Localization of Spinwaves in the Quantum Hall Ferromagnet.

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The quantum Hall ferromagnet at filling fraction $\nu = 1$ has some unusual properties due to the remarkable identity between the topological density of a spin distortion and the associated electrical charge density. We investigate the localization of spinwaves by coupling to a scalar disorder potential via their topological density. A low energy description of the system in terms of a non-linear sigma model of unitary supermatrices is derived. All states of this model are localized in two dimensions. A possible experimental signature of these effects in photo-luminescence is suggested.

The ground-state of a two-dimensional electron gas(2DEG) at exact filling of a single Landau level (filling fraction $\nu = 1$) is strongly ferromagnetic. The properties of this quantum Hall ferromagnet(QHF) are profoundly affected by the topological nature of the quantized Hall plateau. In a quantum Hall state, there is a commensuration between the magnetic flux through the 2DEG and the electrical charge density. This leads to the identification of the topological charge density of a spin distortion with an associated electrical charge density[1] (the Berry phase induced by a spin distortion may be reproduced by the Aharonov-Bohm phase induced by a fictitious magnetic flux). As a consequence, the elementary excitations formed as the filling fraction is moved slightly away from $\nu = 1$ are electrically and topologically charged objects known as Skyrmions[1,2]. In addition, spinwaves may couple to a scalar disorder potential via their topological density[3]. The geometrical structure of this disorder interaction is very similar to minimal coupling of quantum particles to a random flux and the spinwave system provides a novel realization of this intently studied problem[4]. In this paper, we investigate the localization of spinwaves in circumstances where the disorder potential is sufficiently weak that the ground-state remains ferromagnetic and coupling to spinwaves is the dominant effect. In the first section, we use diagrammatic techniques to demonstrate a decoupling of charge-density fluctuations from fluctuations in exchange-energy. In the second, we use supersymmetry[5] to construct a low energy description of the system in terms of a non-linear sigma model of unitary super-matrices. All states of this model are localized in two dimensions[6]. A possible experimental signature of these effects in photo-luminescence is suggested.

Our starting point is a spinwave expansion about the continuum field theory of the QHF proposed by Sondhi *et al*[1,3]. The effective action and electrical current density for small fluctuations, $\mathbf{l} = (l_1, l_2, 0)$, about the ferromagnetic ground-state, $\bar{\mathbf{n}} = (0, 0, 1)$, are given by

$$S = \int d^2x dt \left[\frac{1}{2} \bar{l} \left(\frac{\bar{\rho}}{2} \partial_t - \rho_s \nabla^2 - \bar{\rho} g B \right) l - J_0(\mathbf{x}) U(\mathbf{x}) \right], \quad (1)$$

$$J_\mu = i \frac{e\nu}{8\pi} \epsilon^{\mu\nu\lambda} \partial_\nu \bar{l} \partial_\lambda l. \quad (2)$$

$\mathbf{n} = (l_1, l_2, \sqrt{1 - |\mathbf{l}|^2})$ is an O(3)-vector order parameter describing the local polarization of the quantum Hall system. The first part of Eq.(1) is the usual Schrödinger effective action for spinwaves in a continuum ferromagnet. We use the complex notation, $l = l_1 + il_2$, $\bar{l} = l_1 - il_2$. $\bar{\rho}$ is the electron density ($\bar{\rho} = \nu/2\pi l_B^2$, where l_B is the magnetic length), ρ_s is the spin stiffness and g is the Zeeman coupling, into which we have absorbed the electron spin and the Bohr magneton for ease of notation. The second term Eq.(1) is due to the identity of charge and topological charge embodied in Eq.(2). It describes the interaction of spinwaves with a scalar disorder potential, $U(\mathbf{x})$. The Coulomb self-interaction of the spinwave charge density has been neglected, since it is higher order in both derivatives and in the spinwave field. Throughout the bulk of the calculations presented in this paper, we will ignore the Zeeman term in Eq.(1). It is a simple matter to re-introduce it at the end of our calculations.

