Essays on Industrial Organization: Reference Dependence, Prominence and Search, and Advertising

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DECLARATION OF AUTHORSHIP

I, Jidong Zhou, confirm that the work presented in this thesis "Essays on Industrial Organization: Reference Dependence, Prominence and Search, and Advertising" is my own. Where information has been derived from other sources, I confirm that this has been indicated in the thesis.

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Abstract

This thesis consists of four essays on three topics in industrial organization with an emphasis on consumer behavior.

Chapter 1: Introduction. This chapter is a non-technical summary of this thesis.

Chapter 2: Reference Dependence and Market Competition. This chapter examines the implications of reference dependence with loss aversion in a competitive market where consumers consider products sequentially and regard the first product they have considered as the reference point. We find that consumer reference dependence can induce firms to randomize prices, to differentiate their advertising intensities even if firms are \textit{ex ante} identical, and to differentiate their product qualities even if improving quality is costless.

Chapter 3: Prominence and Consumer Search. This chapter examines the pricing and welfare implications of prominence in a search model where firms compete in price and consumers are “biased” to consider the prominent product first. We find that the prominent product will be cheaper than others, and making one product prominent will usually increase industry profit but lower consumer surplus and total welfare.

Chapter 4: Prominence in Search Markets: Competitive Pricing vs Central Pricing. This chapter develops Chapter 3 by considering multiple prominent products and compares it with a central-pricing model where a multi-product firm chooses the prices of all products. The implications of prominence are almost reversed in the central-pricing case: prominent products are now more expensive than others, and making some products prominent can improve all market players' surplus.

Chapter 5: Advertising, Misperceived Preferences, and Product Design. This chapter studies a kind of advertising which highlights one (or few) attribute(s) of a multi-attribute product. We propose that this kind of advertising can mislead some naive consumers to overestimate the relative importance of the advertised attribute. In a monopoly market where naive consumers coexist with sophisticated consumers, we investigate how the firm can take advantage of consumers through advertising and product design, and how naive consumers can impose negative externalities on sophisticated consumers.
To my parents and my wife
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Chapter 1

Introduction

This thesis consists of four essays on three topics in industrial organization with a considerable emphasis on consumer behavior. Chapter 2 investigates how reference-dependent consumer preferences will affect market competition. Chapters 3 and 4 study the market implications of prominence in a non-random search model by supposing that consumers are “biased” to sample prominent options prior to others. Chapter 5 explores how firms can take advantage of consumers through single-attribute advertising and product design.

This introduction presents a non-technical summary of these four chapters.

Chapter 2: Reference Dependence and Market Competition. Since the seminal work by Kahneman and Tversky (1979), it has been well established that people’s preferences are often influenced by some reference point and characterized by loss aversion—a loss relative to the reference point looms larger than a same-size gain. (See, e.g., Kahneman and Tversky (2000).) Although lots of research has studied the impact of reference dependence on consumer choices, few articles have studied how firms might respond to such non-standard consumer behavior, especially in a competitive environment. (One exception is Heidhues and Köszegi (2007).) Chapter 2 contributes in this direction.

Our model is motivated by the empirical fact that the option which is considered (or tried) first by people could be favored disproportionately even if there is little cost involved in moving to options considered later. One explanation, based on reference dependence, is that people tend to regard the first option as the reference point when they come to value later ones, and they display loss aversion in the sense later options’ relative disadvantages are weighed more than their relative advantages.

Specifically, we consider a model where two firms supply horizontally differentiated products and consumers consider (or try) products sequentially. We suppose that consumers tend to regard the first product they have considered as the reference point, and they are excessively averse to paying a price higher than the reference price or having a product less well matched than the reference product. The main question we investigate
is how firms will strategically adjust their prices and product attributes to manipulate consumers’ reference points in a competitive environment.

We find that the firm whose product is more likely to be taken as the reference point by consumers will randomize its price: it will either charge a relatively low price to make consumers more price sensitive and then occupy a large market share, or charge a relatively high price to skim high-value consumers who have a strong taste for its product and will thus become less price sensitive due to loss aversion in the taste dimension. We find that this firm occupies a larger market share on average and earns a higher profit than its rival. The main welfare finding is that more severe loss aversion could harm firms and benefit consumers through intensifying price competition, but it usually leads to lower total welfare if consumers have inelastic demand.

Consumer reference dependence will also affect firms’ advertising strategies. We find that ex ante identical firms will tend to differentiate their advertising intensities if advertising only affects the order in which consumers consider products. One possible equilibrium is that one firm advertises while the other keeps “silent”. Finally, we show that, with consumer reference dependence, the firm whose product is less likely to be taken as the reference point may have an incentive to supply a lower-quality product even if improving quality is costless.

Broadly, Chapter 2 contributes to the emerging literature on behavioral industrial organization that mainly explores the market response to biased consumer behavior (see Ellison (2006), for instance).

**Chapter 3 and 4: Prominence in Search Markets.** People often face many options in the market and need to search to find a satisfactory one. In many circumstances, some options are more “prominent” than others in the sense they are more likely to be sampled before others. In a costly search environment, all else equal, such “biased” search order will earn the prominent option extra market share. Sellers are thus willing to pay for their products to be displayed prominently. For example, internet search engines make money through selling sponsored links, and manufacturers pay supermarkets for access to prominent display positions on the shelf.

Although prominence is commonplace, we know little about its impact on market performance. For example, all else equal, will prominent products be more expensive or cheaper than others? How will industry profit, consumer surplus and overall welfare change when some products become prominent in the market? These two chapters try to answer these questions in a sequential search model with differentiated products. The main deviation of our model from the traditional search model is that consumer search order is systematically influenced by the relative prominence between products, and consumers will sample prominent products before others.
We examine the implications of prominence in two possible market structures: the competitive-pricing case where each firm only sells one product and the central-pricing case where a multi-product firm sells all products. The pricing and welfare implications are contrasting in these two cases. In the competitive-pricing case, prominent products will be cheaper than others; while in the central pricing case, prominent products will be more expensive. In the competitive-pricing case, making some products prominent will usually enhance industry profit but lower both consumer surplus and total welfare; while in the central-pricing case, making some products prominent can boost all market players' surplus. Our result concerning total welfare depends on the balance between search efficiency and output efficiency. Making some products prominent will result in inefficient consumer search since it causes non-uniform prices among products, so prominence can improve efficiency only if it can increase total output.

Chapter 3 is a joint paper with Mark Armstrong and John Vickers. It deals with the competitive-pricing case with one prominent product. It also considers two extensions. First, if there is systematic quality difference but little cost difference among products, prominence can act as efficient search guidance because the firm with the highest quality is willing to pay the most to become prominent. Second, heterogeneous search costs among consumers will give the prominent firm extra monopoly power such that it could charge a higher price than others.

Chapter 4 deals with the competitive-pricing case with multiple prominent products, and it also analyses the central-pricing case. An important difference which emerges in the case with multiple prominent products is that consumers’ optimal stopping rule becomes non-stationary. In light of the complexity of this stopping rule, we also consider a simpler behavioral rule in the spirit of satisficing behavior (Simon (1955)). We find that the main price results still apply, so our price prediction actually relies little on consumer rationality.

From a pure theory perspective, these two chapters contribute to the search literature by exploring a non-random search model (see also Arbatskaya (2007), and Perry and Wigderson (1986)).

Chapter 5: Advertising, Misperceived Preferences, and Product Design.
We often see advertising or marketing activities that highlight one (or few) attribute(s) of a complex multi-attribute product (e.g., huge amounts of megapixels in digital cameras and super CPU speeds of computers). In Chapter 5, we propose that, through some psychological channels, this kind of advertising may mislead some naive or less knowledgeable consumers to overestimate the relative importance of the advertised attributes. In a monopoly market where naive consumers coexist with sophisticated consumers who are immune to advertising, we investigate how the firm can take advantage of consumers through this kind of single-attribute advertising and how this affects product design.
Interestingly, this analysis reveals that the presence of naive consumers imposes negative externalities on sophisticated consumers.

We consider a monopoly firm that produces a two-attribute product. When the firm advertises only one attribute, consumers will be segmented into two groups. In this case, although the firm cannot identify the type of each consumer, it can design two variants of the product to screen consumers. We show that the design of both products will be distorted relative to the socially efficient design. The product designed for naive consumers will be distorted so that the quality of the advertised attribute is too high but that of the unadvertised one is too low. Such a distortion reflects the misperceived preferences of naive consumers. The product designed for sophisticated consumers, however, will be distorted in the opposite directions as the consequence of adverse selection. Moreover, when consumers have heterogeneous reservation utilities, a greater number of sophisticated consumers will be excluded from the market due to the existence of naive consumers and the single-attribute advertising. Therefore, single-attribute advertising will harm all consumers whether they are naive or not.

Although the three topics in this thesis are separate, there is a common behavioral element—"salience"—in the consumer side of all models. First, the relative salience of different aspects of an object may affect people's valuation of the object, and the more salient aspect could be over weighed. The advertising effect in Chapter 5 is in this vein. The behavioral assumption—reference-dependent preferences with loss aversion—in Chapter 2 can also be understood as the consequence of a possible behavioral regularity that relative changes are more salient than absolute values and losses are more salient than gains. (See Kahneman (2003) for a relevant discussion.) Second, salience can also affect how people process information. For example, a salient stimulus might be processed prior to others. This provides a behavioral interpretation for the "biased" search order in Chapters 3 and 4.
Chapter 2

Reference Dependence and Market Competition

2.1 Introduction

People often encounter and consider options sequentially. There has been lots of evidence that the order in which people consider options can significantly influence their choice behavior, and the options that are considered or tried first tend to be favored disproportionately. This can happen even if there are no apparent costs involved in moving to later options. For example, Madrian and Shea (2001) identify a default effect with employee savings plans. They find that participation in such schemes is significantly higher under automatic enrollment, and a substantial fraction of participants under automatic enrollment choose both the default contribution rate and fund allocation even though few employees hired before automatic enrollment picked this particular outcome.¹ Ho and Imai (2006) and Meredith and Salant (2007) observe that being listed first on the ballot paper can significantly increase a candidate’s vote share.² Hartman et al. (1991) document that about 60 percent of consumers who had experienced a highly reliable electrical service regarded the current option as the best one when offered a menu with other five options. While only 5.7 percent expressed a preference for the low reliability option currently being experienced by the other group of consumers, though it came with 30 percent reduction in rates. The low reliability group of consumers exhibit similar preferences for their current

¹A similar default effect in automobile insurance purchases is documented by Johnson et al. (1993). In the early 1990s, both New Jersey and Pennsylvania introduced a limited-right policy which has a reduced right to sue and a lower premium. New Jersey set the limit-right policy as the default, but Pennsylvania set the full-right one as the default. As a result, only about 20 percent of New Jersey drivers chose to acquire the full right to sue, while about 75 percent of Pennsylvanians retained the full right to sue.

²Meredith and Salant (2007) also find that candidates perform significantly worse when listed behind higher quality candidates, and this effect comes primarily from the immediately preceding candidate.
option.\(^3\)

One explanation, which does not resort to exogenous search costs or switching costs, is the reference-dependence effect (Kahneman and Tversky (1979,91)). First, people’s preferences might be reference dependent, and what they consider or try first might become the reference point when they come to value later options. Second, people may exhibit loss aversion in the sense that the relative disadvantages of later options are weighed more than their relative advantages. Then the early option may outperform later ones even if it does not have any systematic quality advantage and even if moving to later options has no apparent costs. (Henceforth, we refer to reference-dependent preferences and loss aversion together as “reference dependence”.) We can also imagine many other examples in which reference dependence plays a similar role. For instance, a consumer once saw an advert of a laptop computer with a nice wide screen. Since then she has been thinking of possessing it. But when she finally goes to the computer store to buy it, she finds many other models there. Some may not have the nice wide screen but have other attractive attributes (e.g., nice sound device). Now she needs to decide whether to buy other models. In such a situation, the model brought to her attention first by adverts might become the reference point and influence her eventual choice.\(^5\)

Although lots of research has studied how reference dependence could influence consumer choices (see, e.g., Kahneman and Tversky (2000), and Della Vigna (2007)), fewer articles have investigated the supply side’s response to such biased consumer behavior, especially in a competitive market. This paper contributes in this direction. The main question we will investigate is how firms will strategically adjust their prices and product attributes to manipulate consumers’ reference points in a competitive environment, and what the impact of this strategic behavior on the market is.

Specifically, we consider a duopoly model with differentiated products where consumers consider or try products sequentially. We suppose that consumers regard the first product they have considered as the reference point when they value the second one, and they exhibit loss aversion in both the price dimension and the product dimension: they are excessively averse to paying a price higher than the reference price or to having a product less well matched than the reference product. Another ingredient of our model is that one firm might be more “prominent” than the other in the sense that the prominent product is

\(^3\)Differences in income and electricity consumption between the two groups were minor and did not appear to influence their results significantly.

\(^4\)There is lots of experimental evidence for this kind of status quo bias (see, e.g., Samuelson and Zeckhauser (1988), and Kahneman et al. (1991)).

\(^5\)The reference-dependent effect does not necessarily require that people possess the reference product physically for a long time, though it might be more pronounced in that case. For example, in most of experimental studies on the status quo bias and the endowment effect, the time of possessing the object is rather short and sometimes subjects only possess the object mentally. However, even in such situations, subjects seem to be attached to the object strongly.
considered first and so taken as the reference product by more consumers. The prominent product could be the default option, the product which is more heavily advertised, the product which is recommended or displayed more noticeably in the store, and the product which enters the market earlier and consumers first hear of.\(^6\)

Sections 2.2 and 2.3 investigate the pricing and welfare implications of consumer reference dependence. Section 2.2 considers the case where all consumers take the same product as the reference product. Such a relatively simple setting helps illustrate the key feature of our model: firms’ price choices have a direct bearing on consumers’ price sensitivity. If the reference firm charges a lower price than its rival, loss aversion in the price dimension makes consumers more price sensitive; if the reference firm charges a higher price, loss aversion in the product dimension makes the marginal consumer who must have a strong state for the reference product less price sensitive. Graphically, the reference firm’s demand curve has an inward kink at its rival’s price. In contrast, the other firm’s demand curve has an outward kink. With this new function for price, the reference firm has an incentive to randomize its price. It will either charge a lower price than its rival to earn a large market share or charge a higher price to focus on high-value consumers. But the other firm will charge a medium price constantly. We further show that, all else equal, the reference product is on average more expensive and occupies a larger market share, and the reference firm also earns a higher profit.

Section 2.3 considers a more flexible setting which allows for heterogenous reference products among consumers. There the prominent firm plays the same role as the reference firm and similar results hold. One implication of our price result is that the prominent firm (e.g., the firm advertising more heavily) will charge a more volatile price in a single product market (e.g., put its product on sale more frequently) or charge more dispersed prices across several independent product markets. Our price result contrasts with the view that loss aversion tends to eliminate price variation in the market (e.g., Heidhues and Kőszegi (2007)). We will discuss this difference in detail in the literature review part.

The main welfare findings are: (i) More severe loss aversion could intensify price competition, this harming both firms and benefitting consumers. However, it usually leads to lower total welfare in our setting with inelastic demand. This is mainly because more severe loss aversion tends to enlarge the price difference between products and thus induce a worse matching of consumers along the personal taste dimension. (ii) Although the prominent product is on average more expensive, it may be better for consumers to

\(^6\)In our model, the reference point is an individual product. An alternative specification of the reference point could be a weighted average of all products a consumer has considered before making a purchase decision. A product is more prominent if consumers put more weight on it. Our main results carry over to that case qualitatively. However, the assumption that the reference point is from the market rather from outside (e.g., some “ideal” product in a consumer’s mind before she enters the market) is important. We will discuss this point later in more detail.
consider it first, because doing so will prevent them from being over “addicted” to the low price at the expense of taste satisfaction.

Section 2.4 examines the impact of consumer reference dependence on firms’ advertising (or other marketing) strategies. The main result is, if there is an advertising competition prior to the price competition and if advertising only increases product prominence in the sense that the more heavily advertised product will be taken as the reference product by more consumers, then ex ante identical firms want to differentiate their advertising intensities. In particular, when advertising is not too costly, one equilibrium is that one firm advertises while the other keeps “silent”. This offers one justification for asymmetric product prominence without resorting to other exogenous reasons.

Section 2.5 explores how consumer reference dependence will shape firms’ product quality choices. We find that a relative increase of the prominent firm’s product quality could benefit both firms by making consumers in aggregate less price sensitive and thus relaxing price competition. Therefore, if there is a quality choice stage prior to the price competition, the less prominent firm may have an incentive to supply a lower-quality product than its prominent rival even if it is costless to improve quality. This offers an alternative explanation for quality differentiation in the market.7

Related Literature:

Reference dependence is the most extensively studied framing effect. It was first formally proposed in Kahneman and Tversky’s (1979) prospect theory in the context of choice with uncertainty,8 and it was further developed in the riskless environment by Tversky and Kahneman (1991). The research of psychology and behavioral economics has accumulated abundant experimental and field evidence for it, and it has also been applied to explain many economic anomalies such as the endowment effect, the status quo bias, and small-stake risk aversion. (See its extensive applications and relevant evidence in, e.g., Kahneman and Tversky (2000), and Della Vigna (2007).)

There is some research which investigates a firm’s response to consumer reference dependence. Most of the work in this direction focuses on how a monopoly firm will make its dynamic pricing decision when consumers tend to regard the historical price as the reference price (see, e.g., Fibich et al. (2007) and references cited therein).9 A main result

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7In the conventional literature on vertical product differentiation (Shaked and Sutton (1982), for instance), due to a different reason (consumers’ heterogeneous preferences over quality), firms also want to differentiate their qualities to soften price competition.

8Besides reference-dependent preferences and loss aversion, the other two elements of prospect theory are diminishing sensitivity (which implies risk aversion in the domain of gains and risk seeking in the domain of losses) and nonlinear decision weights (in the environment with uncertainty), but they are irrelevant in our model.

9Putler (1992) is an early theoretical attempt to introduce the reference-price effect into consumer demand theory. Gilboa and Schmeidler (2001) study the effect of satisficing behavior and adjustable aspiration levels on consumers’ dynamic choice, which bears some resemblance to the effect of reference-
is, if the effect of a loss is greater than a corresponding gain (i.e., if loss aversion prevails), then the firm should charge a constant price; in contrast, if the effect of a gain is greater, the firm should price cyclically.

However, little research studies consumer reference dependence in competitive markets. Clearly, competition is important for investigating the market implications of consumer biases. In addition, it is also desirable to take into account reference dependence in non-price product dimensions. Our work makes a step in this direction. A related recent paper is Heidhues and Kőszegi (2007). Following the basic idea in Kőszegi and Rabin (2006), they use consumers’ rational expectations of possible transaction results as the reference point. They then argue that loss aversion might give rise to “focal pricing” in the sense that firms may not adjust their prices even if their costs have changed and firms with different costs may charge the same price. A simple argument can go as follows. Suppose that consumers expect to pay some fixed price before they enter the market. Then, due to loss aversion, each firm’s demand will become more price responsive if its actual price is higher than that expected fixed price, and so at that price the demand curve has an outward kink. This could drive all firms to actually charge that fixed price for a range of cost conditions. The main difference between their model and ours is that consumers in their model take the expectation as the reference point, so no individual firm’s actual decision can influence it; while our reference point is some product in the market, so firms can manipulate it directly. This difference leads to a quite different prediction: our model suggests that reference dependence could cause price variation in the market in contrast to leading to price stickiness. Of course, which assumption about the reference point is more reasonable mainly depends on the context we are considering. In addition, we also investigate the impacts of consumer reference dependence on firms’ advertising and product quality choices.

Other recent papers which study the implications of the reference-dependence effect (in a broader sense) include, for example, Compte and Jehiel (2003) (prior offers as reference points in sequential bargaining), Eliaz and Spiegler (2007) (the default alternative as the reference point in forming consideration sets), Hart and Moore (2007) (contracts as reference points in ex post trading relationship), and Rosenkranz and Schmitz (2007) (reserve prices in auctions as reference points in deciding on bidding strategies).

The reference-dependence effect in our model can be regarded as a particular kind of switching costs: moving from the reference product to the other involves psychological preferences. However, neither paper studies how firms might respond to this non-standard consumer behavior.

Some empirical research (Hardie et al. (1993), for instance) suggests that loss aversion is even more severe in the product dimension than in the price dimension.

See also their companion paper Heidhues and Kősze (2005) which, among other results, shows a similar price-stickiness result in a static monopoly setting.
costs if the latter is relatively inferior in some aspects. But it is rather different from the traditional switching costs in both specifications and consequences (see, e.g., Farrell and Klemperer (2007)). We will discuss this difference more in the end of Section 2.5.

Broadly, this paper also contributes to the emerging literature on behavioral IO which mainly explores the market response to biased consumer behavior. For instance, Della Vigna and Malmendier (2004), Eliaz and Spiegler (2006), Gabaix and Laibson (2006), and Grubb (2006) study how firms might take advantage of consumers' limited abilities to forecast their future preferences. Armstrong and Chen (2007), Rubinstein (1993), and Spiegler (2006a,b) investigate how the heuristic decision making of consumers could induce firms to confuse consumers. Chen et al. (2005) and Shapiro (2006) examine the market implications of consumers' limited memory. See also the survey paper by Ellison (2006).

2.2 Single Reference Product

2.2.1 Model

There are two firms (1 and 2) in an industry, each supplying a distinct product at constant common unit cost, which we normalize to zero. They set prices \( p_1 \) and \( p_2 \) simultaneously. Consumers have diverse tastes for different products. We model this scenario via the Hotelling linear city. A consumer's taste is represented by the parameter \( x \) which is distributed on the interval \([0,1]\) according to a cumulative distribution function \( F(x) \) which is differentiable and has a positive density \( f(x) \). Firm 1 is exogenously located at the endpoint \( x = 0 \) and firm 2 is at the other endpoint \( x = 1 \). For a consumer at \( x \), the match utility of product 1 is \( u - x \), and that of product 2 is \( u - (1 - x) \), where \( u \) is the gross utility of the product and is assumed to be sufficiently large such that the whole market is covered in equilibrium.\(^\text{12}\) Consumers have unit demand for one product, and the number of consumers is normalized to one.

We introduce consumer reference dependence by considering a sequential-consideration scenario: consumers consider or try products one by one, and a product's price and match utility are discovered when it is considered or tried. We assume that consumers will take the first product they consider as the reference point. When they come to the second one, they will value its relative advantage (lower price or higher match utility) in the standard way, while they will over weigh its relative disadvantage (higher price or lower match utility) in the spirit of loss aversion. We also assume that consumers do not intentionally choose the order in which they consider products, and they may just follow

\(^{12}\text{Alternatively, we can assume that a product's match utility is a random draw from some common distribution and its realization is independent across consumers and products. Our following analysis still applies to this setting by modifying notation slightly.}\)
some natural presentation order of products or be guided by firms' marketing activities. (See a discussion about more sophisticated consumers in Section 2.3.) In this section, for simplicity we further suppose that all consumers will consider product 1 first (which is the default option, for instance), and we call it the reference product. (We will treat a more flexible setting in Section 2.3.)

One point deserves mention before proceeding. Although sequential consideration is a reasonable scenario to think about reference dependence, our model is actually not restricted to this interpretation. What we need is that consumers somehow take some product in the market as the reference point when they evaluate others. The details on why some product becomes the reference point is not crucial to most of our following analysis.

Consumer preferences are specified as follows. Given the prices $p_1$ and $p_2$, a consumer at $x$ values product 1, the reference product, in the standard way:

$$v - x - p_1;$$

her valuation of product 2 is

$$v - (1 - x) - p_2 - (\lambda - 1) \max\{0, p_2 - p_1\} - (\lambda - 1) \max\{0, 1 - 2x\},$$

where the first three terms represent the standard intrinsic surplus of product 2 and the other two terms capture the potential reference-dependent "loss utility" in each dimension. $\lambda > 1$ is the loss-aversion parameter and measures the strength of the reference-dependence effect. If $\lambda = 1$, we return to the orthodox Hotelling model. An implicit assumption here is that the reference-dependent "loss utility" occurs separately in the price dimension and the product dimension. It is psychologically reasonable and has been well supported in the literature of prospect theory (see, e.g., Tversky and Kahneman (1991)). For simplicity, we have assumed the same degree of loss aversion in both dimensions. Considering asymmetric degrees of loss aversion in the two dimensions will not affect most of our main results qualitatively.

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13Our one-shot model may be more suitable for infrequently purchased products. For frequently purchased products, unless consumers have relatively poor memory, the reference point might also be influenced by the historical purchase.

14The strength of the reference-dependence effect could be affected by the time lag between considering options. If the time lag is too long, people may have forgotten the first option when they value the second one; if it is too short, people may have not adapted themselves to the first option when the second one comes. Hence, the effect might be most pronounced when the time lag is appropriate. Presumably, the effect would be also more pronounced if consumers encounter alternative options somehow unexpectedly. If people have been expecting to consider other options when they encounter the first one, they may not attach to it too much and so the reference dependence effect might be weak.

15Our welfare results could be affected if the degrees of loss aversion in the two dimensions differ sufficiently. We will discuss this issue in Section 2.3.
To highlight how reference dependence could benefit the reference product, we first focus on the case with a symmetric distribution of consumers (i.e., \( F(1 - x) = 1 - F(x) \)). That is, there is no systematic quality difference between the two products. (We will discuss the impact of asymmetric qualities in Section 2.5.) We also assume away any possible explicit costs involved in moving from one product to the other. Introducing such costs will bring firm 1 with an extra advantage.

Now we are ready to derive each firm’s demand function. We claim that, if firm 1 charges \( p_1 < p_2 \), its demand function is

\[
q_1(p_1 < p_2) = F \left( \frac{1}{2} + \frac{\lambda}{2}(p_2 - p_1) \right). \tag{2.1}
\]

This is because all consumers with \( x \leq \frac{1}{2} \) will definitely buy product 1, and those with \( x > \frac{1}{2} \) will buy product 1 only if the gain from product 2’s higher match utility is less than the loss (including the psychological part) from its higher price, i.e., only if \( 2x - 1 < \lambda(p_2 - p_1) \), which leads to

\[
x < \frac{1}{2} + \frac{\lambda}{2}(p_2 - p_1).
\]

It is ready to see that consumers are now more price sensitive than in the orthodox model (which applies when \( \lambda = 1 \)). This is because the attractiveness of firm 1’s lower price has been amplified by consumers’ loss aversion in the price dimension.

When firm 1 charges \( p_1 > p_2 \), all consumers with \( x > \frac{1}{2} \) will buy product 2, and those with \( x < \frac{1}{2} \) will choose product 1 only if the loss (including the psychological part) from product 2’s lower match utility exceeds the gain from its lower price, i.e., only if \( \lambda(1 - 2x) > p_1 - p_2 \). Now those consumers with \( x < \frac{1}{2} \) become less price sensitive, because the unattractiveness of firm 1’s higher price has appeared less important relative to the unattractiveness of product 2’s lower match utility. The corresponding demand function is

\[
q_1(p_1 > p_2) = F \left( \frac{1}{2} + \frac{1}{2\lambda}(p_2 - p_1) \right). \tag{2.2}
\]

(2.1) and (2.2) imply that, around \( p_2 \), firm 1’s demand is more price responsive at \( p_1 < p_2 \) than at \( p_1 > p_2 \), and hence the demand curve has an inward kink at \( p_1 = p_2 \) (see Figure 2.1 below which illustrates the case with uniform \( x \)).

Firm 2’s demand is \( q_2 = 1 - q_1 \). Explicitly, using the symmetry of distribution, we have

\[
q_2(p_2 > p_1) = F \left( \frac{1}{2} + \frac{\lambda}{2}(p_1 - p_2) \right); \quad q_2(p_2 < p_1) = F \left( \frac{1}{2} + \frac{1}{2\lambda}(p_1 - p_2) \right). \tag{2.3}
\]

When \( p_2 > p_1 \), the unattractiveness of firm 2’s higher price will be amplified by loss aversion since consumers regard \( p_1 \) as the reference price. When \( p_2 < p_1 \), the attractiveness
of its lower price to the marginal consumer at $x < \frac{1}{2}$ will be reduced by her extra aversion to product 2's lower match utility. Clearly, around $p_1$, firm 2's demand is more price responsive at $p_2 > p_1$ than at $p_2 < p_1$, which implies that $q_2$ has an outward kink at $p_2 = p_1$ (see Figure 2.1 below).

![Figure 2.1: An Illustration of Demand Curves](image)

In sum, compared to the orthodox case, consumers will become more (less) price sensitive if the reference product is cheaper (more expensive) than the other. Moreover, consumer reference dependence benefits the reference firm but harms the other in the sense that, at any pair of prices $p_1 \neq p_2$, $q_1$ increases but $q_2$ decreases relative to the orthodox case.

Two additional properties of the demand function deserve mention. First, $q_1 > q_2$ if and only if $p_1 < p_2$. In effect, reference dependence does not affect each firm's demanded quantity if they charge the same price. However, it still affects the price sensitivity at that point. Second, at any fixed price pair $p_1 \neq p_2$, both firms' demand curves have the same slope given $q_2 = 1 - q_1$.

### 2.2.2 Equilibrium

Now we derive the Nash equilibrium of the price competition. First of all, both firms charging the same price is not an equilibrium. Given a positive price of firm 2, firm 1's demand has an inward kink at this price, which means that its profit function has a local minimum at this point. Hence, charging the same price will never be firm 1's best response. Second, there is no asymmetric pure-strategy equilibrium either. Suppose $p_1 \neq p_2$ were an equilibrium. Since both firms face the same demand slope at this hypothetical equilibrium point, the firm charging the higher price should have a higher demand. But

---

16This property does not depend on the assumption of a symmetric distribution of consumers.
that is impossible in our symmetric environment. We formalize the above argument in the following proposition. Denote by \( \pi_i(p_1, p_2) = p_i q_i(p_1, p_2) \) the profit function of firm \( i \).

**Proposition 2.1** Given a symmetric distribution of consumers, the price competition has no pure-strategy Nash equilibrium.

**Proof.** Define
\[
q'_i(p_1, p_2) = \frac{\partial q_i(p_1, p_2)}{\partial p_i}; \quad \pi'_i(p_1, p_2) = \frac{\partial \pi_i(p_1, p_2)}{\partial p_i}.
\]

If \( p_1 = p_2 = \hat{p} > 0 \) were an equilibrium, we must have
\[
\lim_{p_1 \to \hat{p}^-} \pi'_i(p_1, \hat{p}) \leq 0 \leq \lim_{p_1 \to \hat{p}^+} \pi'_i(p_1, \hat{p})
\]
where
\[
\pi'_i(p_1, \hat{p}) = q_1(p_1, \hat{p}) + p_1 q'_1(p_1, \hat{p}).
\]

However, we have
\[
\lim_{p_1 \to \hat{p}^-} q'_i(p_1, \hat{p}) = -\frac{\lambda}{2} f(\frac{1}{2}) < \lim_{p_1 \to \hat{p}^+} q'_i(p_1, \hat{p}) = -\frac{1}{2\lambda} f(\frac{1}{2})
\]
which leads to
\[
\lim_{p_1 \to \hat{p}^-} \pi'_i(p_1, \hat{p}) < \lim_{p_1 \to \hat{p}^+} \pi'_i(p_1, \hat{p}).
\]
This is a contradiction, so \( p_1 = p_2 = \hat{p} \) cannot be an equilibrium.

Now suppose \( p_1 > p_2 > 0 \) were an equilibrium. First of all, it is impossible that \( q_2 = 1 \), since firm 1 would then choose \( p_2 - \varepsilon \) to earn a positive profit. Since each firm's demand function is smooth around its own equilibrium price in such an asymmetric equilibrium, we must have
\[
q_1 + p_1 q'_1(p_1, p_2) = q_2 + p_2 q'_2(p_1, p_2) = 0.
\]
However, it is always true that \( q'_1(p_1, p_2) = q'_2(p_1, p_2) \) for \( p_1 \neq p_2 \). Therefore, we have
\[
\frac{q_1(p_1, p_2)}{q_2(p_1, p_2)} = \frac{p_1}{p_2}
\]
On the other hand, when \( p_1 > p_2 \), \( q_1(p_1, p_2) < \frac{1}{2} < q_2(p_1, p_2) \) since \( F(\frac{1}{2}) = \frac{1}{2} \). This again leads to a contradiction, so \( p_1 > p_2 \) can neither be an equilibrium. Using the same logic, we can also exclude the possibility of \( p_1 < p_2 \). \( \blacksquare \)

We will then show that, under regularity conditions, the game has a mixed-strategy equilibrium in which firm 1 charges a low price \( p_1^L \) with probability \( \mu \in (0, 1) \) and a high price \( p_1^H \) with probability \( 1 - \mu \), and firm 2 charges a medium price \( p_2 \) for sure. Given
firm 1’s mixed pricing strategy,

\[ q_2^*(p; \mu, p_1^L, p_1^H) = \mu q_2(p_1^L, p) + (1 - \mu) q_2(p_1^H, p) \tag{2.4} \]

is firm 2’s expected demand function. It has two outward kinks at \( p_1^L \) and \( p_1^H \) which divide it into three segments (see Figure 2 below). The regularity conditions are:

**Assumption 2.1** (i) \( f(x) \) is logconcave.\(^\text{17}\) (ii) for \( \mu \in (0, 1) \) and \( 0 < p_1^L < p_1^H \), each segment of \( q_2^* \) is regular such that the corresponding part of firm 2’s profit function is quasi-concave.\(^\text{18}\)

Notice that the uniform distribution \( (F(x) = x) \) satisfies Assumption 2.1 since then each segment of \( q_2^* \) is linear.

**Proposition 2.2** Given a symmetric distribution of consumers and Assumption 2.1,\(^\text{19}\) there exists a mixed-strategy equilibrium as specified in the above where the quadruplet \((\mu, p_1^L, p_1^H, p_2)\) satisfies the following conditions:

(i) \( p_2 = \arg \max_p p q_2^*(p; \mu, p_1^H, p_1^L) \);
(ii) \( p_1^L = \arg \max_{p \leq p_2} p q_1(p, p_2) \), and \( p_1^H = \arg \max_{p \geq p_2} p q_1(p, p_2) \);
(iii) \( \pi_1(p_1^L, p_2) = \pi_1(p_1^H, p_2) \).

Conditions (i) and (ii) define each firm’s best response given its rival’s strategy, and condition (iii) means that firm 1 is indifferent between charging the low and the high price. A potential complication is, if \( \lambda \) is sufficiently large, firm 1 may occupy the whole market when it charges the low price \( p_1^L \). As we discuss in Appendix A.1, such an equilibrium with a corner solution could occur. However, in the main text of this paper (except in Section 2.5), we focus on the interior-solution equilibrium in which no firm captures all consumers (which requires \( \lambda \) not too large).

**Proof.** Define

\[ z_L = \frac{1}{2} + \frac{\lambda}{2} (p_2 - p_1^L); \quad z_H = \frac{1}{2} + \frac{1}{2\lambda} (p_2 - p_1^H). \tag{2.5} \]

\(^\text{17}\)The logconcavity condition is satisfied by many well-known distributions. See, e.g., Bagnoli and Bergstrom (2005) for a detailed discussion.

\(^\text{18}\)Since logconcave \( f \) implies logconcave \( F \), firm 1’s demand in each side of its kink is logconcave such that the corresponding part of its profit function is logconcave (so quasi-concave). (Remember that firm 1’s whole profit function will never be quasi-concave.) However, a weighted average of two logconcave functions may fail to be logconcave. In our case, though \( q_1(p_1^L, p) \) is logconcave under (i), \( q_2^*(p) \) defined in (2.4) may not be. That is why we need (ii). But one can show that (i) implies (ii) if \( \lambda \) is close to one or if \( \frac{1}{p_1^L} < \frac{1}{p_1^H} \). (The latter condition actually implies concave profit function of firm 2.)

\(^\text{19}\)If Assumption 2.1 fails to be satisfied, we may have other types of mixed-strategy equilibrium. But note that the general existence of equilibrium is no problem according to the Glicksberg Theorem, since each firm’s profit function is continuous and we can restrict each firm’s feasible prices to a compact interval.
They are the locations of consumers who are indifferent between the two products when firm 1 charges \( p^L_1 \) and \( p^H_1 \), respectively. Then the demand functions are

\[
q_1(p^L_1, p^L_2) = F(z_i), \ i = L, H; \quad q^*_2 = \mu [1 - F(z_L)] + (1 - \mu) [1 - F(z_H)].
\]

Let \( F_i = F(z_i) \) and \( f_i = f(z_i) \). Then, in the interior-solution case, condition (i) requires

\[
\mu (1 - F_L) + (1 - \mu) (1 - F_H) = \frac{p_2}{2} \left( \mu \lambda f_L + \frac{1 - \mu}{\lambda} f_H \right). \tag{2.6}
\]

Condition (ii) requires

\[
F_L = \frac{\lambda}{2} p^L_1 f_L \iff \frac{F_L}{f_L} + z_L = \frac{\lambda}{2} p_2 + \frac{1}{2}, \tag{2.7}
\]

\[
F_H = \frac{1}{2\lambda} p^H_1 f_H \iff \frac{F_H}{f_H} + z_H = \frac{1}{2\lambda} p_2 + \frac{1}{2}, \tag{2.8}
\]

where we have used \( p^L_2 = p_2 + (1 - 2z_L)/\lambda \) and \( p^H_2 = p_2 + \lambda(1 - 2z_H) \) from (2.5). The indifference condition (iii) is

\[
p^L_1 F_L = p^H_1 F_H. \tag{2.9}
\]

(2.6)–(2.9) define an interior-solution equilibrium if (a) they have a solution \((\mu, p_2, z_L, z_H)\) with \( \mu \in (0, 1) \) and \( 1 > z_L > \frac{1}{2} > z_H > 0 \), and (b) no firm has global profitable deviation given its rival's strategy. In Appendix A.1, we show that (a) and (b) are indeed satisfied given the symmetric distribution and Assumption 2.1.

We illustrate the equilibrium in Figure 2.2 below which is based on the uniform-distribution case, where \( \pi_i \) is firm \( i \)'s iso-profit curve. The intuition of this mixed-strategy equilibrium is as follows. Given firm 2's price, firm 1 can either charge a lower price to make consumers more price sensitive and then earn a large market share (the mass strategy), or charge a higher price to exploit those consumers who have a strong taste for its product and will thus become less price sensitive due to loss aversion in the taste dimension (the niche strategy).\(^{20}\) Although these two strategies are equally profitable in equilibrium, firm 1 will not adopt either strategy predictably. Otherwise, firm 2 would either be attempted to charge a lower price than \( p^H_1 \) to steal business, or be forced to match \( p^L_1 \) to protect its own market share. Either situation will lower firm 1's profit, so firm 1 has an incentive to randomize its price and keep firm 2 guessing.

\(^{20}\)This argument does not apply to firm 2. Given fixed \( p_1 \), if firm 2 charges a higher price, consumers will become more price sensitive, which will drive firm 2 to lower its price; if firm 2 charges a lower price than \( p_1 \), the marginal consumer who has a strong taste for product 1 will become less price sensitive, which will drive firm 2 to raise its price.
Robustness Discussion:

Readers may wonder whether there are other types of mixed-strategy equilibrium in our model. A sufficient condition for the uniqueness of our equilibrium is, on top of Assumption 2.1, for any possible mixed pricing strategy of firm 1, firm 2's expected profit function will be globally quasi-concave. We do not have primitive conditions for this, but it is satisfied at least by the uniform distribution as we will show in Section 2.3.

Our equilibrium is robust to heterogenous reference points among consumers. For example, when product 1 is more heavily advertised than product 2, more than half consumers may notice and consider product 1 first and others may notice product 2 first. We will investigate such a general setting in Section 2.3, and there we will show that a similar equilibrium exists provided that the two products are not equally noticeable. It is also not difficult to extend our model to the case with more than two firms, if consumers still take some product as the reference point in evaluating others.\(^{21}\) No fundamental changes will take place since how firms' price choices affect the price sensitivity of consumers remains unchanged.

Another issue is about the assumption of symmetric distributions. From the proofs of Propositions 2.1 and 2.2, we see that this assumption can be replaced by a weaker

\(^{21}\) However, if in the sequential-consideration scenario consumers' reference points evolve as the search process goes on, then the situation could become complicated, depending on how the evolution process is specified.
condition: $F(\frac{1}{2}) = \frac{1}{2}$. Beyond this, will our mixed-strategy equilibrium still persist? The following proposition, which is proved in Appendix A.1, tells us that, given the degree of loss aversion, our equilibrium continues to hold provided that the distribution is not too skewed to either endpoint.

Proposition 2.3 Given Assumption 2.1,

(i) for fixed $\lambda$, there exists $\varepsilon > 0$ such that, when $|F(\frac{1}{2}) - \frac{1}{2}| < \varepsilon$, there is no pure-strategy equilibrium and a similar mixed-strategy equilibrium as before exists;

(ii) for fixed $|F(\frac{1}{2}) - \frac{1}{2}| > 0$, there exists $\lambda^* > 1$ such that, when $\lambda < \lambda^*$, there is only a pure-strategy equilibrium with $p_1 > p_2$ if $F(\frac{1}{2}) > \frac{1}{2}$ and $p_1 < p_2$ if $F(\frac{1}{2}) < \frac{1}{2}$.

Part (ii) of this proposition means that, given an asymmetric distribution, if the degree of loss aversion is sufficiently low, pure-strategy equilibrium will emerge. We will illustrate this result in Section 2.5 where we discuss asymmetric qualities between products (which is a special case of asymmetric distributions).

2.2.3 The benefit of selling the reference product

We then investigate whether the reference firm enjoys an advantage over its rival merely due to consumer reference dependence. As we can see from the demand function, if the reference firm charges a higher price than its rival, the shrink of its market share will be mitigated by consumers' loss aversion in the product dimension; and if it charges a lower price, the expansion of its market share will be amplified by consumers' loss aversion in the price dimension. In either case, consumer reference dependence favors the reference firm. Thus, we should expect that the reference firm would earn more the other. In addition, we will also compare the price and market share between the two firms, of which the result is less easy to predict in advance given the mixed-strategy equilibrium. We first introduce a lemma which is proved in Appendix A.2:

Lemma 2.1 Suppose $f(x)$ is symmetric and logconcave on $[0, 1]$.

(i) The function

\[ \phi(x) = \frac{1/2 - F(x)}{(1/2 - x)f(x)} \]

is symmetric on $[0, 1]$. For the uniform distribution, $\phi(x) = 1$. Beyond this special case, $\phi(x)$ strictly decreases on $[0, \frac{1}{2})$ and strictly increases on $(\frac{1}{2}, 1]$.

(ii) The function

\[ A(x) = \frac{F(x)^2}{F(x) + (x - 1/2)f(x)} \]

decreases on $(z_0, \frac{1}{2})$ and increases on $(\frac{1}{2}, 1)$, where $z_0$ satisfies $F(z_0) + (z_0 - 1/2)f(z_0) = 0$. For any $\varepsilon \in (0, \frac{1}{2} - z_0)$, $A(\frac{1}{2} - \varepsilon) > A(\frac{1}{2} + \varepsilon)$. 

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(iii) The function
\[ B(x) = F(x)^2 [2\phi(x) + 1] \]
increases on \([0, 1]\).

Now we are ready to derive the following results:

**Proposition 2.4** Given a symmetric distribution of consumers and Assumption 2.1, in the mixed-strategy equilibrium we have identified,

(i) firm 1 charges the high price more frequently \((\mu < \frac{1}{2})\) and product 1 is on average more expensive than product 2 \((p_1^H = \mu p_1^T + (1 - \mu)p_1^H > p_2)\);

(ii) on average firm 1 occupies a (weakly) larger market share than firm 2 \((q_1^H \leq \frac{1}{2})\), and they share the market equally if and only if the distribution is uniform;

(iii) firm 1 earns strictly higher profit than firm 2.

**Proof.** We first show a preliminary result:
\[
\frac{1}{2} - z_H < z_L - \frac{1}{2}, \tag{2.10}
\]
where \(z_i\) is defined in (2.5). To prove this, we rewrite the indifference condition (2.9) as
\[
\frac{F_L^2/f_L}{F_H^2/f_H} = \lambda^2 \tag{2.11}
\]
by using (2.7) and (2.8). On the other hand, (2.7) and (2.8) also imply
\[
\frac{F_L/f_L + (z_L - 1/2)}{F_H/f_H + (z_H - 1/2)} = \lambda^2. \tag{2.12}
\]
From (2.11) and (2.12), we have \(A(z_L) = A(z_H)\). (2.10) then follows from result (ii) in Lemma 2.1 given \(z_H > z_0\) (which is further implied by (2.8) and the definition of \(z_0\)).

(i) Using (2.7) and (2.8), we rewrite (2.6) as
\[
\mu \left[ 1 - 2F_L - (z_L - \frac{1}{2})f_L \right] + (1 - \mu) \left[ 1 - 2F_H - (z_H - \frac{1}{2})f_H \right] = 0. \tag{2.13}
\]
Let \(g(x) = 1 - 2F(x) - (x - \frac{1}{2})f(x)\). Then we solve
\[
\mu = \frac{g_H}{g_H - g_L}, \tag{2.14}
\]
where \(g_i = g(z_i)\). Notice that, for any \(\varepsilon \in [0, \frac{1}{2}]\), \(g(\frac{1}{2} - \varepsilon) = -g(\frac{1}{2} + \varepsilon) > 0\). Hence, (2.10) implies \(\mu < \frac{1}{2}\). We now prove \(p_1^H > p_2\). It is equivalent to \((1 - \mu)(p_1^H - p_2) > \mu(p_2 - p_1^T)\), which holds if and only if
\[
\frac{-g_L}{z_L - \frac{1}{2}} \lambda^2 > \frac{g_H}{1/2 - z_H}
\]
by using (2.14) and (2.5). Furthermore, using (2.11) and the definition of \( g_i \), we can rewrite this inequality as \( B(z_L) > B(z_H) \), which is true given result (iii) in Lemma 2.1 and \( z_L > z_H \).

(ii) \( q^*_2 = 1 - \mu F_L - (1 - \mu)F_H \leq \frac{1}{2} \) if and only if

\[
(F_L - \frac{1}{2})g_H \geq -(\frac{1}{2} - F_H)g_L.
\]

Using the definition of \( g_i \), this is further equivalent to \( \phi(z_H) \leq \phi(z_L) \). For the uniform distribution, this inequality must be binding and so \( q^*_2 = \frac{1}{2} \). Beyond this special case, result (i) in Lemma 2.1 and (2.10) imply that this inequality holds strictly.

(iii) Given \( p_2 \), firm 1 can at least earn \( \frac{p_2}{2} \) by charging \( p_2 \). Thus, \( \pi_1 > \frac{p_2}{2} \geq \pi_2 \) since \( q^*_2 \leq \frac{1}{2} \). ■

There are two other questions deserving investigation. First, how will price and welfare vary with the degree of loss aversion? Second, given firms’ equilibrium pricing strategies, if consumers realize their own biases and can choose the consideration order freely, is it really in their own interests to consider the reference product first? Due to the tractability issue, we discuss them in the uniform-distribution case in next section.

### 2.3 Heterogeneous Reference Products

This section allows for heterogeneous reference products among consumers. Specifically, we now suppose that \( \frac{1}{2} + \theta \) of consumers will consider product 1 first and take it as the reference product, while \( \frac{1}{2} - \theta \) of consumers will take product 2 as the reference product. Without loss of generality, let \( \theta \in [0, \frac{1}{2}] \). When \( \theta > 0 \), we say product 1 is more "prominent" than product 2, and \( \theta \) indicates the prominence difference between the two products. This flexible setting will allow us to discuss endogenous prominence through advertising competition in Section 2.4.

#### 2.3.1 The general case

Each firm now has two demand sources: those consumers regarding its product as a reference point and those regarding its rival’s product as a reference point. When firm 1 charges \( p_1 < p_2 \), its demand function becomes

\[
q_1(p_1 < p_2) = (\frac{1}{2} + \theta)F \left( \frac{1}{2} + \frac{\lambda}{2}(p_2 - p_1) \right) + (\frac{1}{2} - \theta)F \left( \frac{1}{2} + \frac{1}{2\lambda}(p_2 - p_1) \right).
\]

The first part is the same as before, and the second part is because, among consumers who regard product 2 as the reference product, the marginal consumer is now at \( x > \frac{1}{2} \).
and she will become less price sensitive due to her extra aversion to the less well matched product 1. Similarly,

\[ q_1(p_1 > p_2) = \left( \frac{1}{2} + \theta \right) F\left( \frac{1}{2}, \frac{1}{2\lambda} (p_2 - p_1) \right) + \left( \frac{1}{2} - \theta \right) F\left( \frac{1}{2}, \frac{\lambda}{2} (p_2 - p_1) \right). \]

For \( \theta \in (0, \frac{1}{2}) \), we have

\[
\lim_{p_1 \to p_2} q_1'(p_1 < p_2) = -\frac{1}{2} f\left( \frac{1}{2} \right) \left[ \frac{1}{2} + \theta \right] \lambda + \left( \frac{1}{2} - \theta \right) \frac{1}{\lambda},
\]

\[
\lim_{p_1 \to p_2} q_1'(p_1 > p_2) = -\frac{1}{2} f\left( \frac{1}{2} \right) \left[ \frac{1}{2} - \theta \right] \lambda + \left( \frac{1}{2} + \theta \right) \frac{1}{\lambda},
\]

which means that \( q_1 \) has an inward kink at \( p_1 = p_2 \). Firm 2’s demand function can be treated similarly and it has an outward kink at \( p_2 = p_1 \). Therefore, compared to the single-reference-product case, we should not expect any qualitative changes to take place. The counterparts of Proposition 2.1–2.3 can be proved similarly but with heavier notation. Although the counterpart of Proposition 2.4 has not been established completely, we conjecture it would hold. We will verify it in the uniform-distribution case below, and we can also verify it when \( \lambda \) is close to one.22

A notable exception is \( \theta = 0 \) (in which case the two products are equally prominent). In this symmetric case, the demand functions are smooth and we have a symmetric pure-strategy equilibrium with

\[ p^* = \frac{2}{(\lambda + 1/\lambda) f(1/2)}. \]

Compared to the standard Hotelling model where \( \lambda = 1 \), loss aversion leads to lower equilibrium price since \( \lambda + \frac{1}{\lambda} > 2 \). However, any extent of prominence difference between the two firms will overturn this equilibrium.

