 11.8 | 14.9 | 17.3 | 19.4 | 19.5 | 16.9 | 12.6 | 9.2 | 6.2 |
| $T_{min}$ | 1.5 | 1.2 | 2.3 | 3.9 | 6.2 | 9.1 | 10.5 | 10.7 | 9.0 | 6.1 | 3.2 | 1.0 |

*Note: Average outdoor dry-bulb air temperatures in (2000 to 2009), (Source data MetOffice, 2010), Extract from chart of the UK from irradiation data, (Šuri et al., 2007)*
G.1.3 Sculpture deployment mechanism

Given the thermal context that has been described in the previous section, we sought to animate the sculpture with a movement that bears a bio-mimetic resemblance to the turgor movements exhibited by many plant organisms. Specifically, the response to a change in the direction and magnitude of sunlight that has been shown to alter the angle of leaves relative to the axis aligned between cardinal points or to sun-paths (Ehleringer and Forseth, 1980).

This is a bio-mimetic resemblance in the form of the response not in its means. As a technical object its response is due to the energy exchange between the thermo-hydraulic actuator with solar and ambient thermal conditions and not the biochemical changes in the concentration of soluble salts thought to drive turgor movement in plants (Whippo and Hangarter, 2006).

The form of the sculpture’s response that is desired is for the leaves to present a small area that is perpendicular to the axis between cardinal points north-south when ‘stowed’ and a larger area parallel to this axis when ‘deployed’. While many linkages have been studied (McCarthy, 2000), relatively few exhibit the property of being thin and compact when ‘stowed’ and thick and broad when ‘deployed’. A material or structure with this property has been described as auxetic (Evans and Alderson, 2000). One specific linkage that qualifies for this description is the ‘Sarrus’ hinge, six macro-scale ‘leaves’ linked in this arrangement are shown on the right in Figure G.3.

Figure G.3: Leaves of the Samanea saman and bio-mimicry of the 'Sarrus' hinged leaves

Note: (Left-hand side) Nyctinastic or ‘sleep-movement’ in the plant species Samanea saman. The angle of the leaves changes with the circadian cycle, opening up toward the sky during the day and closing at night. (From Hart, 1990) (p. 130 Simons, 1992), (Right-hand side) Deployment of the six-bar ‘Sarrus’ hinge corresponding to the heat-motor’s actuation displacement (Model by Dr Ayres)
Figure G.4: Model of the heat-motor and solar collector tube shown disassembled

(1) Return spring
(2) Drive coupler
(3) Heat-motor stroke rod
(4) Heat-motor collar and glass tube assembly
(5) Glass collector tube
(6) Heat-motor cylinder
(7) Glass tube seal assembly
(8) Sculpture from six links of sarrus-hinge (shown ghosted)
(9) Wax charge (shown when solidified)

Note: Exploded assembly drawing from parametric model in AutoDesk® Inventor™ (Shown above)
Longitudinal section through the assembled enclosure showing the arrangement of the heat-motor components in detail (Shown below). (Design by Dr Phil Ayres)
The ‘Sarrus’ is a linkage that consists of six bars, its deployment is controlled by varying the angle between two bars opposite one another along a line of symmetry. Examples of this arrangement have been shown to provide a stable and rigid deployment of the other four links (Angeles, 2002). An abstract arrangement of the ‘Sarrus’ was developed virtually using solid modelling software for mechanical and engineering design. This led to the form shown on the right in Figure G.3 with the six bars represented as ‘leaves’ connected at common fixing points but having different profiles.

The virtual simulation of the mechanical translation between ‘stowed’ and ‘deployed’ positions allow us to study the dynamics of the displacement length that is available from the heat-motor’s linear actuation. This was used to match the limits of the ‘Sarrus’s different positions and select the actuation axis. The resolved arrangement is shown in Figure G.4 with the component-level detail of the link between the heat-motor and the angle that controls the ‘Sarrus’s deployment.

The axes that link each of the six bars together generates a complicated geometry, for which a set of purpose-designed fixings with bearing pockets were modelled and CNC milled from blocks of the self-lubricating material Polyoxymethylene (DIN POM). Finally, an adjustable stalk was designed to mount the sculpture so that the leaves could be oriented to align to the east-west axis as required. A series of units were then constructed from this kit-of-parts using the assembly drawings generated by the solid modelling software as shown in Figure G.5. Some clashes and issues of component clearance were found, however these were largely the result of modifications to parts that were fabricated by hand that were not developed in enough detail in the virtual model.

Following assembly, the sculptures were transported to the Kielder forest and prepared for the observation study. The next section discusses the design of the observation campaign for recording the sculpture’s response to the ambient environment that it is exposed to.

**Figure G.5: Kit-of-parts made using CNC fabrication**

*Note:* The sculpture’s parts were fabricated using digital manufacturing technology. A Computer Numerically Controlled (CNC) abrasive water-jet machine cut the components out of 10 mm thick aluminium sheet (shown right-hand side). All parts produced directly from the digital files extracted out of the solid modelling software (shown on left-hand side, Design by Dr Phil Ayres).*
G.1.4 Design of the observation study

This section gives a brief description of the methodology that was used to conduct the observation study. Firstly, its objective and constraints are outlined followed by the measures taken to address them. The objective of the campaign is to establish whether the heat-motor can actuate the sculpture by exploiting solar gains when the ambient temperature is below the critical-point.

To achieve this, the relevant variables to monitor and record are short-wave radiation, ambient air temperature and displacement length. In order to verify the deployment of the 'Sarrus' hinge, a method was needed to capture its complicated geometrical transformation during actuation without physically interfering with the leaves. The campaign faced the constraint that there was no infrastructure at all to support the observation study 'in-situ', including no electrical power. In addition, attendance was limited to infrequent visits during daytime only.

To meet these objectives and constraints a purpose-designed instrumentation package was developed. Figure G.6 shows an overall schematic of the observation system’s components, this consists of three parts; the ‘Sarrus’ hinge (part 16), the heat-motor mechanism that operates it (parts 1 to 3) and the observation system that monitors its behaviour (parts 4 to 15).

A pre-pilot study was conducted to monitor the ‘Greenhouse effect’ in a small glazed enclosure compared with ambient air temperatures. The duration of this preparatory exercise was one year, its purpose was to provide a data-set for making comparisons on a seasonal basis.

The environmental variables could be monitored by implementing a miniature weather station on-site for the duration of the pilot study. However, the periodic measurement of movement posed some challenges because ideally it would be a non-contact surveying method that measured the absolute position of the ‘Sarrus’s linkages in a three-dimensional coordinate system. To address this, an imaging-based technology was adopted to time-lapse capture the scene together with software to measure the images using the technique of close-range photogrammetry.

It was identified during the simulations of the leaf parts in the virtual model that the geometrical transformation of this auxetic structure is quite complicated. This presented uncertainties as to whether a simple system with one fixed camera station could provide adequate coverage for obtaining photogrammetric solutions for all stages of deployment. Practical tests showed that it was difficult to cover each stage of the mechanism’s deployment with fewer than five fixed camera stations. This was mainly because of the changing angles of obscuration and large residuals.

The study was scheduled to span three weeks in order to capture a range of synoptic conditions with the hope that one or more 'test-days' would occur with conditions that approximate clear-skies. During this time access to the site would be very limited, thus ideally all parts of the observation study would need to be automated. To address this constraint, a purpose-designed electronic control system was developed to coordinate the array of time-lapse cameras with the environmental monitoring. An image of the sculpture set-up at the start of the observation campaign is shown in Figure G.7.
Figure G.6: Schematic of sculpture and observation system

Note: Birds-eye view of ‘Sarrus’ leaves in-situ, the heat-motor is pointing due-south (right-hand side part 1) so that when the ‘Sarrus’ is deployed it is split open parallel to the east-west axis, Automatic weather station (Below left), part of the array of fixed camera stations in enclosures (Below right)
Note: Sculpture installed in the Kielder forest, Northumberland (UK) at geo-coordinates Latitude 55°07′59″N by Longitude 02°28′45″W, Altitude 360 m a.m.s.l to the WGS 84 datum. The leaves are in the partially ‘deployed’ position, from observations on 27th August 2006, Image by Sixteen*(makers) © 2006. [Time-lapse study] (WMV, 2.5 Mb)
G.1.5 Results for a diurnal period during Autumn

This section reports the observations made about the heat-motor’s response to prevailing conditions. In general, the synoptic conditions over the three-week monitoring campaign during late-August were cold and overcast. There were occasional breaks in the weather, however only one day experienced spells of sunshine and provided a full set of data.

The findings reported here are from observations made on the 27th August 2006. The data is summarised in the two time-series shown in Figures (G.9 and G.10). The top-half in Figure G.9 shows the level of insolation due to synoptic conditions. The lower-half in Figure G.10 shows the ambient environment and the displacement length from the heat-motor’s actuation.

A numerical integration of the insolation time-series gives a yield of \( Y_i = 54\% \) when compared with a standard clear-sky model, this is a moderate level of sunshine. The ambient air temperature reached a maximum of \( T_{db} = 20.0^\circ C \), this is just below the critical-point temperature \( T_{mp} \). The time-series of displacement clearly shows that the heat-motor was actuated between approximately \( t_1 = 11:45\text{ hrs} \) and \( t_2 = 15:30\text{ hrs} \).

**Note:** Composition of stills taken during observations on the 27th August 2006 (Above left and right, by automated imaging system, camera station 3 of 5). Images processed in Photomodeler® to obtain a photogrammetric solution. Once correctly scaled, oriented with geographic north point and aligned to the nadir-zenith, the position of each leaf’s could be measured.
G.1. Pilot study of heat-motor in glazed enclosure

Figure G.9: External ambient thermal conditions around heat-motor

![External ambient thermal conditions around heat-motor](image)

- Apparent sky temperature ($T_{sky}$)
- Solar insolation modelled† under a clear-sky ($G_cH$)
- Solar radiation measured on a horizontal plane ($G_iH$)

Figure G.10: Time-temperature-displacement profile attained by the enclosed heat-motor

![Time-temperature-displacement profile attained by the enclosed heat-motor](image)

- Ambient temperature ($T_{db}$)
- Heat-motor actuation displacement ($l_a$)
- Enclosure temperature ($T_{enc}$)
- Critical point of wax blend ($T_{mp}$)

Note: Observation date 27th August 2006 in Kielder Forest, Northumberland (UK) at Latitude $55^\circ N$. Data recorder stopped at t=17:45 hrs (UTC).† Model after (Haurwitz, 1945). Daily insolation yield approximately ($Y_i = 54\%$) of a clear-sky.
A comparison between the time-series of the ambient temperatures and the inside of the enclosure shows a large ($\Delta T$) develops around mid-day due to the ‘greenhouse’ effect. The duration of the maximum levels of insolation matches the duration of the maximum actuation displacement. This demonstrates that the energy absorbed from sunlight due to the ‘greenhouse’ effect inside the glazed tube has contributed to supply latent heat to change with phase of wax in the heat-motor despite ambient temperatures barely reaching the critical-point for phase-change.

The deployment of the ‘Sarrus’s leaf from the heat-motor’s actuation was verified by analysing the time-lapse images. These were processed using proprietary photogrammetry software to give a time-series of 3D solutions to relate the angle of deployment to the displacement measurement.

The images in Figure G.8 are from one of the stations taken during the time-lapse, shown from left-to-right in chronological order. These clearly show the sculpture in the ‘stowed’ position during the morning and ‘deployed’ position during the early afternoon. Reading from top-to-below, the top-half shows the photograph while the lower-half shows the analytical geometric solution with meta-data overlaid. These results have been visualised with a photo-matched animation of the time-lapse images, with the time-series data from the instrumentation added to each frame. [Time-lapse study] (WMV, 3 Mb).

G.1.6 Discussion

Using the results from the year-long pre-pilot study together with the three-week observation campaign, we can predict how effective the glazed enclosure is for animating the sculpture. The question is posed on the basis that it is desirable for the sculpture to undergo a duty-cycle on as many days as possible, as well as spread across as much of the year as possible.

To make a prediction of what the impact of the ‘greenhouse effect’ inside the glazed enclosure might have on an annual basis, a comparison is made between the temperature recorded inside and outside the glazed enclosure. The results are shown in Figure G.11 as pairs of histograms. These show the total number of hours when the temperature was in each (1°C) wide band each month. Comparing the value between each pair indicates the difference made by the enclosure, the larger the difference the more duty-cycles there are likely to be due to the ‘greenhouse effect’.

The data is interpreted on the assumption that a threshold can be drawn at the critical-point temperature ($T_{mp} = 20.9\, ^\circ C$) that divides Figure G.11 in half from top-to-bottom. In the bottom-half of Figure G.11 the temperatures are ($> T_{mp}$), so a duty-cycle would be expected if the histogram shows ($> 0\, hrs$). A comparative analysis can be made by differentiating the hourly count between histograms for the temperature outside when ($0\, hrs$) and inside when ($> 0\, hrs$).

Assuming that there are no time-lags, a histogram with ($\geq 1\, hrs$) in a temperature band above the critical point is expected to lead to the sculpture exhibiting a duty-cycle. This study suggests that the glazed enclosure would extend duty-cycle activity a further four months into the coldest months of the year compared with un-glazed exposure.
Based on the data from (2005 to 2006) the number of intervals above the threshold inside the glazed enclosure was (627.5 hrs) vs (295 hrs) outside. This suggests that the enclosure can increase the total number of hours per year that have duty-cycling potential by (≈ 200%). Comparing the histogram pairs for August with the results of the observation study, it is clear that the efficacy of this prediction depends upon the availability of solar irradiation.

