THE IMPACT OF TAXATION AND FINANCIAL FACTORS ON COMPANY INVESTMENT: AN EXAMINATION USING UK PANEL DATA

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This thesis examines the impact of taxation and financial factors on the level of investment in fixed assets by quoted manufacturing companies in the United Kingdom between 1971 and 1986. Its most important theme is that there exist substantial differences between companies in the way that they are affected by both taxation and financial factors. The empirical work therefore uses individual company accounting and stock market data (described in Appendix A) together with a detailed model of the corporation tax system (described in Appendix B) in order to exploit cross sectional as well as time series variation.

Chapters 2 and 3 investigate the role played by taxation in the investment decision. Part of the cross sectional variation in taxation arises through "tax exhaustion", caused by the asymmetric treatment of taxable profit and loss in UK corporation tax and restrictions on the use of the imputation system. Two investment equations, the first based on Tobin's Q and the second on the cost of capital in an Euler equation framework are developed from the same neoclassical model of the firm which explicitly models tax exhaustion and the role played by expectations. Each is a forward-looking model, which could be used for the purposes of simulating the effects of tax reform on investment, whether the reform is announced or unannounced, permanent or temporary. The results confirm that tax does play a role in the determination of investment, although, for various reasons, the precise effect is difficult to quantify. They also suggest that the Q model is a poor means of assessing the impact of taxation on investment and that it is dominated by the second model.

Chapters 2 and 3 also consider the impact of taxation on company financial policy, and, in particular consider various regimes in which the company may find itself which depend on tax exhaustion and agency costs of debt. The stability of these regimes is more complex than commonly argued in the literature. The appropriate definition of the
cost of capital is also developed further, under similar conditions, and a matrix of nine possible values is constructed, depending on the marginal source of finance in this period and the next period.

Chapter 4 discusses the role played by financial factors. A model with legal constraints on financial behaviour and agency costs on debt is developed which predicts that, for all firms, investment depends on the level of cash generated, as well as Tobin’s Q. The importance of cash flow for firms of different size and age is investigated. The results support the hypothesis that cash flow is a significant determinant of investment for all firms. Cash flow has the highest impact for large and new firms.
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CHAPTER 1

Micro data, tax exhaustion and financial factors

1.1 Introduction

This thesis examines the impact of taxation and financial factors on the level of investment in fixed assets by quoted companies in the United Kingdom between 1971 and 1986. Its most important innovation, compared with most previous studies, is to examine the nature of differences between companies in the way that they are affected by both taxation and financial factors. The empirical work presented here therefore uses individual company accounting and stock market data in order to exploit cross sectional as well as time series variation. The model which is developed to analyse investment decisions is also used to examine company financial decisions.

Cross sectional variation in taxation arises partly through the phenomenon of tax exhaustion. The UK corporation tax system, especially during the 1970s and early 1980s, operated a system of generous reliefs and allowances particularly with regard to allowances for the cost of depreciation of fixed assets (capital allowances). The rate of these allowances was much higher than depreciation rates commonly charged by companies against accounting profit, with the result that taxable profit was often considerably lower than accounting profit. However, the asymmetric nature of the corporation tax system is such that, although positive taxable profit incurs an immediate tax charge, negative taxable profit does not lead to an immediate rebate (apart from in some cases in which the loss can be "carried back" to be set against profit from earlier years); rather the loss must be "carried forward", without compensation, to be set against the profit of future years. The term "fully tax exhausted" is used to describe companies in this position. This asymmetry is a common feature of most corporation tax systems; however its importance depends on the definition of the tax base since it is this which determines whether a company makes a taxable profit or loss.
A second form of tax exhaustion arises from the imputation system in the UK and in other European countries. This imputes part of the corporation tax charge paid by a company to be a prepayment of the shareholders' income tax due on dividends received from the company. Essentially, the company withholds part of the income tax due on dividends which it pays (this is known as advance corporation tax or ACT) and it is allowed to reduce its corporation tax charge by the amount withheld. However, if taxable profit is insufficiently high, not all of the amount withheld can be set against corporation tax. The amount not used cannot, in some circumstances, be offset against taxable profit earned in earlier years, but otherwise must again be carried forward without compensation to offset against future corporation tax liabilities. The term "ACT exhausted" is used to describe companies in this position.

Cross sectional variation in financial factors may arise for two groups of reasons. The first is the personal tax rates on income and capital gains which apply to the marginal shareholder of the firm. Since these tax rates differ between individuals, it is likely that they will differ between the relevant shareholders. But the optimal financial policy of the company may well depend on these personal tax rates, and hence may vary between companies. One strand of the literature suggests that 'clientele effects' will be created by differences in tax rates, with some shareholders preferring returns in the form of capital gains and some preferring dividends. This may lead to difference between companies in their dividend payout ratios, with shareholders choosing companies according to their individual tax position1.

A second form of variation may arise through imperfections in capital markets. Essentially, the existence of asymmetric information between managers in the company and outside investors, and incentive problems, whereby existing shareholders and debtholders do not trust the managers to act in the interests of the investors, both combine to make external finance more expensive than internal finance. In extreme cases, it is possible that rationing exists; some companies may not be able to raise external finance whatever rate of return they are willing to pay.

1Although as Miller (1977) argued, this does not necessarily mean that the company is not indifferent to its payout ratio.
The extent of the difference in cost between internal and external finance, or the existence of rationing, may depend on many different factors. Obvious candidates are publicly available accounting data on profit and gearing within the firm. Thus a high profit, low geared firm may find it cheaper to raise debt than a low profit, highly geared firm. Other factors may include particular characteristics of the firm, such as its size or age. At first sight, it may appear that small firms would be more likely to face problems of asymmetric information than large firms, simply because the latter have more analysts following their performance. However, more sophisticated arguments might even point in the opposite direction; the larger the firm, the more dispersed its ownership is likely to be and hence the greater the problems of shareholders exercising control over the managers. Whatever the relative merits of these arguments, the point here is simply that such issues may induce cross section variation between companies in the cost of different forms of finance, which may influence not only optimal financial policy, but also optimal investment policy.

Closely related to this issue of cross sectional variation between companies is the need for careful econometric analysis. Of the huge literature which has examined the impact of tax on investment decisions the vast majority of empirical studies have used time series data (virtually all have ignored tax exhaustion) and estimated equations by ordinary least squares. Not only does the availability of a large panel of data allow the investigation of cross sectional variation and improve the precision with which parameters may be estimated; it also permits fuller investigation of the nature of stochastic shocks to the investment process.

So far the discussion has concentrated on the reasons for cross sectional variation between companies. These issues are discussed further below in section 1.2 which is a brief survey of the relevant literature and section 1.3 which describes the UK corporation tax system in more detail. It is also worth noting the main features of the investment model developed in chapter 3.

