Temperature-dependent molecular absorption cross sections for exoplanets and other atmospheres

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Abstract

Exoplanets, and in particular hot ones such as hot Jupiters, require a very significant quantities of molecular spectroscopic data to model radiative transport in their atmospheres or to interpret their spectra. This data is commonly provided in the form of very extensive transition line lists. The size of these line lists is such that constructing a single model may require the consideration of several billion lines. We present a procedure to simplify this process based on the use of cross sections. Line lists for water, H$_3^+$, HCN /HNC and ammonia have been turned into cross sections on a fine enough grid to preserve their spectroscopic features. Cross sections are provided at a fixed range of temperatures and an interpolation procedure which can be used to generate cross sections at arbitrary temperatures is described. A web-based interface (www.exomol.com/xsec) has been developed to allow astronomers to download cross sections at specified temperatures and spectral resolution. Specific examples are presented for the key water molecule.

Keywords: Atmospheres, composition, Extra-solar planets, Infrared observations, Radiative transfer
1. Introduction

With the growing realization that many, probably most, stars support exoplanets, developing the means to systematically characterize the atmospheres of these planets has become a major scientific priority (Tinetti et al., 2012). Given the likely complex chemistry of these atmospheres and the elevated temperature that is found in the most observable planets, there is a significant demand for spectroscopic data on the probable exoplanet atmospheric constituents.

Recently we have launched a new project, called ExoMol (see www.exomol.com), with the aim of providing molecular transition data appropriate for exoplanet models which is reliable over a wide range of temperatures (Tennyson and Yurchenko, 2012). The ExoMol project involves constructing line lists of spectroscopic transitions for key molecules which are valid over the entire temperature and wavelength domain that is likely to be astrophysically important for these species. Especially for polyatomic molecules, these line lists can become very large with hundreds of millions (Harris et al., 2006; Barber et al., 2006; Voronin et al., 2010; Tashkun and Perevalov, 2011) or even billions (Yurchenko et al., 2011) of individual transitions needing to characterized and stored. A complete linelist for methane, for which so far only a preliminary version is available (Warmbier et al., 2009), can be expected to be even larger. Indeed potential line lists for larger species, such as higher hydrocarbons, for which spectroscopic data is needed for exoplanetary research, are likely to be so large as to potentially make their use impractical.

Molecular line lists are being actively used to model the spectra of exoplanets (e.g. Beaulieu et al., 2011) and cool brown dwarfs with similar tem-
peratures (e.g. Lucas et al. (2010); Cushing et al. (2011)). However, sampling billions of individual transitions to model relatively low resolution astronomical spectra is probably not necessary in many cases. An alternative approach is to represent the molecular absorptions as cross sections generated at an appropriate resolution and temperature. The advantage of this approach is that the data handling issues related to dealing with large data sets largely disappear. The disadvantage is that cross sections are inflexible - a particular cross section set is only valid for a single state of temperature and pressure. Cross sections are therefore often regarded a second choice compared to maintaining a full line list (Rothman et al. 2009).

In this paper we develop a strategy whereby cross sections are provided for the user in a flexible fashion without loss of accuracy or the specificity of using a complete line list. To this end we have provided a web application which, starting from very high resolution cross sections generated for each molecule at a range of temperatures, can provide cross sections at a temperature and resolution specified by the user. Of course this approach is based on the implicit assumption of local thermodynamic equilibrium (LTE) and any non-LTE treatment will continue to have to rely on the explicit use of transition line lists. So far, these cross sections do not consider collisional broadening effects and are therefore, at their highest resolution, appropriate for the zero pressure limit only.