It follows that the exposure of the glazed enclosure to the solar path becomes the main variable because the fluctuations in ambient conditions contribute less to duty-cycling and less frequently. This result supports the proposition that the sculpture can represent an indicator of longer-term changes at the installation site, in particular the growth cycle of the tree crops. The dominant species in Kielder forest is the *Picea sitchensis* (Sitka spruce), the time-scale for silva-culturing a specimen from a sapling to a crop ready for harvest is in the order of a half-century. Given that it grows at the rate of (Δl ≈ 1 m · y⁻¹) there will be a subtle alteration in the seasonal profile of the sculpture’s response on sunny days over the course of generation (p. 114 in Sterry, 2007).

As the trees grow through the harvesting cycle in the coupes adjacent to the installation site, the portion of the sun’s path across the sky that is obstructed by the tree-line as its height increases year-on-year and then abruptly decreases over a half-century period. This characterises the time-scale over which annual patterns of responsive behaviour that the sculpture exhibited would be expected to change accordingly as seasonal access to solar radiation altered.

**Figure G.11: Extending the seasonally activity of the sculpture using the ‘Greenhouse effect’**

*Note:* The distribution of each histogram is based on data collected pre-pilot (2005 to 2006) from Kielder Forest in Northumberland (UK) at Latitude 55°N from both inside and outside the enclosure.
G.1.7 Limitations

The simulation of the sculpture’s deployment is based on the mechanical Degrees-of-Freedom (DoF) in the 'Sarrus' hinge, it does not account for the thermal exchanges that the results of the observation study have shown are central to its behaviour. A more detailed model and simulation would be needed to couple the domain of thermal physics with hydraulic and mechanical domains.

There are several improvements that could be made to the methodology. It should be possible to reduce the number of fixed camera stations required from five to two, while still achieving the same coverage. The complicated geometry might have been surveyed from the vantage points of an aerial view and worm’s eye view. In this study, the lenses were limited to a field-of-view (f.o.v.) of (≈ 60°). The two camera station solution might need wider-angle lenses giving a f.o.v. of (84 to 92)° to achieve the same coverage.

The data-collection could have been more efficient, the processing of the images proved time-consuming. Pre-pilot tests showed that each leaf needed a minimum of three to four points to be successfully registered by the photogrammetry software. While the deployment of the leaves was being simulated virtually, we tried to select positions for these points using the rationale that they should remain un-obscured throughout the range of movements.

Based on this criteria, holes were made at these points in order to coordinate through both sides of each leaf to be seen by camera stations on either side of the sculpture. In addition, circular high-contrast markers were attached around each hole on either side to assist visibility. The implication of this approach for post-capture processing was that all points would have to be marked and registered across each set of images by hand. This time-consuming process might have been overcome by applying a unique coded-target (CT) over each hole instead. Practical tests with CTs carried pre-pilot showed that Photomodeler® software was reliable for accurately detecting and identifying CTs under controlled lighting conditions and near normal angles of obliquity.

**Figure G.12: Self-illuminating coded-targets for feature marking in Photomodeler®**

(a) Daylight illuminated (visible band)  (b) Nocturnal self-illuminating (near-IR band)
However, conditions encountered in the field would present both a wide range of un-controlled lighting conditions and moderate to high angles of obliquity. In the field the lighting conditions varied between bright sunlight and moonless nights, this presented challenges for feature identification even using the high-contrast markers. The limited off-grid power available prevented the use of artificial lighting to maintain adequate illumination. Relying on daylight to illuminate the markers enough to be visible to the imaging system was limited to before and after twilight. In the event, relatively little movement occurred nocturnally and this was only during evenings.

This limitation might have been overcome by a simple passive method using targets that self-illuminate. A set of ‘coded targets’ generated with Photomodeler® were made on phosphorescence coated film as shown in Figure G.12. These were successfully evaluated using but proved expensive and time-consuming to make in enough quantity to use in the pilot. Successful implementation of CTs could have further automated the verification process and made the image processing stage more efficient.

G.1.8 Summary
This pilot study successfully demonstrated how a glazed enclosure can be used to raise the balance of specific enthalpy enough to phase-change wax by exploiting the ‘greenhouse effect’ despite ambient conditions being at or below the actuator’s critical temperature. The objective to increase the number of days when the proprietary heat-motor would undergo a duty-cycle to operate the sculpture was shown to be feasible by this method even when only moderate amounts of solar radiation are available.

This study demonstrates that complicated movable geometries such as the ‘Sarrus’ hinge can be effectively deployed using a heat-motor actuator. The action of the deployment mechanism was shown to change the sculpture’s shape with a geometric transformation that oscillates with and is matched to the availability of sunlight in each diurnal cycle. This was achieved using only ambient environmental energy and without being dependent on access to electrical power or other primary energy sources. On a location as remote as the Kielder’s forest, this offers an advantage over alternative expensive and complicated off-grid power sources.

An nth Dimensional (nD) surveying technique was developed to record the behaviour of the weather responsive sculpture on the installation site. The results of imaging of the sculpture’s deployment proved a rich source of feedback, it supported both to debugging of the mechanical design and the verification of the virtual model for the linkage mechanism.

Post-script
An article that reports the methodology adopted during this pilot study was accepted for publication in the Cybernetics Journal ‘System research in the AEC of built environments’. It elaborates on the feedback loop between digital prototyping ‘as designed’ and the digital observation of ‘as operated’ in-situ behaviour. The article is reproduced in full from the next page.
Environmental feedback in an iterative design process

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Abstract
This paper is an account of a practice-led project carried out during an “architects’ residence” in Kielder in the North of the UK, between (2003 to 2006). It describes how a systems-based approach was applied to cycle through the design-process iteratively. The aim was to fabricate an object embodying a mechanical mechanism that would visibly respond to the diurnal cycle of environmental change found in microclimates in Kielder. An evaluation of the object’s behaviour was carried out based on environmental metrics. The iterative process of building and monitoring prototypes suggests a design methodology that incorporates “as built and operated” evidence into the design process.

Initially, the design of the object was investigated using virtual models that served as descriptions of the finished object due to be placed on sites in Kielder. In an iterative approach to design with physically fabricated prototypes, there is the opportunity to capture and use feedback to develop the design and location of a prototype in the subsequent iteration. This paper discusses the methodology implemented in a case-study project and how information returning across the real-to-virtual threshold became design intelligence used in the next iteration. The collection of empirical data by “design probes” (Sheil and Leung, 2005) was coupled with a computer simulation; two feedback loops were identified, one informing model validation the other, objective validation. An iterative methodology is proposed to revise objectives, to re-model solutions, to re-synthesise outcomes and to re-locate on-site between iterations based on feedback from the ‘as built and operated’ data.

Keywords
Data-driven modelling, design-process, passive-actuation, responsive system

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Introduction

The design research group Sixteen*(makers) have explored digital design and its relationship with digital fabrication through a series of projects including “Shorting the Automation Circuit” (StAC) (Ayres, 2006). The StAC project developed the design of an object using solid modelling CAD software, data was exported and used to fabricate a physical prototype that was then placed in an unused observatory on the UCL campus. The fabricated object was partially exposed to the external environment and the effects of wind direction and magnitude were digitised using sensors and microprocessors embedded within the object, these were interfaced to a PC to “record an informal ‘thumbprint’ of the prevailing physical conditions”. Following StAC and complementary projects Sixteen*(makers) were invited to an “architects’ residence” in 2003 by the curator Peter Sharpe at the Art and Architecture Partnership in Kielder (AAP@K). An “architects’ residence” as distinct from an architect’s “commission” offered the opportunity of an open brief, a long time-scale and the expansive (62,000 ha) territory of Kielder’s forest and reservoir. The StAC project produced one object collecting environmental data that suggested changes to its form in a “search for specificity” (Callicott, 2001, pg. 80), the “architects’ residence” in Kielder has produced a series of objects in successive iterations, a brief description of the stages of each iteration are given below, followed by a presentation of the case-study itself, finally there is a discussion of the advantages of this methodology.

Figure G.13: System diagram of an iterative design-process

Note: The solution to the three-dimensional points + time was made from still images composed and processed using Photomodeler® by EOS systems Inc.
An Iterative design-process

An iterative design-process is suggested here as an approach to design practice with the following initial stages:

1. A definition of objectives and operating characteristics (the statement of requirements)

2. A definition of performance metrics (measures to assess the realisation of the objectives)
   Followed by a cycle of steps to:

3. Describe design intent (design documents, digital modelling and simulations) and provide instructions to manufacture, assemble and locate (exported from digital models)

4. Synthesise (fabricate, assemble and site a physical prototype)

5. Observe (digitally capture site-specific 'as built and operated' behaviour)

   This is illustrated in Figure G.13, beginning with a declaration of (1) and a selection of (2) the first pass through (3) to (5) establishes a design outcome crossing from the virtual to the real. The need for and relevance of (5) to the case-study presented here is reflected in a survey of adaptive kinetic architecture that identified the limited availability of empirical information of the performance of existing systems and how it is necessary for the design and engineering of a kinetic architecture to have accurate models and realistic input values to the parameters that drive their behaviour, while also providing a means for their validation (Sanchez-del Valle, 2005). An iterative design process has similarities with evolutionary strategies that use simulation to evaluate behaviour, behavioural deviations between as-simulated and as-operated may be accounted for by the completeness and accuracy of the virtual simulation environment and uncertainty in the input values it uses as well as tolerances in metrology from sensor measurements of the physical environment (Jakobi et al., 1995) and (Wit, 2003, chap. 3). The case study has an output control element with a passive-response to its environment, therefore the discussion here is restricted to how the model description and its simulation of the environment was informed through feedback.

Kielder Forest

The case-study is a dynamic sculpture that was designed to change its shape between dawn and dusk in response to its local micro-climatic environment. The geometry of the sculpture’s articulation is complex, for which a system was designed to capture its transformation in between a deployed and stowed configuration to understand its dynamic nature. In order to study the response of the sculpture’s to its specific site, environmental micro-climate data was gathered and combined with geometric data in an animated visualisation. Kielder forest experiences a cool northern temperate macro-climate (Latitude 55° N) and is a managed [sic artificial] forest planted in the expansive Cheviot Hills.
The hill and valley meso-climate can experience rapidly changing weather conditions and within the large territory of the forest the variety of microclimates range from exposed windswept hilltops to sheltered hollows deep within the forest coupes.

**Site Micro-Climate Selection**

A site was selected in a forest clearing at the junction of three forest coupes with different plantation dates with unobstructed views to the east and south and varied shelter to the north and north-west. It was anticipated that this would expose the observation site to a variety of weather conditions and that the micro-climate would change as each adjacent forest coupe reaches maturity and is harvested. Although Kielder forest is one of the most remote locations in the UK, it is a popular day-trip destination in the north-east attracting an estimated 230,000 visitors per year. The artificial wilderness forms the physical and visual context for an audience to contemplate this sculpture.

**The Design Objective**

The requirement was to animate the sculpture given the cool temperate thermal conditions and remoteness of the Kielder forest. The design of the mechanism and form of the sculpture together with its site aspect, orientation and exposure in a given micro-climate were to display a varied behaviour for the visitor at different times of the day and over the changing seasons.

**Passive Actuation**

The remoteness of the site motivates an environmental imperative to design the sculpture and its observation system for self-sufficiency, autonomous control and low-energy use. To achieve dynamic movement in this environment it was proposed to use a passive temperature activated actuator containing a phase-change material (PCM) as the hydraulic medium. The hydraulic behaviour of this actuator is determined by net heat gains and losses, net heat gains cause the actuator to warm and if sufficient reach its critical-point and melt, it expands considerably extending the actuator and this transforms the sculpture to a deployed configuration. Sufficient net losses causes the PCM to freeze thus contracting the actuator aided by a return gas-spring, this transforms the sculpture back to a stowed configuration. A standard proprietary actuator was used containing a mineral wax with a melt temperature of approximately 25 degrees Celsius. A typical late-summer day is shown in Figure G.14 indicating the diurnal time-temperature time-series during daylight hours, the time-series plots solar-radiation, solar heated air (actuator surface temperature) and the linear mechanical movement of the actuator. The plot of actuator expansion shows a single 'camel-back' indicating a melting/freezing cycle during which the sculpture transformed from stowed to deployed and returned to the stowed configuration at dusk.

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Environmental Responsive

A very simplified model of temperature/actuator response was used for the first iterations using a linear and proportional relationship between temperature and actuation. The case-study attempts to establish a base-set of parameters to describe the physical environment that will be responded to, consisting of those parameters that theoretically have the most effect on the sculpture’s behaviour: solar-availability was assessed by an elevation survey, the site-specific thermal micro-climate was monitored at three candidate locations at thirty minute intervals for one year to provide a realistic input data-set for the simulation. During a five day pilot trial of iteration three, temperature, global solar radiation and linear movement of the actuator were also monitored at five minute intervals.

The Observational Study

Given these base-set parameters the capture of ‘as built’ and operated behaviour required spatial and temporal correspondence between environmental parameters modelled in simulation and those monitored on the real sculpture to achieve a ‘base-set robust’ response model after (Jakobi et al., 1995). This was feedback that informed the validation of the response model. The geometry description of the sculpture was the framework to coordinate the environmental parameters as ‘meta-data’ between a CAD solid-model in the virtual and survey metrology using time-lapse close-range photogrammetry in the real “as built and operated”.

Discussion

A spatially coincident and temporally synchronised animation that composites simulated with ‘as built’ and operated provides feedback by high-lighting deviations between designed and real, whilst this feedback is descriptive and does not explain how or why a deviation exists its value is when deviations are exposed that are particularly large or unexpected. Since formal design decisions and engineering considerations to achieve the sculpture’s behaviour are strongly influenced by the environmental feedback data captured from “as operated”.