The literature on the impact of taxation on investment has followed two,
closely related, approaches. The first, associated primarily with Jorgensen relates the 'desired' capital stock to the cost of capital and output. The empirical form of this model generally takes an ad hoc approach to the dynamic properties of the investment process, adding lags of investment and other variables to the estimated equation to allow for adjustment costs and various forms of lags which may occur between the decision to invest and the new capital being productive. However, since this is a backward-looking model, and, although possibly data coherent, not derived from an explicit optimising model, it suffers from the Lucas critique in being unable to adequately examine the impact of taxation on investment.

By contrast, the Q model adds adjustment costs to the underlying model and consequently arrives at a theoretically consistent model where the rate of investment depends on Tobin's Q, the ratio of the market value of the company to the replacement value of its capital stock. The problem with the Q model is that its empirical performance is poor. The coefficient on Q is generally extremely low (implying very high adjustment costs), the model exhibits serial correlation and, contrary to the standard model, other variables are significant when added.

One possible reason for the poor empirical performance of Q is its reliance on the assumption of perfect capital markets, so that the market value of the firm can be taken to be the 'true' value, reflecting all currently held expectations of future events. The poor performance of Q is consistent with the stock market being too volatile. The model generated here therefore derives an alternative investment equation from the same underlying model, in which the rate of investment depends on the rate of investment and the marginal product of capital in the following period, as well as the Jorgensen cost of capital. The advantage of this approach is that, like the Q model, it does not suffer from the Lucas critique, but it also does not have to rely on the market value of the firm.

This thesis is organised as follows. This introductory chapter has two main purposes. The first is to briefly review the literature relevant to the issues discussed here. The literature on investment and taxation has already been the subject of several extensive surveys, and only a highly
selective review is presented here, mostly covering papers which examine the role of tax exhaustion or which use panel data in estimation. Some discussion is also presented regarding the literature on the interactions of real and financial decisions of the firms, covering particularly papers which examine the impact of financial factors on the level of investment.

The second purpose of this introductory chapter is to describe the nature of the UK corporation tax system. The lack of relevant information regarding the extent of tax exhaustion prevents this from being simply a description of tax rules, however. Rather, it has proved necessary to construct a model of the corporation tax system in order to provide estimates of tax liabilities and tax exhaustion. This model provides such estimates by applying the rules of the tax system to company accounting data such as the level of profit and investment. The precise equations in the model are presented in Appendix B. This chapter summarises the approach and provides some results indicating the extent to which companies in the sample experienced tax exhaustion.

Chapter 2 presents a detailed theoretical model of the firm. It assumes that companies attempt to maximise the present value of the shareholders' wealth. The principal innovation is that it explicitly models the role played by tax exhaustion. In addition, the role played by leasing, rather than purchasing, fixed assets (the lessor being able to claim capital allowances, but likely to pass on some of the benefit to the lessee) is examined. The model also examines in detail the financial side of the company's activities by including personal tax parameters, constraints on new equity issues and dividends and adding agency costs which arise in issuing debt and which depend positively on the level of debt and negatively on the size of the firm (measured by the capital stock).

The main aim in Chapter 2 is to derive a Q model of investment which explicitly allows for tax exhaustion. This is done, and then estimates of tax-adjusted Q are presented using accounting and market data from a panel of 729 UK manufacturing companies between 1968 and 1986, together with estimates of tax exhaustion provided by the tax model. The data is described in detail in Appendix A. Chapter 2 then presents estimates of
the Q investment equation using this data. It examines in some detail the specification of the model for empirical purposes and also investigates the impact of tax and tax exhaustion on investment.

In addition, various financial regimes which the company might encounter are examined in some detail. The main determinants of the financial regime are corporate and personal taxes, non-negativity constraints on dividends and new equity issues and agency costs of debt. This analysis merges two strands of the literature on financial regimes which have looked separately at the impact of tax exhaustion and the impact of capital market imperfections resulting in agency costs.

Chapter 3 develops the forward-looking investment model of the firm based directly on the Euler equations from the optimisation process. The underlying model is the same as that used in Chapter 2. The chapter first develops a measure of the cost of capital which takes into account tax exhaustion. This allows analysis of the impact of tax exhaustion on the investment incentives facing the firm. The predictions of the model for the effects of different kinds of tax reform are then discussed. In particular, the model can be used to analyse the impact of permanent and temporary, and announced and unannounced, tax reforms. The investment equation is then estimated on the panel of company data, and a comparison is made between the performance of the model with and without tax exhaustion.

Chapter 4 turns attention towards other financial factors and in particular attempts to examine whether companies are in some way constrained in their investment decision the cost of external finance and by lack of internal funds. This chapter once more uses a similar model to that presented in Chapter 2, but concentrates more on identifying the nature of possible constraints. The theoretical model generates the result that cash flow can be a significant determinant of investment for all companies, not just those which are at a particular point in a financing hierarchy. The model is then tested to discover whether cash flow does play a significant role for all firms, and whether it is more important for some firms than others. This chapter also investigates whether these results might be due to factors other than financial constraints.
2. Brief Survey of Literature

As outlined above, the key elements of this thesis are the impact of taxation on firms' investment and financial decisions, the possible impact of financial constraints on firms' investment decisions, and the role of micro data in examining these issues. While the following chapters do investigate the more general question of the effects of taxes on firms' behaviour, there are two reasons why this more general question is not surveyed in any detail here. The first is to keep the survey to a reasonable length by concentrating on the more innovative features of this study. The second is that there already exist several excellent surveys of the relationship between taxes and investment (see, for example, Chirinko (1987) and (1988) and Nickell (1978) for general surveys, Chirinko and Eisner (1983) for a comparison of empirical estimates from several US macroeconomic models of the impact of taxes on investment and Auerbach (1983) for a survey of the literature on the cost of capital which includes discussion of the relationship between taxation and financial policy).

Investment and Taxation

The investigation here is based closely on the main two approaches in the literature, with taxes having an effect on investment via Tobin's Q (Tobin, 1969) or the cost of capital. Of course, as shown first by Abel (1979), both of these approaches are based on essentially the same neoclassical model of firm behaviour (and this is reflected below in that the two approaches are derived from the same model, specified in Chapter 2). This basic neoclassical model has been the basis of most theoretical work on investment since Jorgensen (1963).

The original formulation of the model predicted that investment should depend primarily on the cost of capital and output. The cost of capital, in turn, depends on interest rates, depreciation rates and taxes. Jorgensen and Hall (1967), produced promising results which suggested that the cost of capital was an important determinant of investment. However, Eisner and Nadiri (1968) showed that this result depended on the impact of the cost of capital being jointly estimated with the
impact of output. When the two terms were separated, the impact of the
cost of capital was weak. It is probably fair to say that it is this
latter result which has been dominant ever since.

Heated debates took place in the late 1960s and 1970s between Jorgensen
and many of his contemporaries concerning several aspects of the model.
These issues concerned such factors as the structure of the production
function and hence the role of the elasticity of substitution between
different factors of production (Eisner and Nadiri (1970), Eisner
(1969)), the role and interpretation of distributed lags in the
estimated equations - and, in particular, the nature of adjustment
costs: whether they should be treated as internal (eg. Lucas (1967),
Gould (1968) and Treadway (1969) or external (eg. Eisner and Strotz
(1963) and Precious (1987)), or, more fundamentally whether the effects
attributed to them should be attributed instead to delivery lags and the
irreversibility of investment decisions (eg. Nickell, 1974)).