22When \( \lambda = 1 + \epsilon \) and \( \epsilon \) tends to zero, equilibrium prices can be approximated as

\[
p_1^l \approx p \left[ 1 - \theta \epsilon + \left( 30^{\theta^2} - \frac{10^{1 + \theta}}{2} - A \right) \epsilon^2 \right],
\]

\[
p_1^r \approx p \left[ 1 + \theta \epsilon + \left( 30^{\theta^2} - \frac{10^{1 + \theta}}{2} - A \right) \epsilon^2 \right],
\]

\[
p_2 \approx p \left[ 1 + (20^{\theta^2} - \frac{10^{1 \frac{1}{2}}}{2}) \epsilon^2 \right], \quad \mu \approx \frac{1}{2} - \left( \frac{\theta}{2} + \frac{A}{20^{\theta^2}} \right) \epsilon,
\]

where \( p = 1/f(1/2) \) is the equilibrium price in the standard Hotelling model and \( A = \frac{20^{\theta^2}}{16} f''(1/2) \). The complication in this limit analysis is that we need the second-order price approximations to proceed welfare analysis. This is because, when \( \epsilon \to 0 \), we find \( \lambda \) has no first-order effect on all welfare variables. (But it turns out that, for \( \mu \), the first-order approximation is enough.) All details about the limit analysis in this paper are available from the author.
2.3.2 The uniform-distribution case

In the following, we will proceed our analysis in the uniform-distribution case with asymmetric prominence (i.e., $\theta > 0$). One justification for asymmetric prominence will be provided in Section 2.4 where we consider endogenous prominence through advertising competition.

We first introduce two pieces of notation:

$$ h = \left(\frac{1}{2} + \theta\right)\lambda + \left(\frac{1}{2} - \theta\right)\frac{1}{\lambda}, \quad l = \left(\frac{1}{2} - \theta\right)\lambda + \left(\frac{1}{2} + \theta\right)\frac{1}{\lambda}. $$

Clearly, $h > l$ when $\theta > 0$. Then the two firms’ demand functions can be written as

$$ q_1 = \frac{1}{2} + \frac{i}{2}(p_2 - p_1), \quad q_2 = \frac{1}{2} + \frac{i}{2}(p_1 - p_2), $$

where $i = h$ if $p_1 < p_2$ and $i = l$ if $p_1 > p_2$. So the demand is more price responsive when the prominent product is relatively cheaper. When firm 1 uses the mixed strategy as in Section 2.2, firm 2’s expected demand function is

$$ q^*_2(p_2) = \frac{1}{2} + \frac{\mu h}{2}(p^*_2 - p_2) + \frac{(1 - \mu)l}{2}(p^{H}_1 - p_2), \quad (2.15) $$

for $p_2 \in [p^L_1, p^{H}_1]$.

In this uniform setting, the following condition guarantees the mixed-strategy equilibrium with an interior solution:

$$ 1 < r = \sqrt{\frac{h}{l}} < 3. \quad (2.16) $$

In particular, when $\theta = \frac{1}{2}$, we have $h = \lambda$ and $l = \frac{1}{\lambda}$, so (2.16) requires $\lambda < 3$. For smaller $\theta$, (2.16) is easier to hold. For example, when $\theta$ tends to zero, $h$ and $l$ will coincide, and so (2.16) is always true.

We first derive equilibrium. (Note that, in the uniform setup, all following necessarily conditions are also sufficient.) Given $p_2$, firm 1’s best responses imply

$$ p^L_1 = \frac{1}{2h} + \frac{p_2}{2} \quad p^{H}_1 = \frac{1}{2l} + \frac{p_2}{2}. $$

And the indifference condition requires

$$ p^L_1 \left[\frac{1}{2} + \frac{h}{2}(p_2 - p^L_1)\right] = p^{H}_1 \left[\frac{1}{2} + \frac{l}{2}(p_2 - p^{H}_1)\right]. $$

22
From them, we solve

\[ p_1^L = \frac{1}{2} \left[ \frac{1}{\sqrt{h}} + \frac{1}{\sqrt{hl}} \right] = \frac{1+r}{2h}, \]
\[ p_1^H = \frac{1}{2} \left[ \frac{1}{l} + \frac{1}{\sqrt{hl}} \right] = \frac{r(1+r)}{2h}, \]
\[ p_2 = \frac{1}{\sqrt{hl}} = \frac{r}{h}. \]

With the expected demand function in (2.15), firm 2’s best response implies

\[ 2[\mu h + (1-\mu)l]p_2 = 1 + \mu h p_1^L + (1-\mu)lp_1^H, \]

(2.17)

from which we get

\[ \mu = \frac{1}{1+r}. \]

It is ready to see that \( p_1^L < p_2 < p_1^H \) and \( \mu \in (0, \frac{1}{2}) \). For this solution to be a real equilibrium, we further need that no firm captures all consumers. Simple calculation shows that firm 1’s demands are

\[ q_1^L = \frac{1+r}{4}, \quad q_1^H = \frac{1+r}{4r}, \]

when it charges \( p_1^L \) and \( p_1^H \), respectively. So (2.16) indeed guarantees an interior-solution equilibrium.

Moreover, this is the unique equilibrium. Since firm 2’s demand function is strictly concave for any fixed \( p_1 \) in this uniform setting, its expected demand function is also strictly concave for any mixed pricing strategy of firm 1. Firm 2 will therefore never randomize its price. On the other hand, for any fixed \( p_2 \), firm 1’s demand function is linear in each side of its kink, so it is impossible for firm 1 to have more than two best replies.

We then study the properties of equilibrium. Define

\[ \Delta_L = p_2 - p_1^L = \frac{r-1}{2h}; \quad \Delta_H = p_1^H - p_2 = \frac{r(r-1)}{2h}. \]

Then

\[ \frac{\Delta_H}{\Delta_L} = r > 1, \]

(2.18)

which means that, relative to firm 2’s price, \( p_1^H \) deviates more than \( p_1^L \). On the other hand, firm 1 charges the high price \( p_1^H \) more frequently since \( \mu < \frac{1}{2} \). These two observations
imply that product 1 is on average more expensive than product 2:

$$p_1^1 = \frac{1 + r^2}{2h} > p_2 = \frac{r}{h}.$$  

One can check that firm 1's expected demand is just \( \frac{1}{2} \). That is, on average both firms share the market equally though firm 1 is more prominent.\(^{23}\) But firm 1 earns more than firm 2:

$$\pi_1 = \frac{1}{8} \left[ \frac{1}{\sqrt{h}} + \frac{1}{\sqrt{l}} \right]^2 = \frac{(1 + r)^2}{8h} > \pi_2 = \frac{p_2}{2} = \frac{r}{2h}. \tag{2.19}$$

Thus, similar results as in Proposition 2.4 have been established in this heterogeneous-reference-product setting (but with the uniform distribution).

We now answer those two questions proposed in the end of Section 2.2.

- **Which product should consumers consider first?** In our model, the order in which consumers consider product is specified exogenously or determined by firms' marketing activities such as advertising. We have not yet consider the question that, if consumers realize their own behavioral biases and can choose their own consideration orders freely, how they will behave given that firms adopt the above equilibrium pricing strategies and they are distinguishable in the market.

We first set the welfare criterion. Throughout this paper, we take the view that the psychological “loss utility” only occurs in the decision process, and it affects the ultimate welfare status of consumers only through influencing their choices. Hence, we use the orthodox welfare measurement.\(^{24}\) Also remember that consumers do not know a product’s match utility until they consider it, so consumers are ex ante identical. Since \( p_1^1 > p_2 \), people may conjecture that those considering product 2 first would obtain higher surplus. This conjecture, however, is not true.

Define

$$x_L = \frac{1}{2} + \frac{1}{2} \Delta_L; \quad x_H = \frac{1}{2} - \frac{1}{2\lambda} \Delta_H. \tag{2.20}$$

For a consumer considering product 1 first, if its price is \( p_1^1 \), she will buy product 1 if and only if her location is less than \( x_i \), so her expected surplus is \( v - \alpha_i \), where

$$\alpha_i = \int_0^{x_i} (x + p_1^1)dx + \int_{x_i}^1 (1 - x + p_2)dx$$

is the sum of expected taste loss and expected payment. Thus, given firms’ strategies, the expected surplus of a consumer who considers product 1 first is \( v - \mu \alpha_L - (1 - \mu) \alpha_H \).

\(^{23}\)This result is not a general property as Proposition 2.4 has suggested.

\(^{24}\)Even if we add the “loss utility” to welfare calculation, our following result still holds if \( \lambda \) is relatively small.
Similarly, define

\[ y_L = \frac{1}{2} + \frac{1}{2\lambda} \triangle_L; \quad y_H = \frac{1}{2} - \frac{\lambda}{2} \triangle_H. \quad (2.21) \]

Then, the expected surplus of a consumer who considers product 2 first is \( v - \mu \beta_L - (1 - \mu)\beta_H \), where

\[ \beta_i = \int_0^{y_i} (x + p_i^e)dx + \int_{y_i}^1 (1 - x + p_2)dx. \]

Therefore, considering product 1 first is better if

\[ \mu \alpha_L + (1 - \mu)\alpha_H < \mu \beta_L + (1 - \mu)\beta_H. \quad (2.22) \]

One can verify

\[ \alpha_L - \beta_L = \int_{y_L}^{x_L} (2x - 1)dx - \int_{y_L}^{x_L} \triangle_L dx = \frac{M}{4} \triangle_L^2 > 0, \]

where \( M = (\lambda - 1/\lambda)(\lambda + 1/\lambda - 2) > 0 \). This means that, when firm 1 is charging the low price, considering its product first is actually worse than considering product 2 first. Two conflicting forces work here. On the one hand, if a consumer considers the cheaper product first, she will become excessively averse to paying a higher price due to loss aversion, and so she will be more likely to buy product 1 (i.e., \( x_L > y_L \)), which of course can save expected payment. On the other hand, relative to the socially optimal situation in which consumers should buy the most matched product regardless of the price difference, \( x_L > y_L > \frac{1}{2} \) implies that considering the cheaper product 1 first will result in more severe product-choice distortion and so involve a greater expected taste loss. It turns out that the latter negative effect dominates in our setting. Similarly, we have

\[ \alpha_H - \beta_H = \int_{y_H}^{x_H} (2x - 1 + \triangle_H)dx = \frac{-M}{4} \triangle_H^2 < 0. \]

That is, when firm 1 is charging the high price, considering its product first is actually better.

Now (2.22) becomes \( \mu \triangle_L^2 < (1 - \mu)\triangle_H^2 \), which, according to (2.18), is equivalent to

\[ \frac{\mu}{1 - \mu} < r^2. \]

But this inequality must be true since \( \mu < \frac{1}{2} \) and \( r > 1 \).

Therefore, in the uniform-distribution setting with the orthodox welfare criterion, considering the more prominent product 1 first will yield higher expected consumer surplus. The reason is that considering the more expensive product first will prevent consumers
from being over "addicted" to the low price at the expense of taste satisfaction.\textsuperscript{25,26}

\textbullet \textbf{The impact of loss aversion.} We now turn to examine how price and welfare vary with the degree of loss aversion. We will show that, when the prominence difference between the two firms is relatively small, more severe loss aversion will intensify price competition, harm firms and benefit consumers (which is consistent with the observation at $\theta = 0$); when the prominence difference is relatively large, more severe loss aversion will tend to increase industry profit and harm consumers. In either case, more severe loss aversion leads to lower total welfare in our inelastic-demand setting.

We first present some useful observations (remember $r = \sqrt{\frac{1}{\lambda}}$):

\begin{center}
\begin{tabular}{lccc}
 & $h$ & $l$ & $hl$ & $r$ \\
\hline
$\lambda$ & + & ? & + & +
\end{tabular}
\end{center}

In this table, "+" means the variable in the row increases with the parameter in the column, and "?" means a possible non-monotonic relationship.\textsuperscript{27} (Note that, when $\theta = \frac{1}{2}$, $hl = 1$ is independent of $\lambda$. This caveat applies to all following analysis.)

\textbf{Price.} It is ready to see that both $p_2 = 1/\sqrt{hl}$ and $p_1^L = \frac{1}{2h} + \frac{1}{2}$ fall with $\lambda$. But $p_1^H$ may vary with $\lambda$ non-monotonically. Figure 2.3 below is such an example where the thin line is $p_2$.

![Figure 2.3: Prices and $\lambda$ with $\theta = 0.3$](image)

\textsuperscript{25}We conjecture that this result would even hold for a general distribution, which can be verified at least in the limit case with $\lambda$ close to one. However, if the extent of loss aversion in the price dimension is rather weak, then the result could be reversed. In particular, if there is no loss aversion in the price dimension, then $M = (1 - 1/\lambda)(1 + 1/\lambda - 1) < 0$ and so considering product 2 first is better.

\textsuperscript{26}One implication of our result is that, if consumers are sophisticated and they are able to choose their consideration order freely, our game with \textit{ex ante} symmetric firms has three equilibria. In the symmetric pure-strategy equilibrium (with $\theta = 0$), expecting both firms are charging the same price, consumers will consider products in a random order, which will further sustain the symmetric equilibrium. In the other two asymmetric equilibria (with $\theta = \frac{1}{2}$ and $-\frac{1}{2}$, respectively), expecting one firm is charging a random but on average higher price, consumers will visit this firm first, which will also further sustain the asymmetric equilibrium.

\textsuperscript{27}It is ready to show $\frac{\partial l}{\partial \lambda} = \frac{1}{2} - \theta - (\frac{1}{2} + \theta)\lambda^{-2}$. Thus, $l$ increases with $\lambda$ if and only if $\lambda^2 > \frac{1/2+\theta}{1/2-\theta}$ for $\theta < \frac{1}{2}$.
In particular, from \( p_1^H = \frac{1}{2} + b^2 \), we can see that \( p_1^H \) will fall with \( \lambda \) if \( l \) increases with \( \lambda \), which is true if (see footnote 27)

\[
\lambda^2 > \frac{1/2 + \theta}{1/2 - \theta}.
\] (2.23)

This condition is easier to hold for higher \( \lambda \) and lower \( \theta \). For example, when \( \theta \) tends to zero, it is always true; when \( \theta \) tends to \( 1/2 \), it fails for sure. For higher \( \theta \), firm 1’s profit from exploiting those strong-taste consumers by charging a high price will go down since now fewer of them will visit firm 1 first.

**Profit.** It is clear that \( \pi_2 = \frac{b^2}{2} \) decreases with \( \lambda \). But \( \pi_1 \) could vary with \( \lambda \) non-monotonically. In particular, since \( \pi_1 = \frac{1}{8}(h^{-1/2} + l^{-1/2})^2 \) (see (2.19)), a sufficient condition for \( \pi_1 \) to be decreasing with \( \lambda \) is also (2.23) given \( h \) increases with \( \lambda \). Therefore, it is possible (at least under (2.23)) that more severe loss aversion will intensify competition and harm both firms.

**Consumer surplus and welfare.** Remember that our welfare measurement does not include the psychological “loss utility”. Let \( W \) be total welfare and \( W = v - T \), where

\[
T = \frac{1}{4} \left[ 1 + \mu A_L \Delta L^2 + (1 - \mu) A_H \Delta H^2 \right]
\]

(with \( A_L = h(\lambda + 1/\lambda) - 1 \) and \( A_H = l(\lambda + 1/\lambda) - 1 \)) is the overall taste loss. Notice that, if both firms charge the same price, then each consumer will buy the product she most prefers, which is socially optimal and leads to the minimum taste loss \( 1/4 \). When consumers exhibit reference dependence and \( \theta > 0 \), there exists price difference between products, which will cause distortion in product choice. Specifically, when firm 1 charges \( p_1 \) and firm 2 charges \( p_2 \), the efficiency loss is

\[
\left( \frac{1}{2} + \theta \right)(x_i - \frac{1}{2})^2 + \left( \frac{1}{2} - \theta \right)(y_i - \frac{1}{2})^2 = \frac{1}{4} A_i \Delta i^2
\]

where \( x_i \) and \( y_i \) have been defined in (2.20) and (2.21). Consumer surplus is \( V = v - T - \pi_1 - \pi_2 \).

We consider the simple case with \( \theta = \frac{1}{2} \) first. In this case, one can check \( 4T = 1 = (\lambda - 1)^2/4\lambda \) which goes up with \( \lambda \), and so more severe loss aversion is detrimental to total welfare (and consumer surplus in the light of footnote 30). This welfare result is mainly driven by the fact that \( \Delta i \) increases with \( \lambda \) and larger price gaps imply greater product-choice distortion and so lower efficiency. For \( \theta < \frac{1}{2} \), numerical simulations suggest that

\footnotetext{24}{For \( \theta = \frac{1}{2} \), \( p_2 \) is a constant, and so \( p_1^H \) increases with \( \lambda \).}

\footnotetext{25}{One can show that \( \frac{d\pi_1}{d\lambda} \) has the sign of \( \frac{\lambda(1 + \lambda^{-2})(1 + \lambda^{-2})^2}{2} - \frac{\lambda^2(1 - \lambda^{-2})^2}{4} \). Thus, when \( \theta \) tends to zero, \( \frac{d\pi_1}{d\lambda} < 0 \); when \( \theta \) tends to \( \frac{1}{2} \), \( \frac{d\pi_1}{d\lambda} > 0 \); and for intermediate \( \theta \), \( \pi_1 \) could be non-monotonic with \( \lambda \).}

\footnotetext{26}{However, it is also possible that more severe loss aversion will boost industry profit. This will happen at least when \( \theta \) is close to \( \frac{1}{2} \), because at \( \theta = \frac{1}{2} \), \( \pi_2 \) is a constant and \( \pi_1 \) increases with \( \lambda \).}

\footnotetext{31}{When \( \theta = \frac{1}{2} \), \( \Delta L = \frac{\lambda^2}{2\lambda} \) and \( \Delta H = \frac{\lambda - 1}{2} \) both increase with \( \lambda \). Another subtler effect of \( \lambda \) on efficiency...
$W$ still decreases with $\lambda$ (see Figure 2.4 below where from the bottom to the top $\theta$ ranges from 0.1 to 0.5), but how $V$ varies with $\lambda$ depends on $\theta$.\footnote{We should be cautious of this total welfare result, because our unit-demand model does not reflect output efficiency. If higher $\lambda$ could also give rise to lower prices even in an elastic-demand model, more severe loss aversion could lead to higher total output and so higher efficiency.} Figure 2.5 below indicates that more severe loss aversion will be beneficial to consumers themselves when $\theta$ is relatively low. This is because, when $\theta$ is lower, it is more likely that higher $\lambda$ will decrease all prices (see (2.23)).

![Figure 2.4: $T$ with $\lambda$](image1)

![Figure 2.5: $T + \pi_1 + \pi_2$ with $\lambda$](image2)

**Discussion:**

Now we discuss how asymmetric degrees of loss aversion in the two dimensions will affect our results. The influence can be seen from two extreme cases. (i) If loss aversion occurs only in the price dimension, all analysis applies if we use $h = (\frac{1}{2} + \theta)\lambda + (\frac{1}{2} - \theta)$ and $l = (\frac{1}{2} - \theta)\lambda + (\frac{1}{2} + \theta)$. Clearly, both $h$ and $l$ increases with $\lambda$. One can verify that more severe loss aversion will then always intensify price competition such that all prices and profits decrease with $\lambda$ and consumer surplus increases with $\lambda$. So loss aversion in the price dimension is pro-competitive. (ii) If loss aversion occurs only in the taste dimension, all analysis also applies as long as we use $h = (\frac{1}{2} + \theta) + (\frac{1}{2} - \theta)/\lambda$ and $l = (\frac{1}{2} - \theta) + (\frac{1}{2} + \theta)/\lambda$. Now both $h$ and $l$ decreases with $\lambda$. It is not difficult to check that more severe loss version will now always soften price competition such that all prices and profits increase with $\lambda$ and consumer surplus decreases with $\lambda$. So loss aversion in the taste dimension is anti-competitive. In either case, total welfare in our inelastic-demand setting still goes down with $\lambda$. Our model with symmetric degrees of loss aversion is just a combination of these two extreme cases.

### 2.4 Reference Dependence and Advertising Competition

Till now the prominence difference between products is given exogenously. A natural way to endogenize it is to consider advertising (or other marketing activity) competition. For
example, a consumer might first notice and consider the product whose adverts come to her attention first, and so the more heavily advertised product might be more prominent in the market. In this section, we introduce an advertising competition stage before the price competition and suppose advertising increases product prominence by influencing the order in which consumers consider products. We will show that consumer reference dependence can induce ex ante identical firms to differentiate their advertising intensities. Thus, asymmetric prominence between products could be the outcome of advertising competition.

We now extend our model. Suppose the advertising technology is described by the function $\theta(a_1, a_2) \in [-\frac{1}{2}, \frac{1}{2}]$, where $a_i$ is firm $i$’s advertising intensity, and $\theta$ increases in $a_1$ but decreases in $a_2$. Given firms’ advertising intensities, $\frac{1}{2} + \theta(a_1, a_2)$ of consumers will notice product 1 first and the other $\frac{1}{2} - \theta(a_1, a_2)$ of consumers will notice product 2 first. We also assume that the advertising technology is symmetric: $\theta(x, y) = -\theta(y, x)$, which implies $\theta(a, a) = 0$. Let $c(a_i)$ be the advertising cost function and $c' \geq 0$. The timing of the extended game is as follows. First, firms choose their advertising intensities simultaneously. Second, observing the outcome of advertising competition, they choose prices simultaneously. Third, consumers visit firms to find out price and match utility in the order determined by the advertising competition.

The key step in solving this extended game is to know how $\theta$ affects each firm’s profit. Since the situation is symmetric between $\theta > 0$ and $\theta < 0$, our discussion focuses on $\theta > 0$. We consider the uniform setting in Section 2.3 first (remember $\theta > 0$ there). The useful observations are:

<table>
<thead>
<tr>
<th></th>
<th>$h$</th>
<th>$l$</th>
<th>$hl$</th>
<th>$r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>$+$</td>
<td>$-$</td>
<td>$-$</td>
<td>$+$</td>
</tr>
</tbody>
</table>

$\pi_2 = \frac{p_2}{2}$ increasing with $\theta$ is easy to see since $p_2 = 1/\sqrt{hl}$ rises with $\theta$. We can also show $\pi_1 = \frac{1}{8}(h^{-1/2} + l^{-1/2})^2$ increases with $\theta$. That is, a greater prominence difference between firms will benefit both firms. The intuition is, when more consumers consider product 1 first, firm 1 will rely more on those strong-taste consumers and so charge the high price more frequently, which will further relax the price competition in an environment where prices are strategic complements. Figure 6 below a numerical example with $\lambda = 2$. For a general distribution, we can get the same result at least in the limit case with $\lambda$ close to one.\(^{34}\)

\(^{33}\)This is because the derivative of the bracket term with respect to $\theta$ has the sign of $\frac{1}{h^{1/2}} - \frac{1}{l^{1/2}} > 0$. One can also show that both profit functions are convex in $\theta$. Other results concerning the impact of $\theta$ include: all prices rise with $\theta$ except that $p_1^2$ could vary with $\theta$ non-monotonically; total welfare normally decreases with $\theta$, except when $\theta$ is close to $\frac{1}{2}$; and consumer surplus always decreases with $\theta$. (Both welfare results are based on numerical simulations.)

\(^{34}\)When $\lambda = 1 + \epsilon$ and $\epsilon$ tends to zero, using the approximations in footnote 22, we can approximate equilibrium profits as $\pi_1 \approx \frac{\theta}{2} + (\frac{3}{4} \theta^2 - \frac{1}{4}) \epsilon e^2$ and $\pi_2 \approx \frac{\theta}{2} + (\theta^2 - \frac{1}{4}) \epsilon e^2$. Both of them increase with $\theta$. 
Now let $\pi_i(\theta)$ be firm $i$'s gross profit in the price competition subgame if $\theta \geq 0$. (Remember $\theta$ indicates firm 1's relative prominence.) Due to the symmetry of the environment, we have $\pi_i(\theta) = \pi_j(-\theta)$ if $\theta < 0$. Then, if $\pi(a_1, a_2)$ is firm 1's gross subgame profit function when it chooses $a_1$ and firm 2 chooses $a_2$, we have

$$\pi(a_1, a_2) = \begin{cases} 
\pi_1(\theta(a_1, a_2)), & \text{if } a_1 \geq a_2 \\
\pi_2(-\theta(a_1, a_2)), & \text{if } a_1 < a_2.
\end{cases}$$

If $\pi'_i(\theta) > 0$ for $\theta > 0$ as in the above uniform setting (or the general setting with small $\lambda$), $\pi(a_1, a_2)$ rises with $a_1$ when $a_1 > a_2$ and falls with $a_1$ when $a_1 < a_2$. That is, for fixed $a_2$, $\pi(a_1, a_2)$ reaches its minimum at $a_1 = a_2$. This implies that, if a greater prominence difference boosts each firm's profit, advertising at the same positive intensity can never be an equilibrium outcome. (Otherwise, each firm has a profitable deviation by reducing advertising intensity.)

Possible advertising equilibria are reported in the following proposition:

**Proposition 2.5** Suppose $\pi_i(\theta)$ increases with $\theta$ for $\theta > 0$ (which is at least true in the uniform setting or the general setting with $\lambda$ close to one), and suppose $a^* = \arg\max_a \pi(a, 0) - c(a)$ exists uniquely. In the extended game with advertising competition,

(i) if $a^* = 0$, the pure-strategy advertising equilibrium specifies that both firms do not advertise;

(ii) if $a^* > 0$ and $0 = \arg\max_a \pi(a, a^*) - c(a)$, the pure-strategy advertising equilibria are asymmetric, in which one firm advertises at $a^*$ and the other does not advertise at all.\(^{35,36}\) If $a^* > 0$ but $\arg\max_a \pi(a, a^*) - c(a) > a^*$, there is no pure-strategy advertising

---

\(^{35}\) Given $a^* > 0$, a sufficient condition for $0 = \arg\max_a \pi(a, a^*) - c(a)$ is that $c(a)$ is convex, $\pi(a_1, a_2)$ is concave in each argument, and $\pi_{12} < 0$.

\(^{36}\) If $\pi'_i(\theta) > 0$ for $\theta > 0$, another story which helps justify asymmetric prominence in the market is to consider a platform (e.g., a supermarket) on which firms sell products. Suppose manufacturers determine prices but the platform can extract industry profit by, for example, charging each firm a fixed fee for access to consumers. Then the platform has an incentive to make one product more prominent than the other.
equilibrium;

(iii) if there is any symmetric mixed-strategy advertising equilibrium and \( \frac{\partial \pi(0,0)}{\partial a_1} < \infty \), then each firm does not advertise with strictly positive probability.\(^{37}\)

**Proof.** (i) and (ii): First, we have argued that there is no symmetric pure-strategy advertising equilibrium in which each firm advertises. Second, if there is any asymmetric equilibria with \( a_i > a_j \), then \( a_j \) must be zero. Otherwise, reducing \( a_j \) is always profitable for firm \( j \). It is clear that, if \( a^* = 0 \), the pure-strategy equilibrium is that both firms do not advertise. If \( a^* > 0 \), then both firm not advertising is neither an equilibrium. But if \( 0 = \arg \max_a \pi(a, a^*) - c(a) \), then the pure-strategy advertising equilibrium is that one firm advertises at \( a^* \) and the other does not advertise at all. Otherwise, there does not exist any pure-strategy advertising equilibrium.

(iii) Consider a symmetric mixed-strategy advertising equilibrium in which a firm’s advertising intensity distributes on \([a_{\min}, a_{\max}]\) according to \( G(a) \). Denote by \( \hat{\pi}(a_1) \) firm 1’s expected subgame profit function given that firm 2 uses the mixed advertising strategy \( G \). Then

\[
\hat{\pi}(a_1) = \int_{a_{\min}}^{a_{\max}} \pi(a_1, a_2) dG(a_2).
\]

First, following the above argument, we must have \( a_{\min} = 0 \). Second, we have

\[
\lim_{a_1 \to 0} \hat{\pi}'(a_1) = \lim_{a_1 \to 0} \left[ \int_0^{a_1} \frac{\partial \pi(a_1, a_2)}{\partial a_1} dG(a_2) + \int_{a_1}^{a_{\max}} \frac{\partial \pi(a_1, a_2)}{\partial a_1} dG(a_2) \right] < 0,
\]

since \( \frac{\partial \pi(a_1, a_2)}{\partial a_2} < 0 \) for \( a_2 > a_1 \) and \( \frac{\partial \pi(0,0)}{\partial a_1} < \infty \). Thus, there must exist \( \delta > 0 \) such that the distribution is empty on \((0, \delta)\), i.e., \( G \) must put a mass point on \( a_{\min} = 0 \). \( \blacksquare \)

Whether \( a^* > 0 \) depends on the properties of \( \pi(a, 0) \) and \( c(a) \). Normally, when advertising is less costly, \( a^* > 0 \) should be more likely to happen.\(^{38}\) But there is a coordination problem in this pure-strategy equilibrium with \( a^* > 0 \) since the prominent firm earns more the other. If coordination is hard to achieve, then the symmetric mixed-strategy equilibrium in part (iii) (if it exists) is a more plausible prediction. However, even in that equilibrium, two ex ante identical firms still tend to differentiate their advertising intensities to some extent, which is the effect of consumer reference dependence on advertising we want to emphasize.

\(^{37}\)Exploring more about the mixed-strategy advertising equilibrium requires more structures of \( \theta(a_1, a_2) \) and \( c(a) \). One possible equilibrium is that each firm does not advertise with probability \( \rho \) and advertises at intensity \( \hat{a} > 0 \) with probability \( 1 - \rho \). Such an equilibrium exists if there is \( (\rho, \hat{a}) \) such that \( \{0, \hat{a}\} = \arg \max_{\rho} \rho \pi(x, 0) + (1 - \rho) \pi(x, \hat{a}) - c(x) \).

\(^{38}\)In particular, in the uniform setting, a sufficient condition for \( a^* > 0 \) is that \( c'(0) = 0 \) and \( c''(0) \) is sufficiently small (e.g., \( c(a) = ka^2 \)). The proof is lengthy, and the details are available from the author.
2.5 Reference Dependence and Product Quality

We now return to the setting with exogenous prominence and explore how consumer reference dependence could shape firms’ product quality choices. We will first study the properties of equilibrium with two products differing in their qualities. The main finding is, when mixed-strategy equilibrium occurs, a relative increase of the prominent product’s quality will soften price competition and benefit both firms. We then deduce that the less prominent firm may want to choose a lower quality level than its prominent rival even if improving quality is costless.

Let \( v_i \) be product i's gross utility, and define \( \Delta = v_1 - v_2 \) to be the quality difference between the two products. We assume that firm 1 is exogenously more prominent and \( \theta + \frac{1}{2} \) of consumers will consider it first. Denote by \( \hat{x} \) the solution to \( v_1 - x = v_2 - (1 - x) \). Then the consumer at

\[
\hat{x} = \frac{1}{2} + \frac{\Delta}{2}
\]

is indifferent between the two products if there is no price difference. To make the situation interesting, we focus on mild quality difference \( \Delta \in (-1, 1) \) such that \( \hat{x} \in (0, 1) \). That is, no firm will occupy the whole market if they charge the same price. For tractability, we focus on the uniform setting again. One can check that now the demand functions become\(^{39}\)

\[
q_1 = \hat{x} + \frac{i}{2}(p_2 - p_1), \quad q_2 = 1 - \hat{x} + \frac{i}{2}(p_1 - p_2),
\]

where \( i = h \) if \( p_1 < p_2 \) and \( i = l \) if \( p_1 > p_2 \).

We derive equilibrium first. Now the equilibrium will be either pure-strategy or mixed-strategy depending on the magnitude of quality difference relative to the strength of reference dependence. If there is no consumer reference dependence and \( \Delta > 0 \), clearly firm 1 will charge a higher price than firm 2. When the psychological bias emerges, charging a lower price than its rival will also become an attractive strategy to firm 1, because that will expand its market share substantially (remember firm 1 is more prominent). Hence, we expect that, for fixed quality difference, when the psychological bias becomes stronger gradually, the equilibrium should evolve from a pure-strategy one to a mixed-strategy one.

Let us keep the notation \( r = \sqrt{\frac{h}{t}} \) which increases with \( \lambda \) and \( \theta \), and define

\[
a(x) = \frac{2 - x}{x}; \quad b(x) = 3\sqrt{\frac{1 - x}{5 - 5x - x^2}}.
\]

Then we have the following result:

\(^{39}\)We are implicitly assuming that consumers regard personal taste and product quality together as the product dimension. This assumption is reasonable when consumers only have an overall impression of product satisfaction.
Proposition 2.6 With the uniform distribution and the quality difference $\Delta \in (-1, 1)$, 
(i) we have the mixed-strategy equilibrium with an interior solution (i.e., $q^L_t < 1$) if 
\[ \frac{1}{r} < \frac{a(\hat{x})}{3} < r < a(\hat{x}) \text{ for } \hat{x} \in (0, \sqrt{3} - 1), \] 
(2.24)
and we have the mixed-strategy equilibrium with a corner solution (i.e., $q^L_t = 1$) if 
\[ \begin{cases} 
    r > a(\hat{x}) & \text{for } \hat{x} \in (0, \sqrt{3} - 1) \\
    r > b(\hat{x}) & \text{for } \hat{x} \in (\sqrt{3} - 1, \hat{x}), 
\end{cases} \] 
(2.25)
where $\hat{x} \approx 0.854$ is the solution to $5 - 5x - x^2 = 0$.

(ii) We have the pure-strategy equilibrium with $p_1 > p_2$ if 
\[ \begin{cases} 
    r < 3/a(\hat{x}) & \text{for } \hat{x} \in (1/2, \sqrt{3} - 1) \\
    r < b(\hat{x}) & \text{for } \hat{x} \in (\sqrt{3} - 1, \hat{x}) \\
    \text{any } r & \text{for } \hat{x} > \hat{x}, 
\end{cases} \] 
(2.26)
and we have the pure-strategy equilibrium with $p_1 < p_2$ if 
\[ r < \frac{a(\hat{x})}{3} \text{ for } \hat{x} \in (0, 1/2). \] 
(2.27)

Proof. Here we only derive the condition for the mixed-strategy equilibrium with an interior solution. (See other proofs in Appendix A.3.) If firm 1 uses the mixed strategy, then for $p \in (p^L_t, p^H_t)$ firm 2’s expected demand function is 
\[ q^*_2(p) = 1 - \hat{x} + \frac{\mu}{2} (p^L_t - p) + \frac{(1 - \mu)}{2} (p^H_t - p). \] 
(2.28)
Similar treatment as in Section 2.3 leads to 
\[ p^L_t = \frac{1 + r}{h} \hat{x}, \quad p^H_t = \frac{r(1 + r)}{h} \hat{x}, \quad p_2 = \frac{2r}{h} \hat{x}. \] 
(2.29)
Now (2.17) becomes 
\[ 2 [\mu h + (1 - \mu) l] p_2 = 2(1 - \hat{x}) + \mu h p^L_t + (1 - \mu) l p^H_t. \]
Then 
\[ \mu = \frac{1}{r^2 - 1} \left( \frac{2 - \hat{x}}{3\hat{x}} r - 1 \right). \] 
(2.30)
For $\mu$ to be between zero and one, we need 
\[ \frac{1}{r} < \frac{2 - \hat{x}}{3\hat{x}} < r. \]
Since \( q_1^L = \hat{x} + \frac{1}{2}(p_2 - p_1^L) = \frac{1}{2}(1 + r) \), the condition for having an interior solution is

\[
r < \frac{2 - \hat{x}}{\hat{x}},
\]

which is the counterpart of (2.16) in this asymmetric-quality case. Finally, these two conditions themselves require \( \hat{x} < \sqrt{3} - 1 \).\footnote{This is because \( \frac{1}{r} < \frac{2 - \hat{x}}{\hat{x}} \) implies \( r > \frac{\hat{x}}{2 - \hat{x}} \). Then we need \( \frac{\hat{x}}{2 - \hat{x}} < \frac{\hat{x}}{2} \) which implies \( \hat{x} < \sqrt{3} - 1 \).}

The following graph describes the relationship between equilibrium and the parameter pair \((r, \hat{x})\), where the horizontal axis is \( \hat{x} \) and the vertical one is \( r \).

![Equilibrium Diagram](image)

Figure 2.7: Equilibrium and \((r, \hat{x})\)

In the area below the solid lines, we have pure-strategy equilibrium. In the area above them, we have mixed-strategy equilibrium. This area is further divided by the dashed line \( a(\hat{x}) \). Below that, we have the interior-solution equilibrium, and above that, we have the corner-solution equilibrium. When \( r < \sqrt{3} \) (i.e., below the horizontal dashed line), there is no mixed-strategy equilibrium with a corner solution for any \( \hat{x} \). We observe that, for fixed \( r \), mixed-strategy equilibrium is more likely to occur for smaller quality difference; and for fixed \( \hat{x} < \hat{x} \approx 0.854 \), mixed-strategy equilibrium is more likely to occur for greater \( r \) (so for greater \( \theta \) or \( \lambda \)). (In particular, when there is no quality difference (i.e., when \( \hat{x} = \frac{1}{2} \), we only have mixed-strategy equilibrium.) These two observations illustrate Proposition 2.3.

We now turn to investigate the properties of equilibrium. In the pure-strategy equi-
librium, we can show

\[ p_1 = \frac{2}{i} \cdot \frac{1 + \hat{x}}{3}, \quad \pi_1 = \frac{2}{i} \left( \frac{1 + \hat{x}}{3} \right)^2, \]
\[ p_2 = \frac{2}{i} \cdot \frac{2 - \hat{x}}{3}, \quad \pi_2 = \frac{2}{i} \left( \frac{2 - \hat{x}}{3} \right)^2, \]

where \( i = l \) if \( \hat{x} > \frac{1}{2} \) and \( i = h \) if \( \hat{x} < \frac{1}{2} \). It is clear that \( p_1 \) and \( \pi_1 \) increase with \( \hat{x} \) while \( p_2 \) and \( \pi_2 \) decrease with \( \hat{x} \). Thus, a relative increase of firm 1’s quality will benefit firm 1 but hurt firm 2. This is consistent with the result in the orthodox model with \( \lambda = 1 \).

However, the situation is very different in the mixed-strategy equilibrium. Our discussion is based on the interior-solution case. (In Appendix A.3 we establish the similar results in the corner-solution case.) First, from (2.29) and (2.30), we see that all prices increase with \( \hat{x} \) and \( \mu \) decreases with \( \hat{x} \) (i.e., firm 1 will charge the high price more frequently when its relative quality rises). Second, following the proof of Proposition 2.6, simple calculation yields

\[ \pi_1 = \frac{(1 + r)^2}{2h} \hat{x}^2, \quad \pi_2 = \frac{2r}{3h} (2 - \hat{x}) \hat{x}. \]

It is clear that each firm’s profit increases with \( \hat{x} \) since \( \hat{x} < 1 \). That is, a relative increase of the prominent firm’s quality will benefit both firms.

The results concerning firm 1 are not surprising, and here we try to understand the results concerning firm 2. When firm 1’s relative quality increases, several forces affect firm 2’s pricing incentive. Let us see its first-order condition \( p_2 = \frac{q_2}{\frac{d \hat{q}_2}{dp_2}} \), where \( \hat{q}_2 \) is its expected demand defined in (2.28). (i) Given prices, higher \( \hat{x} \) reduces \( \hat{q}_2 \) directly since firm 2 is then relatively less favored by consumers. This will drive firm 2 to lower its price. (ii) Higher \( \hat{x} \) causes higher prices of firm 1 and lower \( \mu \), which will enhance \( \hat{q}_2 \) and give firm 2 an incentive to raise its price. (Note that this is the standard strategic effect in an environment where prices are strategic complements.) (iii) Lower \( \mu \) also decreases \( \frac{\partial \hat{q}_2}{\partial p_2} = \frac{1}{2} (\mu h + (1 - \mu)l) \) (i.e., makes firm 2’s expected demand less price responsive). This is because consumers in aggregate are less price sensitive when firm 1 charges the high price. This will further motivate firm 2 to raise its price. The first two effects are standard, but the third one is only present in the mixed-strategy equilibrium caused by consumers reference dependence. Our result implies that the latter two positive effects together outweigh the first negative one. In light of this price result, the profit result is not difficult to understand.

We further illustrate the above results in the following two graphs which are based on a numerical example with \( \theta = \frac{1}{3} \) and \( \lambda = 2 \).
They describe how equilibrium prices and profit vary with $\tilde{x}$, respectively. (The thick lines correspond to firm 1, and the dashed parts correspond to the mixed-strategy equilibrium.)

The main implication of the above results is, when the quality difference between the two products is not too large (such that the mixed-strategy pricing equilibrium occurs), the less prominent firm has no incentive to improve its quality slightly even if it is costless to do so. This is because it does not want to trigger the prominent firm to charge a low price more frequently. Put differently, the less prominent firm even has an incentive to reduce its quality even if doing so does not save any costs. We then deduce, if one firm is more prominent than the other and if there is a (simultaneous) quality choice stage before the price competition, choosing the same positive quality level will never be an equilibrium outcome. This is because at $\tilde{x} = \frac{1}{2}$ we must have the mixed-strategy equilibrium and then $\pi_2$ increases with $\tilde{x}$ (i.e., reducing quality is profitable for firm 2). This also implies, at least when the range of feasible quality levels is restricted such that there is no possibility for pure-strategy pricing equilibrium, the less prominent firm will choose a lower quality level than its prominent rival even if improving quality is costless.$^{41,42}$

We summarize the main results in the following proposition:

**Proposition 2.7** (i) With the uniform distribution and product 1’s relative quality “advantage” $\Delta \in (-1, 1)$, the prominent firm 1’s prices and profit always increase with $\Delta$. Firm 2’s price and profit decrease with $\Delta$ in the pure-strategy equilibrium, but increase with $\Delta$ the mixed-strategy equilibrium.

$^{41}$This result could even hold for a broader range of quality levels. Let us see the following simple example. Suppose the free quality feasible set is $v_i \in [0, v + 1]$ (so $\Delta \in [-1, 1]$). Then it is clear that the prominent firm 1 will always pick the highest quality level $v_1 = v + 1$. Firm 2’s problem is thus to choose $\tilde{x}$ between $[\frac{1}{2}, 1]$. From Figure 9, we see that it will choose $x_2 > \frac{1}{2}$ (i.e., $v_2 < v_1$), where $x_2$ is the upper limit value of $\tilde{x}$ such that we have a mixed-strategy pricing equilibrium at $\tau$.

$^{42}$Another implication of our profit results is that, if both firms sell their products through a platform (e.g., a supermarket) and if the platform can extract the whole (or a fixed proportion of) industry profit by charging firms some fixed access fees, then we claim that, for any $\Delta \in (-1, 1)$, the platform will display the higher-quality product more prominently because that will lead to higher industry profit. To prove this result, we need to consider four cases because reversing the relative prominence between products could change the form of the pricing equilibrium. The details are available from the author.
(ii) Suppose there is a quality choice stage prior to the price competition. Then in equilibrium the two firms will never choose the same quality level. Moreover, at least when the range of feasible quality levels is relatively narrow, the less prominent firm will choose a lower quality level than its prominent rival even if improving quality is costless.

Discussion:

The reference-dependence effect in our model can be regarded as a kind of switching cost. But it occurs only if the second product is relatively inferior to the first one in at least one aspect. Readers may wonder whether the results we have derived in this paper could be replicated by using an exogenous cost involved in moving from one product to the other. In that setting, all else equal, the prominent firm will also earn more than the other, but we cannot establish other main results. Suppose the cost is $s$. Then, with the same notation, the demand functions are:

$$q_1(p_1) = \hat{x} + \theta s + \frac{p_2 - p_1}{2}; \quad q_2(p_2) = 1 - (\hat{x} + \theta s) + \frac{p_1 - p_2}{2}. $$

Since they are smooth functions, no firm will randomize its price. If $s$ is appropriate such that we have an interior-solution equilibrium, then the equilibrium prices and profits are:

$$p_1 = \frac{2}{3}(1 + \hat{x} + \theta s), \quad \pi_1 = \frac{2}{9}(1 + \hat{x} + \theta s)^2;$$

$$p_2 = \frac{2}{3}(2 - \hat{x} - \theta s), \quad \pi_2 = \frac{2}{9}(2 - \hat{x} - \theta s)^2. $$

Clearly, making one firm more prominent or improving its product quality will benefit this firm but harm the other, so our results on advertising and quality choice will not emerge either.

2.6 Conclusion

This paper has examined the impacts of consumer reference dependence on market competition. When consumers take some product as a reference point in evaluating others and exhibit loss aversion, the firm whose product is taken as the reference point by more consumers will randomize its price over a high and a low one, while the other firm will charge a medium price constantly. All else equal, the prominent product is on average more expensive and has a larger market share, and the prominent firm earns more than the other. The welfare impact is that consumer reference dependence could harm firms and benefit consumers by intensifying price competition. We also find that a greater prominence difference between firms can soften price competition and improve each firm’s profit. So ex ante identical firms may tend to differentiate their advertising intensities. When firms vary in their prominence, a relative increase of the prominent product’s quality could
also soften price competition, and so the less prominent firm might supply a lower-quality product even if improving quality is costless.

Some related topics deserve future studies. First, it is desirable to explore the impact of consumer reference dependence in a dynamic competition setting. The impact could be different because there the historical purchase may influence the reference point. Second, it is also interesting to investigate how a firm supplying several (vertically) differentiated products could benefit from manipulating the order in which consumers consider or try products (e.g., by recommending some products or by adjusting the product launch strategy). In the product launch case, for instance, the firm can choose to provide a complete choice set immediately or expand the choice set gradually. If the high-quality product has a higher profit margin, it may be a good strategy to launch it first, especially when consumers are overconfident that trying the high-quality product first would not influence their subsequent preferences.

In the end of this paper, we want to further emphasize two points. First, our price result that reference dependence can induce some firm to randomize its price contrasts with the view that reference dependence tends to eliminate price variation. This difference indicates that the market implications of reference dependence may be sensitive to the specification of reference points. In particular, it is crucial whether consumers' reference points are independent of or influenced by firms' actual decisions.

Second, our work also intends to delivery the message that the order in which people consider options deserves more attention in economics, even if there are no explicit costs involved in moving from one option to the other. In traditional economics, the consideration order has no impact on people's choices as long as they face the same choice set eventually. However, with some behavioral biases, it may become an important choice determinant factor. Reference dependence is such a bias and this paper has studied its market implications. Other biases also deserve research. For example, when processing informative signals sequentially, people with confirmatory bias tend to stick to the opinion formed in the early stage (Rabin and Schrag (1999), for instance), and people with limited attention may unconsciously allocate too much effort in processing early signals but leave too little to later ones. In both cases, the early signals might be over weighted in information integration. This may motivate some market players to manipulate the order in which other players receive signals.

\footnote{See Rubinstein and Salant (2006) and Salant (2007) for some research about choice from a list (i.e., an ordered choice set) in the decision theory context.}
Chapter 3

Prominence and Consumer Search*

3.1 Introduction

In many markets consumers must search to find a satisfactory option. Without guidance, consumers may search randomly through the options, and sellers share the market equally if there are no systematic differences between them. However, if one option is somehow more prominent than others, consumers are likely to consider that option first. For example, when using a search engine online, people might first click on links displayed at the top of the page; when deciding what to watch on television, viewers may be biased towards the channels listed at the top of an electronic programme guide; when people visit a supermarket or bookstore, those products displayed in the entrance or other prominent positions might catch their attention first. In these examples, a consumer’s search order is not random but influenced by the way the options are presented.

There is abundant evidence that the way options are presented can significantly influence people’s choices, and more prominent options could be favored disproportionately. For example, Madrian and Shea (2001) identify a significant default effect with employee savings plans. They find that participation in such schemes is significantly higher under automatic enrolment, while a substantial fraction of participants hired under automatic enrolment “choose” both the default contribution rate and the default fund allocation. Ho and Imai (2006) and Meredith and Salant (2007) observe that being listed first on the ballot paper can significantly increase a candidate’s vote share. Einav and Yariv (2006) present evidence that economists with surname initials earlier in the alphabet have more

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successful professional outcomes, and discuss various reasons why such researchers may be more “prominent”. Lohse (1997) investigates experimentally the influence of yellow page advertisement characteristics on consumer information processing behaviour. By tracing subjects’ eye movements, he finds that adverts which are larger, colorful, with graphics, or near the beginning of a heading, are more likely to catch a reader’s attention.