Figure G.14: Sample of a temperature and actuation time-series
Each cycle in the iterative design-process provides an opportunity for a prospective design to be evaluated both qualitatively and quantitatively against the performance of past designs on a given site. This feedback informed the validation of the design objectives. The case-study illustrates an implicit challenge for a passively-powered dynamic sculpture; to achieve a temporal match between the effects of an unpredictable natural energy resource the weather and the desire to drive a dynamic behaviour, since the passive actuator used to operate the mechanism is sensitive within narrow margins of environmental and temporal tolerance the benefit of an iterative design methodology is to support the systematic reduction of deviations between virtual simulation and actual operation through feedback.

Further Work
An engineering model that considers in more detail the heat transfers between the surrounding environment and the passive-actuator receiver over time would improve the efficacy of the model, this could support design decisions for future iterations of the dynamic sculpture designed for the Kielder forest. The data collected during this case-study provides empirical data to assist building and calibrating such a mathematical model if supported by further observational and controlled studies. A generic model suitable for behaviour modelling in different micro-climate conditions may be beneficial to design evaluations for a variety of architectural applications of passive-actuators. Given a model with greater efficacy there is the opportunity to address the challenge of achieving a temporal match between availability and need through the design and control of response modifiers e.g. dynamic insulation, solar radiation shade or concentrators to increase or decrease the rate of heat gains and losses to meet anticipated needs. This would need the development of an intelligent control system that actively monitored the prevailing external site micro-climate while considering the possible and likely conditions for the near-future given specific performance objectives.

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Appendix H

H.1 Pilot study of thermo-hydraulic tri-state shutters

This appendix describes a pavilion that was constructed to demonstrate heat-motors operating a façade of external deployable thermally insulated window shutters. This provides a pilot-study to experiment with matching the passive operating behaviour of the thermal comparator mechanism developed in Chapter 4 to deploy the shutter into the preferred positions in response to different prevailing external conditions. The question that this pilot-study aims to test is can the dual heat-motor operated comparator output three stable states, and mechanically move the shutters into the three desired positions.

The literature review in Chapter 2 surveyed movable components in façades that can adopt two or more different functions depending upon the prevailing external conditions. The ‘Sundows’ were one example of indoor shutters that can alternate between offering insulation and access to daylight by a temperature regulated deployment. It identified that an improvement could be external shutters that switch between providing insulation at night-time, access to daylight during the daytime but also provide shading during periods of excessive solar gains to prevent overheating. A brief outline of each configuration is given, describing how there are seasonal variations and how a selective response would be beneficial.

The demonstration pavilion reported here was completed in late-Spring 2008 and opened to the public at the Bartlett summer exhibition at UCL in 2008 followed by the London Architecture Biennale festival. It was subsequently moved to Trinity Buoy Wharf in London’s docklands where an observational study was carried out during (2008 to 2009). The following sections give a brief outline the design of the pavilion, then the findings of the studies are reported.
H.1.1 The benefits of movable insulated shutters for windows

Windows have an impact on regulating the dynamics in a building’s energy balance, especially in cases where there are extensive areas of façade glazing. The significance of this role depends on many factors, those that contribute to the fabric performance are the window’s orientation, the glass, glazing unit, window frame performance and the construction details around the window’s reveal. This is far from exhaustive, in particular regulating the performance of the windows by selective changes to these factors in response to external conditions can have large impacts on the overall operational performance of a building. The following benefits are outlined for the case of a vertically-inclined south-facing window that serves a notional room.

Passive solar gains captured by glazing can contribute to a building’s space heating requirements, however there can be a large seasonal variation. Winter-time gains from exposure to sunlight can be considerable because the intensity of insolation is highest when the direct-beam component has near-normal inclination, even at the highest elevation \((\theta_{sol} \approx 18^\circ)\). During summer-time, the benefits of accumulating free gains during the longer photo-period needs to be balanced against avoiding the risk of creating a cooling demand by excessive heating. This is despite the lower intensity due to the cosine-effect of direct-beam insolation at the highest elevation \((\theta_{sol} \approx 60^\circ)\).

During night-time windows represent a radiator of heat-losses, both by convective and conductive transfers through the glass and frame. In addition, there are radiant transfers to the surrounding surfaces including the night sky because these all tend to be colder than internal sources. Night-time losses are true for most of the year especially during winter and for most of early-spring and late-autumn. A comparison can be made in terms of fabric between an area of glazing with a weighted average ‘U-value’ of \((U = 2.2 W \cdot m^{-2}\cdot K^{-1})\) and a wall at \((U = 0.35 W \cdot m^{-2}\cdot K^{-1})\) that meets the building regulations detailed in Approved Document part L1A (p.19 in ODPM, 2006a). The thermal performance of the standard double-glazing unit is \(\times (6 \text{ to } 7)\) worse compared with a typical insulated cavity wall. Advances in glazing technology continue to close this performance gap, however neither parity nor the selective function of solar gain admission as well as the rejection of excesses are expected in a mass-market product in the near future (Short, 2007).

A concurrent consideration is the contribution that is made by windows to provide access to daylight. Specifically, the demand on energy consumption for general service illumination can be dramatically reduced by achieving an appropriate daylighting factor for a given activity. This is by offsetting the use of artificial lighting to meet an equivalent level of service illumination. In practice, the effectiveness of this approach depends upon how well the lighting system’s sensing and control regime manages the switching of artificial lights against daylight levels.

The preferences of occupants also play a part in determining the effectiveness of an approach that automatically controls the shutter’s position over the window. When they are present, there are implications of their interaction with an override control that can be either in favour of or act against the provision of daylight, increasing heat-losses and the use of artificial lighting. Reviews of
the literature have reported that the occupant preference is strongly weighted in favour of daylight provision over artificial lighting (Galasiu and Veitch, 2006). So the effective provision of daylighting could yield an advantage when balanced against the drawbacks of large areas of glazing.

Another concurrent benefit from a response to sunlight during summer is to protect the window from exposure to direct sunlight by external shading. On warmer days under clear-skies, this possibly extends into late-spring and early-autumn. This response could benefit both indoor lighting conditions and the building’s thermal performance, by reducing the risk of discomfort glare for occupants from excessive levels of luminance. As well as the risk of unnecessarily adding to the cooling load by preventing an excessive accumulation of gains from insolation.

Between each of these benefits is a mutual exclusivity, this is due to limitations in the glass technology that is currently available. The performance of a window against these three different constraints could be reconciled by adding a movable thermally-insulated shutter mounted externally over the window. This would provide thermal insulation to prevent heat-loss when closed at night. During winter, it would open during daytime to collect beneficial gains as well as admit daylight to reduce demand on artificial lighting. During daytime on hot conditions during summer the shutter would partly close to shade the window from excessive gains while still admitting daylight. The impact of shutters have been studied both theoretically and in surveys of existing buildings across north-America during the middle to late-1970’s by (Langdon, 1980) and (Shurcliff, 1980).

Figure H.1: Annual savings from using external insulating shutter to reduce heat-loss

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Note: This estimate of savings assumes that the shutter is closed for 14 hrs over-night between (17:00 until 07:00) hrs during the six coldest months in a location with a given climate (p. 29 in Shurcliff, 1980). The comparison is with London is ($\approx 1800 \text{ DD} \cdot \text{y}^{-1}$) based on data for the Thames Valley region averaged over (20 y) at a base temperature of ($T = 15.5^\circ \text{C}$) from (Vesma et al., 2010). The curves show that after the thermal performance approaches ($R = 5$) there are diminishing returns. This corresponds to ($U = 0.35 \text{ W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$) in SI units and slightly worse than present-day UK building regulation for an external wall in a dwelling (ODPM, 2006a). On this basis a shutter system could reduce heat-loss by ($47.3 \text{ kWh} \cdot \text{m}^{-2} \cdot \text{y}^{-1}$) per unit window area.
These studies estimated the reduction in space heating demand that would result from the reduction in heat-loss by selectively improving the window’s insulation performance when demand is highest. The savings on space-heating were calculated based on a daily deployment of a 14 hrs interval for a range of climates (Shurcliff, 1980). From these estimates, the savings that may apply for London (UK) based on a comparable heating season is shown on the left in Figure H.1 from the closest match to the number of degree-days (DD) per year.

Based on these assumptions, the benefit for a residential property in London can be estimated. For a living room with a floor area of \(A = 40 \, m^2\) and a south-facing double-glazed window sized at 20\% of the floor area, using an external thermal shutter may reduce the heat-loss in the order of \(\approx 400 \, kW \cdot h \cdot y^{-1}\). The savings in carbon emissions would depend on the fuel-mix used for space-heating. Realising this potential depends upon the behaviour of the occupants, it would be reduced the more the closed shutter is overridden during the heating season.

It is difficult to quantify whether there are benefits from admitting daylight over artificial lighting on a time-basis, solely in order to achieve a level of service illumination appropriate to the accommodation’s use. A technical assessment of the economic benefits could be made about the lighting technology and its control regime. Access to daylight is directly dependent on occupant presence because of the intolerance to low levels of illumination prompts an immediate response. Either shutters are opened during daytime or a light-switch is activated. The difficulty is that the survey evidence about space utilisation and occupancy rates indicates that both tend to be low and can be highly variable (SMG, 2006).

The implications on human health from promoting visual stimulation by exposure to the seasonal variation in photo-period that is not replicated in conventional artificial lighting is also difficult to quantify. Recent work in the field of neuro-physiology from studies on humans has now established that light-sensitivity in the non-image forming parts of human vision is strongly connected to synchronising internal circadian (diurnal) as well as circannual (seasonal) rhythms (Foster, 2010). Qualitatively, this physiological link favours a shutter deployment rationale that maximises the availability of daylight, in addition to the economic benefits over using artificial light sources. A clinical diagnosis for the psychiatric condition of Seasonal Affective Disorder (SAD) is also connected to deficiencies in daylight exposure (Chap. 11 in Foster and Kreitzman, 2009). It may have positive consequences for those susceptible to the clinical symptoms of SAD to maintain synchronisation between the seasonal variation in photo-periodic and human circannual rhythms.

A design-realisation project at UCL carried out by post-graduate students of architecture in 2009 sought to demonstrate the benefits of external shutters by constructing a full-sized pavilion building. The construction of a practical demonstrator offered an opportunity to test the thermal comparator mechanism that was developed in Chapter 4 for operating a practical insulated shutter leaf system in-situ. The following section describes the project and the methodology used to observe the thermal and mechanical behaviour of the comparators.
H.1.2 Demonstration pavilion

For simplicity and low-cost, a standard freight shipping container was reused as the shell for demonstrating a set of bespoke shutters. Urban Space Management (USM) were contracted to adapt their “Container City™” system to provide the structural enclosure from a 20’ long ISO-688 dry-freight shipping container.

One of the container’s side panel of corrugated Corten® was removed and a structural steel section was welded across the head of the opening to provide stiffness. The container’s interior was refurbished in a fit-out with foam insulation and plasterboard lining to the walls and ceiling. The floor and doors were insulated and lined with timber boards on timber battens.

Nine bays of full-height single-glazed fixed-light windows were installed into the opening in a timber-framed curtain wall, with each window served by a shutter leaf. A large variety of window arrangements have been enumerated by (Hochberg et al., 2010) and the complementary options for shutter deployment by (Langdon, 1980) as shown on the left in Figure H.2 (a).

The degree of translation from linear-to-rotational motion in the realised comparator mechanism is shown on the right in Figure H.2 (b), this imposed some limitations on the options for deployment. Within these constraints the output was matched with deployment version ‘c’ in Figure H.2 (a). Specifically, each window bay’s shutter was side-hung and vertically pivoted to rotate between $\theta = (0 \text{ to } 90)^\circ$ of the glazing line. The general arrangement of the bespoke shutters is shown in Figure H.3 labelled (S1 to S9), operated by the nine comparators labelled (CS1 to CS9).

Figure H.2: Arrangements of shutter leaf deployment coordinated with mechanism

(a) Options for deployment (Langdon, 1980)   (b) Comparator mechanism translation
Figure H.3: General arrangement of pavilion showing front elevation, plan and section

Note: Original drawings by Fred Guttfield and Joseph Moorehouse of the Douglas Stephen Partnership. One of the leaves in the door-set at either end of the container was openable to allow visitors to walk-through the pavilion interior where the project was described by graphics and multi-media.
To provide ventilation, three openings above the internal space join it to a vertical stack, as labelled (V1 to V3) in Figure H.3. Air changes are driven by displacement from insolation gains that heat the internal air, the increased buoyancy allows the air to escape through the grille openings by the stack-effect. Fresh air was replenished by being drawn in through background infiltration.

The free-area available for ventilation was regulated by thermally-insulated dampers operating over each grille in the ventilation stack. The operating rationale for the dampers is to open with a proportional response to external ambient air temperature between (13 to 21) °C. This was achieved by a single-action version of the comparator mechanism as labelled (CV1 to CV3).

The repertoire of mechanical responses in the thermal comparator mechanism’s output was enumerated earlier in Chapter 4 by Figure 4.9. Having connected the output using a fixed linkage to the leaf hinge, the angles of the shutter’s deployment are shown in Figure H.4 (a to c). The following briefly describes the correspondence between thermal conditions and each output position. Under cold conditions at night the thermal comparator outputs position (a) and closes the shutter. This provides an air-seal around the perimeter of the leaf and the thermal insulation in the leaf reduces the heat-loss through the window. When thermal conditions are moderate enough to melt the wax in heat-motor (A), the comparator output’s moves to position (b) and opens the shutter to (θ = 90°). This provides the maximum access to daylight and amenity views outside.

