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Bill Hillier, Tao Yang, Alasdair Turner
 The Bartlett School of Graduate Studies
 University College London (UCL)

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This paper is dedicated to the memory of Alasdair Turner, the creator of Depthmap, and one of our co-authors. Alasdair was involved in the early discussions on the questions the paper addresses, but was too ill to be involved in the later stages. So if we've got it wrong, we are to blame, not Alasdair. But we hope the answers we are proposing to the problem of normalising angular measures are wholly in the intellectual spirit of Depthmap, and, once thoroughly tested, can become part of it.

Depthmap embodies a theory of the city, as well as being a method for analysing the city. By solving outstanding problems of the normalisation of measures, most notably syntactic choice (mathematical betweenness), to permit comparison of cities of different sizes, we can gain new theoretical insights into their spatial structuring.

Keywords:
*Space, syntax,
choice, betweenness,
normalisation, city
structure*

1. Introduction

Depthmap as a theoretical tool

Depthmap is a remarkable piece of software. The parts we use to analyse cities – segment analysis – were based on a theory of the city, and quickly led to a better one. The concept of *pervasive centrality* (Hillier, 2009), a key idea in the syntactic perspective on the sustainable city, came from the power of Depthmap to detect delicate local structures which hardly seem to be present (Figure 1), as in the spatial detection of London’s ‘urban village’ structure through low metric radius angular choice. It is unusual, to say the least, to be able to use two definitions of distance in the same measure - angular change for distance within the measure and metric distance for the radius - but this is the kind of thing Depthmap can do, and in this case it led directly to a clarification in our syntactic understanding of the city.

But Depthmap is a work in progress and there is much more to come. One significant area where elements are missing is in the normalisation of the



Figure 1:

London's urban village structure identified by low metric radius angular choice.

key measures of integration and choice for angular systems, which would enable the comparison of cities of different sizes, and their parts and elements. To remind the reader, integration measures the distance from each spatial element to all others in a system (up to a certain radius and given a definition of distance), and so corresponds to mathematical

closeness. Choice measures the quantity of movement that passes through each spatial element on shortest or simplest trips between all pairs of spatial elements in a system (again up to a certain radius and given a definition of distance), and so corresponds to mathematical betweenness. We often say that integration represents the *to-movement* potential of a space, and choice the *through-movement* potential, pointing out also that the two measures correspond to the two basic elements in any trip: selecting a destination from an origin (integration), and choosing a route, and so the spaces to pass through between origin and destination (choice).

So let me remind you of the background to the problem of normalisation. Since *The Social Logic of Space* (Hillier and Hanson, 1984) we have had a robust normalisation of integration in the axial map by means of the D-value – the idea that we could numerically compare the justified graph of each line with a ‘diamond-shape’ graph. The details can be found in SLS, and this is still the basis for ‘integration HH’ for the axial map in Depthmap. The normalisation was not beautiful, but it was effective. A good alternative was proposed by Tecklenburg et al. (1993) which is implemented in Depthmap, but by and large, the D-value normalisation has proved robust and has continued to be used successfully in the axial analysis of cities (see for example Hillier, 2002) and for predicting movement at the design stage of projects.

For choice in the axial map, on the other hand, there never has been a normalisation. One reason for this is probably that it did not seem necessary, since in the axial map, integration had proved itself a far more powerful variable for both analysis and design prediction than choice. It was only with segment analysis, and in particular with segment angular analysis (segment based analysis using least angle change as the definition of distance – the default definition of distance in Depthmap) that choice came into its own as at least as powerful as

integration in both analysis and movement postdiction. This already pointed to the need to be able to normalise choice to compare systems of different sizes – but it also reminded us that in angular analysis we no longer had a normalisation of integration, since the morphological conditions in axial maps that permitted the diamond comparison clearly did not apply either in segment maps or under angular definitions of distance.

In what follows, we will set out what we believe to be the best ways to normalise both choice and integration for the segment angular case. The reason for this being the default definition of distance in Depthmap is that it has proved the best predictor of both vehicular and pedestrian movement, as shown in (Hillier and Iida, 2005), and this has been amply confirmed by its use in projects since then. But we will also argue that, as with previous advances in the mathematical techniques of space syntax, an understanding of normalisation, especially of choice, can lead us to new and deeper understanding of the spatial morphology of cities.

This will also be a story of the interaction between the worlds of research and design. The problems that led us first to identify the urgent need to normalise choice, and then move towards the solution, first arose in the use of Depthmap in design in Space Syntax Limited, and in particular in the need to use the choice measure as a variable in movement prediction models. To put it simply, we found that complex segregated designs were predicting overall higher rates of movement than integrated designs, both within and in the close vicinity of the site. As you can imagine, this was quite an unpleasant surprise, especially as the same choice measure had just proved itself excellent in postdicting existing movement around the site. So why did it seem that we could use choice for postdicting, but not for predicting, movement? It did not seem rational. This was the problem we took back to the university.

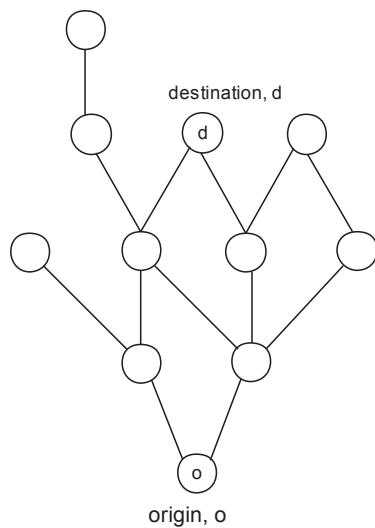
2. Normalising the measures of choice and integration

The measure of choice

As the normalisation problem showed itself most acutely in the choice measure, we will start with the problem of normalising choice, then go on to the relatively simpler – with hindsight! – problem of integration. The choice measure in syntax was first presented descriptively as part of a model of measures in ‘Creating life: Or does architecture determine anything?’ (Hillier et al., 1987). For technical details, we referred the reader to a research report from the Unit for Architectural Studies (as it was then) to the Engineering and Physical Sciences Research Council (as it was then). It is widely thought that we simply used Freeman’s ‘betweenness’ measure, but in fact we had developed our own original measure, giving the same results as Freeman’s measure but with a different method of calculation based on the concept of the j-graph (see below). Both are calculated in Shinichi Iida’s SEGMENT software (Iida, 2006), so can be compared, and there is also a third involving randomisation for computational speed, which is the one implemented in Depthmap.

Figure 2:

A simple justified graph G_1 .



So let me explain our measure of choice, before showing that it gives the same results as Freeman’s. In explaining it, we will first talk about the case of a simple graph using graph distance (a value of 1 being given to the distance between each spatial element and each of its neighbours), and then talk about how this can be adapted to segment graphs using least angle change as the measure of distance. As with the integration measure, the idea of the choice measure came originally from visualising the spatial network as a justified graph. Consider the simple graph shown in Figure 2 in which one space, the root, is marked o , meaning *origin*, and another is marked d for *destination*, indicating that we are interested in movement between these two spaces. First, imagine the graph is a flexible net. We pick up o and d and pull the net tight. All nodes that lie on simplest paths between o and d are then tight, and the others hang loose. We eliminate the loose hanging nodes as they are not on simplest paths between o and d , and then take each level in the tight net and stretch each one to its natural width. The result for G_1 will be Figure 3:

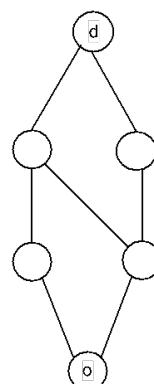


Figure 3:

The ‘spanning net’ between o and d .

We can think of this as a kind of *spanning net* between o and d . To distinguish the spanning net from the usual justified graph, and to emphasise the fact that we are interested in movement from both o to d and from d to o , we rotate the spanning net 90 degrees to give Figure 4.

Now we can see what we are doing, we assign a value of 1 to o , then move the value in the direction of d , at each stage splitting the value according to how many choices we have at each point.

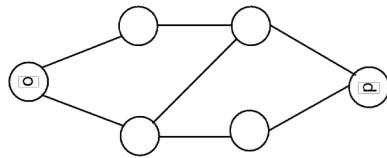


Figure 4:

The 90 degrees rotation of the spanning net between o and d .

the number of bridges. The more parallel nodes at each layer, the wider the bridge.

Now this simple demonstration makes it self-evident that the total value passing through each bridge from an origin to a destination will always be 1, and cannot be more or less than 1 however many or few parallel nodes we have on a bridge. It follows that the *total* choice in the system will only be a function of the *number* of bridges, not the width of the bridges. The number of bridges, in turn, is exactly and only a function of the depth of nodes from each other: the farther nodes are from each other, the higher the total quantity of choice depos-

At the first layer of the top graph in Figure 5 there are two choices, so each gets .5. At the second stage, the top .5 gives all its value to the top node, while the lower .5 splits its value between the two choices available in the next layer. At the third layer, a total value of 1 is received at the destination. We then carry out the same procedure (second graph in Figure 5) in reverse, from d to o , noting that the values acquired by each node are not the same when calculated the other way round. The sum of the two (third graph in Figure 5) is then the total choice acquired by each node from simplest routes between o and d . The choice value of a node will then be the sum of values acquired by each node on simplest routes between all pairs of nodes in the graph.

It is useful to think of each 'layer' in the – horizontally – justified graph as a *w-bridge* from origin to destination, where *w* stands for the number of *parallel nodes* making up the bridge. Thus wherever there is a single node, we will have a 1-bridge, and all movement between the two nodes must pass through it. In the above case we have two 2-bridges. The deeper the spanning net from o to d , the larger

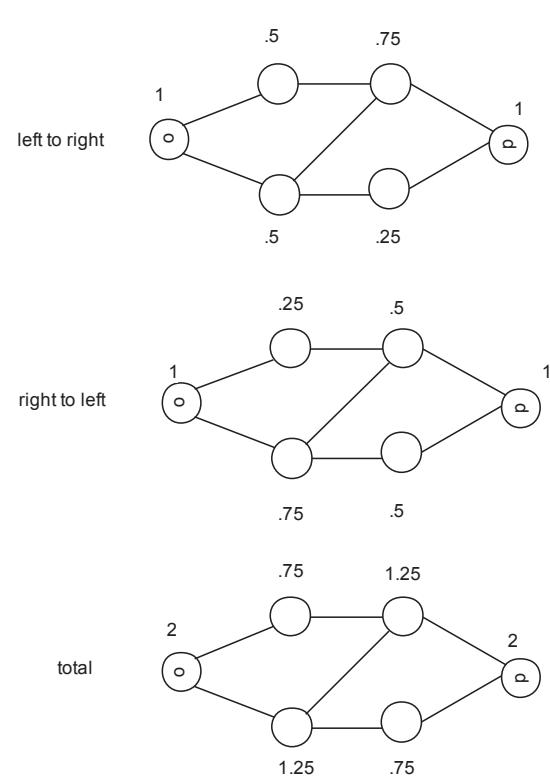


Figure 5:

The way of calculating choice.

ited on the nodes that lie on simplest paths between them. In other words, the total choice in a system is exactly and only a function of the total depth of the system. The more the system is segregated, the more choice it generates.

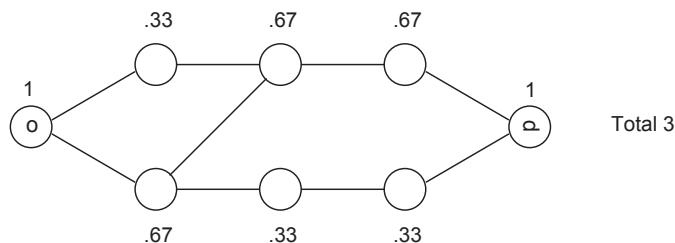
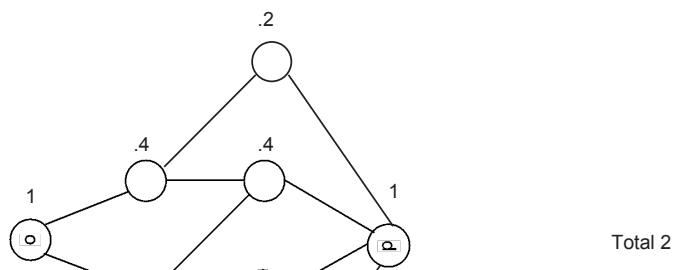
This can be further clarified by making a distinction between the choice *acquired* by each node through the process we have described above, and the choice *contributed* by each node to the system. By choice *contributed*, we mean the total choice deposited by a node on simplest paths to all other nodes in the system. It is evident that this total will be higher the more nodes must be passed through. So the deeper the system from a node, the more choice that node will contribute to the system.

Since the sum of choice *contributed* by all nodes must be equal to the sum of choice *acquired* by all nodes, it follows that a segregated system will have higher total choice than an integrated system with the same number of nodes.

This will be the case however we calculate choice as being the number of routes in a system that pass through a particular space. For example, if we follow Freeman and calculate the proportion of all routes between an *o* and a *d* which pass through each space, we find exactly the same pattern emerges as with the space syntax measure. In Figure 6, we add two nodes to our simple graph, in the first case as *parallel* nodes, in the second as adding a new bridge:

Figure 6:

Two nodes are added to our simple graph, in the first case as parallel nodes, in the second as adding a new bridge, using Freeman's definition of betweenness. The result will be the same with the syntactic choice calculation.