The line lists for water (Barber et al. 2006; Voronin et al. 2010), \( \text{H}_3^+ \) (Neale et al. 1996; Sochi and Tennyson 2010), HCN /HNC (Harris et al. 2002, 2006, 2008) and ammonia (Yurchenko et al. 2011) were used to generate cross sections for these species. For concreteness, this work uses the
main water isotopologue, H$_2^{16}$O, as its working example. Water is known to be a key species in exoplanetary atmospheres and the BT2 line list has been used in studies of exoplanets (Tinetti et al., 2007; Swain et al., 2009; Tinetti et al., 2010a; Baraffe et al., 2010; Tinetti et al., 2010b; Shabram et al., 2011; Barman et al., 2011; Tessenyi et al., 2012) as well in a large variety of planetary (Bykov et al., 2008; Chesnokova et al., 2009; Bailey, 2009), astrophysical (Warren et al., 2007; Dello Russo et al., 2004, 2005; Burgasser et al., 2008; Barber et al., 2009; Lyubchik et al., 2007; Banerjee et al., 2005) and, indeed, engineering (Kranendonk et al., 2007; Lindermeir and Beier, 2012) studies which generally focus on the radiative transport by hot water. The BT2 line list was used as part of the recently updated HITEMP database (Rothman et al., 2010). In that work the size of the line list was reduced using a technique based upon importance sampling at a range of key temperatures. In practice the number of water lines in HITEMP remains large, over 100 million.

The calculation of opacities and other spectral properties due to water vapour at these elevated temperatures can therefore become onerous, and so we present here pre-calculated absorption cross sections for a range of temperatures between 296 K and 3000 K, binned to different resolutions. The highest resolution cross sections are suitable for modelling low-density environments where only Doppler broadening contributes to the line width whereas by binning to a wavenumber grid spacing significantly larger than the pressure-broadened half-width, higher-density environments are described well by the calculated cross sections. However, no attempt is made to include contributions to the opacity from the water vapour continuum or water
dimer absorption.

2. Method

The high-resolution cross section is calculated on an evenly-spaced wavenumber grid, $\tilde{\nu}_i$, defining bins of width $\Delta \tilde{\nu}$. Only Doppler broadening is considered so each absorption line has a Gaussian shape:

$$f_G(\tilde{\nu}; \tilde{\nu}_{0j}, \alpha_j) = \sqrt{\frac{\ln 2}{\pi}} \frac{1}{\alpha_j} \exp \left( -\frac{(\tilde{\nu} - \tilde{\nu}_{0j})^2 \ln 2}{\alpha_j^2} \right)$$  \hspace{1cm} (1)

where the line centre position is $\tilde{\nu}_{0j}$ and the Doppler half-width at half-maximum,

$$\alpha_j = \sqrt{\frac{2k_B T \ln 2 \tilde{\nu}_{0j}}{m c}},$$  \hspace{1cm} (2)

at temperature $T$ for a molecule of mass $m$.

The contribution to the cross section within each bin is a sum over contributions from individual lines:

$$\sigma_i = \sum_j \sigma_{ij}$$  \hspace{1cm} (3)

where

$$\sigma_{ij} = \frac{S_j}{\Delta \tilde{\nu}} \int_{\tilde{\nu}_i - \Delta \tilde{\nu}/2}^{\tilde{\nu}_i + \Delta \tilde{\nu}/2} f_G(\tilde{\nu}; \tilde{\nu}_{0j}, \alpha_j) d\tilde{\nu}$$  \hspace{1cm} (4)

$$= \frac{S_j}{2\Delta \tilde{\nu}} \left[ \text{erf}(x_{ij}^+) - \text{erf}(x_{ij}^-) \right],$$  \hspace{1cm} (5)

where $\text{erf}$ is the error function and

$$x_{ij}^\pm = \frac{\sqrt{\ln 2}}{\alpha_j} \left[ \tilde{\nu}_i \pm \frac{\Delta \tilde{\nu}}{2} - \tilde{\nu}_{0j} \right]$$  \hspace{1cm} (6)
Figure 1: The calculation of the absorption cross section in a wavenumber bin centered on $\tilde{\nu}_i$ due to a single line. The integrated line intensity within the shaded region, of width $\Delta \tilde{\nu}$, contributes to $\sigma_{ij}$.

are the scaled limits of the wavenumber bin centred on $\tilde{\nu}_i$ relative to the line centre, $\tilde{\nu}_{0,j}$, and the line intensity in units of cm$^{-1}$/molecule cm$^{-2}$ is

$$S_j = \frac{A_j g'_j e^{-c_2 E''_j / T}}{8\pi c \tilde{\nu}_{0,j}^2 Q(T)} \left( 1 - e^{-c_2 \tilde{\nu}_{0,j} / T} \right).$$

Here, $g'_j$ and $E''_j$ are the upper-state degeneracy and lower-state energy respectively, $A_j$ is the Einstein $A$ coefficient for the transition and $c_2 \equiv hc/k_B$ is the second radiation constant. For H$_2^{16}$O, the molecular partition func-
Table 1: Temperatures at which calculated H$_2^{16}$O cross sections are provided.