When conditions are hot enough to melt the wax in heat-motor (B), the comparator output’s position (c) and moves the shutter to be partially open at (θ = 45°). This provides shading from sunlight by reducing the window’s acceptance angle and reduces the risk of discomfort glare. This position still admits daylight to offset the need for artificial lighting.

**Figure H.4: Positions of window shutter operated by thermal comparator mechanism**

(a) Cold conditions (Insulation)  (b) Moderate conditions (Daylight)  (c) Hot conditions (Shading)

Note: Comparator mechanism operating a shutter over a single window bay into the three different deployment positions. [Click for animation study] (WMV, 2.7 Mb).
Figure H.5: Demonstration pavilion for external deployable thermally insulated shutters

Note: Installation in the main quadrangle of University College London. Exhibited as part of the 'London Architecture Festival' in June-July 2008. Pavilion shown with the ventilation stacks temporarily removed. The project was reported in the UrbanBuzz book 2009 (pp. 108-11 in O’Rouke, 2009) and in the professional press by the Royal Institute of British Architects Journal (p. 60 in Kucharek, 2009) [Time-lapse of shutter deployment] (WMV, 6.3 Mb)
Figure H.6: Details of container and mechanism installation

(a) Shutter leaf opening over stack ventilator

(b) Close-up of comparator mechanism

(c) Interior of container during daylight study

(d) Complete set of comparator mechanisms in-situ

(e) Close-up of shutter view-glass

(f) Shutters closing at dusk during UCL show 2008
The pavilion was exhibited as part of the London Architecture Festival in 2008 and UCL’s Bartlett School of Architecture end-of-year show as shown in Figure H.5. These section have described the general arrangement of the pavilion and the deployment method for operating the insulated shutters. Moving on from the public exhibition of this project as a technology demonstrator, the following section describes the methodology used to carry out an in-situ observation study of the comparator’s behaviour.

H.1.3 Methodology of observation study

The objective of the study is to validate whether the practical thermal comparator that was developed in Chapter 4 can mechanically operate the full-size shutters into the three different deployment positions in-situ. To achieve this, an observation study is designed to collect data on the actuation displacement of the comparator mechanisms and the opening angle of the shutters on a sample of days under realistic conditions. The observations are made under passive ‘free-running’ conditions without using manual interventions or automated electrical and mechanical overrides.

A rooftop location at Trinity Buoy Wharf in London’s docklands was used to carry out this in-situ study. This allowed the pavilion’s glazed wall to be oriented due-south in a location that enjoyed largely un-obstructed views to the horizon. The flat roof has an asphalt base finished with a highly solar-reflective coating, this is atypical compared with common ground-level finishes. The implications are that there will be a higher global gain from the incident radiation flux into the vertical aperture than the standard model because of the much higher than average surface albedo. The general arrangement of the pavilion and observation instrumentation is shown in Figure H.7 with summary annotations.

The total duration of the study spanned from late July 2008 until September 2009. However, owing to obstacles in setting up and time calibrating the sensors, the completed system could only record data for a total of 88 days during the summer of 2009 between 11th May to 7th August 2009. Measurements from sensors were recorded for the behaviour in three out of the nine comparator enclosures that were constructed, time-lapse imaging was limited to (< 20 days).

The monitoring campaign captured the context of the prevailing meteorological conditions during each test day that leads to the resultant actuation displacement length exhibited by each heat-motor inside the comparator enclosures. The results are drawn from this body of data with the observation days filtered by clear-sky conditions defined by exceeding the threshold of \( Y \geq 67\% \) in the daily accumulated insolation yield.

The comparators were observed operating the shutters on a total of 28 days, of these 23 days were single-action responses and 5 days were dual-action responses. An set of operating profiles were obtained for both types of response, they show the pattern of actuation displacement in both heat-motors over a complete diurnal cycle. Having outlined the methodology for the observation study, the following section gives a summary of the results.
Figure H.7: General arrangement of pavilion and instrumentation for observation study

(a) Pavilion facing due-south on roof-top in “Container City”
(b) Autonomous monitoring system

1. Pyranometer: Total global horizontal solar radiation $G_r = (0 \text{ to } 1500)$ in $W \cdot m^{-2}$
2. Radiation shielded thermoresistive sensor: External ambient air temperature $T_{db} = (-10 \text{ to } 40)$ in $^\circ$C
3. Intervalometer controlled digital imaging: External and internal camera stations (10 min) intervals
4. Thermoresistive sensor: Air temperature inside comparator enclosures $T_{enc} = (-10 \text{ to } 150)$ in $^\circ$C
5. Resistance potentiometers: Heat-motor actuation displacement length $L = (0 \text{ to } 150)$ in $mm$
6. Digital compass: Shutter leaf deployment angle $\theta_s = (0 \text{ to } 90)$ in $^\circ$

Note: Site at Trinity Wharf in London (UK) geo-coordinates Latitude $51^\circ30'29''N$ by Longitude $0^\circ0'53''W$, Altitude 24 m a.m.s.l to the WGS 84 datum. [Time-lapse] (WMV, 3.6 Mb)
H.1.4 Results for comparator exhibiting a single-action response

The prevailing meteorological conditions on this observation day are shown in the top-half of Figure H.9. Specifically, the diurnal insolation and ambient temperature profiles are plotted, the lower-half of Figure H.9 plots the duty-cycle response of the comparator’s two heat-motors.

Comparing the measurements of instantaneous solar radiation with a model for clear-skies by Haurwitz (1945) shows that conditions were bright and largely cloudless in the morning, from noon onwards there was some intermittency in sunlight. Under these conditions, insolation levels fluctuated and occasionally exceeded the model for clear-skies by (10 to 20) %. This is thought to be due to the reflectance off the surfaces between columns of clouds (Muneer, 1997) leading to an increased apparent area \( A \) of the sky in Stefan’s relation \( q_{rad}'' \propto A \cdot T^4 \).

A comparison between the temperature of external ambient air and the thermal comparator’s internal enclosure as plotted in Figure H.9 shows that while ambient remains below the critical point by (1 to 9) °C of the wax in the heat-motor, the balance of internal energy is tipped in favour of phase-change by absorbing insolation. The response was for heat-motor \( A \) with the low-temperature melting-point wax to exhibit a complete duty-cycle within the diurnal cycle. The time-lapse images confirm that the comparator’s output did deploy the shutter mechanism with a single duty-cycle. Contrasting shutter positions are shown in Figure H.8 (a and b).

Figure H.8: The operation of the thermal comparator over a diurnal cycle on a warm day

(a) 04:55 hrs (UTC) ambient \( T_{db} = 19 \) °C
(b) 16:15 hrs (UTC) ambient \( T_{db} = 28 \) °C

(Left-hand image) Shows the thermal comparator driving the shutter closed at dawn. (Right-hand image) Shows the thermal comparator driving the shutter fully open in the afternoon. Observations at Trinity Wharf, London (UK) on 25th July 2009 [Time-lapse study] (WMV, 3.4 Mb)
Figure H.9: Diurnal time-series of comparator operating shutter with single-action

Note: Based on observations made on the 22nd May 2009 during a warm summer day with prolonged sunny spells. Trinity Buoy Wharf, London (UK)
H.1.5 Results for comparator operating shutter with a dual-action response

The results from an observation made on a hot day in mid-summer are shown in Figure H.11 using the same format as Figure H.9 shown previously. Comparing the measured solar radiation with the model of a clear-sky during the whole photo-period shows a very good fit, indicating that conditions were nearly cloudless. The temperature of ambient air did exceed the critical-point of the low melt-point wax in heat-motor (A) within the photo-period. However, the temperature of ambient did not exceed the high melt-point wax in heat-motor (B) within the diurnal cycle.

The lower-half of Figure H.11 shows that the insolation gains made inside the comparator enclosure did raise the temperature above ambient and exceeded the high melt-point of the wax in heat-motor (B). The time-series of actuation displacement in heat-motor (A) and (B) shows that both underwent a complete duty-cycle. The sequence of the comparator’s output to operate the shutter deployment is annotated against this time-series of actuation displacement.

In principle, this verifies the design rationale because the shutter closes to provide insulation during the night, opens to access daylight during the photo-period and provides shading protection during the hottest part of the day. The time-lapse images observing shutter (S2) confirm the evolution of the shutter’s dual-action deployment. A sequence of stills of the deployment positions are shown in Figure H.10 (a to c).

Having given a descriptive presentation of the two different comparator responses, the following section gives general observations on the duty-cycle pattern for all the observations made under (Y ⩾ 67 %) conditions during the monitoring campaign. This is followed by a brief discussion of the limitations of this pilot-study.

Figure H.10: The operation of the thermal comparator over a diurnal cycle on a hot day
Figure H.11: Diurnal time-series of comparator operating shutter with dual-action

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External ambient air temperature ($T_{db}$)
 Insolation modelled under clear-sky ($G_{c,H}$)
 Apparent sky temperature ($T_{sky}$)
 Global radiation flux on horizontal plane ($G_{i,H}$)

---

Air temperature in enclosure ($T_{enc}$)
 Heat-motor ($A$) wax critical-point ($T_{mp}$)
 Actuation displacement ($l_a$) of heat-motor ($A$)
 Actuation displacement ($l_b$) of heat-motor ($B$)

---

Note: Based on observations made on the 2\textsuperscript{nd} June 2009 during a hot summer day with continuous sun. Trinity Buoy Wharf, London (UK)
H.1.6 General observations

On most days during the observation study, neither heat-motor exhibited a duty-cycle that was symmetrical about solar-noon. Both were shifted to later in the day, this can be attributed to the time-lags during the phase-transitions of the wax charges. Across all the observation days in both heat-motors, the rate of the rising-edge during melting tended to be faster than the falling-edge during solidification. This applied both to the blended wax in heat-motor (A) as well as the narrow melt-point wax used in heat-motor (B).

The time-lag of the falling-edge in heat-motor (A) tended to extend well beyond the end of the photo-period. This can be attributed to the low melting-point relative to the typical range of ambient temperatures of the blended wax used in heat-motor (A), so the value of \( \Delta T \) remained relatively small leading to a low rate of heat rejection. The components in the lab work comparator were not thermally isolated so the metal components embodied a substantial heat-sink \( m \approx (12 \text{ to } 15) \text{ kg} \), this lent additional thermal inertia to the rate of heat rejection.

H.1.7 Limitations and the concept-design for a binary-latch

At the time the practical work was carried out there was some uncertainty about the exact actuation displacement that the heat-motors would exhibit under in-situ conditions. This was directly related to the lack of a suitable model and supporting simulation data to use in order to estimate what the range of stagnation temperatures would be attained inside the comparator’s enclosure.
It proved difficult to align the drive bar to a well-defined ‘Open’ and ‘Closed’ position, and do so consistently for each comparator mechanism. Even under controlled ‘Steady state’ conditions in the thermal laboratory, the practical preparation of each motor’s mounting was problematic and it proved difficult to make fine adjustments.

The data for days with single-action response showed that the shutter did not fully deploy to the open position \((\theta \leq 90^\circ)\). This is attributed to the mechanical problem of aligning the zero-offset between the motor’s drive-rod and hinge-line.

The data for days with dual-action response showed that the shading response was found to over-compensate and nearly close the shutter leaf instead of reaching the intended \((\theta = 45^\circ)\) from the glazing-line. There are several reasons that may have contributed to this. Firstly, this is partly attributed to the problem of aligning the zero-offset for the heat-motor \((B)\) consistently.

Secondly, the cylinders used for the high melt-point temperature wax were slightly modified during preparation which increased the cavity size. Therefore, in practice heat-motor \((B)\) was charged with slightly more wax by weight than heat-motor \((A)\). In addition, the end-stops used in the test did not restrict the linear-motion of the mechanism as much as necessary to constrain the deployment angle to \((\theta = 45^\circ)\).

The wide temperature range of melting in the blended wax meant that on days with intermitted insolation when the ambient temperature was close to the critical-point heat-motor \((A)\) exhibited ‘signal hunting’ behaviour as shown in Figure H.12. The impact on the resultant output \((Q)\) was to oscillate the shutter’s position, this may be deemed undesirable to maintain access to daylight. The oscillations could have been compensated for with a mechanical binary-latch to hold the output of heat-motor \((A)\) in the open position until it completely solidified when it could self-reset. This invariably occurred after the end of the photo-period. The following considers the design issues and possible benefits of such a mechanism in outline.

The concept for a simple mechanism to dampen the output oscillations is shown in Figure H.13, it reuses the proprietary heat-motor already described. Figure H.13(a) shows the assembly of a single heat-motor with Figure H.13(b) showing it disassembled at the component-level of detail. The mechanism addresses two of the limitations identified here, firstly it decouples the heat-motor’s stroke-rod from the resultant output, this is the main difference compared with the machines explored in the case-studies that used a direct and proportional connection. Secondly, it maintains a stable output state \((Q)\) by dampening the oscillations due to ‘signal hunting’ behaviour in the heat-motor by using a mechanical binary-latch.