Most fundamentally, however, the main problems arising from usual
formulations of the cost of capital model concern expectations and the
Lucas critique. Investment, by its very nature, is a forward-looking
decision; it must depend on what managers think will happen in the
future. But by forcing any expectational variable to be a function of
its past values as the usual formulation does, any news cannot be
incorporated into the equation. For example, if the government announces
a change in the corporation tax system to come into effect in one year's
time (as happened in the UK in 1984), this cannot be analysed in the
context of the usual cost of capital model. Neither can the model deal
with temporary changes to the tax system. Furthermore, the parameters on
past investment, output and the cost of capital reflect policy variables
rather than just structural parameters and so they suffer from the Lucas
(1976) critique.

The popularity of the Q model in the 1980s arose largely because it
apparently deals with these problems. However although the theoretical
marginal Q model generates an investment equation in which only
structural parameters are estimated (if one is prepared to accept one
particular form of adjustment costs), marginal Q is, in general
unobservable. The empirical specification which equates marginal and
average Q is only valid under strong assumptions of constant returns to scale, perfect competition in the output market and a perfect capital market (Hayashi (1982)). However, it is an important theoretical advantage of the average Q model that the stock market value of the firm (the numerator of average Q) in principle incorporates all currently held expectations about the firm's future performance.

Despite these advantages, however, average Q models have tended to perform badly in their empirical implementation (for typical results, see for example von Furstenburg (1977), Summers (1981), Poterba and Summers (1983) and Poret and Torres (1989)). In general, the explanatory power of the Q model is weak, serial correlation or dynamic structures including the lagged dependent variable are common, and other variables, contrary to the basic Q model, enter the equation significantly. It might be noted also that Abel and Blanchard (1986) attempted to estimate a model based on the Q approach which did not substitute average for marginal Q. However, their results were in line with those found in using average Q.

One other approach to modelling investment behaviour has been to estimate simple, ad hoc, models without any explicit overidentifying restrictions. Some such simple models (for example, Feldstein (1982)) have predicted a powerful role for taxation. However, parameter estimates have been found to be sensitive to small changes of specification (Chirinko (1986)). More important, though, is the absence of any structural interpretation of such models. To this extent, they suffer, like the basic cost of capital models, by being unable to predict the impact of tax reform because it is not known whether the parameters will change with the tax reform.

The approach followed in Chapter 3 of this thesis is to estimate the Euler equations for the optimising model of the firm behaviour. This follows the approach of Pindyck and Rotemberg (1983) and Shapiro (1986). However, while each of these papers included a measure of the rental price of capital, neither allowed for anticipated changes in the tax regime or of the tax position of firms.

It should be also be noted that almost all published empirical work on
investment models has used aggregate data and have therefore not exploited cross section variation between firms. One of the aims of this thesis is therefore to examine whether the poor results found in the estimation of average Q equations may be due to problems of aggregation. It should be noted, though, that previous results using panel data for Q models, at best, are mixed. Salinger and Summers (1983) find that the coefficient on Q in time series regressions for individual US firms takes the expected sign in almost all cases but is statistically insignificant nearly half the time. Hayashi and Inoue (1989), using a panel of Japanese firms find Q to be a significant determinant of investment, but also find that cash flow is also significant. Fazzari, Hubbard and Petersen (1988) find similar results for US firms.

Another failing of the literature on the effect of taxation on investment behaviour is that no paper deals adequately with differences in the structure of the tax incentives between firms. Part of these differences arise because of the different asset mix required for particular investments. In the UK, for example, a firm in a service industry seeking to acquire commercial property receives no capital allowances to match the depreciation of that property while a manufacturing firm purchasing a mixture of plant and machinery and industrial buildings receives a relatively high allowance - it is difficult to attribute these differences solely to differing depreciation rates.

In addition to these factors, the prevalence of tax exhaustion adds to cross sectional variation. This has certainly long been recognised in principle. For example, Domar and Musgrave (1944) and Stiglitz (1969) argued that tax asymmetries of the kind described above would discourage risk taking because the government does not share in the 'downside' risks. Thus, riskier firms would be less likely to undertake investment. However, this argument is too simple, as has been pointed out by Auerbach (1986) in the context of tax systems which would have been neutral with full loss offset. The reason is that the incentives facing firms depend crucially on the dynamics of tax exhaustion. Thus a firm approaching tax exhaustion may have a higher incentive to invest since some allowance can be claimed on the purchase of the asset in the current period, but payment of tax on the returns to the investment in
subsequent periods may be delayed. The calculations in Chapter 3 confirm that this factor can lead to marked variations in the cost of capital even under non-neutral tax systems.

The few empirical papers which have allowed for the impact of tax exhaustion on investment have invariably done so in a very simplified fashion, by setting the tax rate to zero in periods of tax exhaustion (for example, Anderson (1987) and Hayashi and Inoue (1989)). However, as should be clear, this implies the very strong assumption that the firm never expects to resume a tax paying position; if it does expect to resume paying tax the effect of tax exhaustion is only to delay tax effects rather than to cancel them altogether.

The most well known studies which attempt to quantify the extent of tax exhaustion are Cordes and Sheffrin (1983), Auerbach and Poterba (1986) and Altschuler and Auerbach (1987). Cordes and Sheffrin and Altschuler and Auerbach used confidential tax return data to estimate the importance of tax exhaustion in the US; the former have only a single cross section whereas the latter have a panel of around 2800 firms between 1971 and 1982. Auerbach and Poterba used accounting data to perform the same task but concluded that their estimates may have seriously underestimated the magnitude of aggregate loss carryforwards. The most detailed study then, Altschuler and Auerbach, conclude that tax exhaustion is of major importance for US firms. They find that about 50% of firms in 1982 had unused tax benefits that were being carried forward (slightly less than half of these unused benefits were tax losses; the rest were unused tax credits). Altschuler and Auerbach also estimated marginal tax rates on different forms of investment. They did so by using an autoregressive model to forecast future period of tax exhaustion facing firms (this contrasts with the approach used below in which (estimated) actual outturns are used. Using this approach they calculate that the effective average tax rate (essentially the statutory tax rate multiplied by a discount factor to represent the period over which losses are carried forwards) was only 2/3 of the full statutory rate. It should be noted that, at reasonable discount rates this implies very long periods of tax exhaustion. They also confirm that marginal tax rates are complex and may actually be reduced by tax exhaustion, as explained above.
Investment and Financial Factors

Most empirical models of company investment rely on the assumption of perfect capital markets. As is well known, in a world without taxes, one implication of this assumption is that firms are indifferent to funding their investment programmes from internal or external funds (Modigliani and Miller (1958)). However, there is a rapidly growing body of literature examining the possible existence of imperfections in capital markets and their effects on firms' financial and real decisions. The common theme underlying the various contributions is the lack of perfect substitutability between inside and outside financing. The existence of differential information and incentive problems make external finance more costly than internal finance. In this setting the availability of internally generated funds and possibly of assets that can be used as collateral may have an effect on investment decisions.