The correlations in the disorder potential felt by the two-dimensional electron gas in GaAs heterostructures are conveniently modeled as follows:[7]

$$\langle\langle U_{\mathbf{q}} U_{\mathbf{q}'} \rangle\rangle = (2\pi)^2 \delta(\mathbf{q} + \mathbf{q}') \gamma' \frac{e^{-2|\mathbf{q}|d}}{|\mathbf{q}|^2}. \quad (3)$$

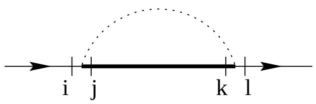
d is the width of the insulating spacer layer separating the electrons from the ionized donor impurities. γ' is a measure of the disorder strength and is related to the area density of donor impurities, n_d; $\gamma' = (e\sqrt{n_d}/2\epsilon)^2 (\nu/8\pi)^2$. Our aim in this paper is to determine the universal features that arise due to the geometrical structure of the interaction. To simplify our explicit calculation, we assume a Gaussian, δ-function correlated distribution for the disorder potential:

$$\langle\langle U_{\mathbf{q}} U_{\mathbf{q}'} \rangle\rangle = (2\pi)^2 \gamma \delta(\mathbf{q} + \mathbf{q}') \quad (4)$$

We return to the more realistic correlations of Eq.(3) towards the end of the paper.

Laid out in this way, the interaction of spinwaves with a disorder potential has many similarities to the problem of non-interacting Schrödinger particles in a random flux[4]. To see this, one should integrate the interaction in Eq.(1) by parts. The result looks like minimal coupling to a vector potential, $A_i = -(e\nu/16\pi)\epsilon_{ij}\partial_j U(\mathbf{x})$, aside from the absence of an $|\mathbf{A}|^2$ term (which leads to broken gauge symmetry in the present case).

The self-energy, calculated in the self-consistent Born approximation, with correlations in the disorder potential given by Eq.(4), is

$\Sigma^R(\mathbf{p},\omega) = \epsilon_{ij}\epsilon_{kl}$  (5)

In this diagram, full lines represent spinwave propogators, cross-bars indicate spatial derivatives of these propogators and the dotted lines represent disorder correlations. The thick line represents a full spinwave propogator including the self-energy; $G^{-1} = G_0^{-1} + \Sigma$. The real part of the self-energy may be absorbed into a renormalization of the spinwave stiffness. The imaginary part is given by

$$\mathcal{I}m\Sigma^R(\mathbf{p},\omega) = \gamma \mathcal{I}m \int \frac{d^2q}{(2\pi)^2} (\mathbf{p}\times\mathbf{q})^2 G^R(\mathbf{q},\omega). \quad (6)$$

To lowest order in the disorder strength, this has the solution

$$\mathcal{I}m\Sigma^R(\mathbf{p},\omega) = \frac{\gamma}{8\rho_s^2}\left(\frac{\bar{\rho}\omega}{2}\right)|\mathbf{p}|^2. \quad (7)$$

The disorder averaged spinwave Green's function is then

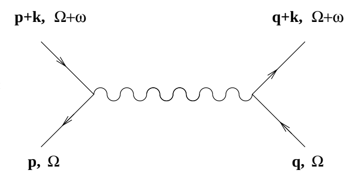
$$\begin{aligned} \langle G^R(\mathbf{p},\omega)\rangle &= \left(\rho_s|\mathbf{p}|^2 - \frac{\bar{\rho}\omega}{2} + i\frac{\gamma}{8\rho_s^2}\left(\frac{\bar{\rho}\omega}{2}\right)|\mathbf{p}|^2\right)^{-1} \\ &\approx \left(\rho_s|\mathbf{p}|^2 - \frac{\bar{\rho}\omega}{2} + i\frac{\bar{\rho}}{2\tau_\omega}\right)^{-1} \\ \tau_\omega &= \frac{4\bar{\rho}\rho_s^3}{\gamma}\left(\frac{\bar{\rho}\omega}{2}\right)^{-2}. \end{aligned} \quad (8)$$

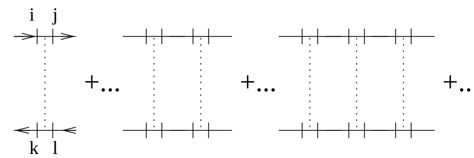
In writing down the final expression for the disorder averaged Green's function, we have removed a factor of $\left(1 + i\gamma\bar{\rho}\omega/16\rho_s^3\right)$ and Taylor expanded the denominator in powers of $\gamma\bar{\rho}\omega/16\rho_s^3$. Since this latter quantity is a small parameter in our perturbative expansion, this Taylor expansion is justified.