The operation of this mechanism is shown in Figure H.14 as a sequence on a day when the heat-motor duty-cycles and its output also oscillates due to intermittent cloud turbidity. To orient the reader, the diurnal cycle starts at the top of the page going down. The state of the heat-motor’s actuation is indicated by the ‘Drive block’ (component 5 in Figure H.13b) and the resultant output \((Q)\) by the position of the circle on the left-hand side.
Figure H.13: The 'self-latching hold-open and automatic release' mechanism

(a) Model of the assembled mechanism showing the heat-motor active and latched open

(b) Model of the mechanism showing the components disassembled

(1) Output stroke-rod and indicator
(2) Return spring for output rod (compression)
(3) Return spring for drive rod (compression)
(4) Transfer block for output
(5) Drive block
(6) Latch strike
(7) Latch spring (opening lever type)
(8) Latch trigger
(9) Thermal actuator stroke rod
(10) Reference frame for output actuation
(11) Thermal actuator assembly
(12) Glazed solar collector enclosure
Figure H.14: Heat-motor oscillations dampened by a latching mechanism

Step | Output | Transfer-mechanism
--- | --- | ---
1 |  |  
2 |  |  
3 |  |  
4 |  |  
5 |  |  
6 |  |  
7 |  |  
8 |  |  
9 |  |  
10 |  |  
11 |  |  
12 |  |  
13 |  |  
14 |  |  
15 |  |  

Note: Animation of mechanism operation [Animation] (WMV, 1.8 Mb)
A brief description through the sequence in Figure H.14 follows, at the start of the photo-period the wax is assumed to be solidified and the heat-motor closed (Step 1). Accumulated gains from solar radiation melt the wax to drive the heat-motor open (Steps 2 to 5) when the ‘Drive block’ engages with the ‘Output transfer block’. This operates a mechanical latch to hold-open the output when the heat-motor reaches the fully open position (Step 6).

Once the latch’s strike is spring-loaded into the set position (Step 6), the output \((Q)\) is held in the logical ‘On’ position until the heat-motor is fully deactivated (Step 15). The implication of the mechanism’s latching feature is that until then, the heat-motor’s actuation displacement may oscillate independently (Steps 6 and 10). The latch will only be released when the heat-motor undergoes a full recovery stroke and releases the trigger in (Steps 13 to 15). The latch then automatically frees the spring-loaded output to return to the logical ‘Off’ position (Step 15).

A simulation that shows how the resulting output \((Q)\) from this binary-latching mechanism would modify the ‘as measured’ actuation displacement of heat-motor \((A)\) is indicated by the dark dotted trace in Figure H.12, the output from heat-motor \((A)\) is taken from the observation study when conditions of intermittent cloud cover prevailed.

This “self-latching hold-open and automatic release” mechanism operates passively without the need for an electro-mechanical latch and strike mechanisms operated by a solenoid or similar externally powered device or control logic. The advantages of developing this mechanism would be to maintain the output \((Q)\) in a stable state by dampening oscillations in the face of moderately changeable synoptic meteorological conditions. In addition, decoupling the latched mechanism’s direct drive from the heat-motor would isolate it from dynamic loads such as the non-trivially high wind loads on the thermal shutters experienced during the observation study. A further advantage of decoupling the ‘Drive block’ from the ‘Output transfer block’ is to accommodate any mechanical overruns in the actuation displacement that are due to attaining temperature above the melting-point rather than using mechanical end-stops.

**H.1.8 Summary**

The objective of this pilot-study was to demonstrate the feasibility of the comparator mechanism to operate the full-size shutter leaves. The results presented in this Appendix have verified that the practical comparator mechanism can perform under *in-situ* conditions. It also showed that the comparator can distinguish between single and dual-action responses, although there were mechanical problems with the linear travel limits that prevented fully open deployment and over-compensated the shading position.

**Post-script**

This demonstration project was widely reported in the professional press including the Architects Journal and the Royal Institute of British Architects Journal, these articles are reproduced in the following sections H.1.8 and H.1.8.
Article in the Architect’s Journal

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Appendix I

I.1 Pilot study of dual hot-boxes

This Appendix contains details of the methodology adopted to carry out a year-long observation study on the behaviour of a practical dual hot-box system. The results of the study are reported in Chapter 7. A brief discussed is given of the design issues surrounding the instrumentation system that was developed to meet the campaign’s objectives.

This Appendix presents detailed plots of diurnal profiles for the passive responsive behaviour of the heat-motors in each of the hot-boxes, these show a sample of days from across the year-long study. This is followed Tabulated summary of the key parameters for each observation day.

Objectives of observation study

The aim of the monitoring campaign was to collect numerical data showing the characteristic duty-cycle behaviour of the heat-motors inside two hot-boxes at different inclination angles. To establish whether or not a seasonal response occurred, the monitoring campaign spanned across a year with the aim of gathering as many days across a year with clear-skies as possible.

The data served to test the complementary numerical model and FEM simulation that has been developed to describe the heat-motor’s behaviour in-silica. To verify and validate the efficacy of the theoretical studies, the observations needed to monitor the seasonal differences in the ambient environment and monitor the seasonal variation of the solar radiation that is available. The following describes the design of a system to meet these different objectives.

(a) Hot-boxes set-up on observation study test-stand (b) Close-up into vertically-inclined aperture
I.1.1 Monitoring campaign

This section focuses on the design and specification of the instrumentation that was used to monitor the system’s variables. These can be divided into those for the hot-box components including the heat-motor, and those of the prevailing ambient environment.

**Hot-box environment**

The air temperature inside each hot-box was monitored with a Pt 100-type thermoresistor. These were mounted on thermally isolated brackets to minimise conductive transfers. To minimise the heating effects from direct irradiation, they were shielded by foil screens. These also reduce the emissive losses from the thermoresistor at long wavelengths to the aperture’s inner surfaces.

To obtain time-temperature response curves in the wax, the method developed in Chapter 3 to measure the core temperature inside the cylinder is reused. The platinum thermoresistors were to BS EN IEC 60751 Class B to give a tolerance of \( T = \pm 0.30^\circ C \) and were calibrated in an ice-bath to \( T = 0^\circ C \) prior to installation. The temperature of the cylinder surface and glass cover was measured with miniature K-type bead thermocouples that were surface-mounted using adhesive mats. Details of in-situ installation are shown in Figure I.2.

**Figure I.1: Measurement of the displacement length from motor actuation**

(a) Spur pinion gear and linear rack transducer

(b) Testing and calibrating heat-motor actuation

**Figure I.2: Surface thermometry in the hot-box with miniature thermocouples**

Note: Thermocouple specification and installation, datasheet © RS components 2009. [Time-lapse observation in-situ] (WMV, 26.9 Mb), Demonstration test [Time-lapse study] (WMV, 3.3 Mb)
The actuation displacement of the heat-motors was measured with a proprietary linear motion transducer. A rotary potentiometer was attached to a spur gear running along a linear track. Figure I.1 shows the arrangement during testing. This transducer measured displacements of $l_{\text{max}} = (0 \text{ to } 150) \text{ mm}$ with a precision of $(l_q = \pm 1 \text{ mm})$. The potentiometer resistance versus displacement was calibrated using time-lapse imaging through the hot-box’s aperture. During the twelve month monitoring campaign the carbon-track type potentiometers did suffer some degradation that led to an increasingly noisy output signal.

**The prevailing ambient environment**

The minimum requirement for verifying the thermal rationale model is to obtain time-series data for $(T_c, h)$ and $(T_c, v)$. However, the compelling reason to monitor the prevailing conditions is to capture the operating context faced by the hot-boxes that drives $(T_c, h)$ and $(T_c, v)$. This determines the criterion for the selection of sensor type, location and rate of data acquisition.

The radiation exposure at each inclination was monitored using a silicon pyranometer. The total global instantaneous short-wave solar irradiation into the horizontal and vertical hot-box apertures was measured independently in the range $G_H, G_V = (0 \text{ to } 1500) \text{ W} \cdot \text{m}^{-2}$. The temperature of ambient air was measured with a platinum thermoresistor in a ventilated radiation shield raised 3.5 m from the ground as shown in Figure I.32(a).

While the model attempts to predict the timing of the actuation duty-cycle, the monitoring campaign has to account for whenever it actually occurs regardless of how premature or belated this may prove to be. The practical difficulty with this is that when each phase-change occurs it may take only minutes and seconds to complete. Drawing on experience from the earlier pilot studies, the rate-of-change in insolation and the typical thermal inertia of the components suggested that a balance between capturing the variation in the variables with the most rapid rates of change and recording capacity, a data acquisition interval of $(t = 30 \text{ s})$ was selected.

It is a different problem to achieve a temporal correspondence between the data recording interval used in the observation study and the time-steps in the simulation of the thermal model. The time interval in the FEM simulation is determined by the stability of the solution to the differential equations whereas the observation study needs to capture the fastest rate-of-change in a variable with a magnitude that affects the solution. To determine what values will achieve this balance *a priori* presents a challenge. During the development of the simulation, a time-step of $(\Delta t = 0.05 \text{ s})$ was found to be the maximum interval that maintained stability and allowed convergence on a solution with an acceptably small error value. Ultimately, this is determined by the need to accommodate whichever variable exhibits the fastest rate-of-change.

We can see from the steepness of gradients in the evolution of insolation in section 5.2 that frequent sampling is needed to validate the energy accumulated from insolation. Instrument data from all the logger channels is recorded at intervals of $(t = 30 \text{ s})$. Thus each diurnal test-day has 2880 data-points.
I.1. Pilot study of dual hot-boxes

The optical surface properties of the materials

In theory, all the surfaces of the hot-box system are a source of 'Black body' radiation emission, even while exposed to the incoming solar radiation. For surfaces with a view-factor toward one another, any difference in temperature leads to an exchange of heat-energy by radiation. The intensity of ‘Black body’ emission is not spectrally continuous, this has implications for modelling the dynamics of this system for which the following assumptions were made.

While the sun’s peak emission occurs at short wavelengths $\lambda_p = 0.5 \mu m$ due to the high surface temperature of the sun’s photo-sphere ($T_\text{sol} = 5770 K$) (p. 81 in Norton, 1989), the temperature attained in the hot-boxes is much cooler. This leads to the peak ($\lambda_p$) in the spectra of the emissions to be at a longer wavelength. Its value can be determined empirically from the surface temperatures ($T$) recorded during the observation campaign and applying Wien’s Law using Equation I.1. The pilot-study found surface temperatures were (246 to 354) K that means the ‘Black body’ radiation emitted by internal surfaces had a peak wavelength of $\lambda_p = (8.2 \text{ to } 11.8) \mu m$, as summarised in Table I.1. There is almost no overlap between the incoming insolation and the outgoing hot-boxes emission as shown on the left in Figure I.3.

\[
\text{Peak wavelength } \lambda_p = \frac{w}{T_{\text{sur}}} \{\text{in } m\} \quad (\text{pp. 737-8 in Incropera and DeWitt, 2007}) \quad (I.1)
\]

$w$ Wien’s constant (pp. 737 in Incropera and DeWitt, 2007) 0.002898 \{in m \cdot K\}

$T_{\text{sur}}$ Temperature of surface \{See Table I.1\} \{in K\}

In reality, the heat-motor’s surface is not a perfect ‘Black body’ emitter which leads to an uncertainty in estimating its emissivity at long wavelengths. The heat-motor’s ‘Black body’ radiation can be imaged as shown in Figure I.4 (a) (the matching view-point in Figure I.4 (b) is shown only to orient the reader) using a bolometer with its sensitivity tuned to the same range of wavelengths that the surface radiates its peak emissive power. In practice, it proved too difficult to calibrate this image against the direct surface measurement for the actual aluminium alloy with a surface layer of natural atmospheric oxidization ($Al_2O_3$). In the range between an ideal ($\varepsilon_c = 1$) and a poor ($\varepsilon_c = 0$) approximation of a ‘Black body’ source, the value of ($\varepsilon_c = 0.3$) was used.

The optical properties of glass are spectrally selective, it transmits most incoming short wavelength radiation from the sun but obscures most outgoing long wavelength ‘Black body’ radiation from the cylinder. The dynamics of heat-transfer by ‘Black body’ radiation between the cylinder and the back of the glass aperture cover are determined by the emissivities of the respective surfaces at the peak wavelength ($\lambda_p$). This is central to heat-rejection during the solidification phase of the duty-cycle.

The subsequent process is the heat-rejection from the outer surface of the glass aperture cover by radiation exchange with the apparent sky vault and surrounding surfaces. The following section describes the method used to survey the characteristics of the two view-factors seen from each of the hot-box apertures.
Table I.1: Observed surface temperatures and peak wavelengths of ‘Black body’ radiation

<table>
<thead>
<tr>
<th>Ambient Effective sky Temperature¹</th>
<th>External hot-box apertures</th>
<th>Internal hot-box surfaces</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Horizontal glass cover</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Vertical glass cover</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Heat-motor (B)</td>
<td>Heat-motor (A)</td>
</tr>
<tr>
<td>T_{sky} (K)</td>
<td>T_{g,h} (K)</td>
<td>T_{c,h} (K)</td>
</tr>
<tr>
<td>λ_p (µm)</td>
<td>λ_p (µm)</td>
<td>λ_p (µm)</td>
</tr>
<tr>
<td>Max 292</td>
<td>10</td>
<td>330</td>
</tr>
<tr>
<td>Min 246</td>
<td>12</td>
<td>258</td>
</tr>
</tbody>
</table>

Notes: From observations made under (Y \geq 67\%) clear-sky conditions during (2011 to 2012) in London (UK), Latitude 51.5° N. The effective temperature of the sky is tabulated from data collected during the monitoring campaign with a Pyrgeometer sensor at the Department of Meteorology by the University of Reading (Reading, 2012).