There may be advantages and disadvantages of both forms of external finance, debt and new equity, which we now discuss in turn, beginning with debt finance. There are different reasons why there may be a conflict between shareholders and debtholders, giving rise to agency costs of debt. In a seminal paper, Jensen and Meckling (1976) suggest that shareholders have an incentive to engage in projects that are too risky and so increase the possibility of financial distress and bankruptcy. If successful, the payoff to the owners of the firm is large. If unsuccessful, the limited liability provision of debt contracts implies that the creditors bear most of the cost. Myers (1977) suggests that if the firm is partly debt-financed, it may underinvest in the sense that it foregoes projects with a positive net present value. This problem is particularly severe when assets in place are a small proportion of the total value of the firm. Other areas of conflict between bondholders and shareholders are represented by the claim dilution resulting from the issue of additional debt and by the possibility that the firm may pay out excessive dividends financed by reduced investment.

Since potential creditors are assumed (by Jensen and Meckling, for example) to understand the incentives facing stockholders and are aware
of the risk of bankruptcy when loans are negotiated, ultimately the owner will bear the consequences of these agency problems in terms of a higher cost of debt. With asymmetric information about borrower quality, rationing may also occur (see, for example, Jaffe and Russell (1976) and Stiglitz and Weiss, (1981)). As a way to control the conflict between bondholders and shareholders and to minimise the agency cost of debt, bond covenants are observed, limiting the discretionary action of the owners regarding dividends, future debt issues, and maintenance of working capital (Smith and Warner (1979)). Debt covenants usually contain a maximum limit on the amount of dividends that can be paid out which depends positively upon accumulated earnings. Restrictions on the minimum value of the ratio between tangible assets and debt, working capital and debt and, finally, between interest payments and cash flow are also common. The greater is the amount of debt in the firm's capital structure, the more severe the incentive problems become, and the more likely it is that the firm will face financial distress and ultimately bankruptcy. Because of the less favourable terms on which debt can be obtained and because of the cost associated with tighter monitoring and bonding activities, agency costs are therefore likely to be increasing in the level of debt. On the other hand it is likely that such costs are a decreasing function of the level of past and present earnings and of assets, particularly if they are liquid, that can be used as collateral.

While agency costs make debt less attractive, the tax deductibility of interest payments make it more attractive. In the absence of such costs, and neglecting tax exhaustion, debt is preferred to retentions if \((1-m)/(1-z) > 1-\tau\), where \(m\) is the marginal personal tax rate on interest income, \(z\) the tax rate on capital gains and \(\tau\) the corporate tax rate (King, 1977). In the UK this inequality has usually been satisfied for most investors; while the highest rate of income tax has generally been above the corporate tax rate, higher rate taxpayers commonly also pay capital gains tax\(^2\) - moreover, a large proportion of equity is now held by financial institutions which have either no further tax liability (pension funds) or only relatively low liability (insurance companies).

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\(^{2}\) The existence of a high allowance for capital gains results in a zero marginal tax rate for investors earning less than £5,000 per year (in 1990) in the form of capital gains.
Turning to new share issues, they may disadvantageous because of transaction costs or asymmetric information and either advantageous or disadvantageous due to taxation. Informal evidence on transactions costs in the UK suggests that there are large fixed costs in issuing new equity (for example, typical transactions costs in raising £5 million would be around £250,000 compared to only £500,000 for raising £50 million). Ignoring issues of tax exhaustion, the tax advantage or disadvantage of new share issues relative to retentions depends on the system of company taxation and personal tax rates. Under a classical system, if the personal tax rate on dividends, \( m \), is greater than that on capital gains, \( z \), as is usually the case, new equity issues are more expensive (see, for example, King, 1977). In an imputation system, like the one in existence in the UK since 1973, the situation is more complex. New share issues are a cheaper source of finance for a full tax paying firm if \( \gamma = \frac{(1-m)}{(1-z)(1-c)} > 1 \), where \( c \) is the rate of imputation. This condition will be satisfied for institutional investors for whom \( m = z = 0 \) and for other investors with a low marginal tax rate on dividends.

Finally, new share issues may be more costly because of asymmetric information. Myers and Majluf (1984) suggest that, if managers have inside information, it may happen that it is so favourable that management, acting in the interest of old shareholders, will not issue new shares because they perceive them as being underpriced. Investors will therefore interpret the decision to issue new shares as a bad signal. In this case new equity finance can only be obtained at a premium, because of the adverse selection problem.

Up to this point in the discussion we have implicitly assumed that management acts in the interest of shareholders. Allowing for the possible divergence of interest between managers and outside shareholders provides an additional rationale for the disadvantage of external finance. If managers have a less than 100% ownership stake in the company, they will be encouraged to use a greater than optimal amount of the firm's resources in the form of perquisites (Jensen and Meckling (1976)). Such activities can be monitored by the outside shareholders, but such monitoring is costly and the insiders will
ultimately bear the cost in terms of a reduced price that prospective outside shareholders are willing to pay for a stake in the firm. This consideration suggests that the cost of outside financing is related to the stake of insiders and to the dispersion of outside ownership (since greater dispersion of ownership will increase the cost of monitoring). One result of this (as suggested by Rozeff (1982) and Easterbrook (1984)) is that firms may be obliged to pay higher dividends than they would otherwise wish to, so that the shareholders force the firm to submit itself to greater scrutiny from outside investors.

This raises the issue of whether factors such as the size of the firm or the age of the firm may influence the relative cost of external finance. While this has not yet been directly addressed in the literature, it seems likely that informational problems will be greater for small firms and young firms, because they are likely to be a small part of an investor's portfolio and as such, the investor may have little knowledge of them. Banks, for example, may have more information about larger and more developed firms. On the other hand, it is possible that smaller firms have a less diverse ownership, or that more of the firm is owned by the managers.

In another group of papers the role and consequences for investment of informational imperfections and control problems are more closely analysed. In this context the amount of net assets that can be used as collateral is a determinant of the agency cost of external finance and has an effect of investment. The particular informational asymmetry and the details about technology differ across papers, but the common theme is that insiders have less incentive to cheat and more incentive to act in the interest of outside investors when their stake in the project is greater (see Bernanke and Gertler (1989), Gertler (1988) and Gertler and Hubbard (1988)). The link between the firm's value and the fraction of entrepreneur wealth invested in the project is also emphasised by Leland and Pyle (1977). Since changes in borrower's net worth are likely to be procyclical, incentive problems may be particularly severe in a recession. This may lead to an asymmetric effect of financial variables on investment during the business cycle. The more precise modelling of the informational asymmetries and of the possibility of bankruptcy is clearly a strength of these models. However, they do not yield an
investment equation that explains how financial factors and expectations about firms' prospects jointly determine investment.