In both cases, the proportions at each bridge must add up to 1, and the total choice in the system is simply a function of the number of bridges. Note that this cannot be ‘normalised’ simply by dividing by the number of nodes. The two systems in Figure 6 have the same number of nodes, and the difference in total choice is purely a function of the degree of segregation in the system as a whole.

This is the problem – and the paradox – of choice. While the quantity of choice acquired by each node will be related to the *integration* of that node into the system, the total choice in the system is a function of the *segregation* of the system. This is the problem we encountered in trying to use choice to form predictions in design. Segregated designs add more total – and average – choice to the system than integrated ones. This was most evident within and close to the location of the design. Clearly here lies a problem that must be solved if we are to use the choice measure successfully in analysis or design.

How then can the problem be solved?

How then can a solution to this problem be found? Simply dividing choice by node count to find the average will not provide the answer, since this omits the increases in total – and average – choice that comes from depth in the system. Nor is it clear how the problem of depth can be mathematically solved, since if we count the destination as a through-movement space then the total choice and total depth are the same. However, this is only the case if we define distance as graph (or topological) distance. Since we know that the distance concept that best captures how people read distance in the system is least angle change distance, the differences in values between choice and angular depth suggest there may be mathematical possibilities to bring the two measures together. In fact, if people in general calculate distance geometrically by trying to ap-

proximate a straight line across a grid, then minimising angular deviation from a straight line between origin and destination would seem to approximate the human conception of distance (Hillier, 2012a).

Suppose then that we simply divide total choice by total angular depth for each segment in the system. This would adjust choice values according to the total angular depth of each segment, since the greater the segregation, the more the choice value will be reduced by being divided by a higher total depth number. This would seem to have the effect of measuring choice in a cost-benefit way (a suggestion first made by the most junior member of our team, Tao Yang): so much choice, but at such and such a cost in angular depth; or, so much through-movement, but at such and such a cost in angularly getting there. At the same time, this would take the total depth component out of the measure and leave something like a pure choice measure. This seems theoretically promising, not least in the sense that it seems to combine our two measures – depth and choice, to and through-movement – in a lifelike way. Let us provisionally call this measure ‘normalised angular choice’, or NACH ($\log CH + 1 / \log TD + 3$ – see mathematical Appendix), and explore its behaviour in real and theoretical systems. Mathematical details are given in Appendix 2.

The behaviour of normalised angular choice, NACH, in real urban systems

The first two tests of a normalised measure must be: whether the means for the system correlate with the size of the system in terms of numbers of segments; and whether the values for the individual segment continue to predict movement. To address the former, we constructed a data table of 50 cities, with the smallest having less than 1000 segments and the largest over 250,000, so varying across orders of magnitude in terms of segment numbers. Figure 7a shows the correlation between the mean

NACH for cities and the size of the city in segment numbers. There is no correlation. The measure passes its first test.

But do the individual values for segments continue to predict actual movement rates as well as choice? In fact, slightly better. We took the standard test cases (the four areas of London used in Hillier and Iida 2005, where movement has been densely observed) and first compared the movement correlations between NACH and simple choice for all four areas together at different radii, and for each area taken separately. The results in each case are very similar scattergrams, but with slightly improved r-squares. Figure 7b shows a typical result. At radius-n for all four areas taken together, NACH

yields an r-square of .633, compared to .626 for log choice. At radius 800 metres we have .613 for NACH compared to .594 for log choice, and for 2000 metres we have .664 for NACH compared to .659 for log choice. For individual areas, we have .734 for Clerkenwell compared to .723 for log choice, .775 for Barnsbury compared to .73, .566 for Brompton compared to .546 and .559 for South Kensington compared to .566. So we can safely say that NACH at least emulates and perhaps slightly improves the predictive power of the choice measure. The similarity of the scattergrams also confirms that NACH really is a choice measure. We have passed our second test.

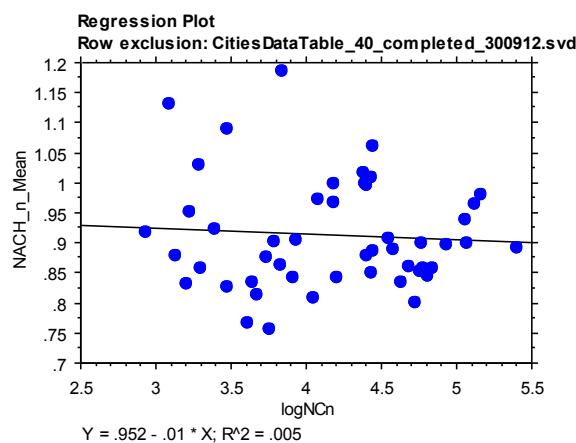


Figure 7a:

The correlation between the mean NACH for cities and the size of the city in segment numbers.

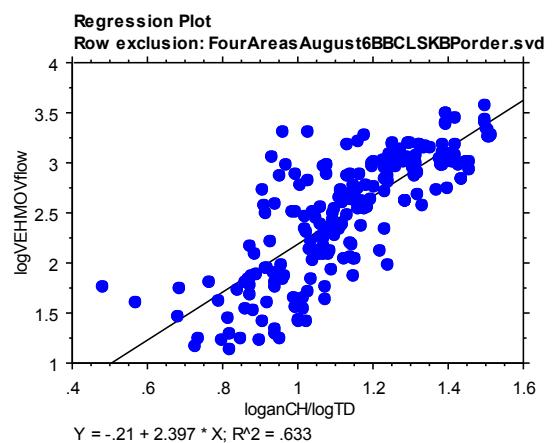


Figure 7b:

The correlation between NACH and vehicular movement for individual segments in four areas of London, with an r-square of .633.

So what does the measure mean and what can we do with it?

But what does the measure really mean? How can we visualise it? The first question is: if NACH does not correlate with size, what does it correlate with? The answer surprised us. By far the most powerful correlation we can find is with simple segment connectivity, as in Figure 8, in spite of the fact that segment connectivity is a very poor predictor of movement, with an r-square of only .019, for example, with vehicular movement in the four standard cases taken together. In fact, this permitted a useful demonstration of the independence of NACH from the size of the system. We added to Figure 8, which represents whole cities between 1000 and 250000 in numbers of segments, data from 11 randomly selected London areas, varying from just over 600 to just over 3000 segments, and found them spread along the same regression line, with virtually no effect on the r-square.

More importantly, the link to segment connectivity allows us to see more clearly what it is that NACH measures. Because it indexes the degrees of deviation from straight line routes from each segment to all others, it is clear that mean NACH in effect measures something like the degree of deviation from a regular grid that the system has when seen from each segment within the grid. This will be reflected in the fact that high mean NACH will often be found in more regularised grids like Barcelona and Manhattan. However, a high mean level is not simply dependent on a grid-like structure. For example, both London and Tokyo, among the least obviously geometric of cities, have higher means than Beijing or Kyoto which are based on rigorous orthogonal grids.

Nevertheless, the mean may not be the only critical property. What in syntax we have always called the *structure* of the system depends on its highest value lines, and in NACH analysis, the maximum value turns out to be very interesting. While in certain

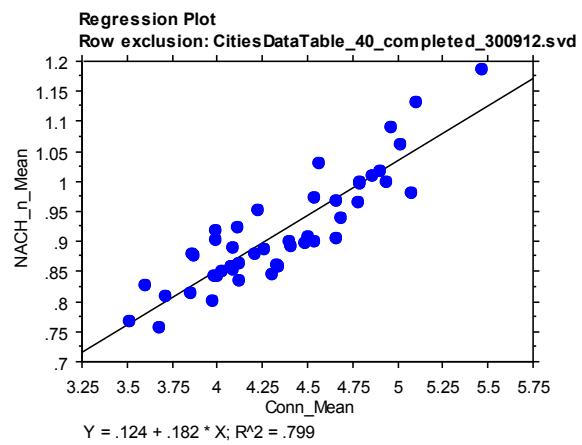


Figure 8:

The correlation between mean segment connectivity and mean NACH for a sample of cities, varying in size from under 1000 segments to over 250000.

small systems, such as Venice, the value can fall around 1.4, to a remarkable degree, the maximum NACH value in cities in general is found in the region between 1.5 and 1.6, or a little more. Why is this the case, what does it mean and how does it arise?

We can begin to clarify the nature of the problem by comparing cities to completely regular theoretical grids. We take three cases: a 60 segment by 60 segment orthogonal grid; the same with a single pair of crossed diagonals, meeting in the centre; and the same diagonals across all the squares formed by the grid. While in real cities the mean NACH value varies from about .7 to 1.2, in all cases the regular grids are higher, with 1.251490 for the regular grid, 1.251340 for the pair of diagonals and 1.27552 for the all diagonals – all higher than any city we have found. However, in terms of maximum values, we find much lower values than those occurring in cities: 1.36564 for the 60x60, 1.51366 for the single pair of diagonals, and 1.39547 for the all diagonals. Even the single pair of diagonals is not enough to create the kinds of highest values we typically find in cities.

It is not much more informative if we compare the values we find in cities with regular square grids with the same number of segments. Mathematically, the maximum value of NACH in such a system goes to a limit of 1.5 as the number of segments, k , goes to infinity. In real systems, a maximum value *below* 1.5 is quite rare. Perhaps a little more unexpectedly, mean NACH converges on max NACH as the square grid expands, and so also goes to a limit of 1.5 as k goes to infinity. But this is only the case for systems well above the size of the planet. A system as big as 400 Tokyos, for example, would have a mean NACH of just over 1.4 and a maximum of 1.427, and no real system has a mean NACH as high as 1.2, let alone 1.5, or a maximum below 1.4. This divergence in contrary directions means that the comparison of real systems with regular theoretical grids is uninformative, except to warn us that something very ungrid-like is going on in real systems!

If we change the shape of the regular grid, however, we do discover an important principle. If we hold the size of the system steady and elongate the grid, we find a slight increase in mean NACH, but a stronger increase in max NACH – so a strong increase in the difference between mean and max. If we then extend the maximally linear form to infinity, the maximum value of NACH goes to 2, and the mean to 1.667. These values are again only found in unrealistically large systems, but they do warn us that one of the ways that high maximum values can be created is by linearisation, and we should be aware of this when looking at such long thin systems as Manhattan. Here, however, the max NACH value is comparatively low, although the mean is high, and the highest maximum values tend to be found in systems like Tokyo, Chicago and London which are manifestly not linear. Yet this may itself be an important clue. Although few large cities are linear to any significant degree, there is a sense in which the foreground grid (in the syntactic sense – see

Hillier 2012b and c) where the highest values occur could be regarded as in a sense linear, or 1-dimensional, in contrast to the 2-dimensional background network. But here we need only note that the fact that mean and maximum values diverge so strongly and in opposite directions in real and theoretical systems, effectively forbids using theoretical grids as a reference system.

How then can we explain why real systems have mean values so much lower than regular theoretical systems, and maximum values so much higher? Intuitively, there would seem to be two ways to create high NACH values in systems: creating additional links by such techniques as a small number of diagonals; and cutting links so as to divert movement to other sequences of segments. From the fact that, compared to grids, we find markedly lower mean NACH values and markedly higher maximum values, it is clear that both are being used to create the spatial structure of the city. *Cities in effect seem to sacrifice mean NACH to create the pattern of high values that we call the structure of the system.* This structure is of course the dual grid with its foreground and background networks, with the near linear geometry of the former and the more right-angled geometry of the latter. The dual foreground and background networks that seem to be found in most cities in fact interdepend: the restriction of the background grid is part of the means by which the strength of the foreground grid is created. In principle then, by understanding how the pattern of low and high values can be created, we seem to stumble upon one of the mathematical principles that lie behind the generic structures of cities.

Theoretically, NACH therefore gives us new insights into the spatial nature of cities. However, there are also great practical advantages in being able to compare numerical values across cities, rather than just colours, which are relative to cities. We can quickly see, for example, that in London the 1.6+ values are all in the centre, clustered along

Oxford Street, while in Tokyo they are at the edge, making lateral links. In Santiago the 1.6+ segments are along the Alameda-Providencia alignment through the Plaza de Italia, and the early stages of the linear route south from the Plaza de Armas. The highest value we have found is in Barcelona, not in fact the *Diagonal*, but the east-west route that skirts the top of the old city and intersects with the *Diagonal* to its east. These aspects are discussed at greater length below.

Local analysis

What then about local analysis? Does the fact that NACH controls for size allow us also to make comparisons across radii, where a crucial factor is that different radii will define systems of different

sizes? First let us examine the behaviour of NACH at local radii.