The most important point to notice here is that the scattering time diverges as the frequency goes to zero. The spectral weight is concentrated in an energy range $1/2\tau_\omega$ of the bare pole at $\bar{\rho}\omega/2 = \rho_s|\mathbf{q}|^2$, such that $1/2\tau_\omega \ll \bar{\rho}\omega/2$. This should be compared to the case of electrons scattering from a random scalar potential[5], where the bare pole of the Green's function is near to the Fermi energy, E_F, and the scattering rate, $1/\tau$, is constant. The validity of the perturbative expansion for electrons depends upon the smallness of the parameter $1/(\tau E_F)$. In the present case, the extra derivatives in the interaction vertex lead to the existence of a small expansion parameter without the existence of a Fermi surface. From a calculational point of view, $\bar{\rho}\omega/2$ plays the role of a frequency dependent Fermi surface and momentum integrals may be carried out in the same way as one would in the case of electrons.

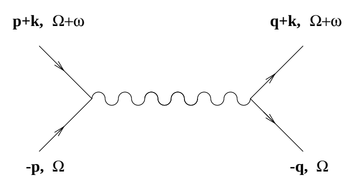
I. DIFFUSONS AND COOPERONS

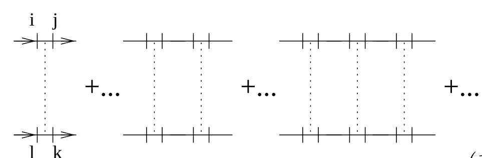
The diffusive and weak localization effects in a disordered system are determined by the diffuson

$D(\mathbf{p},\mathbf{q},\mathbf{k},\Omega,\Omega+\omega) =$ 

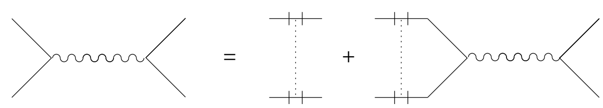
$= \epsilon_{ij}\epsilon_{kl}$  (9)

and Cooperon modes

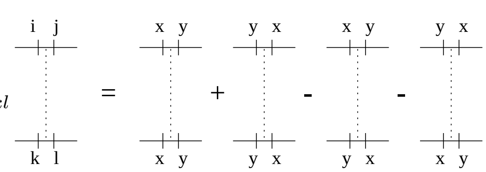
$C(\mathbf{p},\mathbf{q},\mathbf{k},\Omega,\Omega+\omega) =$ 

$= \epsilon_{ij}\epsilon_{kl}$  (10)

respectively. Notice that the interaction vertex is antisymmetric under time reversal and, therefore, occurs with the opposite sign in the ladder summation for the Cooperon, compared with the Diffuson. The ladder approximation to these diagrams, as indicated in Eqs.(9,10), may be summed to give two Dyson's equations that may be represented by the following diagram:

 (11)

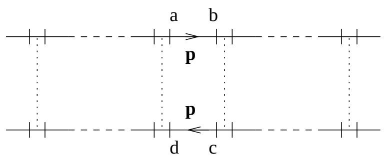
In fact, these equations are rather difficult to handle. The trouble is that the geometrical factors, involving cross-products of momenta, are not easy to factorize. One way to negotiate these difficulties is to write the interaction vertex as the sum of four vertices;

$\epsilon_{ij}\epsilon_{kl}$  (12)

The diffuson and Cooperon diagrams are decomposed similarly into the sum of diagrams;

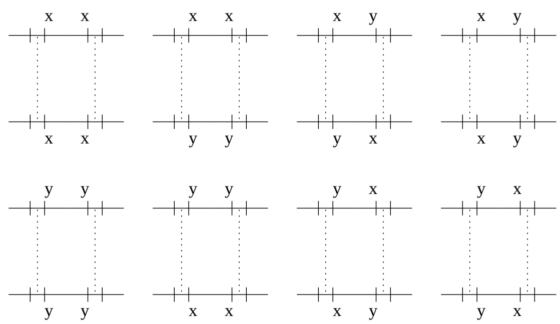
$$D = D_{xxxx} + D_{xxxy} + D_{xxyx} + 13 \text{ other terms}, \tag{13}$$

where the suffices label the derivatives on the external legs of the contributing diagrams. We wish to evaluate these diagrams in the limit of small momentum, $\mathbf{k}$, and frequency, ω. Consider the momentum integral over some internal rung of a ladder diagram when $\mathbf{k} = 0$;



$$= ... \int \frac{d^2p}{(2\pi)^2} p_a p_b p_c p_d \mathcal{G}(\mathbf{p}, \Omega + \omega)\mathcal{G}(\mathbf{p}, \Omega)... \tag{14}$$