Sellers are willing to pay for their products to be displayed in a prominent position. For example, internet search engines make money through selling sponsored links in response to search enquiries. Manufacturers pay supermarkets for access to prominent display positions (e.g., at eye level, or at the ends of aisles), publishers pay bookstores substantial fees for a book to be the “book of the week”,¹ more prominent adverts are more expensive in yellow pages directories, and eBay offers sellers the option to list their products prominently in return for an extra fee. Moreover, when a product is made prominent—such as a book at the entrance to a bookshop—it is often sold at a discount. Indeed, the word “promote” can mean to make prominent and to offer at a discount.²

This discussion suggests that prominence plays an important role in affecting consumer choices and product prices, and its impact on market performance deserves investigation. In this chapter, we examine the impact of prominence in a framework where consumers search sequentially through their available options. To model “prominence”, we suppose that the prominent firm is the first firm to be sampled by consumers. If consumers are not satisfied with this initial offer, they will go on to search randomly among the remaining firms. We explore the following questions: Will a prominent firm charge a higher or lower price than its rivals? How does profit, consumer surplus and overall welfare change when a firm is made prominent?

In section 3.2 we present a benchmark model in which there are no systematic differences between firms or between consumers. In section 3.2.1, we consider the simplest setting where there is an infinite number of firms, where it turns out that prominence has no impact on price and welfare, and merely redistributes demand and profit towards the favoured firm. However, as we show in section 3.2.2, prominence does matter when there are finitely many suppliers. We find that the prominent firm will charge a lower price than its non-prominent rivals. Essentially, the prominent firm faces more elastic demand than its rivals. In addition, relative to the situation with random consumer search, the prominent firm’s price falls while non-prominent firms’ prices rise. We find that, after introducing a prominent firm, industry profit will typically increase. This means that, if

¹ Journalist Libby Purves, writing in The Times on 30 May 2006, says: “That WH Smith’s “book of the week” title has been bought and paid for. The publisher handed over £50,000. Waterstone’s Book of the Week accolade is £10,000 [...] Smaller sums buy other levels of prominence.”

² According to the Oxford English Dictionary, one definition of “promote” is “to advance the interests of, move to a stronger or more prominent position.” And “on promotion” is defined as “at a reduced price or on special offer as part of a campaign to promote sales.”
a platform can extract industry profit, it will choose to make a firm prominent. However, introducing a prominent firm will lead to lower consumer surplus and lower total welfare. This is because, when a firm is made prominent, market prices are no longer uniform. Since the prominent firm charges a lower price than non-prominent firms, too many consumers are induced to buy from the prominent firm than is efficient. On top of this, we find that making a firm prominent will exclude more consumers from the market—that is to say, the price increase by non-prominent firms is more significant than the prominent firm’s price reduction—and this is a second source of inefficiency.

The remainder of this chapter deals with asymmetries, first on the supply side and then on the demand side. In section 3.3, we explore another effect of prominence in an extension where firms differ in their average quality. If the cost difference is small compared to the quality difference, we find that the firm with the highest quality is willing to pay the most to become prominent, and making it prominent will boost industry profit as before, but also boost consumer surplus and welfare. In effect, prominence now acts to guide consumers towards better products. The high-quality firm will set a high price, but consumers still benefit from encountering this firm first. In section 3.4, we briefly discuss the impact of prominence when consumers differ in their search costs. Here, we find that the prominent firm could offer a higher price than its rivals. The reason is that this firm may now face a less elastic demand than its rivals, since it holds a monopoly over those consumers with high search costs. Rational consumers should therefore avoid the prominent product if they have the ability to do so.

Our model assumes that all consumers sample the prominent option first. There are at least three ways to think about this assumption. First, consumers may be exposed to options in an exogenously restricted order, and they have no ability to avoid the prominent product. For instance, if we go to a travel agent to buy airline tickets or a financial advisor to buy a savings product, the advisor may tell us the options one by one. Second, consumers could suffer from bounded rationality of some form and be susceptible to manipulation by marketing ploys. In the psychological literature, it is well documented that a salient stimulus can more effectively catch people’s attention, and this reaction, to some degree, is independent from the economic importance of the stimulus. Third, consumers could be fully rational: they choose to visit the prominent firm first because they expect this firm to make the best offer, and this expectation is correct in equilibrium. While

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3See, for instance, Fiske and Taylor (1991). If we adopt this bounded rationality interpretation, some readers may wonder why consumers are nevertheless able to calculate the optimal stopping rule (as assumed in this paper). However, we could think of the optimal stopping rule as being used for tractability rather than as a precise description of consumers’ real behavior. In fact, our main results continue to hold qualitatively if we adopt an alternative behavioral stopping rule (e.g., consumers exhibit satisficing behavior and stop searching once their exogenous aspiration levels are met). The reason is that in our models in sections 3.2 and 3.3 the optimal stopping rule is stationary, regardless of the number of firms or whether one firm is prominent or not. (See a formal argument in next chapter.)
our approach is largely neutral with respect to these three possibilities, it is a bonus that most of our results admit the rational-consumer interpretation. In the models presented in sections 3.2 and 3.3, consumers are indeed better off if they choose to go first to the prominent firm, even when they have the ability to avoid it.

This chapter draws on the rich literature on consumer search. In particular, our model is related to the branch of the search literature concerned with product differentiation, where consumers must search both for price and product fitness. An early contribution to this literature is Weitzman (1979), and this was later developed and applied to a market context by Wolinsky (1986). We use Wolinsky's framework as the starting point for our model. Wolinsky's model is developed further by Anderson and Renault (1999), who, among other results, discuss how equilibrium prices are affected by changes in the degree of product differentiation.\footnote{Anderson and Renault (1999) assume that all consumers make a purchase. In our framework, this assumption would eliminate any impact of prominence on total output, which turns out to be an important ingredient in our welfare analysis.} Compared to the homogeneous product search model, models with product differentiation often better reflect consumer behavior in markets with nonstandardized products. Moreover, they avoid the well-known modeling difficulty suggested by Diamond (1971), who showed with homogeneous products and positive search costs that rivalry between firms had no impact on price. In search models with product differentiation, there are some consumers who are ill-matched with their initial choice of supplier and then search further, so that the pro-competitive benefit of actual search is present.\footnote{Another way to "get around" the Diamond Paradox is to stay with homogeneous products, but to allow some consumers to have zero search costs. We discuss this alternative framework briefly in section 3.4.}

While there is much evidence concerning the impact of prominence (generally conceived) on choice, not much of this work has examined market situations in which prominence affects subsequent competition between firms. In particular, there is little analysis of how prominence might affect a firm's pricing policy. Nevertheless, there is a small literature on this topic. For instance, Perry and Wigderson (1986) suppose consumers deal with a finite number of suppliers in a known, pre-determined order. (For instance, a driver may be looking for petrol along a road.) There is no scope for going back to a previous offer (unlike in our model), and so the final supplier holds a monopoly position over those consumers who wait that long. Their model assumes that consumers differ in their willingness-to-pay for the product, however, which implies that the final suppliers could be left with only the low-value consumers. The paper argues that in equilibrium the observed prices could be non-monotonic in the rank order of the supplier.

Arbatskaya (2007) also considers a completely ordered search model with a homogeneous product. Since consumers only care about price, in equilibrium the prices must
decline with the order in which they are sampled, otherwise no rational consumer would have an incentive to sample products in unfavorable positions. This framework is briefly discussed in section 3.4 below.\textsuperscript{6} Both Perry and Wigderson (1986) and Arbatskaya (2007) present a positive analysis of the impact of search order on equilibrium prices, and there is no discussion of how a non-random search order affects industry profit, consumer surplus or welfare compared with random searching.\textsuperscript{7}

Another strand of the literature to which our work relates is advertising. Indeed, a major purpose of advertising is to make a product more “prominent”. For instance, Robert and Stahl (1993) analyze a rich model where consumers search for a low price for a homogenous product, and firms can also advertise their price to a subset of consumers. They find that in equilibrium firms randomize between setting a high, unadvertised price and lower, advertised prices. Thus, the more prominent firms set lower prices, as in our model. A similar effect is found in Bagwell and Ramey (1994), although for very different reasons. In their paper, firms are identical \textit{ex ante} and attract consumers by means of advertising (which is not directly informative). Firms have economies of scale, so that a firm facing greater demand has a lower marginal cost. Consumers follow the rule-of-thumb whereby they buy from the firm which advertises most heavily. Because of economies of scale, this firm will have a lower price than its rivals. Thus, the consumer response to advertising is indeed rational even though the advertising messages are not directly informative, and the more prominent firm sets a lower price. More recently, and closer in spirit to our approach, Hann and Moraga-Gonzalez (2007) propose a model of search and advertising where the search model involves product differentiation as in Wolinsky (1986). A consumer’s search order is potentially non-random, and a consumer’s likelihood of sampling a firm is proportional to that firm’s advertising intensity. (Adverts do not contain price information, and merely “persuade” consumers to sample that product first.) In symmetric equilibrium, all firms set the same (deterministic) prices and advertise with the same intensity, and so no firm is more prominent than any other. So consumers end up searching randomly and advertising is pure waste.

Finally, our work is related to the work on auctions for being listed prominently on online search engines. The two papers by Chen and He (2006) and Athey and Ellison (2007) are especially relevant since they include a model of the consumer side of the market, and consumers search sequentially through the suggested links to find a good

\textsuperscript{6}Hortaçsu and Syverson (2004) construct a related empirical search model, where investors sample different mutual funds with unequal probabilities, to explain the price dispersion in the market for mutual funds. But they did not explore theoretical predictions of their model, and there is also no empirical conclusion about the relationship between sampling probability and price.

\textsuperscript{7}In effect, the setup of Arbatskaya (2007) is not suitable for welfare comparison. In the homogenous product setup, random searching implies monopoly pricing as suggested by Diamond (1971). Then prominence will always (weakly) improve efficiency. But the Diamond result is usually regarded as an extreme result due to the modeling issue, so the welfare comparison provides no insight.
match for their needs. Links differ in "quality" in the sense that a high-quality link is more likely to generate a good match with any consumer. There is equilibrium behaviour on both sides: consumers optimally search for a good match by moving through the links in the order presented since they anticipate that high-quality links will be placed higher up the listing; higher-quality links have a greater incentive to be placed higher in the list than lower-quality links given the consumer search order, since a link's payoff is proportional to the number of good matches with consumers. In particular, the ordered search facilitated by position auctions is beneficial for overall efficiency. (We make a similar point in section 3.3.) However, in other respects our model is quite different. In particular, there is no price competition in Chen and He (2006) and Athey and Ellison (2007), and so no role for prominence to affect market prices. ⁸

3.2 A Model of Prominence

Our underlying model of consumer choice is based on the framework developed by Wolinsky (1986). There are \( n \geq 2 \) firms, each of which supplies a single product at constant common unit cost, which we normalise to zero. There is no systematic quality difference among firms. Our aim is to extend this established framework to allow one product to be displayed more prominently than the others.

The number of consumers is normalized to one, and each consumer wishes to purchase one unit of one product from the market. The value of a firm’s product is idiosyncratic to consumers. Specifically, \((u_1, u_2, \cdots, u_n)\) are the values attached by a consumer to each of the \( n \) products, and \( u_i \) is assumed to be independently drawn from a common distribution \( F(u) \) on \([u_{\text{min}}, u_{\text{max}}]\) which has a positive density \( f(u) \). We also assume that all match values are realised independently across consumers. The surplus from buying firm \( i \)'s product is \( u_i - p_i \), where \( p_i \) is this firm’s price. If all match values and prices are known, a consumer will choose the product with the highest positive surplus \( u_i - p_i \). If \( u_i - p_i < 0 \) for all \( i \), she will leave the market without buying anything. In such a setting, making a firm more prominent has no impact.

Initially, however, we assume consumers have imperfect information about all prices and match values, and they must gather information through a sequential search process. By incurring a search cost \( s > 0 \), a consumer can discover any product’s price and match value. ⁹ We assume that the search process is without replacement and there is costless

⁸Chen and He (2006) do have prices charged by advertisers, but the structure of consumer demand in their model means that the Diamond Paradox is present, and all firms set monopoly prices.

⁹Note that we do not suppose that the prominent firm can be sampled at zero (or reduced) cost. In some situations—for instance, when a book is prominently displayed at the entrance to the store—it would be natural to assume that there was a reduced search cost to evaluate this option. However, in other situations—such as when the suggested links from a search engine are merely re-ordered—it is less natural to suppose that the prominent option has a lower search cost. All of our results concerning the impact
recall (i.e., a consumer can return to any firm she has visited without extra cost).

If all firms are equally prominent, we have the Wolinsky model where consumers randomly sample from firms. When one firm is more prominent than its rivals, we assume that all consumers will sample this firm first and then, if unsatisfied with this prominent product offering, they will go on to search randomly among the other firms.\(^\text{10}\) All firms maximize their profit, and they simultaneously set their prices \(p_i\) \((i = 1, 2, \ldots, n)\) conditional on whether they or their rivals (or neither) are prominent and their expectations of consumer behaviour.

### 3.2.1 An infinite number of suppliers

Consider first the relatively simple case with an infinite number of firms. Let \(p_\infty\) be the symmetric equilibrium price in the random search case. Expecting this uniform price, consumers will adopt a stationary stopping rule. They will stop searching when they find an offer where the net surplus \(u - p\) is greater than a reservation utility \(a - p_\infty\), where \(a\) satisfies

\[
\int_a^{u_{\text{max}}} (u - a) dF(u) = s, \tag{3.1}
\]

so that the incremental benefit from one more search is equal to the search cost.\(^\text{11}\) Therefore, in equilibrium a consumer will buy the first product which generates match utility of at least \(a\). It is clear that \(a\) in (3.1) is decreasing in \(s\), i.e., the higher the search cost, the sooner consumers will cease searching.

Now consider an individual firm’s pricing decision. If a firm chooses price \(p\) instead of \(p_\infty\), a consumer who samples it will buy its product if

\[u - p > a - p_\infty,\]

since she still expects that the other firms charge \(p_\infty\). Here, \(a - p_\infty\) is the reservation surplus when a consumer deals with this firm. Thus, this firm will aim to maximize

\[p [1 - F(p + a - p_\infty)].\]

Under regularity conditions (e.g., if the hazard rate \(f(u)/(1 - F(u))\) is increasing in \(u\)),

---

\(^{10}\) The assumption that all consumers sample the prominent firm first is for simplicity. Relaxing it by assuming that only a fraction (greater than \(1/n\)) of consumers do so will not change our results qualitatively.

\(^{11}\) This optimal stopping rule is well-known in the search literature—for instance, see Kohn and Shavell (1974), and Weitzman (1979).
the first-order condition determines the optimal price, so that\(^\text{12}\)

\[
p_\infty = \frac{1 - F(a)}{f(a)}. \tag{3.2}
\]

(This is expression (18) in Wolinsky (1986).) Provided the hazard rate is increasing, \(p_\infty\) in (3.2) decreases with \(a\), and hence \(p_\infty\) increases with the search cost \(s\). When \(s\) tends to zero (in which case \(a\) tends to \(u_{\max}\)), it follows that \(p_\infty\) also tends to zero. In particular, as emphasized in Wolinsky (1986), we do not see the “Diamond paradox” in this framework.

In equilibrium, \(a - p_\infty\) is a consumer’s expected surplus, including her search costs, from participating in the market.\(^\text{13}\) Thus, a consumer finds it worthwhile to engage in search whenever \(a - p_\infty \geq 0\), which requires that the search cost \(s\) not be too large. In equilibrium, industry profit is \(p_\infty\) (although each individual firm makes negligible profit) while total welfare—industry profit plus consumer surplus—is \(a\).

Consider next the case where firm 1, say, is made prominent and so is sampled first by all consumers. Since all the non-prominent firms are symmetrically placed, we focus on equilibria where the prominent firm charges \(p_1\) and all non-prominent firms charge the same price \(p_2\). We are interested in an active search market where some consumers search beyond firm 1.\(^\text{14}\) Once consumers have rejected the prominent firm’s offering, they will behave as in the random search case just described. Hence, each non-prominent firm faces the same decision problem as in the random search case, and \(p_2 = p_\infty\). Therefore, when a consumer considers the offer from the prominent firm, her reservation surplus is just \(a - p_\infty\). This implies that the prominent firm will also charge \(p_1 = p_\infty\). In sum, introducing prominence has no impact on market prices when there are infinitely many firms, and it merely redistributes consumer demand and profit between firms. Specifically, in equilibrium firm 1’s demand is \(1 - F(a)\) and its profit is

\[
p_\infty(1 - F(a)), \tag{3.3}
\]

while each non-prominent firm again earns negligible profit. We observe that (given the increasing hazard rate condition) this profit (3.3) increases with the search cost, and so

\(^{12}\) As usual in search models, there also exists an uninteresting equilibrium where consumers expect all firms to set very high prices which leave them with no surplus, consumers do not participate in the market at all, and so firms have no incentive to reduce their prices. We do not consider this equilibrium further in this chapter and next one.

\(^{13}\) From (3.1), if all firms’ prices are equal to \(p_\infty\), and if a consumer has found a product with match utility exactly equal to \(a\), she is indifferent between consuming this product and searching further, and so \(a - p_\infty\) is her expected surplus from participating in the market. Her expected surplus from the match achieved exceeds \(a - p_\infty\) by an amount that equals her expected search costs.

\(^{14}\) Similarly to footnote 12, there is another, less interesting equilibrium in which buyers expect that the price charged by non-prominent firms is very high so they never search beyond firm 1, and firm 1 sets the monopoly price. Since they do not expect consumers to visit them, the non-prominent firms have no incentive to deviate from this weakly dominated strategy. We do not consider such equilibria further.
a firm is willing to pay more to become prominent in a market where consumers incur higher search costs. Since market prices are not affected by prominence, we deduce that prominence has no impact on industry profit, consumer surplus, or total welfare. In the next section we show that this "neutrality" fails with a finite number of suppliers.

**Uniform example:** To illustrate these results, consider the case where \( u_i \) is uniformly distributed on the interval \([0, 1]\) (so that \( F(x) = x \)). Then (3.1) and (3.2) imply

\[
a = 1 - \sqrt{2s} ; \quad p_\infty = \sqrt{2s}
\]

and the value of prominence in (3.3) is \(2s\). The market is active provided that \(a - p_\infty\) is positive, i.e., if

\[
s < \frac{1}{2} , \text{ or } a > \frac{1}{2} .
\]

**3.2.2 A finite number of suppliers**

Having set out the benchmark case with an infinite number of firms, we now analyse the case where \(n\) is finite. First consider the random search case, i.e., where no firm is prominent. We focus on symmetric equilibria where each firm sets some price \(p_0\). Expecting this uniform price, the consumer should adopt the following optimal stopping rule\(^{15}\):

1. If \(p_0 \geq a\), where \(a\) is given in (3.1), the consumer should not participate in the market;

2. If \(p_0 \leq a\) the consumer should participate in the market and stop searching whenever she finds a product with \(u_i - p_i \geq a - p_0\), where \(p_i\) is this product's actual price; if no such product is found among the \(n\) options, she buys the product with the highest positive surplus. If all products have negative surplus, the consumer buys nothing.

For simplicity, from now on suppose that \(u\) follows the uniform distribution on \([0, 1]\).\(^{16}\)

To guarantee an active search market, assume condition (3.5) holds. Given the stopping rule, we claim that if a firm deviates to a price \(p\) while other firms offer the equilibrium

\(^{15}\)This strategy can be understood by backward induction. When only one un-sampled firm remains, it is clear that the stated stopping rule is optimal. Now keep the inductive assumption and consider the situation with more than one un-sampled firm remaining. If the available net surplus so far is less than \(a - p_0\), then searching one more firm is always desirable no matter what will happen after that. If the available net surplus so far is greater than \(a - p_0\), expecting that she will stop searching whatever she will find in next firm (because of the inductive assumption), a consumer will actually stop searching now.

\(^{16}\)Our main results will hold for a more general distribution. See Appendix C.11 in next chapter for the details.
price \( p_0 \) its demand is

\[
q_0(p) = \frac{h_0(1 - a + p_0 - p)}{\text{fresh demand}} + r_0 \quad \text{(3.6)}
\]

where

\[
h_0 = \frac{1}{n} \sum_{k=0}^{n-1} a^k = \frac{1}{n} \cdot \frac{1 - a^n}{1 - a}
\]

(3.7)

is the number of consumers who sample this firm’s product, and

\[
r_0 = \int_{p_0}^{a} u^{n-1} du
\]

(3.8)

is the number of consumers who buy from this firm after sampling all firms.

To understand (3.6), consider the two sources of firm \( i \)'s demand. First, a consumer may come to firm \( i \) after sampling \( k \) other firms but without finding a satisfactory product (i.e., their match values are less than \( a \)). The probability of this event is \( \frac{1}{n} a^k \) since a consumer will choose any search order with equal probability. Summing up these probabilities over \( k = 0, ..., n-1 \) leads to \( h_0 \). As usual in search models, a firm cannot affect the number of consumers who choose to sample its product—consumer search decisions are based on their expectations of prices, not the actual prices—and so \( h_0 \) does not depend on the firm’s price. After sampling the firm, a consumer will buy firm \( i \)'s product immediately provided \( u_i - p \geq a - p_0 \), which occurs with probability \( 1 - a + p_0 - p \). This explains the first term in (3.6). We call this portion of a firm’s demand the “fresh demand”. Second, a consumer may find that the net surplus of all \( n \) products is less than \( a - p_0 \) (so she never stopped), and then returns to firm \( i \) if it provides the highest positive surplus. The probability of this event is

\[
\Pr \left( \max \{0, u_j - p_0\} < u_i - p < a - p_0 \right) = \int_p^{p + a - p_0} (u_i - p + p_0)^{n-1} du_i = r_0,
\]

where the second equality follows from changing the integral variable from \( u_i \) to \( u = u_i + p_0 - p \). We call this portion of a firm’s demand the “returning demand”.

It is perhaps surprising that a firm’s returning demand in (3.8) is independent of its own price.\(^{17}\) In general, reducing a firm’s price has two effects on its returning demand: (i) more consumers are satisfied with this firm’s offer and so fewer consumers sample all firms, and (ii) the firm’s share of those consumers who sample all firms is increased. With a uniform distribution, these two effects exactly cancel out, and the returning demand is

\(^{17}\) Our demand function is legitimate only when the deviation price \( p \) is not too high. If \( p > 1 - a + p_0 \), then the fresh demand vanishes and the returning demand becomes \( \int_p^{p + a - p_0} u^{n-1} du = \int_{1-a+p_0}^{1} u^{n-1} du \), which is no longer independent of \( p \). Therefore, a firm’s profit function has a kink at \( p = 1 - a + p_0 \) and may fail to be globally concave. A similar issue exists in the prominence case. However, as we argue in footnote 20, this issue does not affect the equilibrium prices derived below.
price-independent.\footnote{For other distributions, the net impact is not zero. Nevertheless, our main insights based on the uniform distribution will still apply provided that the returning demand is less price elastic than the fresh demand. One can check that this happens when the density function increases or does not decrease too fast.} (This can be seen in Figure 3.1 below, where a decrease in firm 1’s price simply shifts the set of consumers who buy its product after sampling all firms’ offers to the left.) In particular, notice that a firm’s returning demand is less price elastic than its fresh demand (which does depend on its price).

When it charges price $p$ a firm’s profit is $p q_0(p)$. In symmetric equilibrium, each firm should have no incentive to deviate from $p_0$, which yields the first-order condition for $p_0$:

$$h_0(1 - a - p_0) + r_0 = 0.$$  

The first term is the marginal profit from fresh buyers, and the second term is the marginal profit from returning buyers. We rewrite this as

$$p_0 = 1 - a + \frac{r_0}{h_0}. \tag{3.9}$$

Notice that, in equilibrium, each firm’s fresh demand is $h_0(1 - a)$, so $r_0/h_0$ is proportional to the ratio of returning demand to fresh demand. When the number of suppliers $n$ becomes large, one can verify that $r_0/h_0$ tends to zero. As a result, when $n$ tends to infinity $p_0$ converges to $p_\infty = 1 - a$ as in (3.4).

In equilibrium, each firm’s demand is

$$q_0 = h_0(1 - a) + r_0 = h_0 p_0, \tag{3.10}$$

where the second equality follows from the first-order condition (3.9). Thus, total demand in the market is $Q_0 = n h_0 p_0$. On the other hand, total demand must also equal $1 - p_0^n$, since $p_0^n$ is the fraction of consumers who find each product’s utility is lower than its price and who eventually leave the market without purchase. Hence, we have the following formula for the equilibrium price $p_0$:

$$\frac{1 - a^n}{1 - a} = \frac{1 - p_0^n}{p_0}. \tag{3.11}$$

(One can check that (3.11) and (3.9) are indeed equivalent.) One can see there is a unique solution to (3.11) with $p_0 \in (1 - a, 1/2)$, given assumption (3.5).\footnote{The right-hand side of (3.11) is a decreasing function of $p_0$ when $p_0$ is positive. If $p_0 = 1 - a$, the right-hand side is greater the left-hand side given that $a > 1/2$. If $p_0 = 1/2$, the right-hand side is $(1 + \cdots + (1/2)^{n-1})$ which is less than the left-hand side $(1 + a + \cdots + a^{n-1})$. Thus, there is a unique solution to (3.11), where $p_0 \in (1 - a, 1/2)$.} Thus, $p_0 < a$ and

\footnote{For $p_0$ to be the equilibrium price, we need to ensure that each firm has no profitable deviation. The difficult issue is that, as mentioned in footnote 17, the demand function needs to be modified if a firm deviates to a too high price, and the profit function may be no longer globally concave. Nevertheless, the...}
the market is indeed active. Since the left-hand side of (3.11) increases with \( a \) while the right-hand side decreases with \( p_0 \), it follows that the equilibrium price falls with \( a \), i.e., it is increasing with the search cost \( s \).

We now turn to the case where firm 1 is made prominent. Since all the non-prominent firms are symmetric, suppose the prominent firm charges \( p_1 \) and all non-prominent firms charge the same price \( p_2 \). Define \( \Delta = p_2 - p_1 \) to be the price difference (if any) between the two kinds of supplier. If a consumer has seen firm 1’s offer, her optimal stopping rule is similar to that in the random search case since she expects all non-prominent firms to charge the same price \( p_2 \). That is to say, a consumer will stop searching when she finds a product which yields net surplus greater than \( a - p_2 \).

![Figure 3.1: Consumer Demand When Firm 1 is Prominent](image)

The pattern of consumer demand when there are just two firms is depicted in Figure 1. Here, consumers accurately predict that the second firm will be charging price \( p_2 \). If the net surplus from the first product, \( u_1 - p_1 \), is greater than \( a - p_2 \) (where \( a = 1 - \sqrt{2s} \)) the consumer buys this product immediately. (This is firm 1’s “fresh demand”.) If the net price defined in the first-order condition is the equilibrium price even if we take this issue into account. In the random search case, a firm’s profit function is \( \pi(p) = p \int_{p_0}^{1 - p_0} u^{a-1} du \) for a high deviation price \( p \in [1 - a + p_0, 1] \). If we can show that profit is decreasing on this price interval, then we are done. First, since \( u^{a-1} \) is logconcave, the integration term is logconcave in \( p \). So \( \pi(p) \) is logconcave (so quasi-concave). Second, we claim \( \pi'(p_0) = \int_{p_0}^{1} u^{a-1} du - p_0 < 0 \) (i.e., \( (1 - p_0)/p_0 < n \)). This is true because, from (3.11), \( (1 - p_0)/p_0 = (1 - a)/(1 - a) < n \). Finally, since \( \pi(p) \) is quasi-concave and \( \pi'(p_0) < 0, \pi'(p) < 0 \) for any \( p \geq p_0 \). Thus, \( \pi(p) \) decreases with \( p \) on \([1 - a + p_0, 1]\). The prominence case, though more complicated, can be treated similarly.
surplus is below this threshold level, the consumer samples the second firm, and then picks the option from the two with the greater net surplus (if one is positive). This generates firm 1’s “returning demand”. If neither option yields a positive surplus, the consumer does not buy at all (the shaded region in the figure).

The prominent firm’s demand when it charges $p$ and all non-prominent firms charge $p_2$ is

$$q_1(p) = (1 - a + p_2 - p) + r_1 ,$$  \hspace{1cm} (3.12)

where

$$r_1 = \int_{p_2}^{a} u^{n-1} \, du$$

is its returning demand. A non-prominent firm’s demand when it charges $p$ (and all other non-prominent firms choose $p_2$ and the prominent firm chooses $p_1$) is

$$q_2(p) = h_2 (1 - a + p_2 - p) + r_2 ,$$  \hspace{1cm} (3.13)

where

$$h_2 = \frac{a - \Delta}{n-1} \sum_{k=0}^{n-2} a^k = \frac{a - \Delta}{n-1} \cdot \frac{1 - a^{n-1}}{1 - a}$$

is the number of consumers who sample this firm and

$$r_2 = \int_{p_2}^{a} u^{n-2}(u - \Delta) \, du$$

is its returning demand. Note that each firm’s returning demand is independent of its own price $p$, as is the number of consumers who choose to sample its product. In addition, we see that $r_1 \geq r_2$ if and only if $\Delta = p_2 - p_1 \geq 0$.

By definition, all consumers sample the prominent product. They will buy this product immediately if $u_1 - p \geq a - p_2$, which has probability $1 - a + p_2 - p$, and this explains the first term in (3.12). Its returning buyers are those who find that the net surplus of all products is lower than $a - p_2$, but firm 1 provides the highest positive net surplus. The number of these consumers is

$$\Pr \left( \max_{j \geq 2} \{0, u_j - p_2\} < u_1 - p < a - p_2 \right) = \int_{p}^{p + a - p_2} (u_1 - p + p_2)^{n-1} \, du = r_1 .$$

For a non-prominent firm $i$ charging $p$, a consumer will sample it if she initially rejects the prominent firm, which has probability $\Pr(u_1 - p_1 \leq a - p_2) = a - \Delta$, and has visited other $k \leq n - 2$ unsatisfactory firms, which has probability $\frac{1}{n-1} a^k$. Summing these probabilities over $k = 0, \ldots, n-2$ yields $h_2$. She will buy immediately at this firm if its net surplus is greater than $a-p_2$. This explains the first term in (3.13). Its returning buyers
number

\[
\Pr \left( \max_{j \neq i, 1} \{0, u_i - p_1, u_j - p_2\} < u_i - p < a - p_2 \right) = \int_{p}^{p + a - p_2} (u_i - p + p_2)^{-2} (u_i - p + p_1) du_i = r_2 .
\]

The prominent firm’s profit when it deviates to \(p\) is \(p q_1(p)\), and each non-prominent firm’s profit when it deviates to \(p\) is \(p q_2(p)\). In equilibrium, no firm should want to deviate from the equilibrium price, so we get two first-order conditions for \(p_1\) and \(p_2\):

\[
1 - a + p_2 - 2 p_1 + r_1 = 0 ,
\]

\[
h_2 (1 - a - p_2) + r_2 = 0 .
\]

We can solve this pair of simultaneous equations to give

\[
p_1 = 1 - a + \frac{1}{2} \left( r_1 + \frac{r_2}{h_2} \right),
\]

\[
p_2 = 1 - a + \frac{r_2}{h_2} .
\]

Given assumption (3.5), within the square \([0, a]^2\), the pair of equations (3.15)–(3.16) has a unique solution within the region \((p_1, p_2) \in (1 - a, 1/2)^2\). This is proved in Appendix B.1. When \(n\) tends to infinity, both \(p_1\) and \(p_2\) converge to \(p_\infty = 1 - a\).

Using the first-order conditions, the prominent firm’s equilibrium demand is

\[
q_1 = 1 - a + \Delta + r_1 = p_1 ,
\]

while each non-prominent firm’s demand is

\[
q_2 = h_2 (1 - a) + r_2 = h_2 p_2 .
\]

Thus, total demand in the market is \(Q_1 = p_1 + (n - 1) h_2 p_2\). On the other hand, total demand must equal \(1 - p_1 p_2^{n-1}\), since \(p_1 p_2^{n-1}\) is the fraction of consumers excluded from the market (see Figure 3.1 for an illustration of the two-firm case). Therefore, we have the following equation relating \(p_1\) and \(p_2\) to \(a\):

\[
p_1 + \frac{1 - a^{n-1}}{1 - a} (a - \Delta) p_2 = 1 - p_1 p_2^{n-1} .
\]

3.2.3 The impact of prominence

In this section we present five results describing the impact of making a firm prominent on market outcomes. (The proofs of each result are presented in the Appendix B.) The first question is how making a firm prominent influences the equilibrium prices:
Proposition 3.1 (i) The prominent firm charges a lower price than non-prominent firms.
(ii) The prominent firm’s price is lower than with random search while the prices of non-prominent firms are higher. In sum:

\[ p_1 < p_0 < p_2. \]  
(3.20)

The intuition for this result is as follows. If \( p_0 \), \( p_1 \), and \( p_2 \) are not too far apart from each other, then, compared to the random search case, the prominent firm’s demand consists of proportionally more fresh demand while each non-prominent firm’s demand consists of proportionally more returning demand.\(^{21}\) Since fresh demand is more price sensitive than returning demand, a prominent firm faces more elastic demand than a firm in the random-search environment, which in turn faces more elastic demand than a non-prominent firm.

It is useful to consider two polar cases. When \( a \approx 1 \) (i.e., \( s \approx 0 \)), consumers sample all firms before they purchase, and so prominence has no impact and all prices converge to the full-information price \( \bar{p} \), say, which satisfies \( n\bar{p} = 1 - \bar{p}^n \). (This formula for \( \bar{p} \) is obtained from (3.11) by letting \( a \to 1 \).) At the other extreme, when \( a \approx 1/2 \), all prices converge to the same price \( 1/2 \), the monopoly price. Here, the high search cost makes a consumer willing to buy whenever she finds a product with positive surplus, and so each firm (prominent or not) acts as a monopolist. Thus, the price difference \( \Delta \) caused by prominence will vanish when the search cost is too high or too low, and it is most pronounced when the search cost is at an intermediate level.

The next result describes the impact of prominence on total demand and on the search intensity:

Proposition 3.2 (i) Total output is lower when a firm is made prominent, and (ii) the average number of searches made by consumers is smaller when a firm is made prominent.

Proposition 3.1 showed that the impact of making one firm prominent was to make that firm’s price fall and to raise the price offered by non-prominent firms. As such, the impact on overall demand is not clear \textit{a priori}. However, the first part of Proposition 3.2 shows that the effect of the higher prices from the non-prominent firms is more marked than the impact of the price reduction by the prominent firm, and overall output falls with prominence.

Proposition 3.2 demonstrates two contrasting effects of prominence: output falls and the total occurrence of search costs also falls. While the second factor has benefits in

\(^{21}\)In equilibrium, the prominent firm’s ratio of fresh to returning demand is \( (1 - a + \Delta)/r_1 \), a non-prominent firm’s ratio of fresh to returning demand is \( (1 - a)/r_2 \), while each firm’s ratio of fresh to returning demand when no firm is prominent is \( (1 - a)/r_0 \). When all prices are similar, \( r_0, r_1 \) and \( r_2 \) are also similar. Therefore, since \( 1 > r_0 = \frac{a}{n}(1 + \ldots + a^{n-1}) > \frac{a}{n}(1 + a + \ldots + a^{n-1}) \approx h_2 \), the claim in the text is valid.
terms of reducing search costs, it also means that the average match utility is reduced. In fact, as explained in the next result, with prominence there is too little search relative to the efficient benchmark.

**Proposition 3.3** Welfare is reduced when a firm is made prominent.

The intuition behind this result goes as follows. For a given level of total demand, welfare is maximized if each product has the same price. If the market has a uniform price, the consumer’s stopping rule is independent of the price and this is socially efficient because the consumer and the social planner face the same trade off between search costs and match utility. However, when a firm is made prominent, this induces non-uniform prices in the market. Therefore, keeping total demand constant, prominence induces sub-optimal search behaviour. To be precise, when \( \Delta = p_2 - p_1 > 0 \), those consumers with \( u_1 \in [\alpha - \Delta, \alpha] \) will not search beyond firm 1 even though it would be socially efficient for them to do so. Moreover, when \( \Delta > 0 \) too many of the returning buyers end up buying from firm 1. A second, reinforcing reason why welfare falls with prominence is that total output falls (see Proposition 3.2). In sum, making a firm prominent means that output is reduced and this output is poorly distributed across consumers.\(^22\)

We next investigate how this welfare loss is distributed across consumers and industry. It turns out that consumers in aggregate are always made worse off with prominence (see an explanation in the proof part):

**Proposition 3.4** Consumer surplus is reduced when a firm is made prominent.

Finally, we turn to the impact of prominence on profit:

**Proposition 3.5** (i) The prominent firm earns more than a non-prominent firm, and it also earns more than it would with random search, and (ii) industry profit is higher if one firm is made prominent except when \( n = 2 \) and \( \alpha \) is relatively small.

Part (i) of this result is not surprising. For instance, the prominent firm could choose to set the non-prominent firms’ equilibrium price, in which case it still makes more profit than its rivals since it has greater demand. But it can do still better than this by choosing a lower price than its rivals. Part (ii) requires a more delicate analysis, since the impact

\(^{22}\)There is a clear parallel with the welfare effects of price discrimination. If a firm sets different prices for units which cost the same to produce, then total output is sub-optimally distributed across consumers. Therefore, price discrimination can only improve welfare if it induces total output to rise. (See Varian (1985), for instance.) One difference with the (monopoly) price discrimination setting is that monopoly profit is sure to rise with non-uniform pricing allowed (since the firm could set uniform prices if it wishes), whereas in the prominence model prices are determined at equilibrium, and it is not obvious that industry profit rises with prominence. Indeed, as we show in Proposition 3.5, the impact of prominence on industry profit is ambiguous. (The parallel will be more apparent when we come to the central-pricing case in next chapter.)
of the price cut by the prominent firm must be weighed against the price rise by the non-prominent firms. In effect, part (ii) shows that the price cut usually has less of an impact on industry profit than the price rise. This is not surprising in the light of our earlier result that total demand falls when a firm is made prominent.

Proposition 3.5 is silent about the impact of prominence on the non-prominent firms’ profit. In fact, this impact is ambiguous due to three effects: (i) a non-prominent firm suffers from being pushed further back in each consumer’s search order and (ii) from the lower price offered by the prominent firm, but (iii) it benefits from the fact that its other non-prominent rivals raise their price. Let \( \pi_2 \) denote a non-prominent firm’s profit when one firm has been made prominent and let \( \pi_0 \) denote a firm’s profit in the case of random search. First, when \( n = 2 \), it is clear that \( \pi_2 < \pi_0 \) since there is no countervailing benefit (iii). Moreover, for fixed \( a < 1 \), \( \pi_2 < \pi_0 \) always holds for sufficiently large \( n \) because 
\[
\lim_{n \to \infty} \frac{\pi_2}{\pi_0} = \lim_{n \to \infty} \frac{b a}{K_0} = a < 1.
\]
Thus, with either two firms or with many firms a non-prominent firm earns less than it would in a random search environment. By contrast, though, for fixed \( n \geq 3 \), we can show that \( \pi_2 > \pi_0 \) whenever \( a \) is sufficiently close to 1. (See Appendix B.7.) Therefore, when the search cost is very low all firms are better off when one is made prominent.

One question we have not yet discussed is how the impact of prominence is affected by the search cost and the number of firms in the market. A complete investigation of this issue would be lengthy, and here we merely present some numerical examples. Figure 3.2 reports how the impact varies with \( a \) when \( n = 5 \), where the upper (thin) line is the difference in industry profit, the bottom (thick) line is difference in consumer surplus, and the central (medium) line is difference in welfare, all calculated as we move from the random search case to the case with prominence. All variables vary non-monotonically with \( a \). In fact, this pattern holds quite generally. As we have pointed out, \( p_1 \) and \( p_2 \) coincide with \( p_0 \) when \( a \) tends to \( 1/2 \) or 1, so the impact of prominence vanishes at the extremes of \( a \). Figure 3.3 reports how the impact on profit, consumer surplus and welfare varies with \( n \) when \( a = 0.7 \) (i.e., when \( s = 0.045 \)). The non-monotonic pattern for industry profit and consumer surplus seen in Figure 3.3 seems quite widespread according to further numerical simulations. It is natural that the impact of prominence becomes less pronounced as the number of firms becomes large, since we are converging to the infinite firm case where prominence has no impact on industry profit, consumer surplus or welfare. However, it is less clear why it is common for industry profit to rise (and consumer surplus to fall) with \( n \) when the number of suppliers is relatively small. In both figures, it appears that the impact of prominence on overall welfare is small relative to the distributional impact on profit and consumer surplus separately.
Figure 3.2: Impact with $a$ ($n = 5$)  
Figure 3.3: Impact with $n$ ($a = 0.7$)

We end this section by pointing out the implications of these results in situations where there is a profit-maximizing platform through which firms sell their products to consumers. If the platform can extract the whole industry profit by, for example, charging each firm a fixed fee for access to its consumers, Proposition 3.5 tells us that it (usually) has an incentive to make one supplier more prominent than the others. However, if the platform can extract total welfare by charging both firms and consumers, Proposition 3.3 tells us that it has no incentive to do so.

3.3 Asymmetric Firms

Until now, our analysis leads to a somewhat pessimistic view of the benefits of prominence, at least from the viewpoint of consumers and overall welfare. This assessment might change in a setting in which firms have differing product qualities, for then prominence could be used to guide consumer search towards better products.\(^\text{23}\)\(^\text{24}\) In this section we investigate this possibility, assuming that consumers cannot discern a firm’s average quality directly.\(^\text{25}\) Since this analysis is considerably more involved than with the benchmark symmetric case, for simplicity we suppose there are infinitely many suppliers. As in the symmetric firm case, this assumption implies that prominence has no impact on equilibrium prices. However, in contrast to the symmetric case, this “pricing neutrality” nevertheless allows for a significant impact of prominence on industry profit, consumer surplus and welfare.\(^\text{26}\)

\(^{23}\)This point is emphasized in Chen and He (2006) and Athey and Ellison (2007). As in the model we present in this section, the highest quality firm in their models (i.e., the firm with the highest probability of making a good match with each consumer) is willing to bid the most to be listed first, and so consumers have an incentive to click on the sponsored links in the order they appear. As with our model, this means the resulting order search is efficient.

\(^{24}\)The analysis in this section can simply be adapted to allow firms to offer symmetric products but to have different unit costs. In this case the firm with the lowest cost will have the most to gain from becoming prominent.

\(^{25}\)This assumption contrasts with Weitzman (1979), who assumes that consumers know the distribution of payoffs for each option in advance.

\(^{26}\)With a finite number of firms, the “search-guidance" effect will interact with the price effect analyzed in section 3.2.2 which also influences welfare. The results in this section apply at least to the case with large but finite $n$ since the price effect there is weak. In addition, with a finite number of firms a major
Suppose that firms are distinguished by the parameter $\alpha$, where a higher $\alpha$ represents a higher-quality firm (on average). Suppose that a consumer’s match utility $u$ from a type-$\alpha$ firm is uniformly distributed on the interval $[0, \alpha]$. For simplicity, suppose that firms have the same marginal cost of supply even though they differ in quality. This situation might apply to novels, where the marginal cost of production need not be strongly related to quality.) Suppose that $\alpha$ is itself uniformly distributed, on the interval $[1 - \frac{\varepsilon}{2}, 1 + \frac{\varepsilon}{2}]$. Thus, $\varepsilon$ measures the degree of firm heterogeneity present in the market, and the symmetric firm model corresponds to the degenerate case $\varepsilon = 0$.

Let $p_{\alpha}$ denote the equilibrium price of the type-$\alpha$ firm. Consumers are assumed to know the distribution of $\alpha$ in the population of firms, as well as the equilibrium (but not actual) prices $p_{\alpha}$. If there is no prominent firm, they will search among firms randomly; if one firm is made prominent, they are assumed to consider its offer first. In either case, they will use a stationary stopping rule, and they will buy a product if and only if the net surplus, $u - p$, is greater than some threshold $y$. Given the equilibrium prices $p_{\alpha}$, $y$ satisfies the indifference condition

$$\frac{1}{\varepsilon} \int_{1 - \frac{\varepsilon}{2}}^{1 + \frac{\varepsilon}{2}} \left( \frac{1}{\alpha} \int_{p_{\alpha} + y}^{\alpha} (u - p_{\alpha} - y) \, du \right) \, d\alpha = s \quad (3.21)$$

Here, the left-hand side of (3.21) is the expected benefit from one more search. For this expression to be valid, we must have all types of firm be active in equilibrium (i.e., $\alpha - p_{\alpha} > y$ for all $\alpha$). This requires there not be too much quality variation, and we derive an explicit ceiling on $\varepsilon$ below. In addition, we continue to assume (3.5), so that $s < \frac{1}{8}$. As usual, $y$ is also each consumer’s expected net surplus from following the optimal stopping rule described above.

### 3.3.1 Equilibrium prices and the value of being prominent

We first characterize the equilibrium prices given a candidate stopping rule $y$. Regardless of whether there is prominence for one firm or not, given the reservation surplus $y$, a type-$\alpha$ firm’s profit is proportional to $p \left( 1 - \frac{y_{\alpha} + y}{\alpha} \right)$ when it sets price $p$. The equilibrium

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27This analysis can easily be adapted to handle situations where the cost depends on $\alpha$. The results reported below remain valid provided that cost does not vary by “too much”. In other cases, however, it might happen that prominence harms market efficiency by inducing inefficient search. That is because the firm having the greatest incentive to become prominent may not be the highest gross surplus provider.

28It is not difficult to extend the following analysis to other distributions for $\alpha$. 

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price is chosen to maximize this profit, and so

\[ p_\alpha = \frac{1}{2}(\alpha - y) \]  \hspace{1cm} (3.22)

In particular, a higher-quality firm will set a higher price in equilibrium.

Substituting the prices (3.22) into (3.21) shows that \( y \) satisfies

\[ E_\alpha \left[ \frac{(\alpha - y)^2}{8\alpha} \right] = s \]  \hspace{1cm} (3.23)

where \( E_\alpha \) is the expectation operator using the distribution for \( \alpha \). Given the uniform distribution for \( \alpha \), expression (3.23) becomes

\[ \eta_\epsilon y^2 - 2y + 1 - 8s = 0 \]

where \( \eta_\epsilon \equiv E_\alpha \left[ \frac{1}{\alpha} \right] = \frac{1}{2} \ln \frac{1 + \epsilon/2}{1 - \epsilon/2} \). Here, \( \eta_\epsilon \) increases with \( \epsilon \) and \( \eta_\epsilon \to 1 \) as \( \epsilon \to 0 \). The relevant root of the above quadratic is

\[ y = \frac{1 - \sqrt{1 - \eta_\epsilon(1 - 8s)}}{\eta_\epsilon} \]  \hspace{1cm} (3.24)

When \( \epsilon \) becomes small \( y \) tends to \( 1 - \sqrt{8s} \), which is the reservation net surplus \( (\alpha - p_\alpha) \) in the case with symmetric firms in (3.4). The stopping rule in (3.24) has intuitive properties. First, as usual, \( y \) is decreasing in \( s \). In particular, as \( s \to \frac{1}{8}, \ y \to 0 \) and consumers will choose the first product which gives them a positive net surplus. Second, \( y \) is increasing in \( \eta_\epsilon \) and therefore increasing in \( \epsilon \). That is, the greater the degree of firm heterogeneity, the more choosy consumers will be. From (3.22), these two properties imply that equilibrium prices will increase with \( s \) but decrease with \( \epsilon \).

We need to verify that all types of firm are active in the market, since these calculations were predicated on that being so. This requires \( \alpha - p_\alpha > y \) for all \( \alpha \), and from (3.22) this is equivalent to \( y < 1 - \frac{\epsilon}{2} \).\textsuperscript{29} This requirement determines a maximum feasible level for \( \epsilon \), say \( \hat{\epsilon} \), where

\[ \frac{\hat{\epsilon}}{2} + \frac{1 - \sqrt{1 - \eta_\epsilon(1 - 8s)}}{\eta_\epsilon} = 1 \]

For example, when \( s = \frac{1}{32} \), one can check that \( \hat{\epsilon} \approx 0.95 \). Therefore, as long as the quality variation among firms is not too great, so \( \epsilon < \hat{\epsilon} \), all firms will be active in equilibrium and (3.21) is justified. When the search cost is very low (\( s \approx 0 \)), one can check that there can be very little heterogeneity if our analysis is to be valid, so that \( \epsilon \approx 0 \) in that case.

\textsuperscript{29}In addition, we require \( \eta_\epsilon < \frac{1}{1 - 8s} \) to guarantee that \( y \) in (3.24) is a real solution. It turns out that the requirement that \( y < 1 - \frac{\epsilon}{8} \) is the tighter constraint. If \( \eta_\epsilon = 1/(1 - 8s) \) then \( y = 1/\eta_\epsilon \) in (3.24), and \( 1/\eta_\epsilon > 1 - \frac{\epsilon}{2} \) for all \( \epsilon > 0 \).
Within this framework, if a firm is not prominent its profit is zero; if it is prominent, its profit is
\[ \rho_\alpha \left(1 - \frac{y + p_\alpha}{\alpha} \right) = \frac{(\alpha - y)^2}{4\alpha} \]
which increases with \( \alpha \). We deduce that the highest-quality firm has the most to gain from becoming prominent. If there is a procedure to endogenize prominence, the highest-quality firm is therefore likely to become the prominent seller. For example, if a platform
is selling (or auctioning) the prominent position, the highest-quality firm is willing to pay the most. In this case, prominence becomes a signal of high quality (and high price).