Table 1 plots the mean and maximum NACH for the 50 cities at radii from 500 metres to n . With the exception of 500 metres, the mean consistently decreases slightly with higher radius, perhaps reflecting the fact that a lower radius is more likely to define a more completely urbanised system. At the same time, the maximum consistently increases slightly with higher radius, perhaps reflecting the fact that within the smaller areas defined by lower radii there is less scope to develop consistently high values. In both cases, the differences seem then to reflect real properties of the system, but at the same time, the values are small in comparison with the differences between the minima, maxima and means, and so are close enough to each other to

TABLE 1

Descriptive Statistics							
Row exclusion: CitiesDataTable_40_completed_120612.svd							
	Mean	Std. Dev.	Std. Error	Count	Minimum	Maximum	# Missing
NACH_500_Mean	.958	.066	.009	49	.831	1.101	0
NACH_1000_Mean	.985	.074	.011	49	.809	1.149	0
NACH_2000_Mean	.980	.086	.012	49	.783	1.172	0
NACH_3000_Mean	.971	.090	.013	49	.769	1.177	0
NACH_5000_Mean	.958	.092	.013	49	.760	1.180	0
NACH_n_Mean	.919	.097	.014	49	.757	1.187	0

TABLE 2

Correlation Matrix						
Row exclusion: CitiesDataTable_40_completed_120612.svd						
	NACH_500_Mean	NACH_1000_Mean	NACH_2000_Mean	NACH_3000_Mean	NACH_5000_Mean	NACH_n_Mean
NACH_500_Mean	1.000	.844	.804	.792	.780	.753
NACH_1000_Mean	.844	1.000	.986	.977	.962	.866
NACH_2000_Mean	.804	.986	1.000	.998	.990	.900
NACH_3000_Mean	.792	.977	.998	1.000	.997	.917
NACH_5000_Mean	.780	.962	.990	.997	1.000	.939
NACH_n_Mean	.753	.866	.900	.917	.939	1.000

49 observations were used in this computation.

permit direct comparison of radii within and across cases, provided care is taken to account for the less consistent behaviour of systems at a radius of less than a kilometre. This table of means could also of course be used as a reference point when assessing the values for individual systems.

Table 2 is a correlation matrix for NACH at radii from 500 metres to n . Remembering that these are r values rather than r -squared, so the 'real' correlations will be substantially lower (for example, .753 will be .567), it is clear that in the mid range from 2000 to 5000 metres, the pattern of values is very similar, and it is safe to use radius 2000, which day-to-day analysis often shows as a significant radius, as fairly representative of the mid range between local and global.

Local analysis is then made much more powerful by the fact that NACH is relatively independent of size. For example, we can compare NACH across radii, and identify the radius at which the values of key segments – say a local shopping street - are maximised, as well as the range of radii where values are maintained at a high level. The key technique may be a plot of the NACH values for a segment or a system across a range of radii, and analytic experience is already suggesting that it may be possible to find general thresholds for different types of activity to establish themselves in urban grids. For example, shops do not seem to begin to group below a peak radius value of about 1.2, while a peak of 1.3 seems to be associated with continuous shops. A peak of 1.4 seems to be a significant local centre, while 1.5 is likely to be a main centre. Being able to compare the numbers is the critical asset.

What about integration?

What then of integration? How can segment angular integration be normalised? Without the complication of the 'paradox of depth', as found in choice, the question is a simpler one. Again the key lies in the

justified graph. If we imagine the j-graph of an urban segment, and count the accumulated segments in the graph as the radius (in segments) is increased, the increase will follow a certain slope. Yang, in his work on the spatial definition of urban areas, had already proposed that points of inflection in the slope represent spatial discontinuities in the urban network which are useful in identifying spatial areas (Yang and Hillier, 2007). But later work on the current project also shows the general background increase in node count approximates a rate of $NC^{1.2}$. This suggests that if we simply calculate $TD/NC^{1.2}$, and take the reciprocal, $NC^{1.2}/TD$ to give higher values for more integration, then we are normalising integration by comparing the system we have to the urban average. We can call this measure 'normalised angular integration', or NAIN. Mathematical details are given in Appendix 2.

As before, we can test this in two ways. Firstly, in Figure 9a we plot the mean angular depth of systems against size in numbers of segments. We find a marked, though by no means strong, tendency for mean depth to increase as size increases with an r-square of .215. However, if we substitute NAIN for mean depth, as in Figure 9b, the increase with size all but disappears. We then test the measure for its ability to predict movement, this time comparing it to the current angular integration measure in Depthmap. This is found by re-dividing mean angular depth by node count (so emulating the *relative asymmetry* measure in *The Social Logic of Space*), and taking the reciprocal to have high values for high integration. We find that in all four areas taken separately, the r-square for NAIN approximates that for the existing integration measure (.83, .684, .549 and .537), but that when we put all four systems together, with their differently sized reference systems, NAIN puts the systems more or less on the same regression line, while the existing measure does not (Figure 10a and b). NAIN then passes its two tests well.

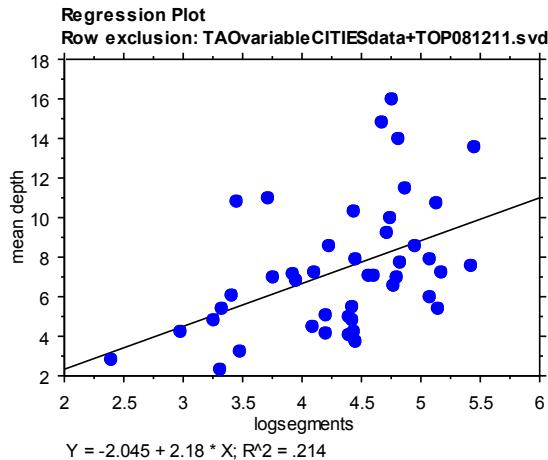


Figure 9a:

The correlation between mean depth and log of number of segments for a sample of whole cities.

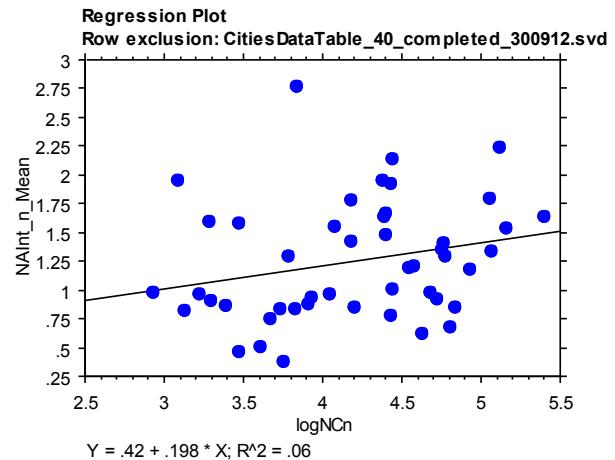


Figure 9b:

The correlation between NAintegration and log of number of segments for a sample of whole cities.

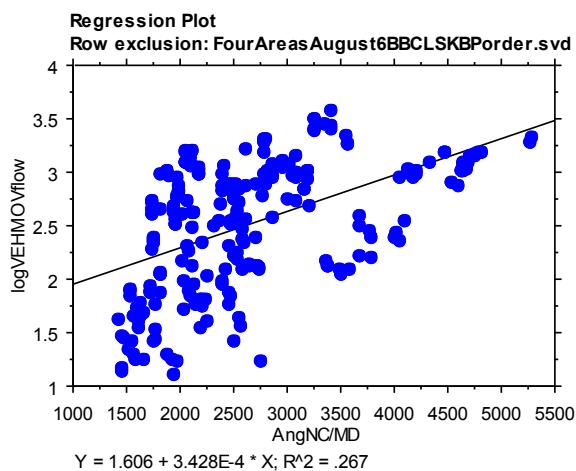


Figure 10a:

The correlation between current Depthmap integration and vehicular movement for our four areas.

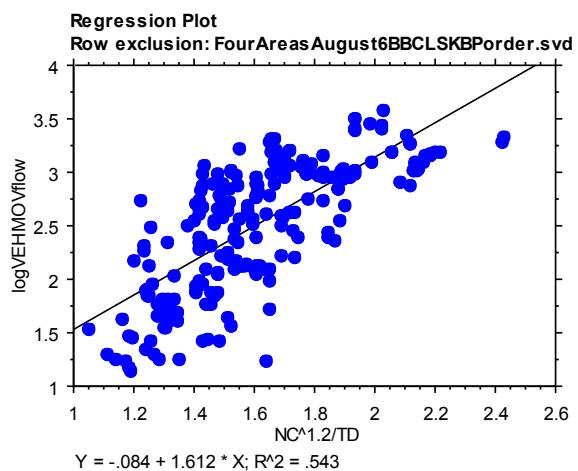


Figure 10b:

The correlation between NAintegration and vehicular movement for our four areas.

TABLE 3

Descriptive Statistics							
	Mean	Std. Dev.	Std. Error	Count	Minimum	Maximum	# Missing
NAInt_500_Mean	1.321	.266	.039	47	.676	1.862	0
NAInt_1000_Mean	1.256	.323	.047	47	.585	2.071	0
NAInt_2000_Mean	1.229	.377	.055	47	.445	2.316	0
NAInt_3000_Mean	1.226	.406	.059	47	.402	2.436	0
NAInt_5000_Mean	1.230	.439	.064	47	.377	2.568	0
NAInt_n_Mean	1.250	.512	.075	47	.375	2.764	0

TABLE 4

Correlation Matrix						
	NAInt_500_Mean	NAInt_1000_Mean	NAInt_2000_Mean	NAInt_3000_Mean	NAInt_5000_Mean	NAInt_n_Mean
NAInt_500_Mean	1.000	.975	.931	.905	.876	.792
NAInt_1000_Mean	.975	1.000	.986	.971	.949	.869
NAInt_2000_Mean	.931	.986	1.000	.996	.985	.918
NAInt_3000_Mean	.905	.971	.996	1.000	.995	.940
NAInt_5000_Mean	.876	.949	.985	.995	1.000	.965
NAInt_n_Mean	.792	.869	.918	.940	.965	1.000

Looking at the behaviour of NAIN under radius, Table 3 shows that means are remarkably stable, and maxima increase with radius, more steeply than NACH but perhaps for the same reasons. Table 4 shows the correlations between radii, with a similar pattern to NACH.

Comparing streets and cities

With these two new normalisations of the basic syntax measures, we hope we have shown that we can indeed not only compare cities, but streets in one city with streets in others. We can even compare a street with a city, since the numbers indexing both share the same scale and mean the same thing. It allows us to see things that were not visible to the naked eye before.

However, of greater significance is what the measures tell us about the structures of cities in general, and how we might use them to learn more. We first note that the degree of difference between the means of the two normalised measures, as in Figure 11a, and the maxima, as in Figure 11b, are enough to show that they are measuring different things. But far more striking are the differences between the means and maxima within the two measures. For NAIN, the mean is a pretty reliable predictor of the maximum (Figure 12a), but for NACH (Figure 12b) this is not the case. On the contrary, mean and maximum are more or less independent of each other. For the reasons given earlier, we believe that the maximum is an indicator of the degree of *structure* in a system, and that this allows us to distinguish

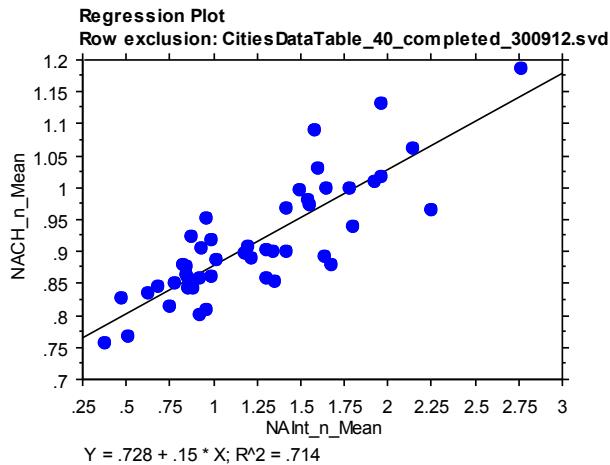


Figure 11a:

The correlation between mean NAIN and mean NACH for cities.

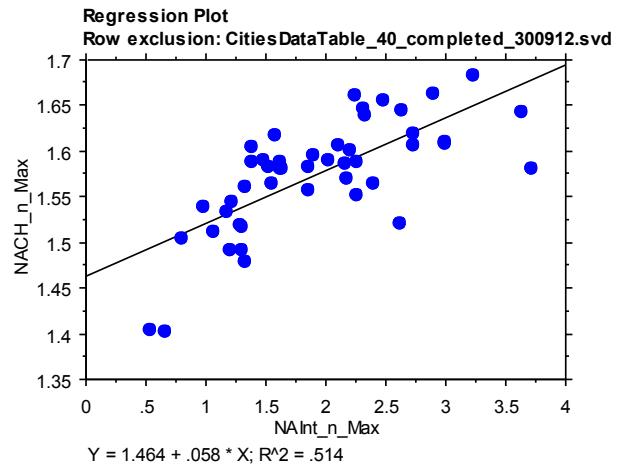


Figure 11b:

The correlation between max NAIN and max NACH for cities.

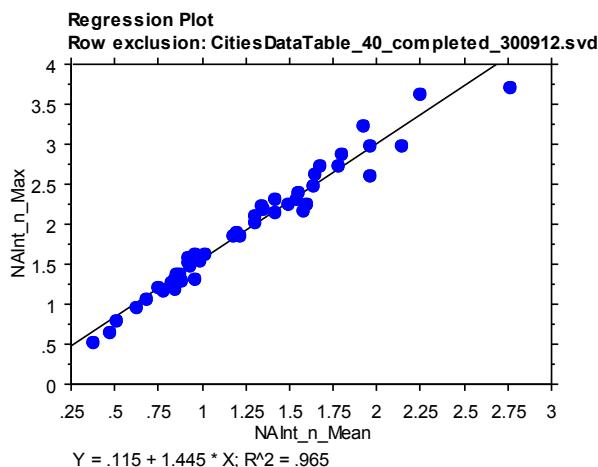


Figure 12a:

The correlation between mean and max NAIN for cities.