These considerations suggest that credit availability, cash flow and collateralisable assets may all have an effect on investment decisions. The literature on these issues has been conducted in the context of models with different structures concerning information and technology. One group of papers adds financial considerations to standard investment models based on the assumption of convex adjustment costs, usually estimated usually in their Q form. For example, credit rationing with an exogenously given ceiling can be easily added to Q models. If there are tax advantages to debt, firms will borrow up to capacity. Under the standard assumptions, outlined above, marginal Q will continue to equal average Q, with the caveat that the present value of the interest payments net of new debt issued should be added to the market value of shares in defining average Q. The present value of these flows can be approximated by the current value of the stock of debt. One could also assume that the maximum amount of debt is a fixed proportion of the capital stock (Summers, 1981) with basically the same result.

A more common approach is to include an additional cost term in the objective function, increasing in the level of debt, that summarises the agency or financial distress cost of debt, as in Auerbach (1984), Chirinko (1987), Hayashi (1985) and King (1987); Steigum (1983) and Bernstein and Nadiri (1986) make the cost of borrowing an increasing function of the debt/equity ratio. In this case, an internal solution for debt can be obtained. If the agency cost of debt is linear homogeneous in its arguments and the change (as opposed to the level) of debt does not enter the agency cost function, marginal Q again equals average Q. If the change in debt does appear in the agency cost function and the latter is not linear homogeneous, the difference between marginal and average Q depends upon the present and future values of the change and level of debt (Chirinko, 1987).

When personal taxation is taken into account and if capital gains are taxed less heavily than dividends, it is possible to distinguish between three financing regimes (see, for example, Auerbach (1984) and Hayashi (1985)). In regime 1, investment can be financed at the margin by
retentions, positive dividends are paid and no new shares are issued. In regime 3, the firm issues new shares and pays no dividend. In the intermediate case, regime 2, both dividends and new share issues are zero and the marginal source of finance is debt. A relationship between investment and tax-adjusted average Q, can be derived only in the regimes 1 and 3 (Hayashi (1985)). In regime 2 no such relationship exists and investment equals cash flow plus new debt issued. In this context, an increase in cash flow makes the probability that investment is financed at the margin by retentions more likely and this can be shown to increase investment. However, in this model conditional on Q, cash flow does not have an additional explanatory power in the regimes 1 and 3. In regime 2, increases in cash flow (and debt) translate into a one to one increase in investment and Q does not matter.

Fazzari, Hubbard and Petersen (1988) extended this analysis by including a premium for issuing new shares, based on the adverse selection argument put forward by Myers and Majluf (1984). The existence of this premium increases the cost differential between internal finance and new equity and it increases the likelihood that the firm will find itself at the point of discontinuity where all profits are retained, no dividends are paid and firm's future prospects are not good enough to induce it to issue new shares.

If a firm is in regime 1, the equilibrium value of marginal q is less than 1, which is the basis of the 'tax capitalisation' hypothesis associated, among others, with Auerbach (1979a) and (1979b) and Bradford (1981). King (1987) uses this result to derive a model of takeovers and mergers: essentially if equity is valued at less than the replacement cost of the assets held by the firm, a firm wishing to expand would find it cheaper to purchase assets by taking over another firm, rather than by doing so directly. In this paper, King assumes that a clientele effect holds in which for the marginal investor \((1-m)/(1-z)\)\(=\)\((1-\tau)\). Investors for whom \((1-m)/(1-z)\) > \((1-\tau)\) will prefer to purchase debt rather than equity, and those for whom \((1-m)/(1-z)\) < \((1-\tau)\) will prefer to purchase debt. This is therefore a Miller (1977) equilibrium, in which the firm is indifferent to issuing debt or equity, but there is a global optimum depending on the wealth and tax rates of investors.
An alternative method of deriving an internal optimum for debt is to rely on the possibility of tax exhaustion. Thus, De Angelo and Masulis (1981) and Mayer (1986) show that, under a classical corporation tax, the possibility of tax exhaustion tends to reduce the value of $\tau$, to $\tau^*$ say. If initially $(1-m)/(1-z) > (1-t)$, an increase in debt will reduce $\tau^*$ because of the deductibility of interest payments. Eventually an optimum is reached where $(1-m)/(1-z)=(1-\tau^*)$. This is not a Miller equilibrium: the optimum is reached only at a unique level of debt. Keen and Schiantarelli (1988) have, however, shown that under an imputation system the possibility of tax exhaustion cannot lead to an internal optimum for debt and for new equity issues unless $\tau = 0$, which implies that the firm is permanently tax exhausted with certainty - effectively there is no tax.

However, the Keen and Schiantarelli paper only considered whether tax exhaustion on its own could lead to an internal optimum. It did not allow for the possibility of other constraints on the firm, or for the possibility that the firm encounters agency costs of debt or new equity. Chapter 2 includes a discussion which merges these two strands of the literature to examine the financial regimes in which a firm may operate under an imputation system, with the possibility of tax exhaustion, and with the possibility of agency costs on debt.

3. Modelling the UK corporation tax

Since the 1960s the UK has witnessed a series of reforms to its corporation tax system. These include the creation of corporation tax as an independent tax in 1965, the gradual introduction of generous capital allowances in the early 1970s, the introduction of the partial imputation system in 1973 and stock relief in 1975, and the 1984 reforms which reduced the tax rate and capital allowances and abolished stock relief. A number of these reforms were designed with the specific intention of influencing the investment and financial decisions of firms.
These changes in the statutory tax system have been one reason why incentives facing firms in the UK have varied over time. One additional factor is the asymmetric treatment of taxable profits and losses and the possibility of not being fully able to utilise the imputation system because taxable profits are too low. These two forms of tax exhaustion have already been described. It should be noted here both that they were more common when the statutory tax system had relatively generous allowances (and so taxable profit was lower), and that they introduce a second source of variation in the incentives facing firms in both time series and cross section dimensions.

In order to study the impact of cross section variation in taxes on firms' decisions, it is necessary to use individual firm data. The most obvious source of such data is company accounting records. Unfortunately, however, company accounts generally do not record actual tax payments or liabilities; instead, they record some notional tax charge estimated by accountants. The most important feature of this notional tax charge is that it includes 'deferred' tax. The general principle here is that accountants wish to make provision for any tax which will eventually become due on the accounting 'profit' earned in the current period. If accountants' profit exceeds taxable profit (because, for example, of accelerated depreciation provisions under the tax system), the tax rate is applied to the former in order to calculate the charge recorded in accounts - the difference between that and the actual tax liability is deferred until some later date.