3.3.2 The impact of prominence

One issue is whether it is in a consumer's interest to sample the prominent firm first in those situations in which consumers are not forced to do so. As just mentioned, the prominent firm is predicted to offer a high-quality product (on average), but also to set a high price. To understand a consumer's incentives, suppose hypothetically a consumer can choose the type \( \alpha \) of the first firm sampled, and denote by \( v(\alpha) \) the payoff to her if she chooses to go first to a type-\( \alpha \) firm. Then
\[ v(\alpha) \equiv \frac{1}{\alpha} \int_{y+p_\alpha}^{\alpha} (u - p_\alpha) \, du + (1 - \varphi_\alpha) y - s , \tag{3.25} \]
where
\[ \varphi_\alpha \equiv 1 - \frac{y + p_\alpha}{\alpha} \]
denotes the probability that a consumer buys the type-\( \alpha \) firm's product when she samples it. To understand expression (3.25), note that the first term represents net surplus in the event the match value is above the threshold \( y + p_\alpha \), while if the match value is below the threshold (which occurs with probability \( 1 - \varphi_\alpha \)), the consumer starts from scratch with random search, which we know yields her the expected payoff \( y \). Finally, the consumer must pay the search cost \( s \) to sample the first product.

After substituting the price (3.22) into (3.25), this formula simplifies to
\[ v(\alpha) = \frac{y^2 + \alpha^2 + 6\alpha y}{8\alpha} - s . \]
(As required, one can verify using expression (3.23) that the expected value of \( v(\alpha) \) over all \( \alpha \) just equals \( y \).) Since \( v(\alpha) \) increases with \( \alpha \), it follows that if a consumer could choose the type of the firm first sampled, she would choose the highest-quality firm. Since the prominent firm is the highest-quality firm, we deduce that a consumer has a strict incentive to visit that firm first, even if she has a choice to ignore the prominent firm. This argument also shows that consumers are better off when there is a prominent firm.
compared to the case where search is random: the firm that is willing to pay most for the privilege of being prominent is also the firm that consumers most want to visit first. That contrasts with the symmetric firm case, where we showed that consumers were worse off when there was a prominent firm (Proposition 3.4).

Consider next the impact on industry profit. Let $\Pi_0$ denote equilibrium industry profit when consumers search randomly. Using a similar argument to that for consumer surplus just above, if all consumers sample a type-$\alpha$ firm first, industry profit, denoted $\Pi(\alpha)$, is

$$\Pi(\alpha) = \varphi_{\alpha} p_{\alpha} + (1 - \varphi_{\alpha}) \Pi_0.$$ 

The value of $\Pi_0$ can be obtained by noting that $E_{\alpha}\Pi(\alpha) = \Pi_0$, so that

$$\Pi_0 = \frac{E_{\alpha}[\varphi_{\alpha} p_{\alpha}]}{E_{\alpha}[\varphi_{\alpha}]}$$

and industry profit with random search is just a weighted average of market prices. Incremental industry profit when the type-$\alpha$ firm is made prominent is $\Pi(\alpha) - \Pi_0 = \varphi_{\alpha}(p_{\alpha} - \Pi_0)$, which is positive whenever the type-$\alpha$ firm’s price is above the market average price. In particular, since $p_{\alpha}$ increases with $\alpha$, this is true if the highest-quality firm is made prominent. The intuition is simple: compared to random search, the prominence case distributes more demand to the firm with the highest profit margin. This result implies that, as in the symmetric-firm case, the platform wants to implement a prominent position if it can extract the whole industry profit.

We summarize these results in the following:

**Proposition 3.6** In the asymmetric environment with an infinite number of firms (and with symmetric costs), a firm with a higher average quality charges a higher price, and the highest-quality firm has the greatest incentive to become prominent. Making this firm prominent boosts industry profit and consumer surplus (and hence total welfare) relative to the situation in which no firm is prominent.

### 3.4 Asymmetric Consumers

In this section we briefly discuss the impact of prominence when consumers differ in their cost of search. Here, a new role for prominence emerges, which is that the prominent firm can exploit the high search cost consumers by setting a high price.\(^{30}\) We can understand this effect most easily in the context of a homogenous product market. Arbatskaya (2007) analyzes this situation in a fairly general framework with $n$ completely ordered firms.

\(^{30}\)Athey and Ellison (2007) also have heterogeneous search costs. However, since there is no product-market competition in their model, there is no scope for exploiting the high search cost consumers.
However, for our purposes the basic point can be illustrated very simply.\textsuperscript{31} A stark model in which consumers have different search costs is in Varian (1980), where $n$ identical firms compete to offer a homogenous product to consumers. A fraction $\lambda$, say, of consumers do not have any search cost and know the prices of all firms, and so buy from the lowest-price supplier. The remaining consumers have an infinite cost of searching beyond their initial sample, and they buy from the first firm they encounter (provided that that firm’s price is no higher than their reservation utility). In Varian’s model, the first firm a consumer sees is random and firms compete by offering random prices. (A firm must trade off the need to compete for the informed consumers with the profit obtained by exploiting the uninformed consumers.) If the reservation utility for a unit of the homogenous product is $v$ and production is costless, Varian shows that in symmetric equilibrium the expected price paid is $p_0 = (1 - \lambda)v$, which is the weighted average of the competitive price ($p = 0$) and the monopoly price ($p = v$).

Suppose now that one firm is made prominent in this market, in the sense that all consumers see its price first, and that there are at least two other firms. This firm therefore knows that it will serve the entire market of uninformed consumers, while the other firms know that they will not meet any of these consumers. So the non-prominent firms can compete only for the informed consumers, and so in Bertrand fashion they will offer the competitive price $p_2 = 0$. Thus, making a firm prominent exerts a competitive externality on the remaining firms, since they are left with only the fully-informed consumers. It is more profitable for the prominent firm to supply only the uninformed consumers, in which case it should charge the monopoly price $p_1 = v$. In sum, in this framework with heterogenous search costs, making a firm prominent causes that firm to raise its price, while non-prominent firms are forced to reduce their price relative to the random search case, so that

$$p_1 > p_0 > p_2.$$  \hspace{1cm} (3.26)

(Under our assumption that each consumer has unit demand there is no impact on industry profit, aggregate consumer surplus, or welfare.) Here, prominence has the opposite impact on prices compared to our earlier model when all consumers had the same search cost (see expression (3.20) above).

In a less extreme model where consumers have intermediate search costs, we still expect to see the same relative prices as in (3.26). In particular, Arbatskaya (2007) shows that equilibrium prices fall monotonically as consumers move along the list of firms. Intuitively, those consumers who immediately stop at the prominent firm are more likely to have a high search cost than other consumers, and so we expect that the prominent firm faces

\textsuperscript{31}This simplified framework also avoids one awkward point in Arbatskaya (2007), which is that an inactive firm offering a particular price must be assumed in order to avoid the Diamond Paradox.
less elastic demand than its rivals. Of course, since consumers here get a worse deal when they buy from the prominent firm (unlike in our other models), in many situations we expect that consumers will learn this feature of the market, and perhaps start to avoid the prominently displayed products if they have a choice to do so. (See a similar point in next chapter when we discuss the central-pricing model.)

This argument relies very much on assuming a homogeneous product in the market. As we have said, in many markets it is more natural to assume a degree of product differentiation. In a richer model where (i) consumers have different search costs and (ii) there is product differentiation à la Wolinsky, the prominent firm's price could be higher or lower than its rival's prices, depending on the relative importance of (i) and (ii). In particular, we expect that our earlier prediction that the prominent firm will offer a lower price remains valid when search costs do not vary too much.

3.5 Conclusion

This chapter has examined the implications of biasing each consumer's search order, so that consumers encounter some prominent firm first. In an environment without systematic quality differences, we find that the prominent firm will charge a lower price than non-prominent firms, and the prominent firm will set a lower price than if no firm is prominent while non-prominent firms will set higher prices. We also find that making a firm prominent will typically reduce average search intensity, increase industry profit, but lower consumer surplus and welfare. In a richer environment in which firms differ in quality, the firm with the highest average quality most wants to become prominent if the cost variation among firms is insignificant, and making it prominent can increase industry profit, consumer surplus and welfare. In this situation, prominence acts to guide consumers efficiently towards better products.

A feature of these models—with the exception of the model with heterogenous search costs in section 3.4—is that rational consumers will prefer to sample the prominent product first, even when they need not do so. In the benchmark model of section 3.2.2, consumers expect the prominent firm to charge a lower price than others so they sample it first; predicting this consumer behaviour, the prominent firm does indeed have an incentive to charge a lower price. In theory, even without exogenous prominence, this kind of asymmetric equilibrium might occur. However, there are many such equilibria, and without guidance it will be hard for consumers to coordinate their expectations on one favoured firm. In this sense prominence functions as a coordination device.\textsuperscript{32} In models in which

\textsuperscript{32}A similar feature is seen in Bagwell and Ramey (1994). However, in the earlier paper, this coordination is good for consumers, since they obtain a lower price. In our model, prominence is often bad for consumers in aggregate.
the prominent firm offers a worse deal to consumers (as in the heterogeneous search cost model), by contrast, for the model to be convincing consumers must either have some form of bounded rationality or they must be exogenously compelled to sample this firm first.

The topic of prominence deserves further research. For instance, making a firm prominent will have an impact on product variety in a free-entry market. In situations where non-prominent firms enjoy lower profit compared to the random search case, we expect that prominence will reduce the free-entry number of firms. But this will not necessarily harm efficiency since free entry may result in excess entry in the random search case.\textsuperscript{33}

It would also be useful to study the situation with multiple prominent products and the impact of prominence on a firm’s choice of product quality.

The fact that prominence as well as pricing can affect consumer behaviour is important for business strategy and public policy in settings ranging from the presentation of choices about savings plans to those concerning healthy eating, promotional marketing and its regulation, the operation of commission schemes for sales agents, and so on. Our analysis has used a consumer search framework with product differentiation and imperfect competition to examine interactions between prominence and pricing, and some implications for profits and welfare. With suitable adaptation, such a framework might have application to a variety of circumstances where market participants or public authorities seek to influence consumer choice by framing the ways that choices are presented.

\textsuperscript{33}See Proposition 3 in Anderson and Renault (1999).
Chapter 4

Prominence in Search Markets: Competitive Pricing vs Central Pricing

4.1 Introduction

As we have argued in the previous chapter, prominence is commonplace in the market. It can significantly affect consumer choices in a costly search environment by causing “biased” search order. Firms realize its function and are willing to pay for becoming prominent. A leading example is the online paid placement (Athey and Ellison (2007), Borgers et al. (2007), Chen and He (2006), Edelman et al. (2007), Varian (2007)). This chapter further investigates the market implications of prominence. Our aim is twofold. Firstly, we extend the previous chapter by considering multiple prominent products. This extension allows us not only to examine how price and welfare vary with the number of prominent products but also to discuss endogenous prominence (e.g., in the case where a platform such as a search engine can choose how many prominent positions to sell).

Secondly and more importantly, we compare the implications of prominence between different market structures. Chapter 3 has considered the competitive-pricing situation where each firm sells one product. Nevertheless, in the market, there also exist multi-product firms which sell many differentiated products (or brands) and control all prices (at least to some extent). For example, a restaurant supplies many dishes and sets all prices by itself; a supermarket or other retailer sells several brands of a product and it often has some discretion to influence retailing prices. In such cases, consumers also need to search to find a satisfactory product, and we also often see prominent products such as the dishes recommended in a special menu and the brands displayed on the gondola ends. Therefore, it is desirable to study the impact of prominence in a central-pricing
model where one multi-product seller supplies several differentiated products and chooses all prices by itself. Surprisingly, we find that the impact of prominence in this situation is almost opposite to that in the competitive-pricing case.

Section 4.2 deals with the competitive-pricing model with multiple prominent products. We suppose consumers will sample among prominent products first, and if they are not satisfied with these products, they will go on to search among non-prominent products. We find that prominent products are always cheaper than non-prominent ones. This generalizes the price result in Chapter 3. However, now making several products prominent may raise all products' prices (and so prominence can be totally anti-competitive), which will never happen in the single-prominent-product case where making one product prominent will always lower its price. We also find that the relationship between welfare variables and the number of prominent products is non-monotonic. This is easy to understand since the case without prominent product is the same as the case where all products are prominent. Our task is to specify this non-monotonic relationship. We show that, at least in the case with a small search cost, industry profit will first increase and then decrease with the number of prominent products, while consumer surplus and total welfare will vary in the opposite way. In particular, when the search cost is relatively small, industrial profit will reach its maximum while consumer surplus and total welfare will reach their minimums when about half of products become prominent. In light of these results, we further discuss the optimal number of prominent positions for a platform.

Section 4.3 analyzes the central-pricing model. We find that prominent products will now be more expensive than others. This is because, due to the search order, raising the price of prominent products will be more likely to drive consumers to buy non-prominent products, while raising the price of non-prominent products will be more likely to drive consumers to leave the market. Moreover, relative to the case without prominent product, the firm will raise the price of prominent products but lower that of non-prominent ones. As far as welfare implications are concerned, the firm will always earn more by making some products prominent, and both of consumer surplus and total welfare can also be enhanced (at least in the case with a small search cost). The total welfare effect is determined by search efficiency and output efficiency. If total output is given, a uniform price among products will lead to socially efficient search behavior. Now the prominent products are more expensive than others, which will induce consumers to search too much and result in search inefficiency. On the other hand, introducing prominent products will also change total output. In the central-pricing case, we find total output will go up because of the lower price of non-prominent products. This positive output effect can dominate the negative search effect. While in the competition case, total output will go down after introducing prominent products, and so total welfare must go down.
An important feature which emerges in the model with multiple prominent products is that consumers' optimal stopping rule (given the restricted search order) will become non-stationary. That is, as the search process goes on, consumers need to revise the cutoff reservation surplus level. This requires consumers to remember their positions in the search process as well as to form expectation of the prices of un-sampled products. Such a decision process is usually demanding for ordinary consumers. Therefore, in this chapter, we also consider a simple behavioral stopping rule in the spirit of satisficing behavior proposed by Simon (1955). We suppose consumers are equipped with some exogenous "aspiration level" and they will stop searching as long as this aspiration level is met. We find that our main price result still holds that the prominent product will be cheaper than others in the competition case but more expensive in the central-pricing case. This indicates that our price prediction actually relies little on consumer rationality.

Our model draws on the rich literature on search in the market. Most of papers in this literature focus on the homogeneous product market and aim to explain price dispersion. (See, e.g., the survey by Baye et al. (2006).) Our model is more related to the branch on search with differentiated products. It is initiated by Wolinsky (1984,86) to investigate the impact of imperfect information on market competitiveness and product variety. It has been recently developed and applied by Anderson and Renault (1999, 2000), Wolinsky (2005), and others. All these papers consider a random sequential search process, while we introduce partially restricted search order to explore the impact of prominence. Our central-pricing model is related to Salop (1977) which also considers a multi-product (or multi-store) monopoly with consumer search. But that paper considers a homogenous product and explores how the monopoly can use dispersed prices among its multiple sublets as a sorting device to discriminate over consumers with heterogenous search costs. (Other related literature has been reviewed in Chapter 3.)

4.2 Competitive Pricing

Our model generalizes the previous chapter to allow for more than one prominent product. There are $n \geq 2$ firms, each of which supplies a single product at constant common unit cost, which we normalize to zero. There are no systematic quality differences among products, but some products are more prominent than others. Let $A = \{1, \cdots, m\}$ be the set of prominent products and $B = \{m + 1, \cdots, n\}$ be the set of non-prominent products. We call them prominent pool $A$ and non-prominent pool $B$, respectively. Firms maximize their profit, and they simultaneously set their prices $p_i$ ($i = 1, 2, \cdots, n$) conditional on the relative prominence between their products and their expectations of consumer behavior as specified below.
There are a large number of consumers with measure normalized to one. Each consumer has a unit demand, and the value of a firm's product is idiosyncratic to consumers. Specifically, \((u_1, u_2, \cdots, u_n)\) are the values attached by a consumer to different products, and \(u_i\) is assumed to be independently drawn from a common distribution \(F(u)\) on \([u_{\text{min}}, u_{\text{max}}]\) which has a positive and differentiable density function \(f(u)\). We also assume that all match utilities are realized independently across consumers. The surplus from buying one unit of firm \(i\)’s product is \(u_i - p_i\). If all match utilities and prices are known, a consumer will choose the product providing the highest surplus. If \(u_i - p_i < 0\) for all \(i\), she will leave the market without buying anything.

Initially, however, we assume consumers have imperfect information about the actual price and match utility of each product, but they can gather information through a sequential search process. The key component of a sequential search model is the stopping rule. In this chapter, we mainly focus on the setting where a consumer can find out a product’s price and match utility by incurring a search cost \(s > 0\), and she will adopt the optimal stopping rule based on her expectation of equilibrium prices. In some places of this chapter, we will also consider a simple behavioral stopping rule in the spirit of satisficing behavior: consumers are equipped with some exogenous aspiration level and they will stop searching as long as the aspiration level is met. By comparing the results under different stopping rules, we can see the requirement of our results on consumer rationality. The disadvantage of using this behavioral stopping rule is that we then do not have a sound welfare criterion and so cannot do meaningful welfare analysis.

The effect of prominence on consumer behavior is reflected through consumers' search order. If all products are equally prominent, we assume that consumers will sample products randomly in each step. If some products are more prominent than others, consumers will sample those products before others. But no matter in the prominent pool or the non-prominent one, consumers sample products randomly.\(^1\) In the following, we refer the random search case to the first situation and the prominence case to the second one. In both cases, we assume that the sampling process is without replacement and there is costless recall (i.e., a consumer can return to any firm she has visited without extra cost).

### 4.2.1 The demand system

We derive the demand in the prominence case first. Then the demand in the random search case can be derived by letting \(m = 0\) or \(n\). Since all prominent products and all non-prominent products are symmetric, we focus on the equilibrium where they are charged at \(p_A\) and \(p_B\), respectively. Denote by \(\Delta = p_B - p_A\) the price difference (if any).

\(^1\)It will be more involved to treat a finer ordered search model in which all products are completely ranked. However, if the valuation distribution is uniform, we can show a similar price result as in our model (i.e., the prices rise with the order in which products are sampled).
We first consider consumers' optimal stopping rule. Let $a$ be the solution to

$$\int_a^{u_{\max}} (u - a) dF(u) = s.$$  

Thus, if there is no price difference among products and if a consumer has found a product with utility $a$, she is indifferent between sampling one more product and just buying this product. As long as the search cost is not too high, $a$ exists uniquely and decreases with $s$. In particular, when the search cost tends to zero, $a$ tends to $u_{\max}$ and the consumer will sample all products. Throughout this chapter, we assume the search cost is relatively small such that both $p_A$ and $p_B$ are not greater than $a$ in equilibrium, which is a sufficient condition for an active search market.\(^2\)

When $m \geq 2$, the optimal stopping rule crucially depends on whether a consumer expects $p_A < p_B$ or $p_A > p_B$. If $p_A < p_B$, as we shall show below, the stopping rule is actually stationary in each product pool (but not across pools). Nevertheless, if $p_A > p_B$, the stopping rule is no longer stationary in the prominent pool. This is because, the more a consumer approaches to the end of the prominent pool, the more attractive the low price in the non-prominent pool is, and so the less willing she is to stop searching. Therefore, in principle our model with $m \geq 2$ may have multiple equilibria depending on consumers' expectation of prices. This is an important difference emerged in the case with multiple prominent products. (Remember that in the case with $m = 1$, the price of the prominent product does not affect the stopping rule as long as consumers enter the market, and hence the stopping rule is stationary.)

Considering the case with non-stationary stopping rule will greatly complicate our analysis. Fortunately, as we will show below, at least in the uniform-distribution case, $p_B > p_A$ fails to be an equilibrium outcome. Therefore, in the following, we focus on the case with $p_A \leq p_B$. Define

$$z_A = a - p_A, \quad z_B = a - p_B,$$

and so $z_A \geq z_B$. As showed in the following stopping rule, they are the respective cutoff reservation surplus levels in pool $A$ and pool $B$.

**The Optimal Stopping Rule:**

**Phase 1:** In the prominent pool $A$, stop searching if the highest available net surplus so far has been greater than $z_A$; otherwise, search on whenever there are products remained un-sampled in pool $A$.

**Phase 2:** After searching all products in pool $A$, if the highest net surplus is greater than $z_B$, then pick the best one in $A$. If the highest net surplus in pool $A$ is lower than $z_B$, then repeat the process in pool $B$.

---

\(^2\)If $p_A \leq a$, then $\int_{p_A}^{u_{\max}} (u - p_A) dF(u) \geq s$, which implies that entering the market is always desirable.
$z_B$, then enter pool $B$.

**Phase 3:** After entering $B$, stop searching whenever the highest net surplus so far is greater than $z_B$. Otherwise, search on if there are products remained un-sampled in pool $B$.

**Phase 4:** After searching all products, if the highest net surplus is positive, then pick the best product among all. Otherwise, leave the market without buying anything.

The stopping rule in pool $B$ is standard, and here we explain the stopping rule in pool $A$. Denote by $v_i$ the highest net surplus after searching $i \leq m$ products in pool $A$. If a consumer comes to the last product in pool $A$ and find $v_m < z_B$, then entering $B$ is always desirable because the benefit from searching one more product in pool $B$ is larger than the unit search cost. (Recall the definition of $a$.) If $v_m \geq z_B$ and she entered pool $B$, she would sample only one more product according to her stopping rule in $B$, so she should not enter $B$. Thus, the consumer should enter pool $B$ if and only if $v_m < z_B$. In the case $v_m \geq z_B$, clearly she should pick the best product in pool $A$. Now consider the situation when she comes to the penultimate product in pool $A$. If the highest surplus so far is $v_{m-1} < z_A$, sampling the last product in $A$ is always desirable. Otherwise, she should stop searching now. That is because even if she searched on, she would not enter $B$ since $z_A \geq z_B$. This argument can go backward further and finish explaining of the stopping rule.

Now we derive the demand function. We claim that a prominent firm’s demand, if it deviates to a price $p$ while other firms keep charging their equilibrium prices, is

$$q_A(p) = h_A [1 - F(a - p_A + p)] + \hat{r}_A(p) + r_A(p), \quad (4.1)$$

where

$$h_A = \frac{1}{m} \cdot \frac{1 - F(a)^m}{1 - F(a)}$$

is the number of consumers who come to this firm fresh,

$$\hat{r}_A(p) = \int_{a-\Delta}^{a} F(u)^{m-1} f(u + p - p_A) du$$

is the number of consumers who return to this firm after sampling all prominent products, and

$$r_A(p) = \int_{p_B}^{a} F(u - \Delta)^{m-1} F(u)^{n-m} f(u + p - p_B) du$$

is the number of consumers who return after sampling all products.

To understand (4.1), consider three possible sources of a prominent firm’s demand. Let $i$ be this firm’s index. (i) A consumer may come to firm $i$ after searching $k \leq m - 1$
prominent products but without finding a satisfactory one (i.e., all of them have net surplus less than \( z_A = a - p_A \)). This probability is \( \frac{1}{m} F(a)^k \). Summing up these probabilities over \( k = 0, \ldots, m - 1 \) leads to \( h_A \). For such a consumer, she will stop at this prominent firm if \( u_i - p \geq z_A \), of which the probability is \( 1 - F(a - p_A + p) \). This explains the first term in (4.1). We call this portion of firm \( i \)'s demand the "fresh demand". (ii) If this consumer finds that all prominent products’ net surplus less than \( z_A \) but product \( i \) is the best one and has net surplus greater than \( z_B \), then she will return to it without entering pool \( B \). (If firm \( i \) happens to be the last firm in pool \( A \), she just stops at it.) The probability of this event is

\[
\Pr \left( \max_{j \neq i, j \in A} \{z_B, u_j - p_A\} < u_i - p < z_A \right) = \int_{p + z_B}^{p + z_A} F(u - p + p_A)^{m-1} dF(u),
\]

which equals \( \tilde{r}_A(p) \) by changing the integral variable. We call this portion of demand the "midway returning demand". (iii) The last possibility is, after sampling all products (because each product has net surplus less than \( z_B \), this consumer goes back to product \( i \) if it is the best one with positive surplus. The probability of this event is

\[
\Pr \left( \max_{j \neq i, j \in A, j \in B} \{0, u_j - p_A, u_i - p_B\} < u_i - p < z_B \right) = \int_{p}^{p + z_B} F(u - p + p_A)^{m-1} F(u - p + p_B)^{n-m} dF(u),
\]

which equals \( r_A(p) \) by changing the integral variable. We call this portion of demand the "final returning demand".

Secondly, we claim that a non-prominent firm’s demand, if it deviates to a price \( p \) while other firms stick to their equilibrium prices, is

\[
q_B(p) = h_B \left[ 1 - F(a - p_B + p) \right] + r_B(p),
\]

where

\[
h_B = \frac{F(a - \Delta)^m}{n - m} \cdot \frac{1 - F(a)^{n-m}}{1 - F(a)}
\]

is the number of consumers who come to this non-prominent firm fresh, and

\[
r_B(p) = \int_{p_B}^{a} F(u - \Delta)^m F(u)^{n-m-1} f(u + p - p_B) du
\]

is the number of consumers who return to it after searching all products. Notice that, if

\[^3\text{Notice that } \frac{1}{m} \text{ is just the probability that this prominent product is on the } (k + 1)_{th} \text{ position in the consumer’s search process.}\]
\( m = 0, q_B(p) \) is just the demand function in the random search case.

A similar explanation goes as follows. Let \( j \) be this non-prominent firm’s index. For a typical consumer, she will come to firm \( j \) fresh if she has left pool \( A \) (because all prominent products have net surplus less than \( z_B \)) and has sampled \( k \leq n - m - 1 \) products additionally in \( B \) but has not found a satisfactory one. This probability is \( F(a - \Delta)^m (1 - m)^{n-m} F(a)^k \). Summing up these probabilities over \( k = 0, \cdots, n - m - 1 \) yields \( h_B \). Then she will buy at firm \( j \) immediately if \( u_j - p > z_B \), of which the probability is \( 1 - F(a - p_B + p) \). Otherwise, she will search on. She will return to firm \( j \) if all products’ net surplus is less than \( z_B \) but product \( j \) offers the highest positive net surplus. The probability of this event is

\[
\Pr \left( \max_{i \neq j, i \in A, j \in B} \{0, u_i - p_A, u_i - p_B\} < u_j - p < z_B \right) \\
= \int_{p}^{p + z_B} F(u - p + p_A)^m F(u - p + p_B)^{n-m-1} dF(u),
\]

which equals \( r_B(p) \) by changing the integral variable.

Two points deserve mention. First, calculating total demand in two different ways yields the equation

\[
m q_A(p_A) + (n - m) q_B(p_B) = 1 - F(p_A)^m F(p_B)^{n-m}, \tag{4.3}
\]

where \( F(p_A)^m F(p_B)^{n-m} \) is the number of consumers who eventually leave the market without buying anything. Second, how a firm’s total returning demand varies with its actual price only depends on the density function \( f \). In particular, for the uniform distribution, a firm’s total returning demand is independent of its actual price, and so the fresh demand is more price responsive than the returning demand. All else equal, a higher fraction of returning demand makes a firm more want to raise its price.

We now consider the behavioral stopping rule in the spirit of satisficing behavior:

**The Behavioral Stopping Rule:** A consumer will stop searching if and only if the available net surplus so far is greater than some exogenous aspiration level \( z \in (0, 1) \). If all products have been sampled, she will pick the one with the highest positive surplus; if all products have negative surplus, she will leave the market without buying anything.\(^4\)

This stopping rule does not require consumers to remember their positions in the search process or to form expectation of the price and match utility of un-sampled products. If

---

\(^4\)One shortcoming of the behavioral stopping rule is the arbitrariness of \( z \). That is why most of search models use the optimal stopping rule which will generate an endogenous \( z \). However, in our model, as we have argued, the optimal stopping rule associated with \( m \geq 2 \) will lead to a non-stationary \( z \).
we use this stationary stopping rule, then each prominent firm’s midway returning demand will disappear. Using the same reasoning as the above, one can verify that the demand functions are:

\[ q_A(p) = h_A [1 - F(z + p)] + r_A(p), \]  

(4.4)

where

\[ h_A = \frac{1}{m} \cdot \frac{1 - F(z + p_A)^m}{1 - F(z + p_A)}; \quad r_A(p) = \int_{pB}^{z+pB} F(u - \Delta)^{m-1} F(u)^{n-m} f(u + p - p_B) du. \]

And

\[ q_B(p) = h_B [1 - F(z + p)] + r_B(p), \]  

(4.5)

where

\[ h_B = \frac{F(z + p_B)^m}{n - m} \cdot \frac{1 - F(z + p_B)^{n-m}}{1 - F(z + p_B)}; \quad r_B(p) = \int_{pB}^{z+pB} F(u - \Delta)^{m} F(u)^{n-m-1} f(u + p - p_B) du. \]

4.2.2 Equilibrium prices

Now we derive the equilibrium prices by assuming the uniform valuation distribution on [0, 1]. (See Appendix C.11 about the case with a more general distribution.) Consider the optimal stopping rule first. Then \( a \) is the solution to

\[ \int_{a}^{1} (u - a) du = s, \]

so \( a = 1 - \sqrt{2}s \). The following assumption guarantees equilibrium prices \( p_A \) and \( p_B \) to be less than \( a \) (and so an active search market):

**Assumption 4.1** The search cost is not too high: \( 0 < s < \frac{1}{8} \iff \frac{1}{2} < a < 1. \)

For expositional convenience, we introduce a piece of notation:

\[ K_i = \frac{1 - a^i}{i(1 - a)} \]

Then a prominent firm’s demand function is\(^5\)

\[ q_A(p) = h_A (1 - a + p_A - p) + r_A + r_A, \]

\(^5\)A similar issue as in the previous chapter exists: if \( p \) is too high, then the fresh demand will be zero and the returning demand will depend on \( p \). However, we can show that, at least when the search cost is relatively small or \( n \) is relatively large, the equilibrium derived below will not be overturned by the global deviation problem.
where
\[ h_A = K_n, \quad \hat{r}_A = \int_{a-\Delta}^{a} u^{m-1} du, \quad r_A = \int_{p_B}^{a} (u - \Delta)^{m-1} u^{n-m} du. \]

Notice that, when \( \Delta \) tends to zero, the midway returning demand \( \hat{r}_A \) will vanish. The firm wishes to maximize \( q_A(p) \). Then the first-order condition in symmetric equilibrium is
\[ h_A (1 - a - p_A) + \hat{r}_A + r_A = 0. \tag{4.6} \]

Notice that, if \( m = n \), this equation with \( p_A = p_B \) is the first-order condition in the random search case.

A non-prominent firm’s demand function is
\[ q_B(p) = h_B (1 - a + p_B - p) + r_B, \]
where
\[ h_B = K_{n-m}(a - \Delta)^m, \quad r_B = \int_{p_B}^{a} (u - \Delta)^m u^{n-m-1} du. \]

The first-order condition of this firm is then
\[ h_B (1 - a - p_B) + r_B = 0. \tag{4.7} \]

Notice that, if \( m = 0 \), this equation with \( p_A = p_B \) is also the first-order condition in the random search case.

From (4.6)-(4.7), we can see that the equilibrium demands for a prominent product and a non-prominent product are \( q_A = h_A p_A \) and \( q_B = h_B p_B \), respectively. Combining them with (4.3), we have the following useful equation:
\[ \frac{1-a^m}{1-a} p_A + \frac{1-a^{n-m}}{1-a} (a - \Delta)^m p_B = 1 - p_A p_B^{n-m}. \tag{4.8} \]

**Proposition 4.1** Given Assumption 4.1, on the area \([0, a]^2\), (4.6)-(4.7) have a unique solution \((p_A, p_B) \in (1-a, \frac{1}{2}]^2\), and \( p_A < p_B \).

**Proof.** We prove the existence and uniqueness in Appendix C.1. Here we show \( p_A < p_B \). Notice that
\[ \hat{r}_A + r_A - r_B = \int_{a-\Delta}^{a} u^{m-1} du + \int_{p_B}^{a} \Delta(u - \Delta)^{m-1} u^{n-m-1} du \]
has the sign of \( \Delta \) when \( p_B < a \). Also notice that \( h_A > h_B \) is always true in equilibrium (a consumer who comes to a firm in pool \( B \) must have visited a firm in pool \( A \)), so we have
\[ \Delta = p_B - p_A = \frac{r_B}{h_B} - \frac{\hat{r}_A + r_A}{h_A} > \frac{1}{h_A} (r_B - \hat{r}_A - r_A). \]
(We have used (4.6)–(4.7) in deriving the second equality.) Since the last term has the sign of $-\Delta$, then $\Delta > 0$ follows. ■

The reason for $p_A < p_B$ is, due to the consumer search order, each prominent firm's demand consists of more fresh demand proportionally, and as we have known, the fresh demand is more price sensitive than the returning demand in the uniform-distribution setting.\footnote{In effect, the result that the fresh demand is more price sensitive will hold in a more general setting (see Appendix C.11). The intuition is as follows. When a firm raises its price, its fresh demand will decrease for sure since more consumers will then search on. But these consumers become potential returning consumers, so raising a firm’s price has an extra positive effect on its own returning demand.}

Before proceeding, we discuss the issue of multiple equilibria. Remember that our demand functions are based on consumers’ expectation of $p_A < p_B$, and we have confirmed that $p_A < p_B$ is indeed an equilibrium outcome. Nevertheless, we have not yet discussed other possible equilibria associated with different expectations. In particular, we are concerned about whether $p_A > p_B$ could also be an equilibrium outcome. The following proposition, which is proved in Appendix C.2, excludes this possibility in our uniform-distribution setting.

**Proposition 4.2** In the uniform-distribution setting, there is no equilibrium with $p_A > p_B$.

**Equilibrium prices with the behavioral stopping rule.** Using (4.4)–(4.5), it is ready to derive
\[
p_i = \frac{1}{2} \left( 1 - z + \frac{r_i}{h_i} \right), \quad i = A, B,
\]
in the uniform-distribution setting. Along the same logic in the proof of Proposition 4.1, we can prove $p_A < p_B$. In this case, we do not need to worry about the multiple-equilibrium issue since the stopping rule is exogenous. Therefore, the result that prominent firms will charge a lower price than non-prominent firms relies little on consumer rationality.

We return to the analysis with the optimal stopping rule. The following are three special cases in which the price difference $\Delta$ will vanish.

(i) *When $n \to \infty$, both $p_A$ and $p_B$ converge to $1 - a$.*\footnote{If we use the behavioral stopping rule, $p_i \to \frac{1}{2}$ as $n \to \infty$.} Notice that
\[
\frac{r_B}{h_B} < \int_{p_B}^{a} \left( \frac{u}{a} \right)^{n-a-1} du.
\]
The right-hand side tends to zero as $n \to \infty$, so $p_B = 1 - a + \frac{r_B}{h_B}$ tends to $1 - a$. Since $1 - a < p_A < p_B$, $p_A$ tends to $1 - a$ too. Note that $1 - a$ is also the equilibrium price in
the random search case as \( n \to \infty \) (see below for the detail). So this limit result implies that prominence has little impact on the market prices if the market has many suppliers.

(ii) When \( a \to 1 \) (i.e., when the search cost tends to zero), both \( p_A \) and \( p_B \) converge to the full information price \( \tilde{p} \) that satisfies

\[
np = 1 - \tilde{p}^n.
\]

It is straightforward to verify that \( p_A = p_B = \tilde{p} \) satisfy the first-order conditions when \( a \) approaches one.

(iii) When \( a \to \frac{1}{2} \), both \( p_A \) and \( p_B \) converge to the monopoly price \( \frac{1}{2} \). This is just because both \( p_A \) and \( p_B \) are between \( 1 - a \) and \( \frac{1}{2} \). The intuition is that the high search cost makes consumers willing to stop searching whenever she finds a product with positive surplus, and so each firm is acting as a monopoly.

Chapter 3 has shown that, compared to the random search case, introducing one prominent firm will make all non-prominent firms raise their prices but make the prominent firm lower its price. Will this effect persist when there are more prominent firms?

First of all, we introduce the equilibrium price \( p_0 \) in the random search case:

\[
\frac{1 - a^n}{1 - a} = \frac{1 - p_0^n}{p_0}.
\]  

(4.9)

This is from (4.8) by letting \( m = 0 \) or \( n \). Second, we introduce the following lemma which is proved in Appendix C.3:

**Lemma 4.1** Given Assumption 4.1, we have\(^8\)

\[
\lim_{n \to \infty} \frac{p_0 - (1 - a)}{p_A - (1 - a)} = a^m + aK_m; \quad \lim_{n \to \infty} \frac{p_0 - (1 - a)}{p_B - (1 - a)} = a^m.
\]

Now we give the result concerning the relationship between \( p_A, p_B \) and \( p_0 \) when consumers adopt the optimal stopping rule.

**Proposition 4.3** Given Assumption 4.1,

(i) \( p_0 < p_B \) for sure, and \( p_A < p_0 \) if

\[
\left(1 - \frac{\Delta}{a}\right)^m > 1 - \frac{\Delta}{p_B}.
\]  

(4.10)

Particularly, if \( m = 1 \), \( p_A < p_0 \) must be true.

(ii) For fixed \( n \) and \( 2 \leq m \leq n - 1 \), there exists \( \varepsilon_1 > 0 \) such that \( p_A < p_0 \) if \( a > 1 - \varepsilon_1 \), and there exists \( \varepsilon_2 > 0 \) such that \( p_A > p_0 \) if \( a < \frac{1}{2} + \varepsilon_2 \).

---

\(^8\) If we use the behavioral stopping rule, the first limit result is \( \infty \) and the second one is \( \left(\frac{1 + \varepsilon}{2}\right)^m \).
(iii) Let \( \tilde{m} > 1 \) be the nearest integer to the solution to \( a^\tilde{m} + aK_m = 1 \). For fixed \( a \), there exists \( N \) such that, for \( n > N \), \( p_A < p_0 \) if and only if \( m < \tilde{m} \).

**Proof.** (i) Since \( \Delta > 0 \), the left-hand side of (4.8) is less than

\[
\frac{1 - a^n}{1-a} p_B + \frac{1 - a^{n-m}}{1-a} a^m p_B = \frac{1 - a^n}{1-a} p_B,
\]

while the right-hand side of (4.8) is greater than \( 1 - p_B^n \). Thus,

\[
\frac{1 - a^n}{1-a} > \frac{1 - p_B^n}{p_B}.
\]

Comparing it to (4.9) yields \( p_0 < p_B \).

Notice that (4.10) is equivalent to \( (a - \Delta)^m p_B > a^m p_A \). If this inequality holds, then the left-hand side of (4.8) is greater than \( \frac{1 - a^n}{1-a} p_A \). Meanwhile, the right-hand side of (4.8) is less than \( 1 - p_A^n \) since \( p_A < p_B \). So

\[
\frac{1 - a^n}{1-a} < \frac{1 - p_A^n}{p_A},
\]

which implies \( p_A < p_0 \). If \( m = 1 \), (4.10) is always true since \( p_B < a \) under Assumption 4.1.

(ii) Since

\[
\left(1 - \frac{\Delta}{a}\right)^m > 1 - \frac{m\Delta}{a},
\]

\( p_A < p_0 \) if \( p_B < \frac{a}{m} \). When \( a \to 1 \), \( p_B \) tends to the full information equilibrium price \( \tilde{p} = (1 - \tilde{p}^n)/n < \frac{1}{m} \), and so \( p_A < p_0 \).

When \( a \to \frac{1}{2} \), we have \( \Delta \to 0 \), so

\[
(a - \Delta)^m p_B \approx (a^m - m a^{m-1} \Delta)(p_A + \Delta)
\]

\[
\approx a^m p_A + a^{m-1}(a - mp_A) \Delta.
\]

Then the left-hand side of (4.8) can be approximated by

\[
\frac{1 - a^n}{1-a} p_A + \frac{1 - a^{n-m}}{1-a} a^{m-1}(a - mp_A) \Delta.
\]

On the other hand, when \( \Delta \to 0 \), we have

\[
\tilde{p}_A^n \tilde{p}_B^{n-m} = \tilde{p}_A^n (p_A + \Delta)^{n-m}
\]

\[
\approx \tilde{p}_A^n + \tilde{p}_A^{n-1}(n-m) \Delta.
\]
Therefore, when $\Delta \to 0$, (4.8) implies
\[
\frac{1 - a^n}{1 - a} p_A - \left(1 - p_A^a\right) \approx \Delta \left[ \frac{1 - a^{n-m}}{1 - a} a^{m-1}(mp_A - a) - (n-m)p_A^{n-1} \right].
\]

When $a$ tends to $\frac{1}{2}$ (so $p_A$ also tends to $\frac{1}{2}$), the square-bracket term approaches to
\[
\frac{1}{2n-1} \left[(m-1)(2^{n-m} - 1) - (n-m)\right],
\]
which must be nonnegative when $m \geq 2$ and $n - m \geq 1$. Therefore, when $m \geq 2$ and $a \to \frac{1}{2}$, we have
\[
\frac{1 - a^n}{1 - a} > \frac{1 - p_A^n}{p_A},
\]
so $p_A > p_0$.

(iii) This is immediately from the first result in Lemma 1. Notice that, when $m = 1$, $a^m + aK_m = 2a > 1$ under Assumption 4.1, and $a^m + aK_m$ decreases with $m$ and tends to zero as $m \to \infty$, so $\hat{m} > 1$ exists uniquely. ■

Part (i) of the proposition says that introducing prominent products will always induce non-prominent firms to raise their prices. This is because their demand now includes more returning demand proportionally. But whether prominent firms will lower their prices is uncertain (except when $m = 1$). There are two forces working here. On the one hand, now each prominent firm has more fresh demand proportionally than before. This tends to make them lower prices. On the other hand, price competition here involves strategic complements, so the higher price of non-prominent products induces them to raise prices. The final outcome depends on which force is stronger. Two limit results about this are reported in part (ii)–(iii). Part (ii) says that, when the search cost is sufficiently small, the prominent firms will always lower their prices; while they will raise their prices if the search cost is sufficiently large. Part (iii) says that, when the market is sufficiently competitive, the prominent firms will actually raise their prices if $m$ is relatively large, and vice versa.

When $m \geq 2$, an interesting result beyond the single-prominence case has emerged: introducing more than one prominent product could lead all firms to raise their prices. We have proved that it will take place when the search cost is relatively large or when the market is rather competitive and the number of prominent firms is not too small. Actually, numerical simulations show that this can even take place under mild conditions (see an numerical example below). The basic reason is that the existence of prominent products induces non-prominent firms to raise their prices, which further drives prominent firms to do so.

The result with the behavioral stopping rule. If we use the same aspiration level $z$ in both the prominence case and the random search case, then we always have
$p_A < p_0 < p_B$, which is proved in Appendix C.4. This suggests that the result that prominence can cause all firms to raise their prices relies on the optimal stopping rule.

Now we study how $m$ affects the market prices. The first simple observation is, when $m = n$, $p_A$ will return to $p_0$, so $p_A$ should be non-monotonic with $m$. Since $p_B$ has no definition when $m = n$, this observation does not apply. A general comparative static analysis with respect to $m$ is hard to proceed. In the following, we focus on the limit cases $n \to \infty$ and $a \to 1$ with the optimal stopping rule.

**Proposition 4.4** Given Assumption 4.1,

(i) for fixed $a$ and $\tilde{m} \geq 1$, there exists $N$ such that $p_A$, $p_B$, and the price gap $\triangle$ all increase with $m$ at $\tilde{m}$ for $n > N$;

(ii) for fixed $n$, there exists $\hat{a} < 1$ such that, for $a > \hat{a}$, $p_A$, $p_B$, and the price gap $\triangle$ all increase with $1 \leq m \leq n - 2$, and $p_A$ increases with $m$ even at $m = n - 1$.

**Proof.** See Appendix C.5. ■

The first result says that, when $m$ is sufficiently small relative to $n$, the prices and the price gap always increase with $m$. The second result means that, when the search cost is sufficiently small, the prices and the price gap always increase with $m$ even if $m$ is close to $n$. Note that $p_A$ increasing at $m = n - 1$ does not conflict with $p_A = p_0$ at $m = n$, because $p_A < p_0$ for a sufficiently large $a$ and $1 \leq m \leq n - 1$. For intermediate $n$ and $a$, it is hard to get analytical results, but numerical simulations suggest that $p_B$ always increases with $m$ but $p_A$ may not. For example, as $a \to \frac{1}{2}$, $p_A > p_0$ at any $2 \leq m \leq n - 1$ (part (ii) in Proposition 4.3), but $p_A < p_0$ at $m = 1$ and $p_A = p_0$ at $m = n$, so $p_A$ must be non-monotonic with $m$ as $a \to \frac{1}{2}$.

The following graph presents an example of the relationship between equilibrium prices and $m$ when $a = 0.7$ and $n = 8$. The thick solid line is $p_A$, the thick dashed line is $p_B$, and the thin dashed line is $p_0$. We can see that $p_B$ increases with $m$, but $p_A$ decreases first when just one prominent firm emerges and then increases with $m$ and even exceeds $p_0$ if $m \geq 3$.\(^9\) However, from $m = 5$ to $m = 8$, $p_A$ falls again.

\(^9\)Our asymptotic condition for $p_A > p_0$ in Proposition 4.3 works well in this example: $a^m + aK_m = 0.7^m + \frac{3}{2}(1 - 0.7^m)/m$ which is greater than one for $m = 1$ and $2$ but less than one for $m \geq 3$. 

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Now we give some intuition about how equilibrium prices vary with \( m \). Consider the non-prominent firms first. When there are more prominent firms, fewer consumers will visit pool \( B \), so each non-prominent firm’s fresh demand should decrease. On the other hand, larger \( m \) also means that, if consumers come to pool \( B \), they must be less satisfied with products in pool \( A \) as a whole, which gives non-prominent firms more advantage in competing for the returning consumers. Therefore, we should expect that a non-prominent firm’s returning demand decreases less fast with \( m \) than its fresh demand, i.e., the relative proportion of returning demand should increase with \( m \). That is why \( p_B \) tends to rise with \( m \). For a prominent firm, initially its price goes down because of the abrupt increase of fresh demand. Eventually, as \( m \) goes up to \( n \), this effect will tend to vanish. In the middle of this process, higher \( p_B \) will contribute to the increase of \( p_A \).

### 4.2.3 Welfare

This part examines the welfare implications of prominence in the competitive-pricing model. Chapter 3 has shown that introducing one prominent firm will usually improve industry profit but harm consumer surplus and total welfare. Here we want to know whether this effect will persist as we introduce more prominent products. Due to the complication of the problem with \( m \geq 2 \), we only have analytical results in the limit cases with \( n \to \infty \) and \( a \to 1 \). Our welfare analysis will also focus on the case with the optimal stopping rule since there is no a sound welfare criterion associated with the behavioral stopping rule.

We first give the expressions for relevant welfare variables. Total output is

\[
Q_m = 1 - p_A^m p_B^{n-m},
\]

and industry profit is

\[
\Pi_m = p_B Q_m - \triangle \cdot mh_A p_A,
\]
where \(m_{hAP} \) is the output supplied by all prominent firms. We subtract the second term because prominent firms are charging a lower price than others. The expression for total welfare is

\[
W_m = a(1 - a^n) + w(\triangle, m) + n \int_{p_B}^{a} (u - \triangle)^m u^{n-m} du,
\]

(4.11) \where

\[
w(\triangle, m) = m \int_{a-\triangle}^{a} u^m du - a(1 - a^{n-m}) [a^m - (a - \triangle)^m].
\]

(See Appendix C.11 for how to derive this expression and its intuition.) Consumer surplus is \(V_m = W_m - \Pi_m\). If \(m = 0\) and \(p_A = p_B = p_0\), we get the expressions for \(Q_0\), \(\Pi_0\), \(W_0\), and \(V_0\) in the random-search case.

We now investigate the relationship between welfare and \(m\). The first simple observation is that making all firms prominent is the same as no prominence at all, so all welfare variables should vary with \(m\) non-monotonically. In the following, we try to specify this non-monotonic relationship in the limit cases. Let us discuss total output first. Remember \(\bar{p}\) is the full-information price and it satisfies \(n \bar{p} = 1 - \bar{p}^n\).

**Proposition 4.5** Given Assumption 4.1,

(i) for fixed \(a\) and \(\bar{m} \geq 1\), there exists \(N\) such that \(Q_m\) decreases with \(m\) at \(\bar{m}\) and \(Q_m < Q_0\) for \(n > N\);

(ii) for fixed \(n\), there exists \(\bar{a} < 1\) such that, for \(a > \bar{a}\), total output \(Q_m\) decreases with \(m\) if and only if

\[
m^2(n - 1)\bar{p} + (n - 2m)(2n - 1 - \bar{p}^{n-1}) > 0,
\]

and \(Q_m < Q_0\) for any \(1 \leq m \leq n - 1\); in particular, if \(m \leq \frac{n}{2}\), \(Q_m\) always decreases with \(m\), and if \(m > \frac{n}{2}\), \(Q_m\) could increase with \(m\) and this must take place if \(m > \frac{\sqrt{5} n}{4 + \sqrt{2}} \approx 0.586n\).