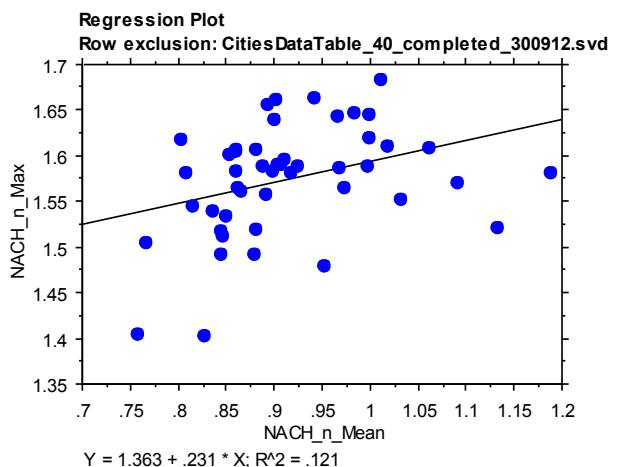


Figure 12b:

The correlation between mean and max NACH for cities.

between cities which have a high maximum and so are highly structured, and those with high means, reflecting grid-like properties but not necessarily in association with high structure. For example, cities like Mexico City (central areas) and Denver have a high mean but are comparatively low in structure, while Tokyo and London are relatively low on mean due to the comparative lack of geometric regularity but are very high on structure. Barcelona and Chicago are unusual in being high in both. We believe that these are very fundamental differences in the spatial structure of cities, and we explore these ideas at greater length below.

But a word of warning. NACH means and maxima make most sense for continuous urban systems. If you expand the city to include partially unurbanised regions, for example expanding London to the M25, then global values will in general be rather lower, reflecting the lack of development in some areas. This will also lead to problems in local analysis of areas, in particular in the peripheral

parts of cities where urbanisation, in the sense of a foreground and background network of spaces, does not yet exist. This problem is dealt with in more detail in Appendix 1.

However, there is an unexpected benefit! As in Figure 13, the new measures, especially NACH, produce some of the prettiest pictures yet, and in our experience this is at its best if the blue threshold in Depthmap is set at .8 and the red at 1.4. This also, of course, facilitates comparisons between cities.

3. Comparative analysis of cities

New images of cities

Now we have the ability to compare cities based on normalised variables, we would like to suggest a new technique for seeing cities in terms of these variables *all at once*, and at the same time to explore what the variables mean in terms of urban spatial structure. We call them *star models*.

Figure 13:

The NACH pattern of Barcelona.



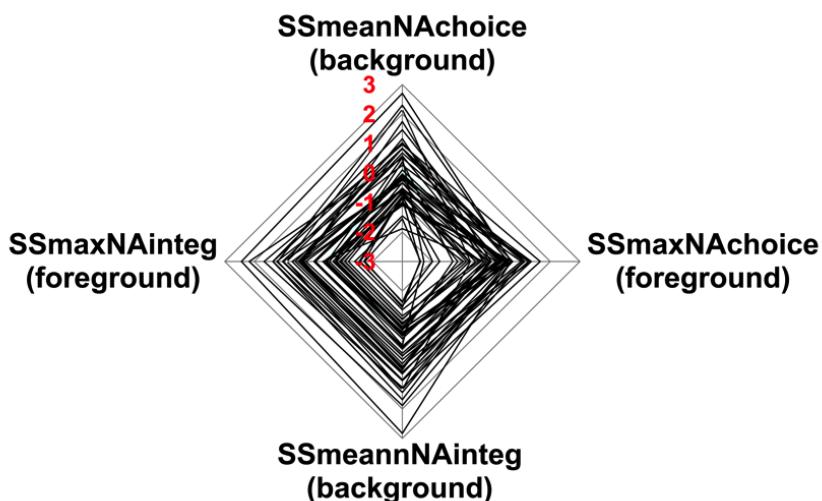


Figure 14:

A four-pointed star model of 50 cities.

Figure 14 is a four-pointed star model of 50 cities. The high and low points on the vertical axis are the mean NACH (top) and mean NAIN (bottom) at radius n for each city, and the left and right points on the horizontal axis are their maximum NACH (right) and maximum NAIN (left). Each measure is a standard score (so on the same scale) varying about 0, with the negative minimum at the centre and the positive maximum at the edge. Because the great majority of spaces in cities are in the *background* network, the means represent the *to-* and *through-movement* potential of the *background* network (areas can be integrated without allowing much *through-movement*), and the maxima represent the *to-* and *through-movement* potentials of the *foreground* network. The mean and max NAIN measures tend to co-vary, so the angle of the line

between them remains fairly constant, while the mean and max NACH measures do not co-vary, and show radically different patterns from city to city.

So mean and max NAIN show the ease of accessibility in the foreground (max) and background (mean) networks in the usual syntactic sense, while mean and max NACH index the degree of *structure* in the system: the mean NACH the degree to which the background network forms a continuous grid with direct connections, rather than being broken up into discontinuous sub-areas; while max NACH represents the degree to which the foreground grid *structures* the system by deformations and interruptions of the grid. We can now use these concepts and the star model to discuss the global forms of individual cities. In what follows, we confine our attention to the radius n values.

Figures 15a and b show the star images for Manhattan and Chicago, the two highest cities on NAIN. In Manhattan, mean NACH is also the highest in the sample, but on max NACH it is surprisingly weak, with no more than an average value. This could be partly explained by the comparatively small size of the system, but this seems unlikely to be the whole explanation. There is no theoretical reason why a system of this size should not have a much higher value, and in fact much smaller systems (for example, the City of London in 1676) do have substantially higher values. In fact, the only value above 1.58 is the section of Broadway that skirts the south-west corner of Central Park, and then at 1.57 or above is a further mid-town section of Broadway either side of

the intersection with 42nd Street, and the sections of Fifth Avenue below and above the intersection with Broadway. 1.56 then adds the whole of Fifth Avenue below Central Park, the downtown continuation of Broadway, and two other north-south alignments. But above 1.5, we find seven other north-south alignments. The system is then strongly integrated, including the highest mean NACH in the sample, but is not strongly differentiated in terms of structure, and the advantage of Broadway and Fifth Avenue is relatively mild. So the system is very strong on movement in the background network, but relatively weakly structured in the foreground network. The numbers say much more than the colours. We will see below what this might mean in urban terms.

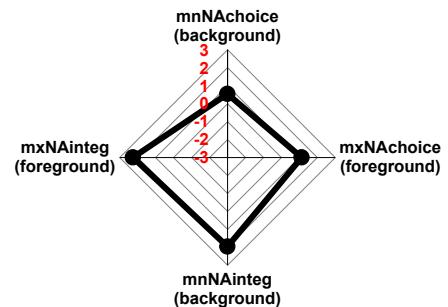
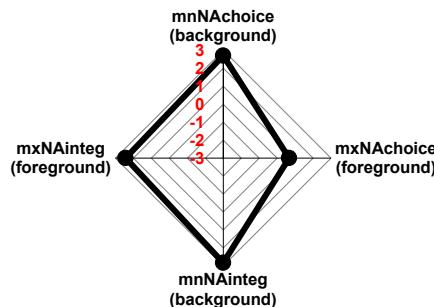
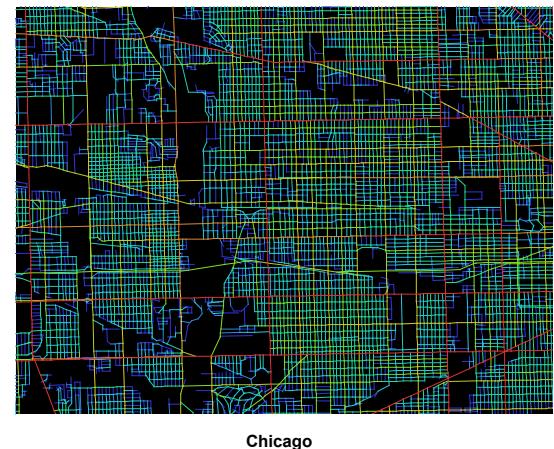
Figure 15

15a (left):

The star model of Manhattan.

15b (right):

The star model of Chicago.



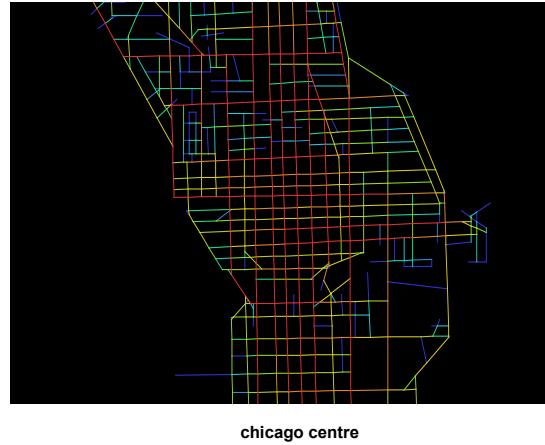
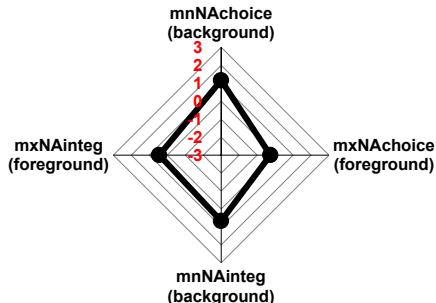
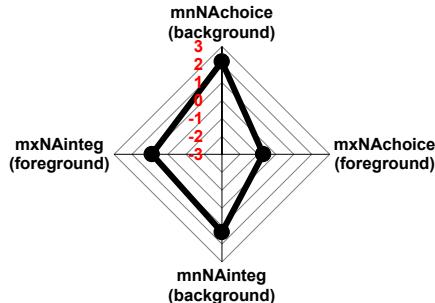


Figure 15

15c (left):
The star model of Chicago Centre.

15d (right):
The star model of Charleston.



In contrast, Chicago, which is second only to Manhattan in mean and maximum integration, is much weaker on mean NACH (15th in the sample), though much stronger on max NACH. The reasons for both can be seen on the map. Discontinuities between areas lead to much weaker area-to-area connections in the background network, so that area-to-area relations depend on a strong foreground network; again this can be seen on the map, which goes above 1.65. Remember, we are now comparing Manhattan Island, (6296 segments), with the whole of Chicago (136,988 segments), and finding they can be structurally compared.

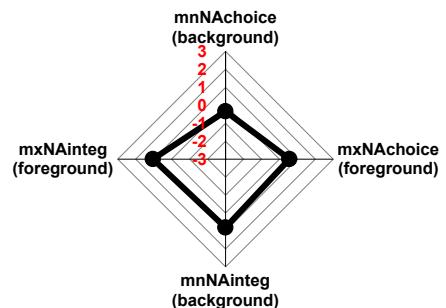
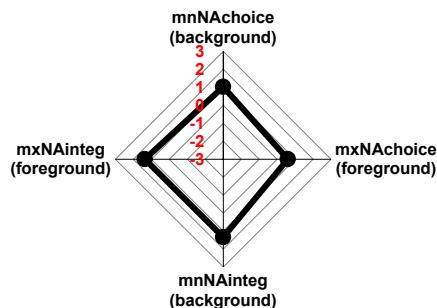
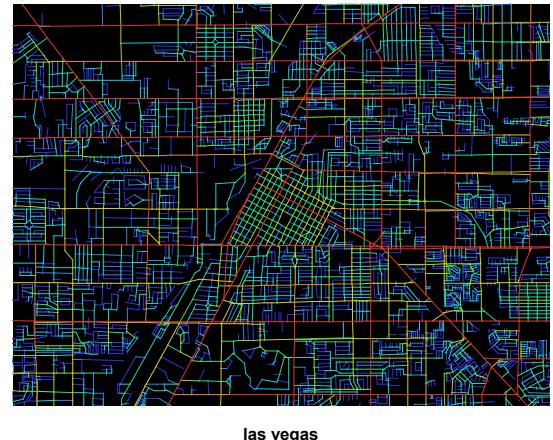
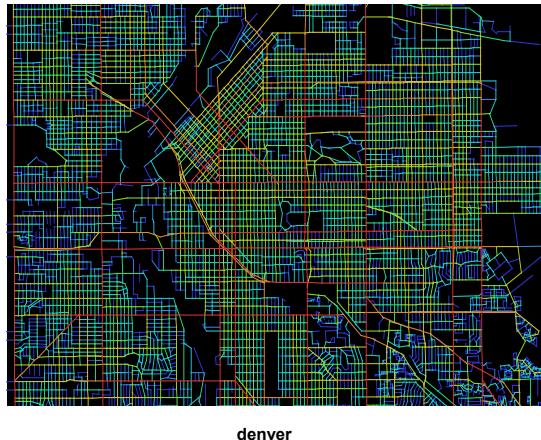
However, if we look just at the centre of Chicago in isolation from the rest of the system, (Figure 15c), we find less integration than in Manhattan but a very similar structure: very high mean NACH in the background grid and relatively weak structure in the foreground grid – although Michigan Avenue is correctly highlighted as the strongest alignment. Perhaps it is only with growth that this ‘democratic’ pattern of a strong background and weakly differentiated foreground is changed. This possibility is supported by Charleston (Figure 15d), which again is less integrated than the centre of Chicago, but has the same pattern of strong background and weak foreground.