<table>
<thead>
<tr>
<th>Temperature (K)</th>
<th>296 K</th>
<th>400 K</th>
<th>500 K</th>
<th>600 K</th>
<th>700 K</th>
<th>800 K</th>
<th>900 K</th>
<th>1000 K</th>
<th>1200 K</th>
<th>1300 K</th>
<th>1400 K</th>
<th>1600 K</th>
<th>1800 K</th>
<th>2000 K</th>
<th>2200 K</th>
<th>2400 K</th>
<th>2600 K</th>
<th>2800 K</th>
<th>3000 K</th>
</tr>
</thead>
</table>

The absorption cross section, $Q(T)$, was obtained from the tabulated values of Vidler and Tennyson (2000).

Note that in the limit of $\Delta \tilde{\nu} \gg \alpha_j$, eqn (4) reduces to

$$\sigma_{ij} \approx \frac{S_j}{\Delta \tilde{\nu}} \int_{-\infty}^{+\infty} f_G(\tilde{\nu}; \tilde{\nu}_0; j, \alpha_j) d\tilde{\nu} = \frac{S_j}{\Delta \tilde{\nu}}, \quad (8)$$

whereas for $\Delta \tilde{\nu} \ll \alpha_j$,

$$\sigma_{ij} \approx S_j f_G(\tilde{\nu}_i; \tilde{\nu}_0; j, \alpha_j). \quad (9)$$

However, the exact expression in all calculations of the cross sections presented in this work.

3. Results

The absorption cross section of H$_2^{16}$O was calculated between 10 cm$^{-1}$ and 30000 cm$^{-1}$ across the temperature range 296 K – 3000 K (Table 1), using the wavenumber grid-spacing given in Table 2.

For comparison with experimental spectra, low-resolution cross sections were produced by binning the high-resolution cross sections to the following
Table 2: Summary of the grid spacings, $\Delta \tilde{\nu}$ for the cross sections calculated in different wavenumber regions

<table>
<thead>
<tr>
<th>wavenumber range /cm$^{-1}$</th>
<th>$\Delta \tilde{\nu}$/cm$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 – 100</td>
<td>$10^{-5}$</td>
</tr>
<tr>
<td>100 – 1000</td>
<td>$10^{-4}$</td>
</tr>
<tr>
<td>1000 – 10000</td>
<td>$10^{-3}$</td>
</tr>
<tr>
<td>10000 – 30000</td>
<td>$10^{-2}$</td>
</tr>
</tbody>
</table>

fixed grid spacing across the entire wavenumber range: $\Delta \tilde{\nu} = 0.01, 0.1, 1, 10, 100$ cm$^{-1}$. At these resolutions, the structure due to individual lines is lost and direct comparison can be made with, for example, the experimental water vapour cross sections of the PNNL database (Sharpe et al., 2004). Such a comparison for the $\Delta \tilde{\nu} = 10$ cm$^{-1}$ resolution spectra is shown in Figure 2.

4. Interpolation of cross sections between temperatures

For use in the web-based application described below, cross sections for the molecules given in Table 5 have been calculated using a wavenumber grid spacing of 0.01 cm$^{-1}$ at a range of temperatures in 100 K intervals below 1000 K and 200 K intervals above 1000 K. A cross section at some intermediate temperature between the values at which the stored cross sections have been calculated may be obtained by interpolation. Suppose that $\sigma_i(T_1)$ and $\sigma_i(T_2)$ are calculated cross sections at temperatures which bracket the temperature of the desired cross section: $T_1 < T < T_2$ (we consider interpolation using only $\sigma_i$ calculated at the two temperatures closest to $T$).
One possible interpolation strategy is the linear interpolation

\[ \sigma_i(T) = \sigma_i(T_1) + m(T - T_1), \]  

where \( m = \frac{\sigma_i(T_2) - \sigma_i(T_1)}{T_2 - T_1} \). \( \text{(10)} \)