Figure I.3: Spectra of solar radiation and ‘Black body’ radiation

Note: The spectral distributions shown in graph (a) is for ‘Black body’ radiation from the range of temperatures attained by the surface of the heat-motor cylinder and hot-box enclosures, graph (b) is for solar irradiation. From (p. 2607 in Granqvist, 1981)

Figure I.4: The ‘Black body’ radiation from the surface of a Bayliss heat-motor

(a) Image taken in the thermal band (λ_p = 10 µm)  
(b) Image taken at visible wavelengths (photopic)

Note: Thermal image taken using a FLIR™ bolometer with a peak sensitivity at (λ_p = 10 µm)
Methodology for imaging the aperture’s view-factor

The objective of this method is to obtain the sky view-factor ‘seen’ by each aperture. This supports accounting for the position and elevation of any site-specific obstructions that block direct sunlight from reaching either aperture. The results are used to make adjustments to the model of solar radiation exposure. This section describes a technique to carry out an in-situ survey by reconstructing the scene from an image of a mirror-ball’s reflection.

The effectiveness of this technique depends upon the accurate coordination between camera, mirror ball, hot-box aperture and surrounding scene. Ideally, the mirror-ball is positioned in the centre of each aperture with the camera’s nodal-point and optical axis aligned plumb along the zenith-to-nadir axis as well as coincident with the centre of the mirror-ball. In order to superimpose the coordinates of the sun’s path onto the reflection, a sufficient number of ground truths are needed to fix the six degrees of freedom. One or more reference points are needed of known azimuth angle in order to coordinate the rotation axes with the sun’s path across the sky.

In practice, it proved difficult to align the camera’s optical axis plumb above the nadir point as well as coincident through an opaque sphere. Without this, accurate measurement of the reflections from the sphere in the context of solar obstructions would be hindered. To overcome this, a three-dimensional target was made with the same dimensions as the mirror-ball to set-up the camera. All of the alignment indicators needed could be drawn or applied to the target as shown in Figure I.5 (b and d). This also provides a platform to project a LASER to mark reference points onto the surrounding scene. Once the target has been leveled and aligned to a cardinal point with a compass, points were marked in the scene at known elevation and azimuth angles.

Once the target is substituted with the ‘mirror ball’, the marked points are reflected from known positions and can be used as ground-truths for orientation and elevation. The \((X-Y)\) axes in the image can be rotated about the sphere’s centre to align with the cardinal North-point simply using the magnetic compass’s needle seen in the image. The local deviation between magnetic and geographic grid North was accounted for by referring to isogonic charts specific to the site. Two sources were compared, the UK’s isogonic chart (BSG, 2011) for magnetic declination and NASA’s global model. For the relevant Epoch, the difference was found to be small at between \((+2°44′′W)\) and \((+1°36′′W)\) from the two sources respectively for July 2012.

Once the coordination and alignment has been successfully established, it is straightforward to plot the sun’s path across the sky by using angle projection from a time-series of the sun’s position in coordinates of azimuth and altitude. Likewise, any obstructions to the direct sunlight can be accounted for by subtracting the outline of the scene from the sun’s path in the image. The same technique was applied in the IR-band to survey the intensity of thermal emissions and the corresponding view-factor they present to each aperture. However, swapping out cameras required markers that have a high-contrast in the IR-band were applied to the target to aid framing and focusing the thermal imaging equipment.
I.1. Pilot study of dual hot-boxes

This methodology could be applied using the mirror-ball as a proxy for the hot-box’s aperture to survey prospective sites. A bi-directional link between the model and its simulation could be interactive with the output from the image analysis to evaluate the predicted pattern of duty-cycles for numerous potential locations and aperture orientations.

To obtain an orthographic projection, the camera would ideally image the sphere with parallel lines of reflection using a long focal-length. The limitation implementing this method with the available equipment was the difficulty imaging a good approximation of this due to the moderate focal-lengths of both the thermal (15 mm) and visible cameras (10 mm). Compensations could be made using a projection sheet, however, a much higher resolution was needed in the thermal than could be obtained with the available equipment.

Figure I.5: Method for aligning the camera’s optical axis with the sphere’s centre

(a) Reflection image from mirror-ball (IR-band)  
(b) Target for aligning camera (IR-band)

(c) Reflection image from mirror-ball (visible-band)  
(d) Target of equal dimension to ball
I.1.2 Notes on comparison between simulated vs observed

The data-reduction applied to visualise the results in Chapter 7 obscures some of the insights gained into the evolution of the key variables, this section shows the hot-box’s operating context in order to relate the actuation displacement exhibited by each heat-motor with the corresponding exposure to solar radiation and the prevailing environment. Diurnal profiles comparing the ‘as predicted’ with the ‘as measured’ results for each operating variable: insolation, temperature and actuation displacement are plotted in Figures I.6 to I.31 for each Day Number (dn) as six concurrent time-series.

To orient the reader, each (dn) is divided into two columns, with data for heat-motor (B) in the horizontal hot-box on the left and data for heat-motor (A) in the vertically-inclined hot-box on the right. These should be read from the top row downward, firstly the profile of solar radiation intensity that is incident on the aperture, then its effects on the cylinder’s surface temperature and finally the actuation displacement exhibited by the motor.

Solar radiation - Notes

Reading the top right-hand corner in Figures I.6 to I.31 shows that the fit between ‘as measured’ and ‘as predicted’ for total global solar insolation level is good across all (dn’s). On days with intermittent sunshine, the ‘as measured’ exceeds the predicted due to an increase in the effective area of the sky and cloud reflections. There are also some sharp drops in the level of intensity from the obstructing trees in the morning and a building in the evening.

Reading the top left-hand corner in Figures I.6 to I.31 for the vertical plane, the measurements are generally slightly lower and substantially less during winter months at low elevations when the ‘as measured’ falls approximately 20 % below the ‘as predicted’. This over-estimate was not expected because the model ignored the ground-reflected component and the surface of the observation site is mostly grass assumed to have a modest albedo of $r = 0.3$ (Based on data in Oke, 1987).

Calculation of daily insolation yield

The daily insolation ‘Yield’ ($Y_i$) for an observation day has been calculated using Equation (I.2). Yield for a given Day Number (dn) is the difference between the measured integral of accumulated insolation per day and the predicted expressed as a percentage.

$$Y_i = \frac{1}{h} \int_{t_0}^{t_1} \frac{G_i}{G_c} \cdot t \, \{\text{in } \%\} \quad (I.2)$$

Where:

- $G_c$ Instantaneous solar irradiation modelled on a horizontal plane under a clear-sky (After Haurwitz, 1945)
- $G_i$ Instantaneous solar irradiation measured on a plane
- $t$ The sample interval
- $t_0, h$ Start of exposure at dawn
- $t_{1, h}$ End of exposure at dusk

$$Y_{i, dn} = \frac{1}{h} \int_{t_0}^{t_1} \left( \frac{G_i}{G_c} \right) \cdot t \, \{\text{in } \%\} \quad (I.2)$$

Where:

- $G_c$ Instantaneous solar irradiation modelled on a horizontal plane under a clear-sky (After Haurwitz, 1945)
- $G_i$ Instantaneous solar irradiation measured on a plane
- $t$ The sample interval
- $t_0, h$ Start of exposure at dawn
- $t_{1, h}$ End of exposure at dusk
I.1. Pilot study of dual hot-boxes

A yield approaching \( Y_i = 100\% \) indicates nearly cloud-free conditions throughout the photo-period. Given the maritime climate experienced by the British Isles, not all days during the monitoring campaign enjoyed perfectly clear-skies throughout each photo-period. As stated at the outset, due to the difficulties of modelling the stochastic effects of cloud turbidity on the simulation of the first-order model, validation is restricted to observations made under approximately clear-skies.

While few days were expected to approach perfect conditions \( Y_c = 100\% \), the difficulty with qualifying a threshold value for filtering the data-set was distinguishing what part of the deficiency \( Y_c - Y_i < 100\% \) was attributed to site obstructions that blocked the sun’s direct beam and what part the effects of cloud turbidity. Both of these are variable for a given \( (dn) \).

Fortunately, the research engineer made a manual record from a visual check on the amount of cloud cover for each \( (dn) \) in a diary of the whole observation study. This was drawn upon to give \( (Y_i \geq 67\%) \) as an approximate threshold for clear-sky conditions. Applying the filter to the observation study data selected twenty-six days spread evenly across the seasons.

Temperature - Notes

Reading the middle rows in Figures I.6 to I.31 across all \( (dn's) \), this shows that the cylinder's surface temperature has time-lags during phase-transitions that are asymmetrical unlike the 'as predicted'. Specifically, the rate of temperature rise during the melting process in the morning is more rapid than the rate of solidification processes during the afternoon and early-evening. The solidification was observed to take much longer than the simulations predicted. This is consistent both for hot-boxes at either inclination.

It is interesting to note that close scrutiny of the 'as measured' time-series for the cylinder's surface temperature in Figures I.6 to I.31 reveals supercooling during solidification. The evidence for this is the small dip below the wax’s melting-point just as the curve’s falling-edge reaches the melting-point temperature. This phenomenon occurs on nearly every day when a heat-motor undergoes a phase-transition. Since the FEM model and its simulation does not account for either super-heating or supercooling effects, this phenomenon is absent from the 'as predicted' curves.

Actuation

The heat-motor in the vertical hot-box did actuate every day as predicted. Except during the months (5 to 7) due to a gradual mechanical problem with the seal when actuation was partial. However, the rising-edge was consistently later than predicted by \( (2 \text{ to } 4) \) hrs.

The heat-motor in the horizontal hot-box did actuate on a seasonal basis as predicted, although extending before and after the start and end cut-offs that were predicted. The start of the rising-edge was consistently \( (1 \text{ to } 2) \) hrs earlier than predicted.

In both horizontal and vertically hot-boxes, the 'as measured' variability in actuation displacement above the phase-change threshold from excessive temperature rises is more pronounced than predicted. This suggests that the coefficient of expansion for wax in the liquid-state using in the FEM may be slightly lower for \( n \)-octadecane that reported in the literature.
Figure I.6: Diurnal evolution on Day 006 in winter: Observation on 6th Jan 2012

(a) Solar radiation exposure into aperture at short wavelengths ($\lambda_p = 0.5 \mu m$)

(b) Profile of thermal actuator’s surface temperature

(c) Resultant motor actuation displacement

Notes: Start+159 days. Cold, clear conditions to start.
Figure I.7: Diurnal evolution on Day 027 in Winter: Observation on 27th Jan 2012

(a) Solar radiation exposure into aperture at short wavelengths ($\lambda_p = 0.5 \mu m$)

(b) Profile of thermal actuator's surface temperature

(c) Resultant motor actuation displacement

Notes: Start+180 days. Bright but cold.
I.1. Pilot study of dual hot-boxes

Figure I.8: Diurnal evolution on Day 042 in Winter: Observation on 11th Feb 2012

(a) Solar radiation exposure into aperture at short wavelengths ($\lambda_p = 0.5 \mu m$)

(b) Profile of thermal actuator’s surface temperature

(c) Resultant motor actuation displacement

Notes: Start+195 days. Very cold ground frost, laying snow, clear sky start.
I.1. Pilot study of dual hot-boxes

Figure I.9: Diurnal evolution on Day 057 in Winter: Observation on 26th Feb 2012

(a) Solar radiation exposure into aperture at short wavelengths ($\lambda_p = 0.5 \mu m$)

(b) Profile of thermal actuator’s surface temperature

(c) Resultant motor actuation displacement

Notes: Start+210 days. Cold night, clear skies start.
I.1. Pilot study of dual hot-boxes

Figure I.10: Diurnal evolution on Day 065 in Spring: Observation on 5th Mar 2012

(a) Solar radiation exposure into aperture at short wavelengths ($\lambda_p = 0.5 \mu m$)

(b) Profile of thermal actuator's surface temperature

(c) Resultant motor actuation displacement

Notes: Start+218 days. Cold, windy, some cloud bright spells.
I.1. Pilot study of dual hot-boxes

Figure I.11: Diurnal evolution on Day 075 in Spring: Observation on 15th Mar 2012

(a) Solar radiation exposure into aperture at short wavelengths ($\lambda_p = 0.5 \mu m$)

(b) Profile of thermal actuator’s surface temperature

(c) Resultant motor actuation displacement

Notes: Start+228 days. Cold start but clearing clear sky to lunch. Closest test-day to the date of spring equinox (equal nights).
Figure I.12: Diurnal evolution on Day 097 in Spring: Observation on 6\textsuperscript{th} Apr 2012

(a) Solar radiation exposure into aperture at short wavelengths ($\lambda_p = 0.5 \mu m$)

(b) Profile of thermal actuator’s surface temperature

(c) Resultant motor actuation displacement

Notes: Start+250 days. Bright start but cold.
I.1. Pilot study of dual hot-boxes

Figure I.13: Diurnal evolution on Day 107 in Spring: Observation on 16th Apr 2012

(a) Solar radiation exposure into aperture at short wavelengths ($\lambda_p = 0.5 \mu m$)

(b) Profile of thermal actuator’s surface temperature

(c) Resultant motor actuation displacement

Notes: Start+260 days. Bright start cold.
I.1. Pilot study of dual hot-boxes

Figure I.14: Diurnal evolution on Day 121 in Spring: Observation on 30<sup>th</sup> Apr 2012

(a) Solar radiation exposure into aperture at short wavelengths ($\lambda_p = 0.5\,\mu m$)

(b) Profile of thermal actuator's surface temperature

(c) Resultant motor actuation displacement

Legend:
- Green line: Observed $G$, $T$, and $L$ for heat-motor (A)
- Green dashed line: Simulation of models for $G$, $T$, and $L$
- Red line: Observed $G$, $T$, and $L$ for heat-motor (B)
- Red dashed line: Simulation of model for $G$, $T$, and $L$

Notes: Start+274 days. Bright warm, some occasional cloud.
Figure I.15: Diurnal evolution on Day 133 in Spring: Observation on 12th May 2012