Figure 16**16a (left):**

The star model of Denver.

16b (right):

The star model of Las Vegas.

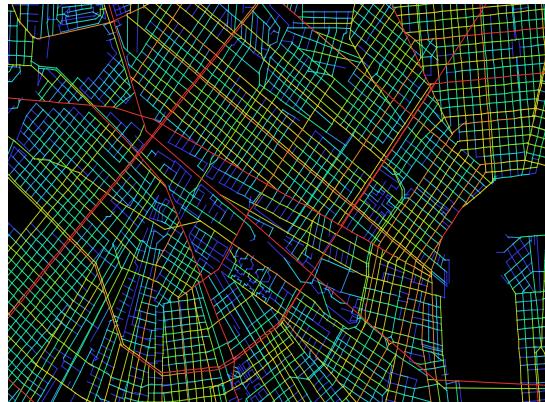


Back to large-scale cities, Figures 16a, b, c and d respectively illustrate the star models of Denver, Las Vegas, Atlanta and New Orleans. Denver is less integrated than Chicago, but reverses its structural priorities; its background mean NACH networks are a little stronger than Chicago's, as can be seen in the continuity of yellow lines in the background grid, but the foreground network is a great deal weaker at the whole city scale. Las Vegas is less again in-

tegrated than Denver, and you can easily see how strongly the background network is fragmented into the areas defined by the foreground grid. The background network of Las Vegas is in fact weaker than any of the previous examples, and while its foreground network is no stronger than Denver's, it is much stronger than its background network. In a sense, the foreground network seems to segregate the background network.



atlanta



new orleans

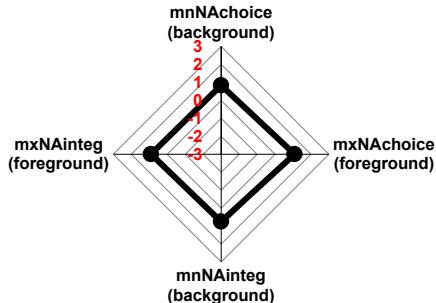
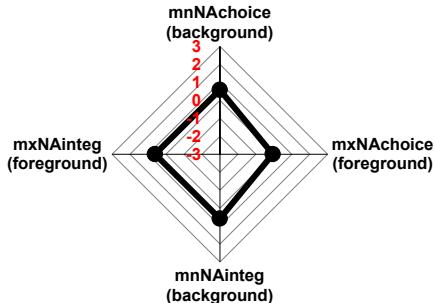


Figure 16

16c (left):

The star model of Atlanta.

16d (right):

The star model of New Orleans.

Atlanta is less integrated than any of the previous cities and has a weaker foreground structure that is compensated by a stronger background structure, comparable to, though less obvious than, Denver. New Orleans is only a little more integrated than Atlanta, but both its background and foreground structures are stronger than the other American

cases apart from Chicago. In fact, New Orleans has the sixth highest foreground structure in the sample, and the ninth highest background structure. The star shape begins to approximate a diamond.

Leaving the USA, but remaining with the theme of how different grid cities can be compared to each other, Beijing (Figure 17a) is less integrated than

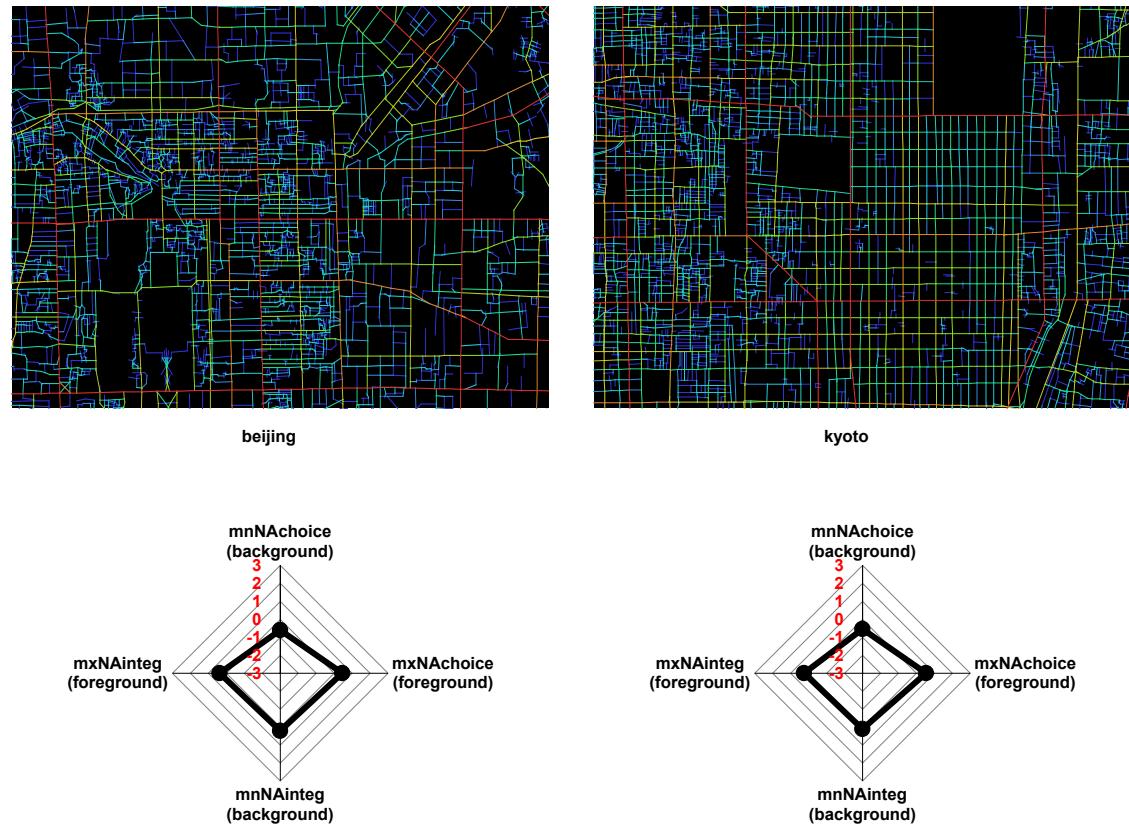
Figure 17

17a (left):

The star model of Beijing.

17b (right):

The star model of Kyoto.



any of the American cities, and relatively low on both background and foreground structure. Beijing has a relatively uniform grid, but in contrast to Manhattan, the background areas defined by the grid are comparatively isolated from each other by the foreground grid, giving a strikingly low background value compared to Manhattan. Kyoto (Figure 17b) is numerically very similar to Beijing, though with

different grid structures in different areas: a uniform small-scale central grid, a larger-scale grid away from the centre, and a more organic peripheral grid. However, the principles of relative weak background and foreground structure remain consistent, though both are relatively stronger on foreground than background.

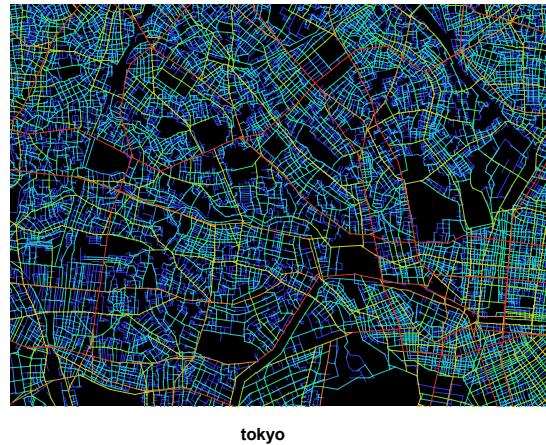
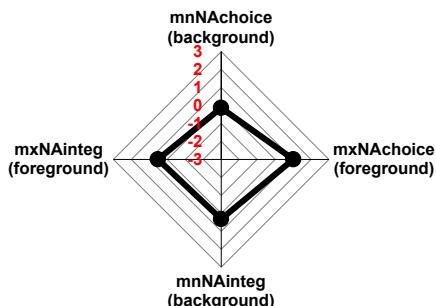
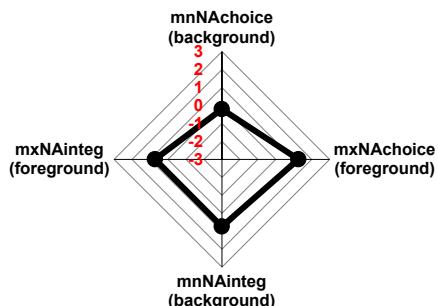


Figure 18

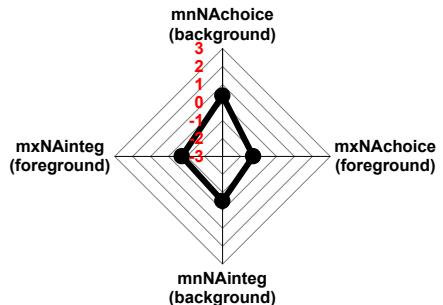
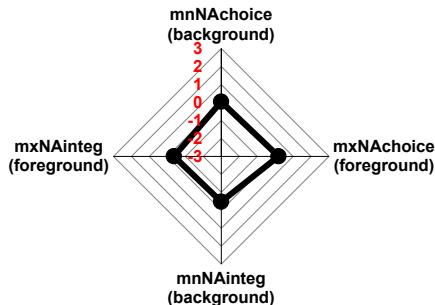
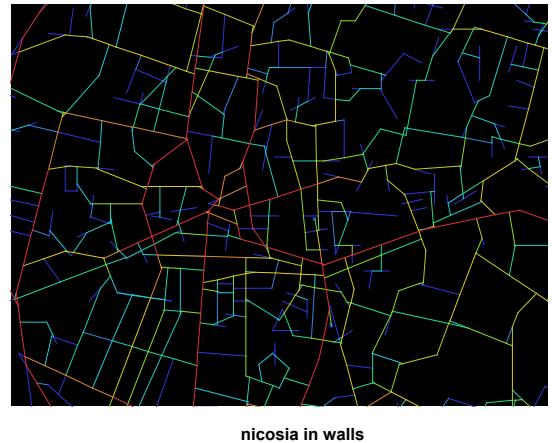
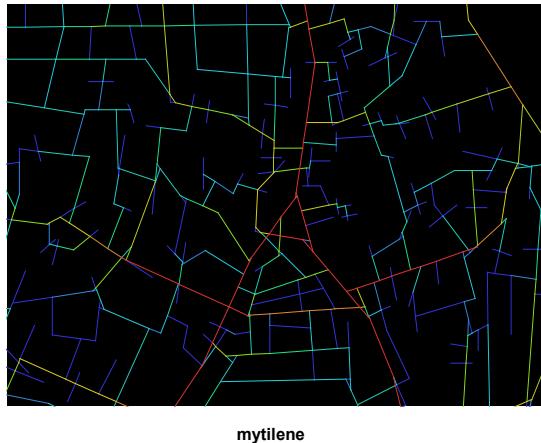
18a (left):
The star model of Tokyo.

18b (right):
The star model of London.



Figures 18a and b then show two irregular grids, Tokyo and London (within the North and South Circular roads, the smaller of our two versions of London in the database). Tokyo is our first predominantly organic grid and also our second largest, with 250,892 segments - over ten times as many as New Orleans. In spite of its lack of regular grid geometry, it has higher NAIN than either Beijing or Kyoto, and, more remarkably, the fourth highest

maximum NACH in the sample, as well as a higher background value than either Beijing or Kyoto. So we see that neither measures of integration or structure are dependent on resemblance to a geometrically regular grid. It is about how things are connected! This is confirmed by London which has a very similar numerical pattern to Tokyo, including the eighth highest value in the sample on foreground grid structure.

Figure 19**19a (left):***The star model of Mytilene.***19b (right):***The star model of Nicosia.*

Turning to much smaller cities and towns, again we find structural variation. Mytilene (Figure 19a), for example, is stronger on choice than integration, and has stronger foreground than background structure. Nicosia (Figure 19b) reverses this, with a much stronger background than foreground struc-

ture. But this conceals the fact that if we take the north-east Turkish quarter on its own, we find one of the lowest background structures in the sample, while if we take the south-west Greek quarter, we find one of the highest.