The angular integration gives zero if $a \neq b = c = d$ (or a permutation of this). The remaining non-zero rungs are

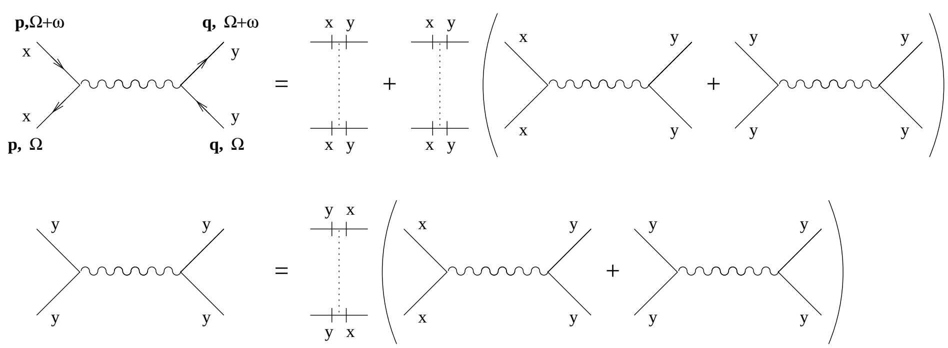


(15)

Only elements of D corresponding to these rungs are non-zero. In addition, rotational symmetry requires that

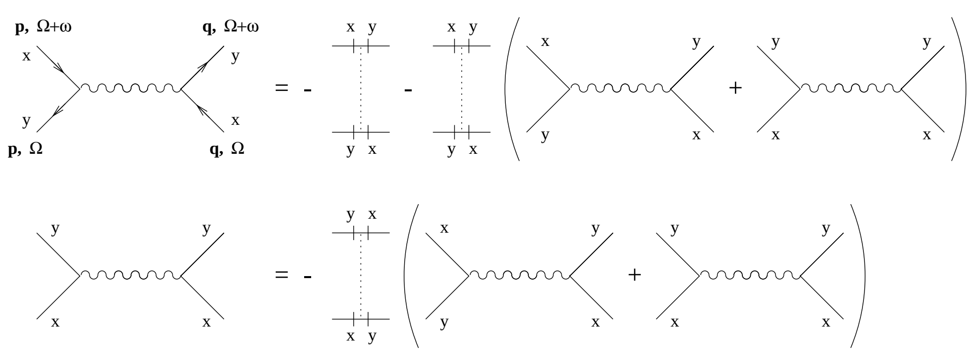
$$D_{xxxx} = D_{yyyy}, \quad D_{xxyy} = D_{yyxx},$$
$$D_{xyxy} = D_{yxyx}, \quad D_{xyyx} = D_{yxxy}. \tag{16}$$

Using these symmetries in Eq.(11), with $\mathbf{k} = 0$, the Dyson's equation decouples into two pairs of coupled equations,



(17)

relating D_{yyyy} to D_{xxyy} and



(18)

relating D_{xyyx} to D_{yxyx}. The components, D_{xyxy}, *etc.* of D, describe the propogation of charge-density fluctuations (the fields on the left-most legs occur in the combinations $\partial_1\bar{l}\partial_2 l$ and $\partial_2\bar{l}\partial_1 l$, the charge-density being proportional to $\epsilon_{ij}\partial_i\bar{l}\partial_j l$) and the components D_{xxxx}, *etc.* describe the propogation of exchange-energy fluctuations (the fields on the external legs occur in the combinations $\partial_1\bar{l}\partial_1 l$ and $\partial_2\bar{l}\partial_2 l$, the exchange-energy being proportional to $\partial_i\bar{l}\partial_i l$). The decoupling of the Dyson's equation, therefore, amounts to decoupling of charge and energy propogation. An alternative way of separating a Dyson's equation such as Eq.(11) is presented in Ref.[8]. Written in the above manner, the physical eignemodes of the scattering process are apparent. Using the ansatz

$$D_{abcd}(\mathbf{p},\mathbf{q},\mathbf{k},\Omega,\omega) = (p+k)_a p_b (q+k)_c q_d \tilde{D}_{abcd}(\mathbf{k},\omega,\Omega) \tag{19}$$