(a) Solar radiation exposure into aperture at short wavelengths ($\lambda_p = 0.5 \mu m$)

(b) Profile of thermal actuator’s surface temperature

(c) Resultant motor actuation displacement

Notes: Start+286 days. Bright start, some clouds pm.
I.1. Pilot study of dual hot-boxes

Figure I.16: Diurnal evolution on Day 151 in Spring: Observation on 30th May 2012

(a) Solar radiation exposure into aperture at short wavelengths ($\lambda_p = 0.5 \mu m$)

(b) Profile of thermal actuator’s surface temperature

(c) Resultant motor actuation displacement

Notes: Start+304 days. Warm, mixed overcast.
I.1. Pilot study of dual hot-boxes

Figure I.17: Diurnal evolution on Day 165 in Summer: Observation on 13th Jun 2012

(a) Solar radiation exposure into aperture at short wavelengths ($\lambda_p = 0.5 \mu m$)

(b) Profile of thermal actuator's surface temperature

(c) Resultant motor actuation displacement

Notes: Start+318 days. Clear bright am, cloudy still bright pm.
Figure I.18: Diurnal evolution on Day 177 in Summer: Observation on 25th Jun 2012

(a) Solar radiation exposure into aperture at short wavelengths ($\lambda_0 = 0.5 \, \mu m$)

(b) Profile of thermal actuator's surface temperature

(c) Resultant motor actuation displacement

Notes: Start+330 days. Bright start, hazy cloud. Closest test-day to summer solstice, the longest photo-period.
Figure I.19: Diurnal evolution on Day 183 in Summer: Observation on 1st Jul 2012

(a) Solar radiation exposure into aperture at short wavelengths ($\lambda_p = 0.5 \mu m$)

(b) Profile of thermal actuator’s surface temperature

(c) Resultant motor actuation displacement

Notes: Start+336 days. Bright but cloudy warm, some dull spells.
I.1. Pilot study of dual hot-boxes

Figure I.20: Diurnal evolution on Day 206 in Summer: Observation on 24th Jul 2012

(a) Solar radiation exposure into aperture at short wavelengths ($\lambda_p = 0.5 \mu m$)

(b) Profile of thermal actuator’s surface temperature

(c) Resultant motor actuation displacement

Notes: Start+358 days. Bright clear am, very hot.
I.1. Pilot study of dual hot-boxes

Figure I.21: Diurnal evolution on Day 215 in Summer: Observation on 3\textsuperscript{rd} Aug 2012

(a) Solar radiation exposure into aperture at short wavelengths ($\lambda_p = 0.5 \, \mu m$)

(b) Profile of thermal actuator’s surface temperature

(c) Resultant motor actuation displacement

- Observed $G$, $T$ and $L$ for heat-motor (A)
- Simulation of model for $G$, $T$ and $L$
- Simulation of model for $G$, $T$ and $L$

Notes: Start+384 days. Bright start, rain, mixed clouds.
Figure I.22: Diurnal evolution on Day 232 in Summer: Observation on 19\textsuperscript{th} Aug 2012

(a) Solar radiation exposure into aperture at short wavelengths ($\lambda_p = 0.5 \mu m$)

(b) Profile of thermal actuator’s surface temperature

(c) Resultant motor actuation displacement

Notes: Start+42 days. Clear hazy sun, then flash heavy rain.
I.1. Pilot study of dual hot-boxes

Figure I.23: Diurnal evolution on Day 244 in Autumn: Observation on 1st Sep 2011

(a) Solar radiation exposure into aperture at short wavelengths ($\lambda_0 = 0.5 \mu m$)

(b) Profile of thermal actuator's surface temperature

(c) Resultant motor actuation displacement

Notes: Start+56 days. Sunny start.
Figure I.24: Diurnal evolution on Day 258 in Autumn: Observation on 15th Sep 2011

(a) Solar radiation exposure into aperture at short wavelengths ($\lambda_p = 0.5 \mu m$)

(b) Profile of thermal actuator’s surface temperature

(c) Resultant motor actuation displacement

Notes: Start+61 days. Sunny and dry. Cold morning. Closest test-day to the date of the autumnal equinox (equal nights).
I.1. Pilot study of dual hot-boxes

Figure I.25: Diurnal evolution on Day 273 in Autumn: Observation on 30th Sep 2011

(a) Solar radiation exposure into aperture at short wavelengths ($\lambda_p = 0.5 \, \mu m$)

(b) Profile of thermal actuator's surface temperature

(c) Resultant motor actuation displacement

Notes: Start+61 days. Clear-sky day.
I.1. Pilot study of dual hot-boxes

Figure I.26: Diurnal evolution on Day 288 in Autumn: Observation on 15th Oct 2011

(a) Solar radiation exposure into aperture at short wavelengths ($\lambda_p = 0.5 \mu m$)

(b) Profile of thermal actuator’s surface temperature

(c) Resultant motor actuation displacement

Notes: Start+76 days. Clear skies day and night.
I.1. Pilot study of dual hot-boxes

Figure I.27: Diurnal evolution on Day 295 in Autumn: Observation on 22nd Oct 2011

(a) Solar radiation exposure into aperture at short wavelengths ($\lambda_p = 0.5 \mu m$)

(b) Profile of thermal actuator’s surface temperature

(c) Resultant motor actuation displacement

Notes: Start+83 days. Cold night, clear-sky day.
Figure I.28: Diurnal evolution on Day 317 in Autumn: Observation on 13\textsuperscript{th} Nov 2011

(a) Solar radiation exposure into aperture at short wavelengths ($\lambda = 0.5 \mu m$)

(b) Profile of thermal actuator’s surface temperature

(c) Resultant motor actuation displacement

Notes: Start+105 days. Bright clear sky start, remained clear.
Figure I.29: Diurnal evolution on Day 329 in Autumn: Observation on 25\textsuperscript{th} Nov 2011

(a) Solar radiation exposure into aperture at short wavelengths ($\lambda_p = 0.5 \mu m$)

(b) Profile of thermal actuator’s surface temperature

(c) Resultant motor actuation displacement

Notes: Start+117 days. Sunny clear start but cold.
I.1. Pilot study of dual hot-boxes

Figure I.30: Diurnal evolution on Day 344 in Winter: Observation on 10th Dec 2011

(a) Solar radiation exposure into aperture at short wavelengths ($\lambda_p = 0.5 \mu m$)

(b) Profile of thermal actuator’s surface temperature

(c) Resultant motor actuation displacement

Notes: Start+132 days. Cold bright clear sky.
I.1. Pilot study of dual hot-boxes

Figure I.31: Diurnal evolution on Day 356 in Winter: Observation on 22\textsuperscript{nd} Dec 2011

(a) Solar radiation exposure into aperture at short wavelengths ($\lambda_0 = 0.5 \mu m$)

(b) Profile of thermal actuator’s surface temperature

(c) Resultant motor actuation displacement

<table>
<thead>
<tr>
<th>Line Style</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Green</td>
<td>Observed $G$, $T$ and $L$ for heat-motor (A)</td>
</tr>
<tr>
<td>Green dashed</td>
<td>Simulation of models for $G$, $T$ and $L$</td>
</tr>
<tr>
<td>Red</td>
<td>Observed $G$, $T$ and $L$ for heat-motor (B)</td>
</tr>
<tr>
<td>Red dashed</td>
<td>Simulation of model for $G$, $T$ and $L$</td>
</tr>
</tbody>
</table>

Notes: Start+144 days. Cold but nearly clear sky condition. Closest test-day to winter-solstice, shortest photo-period length.
<table>
<thead>
<tr>
<th>Date of Observation (Season)</th>
<th>Solar Energy Yield</th>
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### I.1. Pilot study of dual hot-boxes

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## Pilot Study of Dual Hot-Boxes

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### I.1. Pilot study of dual hot-boxes

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<td>0.82</td>
<td>3.89</td>
<td>0.21</td>
</tr>
<tr>
<td>09-Dec-2011</td>
<td>70</td>
<td>0.88</td>
<td>4.16</td>
<td>0.21</td>
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</table>

(continued from previous page)
I.1. Pilot study of dual hot-boxes

continued from previous page

<table>
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<tr>
<th>Date of Observation (Season)</th>
<th>Solar Energy (MJ·m⁻²)</th>
<th>Minimum Actuation Energy (MJ·m⁻²)</th>
<th>Measured insolation on vertical Hot-box Heat-motor (A)</th>
<th>Measured insolation on horizontal Hot-box Heat-motor (B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12-Dec-2011</td>
<td>67.58</td>
<td>0.89</td>
<td>4.23 [coeff. 0.21]</td>
<td>N/A</td>
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</tbody>
</table>

Note: Data from observations in London (UK) Latitude 51.5°N during (2011 to 2012) on days with Yield (Y_i ≥ 67%) clear-skies. ¹ The classification of seasons in the British Isles is made according to (p. 402 in Manley, 1974). ² Observations tabulated by ascending day-number then year. ³ Yield is based on comparison between model of clear-sky by (Haurwitz, 1945) and integration of daily observation of (G_H) with (G_c). ⁴ Based on conditions of thermal equilibrium at dawn (α_sol = 0 ± 1°). Notes on table symbols: t₀_h and t₀_v refers to the start of exposure to direct insolation in horizontal and vertical inclinations respectively. t₁_h refers to the time when heat-motor B in the horizontal hot-box undergoes an actuation displacement (l_h = 0 → > 0.06 m) or at the end of an isothermal in the cavity temperature during melting, t₁_v refers to the time when heat-motor A in the vertical hot-box undergoes an actuation displacement (l_v = 0 → > 0.06 m) or at the end of an isothermal in the cavity temperature during solidification.

I.1.3 Parameters for heat-diffusion model

The Finite Element Method (FEM) that was used in this work was originally developed in a Ph.D by Humphries (1974) to study the ‘Performance of finned thermal capacitors’. The code listing was transposed from the original Fortran V (1974) listings contained in the Appendices of his thesis into MatLab® R14. The FEM implementation for phase-change has been adapted into a simulation of the dual heat-motor system reported in this study.

The input to the ‘feed forward’ scheme used by the FEM in each simulation is drawn from the measurements in a (DD−MM−YYYY.dat) file. Each file contains one days instrumentation data with (24 × 60 × 60/τ = 2880) rows of data-points, columns for each data-type. All the data collected during the observation study is compiled in a database of files in this format with consistent fields matched to each sensor.

A table of observations summarises all the meta-data about each observation day, an extensive list of parameters, flags, calibration offsets and contextual data about synoptic conditions. The primary key used is the date in the format for the file. The table of observations is interrogated by both the simulation engine and the plotting of results. Table I.3 lists the model and simulation parameters and the typical values used.
Table I.3: Typical values for the parameters used when simulating the model

<table>
<thead>
<tr>
<th>Parameter Name</th>
<th>Typical Value</th>
<th>SI Units</th>
<th>Description of the parameter</th>
</tr>
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<tr>
<td>Aluminium extrusion</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>alu_cr</td>
<td>2.750</td>
<td>$m^{-2} \cdot K \cdot W^{-1}$</td>
<td>Thermal contact resistance of cylinder with air (p. 102 in Incropera and DeWitt, 2007)</td>
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<tr>
<td>alu_gam</td>
<td>23.60</td>
<td>$10^{-6} \cdot m^{-2} \cdot K^{-1}$</td>
<td>Thermal expansion coefficient (Kaufman, 2004)</td>
</tr>
<tr>
<td>alu.poi</td>
<td>0.33</td>
<td>coefficient</td>
<td>Poisson’s ratio (Kaufman, 2004)</td>
</tr>
<tr>
<td>alu.fri</td>
<td>0.30</td>
<td>coefficient</td>
<td>Coefficient of friction (Kaufman, 2004)</td>
</tr>
<tr>
<td>Clear annealed float glass</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>g_w</td>
<td>0.45</td>
<td>$m$</td>
<td>Aperture width</td>
</tr>
<tr>
<td>g_l</td>
<td>0.29</td>
<td>$m$</td>
<td>Aperture length</td>
</tr>
<tr>
<td>g_area</td>
<td>0.13</td>
<td>$m^2$</td>
<td>Aperture area</td>
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<tr>
<td>g_thick</td>
<td>0.004</td>
<td>$m$</td>
<td>Aperture thickness</td>
</tr>
<tr>
<td>g_m</td>
<td>1.305</td>
<td>$kg$</td>
<td>Mass of cover sheet</td>
</tr>
<tr>
<td>g_h_Ext</td>
<td>5.0</td>
<td>$W \cdot m^{-2} \cdot K^{-1}$</td>
<td>Convection coefficient external face</td>
</tr>
<tr>
<td>g_h_Int</td>
<td>1.0</td>
<td>$W \cdot m^{-2} \cdot K^{-1}$</td>
<td>Convection coefficient internal face</td>
</tr>
<tr>
<td>g_TK</td>
<td>1.05</td>
<td>$W \cdot m^{-1} \cdot K^{-1}$</td>
<td>Thermal conductivity</td>
</tr>
<tr>
<td>g_Cp</td>
<td>500</td>
<td>$J \cdot kg^{-1} \cdot K^{-1}$</td>
<td>Specific heat capacity</td>
</tr>
<tr>
<td>g_abs</td>
<td>0.1</td>
<td>coefficient</td>
<td>Absorbtion coefficient</td>
</tr>
<tr>
<td>g_scat</td>
<td>0.1</td>
<td>coefficient</td>
<td>Scattering coefficient</td>
</tr>
<tr>
<td>g_t_sw</td>
<td>0.9</td>
<td>coefficient</td>
<td>Transmission ($\lambda_p = 0.5 \mu m$) (Pilkingtons, 2009)</td>
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<tr>
<td>g_e_lw</td>
<td>0.873</td>
<td>coefficient</td>
<td>Surface emissivity ($\lambda_p = 10 \mu m$) (BSI, 2001)</td>
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<tr>
<td>Rigid phenolic insulation (Kingspan Therma™)</td>
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<tr>
<td>ins_K</td>
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<td>$W \cdot m^{-1} \cdot K^{-1}$</td>
<td>Thermal conductivity</td>
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<tr>
<td>ins_Thick</td>
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<td>$m$</td>
<td>Thickness of panel</td>
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<tr>
<td>ins_Rho</td>
<td>32.0</td>
<td>$kg \cdot m^{-3}$</td>
<td>Density of insulation panel</td>
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<tr>
<td>ins_e</td>
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</table>