In order to estimate actual tax liabilities, it has therefore proved necessary to develop a model of the tax system. The aim is simply to apply the rules of the tax system to relevant accounting data on, for example, profit, investment and interest and dividend payments. This procedure permits estimates to be drawn, for each company in each year in which data is available, not only of the tax liability but also whether the company is either fully tax exhausted or ACT exhausted and the size of the losses and unrelieved ACT carried forward. It should be noted immediately that such an approach cannot give precise estimates of these items. The main difficulty is that, although it is in principle possible to model any complexity in the tax system, often the required data is not available, even in company accounts.
Some comparison with alternative methods of estimating tax liabilities should be made. The most common is simply to calculate the liability for some 'typical' firm (e.g. OECD (1985) and King (1986)). The problem with this approach is that the rich cross sectional variation captured by the approach used here is lost, since there is no way of distinguishing firms in different tax positions. A second approach is used by the Inland Revenue (1982), who have developed a model for forecasting purposes based on actual tax returns. This approach is, unfortunately, unavailable since the data used is held to be confidential. In any case, since the Inland Revenue model needs to estimate accounting measures given tax returns and knowledge of the tax system, it does not have any real advantage over the model described here (a better model would combine both sources of data). In addition, Higson (1986) has pointed out that, under certain circumstances, tax liabilities can be estimated directly from the tax data in accounts. While this gives reasonable estimates of tax liabilities for a number of years, it is not as general as the model described in Appendix B (in particular, it cannot be used for forecasting the tax positions of firms). Nevertheless, this method has been used as a means of checking the estimates provided by the model used here, and the measures of the cost of capital and tax-adjusted Q from chapters 2 and 3 have also been computed using the estimates provided from this methodology. The general result of this comparison is that, where comparisons can be made, the results from the model used here appear reasonable. Finally, it should be noted that the kind of model described in Appendix B has been attempted elsewhere, although not in as great detail (see, for example, Goudie (1984), Robson (1985) and Leavis and Morgan (1985)).

The remainder of this section details the main features of the UK corporation tax system since 1968 and presents some results from the model giving estimates of the scale of tax exhaustion over the period 1971 to 1986. The data covers 729 UK quoted manufacturing companies. Each company has at least 4 years of data and 125 companies have data for all 16 years. The data is described in detail in Appendix A.

Table 1.1 gives the main features of the UK corporation tax system. It can be seen that apart from a relatively stable period for 10 years in
the middle of the period, there has been substantial change. In particular, the system began and ended as an approximation to economic profit, with allowance rates roughly equal to economic depreciation, but the stable period could be characterised as being closer to a tax on company cash flow. The classical system was replaced by the imputation system in 1973.

Table 1.1 Main Features of the UK Corporation Tax, 1968-1986

<table>
<thead>
<tr>
<th>Year</th>
<th>Tax Rate</th>
<th>Rate</th>
<th>FYA</th>
<th>WDA</th>
<th>FYA</th>
<th>WDA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1968</td>
<td>45</td>
<td>-</td>
<td>0</td>
<td>20</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>1969</td>
<td>42.5</td>
<td>-</td>
<td>0</td>
<td>20</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>1970</td>
<td>40</td>
<td>-</td>
<td>60</td>
<td>25</td>
<td>30</td>
<td>4</td>
</tr>
<tr>
<td>1971</td>
<td>40</td>
<td>-</td>
<td>80</td>
<td>25</td>
<td>30</td>
<td>4</td>
</tr>
<tr>
<td>1972</td>
<td>40</td>
<td>-</td>
<td>100</td>
<td>25</td>
<td>40</td>
<td>4</td>
</tr>
<tr>
<td>1973</td>
<td>52</td>
<td>30</td>
<td>100</td>
<td>25</td>
<td>40</td>
<td>4</td>
</tr>
<tr>
<td>1974</td>
<td>52</td>
<td>33</td>
<td>100</td>
<td>25</td>
<td>50</td>
<td>4</td>
</tr>
<tr>
<td>1975</td>
<td>52</td>
<td>35</td>
<td>100</td>
<td>25</td>
<td>50</td>
<td>4</td>
</tr>
<tr>
<td>1976</td>
<td>52</td>
<td>34</td>
<td>100</td>
<td>25</td>
<td>50</td>
<td>4</td>
</tr>
<tr>
<td>1977</td>
<td>52</td>
<td>34</td>
<td>100</td>
<td>25</td>
<td>50</td>
<td>4</td>
</tr>
<tr>
<td>1978</td>
<td>52</td>
<td>33</td>
<td>100</td>
<td>25</td>
<td>50</td>
<td>4</td>
</tr>
<tr>
<td>1979</td>
<td>52</td>
<td>30</td>
<td>100</td>
<td>25</td>
<td>50</td>
<td>4</td>
</tr>
<tr>
<td>1980</td>
<td>52</td>
<td>30</td>
<td>100</td>
<td>25</td>
<td>50</td>
<td>4</td>
</tr>
<tr>
<td>1981</td>
<td>52</td>
<td>30</td>
<td>100</td>
<td>25</td>
<td>75</td>
<td>4</td>
</tr>
<tr>
<td>1982</td>
<td>52</td>
<td>30</td>
<td>100</td>
<td>25</td>
<td>75</td>
<td>4</td>
</tr>
<tr>
<td>1983</td>
<td>50</td>
<td>30</td>
<td>100</td>
<td>25</td>
<td>75</td>
<td>4</td>
</tr>
<tr>
<td>1984</td>
<td>45</td>
<td>30</td>
<td>75</td>
<td>25</td>
<td>50</td>
<td>4</td>
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<tr>
<td>1985</td>
<td>40</td>
<td>30</td>
<td>50</td>
<td>25</td>
<td>25</td>
<td>4</td>
</tr>
<tr>
<td>1986</td>
<td>35</td>
<td>29</td>
<td>0</td>
<td>25</td>
<td>0</td>
<td>4</td>
</tr>
</tbody>
</table>

Notes:
1. 'FYA' refers to that percentage of capital expenditure which can be treated as an immediate expense and 'WDA' the percentage rate at which the remainder can be written off against tax. The 'WDA' for plant and machinery is on a 'reducing balance' or exponential basis, and that for industrial buildings is on a 'straight line' basis.
2. Commercial buildings have never received any capital allowances.
3. Stock relief was introduced in 1975, retroactive to 1973. The system was changed in 1981 and abolished in 1984.
4. The partial imputation system was introduced in 1973. Before that a classical system was in operation.
Table 1.2 confirms that tax exhaustion was most important during the period of generous allowances. The precise description of full tax exhaustion and ACT exhaustion are left to Chapter 2. The basic definitions are that, in the former case, the firm is in the position of carrying forward unused tax losses to offset against subsequent profits, and in the latter case, it is carrying forward unrelieved ACT since its gross dividend payment is higher than taxable profit. Firms which have been able to carry taxable losses or unused ACT back to offset against earlier profits are therefore excluded from this definition (because for them the tax system is essentially acting symmetrically in that they are able to receive an immediate rebate on their losses, or are able to claim all the benefit of the ACT offset).