However, we find a more accurate approach is to estimate the temperature dependence to be of the form

\[ \sigma_i(T) = a_i e^{-b_i/T}, \]  

where the coefficients \( a_i \) and \( b_i \) at each wavenumber bin may be calculated from

\[ b_i = \left( \frac{1}{T_2} - \frac{1}{T_1} \right)^{-1} \ln \frac{\sigma_i(T_1)}{\sigma_i(T_2)} \quad \text{and} \quad a_i = \sigma_i(T_1)e^{b_i/T_1}. \]  

\( \text{(12)} \)

The largest values of the interpolation residual error in the region 1000 - 20000 cm\(^{-1}\), calculated as \( \delta \sigma_i = \sigma_{i,\text{calc}} - \sigma_{i,\text{interp}} \), are found to be associated with the \( \nu_2 \) band - as an illustration, this is plotted in Figure 3 at 350 K. The maximum value of the interpolation residual across this wavenumber region at a range of temperatures and wavenumber binning intervals is given in Table 3 expressed as a percentage of the corresponding absorption cross section:

\[ \delta \sigma_{\text{max}}^\% = \max \left( \frac{|\sigma_{i,\text{calc}} - \sigma_{i,\text{interp}}|}{\sigma_{i,\text{calc}}} \right) \times 100. \]  

\( \text{(13)} \)

In all cases, \( \delta \sigma_{\text{max}}^\% \) is found to be less than the estimated uncertainty in the \textit{ab initio} line intensities that the cross section calculation is based on. Interpolation is performed on the 0.01 cm\(^{-1}\) grid before binning to a coarser wavenumber grid, if required.

Finally we note that Hargreaves et al. (2012) recently presented an experimental ammonia spectrum recorded at a range of temperatures at 100 K
Table 3: Maximum interpolation errors in the H$_2^{16}$O cross section as a function of wave number grid spacing and temperature

<table>
<thead>
<tr>
<th>$\Delta \tilde{\nu}$</th>
<th>0.01 cm$^{-1}$</th>
<th>0.1 cm$^{-1}$</th>
<th>1.0 cm$^{-1}$</th>
<th>10.0 cm$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>350 K</td>
<td>1.64 %</td>
<td>1.34 %</td>
<td>1.07 %</td>
<td>1.10 %</td>
</tr>
<tr>
<td>1100 K</td>
<td>1.00 %</td>
<td>0.50 %</td>
<td>0.49 %</td>
<td>0.46 %</td>
</tr>
<tr>
<td>2500 K</td>
<td>0.66 %</td>
<td>0.40 %</td>
<td>0.37 %</td>
<td>0.36 %</td>
</tr>
</tbody>
</table>

intervals. We suggest that our proposed interpolation scheme would also be appropriate for interpolating their data.

5. Web based application

Calculated absorption cross sections can be obtained from the interface at the url [http://www.exomol.com/xsecs](http://www.exomol.com/xsecs). The user of this web-based interface can select a wavenumber range, temperature and wavenumber grid spacing; using these parameters the interface software first obtains a high-resolution cross section at the desired temperature by the interpolation procedure described in the previous section on the pre-calculated spectra, and then bins this interpolated cross section to the requested wavenumber grid.

Cross sections are returned as a list of floating point numbers in a text file, separated by the Unix-style newline character, LF (\n', 0x0A). The wavenumber grid can be generated from the linear sequence

$$\tilde{\nu}_i = \tilde{\nu}_{\text{min}} + i \Delta \tilde{\nu}; \quad i = 0, 1, 2, \cdots, n - 1$$

(14)
where the total number of points in the requested cross section is

\[ n = \frac{\tilde{\nu}_{\text{max}} - \tilde{\nu}_{\text{min}}}{\Delta \tilde{\nu}} + 1. \]  

We also provide an XML file in XSAMS format (Dubernet et al., 2009), compatible with the standards of the VAMDC project (Dubernet et al., 2010). This file may be thought of as a ‘wrapper’ to the cross section data, providing contextual metadata such as the molecular identity and structure, temperature of the calculation, and wavenumber limits and grid spacing. An example of the format is given in Table 4.

Cross section files have been generated for the polyatomic line lists currently available on the ExoMol website. These are listed in Table 5. The table also specifies the maximum wavenumber (\(\tilde{\nu}_{\text{max}}\)) and maximum temperature (\(T_{\text{max}}\)) for each species; we strongly caution against relying on the cross sections or indeed the underlying line lists at temperatures greater than those given. Further cross sections will be provided as line lists for new species as they become available.