**Proof.** See Appendix C.6. ■

The first result says that, when \(m\) is sufficiently small relative to \(n\), total output must decrease with \(m\). The second one says that, when the search cost is sufficiently small and \(n\) is fixed, \(Q_m\) must decrease with \(m\) first and then increase, and the switch point is between \(\frac{n}{2}\) and about 0.586\(n\). The intuition is as follows. Let us decompose the impact of \(m\) on total output into two parts. First, \(m\) will affect the market prices. As we have already known, \(p_A\) and \(p_B\) can both increase with \(m\) (at least when \(m\) is small relative to \(n\) or when the search cost is small), so \(m\) has a negative effect on total output. Second, larger \(m\) shifts more consumers to prominent firms. Since they are charging a lower price, the rise of \(m\) also has a positive effect on total output. These two effects together lead to our non-monotonic result.
From Proposition 4.3, we have known that, when \( m \geq 2 \) and \( a \rightarrow \frac{1}{2} \), we both \( p_A \) and \( p_B \) will be greater than \( p_0 \). Thus, \( Q_m < Q_0 \) also holds in that limit case. In effect, we have not found any numerical example for \( Q_m > Q_0 \), so we conjecture that \( Q_m < Q_0 \) would hold generally for \( 1 \leq m \leq n - 1 \).

We now turn to the impact of \( m \) on welfare.

**Proposition 4.6** Under Assumption 4.1,

(i) for fixed \( a \) and \( m \geq 1 \), there exists \( N \) such that, for \( n > N \), industry profit increases at \( m \) and total welfare and consumer surplus decrease at \( m \), \( \Pi_m > \Pi_0 \), \( W_m < W_0 \), and \( V_m < V_0 \);

(ii) for fixed \( n \), there exist \( \hat{a} < 1 \) such that, for \( a > \hat{a} \), industry profit increases with \( m \) and total welfare and consumer surplus decrease with \( m \) if and only if total output decreases with \( m \), \( \Pi_m > \Pi_0 \), \( W_m < W_0 \), and \( V_m < V_0 \).

**Proof.** See Appendix C.7. ■

These results confirm the welfare results in the previous chapter (if \( m \) is small relative to \( n \) or if the search cost is low). The second result also implies that, when the search cost is small, industrial profit will reach its maximum and total welfare and consumer surplus will reach their minimums when \( m \) is between \( \frac{n}{2} \) and \( 0.586n \).

We know that total welfare is mainly determined by total output \( Q_m \) and the price gap \( \Delta \). On the one hand, since the production cost is zero, every consumer should be served. Hence, higher \( Q_m \) means higher output efficiency. On the other hand, since consumers’ search behavior is socially efficient in the uniform-price case, \( p_A < p_B \) makes too few consumers search beyond, and too many consumers return to, the prominent pool. Thus, larger \( \Delta \) tends to result in less efficient search behavior. Since \( Q_m \) decreases and \( \Delta \) increases with \( m \) when \( m \) is small relative to \( n \), total welfare should also decrease. When the search cost is sufficiently small, we should expect that the search effect is of second order but the output effect is of first order (our proof confirms this), and so the impact of \( m \) on total welfare should be entirely determined by the latter one.

The following graphs give a numerical example of the relationship between welfare and \( m \) (\( a = 0.7 \) and \( n = 8 \)). The dashed line is consumer surplus.
4.2.4 Endogenous prominence with a platform

In many situations where prominence plays a role, there exists a platform such as the search engine and the yellow page on which firms advertise their products and consumers search. The platform can make profit from selling prominent positions. Two issues are important for the platform: how to sell prominent positions and how many prominent positions should be sold? Auction is now a popular way to sell online paid placements. The yellow page, however, just posts the rates of advert slots with different display properties.\textsuperscript{10} In this part, we focus on the second selling mechanism and aim to discuss the optimal number of prominent positions for a platform. We suppose that it is the seller instead of the platform who controls the product price. We consider the following two cases:

**Charging all positions.** For simplicity, we further suppose that the platform is a monopoly and it can extract the whole (or a fixed proportion of) industry profit. Then the optimal number of prominent positions $m^*$ should maximize $\Pi_m$. Propositions 4.5–4.6 imply the following result:

**Corollary 4.1** If the platform can extract the whole (or a fixed proportion of) industry profit, for a sufficiently small search cost, it will establish $m^*$ prominent positions where $m^* \in \left( \frac{n}{2}, \frac{\sqrt{2n}}{1 + \sqrt{2}} \right)$, which will lead the lowest total welfare and consumer surplus.\textsuperscript{11}

Numerical simulations suggest that $m^*$ does not vary by too much even for mild $a$ (e.g., see Figure 4.2). Of course, $m^*$ will become smaller if we consider the cost of establishing prominent positions. $m^*$ might also be restricted by the available space for prominent positions. In addition, the effect of prominence may also become weaker when more prominent positions are established, which could further reduce $m^*$.

\textsuperscript{10}See http://yp.crgis.com/yellowproducts.aspx for an example.

\textsuperscript{11}Propositions 4.5–4.6 also imply that any fixed $m$ cannot be optimal as $n \to \infty$. This is because any fixed $m$ is negligible relative to infinite $n$. A more meaningful question is what fraction of products should be prominent as $n \to \infty$. Suppose $m = \gamma n$. What $\gamma$ will maximize $\Pi_m$ when $n \to \infty$? We have not yet solved this problem.
**Charging only prominent positions.** To avoid the complication caused by the strategic interaction between firms, we focus on the limit case with \( n \to \infty \). Then, if a firm does not buy a prominent position, its profit will be negligible; if it buys one, its gross profit will be \( \pi_A = \hat{h}_A p^2_A \). Thus, in an ideal situation, the monopoly platform can earn \( m\pi_A \) by selling \( m \) prominent positions. Intuitively, a greater number of prominent positions will make each of them less valuable and so \( m\pi_A \) might vary with \( m \) non-monotonically. However, in our limit case, we have \( m\pi_A = (1 - a)(1 - a^m) \) which always increases with \( m \). Hence, if there are no other constraints, the platform will make \( m \) as large as possible. This result does not make much sense.

As a remedy, we introduce a unit cost \( c \) for establishing each prominent position which is also reasonable in many cases. Then, if we neglect the integer problem, \( m^* \) should solve the first-order condition:

\[
(1 - a) a^m \ln \frac{1}{a} = c,
\]

if \( c < (1 - a) \ln \frac{1}{a} \). The following graph gives some numerical examples where \( c \) equals 0.06, 0.02, or 0.01. (The lower curve is associated with a higher \( c \).) Notice that \( m^* \) may vary non-monotonically with \( a \). That is, a higher search cost does not always induce the platform to set more prominent positions.\(^{13}\)

![Graph](image)

**Figure 4.4:** \( m^* \) and \( a \) as \( n \to \infty \)

\(^{12}\)When \( c \) is too high or the search cost is too small (i.e., when \( c > (1 - a) \ln \frac{1}{a} \)), \( m^* = 0 \).

\(^{13}\)Denote by \( \zeta(m; a) \) the left-hand side of the first-order condition, which is the marginal value of a prominent position for the platform. How this marginal value varies with \( a \) determines the relationship between \( m^* \) and \( a \). One can show

\[
\zeta_a(m^*; a) = \frac{c}{a} \left( m^* + \frac{1}{\ln a} - \frac{a}{1 - a} \right).
\]

Since \( m^* \) is bounded by \( \frac{1}{a} \) (otherwise the platform is making a loss), the term in the bracket must be negative if \( a \) is sufficiently large. In contrast, if \( a \) is small, the term in the bracket could be positive when \( m^* \) is relatively large (which happens when \( c \) is relatively small). Therefore, for small \( c \), \( m^* \) varies with \( a \) non-monotonically; and for large \( c \), \( m^* \) always decreases with \( a \).
4.3 Central Pricing

We now turn to the central-pricing model in which a multi-product firm chooses the prices of all products.\footnote{Our monopoly setup abstracts from the potential competition between multi-product sellers, so it is more suitable for the situation where there is substantial differentiation (e.g., physical distance) between sellers.} For example, the restaurant offers a menu of dishes and sets the prices by itself. The local supermarket usually sells several brands of a product and it also has some discretion to influence their retailing prices. Many other retailers have the similar situation. In these examples, we also often see some prominent options (e.g., the brands displayed on the gondola ends and the dishes recommended in a special menu).

We use the same framework as in the competitive-pricing case except that now all prices are decided centrally. Notably, we keep the assumption that consumers will sample prominent products first (because of some form of bounded rationality or exogenously restricted search order). Our aim is to investigate whether the impact of prominence here is different from that in the competition case. Intuitively, given the specified search order, raising the price of prominent products will be more likely to drive consumers to buy non-prominent products, while raising the price of non-prominent products will be more likely to drive consumers to leave the market. Thus, we should expect that the monopoly has more incentive to raise the price of prominent products, and so prominent products will be more expensive than others in equilibrium. In the following, we verify this prediction first and then examine the welfare implications.

Compared to the competition case, we now have two extra issues deserving discussion. First, if consumers expect that prominent products are more expensive, their optimal stopping rule (given their restricted search order) will be non-stationary in the prominent pool. This has been shown in Appendix C.2. Such a stopping rule will complicate our analysis a lot. The exception is $m = 1$, in which case we have the stationary stopping rule. In the following, we mainly deal with the case with $m = 1$, and we will extend the main price result to the case with $m \geq 2$ by assuming the behavioral stopping rule.

Second, when we use the optimal stopping rule, we should be more careful in specifying consumers’ off-equilibrium expectations. In the competition case, the reasonable assumption is that consumers will keep the equilibrium expectation even if they encounter unexpected prices, since firms make pricing decisions independently and simultaneously. We call this “passive belief”. However, in our central-pricing case, consumers might be cautious to a deviation price and then contemplate that the monopoly is also adjusting other prices accordingly. However, this kind of wary belief may be too demanding for ordinary consumers, and it will also complicate our analysis too much. Hence, we focus on the passive belief, i.e., consumers always keep their equilibrium beliefs.\footnote{The distinction of these two kinds of belief has an analogy in the literature on secret contracts (see,}
wonder whether the monopoly can announce and commit all prices up front. (Note that the monopoly always owns more if it is able to do so.) In the following, to be consistent with the competition setting, we will mainly focus on the case with imperfect price information but with passive consumer beliefs. In the end of this section, we will discuss how our results might change if the monopoly can announce prices in advance.

4.3.1 Equilibrium prices

The case with \( m = 1 \). In this case, we have similar price results no matter which stopping rule is used since both of them are stationary. We focus on the optimal one for the purpose of welfare analysis. Let product 1 be the prominent product, and consider the equilibrium where the monopoly firm charges product 1 at \( P_A \) and other products at \( P_B \).\(^{16}\) Keep the notation \( \Delta = P_B - P_A \), and \( a \) still solves

\[
\int_{u}^{u_{\text{max}}} (u - a) dF(u) = s.
\]

If a consumer has decided to enter the market, she will stop searching if and only if the highest net surplus so far is greater than \( a - P_B \). As before, to have an active search market, we keep assuming that \( P_i \leq a \) in equilibrium. Basically, this requires that the search cost is not too high and \( n \) is not too large.\(^{17}\) (We will specify the condition when it becomes possible.)

With the prices \( P_A \) and \( P_B \), the monopoly firm earns

\[
\Pi_1 = P_A q_A + P_B [1 - F(P_A)F(P_B)^{n-1} - q_A]
\]

\[
= P_B [1 - F(P_A)F(P_B)^{n-1}] - \Delta \cdot q_A,
\]

where

\[
q_A = 1 - F(a - P_B^* + P_A) + \int_{P_B}^{P_B + a - P_B^*} F(u) u^{n-1} f(u - \Delta) du
\]

is the demand for product 1 (this is from (4.1) by letting \( m = 1 \)), and \( 1 - F(P_A)F(P_B)^{n-1} \) is total demand. Note that \( P_B^* \) is consumer’s fixed expectation of \( P_B \), which reflects our assumption of passive beliefs. In equilibrium, of course \( P_B^* = P_B \). The following proposition

\(^{16}\)In the central-pricing model, we are implicitly assuming the existence of equilibrium in which symmetric products are charged at the same price. In a general setup, we do not know the primitive conditions for this. But in the uniform setup, it can be verified under some conditions we will specify below.

\(^{17}\)A larger \( n \) will induce the monopoly firm to raise its prices if it believes that consumers will enter and search. However, expecting this, for a fixed search cost, consumers are less likely to enter the market in the beginning. That is why a larger \( n \) needs to be associated with a lower search cost to have an active search market. An alternative way to get around this problem is to introduce heterogenous reservation utilities among consumers and assume they are realized independently from the product match utility. The price of using such a setting is the complication in welfare analysis.
confirms our initial conjecture that the prominent product will be more expensive. As a result, the impact of prominence on the market price is opposite to that in the competition case.

**Proposition 4.7** If there is only one prominent product, in the equilibrium with an active search market, we have $p_A > p_B$.

**Proof.** If $p_A < p_B$ would hold in equilibria, then charging product 1 at $p_B$ and charging some non-prominent product at $p_A$ is a profitable deviation. To see that, notice that such a deviation will not change total demand, so it leads to a new profit level

$$p_Aq' + p_B \left[ 1 - F(p_A)F(p_B)^{n-1} - q' \right],$$

where $q'$ is the new demand for that non-prominent product with price $p_A$. Given consumers’ passive beliefs (so their stopping rule remains), we have $q' < q_A$ due to the restricted search order. Thus, this new profit level is higher than the original one in (4.12). This is a contradiction, so $p_A \geq p_B$.

Suppose $p_A = p_B = p$, then they should satisfy two necessary conditions:

$$\frac{\partial \Pi_1}{\partial p_A} \bigg|_{p_A=p_B=p} = q_A - pf(p)F(p)^{n-1} = 0,$$

$$\frac{\partial \Pi_1}{\partial p_B} \bigg|_{p_A=p_B=p} = (n-1)q_B - (n-1)pf(p)F(p)^{n-1} = 0,$$

where $q_B$ is the equilibrium demand for any non-prominent product. Clearly, they require $q_A = q_B$, but that is impossible given $p_A = p_B$. Thus, $p_A > p_B$. ■

An intuitive explanation goes as follows. When the monopoly raises the price of a product, it earns more from consumers who still buy this product, it may gain or lose from those who switch to buy other products depending on the relative prices, and it loses from those who thereby leave the market. Suppose $p_A = p_B$ and the search market is active, and let us compare slightly raising the price of a prominent product with slightly raising the price of a non-prominent product. The first positive effect is larger for the prominent product since it has a larger demand, the second effect is negligible for both since $p_A = p_B$, and the third negative effect is smaller for the prominent product since consumers visit it first and so have more opportunities to find other satisfactory products after leaving it. Therefore, adjusting up the price of the prominent product is more profitable.

We proceed to compare the prominence case with the random search case. For tractability, we focus on the case with the uniform valuation distribution on $[0,1]$. As we have pointed out in footnote 17, to have an active search market, when the number
of products increases, we must have a lower search cost. The following is the specific condition in the uniform setting:

**Assumption 4.2** $a \geq \hat{a}_n$, where $\hat{a}_n$ increases with $n \geq 2$ from about 0.614 to 1.\(^{18}\)

From (4.12), it is ready to derive two necessary equilibrium conditions:\(^{19}\)

\[
\frac{\partial \Pi_1}{\partial p_A} = q_A - p_B^n + \Delta = 0, \tag{4.13}
\]

\[
\frac{\partial \Pi_1}{\partial p_B} = 1 - (n + 1)p_A^{n-1} - q_A + p_B^n - a^{n-1}\Delta = 0,
\]

where $q_A = 1 - a + \Delta + (a^n - p_B^n)/n$. Adding them together yields

\[
1 - (n + 1)p_A^{n-1} + (1 - a^{n-1})\Delta = 0. \tag{4.14}
\]

So (4.13) and (4.14) define the equilibrium prices.

If there is no prominent product, the equilibrium price $p_0$ should maximize $p(1 - p^n)$ given consumers’ expectation of symmetric prices. Thus, $p_0$ solves

\[
1 - (n + 1)p_0^n = 0. \tag{4.15}
\]

Notice that $p_0$ (so the corresponding profit $\Pi_0$) is independent of the search cost (as long as the search market is active). It is ready to check that $p_0$ increases with $n$ from $\sqrt[3]{\frac{2}{3}} \approx 0.58$ to 1. It is also clear that, when $a$ tends to one, both $p_A$ and $p_B$ defined in (4.13) and (4.14) will converge to $p_0$.

**Proposition 4.8** If there is only one prominent product, in the uniform-distribution setting with Assumption 4.2,

(i) on $[0, 1]^2$, the solution to (4.13) and (4.14) exists uniquely and $p_1 < a$;

(ii) $p_B < p_0 < p_A$, and the firm serves more consumers in the prominence case than in the random search case.

**Proof.** See Appendix C.8. \(\blacksquare\)

Thus, relative to the random search case, the firm will raise the prominent product’s price but lower other non-prominent products’ price, and the prominence case leads to

\[^{18}\text{Explicitly, } \hat{a}_n \text{ solves the equation}
\]

\[
\frac{3a - 1}{2} + \frac{1 - a^n}{2n} = \sqrt[4]{\frac{1}{n + 1}}.
\]

Note that $\sqrt[4]{\frac{1}{n + 1}}$ increases with $n \geq 2$ from $\sqrt[3]{\frac{2}{3}}$ to 1.

\[^{19}\text{Though a little lengthy, one can verify that these necessary conditions are also sufficient when } n = 2
\text{ or when } a \text{ is close to one. Beyond that, it is hard to verify the sufficiency due to the complication of the objective function.}\]
higher total output. These results also contrast with the competition case. The following graph is a numerical example when $n = 2$, where the thin dashed line is $p_0$.

![Graph showing prices and a in Central Pricing ($n = 2$)](image)

**Figure 4.5: Prices and $a$ in Central Pricing ($n = 2$)**

**The case with** $m \geq 2$. We can establish the similar price results if we use the stationary behavioral stopping rule.$^{20}$

**Proposition 4.9** If there are more than one prominent product and consumers adopt the behavioral stopping rule (but $z$ is not too large), then similar results as in Proposition 4.7 and 4.8 hold.

**Proof.** See Appendix C.9. ■

Unlike in the competition case in which following the guidance of prominence is actually optimal, now prominence is misleading consumers since the firm is charging a higher price for the prominent product. If consumers are rational and can choose their search orders freely, they will avoid to sample the prominent product first. Expecting this consumer search order, the firm actually has no incentive to charge more for the prominent product. Then, in equilibrium firm should charge all products, no matter prominent or not, at the same price and consumers sample randomly. Therefore, the assumption that consumers will somehow sample the prominent product first is crucial in our central-pricing model. A more flexible setting should permit both rational consumers and consumers who will be biased by prominence. In that richer model, our price result still holds as long as the fraction of rational consumers is not too large.

$^{20}$We conjecture that the results would also hold even if we use the non-stationary optimal stopping rule, though a complete proof has not been figured out.
4.3.2 Welfare

Due to the tractability issue associated with the non-stationary stopping rule, we mainly investigate the welfare implications of prominence in the uniform-distribution setting with \( m = 1 \). First of all, the monopoly must earn more in the prominence case, because it can at least charge \( p_A = p_B = p_0 \) to obtain \( \Pi_0 \) (see (4.12)). Therefore, the multi-product monopoly does have incentive to make some product prominent if it is costless to do so.

As far as total welfare is concerned, following the discussion in the competition case, the non-uniform prices in the prominence case always tends to lower efficiency because of consumers’ suboptimal search behavior; while prominence also gives rise to higher output in our central-pricing model, which improves efficiency. Therefore, the final outcome depends on which effect is stronger. We will show that, at least for relatively small search cost, making one product prominent can boost total welfare and consumer surplus. This is opposite to the welfare results in the competition case.

We first derive the expression for consumer surplus difference \( V_1 - V_0 \). Let \( V(p, p) \) be consumer surplus when all products are charged at \( p \). When \( p \) is increased by \( \epsilon \), all buyers have to pay more, which leads to a consumer surplus loss \( \epsilon(1 - p^n) \). On the other hand, \( -\epsilon \frac{d(1 - p^n)}{dp} \) more consumers will be excluded, but these marginal consumers’ surplus is of order \( \epsilon \), so the surplus loss from them is of order \( \epsilon^2 \) and can be ignored for small \( \epsilon \). Thus,

\[
\frac{dV(p, p)}{dp} = p^n - 1 \quad \text{and} \quad V(p_B, p_B) - V(p_0, p_0) = \int_{p_B}^{p_0} (1 - p^n) dp.
\]

Let \( V(\delta) = V(p_B + \delta, p_B) \) be the consumer surplus when all products are charged at \( p_B \) except that product 1 is charged at \( p_B + \delta \). Consider an increase \( \epsilon \) of \( \delta \). First, more consumers are excluded, but as before, this effect is of order \( \epsilon^2 \). Second, buyers of product 1 pay more, so their surplus is reduced by \( \epsilon q_A(p_B + \delta, p_B) \). Third, a fraction (with order of \( \epsilon \)) of consumers are shifted from product 1 to other products, but the surplus change of each shifted consumer is of order \( \epsilon \), and so the last effect is also of order \( \epsilon^2 \). Therefore, \( V'(\delta) = -q_A(p_B + \delta, p_B) \), which is an analogy of Roy’s identity in our search model. Then one can show\(^{21}\)

\[
V(-\Delta) - V(0) = \int_{0}^{\Delta} V'(\delta) d\delta = \Delta(p_B^n - \frac{3\Delta}{2}).
\]

\(^{21}\)Note that \( q_A(p_B + \delta, p_B) = 1 - a - \delta + r_A \), where \( r_A = \int_{p_B}^{p_B + \alpha - \beta} u^{n-1} du \) is the returning demand for product 1 according to (4.12). So

\[
\int_{0}^{\Delta} V'(\delta) d\delta = \frac{\Delta^2}{2} + \Delta(1 - a + r_A).
\]

Then using (4.14) yields our expression.
Therefore,

\[ V_1 - V_0 = V(-\Delta) - V(0) + V(p_B, p_B) - V(p_0, p_0) \]
\[ = \int_{p_B}^{p_0} (1 - p^n) dp + \Delta(p_B^n - \frac{3\Delta}{2}). \]

Since

\[ \Pi_0 = p_0(1 - p_0^n), \]
and

\[ \Pi_1 = p_B(1 - p_A p_B^{n-1}) - \Delta \cdot q_A = p_B(1 - p_B^n) + \Delta^2 \]

where we have used \( p_B^n - q_A = \Delta \) from (4.13), we get

\[ W_1 - W_0 = V_1 - V_0 + \Pi_1 - \Pi_0 = n \int_{p_B}^{p_0} p^n dp + \Delta(p_B^n - \frac{\Delta}{2}). \]

**Proposition 4.10** In the uniform-distribution setting with Assumption 4.2, there exists \( a^* < 1 \) such that, for \( a > a^* \), we have \( W_1 > W_0 \) and \( V_1 > V_0 \).

**Proof.** Let \( \varepsilon = 1 - a \). When \( \varepsilon \to 0 \), we have known that both \( p_A \) and \( p_B \) converge to \( p_0 \). Since \( p_0 \) is independent of \( a \), we can approximate these prices as \( p_i = p_0 + k_i \varepsilon \), where \( k_i \) needs to be determined. By extending (4.13) and (4.14) around \( a = 1 \) and discarding all higher-order terms, we can show \( k_1 = k_2 = 0 \).\(^{22}\) We further claim that a sufficient condition for both \( W_1 > W_0 \) and \( V_1 > V_0 \) is

\[ \frac{1 - a^{n-1}}{n + 1} p_B + \frac{3\Delta}{2} > 0, \]

which is proved in Appendix 4.10. Then, when \( \varepsilon \) is around zero but positive, we have

\[ \frac{1 - a^{n-1}}{n + 1} p_B + \frac{3\Delta}{2} \approx \frac{n - 1}{n + 1} p_0 \varepsilon > 0, \]

which means that \( W_1 > W_0 \) and \( V_1 > V_0 \) for \( a \to 1 \). \( \blacksquare \)

This result is mainly because, when the search cost is small, the search inefficiency caused by the non-uniform prices is of second order, while the output effect is of first order. In effect, numerical simulations suggest that our welfare results would hold for any \( a \) permitted by Assumption 4.2. The following graph is an example when \( n = 2 \), where the solid line is \( W_1 - W_0 \) and the dashed line is \( V_1 - V_0 \).

\(^{22}\) Although the first-order coefficients are zero, the second-order coefficients are not. Denote them by \( l_1 \) and \( l_2 \). One can show that \( l_2 = \frac{(n-1)p_0}{2(1-2p_0)} < 0 \) and \( l_1 = (1-n)l_2 > 0 \).
4.3.3 A discussion on announced prices

Now suppose the firm can announce all prices in advance, but consumer search order is still restricted in the prominence case. Let us focus on $m = 1$. We then need to replace $q_B^0$ in (4.12) by $q_B$ since now all prices are observable. This change will only affect the first-order condition $\frac{\partial \Pi}{\partial p_B} = 0$. Proposition 4.7 remains without any modification, so we still have $p_A > p_B$ under general conditions. In the uniform-distribution setup, $\frac{\partial \Pi}{\partial p_A} = 0$ still leads to (4.13), but $\frac{\partial \Pi}{\partial p_B} = 0$ now leads to

$$1 - (n + 1)p_A p_B^{n-1} - q_A + p_B^n - \Delta = 0.$$  

Adding them together yields

$$1 - (n + 1)p_A p_B^{n-1} = 0.$$  \hspace{1cm} (4.16)

From (4.16) and (4.13), one can check the existence result along the same logic in the proof of Proposition 4.8. (In footnote 5 of Appendix C, we further show that price announcement will raise all prices.) Comparing (4.16) with (4.15), we can see that the prominence case and the random search case have exactly the same total output. Then $p_B < p_0 < p_A$ follows since $p_B < p_A$.

Based on the output result, the welfare analysis is also simple. The firm still earns more than in the case without prominent product since it can at least announce prices $p_A = p_B = p_0$. Total welfare (and so consumer surplus) must go down since prominence causes non-uniform prices (so less efficient search behavior) but does not improve output. That is, in the central-pricing model, the monopoly’s commitment power can reverse our welfare results.
Proposition 4.11 If the firm can announce and commit all product prices in advance, we still have $p_B < p_0 < p_A$ under general conditions. However, in the uniform-distribution setting, introducing one prominent product does not affect total output but lowers consumer surplus and total welfare.

The result in the second part is because price announcement raises all prices in equilibrium such that the output increase caused by prominence does not exist any more. In the imperfect-information case, lowering the actual price of non-prominent products will not reduce the prominent product’s fresh demand since consumers’ search decision is based on their expectations; while in the announcement case, it does decrease the prominent product’s fresh demand, which dampens the firm’s incentive to lower the price of non-prominent products. Thus, in equilibrium all prices will be higher.

4.4 Conclusion

This chapter has further studied the implications of prominence in search markets. We have extended Chapter 3 by considering multiple prominent products. A more important contribution is that we find the impacts of prominence on price and welfare are contrasting between the competitive-pricing case and the central-pricing case. Specifically, prominent products are cheaper than non-prominent products in the competitive-pricing case but more expensive in the central-pricing case. Compared to the random search case, non-prominent products become more expensive in the competitive-pricing case, but they become cheaper in the central-pricing case. As far as welfare implications are concerned, in the competitive-pricing case making some products prominent tends to increase industry profit but lower consumer surplus and total welfare; while in the central-pricing case it may boost all players’ surplus (when price information is imperfect).

Prominence is an important economic factor that could lead to asymmetry among economic agents. Beyond the product market, prominence should also play a role in the labor market. For example, some companies are more famous than others and job seekers could be attracted to apply for their jobs first; and some job candidates could also be more prominent than others and so they are more likely to be considered by the company. Hence, two-sided prominence could exist in the labor search market.
Chapter 5

Advertising, Misperceived Preferences, and Product Design

5.1 Introduction

Many products such as cars, electronics and financial services have a large number of attributes. It is usually a complicated task for ordinary consumers to value these multiattribute products properly. Sometimes people do not realize the existence of some attributes. Sometimes people realize them but do not know their exact functions. A more demanding situation is that whether an attribute is important or not depends on the circumstance in which the product will be used and people need to estimate the probabilities of future contingencies. Such complications open the door for the firm to influence the way consumers value the product. The firm can conceal some inferior product attribute by drawing consumers’ attention to other attributes, or it can make some actually trivial attribute look important.

This chapter studies a particular kind of marketing activities which intend to draw people’s attention to some attribute and make them neglect others to some extent. For example, advertising can serve this purpose by flooding the information about a particular attribute but saying little about others. Firms can also frame the presentation of their products such that some attributes are more salient than others. Examples include the digital camera adverts which highlight huge amounts of megapixels,\(^1\) the computer adverts which highlight super CPU speeds, the car adverts which emphasize quiet engines, and the

\(^1\)Megapixels are of course important for the quality of a digital photograph. But as the consumer guidance provided by Which? suggests: "...Megapixels aren’t the be-all and end-all though – lens quality, sensor quality and sensor size play a big role in how sharp and colour-accurate your pictures are. For example, a 7Mp camera with a great lens and a large sensor could provide better image quality overall than a 10Mp camera with a great lens but small sensor...". Moreover, for many ordinary consumers, relatively low megapixels (like 4Mp) are enough to produce photos with acceptable quality. Other features like shutter delay and batteries deserve more attention for use convenience. See more in Which?’s website: http://www.which.co.uk/.
food adverts which emphasize good taste. Without doubt, this kind of advertising has other possible functions, but in this chapter, we focus on its function in manipulating the way consumers value a product: it may make consumers overestimate the importance of the advertised attribute but underestimate the importance of unadvertised ones. In fact, there is some anecdotal evidence that, misled by sellers' advertising or other marketing activities, some consumers, especially those who are not knowledgeable enough, often end up buying product models with some shining attributes but relatively inferior overall performance.

This advertising effect may be the direct result of consumers' manipulable preferences. For example, Mackenzie (1986) documents experimental evidence that, when subjects are given more time to think about the advertising claims on a watch's water resistance, their ratings of the importance of this attribute significantly increase. It might also be because of consumers' limited attention or limited memory. For example, Gardner (1983) shows in an experiment that the product attribute displayed prominently in adverts is more likely to be recalled and so is weighted more heavily in the subsequent product evaluation. Another possible reason is consumers' manipulable beliefs in an uncertain environment. For example, the advertising which highlights a certain attribute by describing a concrete scenario in which this attribute is very useful, could lead consumer to overestimate the likelihood of similar situations in their future and so overestimate the importance of that attribute. (We will discuss these justifications in detail in Section 5.2.)

It is also fair to say that different consumers respond to this kind of advertising in different ways. Some consumers, especially those who do not have enough relevant product knowledge, are more likely to be influenced by this kind of advertising. We call them "naive consumers". But other consumers may be more knowledgeable and experienced and so are immune to this kind of advertising. We call them "sophisticated consumers". This consumer heterogeneity is consistent with the observation that we often see the coexistence of the single-attribute advertising and other product models which have mediocre quality of the advertised attribute (but may have high quality in other attributes). We aim to investigate the impacts of the single-attribute advertising when there are both naive consumers and sophisticated consumers in the market.

We consider a model where a monopoly firm produces a two-attribute product. It can

\footnote{Recently there is also a worry about food advertising which overemphasizes organicity.}

\footnote{Two early papers on imperfect consumer knowledge concerning the function of product attributes are Auld (1972) and Colantoni et al. (1976). However, they did not consider the supply side's response.}

\footnote{The following is an example from Freecom's website: "... Imagine what can happen when you're late for a flight, running to the gate, and your external hard drive accidentally falls from your notebook case or out of your jacket pocket onto the floor... it breaks. Hundreds of hours of work, gigabytes of spreadsheets, documents, photo's... all gone. Not anymore! Freecom introduces the ToughDrive. The ideal external storage solution for people on the move. Fitted with an internal anti-shock frame and a unique soft silicon cover, it can withstand bumps and drops so your drive is still fully functional after it has dropped."}
offer one variant of this product or two depending on its marketing strategy. If the firm advertises two attributes of the product or does not advertise at all, we assume that the two attributes are equally salient and so all consumers will share the same preferences which, according to the viewpoint of an expert, reflect true functionality. If the firm only advertises one attribute, consumers will be segmented into two groups: naive consumers and sophisticated consumers. But the type information is private and the firm only knows the fraction of each group of consumers. In this situation, the firm might want to offer two appropriately designed variants to screen consumers. We will answer the following questions: If the firm advertises one attribute and tries to screen consumers, how will it design the two variants? What is the impact of the advertising on the surplus of each type of consumers and total welfare? How will the presence of naive consumers affect the welfare status of sophisticated consumers?

To illustrate our main points in a simple way, we further suppose that both attributes are equally important to consumers if there is no advertising, but it is less costly to improve attribute 1's quality. Hence, the quality of attribute 1 should be higher than that of attribute 2 in the first-best situation, and the firm has no incentive to advertise attribute 2. Given attribute 1 is advertised, we find that, compared to the socially efficient design, the quality of attribute 1 designed for naive consumers is distorted upward, but that of attribute 2 is distorted downward. Such a distortion reflects the misperceived preferences of naive consumers since they now overestimate the relative importance of attribute 1. Meanwhile, the product designed for sophisticated consumers will also be distorted due to the adverse-selection effect. But it is distorted in the opposite directions, i.e., the quality of attribute 1 is distorted downward and that of attribute 2 is distorted upward. Although such a distortion lowers sophisticated consumers' willingness to pay, it helps the firm exploit naive consumers more since such an alternative product is rather unattractive to them. In sum, when attribute 1 is advertised, relative to the socially optimal design, the product for naive consumers has too imbalanced attribute qualities while that for sophisticated consumers has too balanced attribute qualities.

As far as consumer surplus is concerned, we find that, according to their misperceived preferences, naive consumers will earn positive surplus; while according to the viewpoint of an expert, they have negative surplus and so actually have been exploited by the firm. Taking the latter as the welfare criterion, we suggest that setting an appropriate minimum quality standard for the unadvertised attribute is helpful at least for naive consumers. However, sophisticated consumers always earn zero surplus in our basic model with homogeneous consumer reservation utility. This is because they never want to mimic naive consumers (to get negative surplus) and so no information rent is left to them. If we measure total welfare according to the expert criterion, the single-attribute advertising
always reduces welfare since it causes product-design distortion to all consumers.

Since sophisticated consumers always get zero surplus no matter naive consumers exist or not, our basic model cannot reflect the potential externality (in terms of surplus) imposed by naive consumers on sophisticated consumers. To illustrate this potential externality, we introduce heterogenous reservation utilities among consumers as in Rochet and Stole (2002). We then find that, in order to extract more surplus from naive consumers, the firm will distort both the price and design of the product for sophisticated consumers such that more of them will be excluded from the market. This is the externality imposed by naive consumers on sophisticated consumers.

Related Literature:

A related but independent paper on multi-attribute product design is Bar-Isaac et al. (2007). They argue that, in a multi-attribute product market, a fall of the information-gathering cost could harm efficiency. Their basic idea goes as follows. Consider a two-attribute product. The firm can invest in the average quality of each attribute, but their realizations are stochastic. Consumer can only find out one attribute's realized quality by paying an information-gathering cost. Consumers have heterogenous preferences in the sense that some consumers care more about one attribute while others care more about the other. Now if it becomes easier to access information of one attribute, then more consumers will shift to check this attribute, which will induce the firm to invest more in improving this attribute's quality and so harm consumers who care more about the other attribute. This negative effect could outweigh the benefit from lower information-gathering costs. Both their model and our model intend to illustrate a similar idea: in a multi-attribute product market, when some consumers become more sensitive to one attribute's quality (because of reduced information-gathering costs in their model and misperceived preferences in our model), the firm might invest more in that attribute relative to the other, which will harm other consumers who have opposite preferences. The main difference is that the firm in their paper only produces one product and so there is no screening.

Our screening model has the similar structure as the traditional second-degree price discrimination model (Mussa and Rosen (1978), for instance), but our product is two-dimensional and the information structure is endogenous. Lewis and Sappington (1994) also consider endogenous information structure and screening. Specifically, they consider a model where the firm can provide potential consumers with product information, for example, by allowing them to experiment with the product, which can help consumers learn about their true preferences. The market will then be segmented since different consumers may find different preferences, and the firm can possibly extract more surplus

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5 This idea is also related with the literature on multitask incentive (e.g., Holmstrom and Milgrom (1991)).
through price discrimination. Although we also consider endogenous segmentation in the market, our reason is preference manipulation. This difference will lead to distinct welfare judgement. More importantly, since the product in our model is two-dimensional, it is more interesting to investigate the product design problem. Moreover, we explore the welfare interaction between different types of consumers.\(^6\)

This chapter is also related to Gabaix and Laibson (2006). They explore the market implications of limited attention or unawareness (which is one possible justification for our advertising effect as we will argue in next section) in a different scenario. They assume that the product price consists of two parts: the basic price and the add-on price (e.g., the penalty charge of credit cards). The former is public, while the latter is hidden if the firm does not advertise it. Some consumers are naive in the sense that, if no firm advertises the add-on price, they only realize the existence of the public price. But once they arrive at a firm, they are “forced” to pay both. They show that the existence of a relatively large fraction of naive consumers will lead competitive firms not to advertise the hidden price even if it is costless to do so. Naive consumers are exploited in equilibrium, while sophisticated consumers benefit from naive consumers since competition induces firms to transfer the profit from naive consumers to them.\(^7\)

The rest of this chapter is organized as follows. Sections 5.2 discusses two psychological justifications for our advertising effect. Section 5.3 presents the basic model, and it is then analyzed in Section 5.4. Section 5.5 extends the basic model by introducing heterogenous reservation utilities to investigate the externality imposed by naive consumers on sophisticated consumers. Section 5.6 concludes and discusses other possible applications of our basic idea. Appendix D includes some proofs and an extension with continuous advertising effect.

5.2 Behavioral Foundations of the Advertising Effect

The main assumption of this chapter is, when advertising or other marketing activities make some attribute of a product relatively more salient, people may regard it more important relative to other attributes. This view is related with but different from the persuasive view of advertising which simply assumes that advertising will increase people’s willingness to pay. (See, e.g., the survey on advertising by Bagwell (2007)). We emphasize

\(^6\)A more recent paper on this topic but without considering screening is Johnson and Myatt (2006). Our model is also loosely related to the literature on multi-dimensional nonlinear pricing. But that problem is usually difficult to solve (see, e.g., Armstrong (1996), and Rochet and Chone (1998)). To avoid that technical difficulty, we assume that the two taste parameters in both dimensions are perfectly negatively correlated.

\(^7\)Two other related papers are Rubinstein (1993), and Eliaz and Spiegler (2006). Both of them consider how the firm could discriminate over diversely sophisticated consumers.
that, although advertising influences the way people evaluate the product, whether it can raise their willingness to pay also relies on product design. In the following, we propose two psychological channels which can justify our advertising effect.\footnote{As pointed out by Bagwell (2007), there is no a good micro foundation for the persuasive view of advertising in the economics literature. Our psychological channels also offer justifications for it to some extent.}

5.2.1 Limited attention

Attention is a scarce resource of human beings. It has been long realized that limited attention is an important determinant factor of behavior.\footnote{Herbert Simon (1997, page 374-75) says: "... in a procedural theory, it may be very important to know under what circumstances certain aspects of reality will be heeded and others ignored ... focus of attention is a major determinant of behavior." In the flood-insurance example, he remarks: "... it appears that insurance is purchased mainly by persons who have experienced damaging floods or who are acquainted with persons who have had such experiences, more or less independently of the cost/benefit ratio of the purchaser." In the voting example, he comments: "... A voter who attends to the rate of inflation may behave quite differently from a voter who attends to the federal deficit."} Moreover, people often cannot fully control how to divide their attention among different tasks. Those vivid and salient stimuli are usually able to catch more attention even if their economic importance is low. (See, e.g., Kahneman (1973), Fiske and Taylor (1991), and Pashler (1998).) Thus, for a multi-attribute option, it is reasonable to infer that those attributes that are salient because of advertising or other framing effects, could catch more attention, and those less salient attributes could be neglected to some extent. If some attribute is neglected, it is hard to imagine that it could play any role in people's valuation of the whole option. In contrast, those salient attributes may thereby appear relatively more important.\footnote{This argument is also consistent with the information integration theory (Anderson (1981)) which suggests that people's valuation of an option is the weighted average of valuations of all attributes. The sum of weights is fixed. When people put more weights on some attribute, they will put less on others.}

Recently behavioral economists have started to be interested in the economic implications of limited attention. Their works suggest that our behavioral assumption is consistent with people's real market behavior. For example, Hossain and Morgan (2006) document the evidence that, on eBay auctions, for the same reserve price, low opening bid and higher shipping cost will lead to a greater number of bidders and higher revenues. The leading explanation for this is that bidders may pay less attention to and so is less sensitive to the shipping cost.\footnote{Chetty et al. (2006) provide related evidence. In a grocery store in California, compared to the situation where sales tax is excluded from the price tag, showing the tax-inclusive price (e.g., $4.99 + Sales Tax = $5.36) reduces demand nearly as much as the effect of a price increase of an equivalent amount. But their survey research indicates that people are quite knowledgeable about sales taxes when their attention is drawn to the subject. Therefore, they conclude that customers may neglect the tax to some degree when it is excluded from the price tag.} There is also evidence that firms or the government has incentive to respond to people's limited attention. For example, Della Vigna and Pollet (2006) show that, on Friday, investors are more inattentive to earnings announcement (maybe because weekends distract them temporarily) and so firms are more likely to announce...
bad news. Eisensee and Stromberg (2007) present evidence that a natural disaster is less likely to receive relief from US government if at the same time some disaster unrelated exciting events (e.g., the Olympics) are going on and intensively reported in the media. One possible reason is that those exciting events may distract the public’s attention from the disaster, and so the government’s act to the disaster is less effective in delivering favorable publicity. Politicians know this and then act accordingly.\(^{12}\)

### 5.2.2 Probability misjudgment and support theory

In many cases, the relative importance of a product attribute varies with the circumstance in which the product will be used, and so its expected relative importance depends on the probability judgement of the future uncertainty. However, it is well known that people’s intuitive judgement of probability is subjected to numerous biases (see, e.g., Kahneman, Slovic, and Tversky (1982)). Here we propose that our behavioral assumption could also be justified by misperception in probability judgement in the spirit of support theory which was formally developed by Tversky and Koehler (1994), and Rottenstreich and Tversky (1997).

The main idea of support theory is that the judged probability of an event will be discounted if its explicit components are not mentioned. In contrast, unpacking the description of the event can increase its judged probability. That is, how to describe an event will affect people’s probability judgement of it. For example, the judged probability of “a typical person’s death is caused by heart disease, cancer, or other natural causes” could be higher than the probability of “a typical person’s death is caused natural causes”. This is possibly because an explicit description of death causes could remind people of these possibilities they might have overlooked. In an interesting paper by Johnson et al. (1993), they document experimental evidence that subjects who are offered health insurance that covers hospitalization for any disease or accident are willing to pay a higher premium than subjects who are offered health insurance that covers hospitalization for any reason, and subjects are also willing to pay more for flight insurance that explicitly lists certain events covered by the policy (e.g., death resulting from an act of terrorism or mechanical failure).\(^{13}\)

Let us consider a two-attribute product. Suppose attribute 1 is more important in a category of scenarios (A), while attribute 2 is more important in another category of

\(^{12}\)See more evidence surveyed in Della Vigna (2007). See, e.g., Gabaix et al. (2006) and Hirshleifer et al. (2004) for recent theoretical works on limited attention.

\(^{13}\)Fischhoff et al. (1978) find that subjects’ judged probability of the hypothesis—“The cause of car failure is something other than the battery, the fuel system, or the engine”—increases from 0.22 to 0.44 when it is broken up into more specific causes (e.g., the starting problem, the ignition system). Rottenstreich and Tversky’s (1997) find that the probability that a randomly selected death was a murder is higher when it is described as “homicide by an acquaintance or stranger” than when it was merely described as “homicide”.

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scenarios (B). There are several models of this product: some of them have superior attribute 1 but relatively inferior attribute 2, while others are designed in the opposite way. Now if advertising or the salesman describes some specific A-scenarios, support theory predicts that consumers may then overestimate the relative likelihood of A and prefer models with superior attribute 1. (See the example in footnote 4.) In some cases, it is also possible that an advert which only mentions abstractly the high quality of some attribute (e.g. 9 megapixels) will trigger consumers to come up with scenarios in which this attribute is important.

5.3 Model

There is a monopoly firm which produces a two-attribute product. The quality of attribute i is $x_i$, and it is observable. The firm has separable cost function $c_1(x_1) + c_2(x_2)$, where $c_i(x)$ is an increasing and convex function with $c_i(0) = c'_i(0) = 0$. There is a continuum of consumers with measure one, and each of them has unit demand and zero reservation utility. Initially, all consumers are assumed to share the same preferences which are represented by the utility function\(^\text{14}\)

$$x_1 + x_2,$$

so each attribute is equally important for the consumer.\(^\text{15}\) To motivate the idea that the firm has incentive to manipulate consumers' preferences, we assume\(^\text{16}\)

$$c_1(x) = c(x),$$
$$c_2(x) = kc(x), \quad k > 1.$$ 

Since improving the quality of attribute 2 is more costly (at $x_1 = x_2$), we expect that the firm would wish consumers to regard attribute 1 as the more important attribute.

In our basic model, we suppose there are only two types of consumers. When attribute 1 is framed to be more salient (e.g., by advertising), naive consumers (with measure $\alpha$) will overestimate the relative importance of attribute 1, which is reflected by their misperceived

\(^{14}\)For highlighting that advertising can differentiate consumers, we assume consumers are originally homogeneous. Considering initially heterogenous consumers will complicate our analysis but add no fundamental insights.

\(^{15}\)Assuming unequally important attributes will not influence our main results fundamentally, but it could affect which attribute the firm will advertise. The details on that general setup are available from the author.

\(^{16}\)This particular relationship between $c_1(x)$ and $c_2(x)$ is innocuous. We have considered a more general relationship, and our main results remain qualitatively as long as the cost function is still separable.
utility function

\[(1 + \theta)x_1 + (1 - \theta)x_2,\]

where \(\theta \in (0,1]\) indicates the strength of the advertising effect. In particular, the case with \(\theta = 1\) can be interpreted as the situation in which naive consumers are overwhelmed by the salient attribute 1 such that they are totally unaware of attribute 2. If attribute 2 is framed to be more salient, \(\theta\) will be negative. Sophisticated consumers (with measure \(1 - \alpha\)) are totally immune to advertising, so their utility function remains unchanged. When both attributes are equally salient, we assume that even naive consumers keep their original (correct) preferences. Thus, the firm has no incentive to advertise both attributes in our setting. Consumer’s type information is private, and the firm only knows the fraction of each group of consumers.

The firm first chooses its advertising strategy (whether to advertise and which attribute to advertise) and designs the product. If it does not advertise, then only one variant of the product is provided and it is denoted by \((x_1^0, x_2^0, p_0)\), where \(p_0\) is its price. If it advertises, then the firm can produce two variants to cater to different consumers. They are denoted by \(N = (x_1^1, x_2^1, p_n)\) and \(S = (x_1^s, x_2^s, p_s)\). The product \(N\) is designed for naive consumers, and the product \(S\) is designed for sophisticated consumers. After being exposed to the advertisement and observing the quality and price of each product, consumers make their purchase decisions. One point deserving mention is, if it involves a sufficiently high fixed cost to design and produce an extra variant of the product, the firm will not use advertising to differentiate consumers. To focus on our main point, we just simply assume away this potential fixed cost.

5.4 Analysis

We first solve a simple case where all consumers have preferences

\[(1 + \theta)x_1 + (1 - \theta)x_2.\]

Then the monopoly firm will set the price \(p = (1 + \theta)x_1 + (1 - \theta)x_2\) to extract the whole surplus. Therefore, the optimal quality combination is solved in the following maximization problem:

\[
\max_{x_1, x_2} (1 + \theta)x_1 + (1 - \theta)x_2 - c(x_1) - kc(x_2).
\]

It is ready to see

\[c'(x_1) = 1 + \theta; \; kc'(x_2) = 1 - \theta.\]
Clearly, at optimum $x_1$ increases and $x_2$ decreases with $\theta$ since cost functions are convex. Denote by $\Pi(\theta)$ the optimal monopoly profit in this case. A useful property is that $\Pi(\theta)$ is convex in $\theta$ and it goes up with $\theta$ when $\theta > \hat{\theta}$ and goes down with $\theta$ when $\theta < \hat{\theta}$, where

$$\hat{\theta} = \frac{1 - k}{1 + k} \in (-1, 0).$$  \hspace{1cm} (5.1)

This can be easily verified by using the envelope theorem.\textsuperscript{17} It is also easy to check that $\Pi(\theta) > \Pi(-\theta)$ for any $\theta > 0$ given $k > 1$. Thus, if the strength of the advertising effect $|\theta|$ is the same no matter which attribute is advertised, the firm will never advertise attribute 2.\textsuperscript{18} In the following, we keep this assumption and so the firm will only advertise attribute 1 if it advertises.