Table 1.2 Importance of tax exhaustion in the UK

<table>
<thead>
<tr>
<th>Year</th>
<th>Size of sample</th>
<th>Case 1</th>
<th>Percentage of sample</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1971</td>
<td>190</td>
<td>1.3</td>
<td>0.5</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1972</td>
<td>297</td>
<td>5.0</td>
<td>4.7</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1973</td>
<td>374</td>
<td>4.9</td>
<td>2.8</td>
<td>10.1</td>
<td>10.1</td>
<td>8.5</td>
</tr>
<tr>
<td>1974</td>
<td>409</td>
<td>23.4</td>
<td>19.9</td>
<td>47.9</td>
<td>39.2</td>
<td>11.1</td>
</tr>
<tr>
<td>1975</td>
<td>431</td>
<td>20.1</td>
<td>5.3</td>
<td>36.6</td>
<td>36.6</td>
<td>8.5</td>
</tr>
<tr>
<td>1976</td>
<td>655</td>
<td>18.1</td>
<td>7.1</td>
<td>35.2</td>
<td>10.0</td>
<td>11.1</td>
</tr>
<tr>
<td>1977</td>
<td>673</td>
<td>21.6</td>
<td>7.9</td>
<td>39.7</td>
<td>11.1</td>
<td>7.9</td>
</tr>
<tr>
<td>1978</td>
<td>685</td>
<td>19.7</td>
<td>4.9</td>
<td>37.7</td>
<td>7.9</td>
<td>-</td>
</tr>
<tr>
<td>1979</td>
<td>693</td>
<td>26.3</td>
<td>10.2</td>
<td>44.3</td>
<td>11.0</td>
<td>-</td>
</tr>
<tr>
<td>1980</td>
<td>690</td>
<td>27.2</td>
<td>8.6</td>
<td>46.5</td>
<td>8.4</td>
<td>-</td>
</tr>
<tr>
<td>1981</td>
<td>673</td>
<td>26.1</td>
<td>6.3</td>
<td>43.6</td>
<td>4.7</td>
<td>-</td>
</tr>
<tr>
<td>1982</td>
<td>660</td>
<td>27.2</td>
<td>5.1</td>
<td>41.8</td>
<td>4.5</td>
<td>-</td>
</tr>
<tr>
<td>1983</td>
<td>639</td>
<td>26.1</td>
<td>2.4</td>
<td>40.3</td>
<td>3.8</td>
<td>-</td>
</tr>
<tr>
<td>1984</td>
<td>602</td>
<td>24.1</td>
<td>1.6</td>
<td>35.0</td>
<td>4.0</td>
<td>-</td>
</tr>
<tr>
<td>1985</td>
<td>565</td>
<td>20.7</td>
<td>1.7</td>
<td>28.3</td>
<td>2.1</td>
<td>-</td>
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<tr>
<td>1986</td>
<td>492</td>
<td>14.9</td>
<td>1.3</td>
<td>20.3</td>
<td>1.1</td>
<td>-</td>
</tr>
</tbody>
</table>

Notes.
1. Estimates refer to accounting years ending in the calendar year shown.
Table 1.2 provides some justification for examining the role of tax exhaustion an company investment and financial behaviour. In 1980, for example, it is estimated that 27% of the companies in the sample were fully tax exhausted and 47% ACT exhausted. Even allowing for some margin of error, this suggests that tax exhaustion was at that time an important feature of the tax system. Further, these numbers are not untypical of the proportions in nearby years. Indeed, such was the stock of losses carried forward that it is estimated that 15% of the sample were still fully tax exhausted and 20% ACT exhausted even by 1986, two years after the tax base was dramatically increased.

The analysis in Chapter 3 suggests that the most important aspect of tax exhaustion for the cost of capital is that firms move into and out of positions in which they pay tax (for example, they may gain an allowance from purchasing an asset in one year, but face a delayed tax payment on the return earned a year later). Table 1.2 therefore also gives an indication of the number of companies moving into and out of tax exhaustion. While these proportions of the sample are rather lower, they do indicate that there has been considerable movement into and out of tax exhaustion. Finally, it is interesting to note that, on the estimates of the model, less than 10% of companies were at no point either fully tax exhausted or ACT exhausted.
CHAPTER 2

INVESTMENT, Q AND FINANCIAL REGIMES

2.1 Introduction

Chapters 2 and 3 of this thesis are concerned with the impact of taxation on the investment decision and the financial structure of the firm. They are linked, not just by subject matter, but by a common underlying model. The main innovative feature of this model is that it explicitly allows for variation in the effective tax rate and the effective price of capital goods across firms by introducing the possibility of tax exhaustion. In addition, since the model is tested on UK data, it also incorporates the imputation system of taxing dividend payments, which has been in operation in the UK since 1973 (and which introduces a second form of tax exhaustion)\(^1\). Apart from the detailed analysis of the tax system, the model makes the standard assumption that firms aim to maximise the wealth of the existing shareholders. In order to concentrate on tax effects, it generally assumes that the firm is a price-taker, both for the output price and the price of capital goods. However, the model does include the possibility that the firm faces agency costs in issuing debt, and in Chapter 3, it also allows for the possibility of imperfect competition in the product market.

This chapter first uses the model to analyse the possible financial regimes which the firm may face. As discussed in Chapter 1, it is now standard in the literature to consider three possible regimes, corresponding to the marginal source of finance being retained earnings,

\(^1\)Imputation systems are very common in Europe, being in existence, for example, in Belgium, Denmark, France, Greece, Ireland, Italy and Spain as well as the UK, but have not so far been introduced in the US. Note also that 'ACT exhaustion', defined below, is normally a feature of such systems; this is the role of the *precompte* in France, for example.
debt or new equity (see, for example, Auerbach (1984) and Hayashi (1985)). Allowing for tax exhaustion and the imputation system adds several complications to this basic analysis. The most obvious of these is that the common view that retained earnings are a cheaper source of finance than new equity because of the taxation of dividend payments may not hold under the imputation system. This is because it is quite possible that the effective tax rate on dividends is, in effect, negative$^2$.

This possibility turns the usual 'dividend puzzle' on its head. Under certain values of personal tax parameters, the puzzle is no longer 'why do firms simultaneously issue new shares and pay dividends?', but rather 'why do firms not continuously issue new shares and pay out the funds raised as dividends?'. This question has rarely been addressed in the literature, and when it has been, it is usually dealt with by adding some upper bound to dividend payments. For example, Edwards and Keen (1985) justify such an upper bound by reference to legal restrictions on dividend payments. However, the problem with this approach is that firms are rarely, if ever, observed to be at this upper bound. An alternative explanation is therefore explored here, namely that the possibility of tax exhaustion creates an internal optimum at which firms may be observed issuing a finite amount of new shares while simultaneously paying dividends. However, the role played by tax exhaustion is wider than this, and together with the presence of agency costs can lead to a fourth regime in which the marginal cost of each source of finance is the same.

This chapter goes on to consider the Q model of investment, which can be derived from the basic wealth-maximising approach. Once more, the inclusion of tax exhaustion adds a number of new features to the model.