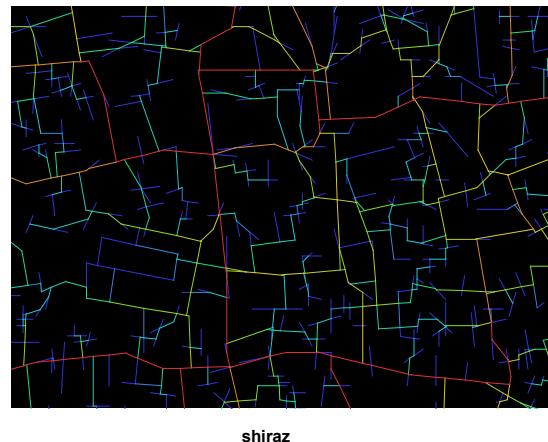
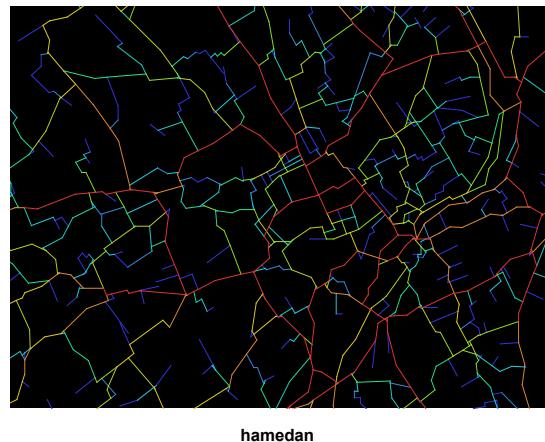


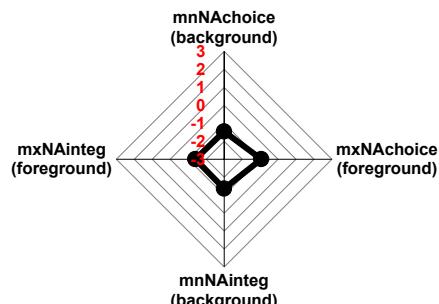
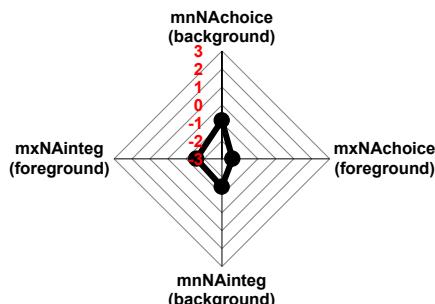
Figure 19

19c (left):

The star model of Hamedan.

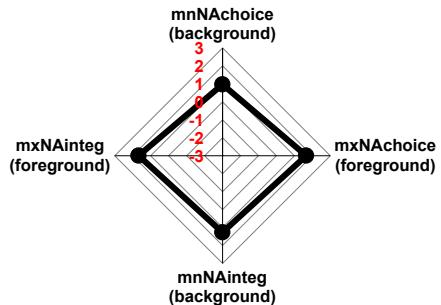
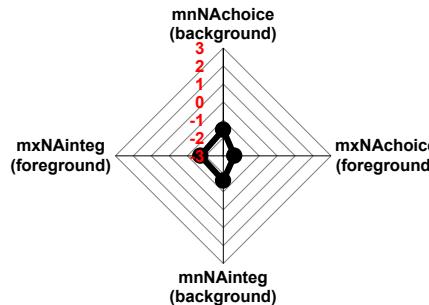
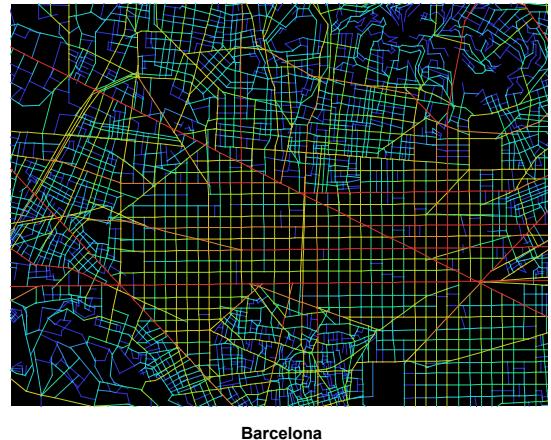
19d (right):

The star model of Shiraz.



So what are the findings concerning Near-Eastern cities, which earlier studies have shown to be much less connected and integrated than Western cities, with shorter lines and higher angle changes. We find Hamedan (Figure 19c) much less integrated, and with a much weaker foreground structure

than any city we have seen so far, but with a much stronger background structure. Shiraz (Figure 19d) is equally low on integration but also on background structure, and in contrast is relatively strong in foreground structure. Near-Eastern cities, then, are just as structurally variable as Western cities.

Figure 20**20a (left):***The star model of Venice.***20b (right):***The star model of Barcelona.*

What are the extreme cases? Venice (Figure 20a) is one, with lower NAIN even than the Near-Eastern cities, and the lowest NACH in both the foreground and background structure. This reflects the fact that Venice has local urbanity in its spaces but lacks either local-to-global or global structure, probably because this was historically provided by the canals (although this can be tested). At the other

extreme is Barcelona (Figure 20b), a near perfect diamond shape, with high values on all variables. The highest value NACH spaces in the foreground structure are not in fact in the *Diagonal*, but in the east-west route passing above the old city and intersecting with the *Diagonal* to the east. It may be that to some extent the high values are affected by the relatively linear form of the city, but this is not clear.

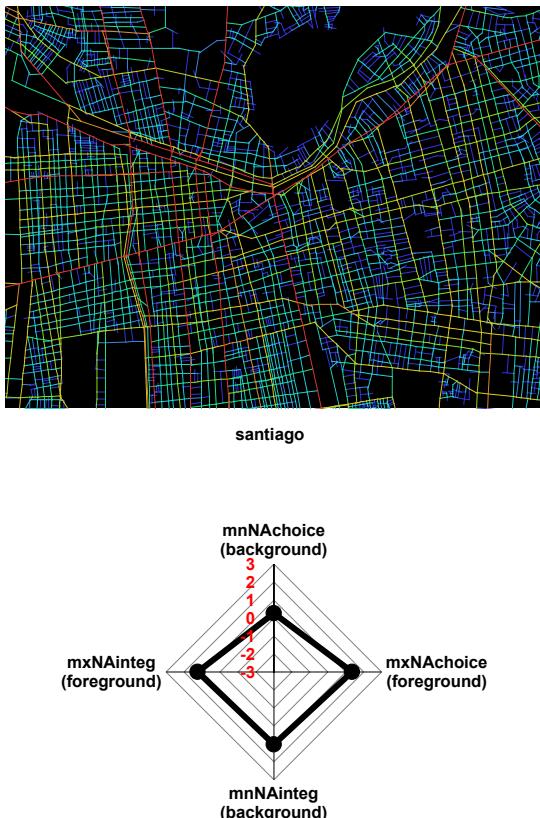


Figure 21: The star model of Santiago.

What then about Santiago? Santiago is second of the cities in foreground structure, reflecting the double centre, with both parts lying on the same east-west alignment. But it is only 18th in background structure, because away from the central areas are relatively large background areas where there is relatively little through-movement potential. However, if you take the area of the historic centre, based on the Spanish grid, we find the background structure rises to over 1, presenting one of the highest values in the sample, and if you take the parts of Providencia adjacent to the main shopping streets, this rises even higher.

4. The global structure of cities

The numbers in the star model then begin to tell us something of the global differences between cities spatially. But what of structure in terms of how the network is connected up? Can the numbers be used to give a more detailed account of structure? Historically, the principal way of describing structure in the topological line map was the ‘integration core’, defined as a percentage of the most integrated lines, say 25% or 10%, or as a percentage of the total value of integration in the system. There is no reason why this should not have been done with choice, although it rarely has been, perhaps in view of the huge numbers involved (using log choice makes the numbers more tractable), but in the main because in the absence of a normalisation, the numbers were so obviously incomparable¹.

With the choice measure normalised, a powerful new possibility appears, allowing us to compare structures across cases. We can specify a value and ask what structure a city has at and above this value. Experience so far suggests that 1.5 (segments having a value of 1.5 or more) and 1.4 are the most interesting and informative systems: 1.5 identifies a dominant global structure, and 1.4 extends this to how it is related to more local organisation. Where it exists, the 1.6 system identifies the choice centre of the system, but little else, though this can itself be highly informative. In what follows, we describe the structure of a series of cities, selected for their interest in these terms, at values from 1.6 to 1.3. Highlighted in the maps, however, are always the 1.4 maps so they can be directly compared. (Figure 22)

What then do we mean by structure? The more powerful and complex representations of the city made available by least angle segment analysis have not so far led to new concepts of global structure, apart from the description of choice patterns as networks, with greater reach throughout the system than the more compressed, deformed wheel structures formed by least angle integration.

Notes:

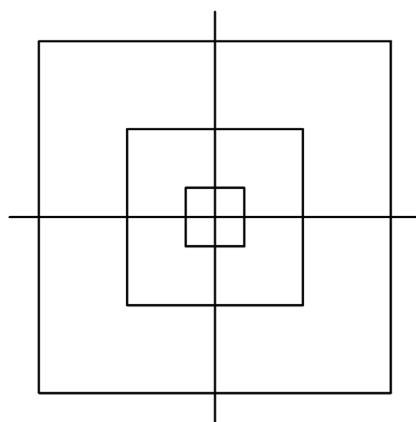
¹ An exception to this was the early suggestion by Peponis et al. (1990) that the choice measure could be used to divide the urban system up into natural areas, a suggestion which the results below indicate may not have yet been exhausted, although less in terms of dividing the city into local areas than in creating global access to local areas.

But if we think of global structure in terms of how parts of the city are accessible to each other and to the world outside, analytic experience increasingly suggests that three main factors are involved: radial structures to link the centre of the city to its edges and to the outside; lateral structures to link the sectors of the city to each other away from the centre; and a ring structure at some scale to make local areas accessible to each other and to the radial and lateral structures. Radials and laterals can of course themselves create a ring system, although probably at a fairly large scale.

We can think of these properties as a kind of *ideal axial geometry* for an urban system, an approximation of which is required for the effective functioning of the system. Without the radials, the centre cannot interact well with the periphery; without the laterals, the parts cannot interact with each other; and without a relatively small-scale rings system, local areas will often be destination spaces rather than spaces to pass through. These properties are configurational in the sense that they reflect and generate function and its primary component, movement, but at the same time they impose a global geometric structure on the city. It is reasonable then to present these properties in the form of an idealised geometric diagram:

Figure 23:

*The geometric model
of an urban system.*



With this in mind, we may begin by exploring a singularly interesting case: Tokyo, our second largest system (Figure 22 - as are all the following cities). The 1.6 system of Tokyo is a single alignment entirely on the periphery of the system, covering around 60% of the urban edge. The 1.5 (which of course includes the 1.6 system) then adds to this a wholly centralised radial system, stronger in the north and east than the south and west, which with one exception does not link to the 1.6 system, and does not form any kind of ring system. The 1.4 system, as the image shows, then completes both the peripheral system (apart from the south east) and the radial system, and creates a multi-level lateral system, at different distances from the centre, and covering almost the whole urban area. This creates a fairly large-scale ring system, forming a frame for local structure, and a denser ring system in the centre. A pattern of smaller scale rings then appears in the 1.3 grid, covering most areas and forming an even denser ring system in the centre.

As with Tokyo, the 1.6 structure of London, Figure 22, (within the North and South Circular roads – the M25 system, our largest system in terms of segment numbers contains large unbuilt areas and so is not fully comparable numerically, but follows the same pattern) forms a single line. However, in complete contrast to Tokyo, this is entirely in the centre: the Oxford Street line between the intersection with Edgware Road and Holborn Circus. The 1.5 grid is then simply a tree-like radial structure, reaching in all directions, but no more than halfway to the edge of the system, and with two large, but asymmetric rings in the centre. The 1.4 grid then further extends the radial structure, both by reaching further outwards and by creating divergent branches, some of which later converge to form asymmetric rings. These convergences seem a distinctive property of the London radial system, and the asymmetry of the rings created means that it does not constitute a useful lateral structure. It is

Figure 22:

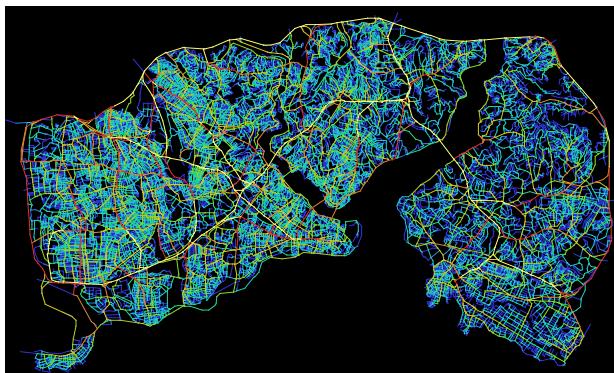
1.4 structures for 12 cities.



TOKYO 1.4



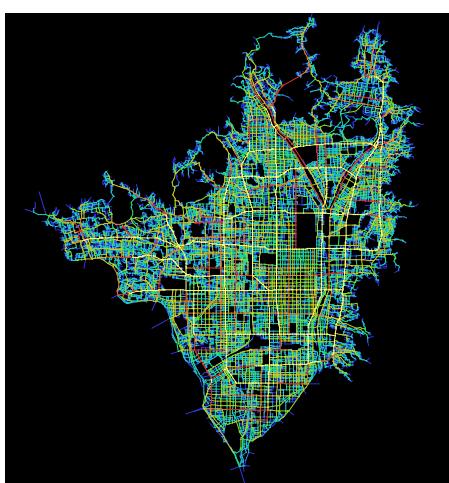
LONDON 1.4



ISTANBUL 1.4



BEIJING 1.4



KYOTO 1.4



SUZHOU 1.4

only with the 1.3 structure that a reasonably scaled ring system appears in the centre, although the rings tend to remain asymmetric, and away from the centre even the 1.3 structure is largely an extension of the tree-like form, with occasional convergence of branches. The contrast is remarkable, with London developing a purely radial structure with few lateral connections, and Tokyo being much stronger on lateral and local ring development. While London is exclusively centralised – and perhaps this is how it acquires the 1.6 structure in the centre of the system – Tokyo is a much more distributed system, with significantly more potential for development away from the centre, especially where the radials meet the laterals.