If the firm does not advertise (so $\theta = 0$), the optimal quality combination is given by

$$c'(x_1^0) = 1; \ kc'(x_2^0) = 1.$$ \hspace{1cm} (5.2)

This is the first-best outcome. Since $k > 1$, we have $x_1^0 > x_2^0$. That is, in the socially optimal situation, the quality of attribute 1 should be higher than that of attribute 2. Denote by $\Pi_0 = \Pi(0)$ the optimal profit when the firm does not advertise.

### 5.4.1 The case without screening

We now consider the situation where the firm advertises but it cannot screen consumers (e.g., because of regulation or the high fixed cost of designing a new variant of the product). Then the firm must decide whether to serve all consumers. First, at optimum, the quality of attribute 1 must be higher than that of attribute 2. Second, given $x_1 > x_2$, the firm can either charge at $p = x_1 + x_2$ to cover the whole market, or charge at $p = (1 + \theta)x_1 + (1 - \theta)x_2$ to serve naive consumers alone. If the firm uses the first strategy, its optimal profit is $\Pi_0$. (This is the same situation as no advertising.) If it uses the second strategy, its optimal profit is $\alpha \Pi(\theta)$. Therefore, given no screening and no advertising cost, advertising will be strictly profitable only if

$$\alpha \Pi(\theta) > \Pi_0,$$

i.e., when the fraction of naive consumers is high enough. Once the firm advertises, it will exclude all sophisticated consumers.

\textsuperscript{17}Some readers may guess that $\Pi'(\theta) > 0$, i.e., it would be always beneficial to the firm to induce consumers to overestimate the relative importance of attribute 1. In fact, whether this is true or not depends on the starting point of consumers’ preferences. If initially consumers almost do not care about attribute 1 (i.e., $\theta \approx -1$ and $x_1 \approx 0$), slightly increasing $\theta$ will decrease their valuation of the whole product. That is why $\Pi(\theta)$ falls with $\theta$ when $\theta$ is negative enough.

\textsuperscript{18}In contrast, if $k < 1$, then the firm will advertise attribute 2.
5.4.2 The case with screening

Now we turn to the main case where the firm advertises attribute 1 and offers two variants to screen consumers. Compared to sophisticated consumers, naive consumers have a higher marginal valuation of the quality of attribute 1 but a lower marginal valuation of the quality of attribute 2. For the same product, which type of consumers having a higher valuation depends on whether $x_1 > x_2$ or $x_1 < x_2$. Although the product designed for naive consumers should have $x_1^n > x_2^n$ in equilibrium, we are not sure whether $x_1^s > x_2^s$ since the product design for sophisticated consumers could be distorted. This means, unlike in the standard unidimensional screening problem, a priori we do not know which type of consumers is the high type.

For expository convenience, we introduce several pieces of notation. Define

$$ r = \frac{\alpha}{1 - \alpha}. $$

Let

$$ v_n = (1 + \theta)x_1^n + (1 - \theta)x_2^n - p_n $$

be a naive consumer’s net surplus (according to her misperceived preferences) if she picks product $N$. Similarly, let

$$ v_s = x_1^s + x_2^s - p_s $$

be a sophisticated consumer’s net surplus if she picks product $S$. Denote by

$$ w_n(x_1^n, x_2^n) = (1 + \theta)x_1^n + (1 - \theta)x_2^n - c(x_1^n) - kc(x_2^n), $$

$$ w_s(x_1^s, x_2^s) = x_1^s + x_2^s - c(x_1^s) - kc(x_2^s) $$

the social surplus of product $N$ (according to the naive consumer’s misperceived preferences) and product $S$, respectively. So $w_i - v_i$ is the profit from a $i$-type consumer. Then the firm’s problem is:

$$ \max_{x_1^n, x_2^n, v_n, v_s} \alpha [w_n(x_1^n, x_2^n) - v_n] + (1 - \alpha) [w_s(x_1^s, x_2^s) - v_s] $$

subject to

$$ v_n \geq 0, \quad (IR^n) $$

$$ v_s \geq 0, \quad (IR^s) $$

$$ v_n \geq v_s + \theta(x_1^n - x_2^n), \quad (IC^n) $$

$$ v_s \geq v_n + \theta(x_2^n - x_1^n). \quad (IC^s) $$

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The IR conditions reflect individual rationality since each consumer’s reservation utility is zero in this basic model, and the IC conditions prevent consumers from cross-buying. A simple observation is that the full-information result (associated with naive consumers’ misperceived preferences) can never be achieved. This is because the full-information result requires \( x_1^* = x_1^0 > x_2^0 = x_2^* \). If that were to be true, then \( v_n > 0 \) since naive consumers can at least buy product \( S \) and get surplus higher than \( v_s \). Then (IC\(^n\)) would bind at optimum, which in turn overturns the full-information result. This argument also indicates that, in order to extract more surplus from naive consumers, the firm has incentive to distort product \( S \) and make it less attractive to naive consumers. Also note that, if “excessive distortion” \( x_1^* < x_2^0 \) happens, the firm can actually extract all surplus by setting \( v_n = v_s = 0 \) (given \( x_1^0 > x_2^0 \)). This is a distinction between our model and the traditional one-dimensional product model (e.g., Mussa and Rosen (1978)) where this result will never hold under regular conditions.\(^{19}\)

If we let \( \Pi_1 \) be the optimal profit in this case, we must have \( \Pi_1 \geq \Pi_0 \). This is because the combination \((x_1^0, x_2^0, p_0)\) defined in (5.2) is always feasible in the current problem. In fact, as we will show below, the strict inequality always holds. Therefore, the possibility of screening will always induce the firm to advertise as long as advertising is not too costly.

Now we start to solve the optimization problem. We first give a useful lemma:

**Lemma 5.1** When attribute 1 is advertised, at optimum \( v_s = 0 \) (i.e., sophisticated consumers get zero surplus) and (IC\(^n\)) is binding.

**Proof.** We first argue that \( x_1^n \geq x_2^n \) at optimum. Suppose \( x_1^n < x_2^n \). Then we show that increasing \( x_1^n \) by \( \varepsilon \) and decreasing \( x_2^n \) by \( \varepsilon \) is a profitable deviation. This deviation makes (IC\(^n\)) easier to hold, and it does not affect all other constraints. On the other hand, naive consumers’ valuation of product \( N \) rises by \( 2\theta\varepsilon \), and the production cost decreases because, for small \( \varepsilon \), we have

\[
\begin{align*}
    c(x_1^n + \varepsilon) + kc(x_2^n - \varepsilon) & \\
    \approx c(x_1^n) + \varepsilon c'(x_1^n) + kc(x_2^n) - k\varepsilon c'(x_2^n) & \\
    < c(x_1^n) + kc(x_2^n).
\end{align*}
\]

(We have used the convexity of the cost function and \( k \geq 1 \).) Therefore, \( w_n \) (so the profit) increases.

\(^{19}\)In this aspect, our model is closer to the countervailing-incentive problem in, e.g., Lewis and Sappington (1989), and Maggi and Rodriguez-Clare (1995), where an agent’s private information is her outside option and the surplus of each type of agent may vary with the type parameter non-monotonically. Then we need deal with more than one individual rationality condition explicitly. This point will be more apparent when we discuss the continuous-type model in the Appendix D.2.
Now suppose $v_s > 0$ at optimum. Then $v_n = 0$ would hold. Otherwise, the firm can increase profit by charging each product slightly more. IC* would also bind. Otherwise, reducing $v_s$ slightly is a profitable deviation. Thus, $v_s = \theta(x_2^n - x_1^n)$. Since $v_s > 0$, we have $x_2^n > x_1^n$. This is a contradiction, so $v_s = 0$.

If $v_n > 0$ at optimum, then (ICn) must be binding. Otherwise, the firm can enhance profit by decreasing $v_n$ slightly. Now suppose $v_n = 0$ at optimum. If (ICn) is slack, then $x_1^n < x_2^n$ since $v_s$ is also zero. Then a profitable deviation exists since maximizing $w_s$ requires $x_1^n > x_2^n$, so (ICn) must be binding. ■

This lemma implies

$$v_n = \theta(x_1^n - x_2^n) \tag{5.3}$$

at optimum, and (IC*) requires

$$x_1^n - x_2^n \geq x_1^s - x_2^s. \tag{5.4}$$

Now our strategy is to first solve a relaxed problem without considering the constraints $v_n \geq 0$ and (5.4), and then check whether the solution will actually satisfy them. Substituting $v_s = 0$ and (5.3) into the objective function, we get

$$\alpha w_n + (1 - \alpha)w_s - \alpha \theta(x_1^n - x_2^n).$$

Maximizing it yields

$$c'(x_1^n) = 1 + \theta, \quad kc'(x_2^n) = 1 - \theta; \tag{5.5}$$

$$c'(x_1^n) = 1 - \theta r, \quad kc'(x_2^n) = 1 + \theta r. \tag{5.6}$$

It is ready to verify that, if $\theta r \leq -\hat{\theta}$ (where $\hat{\theta} < 0$ is defined in (5.1)), then (5.6) implies $x_1^n \geq x_2^n$, and so $x_1^n > x_2^n > x_2^n$. Thus, both $v_n \geq 0$ and (5.4) are satisfied, and what we have got is indeed the optimal solution.

However, if $\theta r > -\hat{\theta}$, then $x_1^n < x_2^n$ (i.e., strict excessive distortion occurs), and so $v_n \geq 0$ will be violated. In this case, we need add the constraint $x_1^n \geq x_2^n$ to the optimization problem (but we still ignore the constraint (5.4)). If this constraint would be slack at optimum (i.e., $x_1^n > x_2^n$), we would have the same solution as before. But we have known that cannot happen at optimum given $\theta r > -\hat{\theta}$. Thus, we must have $x_1^n = x_2^n$. Given this constraint, it is ready to derive that $x_1^n$ is still given by (5.5) and $x_1^n = x_2^n = \bar{x}$ which satisfies

$$c'(\bar{x}) = \frac{2}{1 + k}. \tag{5.7}$$

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It is ready to see that this solution satisfies (5.4), so we are done.

We summarize the solution in the following proposition:

**Proposition 5.1** When attribute 1 is advertised and the firm supplies two variants of the product to screen consumers,

(i) if \( \theta r < -\hat{\theta} \), then at optimum \( v_n = \theta(x_1^* - x_2^*) > 0 \) and \( v_s = 0 \). The optimal quality levels are given by (5.5) and (5.6). Relative to the socially optimal design \((x_1^0, x_2^0)\), \(x_1^*\) is distorted upward and \(x_2^*\) is distorted downward, while the design for sophisticated consumers is distorted in the opposite direction. The firm’s profit is

\[
\Pi_1 = \alpha \Pi(\theta) + (1 - \alpha) \Pi(-\theta r).
\]

(ii) If \( \theta r \geq -\hat{\theta} \), then at optimum \( v_n = v_s = 0 \). That is, the firm extracts the whole surplus. \((x_1^*, x_2^*)\) is still given by (5.5) and \(x_1^* = x_2^* = \hat{x}\), where \(\hat{x}\) is defined in (5.7). The firm’s profit is

\[
\Pi_1 = \alpha \Pi(\theta) + (1 - \alpha) \Pi(\hat{\theta}).
\]

In the following, we discuss the intuition and implications of the above results:

**Product design.** The equilibrium quality levels satisfy

\[
x_1^* > x_1^0 > x_2^* > x_2^0 > x_2^N.
\]

Since naive consumers’ preferences have been manipulated, it is natural that the quality levels of product \(N\) will be distorted. Relative to the socially optimal design, the quality of the advertised attribute is too high and the quality of the unadvertised attribute is too low. Nevertheless, according to naive consumers’ misperceived preferences, there is no distortion at all and the qualities are just “efficient”. This is consistent with the well-known result of “no distortion at the top” in the screening literature, since naive consumers are the high-type consumers in our equilibrium.

The distortion occurring to sophisticated consumers is more interesting. Due to the presence of naive consumers, the product designed for sophisticated consumers has too balanced attribute qualities: the quality of the advertised attribute is too low and that of the unadvertised one is too high. It is also clear that the extent of distortion increases with the relative fraction of naive consumers \(r\) and the strength of advertising effect \(\theta\). When \(\theta r\) is sufficiently large, the attribute qualities of product \(S\) will even coincide. This is sorts of “externality” imposed by naive consumers on sophisticated consumers. Although sophisticated consumers always get zero surplus, this externality is important in the eyes of the social planner since it distorts the resource allocation. Another point is that the degree of distortion in \(S\) is bounded since strict excessive distortion (i.e., \(x_1^* < x_2^*\)) will never happen.
The distortion of $S$ is the result of adverse selection. To prevent naive consumers from buying product $S$, the firm wants to make product $S$ unattractive to them. One way to achieve that aim is to decrease the quality of attribute 1 but improve the quality of attribute 2. Moreover, this is the least costly way in the sense that it avoids a dramatic reduction of sophisticated consumers' willingness to pay for product $S$. The reason why there is no strict excessive distortion is, when $x_1$ becomes close to $x_2$, the firm has almost been able to implement the zero-rent allocation, and so further distortion is unnecessary. However, as we shall see in next section, if zero-rent allocation is not achievable in equilibrium (e.g., because of heterogeneous reservation utilities among consumers), strict excessive distortion might happen.

**Profit.** We now show that the firm can strictly earn more through advertising and screening. Using the convexity of the profit function $\Pi(\cdot)$, if $\theta r < -\hat{\theta}$, we have

$$
\Pi_1 = \alpha \Pi(\theta) + (1 - \alpha) \Pi(-\theta r) \\
> \Pi [\alpha \theta - (1 - \alpha)\theta r] = \Pi_0.
$$

Similarly, if $\theta r \geq -\hat{\theta}$, we have

$$
\Pi_1 = \alpha \Pi(\theta) + (1 - \alpha) \Pi(\hat{\theta}) \\
> \Pi [\alpha \theta + (1 - \alpha)\hat{\theta}] > \Pi_0,
$$

where the second inequality is because $\Pi(\cdot)$ is increasing on $[0, 1]$. Notice that this result holds even for $k = 1$, so the assumption of asymmetric production costs is unnecessary for the firm to manipulate consumer preferences. (But remember that it does matter in determining which attribute the firm will advertise.)

**Consumer surplus.** Naive consumers get non-negative surplus according to their misperceived preferences. However, in the eyes of an expert, they actually get negative surplus. When $\theta r \geq -\hat{\theta}$, $v_n = 0$, and so this is clear to see. When $\theta r < -\hat{\theta}$, a naive consumer's "true" surplus is

$$
\theta(x_1 - x_2) - \theta(x_1^n - x_2^n) < 0
$$

by using (5.8). Therefore, naive consumers are actually exploited by the firm. This seems consistent with our common observation: misled by the firm's marketing activities, some consumers who are not knowledgeable enough to value a product properly, often end up buying models with some shining attributes but relatively inferior overall performance.

An interesting point is, no matter which surplus criterion is used, naive consumers'
surplus always decreases with $\alpha$. This is because $x_1^* - x_2^*$ goes down with $\alpha$, while $x_1^0 - x_2^0$ is independent of $\alpha$. Thus naive consumers suffer from the existence of more of themselves. The intuition is, when $\alpha$ is small, product $S$ is only slightly distorted, and so naive consumers still value this product highly, in which case they must be given a big information rent if the firm wants to prevent them from buying $S$.

In our model, sophisticated consumers always get zero surplus, and so the existence of naive consumers does not affect their welfare status. This is a consequence of our assumption of homogeneous reservation utility. As we shall see in next section, if we introduce heterogeneous reservation utilities among consumers, sophisticated consumers in aggregate will indeed be harmed by the existence of naive consumers.

**Total welfare.** The result on total welfare is simple. First, since all consumers are actually the same in the eyes of the social planner, advertising and screening are purely wasteful. Second, total welfare will decrease with $\alpha$ and $\theta$, since larger of them will push up the degree of product distortion.

**Should screening be permitted?** In our setup with screening, all consumers will be served. If the firm would exclude sophisticated consumers, its profit would be at most $\alpha \Pi(\theta)$. If it would exclude naive consumers, its profit would be at most $(1 - \alpha) \Pi_0$. Both of them are smaller than $\Pi_1$. This is a potential advantage of screening, since sophisticated consumers could be excluded if screening is not permitted. On the other hand, screening may also cause extra product distortion. This happens when $\alpha \Pi(\theta) < \Pi_0$ in which case the firm will supply the first-best product for all consumers if screening is banned. (See the discussion in Section 4.1.) Therefore, when $\alpha < \Pi_0/\Pi(\theta)$ (i.e., when the fraction of naive consumers is relatively small), no permission of screening is better. In contrast, when $\alpha > \Pi_0/\Pi(\theta)$, permitting screening is better, because it can save sophisticated consumers from being excluded.

**Is there any new insight from continuous advertising effect?** As we will show in the Appendix D.2, the setup with continuous advertising effect offers several results which may deserve mention here. First, a fully separating equilibrium like in our two-type model may no longer exist even under regularity conditions. A bunch of relatively sophisticated consumers will be offered the same product if the fraction of them is relatively small. Second, some type of naive consumers will happen to be offered the socially efficient product. This is because the product for the most naive consumer and that for the most sophisticated consumer are distorted in the opposite directions, and for some middle-type consumer, the distortion due to the misperceived preferences and the distortion due to adverse selection just cancel each other out. Third, when consumers are shifted to be more naive in some systematic way, not all consumers suffer in the eyes of an expert. In fact, some naive consumers will then escape from being exploited. All of these points
cannot be reflected in our two-type model.

5.5 The Externality of Naive Consumers

In this section, we aim to show that the presence of naive consumers will reduce the surplus of sophisticated consumers. We first extend our basic model by introducing heterogenous reservation utilities among consumers. For simplicity, we assume that the reservation utility of sophisticated consumers distributes on \([0, \bar{v}]\) according to the cumulative distribution function \(F(v)\), while all naive consumers still have zero reservation utility as before.\(^{21}\) We also assume that the firm will not further screen sophisticated consumers based on their reservation utilities. Then the firm’s problem becomes

\[
\max_{x^n_1, x^n_2, v_n, v_s} \alpha \left[ w_n(x^n_1, x^n_2) - v_n \right] + (1 - \alpha) F(v_s) \left[ w_s(x^n_1, x^n_2) - v_s \right]
\]

subject to

\[
\begin{align*}
v_n & \geq 0, \quad (IR^n) \\
v_n & \geq v_s + \theta(x^n_1 - x^n_2), \quad (IC^n) \\
v_s & \geq v_n + \theta(x^n_2 - x^n_1). \quad (IC^s)
\end{align*}
\]

We focus on the interior-solution case where the optimal \(v_s \in (0, \bar{v})\). This is a plausible case if \(\bar{v}\) is relatively high. Given \(v_s\), the net surplus of an average sophisticated consumer is

\[
V_s = \int_0^{v_s} (v_s - v) dF(v).
\]

(5.9)

It is easy to see that \(V_s\) increases with \(v_s\).

Given the elastic demand from sophisticated consumers, the firm is now able to adjust the price of product \(S\) in a more flexible way. In order to make product \(S\) less attractive to naive consumers, the firm may have incentive to distort both of its price and its qualities. We keep the following regularity condition:

**Assumption 5.1** \(v + \frac{E(v)}{f(v)} \) increases with \(v\).

This assumption holds at least for logconcave \(F\). If there is no advertising, the product design is the first best, and the optimal pricing leads to

\[
w_0 = v_0 + \frac{F(v_0)}{f(v_0)},
\]

(5.10)

\(^{21}\)Similar results as we will derive below can be established under some conditions even if we introduce heterogenous reservation utilities among all consumers. But the analysis will be more complicated.
where $w_0$ is the social surplus of the product and $v_0$ is the surplus for the threshold type of sophisticated consumer. In the following, we aim to show that $v_s$ in the advertising case will be less than $v_0$.

Now we analyze the advertising case. The following lemma helps simply the analysis.

**Lemma 5.2** The incentive compatibility condition ($I^C_n$) must be binding at optimum.

**Proof.** If $v_n > 0$ at optimum, then ($I^C_n$) must be binding. Otherwise, the firm can enhance profit by reducing $v_n$ slightly. Now suppose $v_n = 0$ at optimum. If ($I^C_n$) were to be slack, then $x^*_1 < x^*_2$ since $v_s > 0$. Then we have a profitable deviation by increasing $x^*_1$ and decreasing $x^*_2$. This is a contradiction. So ($I^C_n$) should be binding. ■

This lemma implies

\[ v_n = v_s + \theta(x^*_1 - x^*_2). \] (5.11)

Then ($I^C_n$) requires

\[ x^n_1 - x^n_2 \leq x^n_1 - x^n_2. \] (5.12)

Our strategy to solve the problem is the same as before. We first solve the problem with the constraint (5.11) alone and then check whether the solution to this relaxed problem satisfies the other two constraints $v_n \geq 0$ and (5.12). Substituting (5.11) into the objective function, we get

\[ \alpha w_n + (1 - \alpha)F(v_s)(w_s - v_s) - \alpha [v_s + \theta(x^*_1 - x^*_2)]. \]

Clearly, product $N$ is efficient according to naive consumers’ misperceived preferences, so $x^n_1 > x^0_1$ and $x^n_2 < x^0_2$. If we focus on the interior solution, the other first-order conditions are:

\[ w_s = v_s + \frac{1}{f(v_s)} [F(v_s) + r], \] (5.13)

\[ c'(x^*_1) = 1 - \frac{\theta r}{F(v_s)}, \] (5.14)

\[ kc'(x^*_2) = 1 + \frac{\theta r}{F(v_s)}. \] (5.15)

It is clear that product $S$ is distorted similarly as in the previous basic model (but with a larger degree), i.e., $x^*_1 < x^0_1$ and $x^*_2 > x^0_2$. Therefore, our solution must satisfy (5.12). To satisfy the other constraint $v_n \geq 0$, $x^*_1 - x^*_2$ must not be too negative according to (5.11), which requires relatively small $\frac{\theta r}{F(v_s)}$. As we shall show below, $v_s$ actually goes down with

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$^{11}$They are also sufficient conditions for appropriate $F(v)$ (e.g. the uniform distribution). But Assumption 5.1 alone is not enough unless $f(v) < 0$ or $|F(v) + r|f(v) \leq 2f(v)^2$. 

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\( r \) in this case, and so the right-hand side of (5.11) is decreasing in \( r \). Therefore, to have \( v_n \geq 0 \), we need relatively small \( r \). For example, when \( r \approx 0 \), there is almost no distortion in \( S \), and then we have \( x^*_1 - x^*_2 > 0 \) which further implies \( v_n > 0 \).

However, if \( r \) is relatively large, the condition \( v_n \geq 0 \) will fail to hold, and so we need add it to the optimization problem. Then our problem becomes

\[
\max_{x^*_1, x^*_2, v_s} \alpha w_n + (1 - \alpha) F(v_s) (w_s - v_s) - \alpha [v_s + \theta (x^*_1 - x^*_2)]
\]

subject to

\[ v_s + \theta (x^*_1 - x^*_2) \geq 0. \]

We use the Lagrangian method to solve this problem. If the constraint were to be slack at optimum, we would obtain the same first-order conditions as in the above. Hence, the constraint must be binding (i.e., \( v_n = 0 \)) if the above solution with relatively small \( r \) does not characterize the optimal solution. Given the binding constraint

\[ v_s = \theta (x^*_2 - x^*_1), \tag{5.16} \]

the quality levels are determined by the optimization problem:

\[
\max_{x^*_1, x^*_2} \alpha w_n + (1 - \alpha) F[\theta (x^*_2 - x^*_1)] \cdot [w_s + \theta (x^*_1 - x^*_2)].
\]

Clearly, the quality levels should be independent of \( \alpha \) (and so \( r \)), and \((x^*_1, x^*_2)\) is again efficient according to naive consumers' misperceived preferences (and so the other IC constraint is no problem). One can also check that \((x^*_1, x^*_2)\) must satisfy

\[ c'(x^*_1) = 1 + \theta A, \quad kc'(x^*_2) = 1 - \theta A, \]

where \( A \) satisfies

\[ w_s = v_s + \frac{F(v_s)}{f(v_s)} (1 - A). \tag{5.17} \]

Since \( v_s \) is always positive (remember we are focusing on the interior-solution case), (5.16) requires \( x^*_1 < x^*_2 \), which further requires\(^{23}\)

\[ \theta A < \tilde{\theta}. \]

Therefore, \( A < 0 \) if \( v_n = 0 \) is optimal in equilibrium.\(^{24}\)

\(^{23}\)If the solution does not satisfy \( x^*_1 < x^*_2 \), the constraint \( x^*_1 \leq x^*_2 \) should be explicitly considered and we must have \( x^*_1 = x^*_2 \) (i.e., \( v_n = 0 \)) in equilibrium. We have excluded this possibility in the very beginning since we focus on the case with interior solutions.

\(^{24}\)The trade-off between \( v_n > 0 \) and \( v_n = 0 \) is as follows. If \( v_n = 0 \), then (IC\({}^n\)) requires \( x^*_1 - x^*_2 \) to be
Now let us compare (5.13) and (5.17) with (5.10). First, the existence of product-design distortion makes \( w_s \) no greater than \( w_0 \). Second, if \( v_s \geq v_0 \) were to be true, then the right-hand sides of (5.13) and (5.17) would be greater than the right-hand side of (5.10) because of Assumption 5.1 (remember \( A < 0 \) in (5.17)). This leads to a contradiction. Therefore, we conclude that \( v_s < v_0 \) no matter \( r \) is relatively small or large. Then (5.9) implies that advertising together with the existence of naive consumers will reduce the surplus of sophisticated consumers through both price and product-design distortion. Another point deserving mention is that now excessive distortion \( (x_1^* < x_2^*) \) will happen in the case with relatively large \( r \).

We summarize the above analysis in the following proposition:

**Proposition 5.2** In the model with heterogenous reservation utilities among sophisticated consumers, under Assumption 5.1, the existence of naive consumers (together with advertising) harms sophisticated consumers, and excessive distortion \( (x_1^* < x_2^*) \) will happen if \( v_n = 0 \) in equilibrium (which happens when \( r \) is relatively large).

We further discuss how the change of the fraction of naive consumers affects sophisticated consumers. If the case with \( v_n = 0 \) is optimal, then we have known that \( v_s \) (so \( V_s \)) is independent of \( r \). Now let us focus on the case with \( v_n > 0 \) in equilibrium. Our task is to show that \( v_s \) decreases with \( r \). Based on those first-order conditions (5.13)–(5.15), as we show in the Appendix D.1, \( \frac{\partial v_s}{\partial r} < 0 \) if and only if

\[
B \frac{F(v_s)}{f(v_s)} > C \left[ \frac{r}{F(v_s)} \right]^2,
\]

where

\[
B = \frac{\partial}{\partial v_s} \left[ v_s + \frac{F(v_s) + r}{f(v_s)} \right] ; \quad C = \frac{1}{c'(x_1^*)} + \frac{1}{kc'(x_2^*)}.
\]

However, one can check that this condition is exactly the local concavity condition of the optimization problem, and so it must be satisfied at optimum.

**Proposition 5.3** If \( v_n > 0 \) at optimum (which happens when \( r \) is relatively small), \( v_s \) (so \( V_s \)) decreases with \( r \). That is, the increase of the relative fraction of naive consumers will reduce the surplus of sophisticated consumers. If \( v_n = 0 \) at optimum, \( V_s \) is independent of \( r \).
5.6 Conclusion

This chapter has studied a kind of advertising or marketing activities which highlight one (or few) attribute(s) of a complex multi-attribute product. Based on psychological evidence, we propose that this kind of advertising can manipulate the way consumers value a product. Some naive consumers will overestimate the relative importance of the advertised attribute. We have investigated its implications in a monopoly market where there are also sophisticated consumers who are immune to advertising. If the firm can design different variants of the product to screen consumers, we find that the design of both variants will be distorted but in opposite directions. Moreover, due to the existence of naive consumers, more of sophisticated consumers will be excluded from the market if sophisticated consumers have heterogeneous reservation utilities. That is, naive consumers impose negative externality on sophisticated consumers.

It is desirable to consider the competition case of our model. However, a well-known result in the competitive price discrimination literature (see, e.g., Armstrong and Vickers (2001), and Rochet and Stole (2002)) is that competition may eliminate the distortion caused by the adverse-selection problem, so our product-design distortion upon sophisticated consumers may disappear in a competitive market. But such a result depends on some conditions (e.g., the fully covered market, the symmetry between firms, and no correlation between consumer (quality) preferences and their locations in a spatial model). If these conditions are not satisfied, equilibrium with product distortion still exists, and so our results in the monopoly case could apply.

We believe the idea that the relative salience of aspects of an option affects the way people value the option is rather general and deserves more research. Another possible application is multi-dimensional information disclosure. Consider a two-attribute option, and suppose that the qualities of attributes are independently and stochastically realized and are only observed by the seller. Then the seller decides how to disclose his private information. We also suppose there are some biased audience who will overlook or underestimate the relative importance of an attribute of which the quality information is not explicitly disclosed. Then several interesting results may emerge. First, the disclosure policy on each dimension is no longer independent even if the audience have separable preferences over the two dimensions. Second, even if information disclosure is costless, full revelation will not happen because the seller has incentive to withhold the information of the relatively low-quality dimension. Both results are in contrast to the standard information disclosure result (e.g., Milgrom (1981)). Third, when information disclosure is costly, the seller may over disclose the high-quality information but insufficiently reveal the low-quality information. The overall effect on the amount of information disclosure deserves studies. This is our ongoing research.
Appendices
Appendix A

Appendix of Chapter 2

A.1 Proofs of Propositions 2.2 and 2.3

We prove Proposition 2.2 under the condition $F(\frac{1}{2}) = \frac{1}{2}$ instead of the condition of symmetric distributions. Since our proofs involve asymmetric distributions, we modify firm 2’s demand function first:

$$q_2(p_2 < p_1) = 1 - F\left(\frac{1}{2} + \frac{1}{2\lambda}(p_2 - p_1)\right), \quad q_2(p_2 > p_1) = 1 - F\left(\frac{1}{2} + \frac{\lambda}{2}(p_2 - p_1)\right).$$

We have seen that the key elements in the proof of Proposition 2.1 are $F(\frac{1}{2}) = \frac{1}{2}$ and $q_1'(p_1, p_2) = q_2'(p_1, p_2)$ for $p_1 \neq p_2$. They still hold now, so there is no pure-strategy equilibrium in the single-reference-product case whenever $F(\frac{1}{2}) = \frac{1}{2}$.

Claim A.1 In the single-reference-product case, given Assumption 2.1 and $F(\frac{1}{2}) = \frac{1}{2}$, there exists a mixed-strategy equilibrium in which firm 1 randomizes over $p_1^L$ and $p_1^H$ and firm 2 charges a constant price $p_2$.

Proof. One complication is, when firm 1 charges $p_1^L$, it may occupy the whole market. In that situation, we need deal with the corner-solution case. Define $k_1 = F(\frac{1}{2})/f(\frac{1}{2})$ and $k_2 = F(1)/f(1)$. For logconcave $F$, $k_1 < k_2$.

(1) If $\lambda^2 < (k_2 + \frac{1}{2})/k_1$, we have the mixed-strategy equilibrium with an interior solution. We continue our proof in the main text. We first show that condition (a) holds given $F(\frac{1}{2}) = \frac{1}{2}$. First, since $p_2$ will never be negative, $z_H > 0$ is no problem according to (2.8). From (2.7)-(2.8), we can see that $1 > \frac{2\lambda}{\lambda} > \frac{1}{2} > z_H$ is equivalent to

$$\frac{2}{\lambda}k_1 < p_2 < 2\lambda k_1.$$ 

Note that $z_L < 1$ requires $p_2 < \frac{2}{\lambda}(k_2 + \frac{1}{2})$, which will be implied by $p_2 < 2\lambda k_1$ given the condition $\lambda^2 \leq (k_2 + \frac{1}{2})/k_1$. Now we show that, given (2.7)-(2.8), (2.9) does have a

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solution \( p_2 \in (\frac{2}{3}k_1, 2\lambda k_1) \). If \( p_2 = \frac{2}{3}k_1 \), then (2.7)-(2.8) require \( z_L = \frac{1}{2} \) (i.e., \( p_I^H = p_2 \)) and 
\( 0 < z_H < \frac{1}{2} \), so

\[
\pi_1(p_I^H, p_2) = \max_{p \geq p_2} p q_1(p, p_2) > p_2 q_1(p_2, p_2) = \pi_1(p_I^L, p_2).
\]

Similarly, if \( p_2 = 2\lambda k_1 \), then \( z_H = \frac{1}{2} \) (i.e., \( p_I^H = p_2 \)) and \( 1 > z_L > \frac{1}{2} \), so

\[
\pi_1(p_I^L, p_2) = \max_{p \leq p_2} p q_1(p, p_2) > p_2 q_1(p_2, p_2) = \pi_1(p_I^H, p_2).
\]

Then the continuity of the profit function implies our result. The last step is to show that 
(2.6) has a solution \( \mu \in (0, 1) \). According to (2.14) in the proof of Proposition 2.4, it is
actually true given \( z_L > \frac{1}{2} > z_H \) and \( F(\frac{1}{2}) = \frac{1}{2} \).

We then discuss condition (b). Under Assumption 2.1, \( F(z_1) \) is logconcave in \( p_I^1 \), so
for firm 1, the necessary conditions in (2.7) and (2.8) are also sufficient for optimization.
For firm 2, there is no profitable deviation on \([p_I^L, p_I^H]\) since Assumption 2.1 guarantees
that its profit function on this interval is quasi-concave. But does it have any profitable
development to \( p_2 < p_I^L \) or \( p_2 > p_I^H \)? Under Assumption 2.1, its profit function is also quasi-concave in either case. (Note that this
does not mean that firm 2’s whole profit function is quasi-concave.) Thus, a sufficient condition for neither
case to be a profitable deviation is that \( \frac{\partial \pi^L_2}{\partial p_2} \bigg|_{p_2 \rightarrow (p_I^L)^{+}} > 0 \) and \( \frac{\partial \pi^L_2}{\partial p_2} \bigg|_{p_2 \rightarrow (p_I^L)^{-}} < 0 \),
where \( \pi^L_2 = p q_2(p) \). They are actually true because Assumption 2.1 and \( p_2 \in (p_I^L, p_I^H) \)
implicate \( \frac{\partial \pi^L_2}{\partial p_2} \bigg|_{p_2 \rightarrow (p_I^L)^{+}} \geq 0 \) and \( \frac{\partial \pi^L_2}{\partial p_2} \bigg|_{p_2 \rightarrow (p_I^L)^{-}} \leq 0 \), and the two kinks of \( q^L_2 \) are both outward.

(2) We also have the mixed-strategy equilibrium with an interior solution
when \( (k_2 + \frac{1}{2})/k_1 < \lambda^2 < k_2 f(\bar{z})/F(\bar{z})^2 \) if \( f(1) \leq 2 \), where \( \bar{z} \leq \frac{1}{2} \) is the solution to

\[
\frac{k_2}{k_2 + 1/2} = \frac{F(\bar{z})^2}{F(\bar{z}) + (\bar{z} - 1/2)f(\bar{z})}.
\]  

(A.1)

One can verify that the solution \( \bar{z} \leq \frac{1}{2} \) exists and \( (k_2 + \frac{1}{2})/k_1 \leq k_2 f(\bar{z})/F(\bar{z})^2 \) if and
only if \( f(1) \leq 2 \).\footnote{\textit{Let} \( z_0 \) satisfy \( F(z_0) + (z_0 - 1/2)f(z_0) = 0 \). Then the right-hand side of (A.1) is a decreasing and positive function on \( (z_0, \frac{1}{2}) \) (which varies from \( \infty \) to \( 1/2 \)) and a decreasing and negative function on \( (0, z_0) \). Therefore, \( \bar{z} \in (z_0, \frac{1}{2}) \) when \( k_2 \geq \frac{1}{2} \) (i.e., \( f(1) \leq 2 \)).} The same proof as in the above still applies except that now we
need to show \( p_2 \in (\frac{2}{3}k_1, \frac{2}{3}(k_2 + \frac{1}{2})) \). When \( p_2 = \frac{2}{3}k_1 \), the same proof as before implies
\( \pi_1(p_I^L, p_2) < \pi_1(p_I^H, p_2) \). When \( p_2 = \frac{2}{3}(k_2 + \frac{1}{2}) \), (2.7) implies \( z_L = 1 \), so \( p_I^L = p_2 - \frac{1}{3} = \frac{2}{3}k_2 \).
and we get \( \pi_1(p_1^L, p_2) = \frac{2}{\lambda} k_2 \). Meanwhile, (2.9) implies \( \pi_1(p_1^H, p_2) = 2\lambda \frac{F_H^2}{f_H} \), where \( z_H \) now satisfies
\[
\frac{F_H}{f_H} + z_H - \frac{1}{2} = \frac{1}{\lambda^2}(k_2 + \frac{1}{2}).
\] (A.2)

Then \( \pi_1(p_1^L, p_2) > \pi_1(p_1^H, p_2) \) if \( k_2 > \lambda^2 \frac{F_H^2}{f_H} \) which is further equivalent to
\[
\frac{k_2}{k_2 + 1/2} > \frac{F_H^2}{F_H + (z_H - 1/2)f_H}
\]
by using (A.2). This is true if \( z_H > \tilde{z} \), which is actually implied by \( \lambda^2 < k_2 f(\tilde{z})/F(\tilde{z})^2 \) and (A.1)–(A.2).

**3.** We have the mixed-strategy equilibrium with a corner solution if \( \lambda^2 \geq (k_2 + \frac{1}{2})/k_1 \) **(given \( f(1) > 2 \))** or if \( \lambda^2 \geq k_2 f(\tilde{z})/F(\tilde{z})^2 \) **(given \( f(1) \leq 2 \)).** Under Assumption 2.1, we have the mixed-strategy equilibrium with a corner solution if the following conditions are satisfied:

(i) \( p_1^L = \arg\max_{p \geq p_2} p q_1(p > p_2) \);

(ii) \( p_1^L \) is determined by \( z_L = 1 \), i.e., \( p_1^L = p_2 - \frac{1}{\lambda} \);

(iii) \( p_1^L \) is the best response to \( p_2 \): \( \frac{\partial p_1}{\partial p}(p < p_2) \leq 0 \) at \( p = p_1^L \);

(iv) The indifference condition: \( p_1^L = \pi_1(p_1^H, p_2) \);

(v) \( p_2 \) is the best response to \( (p_1^L, p_1^H, \mu) \):
\[
\frac{p_2}{2} \left[ \mu f(1) + \frac{1 - \mu}{\lambda} f(\tilde{z}_H) \right] = q_2 = (1 - \mu)(1 - F_H).
\] (A.3)

We need to show that the above conditions have a solution with \( p_1^L > 0 \), \( p_2 \leq p_1^H \) and \( \mu \in (0, 1) \).

First, under Assumption 2.1, (i) is again equivalent to (2.8), so we need \( z_H \leq \frac{1}{2} \) (i.e., \( p_2 \leq 2\lambda k_1 \)) for \( p_2 \leq p_1^H \). Second, (iii) requires \( 1 - \lambda f(1)p_1^L/2 \leq 0 \). Using condition (ii), this requires \( p_2 \geq \frac{1}{\lambda}(k_2 + \frac{1}{2}) \) (which also implies \( p_1^L > 0 \)). Therefore, we need to prove \( p_2 \in \left[ \frac{1}{\lambda}(k_2 + \frac{1}{2}), 2\lambda k_1 \right] \). (Note that this interval is not empty given our conditions.)

When \( p_2 \) tends to \( 2\lambda k_1 \), \( z_H = \frac{1}{2} \) (i.e., \( p_1^H = p_2 \)) and condition (iii) is satisfied, so \( p_1^L = \max_{p \geq p_2} p q_1(p < p_2) > \mu q_2(p_2, p_2) = \pi_1(p_1^H, p_2) \). When \( p_2 \) tends to \( \frac{1}{\lambda}(k_2 + \frac{1}{2}) \), we want to have \( p_1^L < \pi_1(p_1^L, q_2) \), i.e., \( k_2 < \lambda^2 \frac{F_H^2}{f_H} \), where \( z_H \) is again determined by (A.2). Reversing the proof in (2) will prove this inequality.

Finally, we prove that condition (v) has a solution \( \mu \in (0, 1) \). When \( \mu = 1 \), the left-hand side of (A.3) is positive but the right-hand side is zero. When \( \mu = 0 \), the left-hand side is \( \frac{p_2}{2} f_H \) and the right-hand side is \( 1 - F_H \). Since \( p_2 < 2\lambda k_1 \), the former is smaller if \( k_1 < (1 - F_H)/f_H \). This is of course true given \( z_H < \frac{1}{2} \) and \( F(\frac{1}{2}) = \frac{1}{2} \). \( \square \)
Claim A.2 In the single-reference-product case, given Assumption 2.1, for fixed $\lambda > 1$, there is $\varepsilon_1 > 0$ such that, when $\| F(\frac{1}{2}) - \frac{1}{2} \| < \varepsilon_1$, there exists a similar mixed-strategy equilibrium as that defined in Proposition 2.2.

Proof. As we have seen, $F(\frac{1}{2}) = \frac{1}{2}$ is only used in proving $\mu \in (0, 1)$. Hence, we only need to revisit that step. (i) The interior-solution case. For $\mu \in (0, 1)$, we need $g_H, -g_L > 0$ in (2.14). Let $\hat{z}$ satisfy

$$1 - 2F(\hat{z}) - f(\hat{z})(\hat{z} - \frac{1}{2}) = 0.$$ 

Then we are done if $z_L > \hat{z} > z_H$, since $\frac{1 - 2F(z)}{f(z)}$ is a decreasing function given logconcave $f$. For fixed $\lambda > 1$, the solution $(p_2, z_L, z_H)$ from (2.7)-(2.9) must satisfy $z_L > \frac{1}{2} > z_H$. Thus, what we need is that $\hat{z}$ is close to $\frac{1}{2}$, which is true if $F(\frac{1}{2})$ and $\frac{1}{2}$ are close enough.

(ii) The corner-solution case. For fixed $\lambda > 1$, we have $z_H < \frac{1}{2}$ and so $(1 - F_H)f_H > (1 - F(\frac{1}{2}))f(\frac{1}{2})$. The latter tends to $k_1$ if $F(\frac{1}{2})$ is close to $\frac{1}{2}$. ■

Claim A.3 In the single-reference-product case, given Assumption 2.1, (i) for fixed $\lambda > 1$, there exists $\varepsilon_2 > 0$ such that, when $\| F(\frac{1}{2}) - \frac{1}{2} \| < \varepsilon_2$, there is no pure-strategy equilibria. (ii) For fixed $\| F(\frac{1}{2}) - \frac{1}{2} \| > 0$, there exists $\lambda^* > 1$ such that, when $\lambda < \lambda^*$, there is a pure-strategy equilibrium with $p_1 > p_2$ if $F(\frac{1}{2}) > \frac{1}{2}$ and $p_1 < p_2$ if $F(\frac{1}{2}) < \frac{1}{2}$.

Proof. (i) We only deal with the case with $F(\frac{1}{2}) > \frac{1}{2}$. (The other one is similar.) First of all, it is ready to check that $p_1 \leq p_2$ cannot even satisfy the first-order conditions given the demand functions associated with $p_1 < p_2$. If $p_1 > p_2$, the demand functions are

$$q_1 = F \left( \frac{1}{2} + \frac{1}{2\lambda} (p_2 - p_1) \right), \quad q_2 = 1 - F \left( \frac{1}{2} + \frac{1}{2\lambda} (p_2 - p_1) \right).$$

If there exists an equilibrium with $p_1 > p_2$, then the necessary conditions are $q_i + p_i \frac{\partial q_i}{\partial p_i} = 0$, which imply

$$\frac{F(z)}{1 - F(z)} = \frac{p_1}{p_2}, \quad \frac{F(z)}{f(z)} = \frac{p_1}{2\lambda},$$

where $z = \frac{1}{2} + \frac{1}{2\lambda} (p_2 - p_1)$. Using $p_2 = p_1 - \lambda(1 - 2z)$, we get

$$\frac{F(z)}{1 - F(z)} = \frac{1}{1 - (1 - 2z)f(z)/2F(z)}.$$  \hspace{1cm} (A.4)

The necessary conditions define a pure-strategy equilibrium with $p_1 > p_2$ if (a) the equation (A.4) has a solution $z < \frac{1}{2}$ and (b) given $p_i$, firm $j$ has no global deviation. In the following, we will show that condition (a) is always true given $F(\frac{1}{2}) > \frac{1}{2}$, while condition (b) will fail if $F(\frac{1}{2})$ is close to $\frac{1}{2}$.

Logconcave $F$ implies that $\frac{f(z)}{F(z)}$ decreases with $z$. Then the right-hand side of (A.4) has the following shape: there exists $z_0 \in (0, \frac{1}{2})$ satisfying $1 - (1 - 2z_0)f(z_0)/2F(z_0) = 0$, 118
such that it decreases from 0 to $-\infty$ when $z \in (0, z_0)$, and it decreases from $+\infty$ to 1 when $z \in (z_0, \frac{1}{2})$. Meanwhile, the left-hand side of (A.4) is an increasing function of $z$, and when $F(\frac{1}{2}) > \frac{1}{2}$, we have $\frac{F(1/2)}{1-F(1/2)} > 1$. Thus, (A.4) must have a solution $z \in (z_0, \frac{1}{2})$. If $F(\frac{1}{2}) \to (\frac{1}{2})^+$, then the solution $z$ to (A.4) tends to $\frac{1}{2}$ and so $p_1 \to p_2$. For fixed $\lambda > 1$, then firm 1 must have a profitable deviation due to its inward demand kink.

(ii) When will (b) be satisfied? We only need to worry about firm 1’s possible deviation. For fixed $F(\frac{1}{2}) - \frac{1}{2} > 0$, (A.4) implies a fixed $z < \frac{1}{2}$, and so $p_1 - p_2 = \lambda(1 - 2z)$ is bounded away from zero. If $\lambda \to 1$, then firm 1’s demand curve tends to be smooth everywhere and so the necessary conditions should also be sufficient under Assumption 2.1. ■

A.2 Proof of Lemma 2.1

(i) The symmetry is easy to see since $f(x)$ is symmetric. Now we prove $\phi(x)$ is strictly decreasing on $[0, \frac{1}{2}]$ for non-uniform distributions. Since $f(x)$ is logconcave and symmetric, it must increase on $[0, \frac{1}{2}]$, and so $F(x)$ is convex on $[0, \frac{1}{2}]$. One can check that, when $x < \frac{1}{2}$, $\phi'(x)$ has the sign of

$$\left(F - \frac{1}{2}\right)\left(\frac{1}{x - 1/2} + \frac{f'}{f}\right) - f$$

which is negative if

$$\left(\frac{1}{2} - F\right)f' + f > \frac{1/2 - F}{1/2 - x}.$$ 

The right-hand side is increasing since $F$ is convex on $[0, 1/2]$. When $x < \frac{1}{2}$, the derivative of the left-hand side has the sign of $ff'' - f'^2$ which must be negative since $f$ is logconcave, and so the left-hand side is decreasing. Moreover, when $x \to \frac{1}{2}$, both sides tend to $f(\frac{1}{2})$. Therefore, the above inequality must hold for $x < \frac{1}{2}$.

(ii) $A(x)$ is positive on $(z_0, 1]$. One can verify that $A'(x)$ has the sign of $(x - \frac{1}{2})(2f'^2 - Ff')$. Since the second term must be positive given logconcave $F$, $A(x)$ decreases on $(z_0, \frac{1}{2}]$ and increases on $[\frac{1}{2}, 1]$. Second, notice

$$A(\frac{1}{2} - \varepsilon) = \frac{(1/2 - \sigma)^2}{1/2 - \sigma - \varepsilon f}, \quad A(\frac{1}{2} + \varepsilon) = \frac{(1/2 + \sigma)^2}{1/2 + \sigma + \varepsilon f},$$

where $\sigma = \frac{1}{2} - F'(\frac{1}{2} - \varepsilon) = F'(\frac{1}{2} + \varepsilon) - \frac{1}{2} > 0$ and $f = f(\frac{1}{2} - \varepsilon) = f(\frac{1}{2} + \varepsilon)$. Then $A(\frac{1}{2} - \varepsilon) > A(\frac{1}{2} + \varepsilon)$ if and only if

$$(\sigma + \frac{1}{4\sigma})(1 + \frac{\varepsilon f}{\sigma}) > \frac{1}{2}.$$ 

Since $\frac{1}{4\sigma} + \sigma \geq 1$, this inequality must be true.

(iii) When $x > \frac{1}{2}$, we have known that $\phi(x)$ is increasing in $x$, so $B(x)$ is increasing
as well. When \( x < \frac{1}{3} \), \( \phi(x) > 1 \) since \( \phi(\frac{1}{3}) = 1 \). One can check that \( B'(x) > 0 \) if 
\[
(2\phi + 1)f/F > -\phi'.
\]
Notice that, for \( x < \frac{1}{3} \),
\[
-\phi' = \frac{\phi - 1}{x - 1/2} + \phi f'/f' < \phi f'/f'.
\]
Thus, it suffices to show \( 2 + \frac{1}{\phi} > Ff'/f^2 \), which however must be true since logconcave \( F \) implies \( Ff' < f^2 \).