Eq.(17) reduces to

$$\begin{aligned}\tilde{D}_{xxyy} &= \gamma + \left[\tilde{D}_{xxyy}\langle\cos^2\theta\sin^2\theta\rangle_\theta + \tilde{D}_{yyyy}\langle\cos^4\theta\rangle_\theta\right] \\ &\quad\times\gamma\int\frac{d^2q}{(2\pi)^2}|\mathbf{q}|^4 G^A(\mathbf{q},\Omega)G^R(\mathbf{q},\Omega+\omega), \\ \tilde{D}_{yyyy} &= \left[\tilde{D}_{xxyy}\langle\cos^4\theta\rangle_\theta + \tilde{D}_{yyyy}\langle\cos^2\theta\sin^2\theta\rangle_\theta\right] \\ &\quad\times\gamma\int\frac{d^2q}{(2\pi)^2}|\mathbf{q}|^4 G^A(\mathbf{q},\Omega)G^R(\mathbf{q},\Omega+\omega)\end{aligned} \tag{20}$$

Evaluating the momentum integrals and performing the angular averages, one obtains

$$\begin{aligned}\tilde{D}_{xxyy}\left(1-i\frac{2\omega\tau_\Omega}{3}\right) &= \frac{4\gamma}{3} + \tilde{D}_{yyyy}\left(1+i2\omega\tau_\Omega\right) \\ \tilde{D}_{yyyy}\left(1-i\frac{2\omega\tau_\Omega}{3}\right) &= \tilde{D}_{xxyy}\left(1+i2\omega\tau_\Omega\right).\end{aligned} \tag{21}$$

To lowest order in ω, these equations have the solution

$$\tilde{D}_{xxyy}(\mathbf{k}=0,\omega,\Omega) = \tilde{D}_{yyyy}(\mathbf{k}=0,\omega,\Omega) = \frac{\gamma\bar{\rho}}{8\tau_\Omega}\left(i\frac{\bar{\rho}\omega}{2}\right)^{-1}. \tag{22}$$

$\tilde{D}_{xxyy}$ and $\tilde{D}_{yyyy}$, therefore, describe massless diffusive modes. A similar calculation for $\tilde{D}_{xyyx}$ and $\tilde{D}_{yxyx}$ produces a result that is finite as $\omega\to 0$. These correspond to massive modes. Expanding the Dyson's equation to lowest order in $\mathbf{k}$, one obtains the result

$$\tilde{D}_{xxyy}(\mathbf{k},\omega,\Omega) = \tilde{D}_{yyyy}(\mathbf{k},\omega,\Omega) = \tilde{D}(\mathbf{k},\omega,\Omega), \tag{23}$$

where

$$\tilde{D}(\mathbf{k},\omega,\Omega) = \frac{\gamma\bar{\rho}}{8\tau_\Omega}\left(i\frac{\bar{\rho}\omega}{2}+D_o|\mathbf{k}|^2\right)^{-1}, \tag{24}$$

and $D_0 = \rho_s\Omega\tau_\Omega$ is the classical diffusion coefficient for the spinwaves. In principle, one should worry that, when $\mathbf{k}\neq 0$, the Dyson's equations no longer decouple into two pairs of equations. This may lead to a shift in the diffusion constant, D_0[4,8]. We find that, in this case, the result is unaffected to lowest order. Finally, summing all of the massless contributions to the diffuson, we find

$$D(\mathbf{p},\mathbf{q},\mathbf{k},\Omega,\Omega+\omega) = (\mathbf{p}+\mathbf{k}).\mathbf{p}(\mathbf{q}+\mathbf{k}).\mathbf{q}\tilde{D}(\mathbf{k},\omega,\Omega), \tag{25}$$

which is the massless propogator for exchange energy fluctuations. The Cooperon diagrams may be evaluated similarly. They are finite in the limit $\mathbf{k},\omega\to 0$ and, therefore, massive. This is a consequence of the time reversal anti-symmetry. There is no contribution to weak localization from these modes.