If we take another very large organic system, Istanbul (Figure 22), we find again a completely different pattern. The 1.6 structure is a short alignment either side of the northernmost bridge crossing the Golden Horn, and the 1.5 structure is, apart from a short spur going north from west of the Golden Horn, an extension of this single line almost to the limits of the urban system in the west and crossing the southern bridge over the Bosphorus in the east, reaching deep into the Asian side of the city. The 1.4 structure then links this alignment to the northern boundary of the city, creating a single very large ring linked to long tree-like elements. The 1.3 structure then continues the pattern of very large rings and tree-like extensions leading into, but not through local areas. Even at 1.2, the local system is more tree-like than ring-like. Again, the contrast with both London and Tokyo is striking.

Moving away from organic cities, in Beijing (Figure 22) the only 1.6 structure occurs where the fourth ring road intersects with Chang An Avenue (and the new CBD is planned). The 1.5 structure is then made up of the western halves of the third and fourth ring roads together with the central part of Chang An Avenue, but also two central north-south roads and two east-west, including the main east-

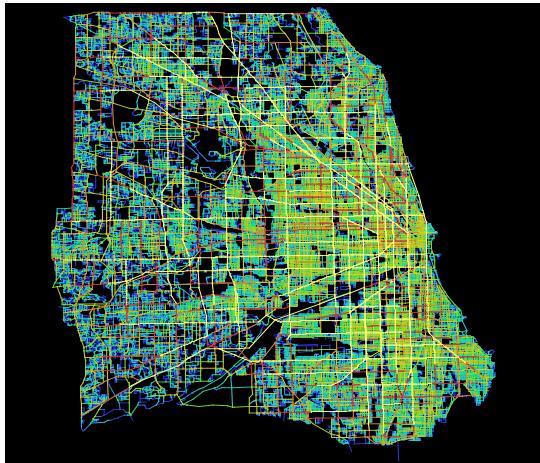
west route through the more organic parts of the city south of Tiananmen Square – so a strongly lateral, but weakly radial structure. With the 1.4 grid, the lateral ring road structure continues to dominate, but with radials now appearing which link the ring roads outwards rather than the centre. A central grid structure is also beginning to appear. The 1.3 grid then consolidates this structure, with radials reaching the edge (though rarely the centre), and a small-scale grid structure appears around the centre.

In Kyoto (Figure 22), which resembles Beijing both numerically and in being based on a regular grid, the 1.6 structure is, in contrast to Beijing, a section of the main shopping street which runs east-west across the whole city. The 1.5 structure then extends this line to most of its length, and adds two rings to the north, with most of the lines forming the rings reaching out as radials towards the edges of the system. In contrast to any of the other cities we have looked at, the 1.5 structure in Kyoto is almost a covering structure for the whole system. The 1.4 structure then extends this by creating a denser grid, with radials reaching towards the edges of the system. The 1.3 system then adds to this a smaller scale ring structure.

In contrast to both of these ancient grid cities, Suzhou (Figure 22), also an ancient grid city but one which has grown spectacularly in the last two decades, has no 1.6 structure, and the 1.5 structure is relatively restricted, focusing on the central cross axis of the historic geometric grid and extending into parts of the ring road running through the ‘extended city’ that surrounds the historic grid, but not reaching the much larger areas that have come into being with the recent rapid expansion of the city. The 1.4 grid then largely confirms and consolidates this centralised structure, with only limited linear links into the new parts of the city. The effective confinement of the structure to the central extended city strongly confirms the local planners’ view of the city as having become ‘five islands’ with the rapid

Figure 22:

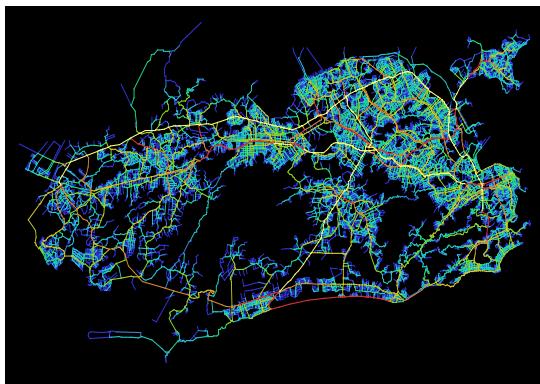
1.4 structures for 12 cities.



CHICAGO 1.4



DENVER 1.4



RIO DE JANEIRO 1.4



SANTIAGO 1.4



ATHENS 1.4



BARCELONA 1.4

recent growth. It is only with the 1.3 structure that the four outer islands begin to develop significant internal structure.

It is interesting to compare these Eastern grid cities with an American one: Chicago (Figure 22). The 1.6 system in Chicago is made up of five discontinuous alignments, two east-west, pointing to, but not reaching, the centre (which of course is at the eastern edge of the system by Lake Michigan), two north-south passing fairly close to, but not touching the centre, and one diagonal north-west, again pointing to, but not reaching the centre. At 1.5, the structure develops all the alignments to form a T-shape on its side, with the intersection adjacent to the centre. The 1.4 structure then includes the main street in the centre (Michigan Avenue), though nothing else in the centre, and adds radials and laterals covering about two thirds of the system, with some degree of small grid creation close to the centre. The 1.3 structure then extends this to cover virtually the whole system with a radial, lateral and small grid system, focused on the central region, but only weakly present in the centre itself.

In Denver (Figure 22), on the other hand, which like Chicago has a single overall grid (apart from the offset centre), the 1.6 system is only the central section of Colefax, the dominant east-west axis running from the plains in the east to the Rockies in the west. The 1.5 grid extends this to cover most of Colefax, and attaches to it a regular grid to the south, covering most of the system, but with only relatively short lines pointing north. It also includes two lines in the offset historic centre including the main street, which attaches to the corner of the main grid of the city. The 1.4 map then extends the south grid, but with very little to the north, suggesting a considerable spatial inequality between the parts of Denver north and south of Colefax. The links from the centre to the north, but not the south, suggest, apart from the main street, a centre which operates more as a destination than a through-movement complex.

Moving to South America, the broken up structure of Rio de Janeiro (Figure 22) has no 1.6 structure, and the 1.5 structure is a single line running east-west in the northern part of the city, linking relatively discontinuous parts of the city together. The 1.4 structure extends this alignment and adds other lines, forming links between the discrete parts of the city, taking the form of a large-scale tree-like pattern with no ring development, and not reaching the historic city. It is only with the 1.3 structure that rings begin to form in the main area, and a single line reaches the main street of the historic city.

Santiago (Figure 22) is a very different story. Its substantial 1.6 structure (including the second highest maximum in the sample) forms a T-shape immediately adjacent to the historic centre, and including east-west alignments forming the two main active centres of the city, as well as a line linking into the historic city, and two lines linking southwards where the bulk of recent development has taken place. The 1.5 structure then extends this, but also adds further radial alignments and laterals so that large ring structures begin to form away from the centre. The 1.4 grid then shows a structure with strong radials and laterals and an overall deformed wheel shape covering most of the system, with large rings in the south but not east and west. The 1.3 structure then consolidates this and creates a fairly small ring system in most parts of the city.

Turning to Europe, in Athens (Figure 22), which is locally geometric but globally organic, and which has the fifth highest maximum NACH in the sample, the substantial 1.6 structure is a north-south line on the east side of the city, passing close to the centre before turning south east towards the southern port of Piraeus. The 1.5 grid then turns this into a multi-directional radial structure focused on the historic centre, where it forms a group of connected near rings. The 1.4 then develops this into a structure with strong radials, fairly strong laterals and some degree of ring development in the centre, although

little growing outwards. The 1.3 structure then creates this pattern across virtually the whole system.

In Barcelona (Figure 22), which has the highest maximum (1.68) in the whole sample, the 1.6 grid is again quite substantial; however, unlike Santiago or Chicago, this does not lead to strong structure in all the senses we have discussed. Dominant is the east-west alignment passing just above the old city and extending well into the eastern part, although a small part of the main diagonal is also included along with two smaller diagonals, and a small part of an east-west line parallel to and north of the main east-west line. The 1.5 then simply extends and consolidates this structure, so retaining a tree-like structure, with only a single ring north of the main point of intersection, and only one short line penetrating into the old city. The 1.4 structure essentially consolidates this structure, adding mainly edge lines and some organic structure in the north, but still with only one line going into, but not through, the old city. We can say that Barcelona has global, but not global-to-local structure.

5. Reflections

Putting the numbers and structures together, we can see that geometrical grids are no less differentiated from each other than they are from organic grids. On the one hand, it seems reasonable to confirm intuition and regard Manhattan as a spatially democratic grid, given both the high mean NACH value of the background grid (first in the database), and the low maximum (29th), pointing to weak differentiation in that the advantages of Broadway and Fifth Avenue in the network are marginal compared to Oxford Street or Alameda-Providencia (the main east-west route in Santiago). Another way of saying this, in view of the consistent pattern of high values, might be that the whole grid tends to the foreground, and the whole system is economically driven, with the means to this being the strength of the grid rather than particular lines.

On the other hand, a geometric grid can also be associated with a low mean NACH value in the background network, as is the case with both Beijing and Kyoto, 37th and 34th respectively on this measure. In both of these cases, the geometric grid, in association with the complex internal structure and paucity of entrances to the internal spaces defined by the grid squares, tends to divide areas from each other and create a clear hierarchy between the foreground and background grid. At the same time, both share Manhattan's relatively low value for structure in the foreground network (18th and 16th respectively). On spatial grounds then, it is perhaps reasonable to see such grids as reflecting the spatialisation of top down social order, rather than economic activity.

More generally, we can say that while geometric grids are often – although, as we have just seen, with striking exceptions – associated with a strong background network, they are not well represented among the high foreground networks. These seem most often to be found, first, where cities have a strong geometric order, but not an overall geometrical form, such as Santiago and Athens. These cities suggest a process of creation which, while not conceptualised 'all at once' as an overarching geometric order, are nonetheless guided in their evolution by step-by-step geometrical reasoning at the level of the area. Santiago and Athens are moderate on background network (17th and 12th) but very strong on foreground network, where they are second and fifth. Both also have NACH core structures which are strong on radiality, laterality and global-to-local ring formation. American cities with many grids offset to each other, such as New Orleans, also seem to fall into this category.

Second, we can find strong foreground structure in 'organic' cities. It has been argued elsewhere that the apparent lack of geometry in such systems as Tokyo and London is superficial (Hillier, 1999; 2012). The foreground and background networks

that characterise such cities have consistent, and consistently different, geometric and metric forms. What makes this geometry hard to see is that it lies in the relations between lines, not at the level of the area. This could reflect a creative process in which growth is led linearly by the creation of the foreground grid, and the background grid then fills in the interstitial areas (although Bloomsbury, the location of this university, is an exception!) This foreground line-led process of growth could in itself be the simple reason for the strength of the foreground network and the less strong background network, and at the same time for the close interrelation between microeconomic activity and the residential network – the urban village phenomenon. So again, growth is geometrically informed but in this case at the level of the line, not the area or the urban whole.

A related analysis might explain Istanbul. Local analysis of Istanbul shows that it has local area structure, but what is missing is global-to-local structure; that is, a global structure which reaches down into the local areas, and this likely could have occurred if growth was not led by the foreground structure but by the background structure, as could be expected where the economic logic of the foreground network was subordinated to the social logic of the background residential process. The similar foreground structure of Rio de Janeiro might be explained by the same logic of a rapidly developing residential process.

In view of the different priorities that cities seem to give to the foreground and background networks, it is interesting to compare them to each other based on the ratio between the two by dividing the mean into the maximum. A high value will mean a strong structure in the foreground compared to a weaker background network, and a low value is one which prioritises the background over the foreground, as Manhattan does. Remembering that these values do not reflect either background or foreground values, only the relation between them, it is striking

that Chicago, at ninth, is the only American city in the top dozen in a list dominated by cities without an overarching geometry, such as Santiago, Barcelona, Tokyo, London and Athens, as well as Kyoto and Beijing (though in these cases the high values come from the low values in the background network, rather than the high values in the foreground network). At the other end of the scale, the order is dominated by American systems, albeit with the smaller ones like Manhattan and Chicago centre in the top places, but also with Atlanta and Denver among six American cities in the top ten. It can then reasonably be said that there is a bias towards the background grid rather than the foreground grid in American cities, a tendency which in clear cases such as Manhattan we identified as spatially and economically democratic.

Could there be a typology of cities? Suggestions for dimensions of variability

It is clear that the fundamental differences between city structures we have brought to light must arise from no less fundamental differences in the constructive processes that over decades or centuries caused them to emerge in their current forms. Simply reading back from the emerged patterns to what the generative processes must have been, three factors seem to stand out.

The first is the scale at which geometry, and hence deliberate thought, is applied to the growing system. In our analyses, we have identified three levels: the level of the whole city, as in Chicago or Beijing; the level of the area, as in Athens and Santiago; and the level of the line, as in London and Tokyo. We should perhaps go further and not only abandon the long-standing distinction between geometric and organic cities, but see it as a continuum in which geometry is applied to the growing city somewhere between the whole city and the line. The second is the aspects of the system to which geometry is first and foremost applied; that is the

foreground or the background network. The third is the reason why it is so applied; that is, whether the reasoning is led by economic or social factors.