### A.3 Omitted Proofs in Section 2.5

#### A.3.1 The pure-strategy equilibrium

Let us consider the case with \( \hat{x} > \frac{1}{2} \) first. In this case, we have an equilibrium with \( p_1 > p_2 \) if (i) \( p_1 = \arg\max_{p \geq p_2} pq_1(p > p_2) \), (ii) \( p_2 = \arg\max_{p \leq p_1} pq_2(p < p_1) \), and (iii) \( p_1 q_1(p_1 > p_2) \geq \max_{p \leq p_2} pq_1(p < p_2) \). Condition (iii) means that firm 1 does not want to deviate to a price lower than \( p_2 \). (We do not need to worry about firm 2 since its demand function is concave.) From (i) and (ii), it is ready to solve
\[
p_1 = \frac{2}{l} \cdot \frac{1 + \hat{x}}{3} > p_2 = \frac{2}{l} \cdot \frac{2 - \hat{x}}{3}
\]
given \( \hat{x} > \frac{1}{2} \). Then
\[
\pi_1 = \frac{2}{l} \left( \frac{1 + \hat{x}}{3} \right)^2; \quad \pi_2 = \frac{2}{l} \left( \frac{2 - \hat{x}}{3} \right)^2.
\]
Now consider firm 1’s potential deviation to \( p_1' < p_2 \). Let \( \pi_1' \) be the corresponding deviation profit. Given \( p_2 \), if we do not consider any constraint, firm 1’s optimal response associated with the demand function \( q_1(p < p_2) \) is
\[
p_1' = \frac{p_2}{2} + \frac{\hat{x}}{h}.
\]
But this price could be greater than \( p_2 \) or too low such that the corresponding demand is greater than 1. This causes complications and we need to discuss the following three cases separately. (a) When \( p_1' \geq p_2 \) (which happens when \( r \leq \sqrt{\frac{3h}{2 - \hat{x}}} \)), the optimal deviation price should be \( p_2 \), so the deviation must be unprofitable. (b) When \( p_1' \) is too low such that the demand is greater than 1 (which happens when \( r \geq \sqrt{3} \)), the optimal deviation price should be
\[
p_2 - \frac{2}{h}(1 - \hat{x}),
\]
which just makes firm 1 win the whole market. Then
\[
\pi'_1 = p_2 - \frac{2}{h} (1 - \hat{x}) = 2 \left( \frac{2 - \hat{x}}{3l} - \frac{1 - \hat{x}}{h} \right),
\]
which is lower than \( \pi_1 \) if and only if \( r^2 (5 - 5\hat{x} - \hat{x}^2) < 9 (1 - \hat{x}) \). (c) When \( p'_1 \) is appropriate (i.e., when \( \sqrt{\frac{3l}{2-\hat{x}}} < r < \sqrt{3} \)), the deviation profit is
\[
\pi'_1 = \frac{1}{2h} \left( \hat{x} + \frac{h}{2} p_2 \right)^2 = \frac{2}{h} \left[ \frac{r^2}{3} + \left( \frac{1}{2} - \frac{r^2}{6} \right) \hat{x} \right]^2.
\]
One can check that \( \pi'_1 < \pi_1 \) if and only if \( r < \frac{3l}{2-\hat{x}} \). The conditions derived in (a)–(c) can be rewritten as (2.26) with the help of Figure 2.7.

The case with \( \hat{x} < \frac{1}{2} \) can be similarly treated. The candidate pure-strategy equilibrium prices are
\[
p_1 = \frac{2}{h} \cdot \frac{1 + \hat{x}}{3} < p_2 = \frac{2}{h} \cdot \frac{2 - \hat{x}}{3}.
\]
Then
\[
\pi_1 = \frac{2}{h} \left( \frac{1 + \hat{x}}{3} \right)^2; \quad \pi_2 = \frac{2}{h} \left( \frac{2 - \hat{x}}{3} \right)^2.
\]
When we consider firm 1’s potential deviation to \( p'_1 > p_2 \), we will not encounter the situation like the above (b), so it is much simpler to derive (2.27). The calculation is straightforward and so omitted.

A.3.2 The mixed-strategy equilibrium with a corner solution

We have the mixed-strategy equilibrium with a corner solution if the following conditions are satisfied:

(i) \( p'_1^H = \arg \max_{p \geq p_2} pq_1(p > p_2) \);

(ii) \( p'_1 \) is determined by \( q'_1 = \hat{x} + \frac{h}{2} (p_2 - p'_1) = 1 \);

(iii) \( p'_1 \) is the best response to \( p_2 \): \( \frac{\partial q_1(p < p_2)}{\partial p} \leq 0 \) at \( p = p'_1 \);

(iv) The indifference condition: \( p'_1 = p'_1 q_1(p'_1^H) \);

(v) \( p_2 \) is the best response to \( (p'_1, p'_1^H, \mu) \):
\[
\frac{p_2}{2} [\mu h + (1 - \mu) l] = q_2^* = (1 - \mu) \left[ 1 - \hat{x} + \frac{l}{2} (p_1^H - p_2) \right].
\]

We prove the existence of solution which satisfies \( p'_1 < p_2 < p'_1^H \) and \( \mu \in (0,1) \) when (2.25) holds. First of all, from (i) and (ii), we have
\[
p'_1 = p_2 - \frac{2}{h} (1 - \hat{x}), \quad p'_1 = \frac{\hat{x}}{l} + \frac{p_2}{2}.
\]
Then the indifference condition (iv) requires

\[ p_2 - \frac{2}{h}(1 - \hat{x}) = \frac{1}{2l} \left( \frac{\hat{x} + l}{2p_2} \right)^2. \]

Given (2.25), one can show that this equation has a solution

\[ p_2 = \left( \frac{2}{h} \left( 2 - \hat{x} \right), \frac{2}{l} \cdot \min \left\{ \hat{x}, \frac{2 - \hat{x}}{3} \right\} \right) \quad (A.5) \]

by noting that \( a(\hat{x}) \) and \( b(\hat{x}) \) cross at \( \hat{x} = \sqrt{3} - 1 \). Explicitly,

\[ p_2 = \frac{2}{l} \left[ 2 - \hat{x} - 2\sqrt{(1 - \hat{x})(1 - 1/r^2)} \right]. \quad (A.6) \]

Next, we prove that condition (v) has a solution \( \mu \in (0, 1) \). Simple algebra shows that condition (v) can be rewritten as

\[ \frac{\mu}{1 - \mu} = \frac{2 - \hat{x}}{hp_2} - \frac{3}{2r^2}. \quad (A.7) \]

To prove that the right-hand side of (A.7) is positive, we consider the case \( \hat{x} < \frac{1}{2} \) and \( \hat{x} > \frac{1}{2} \) separately. In the former case, \( \hat{x} < \frac{2 - \hat{x}}{3} \), so \( p_2 < \frac{2}{l} \) follows from (A.5). In the latter case, \( p_2 < \frac{2}{l}(2 - \hat{x}) \). Both of them imply that the right-hand side of (A.7) is positive. The last step is to check (iii). At \( p = p_1^f \),

\[ \frac{\partial p_1(p < p_2)}{\partial p} = 1 - \frac{h}{2} p_1^f < 0 \]

since \( p_2 > \frac{2}{l}(2 - \hat{x}) \) from (A.5) implies \( p_1^f > \frac{2}{l} \).

Now we discuss the properties of this equilibrium. First, one can show that \( p_2 \) in (A.6) goes up with \( \hat{x} \) under (2.25) and so do firm 1’s prices.\(^2\) Second, firm 1’s profit is just equal to \( p_1^f \) and so increases with \( \hat{x} \), and firm 2’s profit is

\[ \pi_2 = \frac{1 - \mu}{2} \left[ (2 - \hat{x})p_2 - \frac{l}{2} p_2^2 \right] \]

\[ = \frac{h p_2^2}{2} \left[ 1 - \frac{r^2 - 1}{(2 - \hat{x}) + p_2 + r^2 - 3/2} \right], \]

where the second equality follows from (A.7). One can show \( \pi_2 \) rises with \( \hat{x} \) though the proof is lengthy.

\(^2\)It can be shown that \( \frac{\partial \pi_2}{\partial \hat{x}} > 0 \) if and only if \( r^2 > \frac{1}{3} \). If \( \hat{x} > \frac{1}{2} \), it is no problem because \( r > \sqrt{3} \) from (2.25) (see also Figure 2.7). If \( \hat{x} < \frac{1}{2}, \) (2.25) implies \( r^2 > (2\hat{x} - 1)^2 > \frac{1}{3} \).

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Appendix B

Appendix of Chapter 3

B.1 Existence of equilibrium with prominence

Claim B.1 Under assumption (3.5), within the square $[0,a]^2$, (3.15) and (3.16) have a unique solution, and this solution satisfies $(p_1, p_2) \in (1-a, 1/2)^2$.

Proof: By assumption, $a > 1/2$. Fix $p_1 \in [0,a]$, and consider (3.16). Here, $p_2 = 1 - a + t_2$, where

$$t_2 = \frac{r_2}{h_2} = \frac{1}{\frac{1}{n-1} \sum_{k=0}^{n-2} a^k} \int_{p_2}^{a} \frac{u - \Delta}{a - \Delta} u^{n-2} du.$$

Here, $t_2$ is a decreasing function of $p_2$ for $p_2 \in [0,a]$ since the integrand $\frac{u - \Delta}{a - \Delta}$ is positive and decreases with $p_2$ when $p_2 < u < a$. Moreover, since $\frac{1}{n-1} \sum_{k=0}^{n-2} a^k \geq u^{n-2}$ when $p_2 \leq u \leq a$, we have $t_2 < a - p_2$. We then have: (i) If $p_2 = 1 - a$, then $p_2 < 1 - a + t_2$. This is because $t_2 > 0$ given that $p_2 < a$. (ii) If $p_2 = 1/2$, then $p_2 > 1 - a + t_2$. This is because $t_2 < a - p_2$. Therefore, for $p_1 \in [0,a]$ (3.16) has a unique solution for $p_2 \in [0,a]$, say $p_2 = b_2(p_1)$, and $b_2(p_1) \in (1-a, 1/2)$.

Next, from (3.14) we have

$$p_1 = \frac{1}{2} \left[ 1 - a + p_2 + \frac{a^n - p_2^n}{n} \right] \equiv b_1(p_2).$$

One can check that $b_1'(p_2) \in (0, \frac{1}{2})$. One can also check that $b_1(p_2) \in (1-a, 1/2)$ when $p_2 \in (1-a, 1/2)$. A solution to the pair of equations (3.15)–(3.16) involves $p_1 = b_1(b_2(p_1))$, and by a fixed point argument there exists such a $p_1 \in (1-a, 1/2)$. Since $p_2 = b_2(p_1) \in (1-a, 1/2)$, the pair of first-order conditions has at least one solution $(p_1, p_2) \in (1-a, 1/2)^2$.

Finally, consider uniqueness. Substituting $b_1(p_2)$ into (3.16), we have

$$p_2 = 1 - a + \frac{1}{\frac{1}{n-1} \sum_{k=0}^{n-2} a^k} \int_{p_2}^{a} \frac{u - p_2 - b_1(p_2)}{a - p_2 - b_1(p_2)} u^{n-2} du.$$
Since $b_1'(p_2) \in (0, \frac{1}{2})$, $b_1(p_2) - p_2$ decreases with $p_2$. This implies that the right-hand side of the above decreases with $p_2$. Therefore, the solution is unique.

**B.2 Proof of Proposition 3.1**

(i) Since $h_2 < 1$, (3.15)-(3.16) imply that

$$p_2 - p_1 = \frac{1}{2} \left( \frac{r_2}{h_2} - r_1 \right) > \frac{1}{2} (r_2 - r_1) .$$

However, since $r_2 - r_1$ has the same sign as $p_1 - p_2$, the latter must be negative.

(ii) Define

$$A = \frac{1-a^n}{1-a} ; \quad B = \frac{1-a^{n-1}}{1-a} . \quad (B.1)$$

Since $p_1 < p_2$, the left-hand side of (3.19) is less than

$$p_2 + ABp_2 = Ap_2 ,$$

but the right-hand side is greater than $1 - p_2^a$. So we have

$$A > \frac{1-p_2^a}{p_2} .$$

Comparing this to (3.11), we deduce $p_2 > p_0$.

Finally, since $\Delta > 0$ and $a > p_2$, it follows that $(a - \Delta)p_2 > ap_1$. Then the left-hand side of (3.19) is greater than $Ap_1$ but the right-hand side of (3.19) is less than $1 - p_1^a$. So

$$A < \frac{1-p_1^a}{p_1} .$$

Comparing this with (3.11) implies $p_1 < p_0$.

**B.3 Proof of Proposition 3.2**

It is useful first to establish two preliminary results:

**Claim B.2** $p_0 > ap_2 + (1 - a)^2$.

**Proof:** For simplicity, write $K_n = \frac{1}{n} \sum_{k=0}^{n-1} a^k$ (so $h_0 = K_n$ and $h_2 = (a - \Delta)K_{n-1}$). Since $p_2 > p_0$, we have

$$r_2 = \int_{p_2}^{a} u^{n-1}(1 - \frac{\Delta}{u})du < (1 - \frac{\Delta}{a})r_0 ,$$

so

$$\frac{r_2}{h_2} < (1 - \frac{\Delta}{a})\frac{r_0}{h_2} = \frac{r_0}{aK_{n-1}} .$$
Then,

\[
p_0 = 1 - a + \frac{r_0}{K_n} > 1 - a + \frac{aK_{n-1}}{K_n} \cdot \frac{r_2}{h_2} > 1 - a + a \frac{r_2}{h_2} = ap_2 + (1 - a)^2.
\]

The second inequality follows since \( K_{n-1} > K_n \) and the final equality follows from (3.16).

**Claim B.3** \( \Delta < A(p_2 - p_0) \), where \( A \) is given in (B.1).

**Proof:** From (3.11) and (3.19), we have

\[
p_1p_2^{n-1} - p_0^n = p_0 - p_1 + B[\Delta p_2 - a(p_2 - p_0)].
\]

(Recall that \( B \) is defined in (B.1).) On the other hand, we can write the left-hand side of the above expression as

\[
p_1p_2^{n-1} - p_0^n = p_2^{n-1}[(p_2 - p_0)L - \Delta],
\]

where \( L = \frac{1-L}{1-\lambda} \) and \( \lambda = \frac{p_0}{p_2} < 1 \). These two equations imply

\[
\frac{\Delta}{p_2 - p_0} = \frac{A + p_2^{n-1}L}{1 + Bp_2 + p_2^{n-1}}.
\]

(B.2)

Therefore, from (B.3) \( A - \frac{\Delta}{p_2 - p_0} \) has the same sign as

\[
ABp_2 - (L - A)p_2^{n-1} > p_2 - (L - A)p_2^{n-1} > p_2 - (n - 1)p_2^{n-1} > p_2(1 - \frac{n - 1}{2^{n-2}}) \geq 0.
\]

The first inequality follows because \( AB > 1 \). The second inequality follows since \( \lambda < 1 \) implies that \( L < n \), and so \( L - A < n - A < n - 1 \). The third inequality follows because \( p_2 < 1/2 \).

Proof of (i): From section 3.2.2, we know that output with random search, \( Q_0 \), is equal to \( 1 - p_0^n \), while output with a prominent firm, \( Q_1 \), is equal to \( 1 - p_1p_2^{n-1} \). Therefore, expression (B.2) implies that

\[
Q_0 - Q_1 = p_1p_2^{n-1} - p_0^n
= p_2^{n-1}[(p_2 - p_0)L - \Delta]
> p_2^{n-1}(p_2 - p_0)(L - A)
> 0.
\]

Here, the first inequality uses Claim B.3. The second inequality follows from the observation from Claim B.2 that \( p_0 > ap_2 \), so that \( \lambda > a \) in the proof of Claim B.3, and so
Proof of (ii): With random search, the probability that a consumer searches exactly $k$ times, for $1 \leq k \leq n - 1$, is $a^{k-1}(1 - a)$. The probability that a consumers searches all products ($k = n$) is $a^{n-1}$. Using these probabilities to form the expected value of $k$ shows that the expected number of searches with random search is

\[
\frac{1 - a^n}{1 - a}.
\]

On the other hand, with one firm prominent, the probability that a consumer searches just once is $1 - (a - \Delta)$, the probability that a consumer searches $k$ times, where $2 \leq k \leq n - 1$, is $(a - \Delta)a^{k-2}(1 - a)$, while the probability that a consumers search all products is $(a - \Delta)a^{n-2}$. The expected number of searches in this case is therefore

\[
\frac{1 - a^n - \Delta(1 - a^{n-1})}{1 - a}.
\]

### B.4 Proof of Proposition 3.3

Let $W(p_1, p_2)$ denote welfare when the prominent firm charges price $p_1$ and the non-prominent firms charge $p_2$ (and consumers foresee this will be the non-prominent price). Therefore, we wish to evaluate the sign of $W(p_1, p_2) - W(p_0, p_0)$.

Since total demand when all prices are $p$ is $Q_0(p) = 1 - p^n$, it follows that

\[
\frac{dW(p, p)}{dp} = -np^n.
\]

(B.4)

Second, consider the effect on $W(p, p_2)$ of a small increase in $p$ of $\epsilon$. First, more consumers are excluded altogether. (These consumers are depicted on the right-hand boundary of the shaded region in Figure 1.) Since total demand when the prices are $(p, p_2)$ is $1 - pp_2^{n-1}$, this extra exclusion leads to a welfare fall of

\[
\epsilon p \frac{d}{dp}[pp_2^{n-1}] = \epsilon pp_2^{n-1}
\]

since these marginal consumers were originally buying a product with social value $p$. Second, a fraction of consumers are shifted to non-prominent firms from the prominent one, either because they are more likely to search beyond the prominent firm at the start or because they are less likely to return to the prominent firm instead of others after sampling all firms. (These consumers are depicted on the diagonal boundary in Figure

---

1If the uniform price rises by $\epsilon$, total demand falls by $\epsilon Q_0 = \epsilon np^{n-1}$. Since each of the marginal consumers was previously consuming a product with surplus $p$, it follows that total welfare falls by $\epsilon np^n$.  

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3.1.) The number of these marginal consumers is just the increase in total non-prominent firm demand, which from (3.12) is

\[ \varepsilon \frac{d}{dp} \left[ 1 - pp_2^{n-1} - (1 - a + p_2 - p + r_1) \right] = \varepsilon (1 - p_2^{n-1}) . \]

Since these consumers were indifferent between buying from the prominent and non-prominent firms (including the search cost), the impact on these consumers is negligible. On the other hand, industry profit is affected since profit increases by \( p_2 - p \) for each consumer who switches. Therefore, industry profit rises by

\[ \varepsilon (p_2 - p)(1 - p_2^{n-1}) . \]

Putting these two welfare effects together, we obtain

\[ \frac{\partial W(p, p_2)}{\partial p} = p_2 - p - p_2^{n} . \]  \( (B.5) \)

Therefore, using (B.4) and (B.5), the welfare difference between the two regimes is

\[ W(p_1, p_2) - W(p_0, p_0) = [W(p_1, p_2) - W(p_2, p_2)] + [W(p_2, p_2) - W(p_0, p_0)] \]

\[ = \Delta (p_2^n - \frac{\Delta}{2}) - n \int_{p_0}^{p_2} p^n dp . \]

Note that

\[ n \int_{p_0}^{p_2} p^n dp > n p_0 \int_{p_0}^{p_2} p^{n-1} dp = p_0 (p_2^n - p_0^n) > p_0 (p_2^n - p_1 p_2^{n-1}) = \Delta p_0 p_2^{n-1} . \]

The second inequality follows since the prominence case excludes more consumers (Proposition 3.2). Thus

\[ W(p_1, p_2) - W(p_0, p_0) < \Delta (p_2^n - \frac{\Delta}{2}) - \Delta p_0 p_2^{n-1} = \frac{\Delta}{2} [2(p_2 - p_0) p_2^{n-1} - \Delta] < 0 , \]

where the final inequality follows since \( p_2 - p_0 < \Delta \) and \( p_2 < \frac{1}{2} . \)

\section{B.5 Proof of Proposition 3.4}

We use a similar method as in the proof of Proposition 3.3. Let \( V(p_1, p_2) \) denote consumer surplus when the prominent firm charges \( p_1 \) and the non-prominent firms all charge \( p_2 \). We wish to evaluate the sign of \( V(p_1, p_2) - V(p_0, p_0) \).
Since total demand when all prices are $p$ is $1 - p^n$, it follows that

$$\frac{dV(p,p)}{dp} = -(1 - p^n).$$

Similarly, from (3.12)

$$\frac{\partial V(p_1,p_2)}{\partial p_1} = -(\text{demand for prominent firm's product})$$
$$= -(1 - a + p_2 - p + r_1)$$

and so

$$V(p_1,p_2) - V(p_2,p_2) = \int_{p_1}^{p_2} (1 - a + p_2 - p + r_1) \, dp$$
$$= \Delta(1 - a + r_1 + \frac{1}{2}\Delta)$$
$$= \Delta(p_1 - \frac{1}{2}\Delta),$$

where the final equality follows from the first-order condition (3.14). Therefore, we can deduce

$$V(p_1,p_2) - V(p_0,p_0) = [V(p_1,p_2) - V(p_2,p_2)] + [V(p_2,p_2) - V(p_0,p_0)]$$
$$= \Delta(p_1 - \frac{\Delta}{2}) - \int_{p_0}^{p_2} (1 - p^n) dp.$$  \hfill (B.6)

Since $1 - p^n > 1 - p_2^n$ in the above integral, it follows that

$$V(p_1,p_2) - V(p_0,p_0) < \Delta(p_1 - \frac{\Delta}{2}) - (p_2 - p_0)(1 - p_2^n).$$

Claim B.3 tells us that $\Delta < A(p_2 - p_0)$, so if we can show

$$A(p_1 - \frac{\Delta}{2}) < 1 - p_2^n$$

we are done. First, it can be verified that

$$\frac{p_1 - \Delta/2}{1 - p_2^n} < \frac{p_1}{1 - p_1 p_2^{n-1}} \iff 1 > 3p_1 p_2^{n-1}.$$ 

The latter inequality must be true since both prices are less than $1/2$. Therefore, a sufficient condition for the result to hold is that

$$\frac{1 - p_1 p_2^{n-1}}{p_1} > A.$$
However, using (3.19) and the observation that \((a - \Delta)p_2 > ap_1\), we have

\[
\frac{1 - p_1p_2^{n-1}}{p_1} = \frac{1}{p_1} \left[ \frac{1 - a^{n-1}}{1 - a} (a - \Delta)p_2 \right] > 1 + a \frac{1 - a^{n-1}}{1 - a} = A .
\]

We can understand this consumer-surplus result as follows. The last step in the proof implies that the ratio of the prominent firm's market share to all non-prominent firms' market share \(q_1/(n-1)q_2\) is less than \(1/(A - 1)\). On the other hand, Claim B.3 implies that \((p_0 - p_1)/(p_2 - p_0)\) is less than \(A - 1\). Putting them together, we can see that the total output in the prominence case actually costs consumers less if the random-search price would prevail. Moreover, in the random search case consumers can achieve the same search and consumption result as in the prominence case by using an appropriate stopping rule,\(^2\) but they did not choose to do that. Thus, the revealed preference argument implies that consumer surplus must be higher in the random search case.

### B.6 Proof of Proposition 3.5

(i) Write \(\pi_0, \pi_1\) and \(\pi_2\) for the respective equilibrium profits of each firm in the random search case, the prominent firm in the prominence case, and of each non-prominent firm in the prominence case. Then

\[
\pi_1 > p_2(1 - a + r_1) > p_2(h_2(1 - a) + r_2) = \pi_2 .
\]

The first inequality holds since the prominent firm makes smaller profit if deviates from \(p_1\) to the price \(p_2\). (Recall that its demand is given by (3.12).) The second inequality holds since \(h_2 < 1\) and \(r_2 < r_1\). Similarly,

\[
\pi_1 > p_0(1 - a + p_2 - p_0 + r_1) > p_2(h_0(1 - a) + r_0) = \pi_0 .
\]

Here, the second inequality holds since \(h_0 < 1\) and \(p_2 - p_0 > \frac{1}{n} (p_2^0 - p_0^0) = r_0 - r_1\).

(ii) Industry profit when one firm is prominent is \(p_1q_1 + (n - 1)p_2q_2\), which can be written as

\[
p_2 [q_1 + (n - 1)q_2] - \Delta q_1 = p_2 [p_2 - \Delta + B(a - \Delta)p_2] - \Delta p_1 = p_2 [Ap_2 - \Delta(Bp_2 + 1)] - \Delta p_1 .
\]

(Recall the definition of \(A\) and \(B\) in (B.1).) The first equality follows from (3.17)–(3.18),

\(^2\)To be precise, the stopping rule can be constructed as follows: stop at the first firm when \(u_1 > a - \Delta\); after that, stop searching when \(u_1 > a\); if the consumer has sampled all products, her returning policy is based on \((u_1 - p_1, u_1 - p_2)\). That is, the consumer behaves as if she had an illusion about all prices as in the prominence case.
while the second follows after noting that \(A + aB = 1\). From (3.10), industry profit in the random search case is

\[ np_0q_0 = Ap_0^2.\]

Therefore, prominence increases industry profit if and only if

\[ A(p_2^2 - p_0^2) > \Delta [(Bp_2 + 1)p_2 + p_1].\]

From (B.3), this condition is equivalent to

\[ \frac{A + p_2^{n-1}L}{1 + Bp_2 + p_2^{n-1}} < \frac{A(p_2 + p_0)}{(1 + Bp_2)p_2 + p_1}.\]  \(\text{ (B.7)}\)

For \(n = 2\) and \(a = 1/2\) (so that \(A = 3/2, B = 1, L = 2\) and all prices are \(1/2\)), one can check that the left-hand side of (B.7) is \(5/4\) but the right-hand side is \(6/5\), so that (B.7) fails to hold. Therefore, prominence causes industry profit to fall in a duopoly when search costs approach their maximum limit.

We next show that (B.7) holds for \(n \geq 3\) or for \(n = 2\) but \(a\) is relatively large. Inequality (B.7) is equivalent to

\[ Lp_2^{n-1} [(1 + Bp_2)p_2 + p_1] < A [Bp_0p_2 + p_0 - p_1 + p_2^{n-1}(p_2 + p_0)].\]

After dividing both sides of the above by \(p_2\) and using \(p_2 > p_0 > p_1\), a sufficient condition for this inequality is

\[ Lp_2^{n-1} (2 + Bp_2) \leq A [Bp_0 + p_2^{n-2}(p_2 + p_0)],\]

which in turn is true if

\[ \frac{Lp_2^{n-1}}{p_0} (2 + Bp_2) \leq A (B + 2p_2^{n-2}).\]

Note that

\[ \frac{Lp_2^{n-1}}{p_0} = \frac{p_0^{n-1} + p_0^{n-2}p_2 + \cdots + p_2^{n-1}}{p_0} < (n - 1)p_2^{n-2} + \frac{p_2}{p_0}p_2^{n-2} < kp_2^{n-2},\]

where \(k = n - 1 + \frac{1-2(1-a)^2}{a}\). The last inequality follows from Claim B.2 which implies that \(\frac{p_2}{p_0} < \frac{1-2(1-a)^2}{a}\) by noting \(p_0 < \frac{1}{2}\). Therefore, a new sufficient condition for industry profit to rise with prominence is

\[ k\frac{2 + Bp_2}{2 + B/p_2^{n-2}} < A.\]
Since $p_2 < 1/2$, the above inequality is true if
\[
\frac{k}{4 + 2^{n-1}B} < \frac{A}{B + 4}.
\]
When $n = 2$, we have $A = 1 + a$ and $B = 1$, and one can check that this inequality holds for about $a > 0.794$. For $n = 3$, it holds for all $\frac{1}{2} \leq a \leq 1$. For $n > 3$, it holds since the left-hand side decreases with $n$ when $n \geq 3$ and the right-hand side increases with $n$ (which can be seen by noting that $A = 1 + aB$ and $B$ increases with $n$).

**B.7 A result on the non-prominent firm’s profit**

We now show that, for fixed $n \geq 3$, there exists $a^* < 1$ such that a non-prominent firm earns more than in the random search case if $a > a^*$.

Now suppose $a$ tends to one. Let $\varepsilon = 1 - a$, so $\varepsilon$ is close to zero. The idea frequently used in the following is that, with a small search cost, the fresh demand is negligible relative to the returning demand.

First, if we make firm 1 prominent starting from the random-search equilibrium, but keep all prices the same, then a non-prominent firm’s loss of demand is
\[
\frac{(1 - a)(1 - h_0)}{n - 1} \approx \frac{\varepsilon^2}{2},
\]
where $(1 - a)(1 - h_0)$ is firm 1’s demand rise in this process (see (3.6) and (3.12)) and we have used $h_0 \approx 1 - \frac{n-1}{2}\varepsilon$. (Remember $h_0 = \frac{1-a^n}{n(1-a)}$.) Thus, a non-prominent firm’s profit loss in this process is of order $\varepsilon^2$.

Second, from the random-search equilibrium to the prominence equilibrium, all prices actually also change, which will further affect a non-prominent firm’s profit: it will suffer from the prominent firm’s price cut from $p_0$ to $p_1$ but gain from other non-prominent firms’ price rise from $p_0$ to $p_2$. Define $\Delta_1 = p_0 - p_1$ and $\Delta_2 = p_2 - p_0$. These price changes are also close to zero when $\varepsilon \to 0$, so we can ignore the non-prominent firm’s re-optimization problem and fix its price at $p_0$. The following steps aim to show that the effect of these price changes on a non-prominent firm’s demand (so profit) is of order $\varepsilon$.

(i) $\Delta_1$ is of order $\varepsilon$. Let $\bar{p}$ be the full-information price which satisfies $1 - \bar{p}^n = n\bar{p}$. So the equilibrium prices can be approximated by $p_i = \bar{p} + k_i\varepsilon$, $i = 0, 1, 2$. From (3.11), one can show that
\[
k_0 = \frac{(n - 1)\bar{p}}{2(1 + \bar{p}^{n-1})},
\]
so $p_0 - \bar{p}$ is of order $\varepsilon$. From the first-order condition of firm 1: $1 - a + p_2 - 2p_1 + r_1 = 0$, one can also show
\[
2k_1 = (1 - \bar{p}^{n-1})k_2.
\]
Together with \( p_1 < p_0 < p_2 \), we can see that \( p_1 - \bar{p} \) is also of order \( \varepsilon \), which further implies that \( \Delta_1 \) is of order \( \varepsilon \).

(ii) The effect of the price changes on a non-prominent firm’s fresh demand is of order \( \varepsilon^2 \). This is because the fresh demand has the expression \( h_2(1 - a) = h_2 \varepsilon \), and the effect of the price changes \( \Delta_1 \) on \( h_2 \) is of order \( \varepsilon \).

(iii) The effect of the price changes on a non-prominent firm’s returning demand is of order \( \varepsilon \). The change of returning demand is

\[
\int_{p_0}^{a} (u + \Delta_2)^{n-2}(u - \Delta_1)du - \int_{p_0}^{a} u^{n-1} du \approx [(n - 2)\Delta_2 - \Delta_1] \frac{a^{n-1} - p_0^{n-1}}{n - 1},
\]

which is clearly of order \( \varepsilon \).

Finally, we prove

\[
(n - 2)\Delta_2 - \Delta_1 > 0 \iff \frac{\Delta}{\Delta_2} < n - 1.
\]

Using (B.3), we have

\[
\lim_{a^{-1} \to 1} \frac{\Delta}{\Delta_2} = \frac{n(1 + \bar{p}^{n-1})}{1 + \bar{p}^{n-1} + (n - 1)\bar{p}} < n - 1
\]

if and only if

\[
1 + \bar{p}^{n-1} < (n - 1)^2\bar{p}.
\]

Using \( 1 - \bar{p}^n = n\bar{p} \), it suffices to show

\[
\frac{1 + \bar{p}^{n-1}}{1 - \bar{p}^n} < \frac{(n - 1)^2}{n}.
\]

Since \( \bar{p} = \frac{1 - \bar{p}^n}{n} < \frac{1}{n} \), the left-hand side is less than \( \frac{1 + 1/n^{n-1}}{1 - 1/n^n} \), which is further less than the right-hand side because it is decreasing in \( n \) and it is less than the right-hand side at \( n = 3 \). This completes the proof.

\[\text{In this proof, we are assuming that consumers keep their original expectation in the random search case. Using the modified expectation does not change our result at all when } \varepsilon \to 0.\]
Appendix C

Appendix of Chapter 4

C.1 Existence of equilibrium in the competition case

Existence:

Let \( t_B = \frac{r_B}{h_B} \) and rewrite (4.7) as \( p_B = 1 - a + t_B \). Explicitly,

\[
t_B = \frac{1}{K_{n-m}} \int_{P_B}^{a} \left( \frac{u - \Delta}{a - \Delta} \right)^m u^{n-m-1} du.
\]

Clearly, \( t_B \) is a decreasing function of \( p_B \) on \([0, a] \) since \( \frac{u - \Delta}{a - \Delta} \) decreases with \( p_B \) when \( u < a \). Then we show: (i) If \( p_B = 1 - a \), then \( p_B < 1 - a + t_B \). This is because \( t_B > 0 \) under Assumption 4.1. (ii) If \( p_B = 1/2 \), then \( p_B > 1 - a + t_B \). This is because \( t_B < a - p_B \) which can be seen from the expression of \( t_B \) by noting \( K_{n-m} > a^{n-m-1} \). Therefore, for any fixed \( p_A \), on \([0, a] \) (4.7) has a unique solution \( p_B = b_B(p_A) \in (1 - a, 1/2) \).

Let \( t_A = \frac{r_A + r_A}{h_A} \) and rewrite (4.6) as \( p_A = 1 - a + t_A \). We first show that, given \( p_B \in [0, a] \), \( t_A \) is decreasing in \( p_A \). Notice that

\[
\frac{\partial r_A}{\partial p_A} = (m - 1) \int_{p_B}^{a} (u - \Delta)^{m-2} u^{n-m} du < a^{n-m} \left[ (a - \Delta)^{m-1} - p_A^{m-1} \right],
\]

so

\[
\frac{\partial(r_A + r_A)}{\partial p_A} < -(a - \Delta)^{m-1} + a^{n-m} \left[ (a - \Delta)^{m-1} - p_A^{m-1} \right]
= (a^{n-m} - 1)(a - \Delta)^{m-1} - a^{n-m} p_A^{m-1} < 0.
\]

Since the best response \( b_B(p_A) \in (1 - a, 1/2) \), we can focus on \( p_B \in (1 - a, 1/2) \). Given this, similarly as before, we have: (i) If \( p_A = 1 - a \), then \( p_A < 1 - a + t_A \). This is because \( 1 - p_B < a \) implies \( r_A = \int_{1-p_B}^{a} u^{m-1} du > 0 \) and so \( t_A > 0 \). (\( r_A > 0 \) is obvious.) (ii) If
\(p_A = 1/2\), then \(p_A > 1 - a + t_A\) which is equivalent to \(t_A < a - 1/2\). The proof of this goes as follows. When \(p_A = 1/2\),

\[
\tau_A = \int_{p_B}^{a} (u - p_B + 1/2)^{m-1} u^{n-m} du < \frac{a^{n-m}}{m} [(a - p_B + 1/2)^m - 1/2^m],
\]

and

\[
\hat{t}_A = \frac{1}{m} [a^m - (a - p_B + 1/2)^m].
\]

So

\[
t_A < \frac{1}{m h_A} (a^m - 1/2^m) = \frac{1-a}{1-a^m} (a^m - 1/2^m) < a - 1/2.
\]

Therefore, for any \(p_B \in (1-a, 1/2)\), (4.6) has a unique solution \(p_A \equiv b_A(p_B) \in (1-a, 1/2)\).

The continuity of \(b_A(p_B)\) and \(b_B(p_A)\) is no problem. So the above two results, together with the Brower fixed point theorem, imply that, on the area \((0, a)^2\), the system of the first-order conditions has at least one solution \((p_A, p_B) \in (1-a, 1/2)^2\).

**Uniqueness:**

We first show \(b'_A(p_B) \in (0, 1)\). Note that

\[
b'_A(p_B) = \frac{\partial t_A / \partial p_B}{1 - \partial t_A / \partial p_A},
\]

where

\[
\frac{\partial t_A}{\partial p_A} = \frac{1}{K_m} \left[ -(a - \Delta)^{m-1} + (m - 1) \int_{p_B}^{a} (u - \Delta)^{m-2} u^{n-m} du \right]
\]

and

\[
\frac{\partial t_A}{\partial p_B} = \frac{1}{K_m} \left[ (a - \Delta)^{m-1} - (m - 1) \int_{p_B}^{a} (u - \Delta)^{m-2} u^{n-m} du - p_A^{m-1} p_B^{n-m} \right].
\]

Clearly, \(1 - \partial t_A / \partial p_A > \partial t_A / \partial p_B\). Moreover, \(\partial t_A / \partial p_B > 0\), because the square-bracket term is greater than

\[
(a - \Delta)^{m-1} - a^{n-m} [(a - \Delta)^{m-1} - p_A^{m-1}] - p_A^{m-1} p_B^{n-m} > 0
\]

when \(p_B < a\). Hence, \(b'_A(p_B) \in (0, 1)\).

Substituting \(b_A(p_B)\) into (4.7), we get

\[
p_B = 1 - a + \frac{1}{K_{n-m}} \int_{p_B}^{a} \left( \frac{u - p_B + b_A(p_B)}{a - p_B + b_A(p_B)} \right)^m u^{n-m-1} du.
\]

Since \(b'_A(p_B) \in (0, 1)\), \(b_A(p_B) - p_B\) decreases with \(p_B\), which further implies that the term
in the bracket is decreasing in \( p_B \). Thus, the whole right-hand side of the above equation is a decreasing function of \( p_B \). This means that the solution is unique.

C.2 Proof of Proposition 4.2

The proof consists of several steps:

**Step 1: The stopping rule with \( p_A > p_B \).** If consumers expect \( p_A > p_B \) but their search order is still restricted as assumed, then what is their optimal stopping rule? We keep the notation \( \Delta = p_B - p_A \). First of all, once a consumer enters pool \( B \), her stopping rule is standard and the same as in the main text. Now consider the situation before she enters \( B \). Denote by \( z_k \) \((k \leq m)\) the reservation net surplus level when a consumer visits the \( k_{th} \) firm in her search process. That is, she will stop searching at the \( k_{th} \) firm if and only if this firm provides net surplus greater than \( z_k \). According to Kohn and Shavel (1974), \( z_k \) is well defined and unique in our setup. We further claim that \( a - p_A < z_1 < \cdots < z_m = a - p_B \). Here \( z_m = a - p_B \) is easy to understand. Now consider a consumer who comes to the \((m - 1)_{th}\) firm and has the highest net surplus so far \( v_{m-1} \). If \( v_{m-1} \leq a - p_A \), then searching the last prominent firm is always desirable. If \( v_{m-1} \geq z_m = a - p_B \), then she will never search beyond the last prominent firm, and so she should stop searching now since \( a - p_B > a - p_A \). Therefore, \( a - p_A < z_{m-1} < z_m \). Similarly, we can prove \( a - p_A < z_{m-2} < z_{m-1} \) and others. The intuition is, when a consumer more approaches the end of pool \( A \), she has more incentive to search on in pursuit of the lower price in \( B \).\footnote{More precisely, \( z_{m-1} \) can be defined as follows. Let \( \mu = \max(u_m - p_A, z_{m-1}) \) and \( G(\mu) \) be its distribution function. Then \( z_{m-1} = \int_{-\infty}^{\mu} uG(\mu) + \int_{\mu}^{\infty} V_B(\mu)dG(\mu) - s \), where \( V_B(x) \) is the expected surplus from searching pool \( B \) if the consumer has an outside option \( x \). Similarly, other \( z_i \) can also be defined recursively.}

Let us summarize the consumer’s stopping rule with expectation of \( p_A > p_B \) as follows: in pool \( A \), stop at the \( k_{th} \) firm in the search process if and only if the highest net surplus so far is greater than \( z_k \), where \( a - p_A < z_1 < \cdots < z_m = a - p_B \). After entering pool \( B \), stop searching if and only if the highest available net surplus exceeds \( z_B = a - p_B \). After searching all firms, return to the firm with the highest non-negative net surplus.

**Step 2: The demand system.** Since there is no midway returning demand now, the demand function of each firm consists of two parts: fresh demand and (final) returning demand. We consider the returning demand \( r_A \) and \( r_B \) first. One can show that a firm’s returning demand is again independent of its actual price (for local deviation).\footnote{One can check that}

\[
\begin{align*}
    r_A &= \frac{1}{m} \sum_{k=1}^{m} \int_{p_A}^{p_A + z_k} (u + \Delta)^{n-m} u^{m-k} \prod_{i=1}^{k} \min(z_i + p_A, u) du, \\
    r_B &= \int_{p_B}^{u} u^{n-m-1} \prod_{i=1}^{m} \min(z_i + p_A, u - \Delta) du.
\end{align*}
\]

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argue $r_A < r_B$. Let $\Lambda$ be the event that a consumer searches all firms, i.e.,

$$\Lambda = \{u_i - p_A \leq z_i \text{ for all } i \in A, \text{ and } u_j - p_B \leq z_B \text{ for all } j \in B\}.$$ 

For $k \leq m$, let $\Psi_k$ be the event that the $k_{th}$ product in her search process has the highest non-negative net surplus among all products, i.e.,

$$\Psi_k = \{u_k - p_A \geq \max(0, u_i - p_A, u_j - p_B) \text{ for all } i \in A \text{ and } j \in B\}.$$ 

Similarly, let $\Psi_B$ be the event that some non-prominent firm provides the highest non-negative net surplus among all. Then

$$r_A = \frac{1}{m} \sum_{k=1}^{m} \Pr(\Lambda \cap \Psi_k); \quad r_B = \Pr(\Lambda \cap \Psi_B).$$

The $1/m$ term is again because a prominent firm has the equal probability to occupy any of the first $m$ positions in a consumer’s search process. Notice that $\Pr(\Psi_k|\Lambda)$ increases with $k$ and $\Pr(\Psi_m|\Lambda) = \Pr(\Psi_B|\Lambda)$ since $z_1 < \cdots < z_m = z_B$, so $r_A < r_B$. The intuition of this result is actually quite simple: on average, a consumer has a lower valuation of the product when leaving a prominent firm than when leaving a non-prominent firm, so the latter is more likely to win her back.

Now we are ready to write down the demand functions. For a prominent firm, if it charges $p$ while other firms stick to their equilibrium prices, its demand is

$$q_A(p) = \frac{1}{m} \sum_{k=1}^{m} \left[ (1 - z_k - p) \prod_{i=1}^{k-1} (z_i + p_A) \right] + r_A.$$ 

Here, $\prod_{i=1}^{k-1} (z_i + p_A) / m$ is the probability that a consumer will visit this prominent firm as the $k_{th}$ firm in her search process, and $1 - z_k - p$ is the conditional probability that this consumer will buy immediately. For a non-prominent firm, if it charges $p$ while others keep charging their equilibrium prices, its demand is

$$q_B(p) = (1 - z_B - p)K_{n-m} \prod_{k=1}^{m} (z_k + p_A) + r_B.$$ 

Here $K_{n-m} \prod_{k=1}^{m} (z_k + p_A)$ is the likelihood that a consumer will come to this non-prominent firm as a fresh consumer.

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\(^3\)Which position a prominent firm occupies in a consumer’s search process will affect the probability it wins her back as a returning consumer. This is because a consumer, on average, has a higher valuation of the product when it is the $(k + 1)_{th}$ product she left than it is the $k_{th}$ product she left.

\(^4\)More precisely, all $z_i + p_A$ terms should be replaced by $\min(1, z_i + p_A)$ because of the boundary problem. But this does not influence our following analysis.
Step 3: \( p_A > p_B \) is incompatible with equilibrium conditions. Define \( \alpha_k = \prod_{i=1}^{k-1} (z_i + p_A) \) and note \( \alpha_k \geq \alpha_{k+1} \). Then the first-order conditions are

\[
\frac{1}{m} \sum_{k=1}^{m} \alpha_k (1 - z_k - 2p_A) + r_A = 0, \tag{C.1}
\]

and

\[
K_{n-m} \alpha_{m+1} (1 - a - p_B) + r_B = 0. \tag{C.2}
\]

Suppose \( p_A > p_B \) is the solution to these two equations. Then \( p_A > p_B > 1 - a \). The later inequality is from (C.2). Since \( z_k > a - p_A \), we have

\[
1 - z_k - 2p_A < 1 - a - p_A < 0.
\]

Then, (C.1) implies

\[
\bar{\alpha} (1 - a - p_A) + r_A > 0,
\]

where \( \bar{\alpha} = \sum_{k=1}^{m} \alpha_k/m \geq \alpha_{m+1} \). So

\[
K_{n-m} \alpha_{m+1} (1 - a - p_A) + r_B > 0.
\]

We have used \( K_{n-m} < 1 \) and \( r_B > r_A \). This contradicts with (C.2) if \( p_A > p_B \). Therefore, consumers’ initial expectation of \( p_A > p_B \) cannot be confirmed, i.e., \( p_A > p_B \) cannot be an equilibrium outcome.

### C.3 Proof of Lemma 4.1

We first present the first-order condition in the random search case:

\[
h_0(1 - a - p_0) + r_0 = 0, \tag{C.3}
\]

where \( h_0 = K_n \) and \( r_0 = \int_{p_0}^{a} u^{n-1} du \). This is from (4.7) by letting \( m = 0 \). It turns out to be convenient to prove the second limit result first. We claim \( r_B/r_0 \to 1 \) as \( n \to \infty \). On the one hand, \( r_B < r_0 \) since \( p_B > p_0 \) and \( \Delta > 0 \). On the other hand, we have

\[
r_B = \int_{p_B}^{a} \left( 1 - \frac{\Delta}{u} \right)^m u^{n-1} du \quad > \quad \left( \frac{p_A}{p_B} \right)^m \int_{p_B}^{a} u^{n-1} du = \left( \frac{p_A}{p_B} \right)^m \frac{a^n - p_B^n}{a^n - p_0^n} r_0.
\]

Thus,

\[
\left( \frac{p_A}{p_B} \right)^m \frac{a^n - p_B^n}{a^n - p_0^n} < \frac{r_B}{r_0} < 1.
\]
Notice that the left term tends to one as \( n \to \infty \), so \( r_B/r_0 \to 1 \). Then, (4.7) and (C.3) imply
\[
\lim_{n \to \infty} \frac{h_B(1 - a - p_B)}{h_0(1 - a - p_0)} = 1,
\]
which further implies
\[
\lim_{n \to \infty} \frac{p_0 - (1 - a)}{p_B - (1 - a)} = \lim_{n \to \infty} \frac{h_B}{h_0} = a^m.
\]
Similarly, \( r_A/r_0 \to 1 \) as \( n \to \infty \), so
\[
\lim_{n \to \infty} \frac{h_A(1 - a - p_A) + \hat{r}_A}{h_0(1 - a - p_0)} = 1,
\]
which further implies
\[
\lim_{n \to \infty} \frac{h_A(1 - a - p_A) + \hat{r}_A}{1 - a - p_0} = 0
\]
since \( h_0 \to 0 \) as \( n \to \infty \). Using L'Hôpital's rule, we have
\[
\lim_{n \to \infty} \left[ K_m \frac{\partial p_A}{\partial p_0} - (a - \Delta)^{m-1} \left( \frac{\partial p_B}{\partial p_0} - \frac{\partial p_A}{\partial p_0} \right) \right] = 0,
\]
where \( \frac{\partial p_A}{\partial p_0} = \frac{\partial p_A}{\partial p_0} / \partial \Delta \). Therefore,
\[
\lim_{n \to \infty} \frac{\partial p_A}{\partial p_0} = \frac{a^{m-1}}{a^{m-1} + K_m} \lim_{n \to \infty} \frac{\partial p_B}{\partial p_0}.
\]
Noting \( \lim_{n \to \infty} \frac{p_0 - (1 - a)}{p_B - (1 - a)} = 1 \), the first limit result is then implied by the second one.

### C.4 Proof of \( p_A < p_0 < p_B \) with the behavioral stopping rule

In the random search case, the first-order condition implies \( q_0 = h_0 p_0 \). Then \( n q_0 = 1 - p_0^n \) implies
\[
\frac{1 - (z + p_0)^n}{1 - (z + p_0)} = \frac{1 - p_0^n}{p_0}.
\]
(C.4)

In the prominence case, from \( m q_A + (n - m) q_B = 1 - p_A^n p_B^{n-m} \), we get
\[
\frac{1 - (z + p_A)^n}{1 - (z + p_A)} p_A + (z + p_A)^m \frac{1 - (z + p_B)^n-m}{1 - (z + p_B)} p_B = 1 - p_A^n p_B^{n-m}.
\]
(C.5)

Since \( p_A < p_B \), the left-hand side of (C.5) is less than \( \frac{1 - (z + p_B)^n}{1 - (z + p_B)} p_B \), while the right-hand side of (C.5) is greater than \( 1 - p_B^n \). Therefore, we have
\[
\frac{1 - (z + p_B)^n}{1 - (z + p_B)} > \frac{1 - p_B^n}{p_B}.
\]
which together with (C.4) implies \( p_B > p_0 \). Similarly, one can show \( p_A < p_0 \).