II. SUPERSYMMETRIC SOLUTION.

We now develop a low energy theory for the interaction of spinwaves with a weak disorder potential, using supersymmetric techniques. The main subtleties of the current problem are in the handling of the geometrical factors in the interaction. We will concentrate upon these features and refer the reader to the literature[5] for details of the super-symmetry itself. As noted previously, this problem is very similar to that of non-interacting particles in a random flux. An alternative derivation of the results presented here may be made using the techniques of Ref.[4].

We wish to determine the disorder averages of dynamical quantities involving $\langle G^A G^R\rangle$ and, therefore, introduce an eight component superfield,

$$\psi = \left(l^A\bar{l}^A l^R\bar{l}^R\chi^A\bar{\chi}^A\chi^R\bar{\chi}^R\right), \tag{26}$$

in the usual way[5]. The superscripts, A/R, label advanced and retarded sectors, the fields χ are anti-commuting and the overbar indicates complex conjugation. The presence of the commuting and anti-commuting fields in ψ removes the need to write the partition function explicitly in the denominator of correlation functions and allows the disorder average to be performed immediately. The resulting Lagrangian for the superfield, ψ, is

$$\mathcal{L} = \int\left[-i\bar{\psi}\left(-\rho_s\nabla^2-\frac{\bar{\rho}}{2}\tilde{\epsilon}\right)\psi+\gamma\left(\epsilon_{ij}\partial_i\bar{\psi}\partial_j\psi\right)^2\right]d^2r, \tag{27}$$

where $\tilde{\epsilon} = (\Omega+\omega/2)\mathbf{1}+(\omega/2+i\delta)\Lambda$ and Λ is the diagonal supermatrix with elements ± 1 in the advanced/retarded sectors. As is usual in the derivation of the supersymmetric sigma model, we assume that $\omega\ll\Omega$. Disorder averaging may only induce cross correlations between the Green's functions if the difference in frequencies is less than the scattering rate $\bar{\rho}\omega/2\leq 1/2\tau_\Omega, 1/2\tau_{\Omega+\omega}\ll\bar{\rho}\Omega/2$, therefore this approximation is justified. Next, we decouple the quartic interaction, introduced by the disorder average, with a supermatrix field, Q_{ij}, where $i,j\in\{x,y\}$

and each element of the 2×2 matrix Q_{ij} is an 8×8 supermatrix;

$$\exp\left(-\mathcal{L}_{int}[\psi]\right) = \int \exp\left(-\mathcal{L}_{int}[\psi, Q_{ij}]\right) DQ_{ij},$$
$$\mathcal{L}_{int}[\psi, Q_{ij}]$$
$$= \frac{\bar{\rho}}{2\tau_\Omega}\int \mathrm{Str}\left(\left(\frac{\bar{\rho}\Omega}{2\rho_s}\right)^{-1}\epsilon_{ki}\partial_k\bar{\psi}Q_{ij}\partial_j\psi + \frac{1}{32\rho_s}Q_{ij}^2\right) d^2r \quad (28)$$

The supertrace, Str, is as defined in Ref.[5] and summation over repeated spatial indices is implied. Integrating out the superfield, ψ, we obtain the following free energy functional for Q_{ij}:

$$F[Q_{ij}] = \int \mathrm{Str}\left[-\frac{1}{2}(\ln G^{-1}) + \frac{\bar{\rho}}{64\tau_\Omega\rho_s}Q_{ij}^2\right] d^2r, \quad (29)$$

where $G(\mathbf{r},\mathbf{r}',Q)$ is the supermatrix Green's function of the field ψ and satisfies the equation

$$\left(-\rho_s\nabla^2 - \frac{\bar{\rho}}{2}\tilde{\epsilon} + i\frac{\bar{\rho}}{2\tau_\Omega}\left(\frac{\bar{\rho}\Omega}{2\rho_s}\right)^{-1}\epsilon_{ki}\left[Q_{ij}\partial_j\partial_k + \partial_k Q_{ij}\partial_j\right]\right)$$
$$\times G(\mathbf{r},\mathbf{r}',Q) = i\delta(\mathbf{r}-\mathbf{r}'). \quad (30)$$