*continued on next page*
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<table>
<thead>
<tr>
<th>Parameter</th>
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<th>Description of the parameter</th>
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<tr>
<td>p_abs</td>
<td>0.15</td>
<td>coefficient</td>
<td>Absorption coefficient ($\lambda_p = 0.5 \mu m$) (p. 39 in Goswami and Martin, 2005)</td>
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<tr>
<td>p_e_lw</td>
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<td>coefficient</td>
<td>Surface emissivity ($\lambda_p = 10 \mu m$)</td>
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<tr>
<td>ND</td>
<td>2880</td>
<td>Integer</td>
<td>Number of data points per diurnal interval</td>
</tr>
<tr>
<td>EPS</td>
<td>1e-6</td>
<td>N/A</td>
<td>Allowable error in differential solution</td>
</tr>
<tr>
<td>DT</td>
<td>0.05</td>
<td>s</td>
<td>Time interval for each simulation step</td>
</tr>
<tr>
<td>DEN</td>
<td>776</td>
<td>kg $\cdot m^{-3}$</td>
<td>Density of wax n-octadecane</td>
</tr>
<tr>
<td>DEN28_3</td>
<td>891.89</td>
<td>kg $\cdot m^{-3}$</td>
<td>Estimated density of wax at (28.3 °C)</td>
</tr>
<tr>
<td>TK</td>
<td>0.149</td>
<td>J $\cdot m^{-1} \cdot s^{-1} \cdot K^{-1}$</td>
<td>Thermal conductivity of wax</td>
</tr>
<tr>
<td>CP</td>
<td>1700</td>
<td>J $\cdot kg^{-1} \cdot K^{-1}$</td>
<td>Specific heat of wax at constant pressure</td>
</tr>
<tr>
<td>TMELT</td>
<td>28.2</td>
<td>°C</td>
<td>Fusion temperature of wax</td>
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<tr>
<td>HMELT</td>
<td>243500</td>
<td>J $\cdot kg^{-1}$</td>
<td>Heat of fusion of wax</td>
</tr>
<tr>
<td>BETA</td>
<td>0.00081</td>
<td>°C$^{-1}$</td>
<td>Coefficient of volume expansion in wax</td>
</tr>
<tr>
<td>VIS</td>
<td>0.0059</td>
<td>N $\cdot sec \cdot m^{-2}$</td>
<td>Viscosity of wax in its liquid state</td>
</tr>
<tr>
<td>CL</td>
<td>1700</td>
<td>J $\cdot kg^{-1} \cdot K^{-1}$</td>
<td>Specific heat of wax at constant pressure</td>
</tr>
<tr>
<td>TKL</td>
<td>0.149</td>
<td>J $\cdot m^{-1} \cdot s^{-1} \cdot K^{-1}$</td>
<td>Thermal conductivity of liquid wax</td>
</tr>
<tr>
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<tr>
<td>TK1</td>
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<td>J $\cdot m^{-1} \cdot s^{-1} \cdot K^{-1}$</td>
<td>Thermal conductivity of Aluminium</td>
</tr>
<tr>
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<td>919.9</td>
<td>J $\cdot kg^{-1} \cdot K^{-1}$</td>
<td>Specific heat of Aluminium</td>
</tr>
<tr>
<td>H</td>
<td>9.25e-3</td>
<td>m</td>
<td>Overall height of cavity section</td>
</tr>
<tr>
<td>Bs</td>
<td>0.222</td>
<td>m</td>
<td>Length of solid wax element</td>
</tr>
<tr>
<td>BI</td>
<td>0.241</td>
<td>m</td>
<td>Length of liquid wax element at (28.3 °C)</td>
</tr>
<tr>
<td>Bav</td>
<td>0.222</td>
<td>m</td>
<td>Average length when solidified</td>
</tr>
<tr>
<td>mwe</td>
<td>1.3476e-5</td>
<td>kg</td>
<td>Mass of wax in an element</td>
</tr>
<tr>
<td>ΔρCoeff</td>
<td>-6.76e-04</td>
<td>coefficient</td>
<td>Difference in wax density liquid-to-solid</td>
</tr>
<tr>
<td>S</td>
<td>2.5e-4</td>
<td>m</td>
<td>Width of a wax element in the FEM grid</td>
</tr>
<tr>
<td>S1</td>
<td>2.5e-4</td>
<td>m</td>
<td>Height of a wax element in the FEM grid</td>
</tr>
</tbody>
</table>
Calibration and spectral response of solar radiation sensing

The sun is a good approximation of a Black-body source for radiation under clear-sky conditions. A sensor to measure insolation was selected based on a match to the spectral distribution of a radiation with a surface temperature $5700\,K$ as shown in Figure I.32.

**Figure I.32: Solar radiation sensor as installed and spectral response**

(a) Vertical and horizontal-plane pyranometers  
(b) Sensor response, p. 7 in Skye, Instruments (2008)

**Figure I.33: Calibration certificate for pyranometer sensor**

(a) S/n 1002 25044 for total global horizontal radiation  
Model SKL 2650 pyranometer, reproduced without permission from Skye Instruments Ltd © 2002
Appendix J

J.1 Manufacturers and suppliers

<table>
<thead>
<tr>
<th>Manufacturer / Supplier</th>
<th>Contact details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermoforce Ltd (UK)</td>
<td>Wakefield Road</td>
</tr>
<tr>
<td>(Thermostat wax actuator manufacturers)</td>
<td>Cockermouth, Cumbria</td>
</tr>
<tr>
<td></td>
<td>CA13 OHS</td>
</tr>
<tr>
<td></td>
<td>United Kingdom</td>
</tr>
<tr>
<td></td>
<td>Tel: ++44 (0) 1900 823 231</td>
</tr>
<tr>
<td></td>
<td>Fax: ++44 (0) 1900 825 965</td>
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<tr>
<td></td>
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</tr>
<tr>
<td></td>
<td>web: <a href="http://www.thermoforce.co.uk">http://www.thermoforce.co.uk</a></td>
</tr>
<tr>
<td>Bayliss Precision Components Ltd (UK)</td>
<td>Mr Kenneth Bayliss (Technical director)</td>
</tr>
<tr>
<td>(Thermostat wax actuator manufacturers)</td>
<td>Lysander Works</td>
</tr>
<tr>
<td></td>
<td>Blenheim Road, Airfield industrial estate</td>
</tr>
<tr>
<td></td>
<td>Ashbourne, Derbyshire</td>
</tr>
<tr>
<td></td>
<td>DE6 1HA</td>
</tr>
<tr>
<td></td>
<td>United Kingdom</td>
</tr>
<tr>
<td></td>
<td>Tel: ++44 (0) 1335 342 981</td>
</tr>
<tr>
<td></td>
<td>Fax: ++44 (0) 1335 343 860</td>
</tr>
<tr>
<td></td>
<td>e: <a href="mailto:info@bayliss.uk.com">info@bayliss.uk.com</a></td>
</tr>
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<td>web: <a href="http://www.baylissautovents.co.uk">http://www.baylissautovents.co.uk</a></td>
</tr>
<tr>
<td>J Orbesen Teknik ApS (Denmark)</td>
<td>Hr J Orbesen</td>
</tr>
<tr>
<td>(Thermostat wax actuator manufacturers)</td>
<td>Esterhøjvej 57, Asnæs (Sjælland)</td>
</tr>
<tr>
<td></td>
<td>DK 4550</td>
</tr>
<tr>
<td></td>
<td>Denmark</td>
</tr>
<tr>
<td></td>
<td>Tel: ++45 (0) 5965 1717</td>
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continued on next page
### Space Dev Inc : Starsys division (USA)
(High Output Paraffin (HOP) mechanisms)

<table>
<thead>
<tr>
<th>Manufacturer / Supplier</th>
<th>Contact details</th>
</tr>
</thead>
<tbody>
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<td></td>
<td>Fax: ++45 (0) 5965 1286</td>
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<tr>
<td></td>
<td>e: <a href="mailto:info@greenhouse-vent-opener.com">info@greenhouse-vent-opener.com</a></td>
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</tr>
<tr>
<td>Space Dev Inc : Starsys division (USA)</td>
<td>Mr Scott Tibbitts (CEO)</td>
</tr>
<tr>
<td>(High Output Paraffin (HOP) mechanisms)</td>
<td>1722 Boxelder Street</td>
</tr>
<tr>
<td></td>
<td>Louisville</td>
</tr>
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</tr>
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</tr>
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<td></td>
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<tr>
<td></td>
<td>Fax: ++1 (303) 530-2401</td>
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<tr>
<td></td>
<td>e: <a href="mailto:info@spacedev.com">info@spacedev.com</a></td>
</tr>
<tr>
<td></td>
<td>web: <a href="http://www.spacedev.com">http://www.spacedev.com</a></td>
</tr>
</tbody>
</table>

### Rostra Vernatherm LLC
(Thermal actuators)

<table>
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<tr>
<th>Manufacturer / Supplier</th>
<th>Contact details</th>
</tr>
</thead>
<tbody>
<tr>
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<td>Fax: ++45 (0) 5965 1286</td>
</tr>
<tr>
<td></td>
<td>e: <a href="mailto:info@greenhouse-vent-opener.com">info@greenhouse-vent-opener.com</a></td>
</tr>
<tr>
<td></td>
<td>web: <a href="http://www.greenhouse-vent-opener.com">http://www.greenhouse-vent-opener.com</a></td>
</tr>
<tr>
<td>Rostra Vernatherm LLC</td>
<td>Mr Tom Wisyanski (Designer)</td>
</tr>
<tr>
<td>(Thermal actuators)</td>
<td>106 Enterprise Drive</td>
</tr>
<tr>
<td></td>
<td>Bristol</td>
</tr>
<tr>
<td></td>
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<tr>
<td></td>
<td>United States of America</td>
</tr>
<tr>
<td></td>
<td>Tel: ++1 (860) 582-6776</td>
</tr>
<tr>
<td></td>
<td>Fax: ++1 (860) 582-0298</td>
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<tr>
<td></td>
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<td></td>
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</tr>
<tr>
<td></td>
<td>web: <a href="http://www.thermalactuators.com">http://www.thermalactuators.com</a></td>
</tr>
</tbody>
</table>

### Raymot Pty Ltd (Australia)
(Wax thermostatic technology)

<table>
<thead>
<tr>
<th>Manufacturer / Supplier</th>
<th>Contact details</th>
</tr>
</thead>
<tbody>
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<td></td>
<td>web: <a href="http://www.greenhouse-vent-opener.com">http://www.greenhouse-vent-opener.com</a></td>
</tr>
<tr>
<td>Raymot Pty Ltd (Australia)</td>
<td>Raymot Pty Ltd</td>
</tr>
<tr>
<td>(Wax thermostatic technology)</td>
<td>PO Box 736</td>
</tr>
<tr>
<td></td>
<td>Buderim, Queensland</td>
</tr>
<tr>
<td></td>
<td>4556</td>
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<tr>
<td></td>
<td>Australia</td>
</tr>
<tr>
<td></td>
<td>Tel: ++61 (0) 7 5453 4784</td>
</tr>
<tr>
<td></td>
<td>Fax: ++61 (0) 7 5450 1853</td>
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<td></td>
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</tr>
<tr>
<td>Manufacturer / Supplier</td>
<td>Contact details</td>
</tr>
<tr>
<td>-------------------------</td>
<td>----------------</td>
</tr>
</tbody>
</table>
| IGI Wax The International Group (USA)  
(Thermostat wax manufacturers) | Caroline Stephenson  
cstephenson@igiwax.com  
85 Old Eagle School Road, P.O. Box 384  
Wayne  
Pennsylvania 19087  
United States of America  
Tel: ++1 (610) 687 9030  
Fax: ++1 (610) 254 8548  
e: sales@igiwax.com  
web: http://www.igiwax.com |
| Hase petroleum wax company (USA)  
(Fully refined wax manufacturers) | sales@hpwax.com  
Arlington Heights, Chicago  
Illinois  
United States of America  
Tel: ++1 (800) 218 4700  
e: sales@hpwax.com  
web: http://www.hpwax.com |
| Fushun yongning Industrial Co. Ltd (PRC)  
(Thermostat wax manufacturers) | NO.21 Yulin Road Xinfu District  
Peoples Republic of China, Fushun  
Tel: ++86 (413) 2440498 (Liaoning)  
Fax: ++86 (413) 2437193  
Tel: ++86 (512) 57163286 (Kunshan)  
Fax: ++86 (512) 8227976  
e: ningliu@ynwax.com  
web: http://www.yn-wax.com |
| Therm-Omega-Tech, Inc. (USA)  
(Thermal actuators) | 353 Ivyland Road  
Warminster  
PA 18974-2205  
United States of America  
Tel: ++1 (877) 379-8258  
web: http://www.thermomegatech.com |
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