6. Conclusion

High resolution absorption cross sections have been calculated for a number of molecules likely to be important in the atmospheres of exoplanets. The online interface provided at the ExoMol website (www.exomol.com) allows customized cross sections for a given molecular species to be returned at a specified temperature and resolution. Cross sections are only available for those species for which extensive line lists exist. New cross sections will be provided as further species are added to the ExoMol database, see Tennyson and Yurchenko (2012) for example.
Table 4: Sample XSAMS format (Dubernet et al., 2009) XML wrapper for a cross section for H$_2^{16}$O generated from 1000 to 20000 cm$^{-1}$ in steps of 1 cm$^{-1}$ at a temperature of 296 K. In this example, the cross section itself is provided in the file H2O_1000-20000_296K-10.0.sigma as a column of values, here in cm$^2$, one for each of 901 grid points.

<AbsorptionCrossSection envRef="EEXOMOL-1" id="PEXOMOL-XSC-1">
  <Description>
  The absorption cross section for H2O at 296.0 K, calculated at Sun Mar 11 19:50:45 2012, retrieved from www.exomol.com/xsecs
  </Description>
  <X parameter="nu" units="1/cm">
    <LinearSequence count="901" initial="1000." increment="10."/>
  </X>
  <Y parameter="sigma" units="cm2">
    <DataFile>H2O_1000-20000_296K-10.0.sigma</DataFile>
  </Y>
  <Species>
    <SpeciesRef>XEXOMOL-XLYOFNOQVPJNP-FNDQEIABSA-N</SpeciesRef>
  </Species>
</AbsorptionCrossSection>
Table 5: Summary of species for which are cross sections currently available. Also given for each species is the maximum wavenumber ($\tilde{\nu}_{\text{max}}$), the maximum temperature ($T_{\text{max}}$) and the reference to the original line list.

<table>
<thead>
<tr>
<th>Species</th>
<th>$\tilde{\nu}_{\text{max}}$/cm$^{-1}$</th>
<th>$T_{\text{max}}$/K</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>H$_3^+$</td>
<td>10 000</td>
<td>4000</td>
<td>Neale et al. (1996)</td>
</tr>
<tr>
<td>H$_2$D$^+$</td>
<td>10 000</td>
<td>4000</td>
<td>Sochi and Tennyson (2010)</td>
</tr>
<tr>
<td>H$_2$O</td>
<td>20 000</td>
<td>3000</td>
<td>Barber et al. (2006)</td>
</tr>
<tr>
<td>HDO</td>
<td>17 000</td>
<td>3000</td>
<td>Voronin et al. (2010)</td>
</tr>
<tr>
<td>HCN / HNC</td>
<td>10 000</td>
<td>4000</td>
<td>Harris et al. (2002, 2006)</td>
</tr>
<tr>
<td>H$^{13}$CN / H$^{13}$NC</td>
<td>10 000</td>
<td>4000</td>
<td>Harris et al. (2008)</td>
</tr>
<tr>
<td>NH$_3$</td>
<td>12 000</td>
<td>1500</td>
<td>Yurchenko et al. (2011)</td>
</tr>
</tbody>
</table>

It is our intention to make the cross section facility in ExoMol fully interoperable with other spectroscopic databases as part of the VAMDC (Virtual Atomic and Molecular Data Centre) project (Dubernet et al., 2010). Work in this direction will be reported in due course.

Acknowledgements

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Figure 2: Comparison of the calculated H$_2^{16}$O cross section presented in this work (blue) with the experimental cross section of the PNNL database [Sharpe et al., 2004] (green) in the region of the fundamental $\nu_2$ bending mode, at 296 K, both binned to a 10 cm$^{-1}$ wavenumber grid. Also shown is the difference (this work - PNNL) between the two spectra (red).
Figure 3: Calculated absorption cross section (upper pane) and interpolation residual (lower pane) in the region of the fundamental $\nu_2$ bending mode, at 350 K, on a wavenumber grid spacing of 0.01 cm$^{-1}$. The interpolation residual error does not exceed 1.34%.