The saddle point equation for Q_{ij} is

$$Q_{ij} = \frac{8\rho_s}{\gamma}\left(\frac{\bar{\rho}\Omega}{2\rho_s}\right)^{-1}\gamma\int\frac{d^2p}{(2\pi)^2}\epsilon_{ki}p_kp_jG(\mathbf{p},Q_{ij}). \quad (31)$$

This is precisely the self-consistency equation that was solved previously to find $\mathcal{I}m\Sigma$. The solution is

$$Q_{ij} = \epsilon_{ij}V\Lambda\bar{V}, \quad (32)$$

where V is an arbitrary, unitary supermatrix such that $V\bar{V}=1$. The diagonal terms of Q_{ij} are zero at the saddle point and, therefore, correspond to massive modes. The off-diagonal components, however, sit in a Mexican hat potential and have massive longitudinal fluctuations and massless transverse fluctuations. At the saddle point, $Q_{ij}\sim\epsilon_{ki}\partial_k\bar{\psi}\partial_j\psi$, and, therefore, the diagonal elements of Q_{ij} describe charge density fluctuations, $Q_{ii}\sim\epsilon_{ij}\partial_i\bar{\psi}\partial_j\psi$ and the off-diagonal elements describe exchange energy fluctuations, $\epsilon_{ij}Q_{ij}\sim\partial_i\bar{\psi}\partial_i\psi$. This is the same decomposition into massive and massless modes as was found diagrammatically in the preceding section. Let us define

$$Q_{ij} = \epsilon_{ij}Q_s + \mathrm{diag}(Q_{c1},Q_{c2})$$
$$Q_s = \frac{1}{2}\epsilon_{ij}Q_{ij}$$
$$Q_{c1} = Q_{11}$$
$$Q_{c2} = Q_{22} \quad (33)$$

At the saddle point $Q_s = V\Lambda\bar{V}$ and $Q_c = 0$. Expanding the free energy functional, Eq.(29), to quadratic order in fluctuations of Q_{ij} about the saddle point, we obtain

$$S = \frac{\bar{\rho}}{64\tau_\Omega\rho_s}\int d^2r\mathrm{Str}\left[Q_{ij}^2\right]$$
$$-\frac{\bar{\rho}}{8\tau_\Omega^2}\left(\frac{\bar{\rho}\Omega}{2\rho_s}\right)^{-2}\int\frac{d^2p}{(2\pi)^2}\int\frac{d^2q}{(2\pi)^2}$$
$$\times\mathrm{Str}\left[\epsilon_{ki}p_j(p+q)_kG(\mathbf{p})\delta Q_{ij}(\mathbf{q})\right.$$
$$\left.\times\,\epsilon_{nl}(p+q)_mp_nG(\mathbf{p}+\mathbf{q})\delta Q_{lm}(-\mathbf{q})\right]. \quad (34)$$

The $\mathbf{p}$-integral is only finite for the products G^AG^R, which are generated by the components of Q off-diagonal in the advanced and retarded sectors. Henceforth, we use Q to indicate supermatrices with only these off-diagonal components non-zero. Carrying out these integrals and substituting Q_s, Q_{c1} and Q_{c2} from Eq.(33), we find the following effective action for δQ_s:

$$S = \frac{1}{32\rho_s}\int\frac{d^2q}{(2\pi)^2}\left(-i\bar{\rho}\omega/2 + D_0|\mathbf{q}|^2\right)$$
$$\times\mathrm{Str}\left[\delta Q_s(\mathbf{q})\delta Q_s(-\mathbf{q})\right], \quad (35)$$

where

$$D_0 = \rho_s\tau_\Omega\Omega. \quad (36)$$

The corresponding actions for δQ_{c1} and δQ_{c2} contain massive propogators. It is important that one should not simply ignore these massive modes. They may lead to a renormalization of the diffusion constant for the massless modes[8,4]. Here, we find that this is not the case. The cross-terms between δQ_s and $\delta Q_{c1}/\delta Q_{c2}$ are proportional to $|\mathbf{q}|^2$. Integrating out the massive modes induces a $|\mathbf{q}|^4$-term in the δQ_s propogator, which we neglect at small momentum. Keeping only fluctuations of Q over the saddle point manifold, Eq.(35) determines the coefficients of the terms in the sigma model;

$$F[Q_s] = \frac{1}{128\rho_s}\mathrm{Str}\int\left[D_0|\nabla Q_s|^2 - 2i\left(\frac{\bar{\rho}\omega}{2}\right)\Lambda Q_s\right]d^2r \quad (37)$$