To take paradigm cases, we might then suggest that in Manhattan, the city scale spatial geometry is applied to the background network to maximise its value and minimise the difference between it and the foreground network, so spatially equalising economic opportunity and minimising the differences between the foreground and background networks. The top down spatial geometry thus aims to universalise bottom up economic opportunity. This generates the strong background network and the weakly structured foreground network. The strength of Manhattan is in the grid as a whole, not its constituent parts.

London and Tokyo are also economically driven but apply geometric thought at the level of the foreground line, creating a strong emergent foreground structure at the cost of a relatively weak background structure. If Manhattan uses a top down spatial strategy to create spatial equality, London and Tokyo use a bottom up geometric strategy to create spatial difference, with a strongly hierarchical foreground system creating a dense pattern of centres and sub-centres closely related to, but distinct from, the residential background. Beijing and Kyoto use a top down whole city geometric intervention to create spatial dominance of the background network by the foreground, so social rather than economic reasoning dominates. In Istanbul, geometry is applied bottom up to the background network, leaving it locally structured but relatively unrelated to any global structure. So Manhattan, London and Tokyo are economically driven: Manhattan top down, London and Tokyo bottom up. Beijing, Kyoto and Istanbul are socially driven: Beijing and Kyoto top down, Istanbul bottom up. The scaling of geometry, and the focus on foreground or background networks, follow whichever logic is driving the system.

These are not all original interpretations, of course. On the contrary, they tend to confirm what is commonly said. But by clarifying the spatial aspects we are perhaps able to distinguish what might otherwise appear contradictory or paradoxical: the very different structures and functions associated with seemingly homogenous forms such as regular grids and organic patterns. In this sense, we have aimed to follow the space syntax principle to use numbers and structures to clarify what we mean.

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About the authors:

Bill Hillier (b.hillier@ucl.ac.uk), is Professor of Architectural and Urban Morphology in the University of London, Chairman of the Bartlett School of Graduate Studies and Director of the Space Syntax Laboratory in University College London. He holds a DSc (higher doctorate) in the University of London.

As the original pioneer of the methods for the analysis of spatial patterns known as 'space syntax', he is the author of *The Social Logic of Space* (Cambridge University Press, 1984, 1990) which presents a general theory of how people relate to space in built environments, *Space is the Machine* (CUP, 1996), which reports a substantial body of research built on that theory, and a large number of articles concerned with different aspects of space and how it works. He has also written extensively on other aspects of the theory of architecture.

Tao Yang (t.yang@spacsyntax.com) qualified as an architect and urban designer in China. He is studying for a PhD at University College London and joined Space Syntax as a Research Consultant in September 2011.

His research interests focus on urban morphology and digital city modelling. He has been involved in several research projects, including 'Multi-scale Spatial Planning – a Proof of Concept Study for the Suzhou Region', led by Prof Bill Hillier, 2008-2009. 'City History and Multi-scale Spatial Master-planning – International Research Network', led by Prof Alan Penn, 2007-2010 and Road

Centre Line Modeling conducted by Space Syntax, 2005-2006. He has also participated in several consultancy projects for Space Syntax including Arundel House, the Foresterhill Masterplan and the Seville Masterplan.

Alasdair Turner was the Bartlett's Reader in Architectural and Urban Computing. He founded the MSc AAC in 2005, and was course director until 2010. He co-directed the EngD VEIV Centre with Prof Anthony Steed in Computer Science, and is the Bartlett MRes Tutor.

Alasdair researched models of people movement and social interaction, spatial analysis, generative design and neural network models of human behaviour. His main interest was to combine the study of social interaction with the generation of spatial relationships through 'ecomorphic synthesis'; that is, the creation of embodied environments which are co-evolved around people, space and activity, which he developed within the course through the morphogenetic programming module.

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Appendix 1: A note on problems

There are two problems in applying NACH to cities, both of which can be dealt with in a comparatively easy way – in one case leading to a very nice new representation of the city. The first has to do with applying low radius measures at or towards the edge of systems, or in other areas where urbanisation is incomplete. The partial development found in these areas can lead to the creation of small, often linear clusters, which have relatively little angular depth. Figure 24 is such a cluster in Rio de Janeiro. The longer red segment has a NACHr2000 value of 1.49, which would normally be associated with a busy shopping street. While this is a perfectly correct value, and may be genuinely representative of the potential of the space as further development takes place, it is clearly unrealistic in terms of comparison with other segments with similar values within this fully developed urban system.

In fact, the value is the direct product of lack of local urban development and the relative isolation of the local system. The segment has a real angular choice value within the two kilometre system of 64, compared with an average of 2878 for Rio as a whole, and a total angular depth value of 13, compared to an average of 1081 for the whole system. The reason for the lack of angular depth is therefore the lack of 2-dimensional development around the segment. The simplest way of dealing with this is by dividing the system into its fully and incompletely urbanised parts, and since we are dealing with the metrically localised systems defined by the low radius, this can be done by simply eliminating from the system those segments which have node counts at that a radius of less than a fraction of the average for the system at that radius. Experiments suggest that 20% is a reasonable approximation, so in this case the segment has a node count of 15 within two kilometres, compared to a mean of 363 for Rio

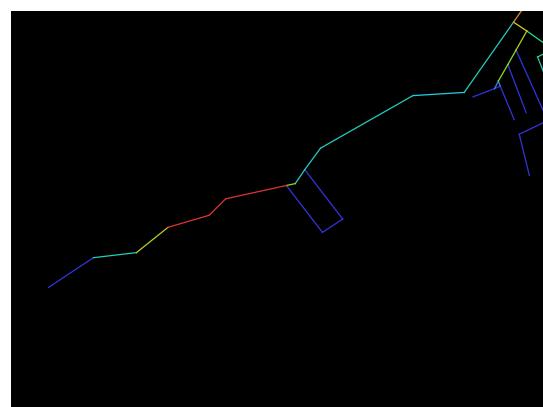


Figure 24:

A small, linear cluster in Rio de Janeiro.

as a whole, which is well below the threshold and so can be eliminated from the system. In fact, the useful thing to do is to leave light representations of the missing segments, so we can see clearly how the system divides itself into the urbanised and unurbanised parts. Figure 25 is such a map of Hamburg which serves the useful purpose of making the distinction in the degree of development of different parts of the city clear. We can then lift this restriction when seeking to examine the potential for development of the incompletely urbanised parts.

The second problem is related and arises mainly when road centre lines are used as the basis for the segment map. This will commonly lead to long segments, particularly on freeways, being divided into a large number of sub-segments linearly connected with little angular change, so giving rise to unrealistically high values at low radii, for the same reasons as in the previous case. This problem can be eliminated by reducing the segment map to its real segments defined by angular change, using a procedure which will be available on the Space Syntax Limited website.

Figure 25:

Map of Hamburg.



Appendix 2 Bill Hillier, Tao Yang

In the paper, we seek to normalise choice by dividing it by total depth:

$$CH_norm_r = \frac{CH_r}{TD_r} \quad (1)$$

where, CH_norm_r denotes normalised choice at radius of r , CH_r indicates choice at r and TD_r means total depth at r . CH_r can be thought to measure a kind of benefit, that is, the possibility that a person standing at a space can be encountered by other persons passing through that space, but need not use energy to go to meet the others at other spaces; and TD_r can be seen as the cost of travelling to all other spaces. To some extent, CH_r over TD_r can be interpreted as the spatial benefit-cost ratio.

We then focus on normalising angular choice by the following equation:

$$NAchoice_r = \frac{\log(ACH_r + 1)}{\log(ATD_r + 3)} \quad (2)$$

where, $NAchoice_r$ denotes normalised angular choice at metric radius of r , ACH_r indicates angular choice at r and ATD_r means angular total depth at r . However, ACH_r will be zero if a segment is a dead end; but to take the logarithm of zero is meaningless. And meanwhile, ATD_r will be -1 given by the Depth-map, if a segment is too long to be selected by the radius of r ; but to take the reciprocal of the logarithm of (-1+2) is also meaningless. Therefore, respectively adding the constants of 1 and 3 to ACH_r and ATD_r can avoid meaningless calculations.

Another equation is used to normalise angular total depth:

$$NAtd_r = \frac{ATD_r}{(NC_r + 2)^{1.2}} \quad (3)$$

where, NAtd_r denotes the normalised angular total depth at metric radius of r, ATD_r indicates angular total depth at r, and NC_r means node count at r. The reciprocal of NAtd can be seen as normalised angular integration (NAintegration). However, NC_r will be -1 given by the Depthmap, if a segment is too long to be selected by the radius of r; but to take the reciprocal of (-1+1) is meaningless. Thus, adding 2 to NC_r can avoid meaningless calculation.

Why do we use the different ways of normalising choice and total depth? The empirical studies suggest that these two variables behave differently with an increase of node count. Based on the case studies, both ACH_r and ATD_r of most individual segments have a power-law relation with NC_r, but their power-law exponents vary within different ranges (Figure 1).

Their power-law relations can be respectively expressed as below:

$$ACH_r = K \times (NC_r)^a \quad (4)$$

where, 'a' is a power-law exponent parameter and K is a scale parameter.

$$ATD_r = M \times (NC_r)^b \quad (5)$$

where, 'b' is a power-law exponent parameter and M is a scale parameter.

For example, in the London case, the power-law exponent 'a' varies from 0.204 to 2.300, with a mean of 1.251; however, the power-law exponent 'b' fluctuates from 1.07 to 1.38, with a mean of 1.19. This suggests that compared to ATD_r, ACH_r tends to be distributed within a much wider range, as NC_r rises.

As the mean 'b' of nearly all the segments of six cities (London, Denver, Beijing, Shanghai, Amsterdam and Chicago) is 0.19 (close to 1.2), ATD_r can be considered to fluctuate around a standard trajectory described by $M \times (NC_r)^{1.2}$, shown in Figure 1a. We can use that standard trajectory ($M \times (NC_r)^{1.2}$) to approximate all the ATD_r values. To a large extent, the differences between the ATD_r values can be assessed by the scale parameter M; and meanwhile, the scale parameter M can be approximated by dividing ATD_r by $(NC_r)^{1.2}$. This explains why we empirically use the equation (3) to normalise ATD_r.

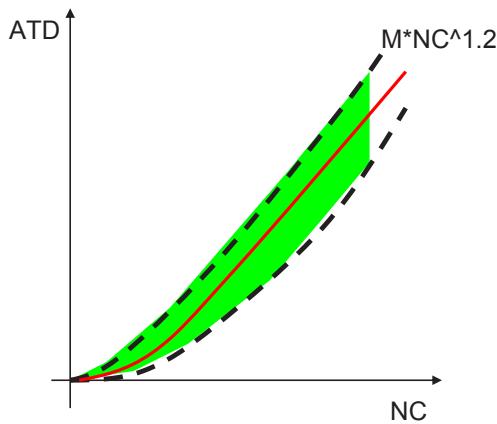
However, the ACH_r values of most of the segments can be approximated by $K \times (NC_r)^a$, as Figure 1b illustrates. The exponent parameter of 'a' affects the value of CH_r much more significantly than the scale parameter of K, because, for example, $(NC_r)^2$ is usually much larger than

$22.38 \times (NC_r)$ (maximum K of 22.38 in the London case). Thus, we can select a standard form with ACH_r of $(NC_r)^a$ (assuming K=1), and compare ACH_r of each segment with ACH_r of that standard form. Thus, we could use the equation of

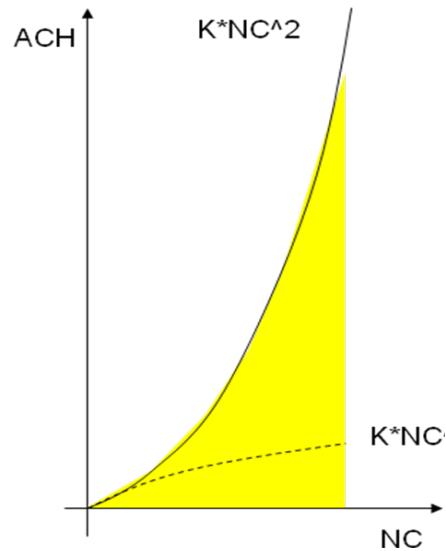
$$\frac{\log(ACH_r + 1)}{\log NC_r}$$

to approximate 'a', and then compare 'a' across different systems and individual segments. This offers one way of normalising ACH.

Since ATD_r varies within a relatively narrow range and fluctuates around $(NC_r)^{1.2}$, $\log(ATD_r + 1)$ can be approximated by $\log NC_r$. This gives another reason why we empirically use the equation (2) to calculate NAchoice.

**Appendix 2 _ Figure 1a:**

ATD_r values fluctuate around a standard trajectory described by $M \times (NC_r)^{1.2}$, so that those values can be approximated by $M \times (NC_r)^{1.2}$. To a large extent, the differences between the ATD_r values can be approximated by the scale parameter M .

**Appendix 2 _ Figure 1b:**

ACH_r values are distributed within the yellow area, and the vertical edge of the yellow area will become wider and wider, with an increase of NC_r . As for most of segments, ACH_r values can be approximated by the trajectories expressed by $K \times (NC_r)^a$. Compared to NC_r , K is usually much smaller. Thus, we can compare ACH_r with a standard form with the ACH_r value of $(NC_r)^a$.