### C.5 Proof of Proposition 4.4

Part (i) is implied by Lemma 4.1 by noting that \( p_0 \) is independent of \( m \). The result about \( \Delta \) is because

\[
\lim_{n \to \infty} \frac{\Delta}{p_0 - (1-a)} = \frac{1}{a^m} - \frac{1}{a^m + a K_m},
\]

which increases with \( m \).

To prove part (ii), we establish the following result first:

**Claim C.1** Let \( \theta = \frac{1-p^{n-1}}{n-1} \) and

\[
\varphi_A = \frac{(\theta - \bar{p}) m + 1 + \bar{p}^{n-1}}{2 - m \bar{p} + \theta}, \quad \varphi_B = \frac{(\theta - \bar{p}) m}{2 - m \bar{p} + \theta},
\]

where \( \bar{p} \) is the full-information equilibrium price. Then, when \( a \to 1 \), equilibrium prices can be approximated by

\[
p_i = \bar{p} + k_i \varepsilon, \quad i = A, B
\]

where \( \varepsilon = 1 - a \), and

\[
k_i = \frac{\bar{p}}{2(1 + \bar{p}^{n-1})} \left[ n(1 - \varphi_i) + m - 1 \right].
\]

**Proof.** Since the procedure is standard, we only give a sketch of the proof. When \( a \to 1 \), we can approximate \( p_i \) as \( \bar{p} + k_i \varepsilon \), where \( \varepsilon = 1 - a \) and \( k_i \) needs to be determined. Extend the first-order conditions (4.6) and (4.7) around \( a = 1 \) by using these approximated prices and discard all higher-order terms, and then we get two equations of \( k_A \) and \( k_B \). Solving them yields

\[
(1 + \bar{p}^{n-1})k_A = \frac{n + m - 1}{2} \bar{p} - \left[ (\theta - \bar{p}) m + 1 + \bar{p}^{n-1} \right] k_{\Delta},
\]

\[
(1 + \bar{p}^{n-1})k_B = \frac{n + m - 1}{2} \bar{p} - (\theta - \bar{p}) m k_{\Delta},
\]

where

\[
k_{\Delta} = k_B - k_A = \frac{n \bar{p}}{2(2 - m \bar{p} + \theta)}.
\]

Using the notation in (C.6), we have

\[
2(1 + \bar{p}^{n-1})k_i/\bar{p} = n - 1 + m - n \varphi_i(m).
\]
It is ready to see $k_\Delta$ is positive and increasing with $m$, so $\Delta$ always increases with $m$. For having $p_i$ increasing with $m$, it suffices to show that $\varphi'(m)$ is less than $\frac{1}{n}$. First, we have

$$
\varphi'_B(m) = \frac{(\theta - \bar{p})(\theta + 2)}{(2 - m\bar{p} + \theta)^2} < \frac{(\theta - \bar{p})(\theta + 2)}{(2 - n\bar{p} + \theta)^2}. \\
\varphi'_B(m) = \frac{(\theta - \bar{p})(\theta + 2)}{(1 + \bar{p}^n + \theta)^2} < \frac{(\theta - \bar{p})(\theta + 2)}{(\theta + 1)^2}.
$$

We have used $n\bar{p} = 1 - \bar{p}^n$ in the second equality. Second,

$$
\varphi'_B(m) < \varphi'_A(m) = \varphi'_B(m) + \frac{\bar{p}(1 + \bar{p}^{n-1})}{(2 - m\bar{p} + \theta)^2} < \frac{(\theta - \bar{p})(\theta + 2) + \bar{p} + \bar{p}^n}{(\theta + 1)^2}. \quad (C.7)
$$

Using the definition of $\theta$, one can show that (C.7) is less than $\frac{1}{n}$ if and only if

$$(n + 1)\bar{p} + \frac{1}{n}\bar{p}^{n-1} < 2.$$

This must be true since $n\bar{p} = 1 - \bar{p}^n < 1$ and $\bar{p} + \frac{1}{n}\bar{p}^{n-1} < 1$.

### C.6 Proof of Proposition 4.5

(i) When $n \to \infty$, equilibrium prices can be approximated by $p_i = 1 - a + l_i \varepsilon$, where $\varepsilon \to 0$ as $n \to \infty$. From Lemma 1, we know that

$$l_A = \frac{l_0}{a^m + aK_m}; \quad l_B = \frac{l_0}{a^m}.$$

Of course, $l_0$ is independent of $m$. Let $l_\Delta = l_B - l_A$ and $b = 1 - a$. Then one can easily show that $Q_m$ is approximated by

$$1 - b^n - (nl_B - ml_\Delta) b^{n-1} \varepsilon.$$

When $m$ is fixed but $n$ is sufficiently large, $nl_B$ totally dominates $ml_\Delta$, so the property of the bracket term is determined by $\frac{b^n}{a^n}$. Therefore, $Q_m$ decreases with $m$.

(ii) When $a \to 1$, $Q_m$ is approximated by

$$1 - b^n - (nk_B - mk_\Delta) b^{n-1} \varepsilon.$$

We need to investigate the property of $nk_B - mk_\Delta$. The results in Claim C.1 imply

$$nk_B - mk_\Delta = \frac{n(n + m - 1)}{2(1 + \bar{p}^{n-1})} \bar{p} - \left(\frac{n(\theta - \bar{p})}{1 + \bar{p}^{n-1} + 1}\right) m k_\Delta.$$
We also have
\[
\frac{\partial m k_{\Delta}}{\partial m} = k_{\Delta} + m \frac{\partial k_{\Delta}}{\partial m} = k_{\Delta} \left( \frac{2m}{n} k_{\Delta} + 1 \right) = \frac{(\theta + 2) n \bar{p}}{2(\theta + 2 - m \bar{p})^2}.
\]

So
\[
\frac{\partial (n k_B - m k_{\Delta})}{\partial m} = \frac{n \bar{p}}{2(1 + \bar{p}^{n-1})} - \left( \frac{n(\theta - \bar{p})}{1 + \bar{p}^{n-1}} + 1 \right) \frac{(\theta + 2) n \bar{p}}{2(\theta + 2 - m \bar{p})^2},
\]

which has the sign of
\[
L = \frac{1}{n(\theta - \bar{p}) + 1 + \bar{p}^{n-1}} - \frac{\theta + 2}{(\theta + 2 - m \bar{p})^2}.
\]

Using the definition of \(\theta\), one can further show that \(L\) has the sign of
\[
m^2(n - 1) \bar{p} + (n - 2m)(2n - 1 - \bar{p}^{n-1}).
\]

When \(2m \leq n\), this is clearly positive, so \(Q_m\) decreases with \(m\). However, if \(2m > n\), the opposite result could happen. Since
\[
\frac{2n - 1 - \bar{p}^{n-1}}{\bar{p}} > 2n(n - 1)
\]

(where we have used \(\bar{p} < 1/n\) and \(\bar{p}^{n-1} < 1\)), a sufficient condition for \(L\) be negative is
\[
2(n - m)^2 < m^2 \iff m > \frac{\sqrt{2}}{1 + \sqrt{2}} n.
\]

Therefore, if \(m > \frac{\sqrt{2}}{1 + \sqrt{2}} n\), \(Q_m\) must increase with \(m\).

In the case without prominence, \(p_0\) is approximated by \(\bar{p} + k_0 \varepsilon\) when \(a \rightarrow 1\). One can show \(2(1 + \bar{p}^{n-1}) k_0 / \bar{p} = n - 1\), and
\[
Q_0 \approx 1 - \bar{p}^n - n \bar{p}^{n-1} k_0 \varepsilon.
\]

\(Q_m < Q_0\) in this limit case is easy to be verified.

### C.7 Proof of Proposition 4.6

Since the approximation procedure is regular, the details are omitted.

(i) Industry profit can be approximated by
\[
\Pi_m \approx (1 - b^n) b + [(1 - (n + 1) b^n) l_B + (m b^n - (1 - a^m)) l_{\Delta}] \varepsilon.
\]
When $n \to \infty$, how $m$ affects the square-bracket term is determined by

$$l_B - (1 - a^m)L = 1 + \frac{1 - a^m}{a^m + aK_m}.$$ 

Clearly, the fraction term increases with $m$, so industry profit goes up with $m$ when $n$ is sufficiently large. Total welfare can be approximated by

$$W_m \approx a - b^{n+1} - \frac{a^{n+1} - b^{n+1}}{n + 1} + (ml_B - nl_B)b^n\varepsilon.$$ 

We have known that, when $n \to \infty$, $\partial Q_m / \partial m$ has the sign of $\partial (ml_B - nl_B) / \partial m$. Therefore, total welfare falls with $m$ when $n$ is sufficiently large. The result on consumer surplus follows naturally.

(ii) When $a \to 1$, industry profit is

$$\Pi_m \approx np^2 + (nk_B - mk_B)(\bar{p} - p^n)\varepsilon,$$

and total welfare is

$$W_m \approx \frac{n}{n + 1} (1 - \bar{p}^{n+1}) + (mk_B - nk_B)\bar{p}^n\varepsilon.$$ 

We have known that, when $a \to 1$, $\partial Q_m / \partial m$ has the sign of $\partial (mk_B - nk_B) / \partial m$, so our results on welfare and $m$ follow. The proofs for $\Pi_m > \Pi_0$ and $W_m < W_0$ in this limit case is straightforward and so omitted.

### C.8 Proof of Proposition 4.8

(i) Rewrite (4.13)–(4.14) as

\[ \Delta = -\frac{1}{2} \left[ 1 - a - p^n_B + \frac{a^n - p^n_B}{n} \right], \]
\[ 0 = 1 - (n + 1)p^n_B + \left[ 1 - a^{n-1} + (n + 1)p^{n-1}_B \right] \Delta. \tag{C.8} \]

They imply

\[ \frac{1 - (n + 1)p^n_B}{A - (n + 1)p^n_B} = \frac{1}{2n} \left[ 1 - a^{n-1} + (n + 1)p^{n-1}_B \right], \tag{C.9} \]

where $A = n(1 - a) + a^n$. Since $A > 1$, the left-hand side is a decreasing function of $p_B$, while the right-hand side is an increasing function of $p_B$. Moreover, when $(1 + n)p^n_B = 1$ (i.e., $p_B = p_0$), the left-hand side is zero and so less than the right-hand side. When $p_B = 0$, the left-hand side $(1/A)$ is greater than the right-hand side $(1 - a^{n-1})$ since $A < n$. Therefore, (C.9) has a unique solution $p_B \in (0, p_0)$. (This, together with (C.8), confirms
\( \Delta < 0. \) Then \( p_A = p_B - \Delta \) is also unique. Explicitly,

\[
p_A = p_B + \frac{1}{2} \left[ 1 - a - p_B^n + \frac{a^n - p_B^n}{n} \right].
\]

One can check \( \frac{\partial p_A}{\partial p_B} = 1 - \frac{1 + n}{2} p_B^{n-1} > 0 \) since \( (1 + n) p_B^{n-1} < \frac{1}{p_0} < \sqrt{3} \), and so \( p_A \) is increasing with \( p_B \) and we have

\[
p_A < p_0 + \frac{1}{2} \left[ 1 - a - \frac{1 - a^n}{n} \right].
\]

Notice that the bracket term decreases with \( a \), so a sufficient condition for all equilibrium prices to be less than \( a \) is \( a > \hat{a}_n \), where \( \hat{a}_n \) solves \( p_0 + \frac{1}{2} \left( 1 - a - \frac{1 - a^n}{n} \right) = a. \)

(ii) From (4.14)–(4.15) and \( \Delta < 0 \), it is ready to see \( p_A p_B^{n-1} < p_0^n \), and so the prominence case serves more consumers. Since \( p_A > p_B \), we get \( p_B < p_0 \). In the following, we show \( p_A > p_0 \). (4.14)–(4.15) imply

\[
p_0^n - p_A p_B^{n-1} = -\Delta \frac{1 - a^{n-1}}{n + 1}.
\]

Suppose \( p_A \leq p_0 \). Then

\[
p_A - p_A p_B^{n-1} \leq -\Delta \frac{1 - a^{n-1}}{n + 1} \Rightarrow n p_B^{n-1} < \frac{p_A^n - p_B^n}{p_A - p_B} \leq p_B^{n-1} + \frac{1 - a^{n-1}}{n + 1}.
\]

So

\[
(n + 1) p_B^{n-1} < \frac{1 - a^{n-1}}{n + 1}.
\]

Then the left-hand side of (4.14) were

\[
1 - (n + 1) p_B^n + \left[ 1 - a^{n-1} + (n + 1) p_B^{n-1} \right] \Delta \\
\geq 1 - (n + 1) p_B^n + \left[ 1 - a^{n-1} + (n + 1) p_B^{n-1} \right] \Delta \\
\geq 1 - \frac{1 - a^{n-1}}{n - 1} (1 - n \Delta) > 0.
\]

This is a contradiction. The first inequality is because \( p_B < 1 \), the second uses (C.11), and the last one uses \( \frac{1 - n \Delta}{n - 1} < \frac{1}{1 - a^{n-\tau}} \) which is because \( -\Delta = \frac{1}{2} (1 - a + \frac{a^n - (n+1) p_B^n}{n}) < \frac{1}{2} \)

from (4.13) and \( a > \frac{1}{2} \) from Assumption 4.2. \( ^6 \)

\( ^5 \) In the case with announced prices, it is ready to check that (C.9) will become

\[
\frac{1 - (n + 1) p_B^n}{A - (n + 1) p_B^n} = \frac{n + 1}{2n} p_B^{n-1}.
\]

So we still have \( p_B \in (0, p_0) \). But now \( p_B \) is larger than that in the case with imperfect price information. Since the expression for \( \Delta \) does not change, that for \( p_A \) also remains. Thus, \( p_A \) in this case is also higher than in the case with imperfect price information.

\( ^6 \) When \( n \geq 4 \), \( \frac{1 - n \Delta}{n - 1} < \frac{1 + n / 2}{n - 1} \leq 1 \). When \( n = 2 \), \( \frac{1 - n \Delta}{n - 1} = 1 - 2 \Delta < 2 \leq \frac{1}{1 - a^{n-\tau}} \) whenever \( a \geq \frac{1}{2} \). When \( n = 3 \), \( \frac{1 - n \Delta}{n - 1} - \frac{1 - 2 \Delta}{3} < \frac{1}{3} \leq \frac{1}{1 - a^{n-\tau}} \) whenever \( a \geq \frac{1}{2} \).
C.9  Proof of Proposition 4.9

First, given the exogenous stopping rule, the existence of optimal solution is no problem. Given \( p_A \) and \( p_B \), it is ready to check that the demand for all prominent products is

\[
Q_A = 1 - F(z + p_A)^m + m \int_{p_B}^{z+p_B} F(u - \Delta)^{m-1} F(u)^{n-m} f(u - \Delta) du,
\]

and the firm’s profit is

\[
\Pi_m = p_B \left[ 1 - F(p_A)^m F(p_B)^{n-m} \right] - \Delta \cdot Q_A.
\]

Now suppose \( p_A < p_B \) were the optimal solution. Then we show that the firm has a profitable deviation: charge a prominent product at \( p_B \), charge a non-prominent product at \( p_A \), and leave other products’ prices unchanged. Since such a price adjustment will not affect total demand which is still \( 1 - F(p_A)^m F(p_B)^{n-m} \), it suffices to show that, after the price adjustment, the demand for those products with price \( p_A \) will become lower. For each of those \( m - 1 \) prominent products whose prices do not change, its returning demand will remain the same as before,\(^7\) but its fresh demand will increase since those consumers who have sampled that prominent product with price \( p_B \) are now more likely to leave that product. Explicitly, this change for product \( k \leq m \) with price \( p_A \) is

\[
\delta_k = \left[ \frac{1}{m} \sum_{i=1}^{m-1} \binom{m-2}{i-1} F_A^{i-1} \right] (F_B - F_A) (1 - F_A),
\]

where \( F_A = F(z + p_A) < F_B = F(z + p_B) < 1 \). The square-bracket term is the probability that a consumer will sample that high-price prominent product before product \( k \).

We then compare the demand for the non-prominent product with a new price \( p_A \) with that for a prominent product before the price adjustment. Again, the returning demand part is the same. So we only need to compare the fresh demand part. Intuitively, due to the restricted search order, the non-prominent product should have a lower demand. Explicitly, this difference is

\[
\delta = \left[ \frac{1}{m} \frac{1 - F_A^m}{1 - F_A} - \frac{F_A^{m-1} F_B}{n-m} \frac{1 - F_B^{n-m}}{1 - F_B} \right] (1 - F_A).
\]

Thus, what we need to show is that

\[
(m - 1) \delta_k < \delta.
\]

\(^7\)This is because the composition of all potential returning consumers is the same as before.
By noting \((m^{-2})/(m^{-1}) = \frac{1}{m-1}\), one can verify that this condition is equivalent to

\[
\frac{F_B - 1 - F_B^{n-m}}{n - m} - \frac{F_B - F_A}{1 - F_A} < \frac{1}{mF_A^{m-1}} \left( 1 - F_A^{m} \right) \frac{1}{1 - F_A}.
\]

The left-hand side is less than \(F_B - \frac{F_B - F_A}{1 - F_A} = \frac{F_A(1 - F_B)}{1 - F_A}\), while the right-hand side is greater than \(\frac{1}{mF_A^{m-1}}\). Thus, the profitable deviation is verified and so we should have \(p_A \geq p_B\) at optimum given the behavioral stopping rule. Using a similar proof as in Proposition 4.7, we can further exclude the possibility of \(p_A = p_B\).

In the uniform-distribution setting, adding up two first-order conditions \(\frac{\partial \Pi_m}{\partial p_i} = 0\) \((i = A, B)\) yields

\[
1 - (n + 1)p_A^n p_B^{n-m} = \Delta \cdot m(z + p_A)^{m-1} \left[ (z + p_B)^{n-m} - 1 \right].
\]

Thus, if \(z\) is not too large (such that \(z + p_B < 1\) in the optimal solution), we have \(p_A^n p_B^{n-m} < \frac{1}{n+1}\) since \(\Delta < 0\). That is, the prominence case excludes fewer consumers than the random search case.

### C.10 Proof of Proposition 4.10

We continue the proof of Proposition 4.10 by showing that a sufficient condition for both \(W_1 > W_0\) and \(V_1 > V_0\) is

\[
\frac{1 - a^{n-1}}{n + 1} p_B + \frac{3\Delta}{2} > 0.
\]  
(C.12)

\[
W_1 - W_0 > p_B(p_0^n - p_B^n) + \Delta(p_B^n - \frac{\Delta}{2}) = -\Delta \left( \frac{1 - a^{n-1}}{n + 1} p_B + \frac{\Delta}{2} \right),
\]

where the inequality uses \(\int_{p_B}^{p_0} p^n dp > p_B \int_{p_B}^{p_0} p^{n-1} dp\) and the equality uses

\[
p_0^n - p_B^n = -\Delta \left( \frac{1 - a^{n-1}}{n + 1} + p_B^{n-1} \right)
\]  
(C.13)

which is further from (C.10). Since \(\Delta < 0\), (C.12) implies \(W_1 - W_0 > 0\).

\[
V_1 - V_0 > (1 - p_B^n)(p_0 - p_B) + \Delta(p_B^n - \frac{3\Delta}{2}) = \frac{n}{n + 1} (p_0 - p_B) + \Delta(p_B^n - \frac{3\Delta}{2}),
\]

which is positive under (C.12) because

\[
\frac{-\Delta}{p_0 - p_B} = \frac{p_0^n - p_B^n}{p_0 - p_B} \left( \frac{1 - a^{n-1}}{n + 1} + p_B^{n-1} \right)^{-1} < \frac{n}{(1 - a^{n-1})p_B + (n + 1)p_B^n} < \frac{n}{(n + 1)(p_B^n - 3\Delta/2)}.
\]

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The first equality is because (C.13), the first inequality uses $\frac{p_0^n - p_B^n}{p_0 - p_B} < np_0^{n-1}$ and $(n + 1)p_0^{n-1} = 1/p_0$, and the last one uses (C.12).

### C.11 Competitive pricing with a general valuation distribution

This part generalizes the price result $p_A < p_B$ and other limit results with $n \to \infty$ in the competitive-pricing case.\(^8\) Let $F = F(a)$ and $K_n = \frac{1 - F_n}{n(1 - F)}$. For expositional convenience, we further introduce a piece of notation:

$$\phi(p, x) = 1 - F(p + x) - pf(p + x).$$

Note that $\phi_1(p, x) - \phi_2(p, x) = -f(p + x) < 0$. We also need the following two assumptions:

**Assumption C.1** $f(u)$ is logconcave and $a > \frac{1 - F(a)}{f(a)} = p_\infty$.

**Assumption C.2** $f(p + x) + pf'(p + x) = -\phi_2(p, x) \geq 0$ for $p, x \geq 0$.

The logconcavity assumption is satisfied by many distributions. (See, e.g., Bagnoli and Bergstrom (2005) for a detailed discussion.) Two well-known results are: (i) logconcave $f(u)$ implies logconcave $F(u)$ and $1 - F(u)$, and (ii) logconcave $1 - F(u)$ is equivalent to increasing hazard rate. The second part of Assumption C.1 requires that the search cost is not too high, which corresponds to Assumption 4.1 in the main text. As we will see below, $p_\infty$ is the equilibrium price when there are an infinite number of firms. The interpretation of Assumption C.2 goes as follows. If there is a monopoly firm which produces a product with valuation distribution $F(u)$ and consumers’ outside option utility is $x$, then $\phi(p, x)$ is the firm’s marginal profit at price $p$. Thus, Assumption C.2 is a sufficient condition for the optimal monopoly price to be decreasing with $x$.

**Equilibrium prices** Let $\pi_A(p) = pq_A(p)$ be a prominent firm’s profit function when other firms keep charging their equilibrium prices, and $\pi_B(p) = pq_B(p)$ be a non-prominent firm’s profit function. One can check that the first-order conditions can be written as

$$h_A \phi(p_A, a - p_A) + \hat{R}_A + R_A = 0,$$

$$h_B \phi(p_B, a - p_B) + R_B = 0,$$

\(^*\)The limit results with $a \to 1$ have not been generalized.
where the $R$-terms are the equilibrium marginal profits from returning consumers.\footnote{One can show that our profit functions are actually concave under Assumption C.1 if there is no boundary problem. Hence, the first-order conditions do define the equilibrium prices if they have solutions on the area $(0,a)^2$.} Explicitly,

$$
\hat{R}_A = -\int_{a-\Delta}^{a} F(u)^{m-1}\phi_2(p_A, u-p_A)du,
$$

$$
R_A = -\int_{p_B}^{a} F(u-\Delta)^{m-1}F(u)^{n-m}\phi_2(p_A, u-p_B)du,
$$

$$
R_B = -\int_{p_B}^{a} F(u-\Delta)^{m}F(u)^{n-m-1}\phi_2(p_B, u-p_B)du.
$$

Now we can see that Assumption C.2 guarantees that the returning demand has positive marginal profit in equilibrium (i.e., the returning demand is less price “sensitive” than the fresh demand). We can rewrite the first-order conditions as

$$
p_A = p_\infty + \frac{1}{f(a)} \frac{\hat{R}_A + R_A}{h_A},
$$

$$
p_B = p_\infty + \frac{1}{f(a)} \frac{R_B}{h_B}.
$$

Then it is not hard to verify that both $p_A$ and $p_B$ tend to $p_\infty$ as $n \to \infty$.

Our demand functions are based on consumers’ expectation of $p_A < p_B$, and now we confirm that this is indeed an equilibrium outcome.

**Proposition C.1** Given Assumptions C.1–C.2, if the system of the first-order conditions have solutions, then one solution must specify $p_A < p_B$.\footnote{The technique used in the uniform case relies on $\hat{R}_A + R_A - R_B$ having the sign of $\Delta$ and does not apply in this general setup.}

**Proof.** Let

$$
\zeta(p_A, p_B) \equiv h_A \phi(p_A, a - p_A) + \hat{R}_A + R_A.
$$

As we will show shortly, if all solutions to (C.16) and (C.17) specify $p_A \geq p_B$, then $\zeta(p_B, p_B) < 0$. This and $\zeta(0, p_B) > 0$ would then imply that $\zeta(p, p_B) = 0$ must have a solution $p < p_B$. This is a contradiction.

Now we confirm $\zeta(p_B, p_B) < 0$ if $p_A \geq p_B$. It is ready to see that

$$
\zeta(p_B, p_B) = h_A \phi(p_B, a - p_B) - \int_{p_B}^{a} F(u)^{n-1}\phi_2(p_B, u-p_B)du.
$$

Using (C.15), we can rewrite it into

$$
\zeta(p_B, p_B) = \frac{h_A}{h_B} R_B - \int_{p_B}^{a} F(u)^{n-1}\phi_2(p_B, u-p_B)du.
$$

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If $\phi_2(p_B, u - p_B) \leq 0$ for $u \in [p_B, a]$, $\zeta(p_B, p_B)$ is negative if
\[ F(u)^{n-1} < \frac{h_A}{h_B} F(u - \Delta)^m F(u)^{n-m-1} \]
for $u \in [p_B, a]$, which is further equivalent to
\[ \frac{F(u)^m}{F(u - \Delta)^m} < \frac{K_m}{K_{n-m} F^m F(a - \Delta)^m}. \]
Notice that $K_m > K_{n-m} F^m$, so it suffices to show
\[ \frac{F(u)}{F(u - \Delta)} \leq \frac{F(a)}{F(a - \Delta)} \]
for $u \in [p_B, a]$. This is true if $\frac{F(u)}{F(u - \Delta)}$ increases with $u$ or equivalently if $\frac{f(u)}{f(a)} \geq \frac{f(u - \Delta)}{f(u - \Delta)}$. Assumption C.1 implies logconcave $F(u)$, and so $\frac{f(u)}{f(a)}$ is a decreasing function. Thus, if $p_A \geq p_B$ (i.e., $\Delta \leq 0$), the sufficient condition we need actually holds, and so $\zeta(p_B, p_B) < 0$.

We have seen that this main result only requires $\phi_2(p_B, u - p_B) \leq 0$ for $u \in [p_B, a]$, which is weaker than Assumption C.2. In fact, if $n$ is sufficiently large, this condition tends to hold for any valuation distribution satisfying Assumption C.1. Therefore, at least for large $n$, $p_A < p_B$ is a quite general result. We now show this point:

**Proposition C.2** Under Assumption C.1, there exists $N$ such that, for $n > N$, we have $-\phi_2(p_B, u - p_B) = f(u) + p_B f'(u) \geq 0$ for $u \in [p_B, a]$.

**Proof.** Since $p_B$ tends to $p_\infty$ as $n \to \infty$, it suffices to show that $f(u) + p_\infty f'(u) > 0$ for $u \in [p_\infty, a]$. If $f'(u) \geq 0$, we are done. If $f'(u) < 0$, then $f(u) + \frac{1-f(u)}{f(u)} f'(u) > 0$ (which is implied by logconcave $1 - F(u)$) implies $f(u) + p_\infty f'(u) > 0$ for $u < a$ since $p_\infty = \frac{1-f(a)}{f(a)} < \frac{1-f(u)}{f(u)}$. ■

The result on the relationship between $p_0$, $p_A$, and $p_B$ in the uniform-distribution case still applies here as $n \to \infty$. The key step is the following lemma (which is a generalization of Lemma 4.1):

**Lemma C.1** Under Assumption C.1, we have
\[ \lim_{n \to \infty} \frac{p_B - p_\infty}{p_A - p_\infty} = 1 - \frac{K_m}{\phi_2^\infty} f(a) > 1, \]
\[ \lim_{n \to \infty} \frac{p_0 - p_\infty}{p_B - p_\infty} = F^m < 1, \]
where
\[ \phi_2^\infty = \phi_2(p_\infty, a - p_\infty) = -f(a) - p_\infty f'(a) < 0. \]
**Proof.** Both $R_A$ and $R_B$ tend to zero as $n \to \infty$, but $R_A/R_B$ tends to one.\(^{11}\) Then (C.14)--(C.15) imply
\[
\lim_{n \to \infty} K_m \phi(p_A, a - p_A) + \hat{R}_A = 0
\]
since $h_B \to 0$. We know that both $p_A$ and $p_B$ tend to $p_\infty$, and so both the numerator and the denominator approach to zero. Using L'Hôpital’s rule and noting $\Delta \to 0$ as $n \to \infty$, we have
\[
\lim_{n \to \infty} \frac{K_m (\phi_1 - \phi_2) \partial p_A/\partial n + \partial \hat{R}_A/\partial n}{(\phi_1 - \phi_2) \partial p_B/\partial n} = \lim_{n \to \infty} \frac{K_m f(a) + F(a)^{m-1} \phi_2^\infty (\partial p_B/\partial p_A - 1)}{f(a) \partial p_B/\partial p_A} = 0,
\]
where we have used
\[
d\hat{R}_A = F'(a - \Delta)^{m-1} \phi_2(p_A, a - p_B) \left( \frac{\partial p_B}{\partial m} - \frac{\partial p_A}{\partial m} \right) - \int_{a-\Delta}^a F(u)^{m-1} (\phi_{12} - \phi_{22}) \frac{\partial p_A}{\partial u} du.
\]
Then we get
\[
\lim_{n \to \infty} \frac{\partial p_B}{\partial p_A} = 1 - \frac{K_m f(a)}{\phi_2^{\infty} F^{m-1}}.
\]
Since $\lim_{n \to \infty} \frac{p_A - p_\infty}{p_B - p_\infty} = \lim_{n \to \infty} \frac{\partial p_B}{\partial p_A}$, our first result follows. A similar but simpler proof generates the second result. \(\blacksquare\)

This lemma immediately implies to the following result:

**Proposition C.3** When $n$ is sufficiently large, $p_0 < p_B$ always holds and $p_A < p_0$ if and only if
\[
F \cdot \left[ F^{m-1} - \frac{K_m}{\phi_2^\infty} f(a) \right] > 1.
\]

All other limit results as $n \to \infty$ derived in the main text can be established in the current general setup by using Lemma C.1. We only show the total welfare result below.

**Welfare** We derive the expression for total welfare first. We need two new random variables for expositional convenience:
\[
u_A = \max\{u_1, \ldots, u_m\}, \quad \nu_1 = \max\{u_1 + \Delta, \ldots, u_m + \Delta, u_{m+1}, \ldots, u_n\}.
\]
Their distribution functions are $F_A(u) = F(u)^m$ and $F_1(u) = F(u - \Delta)^m F(u)^{n-m}$, respectively. Total welfare is then
\[
W_m = \left[ (1 - F^m) + F(a - \Delta)^m (1 - F^{n-m}) \right] \cdot E[u | u \geq a] + \int_{a-\Delta}^a udF_A(u) + \left[ \int_{p_B}^a udF_1(u) - \Delta \cdot m r_A \right] - s T_m,
\]

\(^{11}\)A rigorous proof for this is available from the author.
where

\[ T_m = \frac{1}{1-F} \left[ 1 - F^m + F(a - \Delta)^m (1 - F^{n-m}) \right] \]

is the expected number of searches. A consumer will end up as a fresh buyer if she immediately stops at pool A or pool B. The likelihood of this event is the first square bracket term. So the first term is the expected gross surplus (excluding the search cost) from all fresh buyers. The second one is the expected gross surplus from those midway returning buyers. The third one is the expected gross surplus from all final returning buyers. The reason why we subtract \( \Delta \cdot mr_A \) is that, when we use the order statistics \( v_1 \), each prominent product's utility is artificially added by \( \Delta \), so when they win back returning buyers (of which the probability is \( mr_A \)), \( \Delta \) should be subtracted. Using the fact that \( E[u | u \geq a] = \frac{a}{1-p} = a \) (which is from the definition of \( a \)), one can show

\[
W_m = a \left[ 1 - F^m + F(a - \Delta)^m (1 - F^{n-m}) \right] + \int_{a-\Delta}^{a} u dF_A(u) + \int_{p_B}^{a} (u - \Delta) F(u)^{n-m} dF(u - \Delta)^m + \int_{p_B}^{a} u F(u - \Delta)^m dF(u)^{n-m}.
\]

The expression for total welfare in the uniform-distribution case just follows from this.

Now we show \( W_m - W_0 < 0 \) when \( n \to \infty \). Define a function

\[
W(\delta, p) = a (1 - F^m) + \left( m \int_{a-\delta}^{a} u dF(u)^{m-1} [F(u) - F(a-\delta)] \right) \left( \int_{p}^{a} F(u)^{m-1} f(u) \, du \right).
\]

It is ready to check that \( W_m = W(\Delta, p_B) \) and \( W_0 = W(0, p_0) \). When \( n \) is large, \( W_m - W_0 \) can be approximated by

\[
\frac{\partial W(0, p_0)}{\partial \delta} \Delta + \frac{\partial W(0, p_0)}{\partial p} (p_B - p_0).
\]  

(C.18)

One can show that \(^{13}\)

\[
\frac{\partial W(0, p_0)}{\partial \delta} = m p_0 F(p_0)^{n-1} f(p_0), \quad \frac{\partial W(0, p_0)}{\partial p} = -n p_0 F(p_0)^{n-1} f(p_0).
\]

\(^{12}\)The intuition of this result is clear. In each step of the search process (no matter in the prominent pool or in the non-prominent pool), a consumer has probability of \( 1 - F \) to be a fresh buyer. But the whole probability to being a fresh buyer is the square-bracket term. So our expression for \( T_m \) follows. Alternatively, \( T_m \) can be calculated as \( \sum_{i=1}^{m} (1-F) F_{x-1}^m + m [F_A(a) - F_A(a - \Delta)] F^{n-m} \sum_{i=1}^{m} (1-F) F_{x-1}^m + n F(a - \Delta)^m F^{n-m} \).

\(^{13}\)Readers may wonder why \( \frac{\partial W(0, p_0)}{\partial \Delta} > 0 \). According to our intuition before, larger price difference between \( p_A \) and \( p_B \) will cause less efficient search behavior. But in this partial derivative term, when \( p_B \) is fixed at \( p_0 \), larger \( \delta \) also means lower \( p_A \). This is a positive effect, and so \( \frac{\partial W(0, p_0)}{\partial \Delta} > 0 \) does not conflict with our previous intuition.

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Then (C.18) is negative if and only if
\[ \frac{\Delta}{p_B - p_0} < \frac{n}{m}. \]

From Lemma C.1, we know that \( \lim_{n \to \infty} \frac{\Delta}{p_B - p_0} \) is finite, and so the above condition must hold as \( n \to \infty \).
Appendix D

Appendix of Chapter 5

D.1 Deriving $\frac{\partial v_s}{\partial r}$

Let $f = f(v_s)$ and $F = F(v_s)$. Regard $x^+_i$ as a function of $r$ and $v_s$. From (5.14)–(5.15), we get

$$
\frac{\partial x^+_1}{\partial r} = -\frac{\theta}{F} \frac{1}{c''(x^+_1)^{\prime}}, \quad \frac{\partial x^+_2}{\partial r} = -\frac{\theta}{F} \frac{1}{k c''(x^+_2)^{\prime}},
$$

$$
\frac{\partial x^+_1}{\partial v_s} = \frac{\theta r}{F^2} \frac{1}{c''(x^+_1)^{\prime}}, \quad \frac{\partial x^+_2}{\partial v_s} = \frac{\theta r f}{F^2} \frac{1}{k c''(x^+_2)^{\prime}}.
$$

Remember $w_s = \frac{1}{2}(x^+_1 + x^+_2) - c(x^+_1) - kc(x^+_2)$, so

$$
\frac{\partial w_s}{\partial r} = \frac{\theta r}{F} \left( \frac{\partial x^+_1}{\partial r} - \frac{\partial x^+_2}{\partial r} \right) = -\frac{C \theta^2 r}{F^2},
$$

$$
\frac{\partial w_s}{\partial v_s} = \frac{\theta r}{F} \left( \frac{\partial x^+_1}{\partial v_s} - \frac{\partial x^+_2}{\partial v_s} \right) = \frac{C \theta^2 r f}{F^3},
$$

where we have used the first-order conditions (5.13)–(5.15). From (5.13), we further have

$$
\frac{\partial w_s}{\partial v_s} \frac{\partial v_s}{\partial r} + \frac{\partial w_s}{\partial r} = K \frac{\partial v_s}{\partial r} + \frac{1}{f},
$$

from which one can derive

$$
\frac{\partial v_s}{\partial r} = \left[ \frac{1}{f} + C \frac{\theta^2 r}{F^2} \right] \left[ C \frac{\theta^2 r f}{F^3} - K \right]^{-1}.
$$
D.2 Continuous advertising effect

This extension considers continuous advertising effect. Suppose now $\theta$ distributes on $[0, \bar{\theta}]$, $\bar{\theta} \leq 1$, according to the cumulative distribution function $G(\theta)$ of which the density function is $g(\theta)$. A consumer with a lower $\theta$ is more sophisticated. $\theta$ is private information, and the firm only knows its distribution. This extension confirms our main results in the two-type setup and also provides several new insights which have been mentioned in the main text.

We keep the following regularity assumption:

**Assumption D.1** Both $\theta + \frac{G(\theta)}{g(\theta)}$ and $\theta - \frac{1-G(\theta)}{g(\theta)}$ increase with $\theta$.

This is a standard assumption in the screening literature. One sufficient condition for this is logconcave $g(\theta)$.

According to revelation principle, we can focus on the direct truth-telling mechanism. Let the triplet $(x_1(\theta), x_2(\theta), p(\theta))$ be the product designed for a $\theta$-type consumer. A $\theta$-type consumer's surplus from choosing the product designed for a $\theta'$-type consumer is

$$v(\theta' | \theta) = (1 + \theta)x_1(\theta') + (1 - \theta)x_2(\theta') - p(\theta').$$

Then the truth-telling surplus is $v(\theta) = v(\theta' | \theta)$, and we have

$$v'(\theta) = x_1(\theta) - x_2(\theta).$$

The firm’s problem can be written as

$$\max_{x_1(\theta), p(\theta)} \int_0^{\bar{\theta}} \left[ p(\theta) - c(x_1(\theta)) - kc(x_2(\theta)) \right] dG(\theta)$$

subject to

$$v(\theta) \geq 0,$$

$$v(\theta) \geq v(\theta' | \theta),$$

for all $\theta, \theta' \in [0, \bar{\theta}]$.

**Lemma D.1** IC conditions hold if and only if

$$v'(\theta) = x_1(\theta) - x_2(\theta)$$

---

1In the standard one-dimensional screening problem, this assumption guarantees separating allocation. However, in our setup (like in the countervelling-incentive problem), pooling allocation could happen even with this assumption.
almost everywhere, and

\[ x_1'(\theta) - x_2'(\theta) \geq 0. \]

**Proof.** The proof is standard so omitted. ■

Since *a priori* we do not know whether some products will be excessively distorted such that \( x_1(\theta) < x_2(\theta) \), \( v(\theta) \) could be non-monotonic with \( \theta \). Hence, we cannot replace all participation conditions by \( v(0) = 0 \). This is a common feature in the countervailing-incentive problem. (See, e.g., Lewis and Sappington (1989), and Maggi and Rodriguez-Clare (1995).) But the following lemma can simplify our analysis:

**Lemma D.2** \( v(\theta) \) equals zero on at most an (maybe degenerate) interval.

**Proof.** According to Lemma D.1, IC conditions imply \( v''(\theta) \geq 0 \), so \( v(\theta) \) is weakly convex. Since \( v(\theta) \) is also non-negative, it equals zero on at most an interval. ■

Then we can divide the support of \( \theta \) into three intervals:

\[
v(\theta) = \begin{cases} 
> 0 & \text{if } \theta \in [0, \theta_1) \\
= 0 & \text{if } \theta \in [\theta_1, \theta_2] \\
> 0 & \text{if } \theta \in (\theta_2, \theta].
\end{cases}
\]

(Of course, some intervals may degenerate at optimum.) Therefore, from \( v'(\theta) = x_1(\theta) - x_2(\theta) \), we can recover

\[
v(\theta) = \begin{cases} 
\int_0^{\theta_1} (x_2(t) - x_1(t)) \, dt & \text{if } \theta \in [0, \theta_1) \\
0 & \text{if } \theta \in [\theta_1, \theta_2] \\
\int_{\theta_2}^{\theta} (x_1(t) - x_2(t)) \, dt & \text{if } \theta \in (\theta_2, \theta].
\end{cases}
\]

If we ignore the constraint \( x_1'(\theta) - x_2'(\theta) \geq 0 \), then we can solve the relaxed optimization problem by using the standard procedure. However, one can check that the solution to the relaxed problem may actually violate \( x_1'(\theta) - x_2'(\theta) \geq 0 \). But that does inspire us to conjecture the real solution. We guess that the solution described in the following proposition is the real one. It is easy to see that it satisfies the constraint \( x_1'(\theta) - x_2'(\theta) \geq 0 \) under Assumption D.1. We further verify that it satisfies the sufficient conditions of the relaxed optimization problem. (See, e.g., Maggi and Rodriguez-Clare (1995) for the justification for this procedure.)

**Proposition D.1** Suppose Assumption D.1 holds and remember \( \hat{\theta} = \frac{1-k}{1+k} \in (-1, 0) \).
(i) If $\lim_{\theta \to 0} \frac{1}{g(\theta)} \leq \hat{\theta}$, then $v(0) = 0$, and $v(\theta) > 0$ for $\theta > 0$. The quality levels are fully separated:

\[
\begin{align*}
c'(x_1(\theta)) &= 1 + \theta - \frac{1 - G(\theta)}{g(\theta)}, \\
kc'(x_2(\theta)) &= 1 - \theta + \frac{1 - G(\theta)}{g(\theta)}.
\end{align*}
\]

(ii) If $\lim_{\theta \to 0} \frac{1}{g(\theta)} > \hat{\theta}$, then the quality levels are pooled for part of consumers: for $\theta \in [0, \theta^*)$, we have

\[
\begin{align*}
v(\theta) &= 0, \\
x_1(\theta) &= x_2(\theta) = x^*;
\end{align*}
\]

and for $\theta \in (\theta^*, \tilde{\theta}]$, we have

\[
\begin{align*}
v(\theta) &> 0, \\
c'(x_1(\theta)) &= 1 + \theta - \frac{1 - G(\theta)}{g(\theta)}, \\
kc'(x_2(\theta)) &= 1 - \theta + \frac{1 - G(\theta)}{g(\theta)}.
\end{align*}
\]

where $\theta^*$ solves

\[
\theta^* - \frac{1 - G(\theta^*)}{g(\theta^*)} = \hat{\theta}, \tag{D.1}
\]

and $x^*$ solves

\[
c'(x^*) = \frac{1}{1 + k}.
\]

Note that the condition $\lim_{\theta \to 0} \frac{1}{g(\theta)} > \hat{\theta}$ in part (ii) just guarantees that (D.1) has a positive solution under Assumption D.1. This situation happens when there are too few very sophisticated consumers. One example is $g(\theta) = \frac{2\theta}{\theta^2}$ which satisfies Assumption D.1.

Before presenting the proof, we discuss the implications of this result. First, when $g(0)$ is sufficiently small, a bunch of relatively sophisticated consumers will be offered the same product. This is a distinction of our two-dimensional model. Second, some type of naive consumer will happen to be offered the socially optimal product. Specifically, this happens for $\theta = \frac{1 - G(\theta)}{g(\theta)}$. (Note that this equation always has solutions.) This is because, for some middle-type consumer, the distortion due to the misperceived preferences and the distortion due to adverse selection just cancel each other out. Third, let us consider the impact of consumers becoming more naive. We first specify a precise definition of this
change. Consider two distributions $G(\theta)$ and $H(\theta)$ satisfying
\[
\frac{h(\theta)}{1 - H(\theta)} \leq \frac{g(\theta)}{1 - G(\theta)},
\]  
(D.2)
i.e., $H$ has a lower hazard rate.\(^2\) When the distribution of consumer types varies from $G$ to $H$, we say that consumers are becoming more naive. We call this relation “conditionally stochastic dominance (CSD)” which is slightly stronger the traditional first-order stochastic dominance.\(^3\) One can show that (D.2) is equivalent to
\[
\frac{1 - H(\theta_2)}{1 - H(\theta_1)} \geq \frac{1 - G(\theta_2)}{1 - G(\theta_1)}
\]
for $\theta_1 < \theta_2$. So the meaning of CSD is, conditional on $\theta \geq \theta_1$, $H$ is more likely to have $\theta \geq \theta_2$ than $G$, i.e., $H$ shifts consumers to be more naive in a systematic way. When $H$ replaces $G$, it is ready to see that the scope of pooling allocation expands (i.e., $\theta^*$ becomes larger). But its effect on consumer surplus is less clear. According to the view of an expert, a $\theta$-type consumer’s real surplus is
\[
\hat{v}(\theta) = v(\theta) - \theta [x_1(\theta) - x_2(\theta)].
\]
By recalling $v(\theta) = \int_0^\theta (x_1(t) - x_2(t)) dt$ and $x_1(t) - x_2(t)$ (weakly) increases with $t$, the real surplus must be non-positive for each type of consumer (all types less than $\theta^*$ get zero and others get negative surplus). For the most naive consumer, she suffers when $H$ replaces $G$, since $x_1(t) - x_2(t)$ shrinks (so $v(\theta)$ decreases). While for the consumers whose type is slightly higher than $\theta^*$, they will then get zero surplus instead of the original negative surplus, so they benefit from the change.

**Proof.** Let
\[
w(\theta) = (1 + \theta)x_1(\theta) + (1 - \theta)x_2(\theta) - c(x_1(\theta)) - kc(x_2(\theta))
\]
be the social surplus of product $\theta$ based on misperceived preferences. Then the problem without considering the constraint $x_1'(\theta) - x_2'(\theta) \geq 0$ is
\[
Max_{x_1(\theta), x_2(\theta)} \int_0^\theta [w(\theta) - v(\theta)] g(\theta) d\theta
\]
subject to
\[
v(\theta) \geq 0 \text{ and } v'(\theta) = x_1(\theta) - x_2(\theta).
\]
\(^2\)For example, $g(\theta) = \frac{2}{\theta^2}$ and $h(\theta) = \frac{2}{\theta^2}$ satisfy Assumption D.1 and condition (D.2).
\(^3\)See Maskin and Riley (2000) for a slightly different definition of CSD.
Define the Lagrangian function

\[ L = [w(\theta) - v(\theta)] g(\theta) + \mu (x_1(\theta) - x_2(\theta)) + \eta v(\theta), \]

where \( \mu \) and \( \eta \) are Lagrangian multipliers. (They are also functions of \( \theta \).) The sufficient condition for the optimal solution is that there exists \( \mu \) and \( \eta \) such that \( x_i(\theta) \) and \( v(\theta) \) satisfy

\[
\begin{align*}
\frac{\partial L}{\partial x_1} &= (1 + \theta - c'(x_1)) g(\theta) + \mu(\theta) = 0, \\
\frac{\partial L}{\partial x_2} &= (1 - \theta - kc'(x_2)) g(\theta) - \mu(\theta) = 0, \\
-\frac{\partial L}{\partial v} &= \frac{d\mu}{d\theta} = g(\theta) - \eta(\theta), \\
v'(\theta) &= x_1(\theta) - x_2(\theta),
\end{align*}
\]

and

\[
\begin{align*}
\eta(\theta)v(\theta) &= 0, \quad \eta(\theta) \geq 0, \quad v(\theta) \geq 0, \\
\mu(0)v(0) &= 0, \quad \mu(\theta)v(\theta) = 0, \quad \mu(0) \leq 0, \quad \mu(\theta) \geq 0.
\end{align*}
\]

It is straightforward to check that the solution provided in part (i) and the solution provided in part (ii) on \([\theta^*, \hat{\theta}]\) satisfy the above conditions, so the details are omitted. We now check the solution in part (ii) on \([0, \theta^*]\). We first pin down the associated \( \mu(\theta) \) and \( \eta(\theta) \). Since \( x_1(\theta) = x_2(\theta) = x^* \) on \([0, \theta^*]\), we have \( c'(x_1) = c'(x_2) = \frac{1}{k+1} \), then using the second condition, we get

\[ \mu(\theta) = (\hat{\theta} - \theta)g(\theta). \]

(Remember \( \hat{\theta} = \frac{1 - k}{1 + k} \).) Given such a \( \mu(\theta) \), the first condition and \( \mu(0) \leq 0 \) will also hold. Using the third condition \( \frac{d\mu}{d\theta} = g(\theta) - \eta(\theta) \), we further have

\[ \eta(\theta) = 2g(\theta) - (\hat{\theta} - \theta)g'(\theta). \]

Now what we need to show is \( \eta(\theta) \geq 0 \). If \( g'(\theta) \geq 0 \), we are done since \( \hat{\theta} < 0 \). Suppose \( g'(\theta) < 0 \). From \( \theta - \frac{1-G(\theta)}{g(\theta)} \) increasing in \( \theta \) in Assumption D.1, we have \( \frac{1-G(\theta)}{g(\theta)} < -\frac{2g(\theta)}{g'(\theta)} \). On the other hand, from (D.1), we also have \( \theta - \frac{1-G(\theta)}{g(\theta)} < \hat{\theta} \) for \( \theta \in [0, \theta^*] \). Thus, \( \theta - \hat{\theta} < -\frac{2g(\theta)}{g'(\theta)} \), which just implies \( \eta(\theta) > 0 \). This completes the proof. \( \blacksquare \)
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