D_0 is the classical diffusion constant given by Eq.(36). It may be written in the form $D_0 = \bar{\rho}\tau_\Omega v_\Omega^2/2$, where $v_\Omega^2 = (dE/d\mathbf{p})^2\mid_{E=\Omega/2} = 2\rho_s\Omega/\bar{\rho}$. This energy functional describes the diffusive propogation of exchange energy density fluctuations. All states are localized in this model[5,6], with a localization length

$$\xi = v_\Omega\tau_\Omega\exp\left[\frac{\pi^2D_0^2}{64\rho_s^2}\right]\sim\Omega^{-3/2}\exp\left[\Omega^{-2}\right], \quad (38)$$

which is divergent in the Ω, $\mathbf{q}=0$ limit. This divergence is a natural consequence of the existence of an SU(2) global symmetry (broken to U(1) with the inclusion of the Zeeman coupling). Inclusion of the Zeeman coupling modifies the parameters of the sigma model such that Ω is replaced by $\Omega - gB$. There are no spinwave states below the Zeeman gap and the localization length diverges as the frequency approaches this, $\mathbf{q}=0$, limit. Realistic correlations in the disorder potential, given by Eq.(3), lead to a further modification of the parameters in the sigma model. The scattering time is given by[3]

$$\tau'_\Omega = \frac{4\bar{\rho}\rho_s^2}{\gamma'}\left(\frac{\bar{\rho}\Omega}{2}\right)^{-1}, \quad (39)$$

which again diverges as $\Omega \to 0$ such that $1/\tau_\Omega \ll \bar{\rho}\Omega/2$. Subsequent calculations follow through as before, with some additional complications. Finally, one obtains the same supersymmetric sigma model, Eq.(37), with

$$D'_0 = \rho_s \tau'_\Omega \Omega = \frac{8\rho_s^3}{\gamma'}. \quad (40)$$

The diffusion constant is now independent of Ω and the localization length no longer has an exponential dependence upon frequency, but a power law dependence;

$$\xi' \propto l_B \left(\frac{\Omega}{\rho_s}\right)^{-1/2}. \quad (41)$$

The numerical prefactor in this expression has an exponential dependence upon the disorder strength. Estimates assuming a donor density, n_d, of the same order as the electron density, $\bar{\rho}$, gives prefactors upwards of 10^3. Experimental determination of the diffusion coefficient and localization length may be possible using space/time-resolved photo-luminescence. In the QHF, magneto-excitons and spinwaves are identical[9]. If, in addition, the sub-band wavefunction of valence holes is centered in the same position as the wavefunction of electrons in the two-dimensional electron gas, then excitons have the same Hamiltonian as magneto-excitons[10]. Localization and diffusion of excitons in other systems have been measured using photo-emission spectroscopy[11]. The same techniques may be applicable here.

Above a threshold disorder strength, the groundstate ceases to be ferromagnetic and contains a random distribution of Skyrmions and anti-Skyrmions[3,13]. In certain circumstances, the main effect of this topologically non-trivial groundstate upon the spinwave action is to introduce an effective magnetic field and mass term[12]. Both the magnetic flux and the mass are proportional to the topological density of the background. This is yet another variant of the random flux problem. In this case, the effective random flux may be very large and localization may have further experimental consequences[14].

In summary, we have considered the effect of weak disorder upon the spinwave dynamics in the QHF. We find a spinwave scattering time that diverges as frequency approaches zero, thus ensuring the validity of perturbative and supersymmetric techniques despite the lack of a Fermi-surface. Charge and exchange-energy fluctuations are found to decouple, the former being massive and the latter diffusive. The low energy dynamics of this system are described by a non-linear sigma model of unitary supermatrices. All states in this model are localized. A possible signature of these effects may be found in space/time-resolved photoemission spectroscopy.

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