$^2$For example, in the UK the payment of a dividend by a company which is not tax exhausted to a pension fund will not affect the total tax liability of the company since the additional tax on the dividend (seen as a prepayment of the shareholder's income tax) can be offset against the corporation tax charge. However, because the pension fund is not taxed, it can also reclaim the tax associated with the dividend. The net effect of the dividend is therefore to reduce total taxation.
notably concerning the role played by expectations. In the standard Q model, marginal Q is equal to average Q and can therefore be proxied by the ratio of the observable market value of the firm to the replacement value of the capital stock, where the market value of the firm is assumed to incorporate all currently held expectations of the firm's future performance. However, the presence of tax exhaustion adds two other state variables (different losses carried forward by the firm), the shadow value of which must be deducted from the numerator of average Q to arrive at marginal Q. In the empirical testing of the Q model, then, much attention is paid to the empirical modelling of these terms (which also appear in the measures of the effective tax rate and the effective price of capital goods).

Obviously, the main aim of the empirical work is to assess the impact of taxation on investment via the Q model using a large panel of firms over a reasonably long period. However, part of the importance of using panel data is to allow for cross section variation in effective tax rates, and so to be able to test the importance of tax exhaustion for investment decisions by firms. As discussed below, however, it is not clear that the Q model is well-suited to this task. The main problem is that all tax effects are incorporated into the single variable, Q, along with all other effects concerning, for example, expectations of future demand and supply conditions. Quite large variations in tax parameters can nevertheless be dwarfed by the variation in the stock market value of the firm.

Section 2.2 describes the model used in this chapter and in Chapter 3. Section 2.3 examines the first order conditions for investment and the capital stock to derive a Q model of investment. The role of tax exhaustion in affecting the tax parameters, as well as the role of leasing, is discussed. Section 2.4 turns to the optimal financial policy of the firm in the model. A number of financial regimes are explored, which depend on the interaction of the tax system, especially through tax exhaustion, and agency costs of debt. Section 2.5 considers some of the empirical issues involved in constructing values of Q and tax parameters, especially the complications due to effective tax rates which depend on expected future profitability. Section 2.6 presents some descriptive data on Q and the various tax parameters in the model and
section 2.7 presents the empirical results derived from estimating the Q model on the panel of 729 companies. Section 2.8 briefly concludes.

2.2 Maximisation with Tax Asymmetries and Leasing

The model used here to analyse firms' investment and financial decisions is based on the standard assumption of wealth maximisation for the existing group of shareholders. In characterising the firms' maximisation problem we start from the identity of sources and uses of funds:

\[ D_t = \dot{Y}_t - p_t I_t + V_t^N + B_{t+1} - (1+i)B_t - X_t - A(B_t, K_t) - p_t R^L_t k^L_t \]  

where \( D_t \) denotes dividends paid, \( \dot{Y}_t \) pre-tax profits, \( p_t \) the price of output and \( p_t \) the price of investment goods, \( I_t \) the number of new machines purchased, \( V_t^N \) new equity issues, \( B_{t+1} \) the amount of one period debt issued during period \( t \) and redeemed during period \( t+1 \), \( i_t \) the interest rate, \( X_t \) corporation tax payments, \( A_t \) agency costs on debt, \( R^L_t \) the rental rate on leased machines and \( k^L_t \) their stock (note that \( k^L_t \) may be negative, implying that the firm is a lessor), all in period \( t \).

Agency costs are written as \( A(B_t, K_t) \), where it is assumed that \( A_D > 0 \), \( A_{BB} > 0 \), \( A_K < 0 \). Their introduction can be justified on the basis of the arguments developed in the finance literature concerning asymmetric information about the firm (managers know more than potential investors) and problems of management control (investors want to force managers to act in the interests of the investors, rather than the managers), which were briefly reviewed in Chapter 1. Their main role in this model will be in the discussion of financial structure.

The firm is, for the moment assumed to operate in a perfectly competitive product market and is assumed to face internal costs in adjusting the capital stock, \( G(I_t, I^L_t, K_t) \) which are convex in the total
level of investment goods acquired by the firm through direct purchase or leasing ($I_t^L$). If the firm is a lessee, so that $I_t^L$ is positive, then the difference between leasing and purchasing capital is assumed to be purely financial: thus $I_t$ and $I_t^L$ enter the adjustment cost function in the same way. However, if the firm is a lessor, so that $I_t^L$ is negative, it is possible that $I_t^L$ has a different impact on adjustment costs compared to $I_t$. Note that $K_t$ is the sum of both purchased and leased capital.

Production during period $t$ depends upon the capital stock available at the beginning of the period, $K_t$ and, following Auerbach (1986) and Edwards and Mayer (1987), is subject to a random shock $\alpha_t$ that is not observed at the beginning of the period when decisions are taken. For simplicity $\alpha_t$ is assumed to be i.i.d. with known density function $h(\alpha_t)$. It is assumed that variables factors (in particular labour) are paid their marginal product and hence are optimised out of this model. Hence, the definition of pre-tax profits, $\Pi_t$, is defined from:

$$\Pi_t = U_t(\alpha_t, K_t) - G_t(I_t^L, K_t)$$

where $U_t > 0$. The equation of motion for $K_t$ is

$$K_{t+1} = (1-\delta)K_t + I_t + I_t^L$$

and for $K_t^L$ is

$$K_{t+1}^L = (1-\delta)K_t^L + I_t^L$$

The formalisation of the tax system here builds on the description in Edwards and Keen (1985) and Mayer (1986). Under an imputation system, corporate tax liabilities arise from two sources. First, the firm must pay corporation tax on its operating profit minus current allowances.

3Although the total wage bill becomes relevant when imperfect competition is introduced in Chapter 3.
whenever this taxable profit, $\psi_t$, is positive. $\psi_t$ is defined as:

$$\psi_t = p_t^Y Y_t - \Gamma_t - iB_t - L_t - p_t^R R_t^L$$

where $\Gamma_t$ represents depreciation allowances for tax purposes and $L_t$ losses brought forward from the previous period. More precisely, $\Gamma_t$ equals the sum of first year allowances and depreciation allowances on the tax written down value for tax purposes of the accumulated capital stock at the beginning of the period, $K_t^T$. Thus,

$$\Gamma_t = (1-j)p_t I_t + \delta^T K_t^T$$

where $(1-j)$ is the fraction of investment expenditure that can be subtracted from profits in the year in which it is incurred, $\delta^T$ is the rate at which past investment can be depreciated for tax purposes and

$$K_{t+1}^T = (1-\delta^T)K_t^T +jp_t I_t$$

Losses represent the first non-linearity in the corporate tax system: if $\psi_t < 0$, then generally an immediate tax rebate is not paid, but the taxable loss must be carried forward indefinitely, and without an interest markup, to set against future taxable profits.\(^4\)\(^5\)

Second, under the UK imputation system, the firm makes an advance payment of a fraction of the shareholder’s income tax at a rate $c$ (the imputation rate) on grossed up dividends $D_t/(1-c)$. This payment is

---

\(^4\)Strictly, under the UK corporation tax system, losses can be carried back one period to set against the previous year’s profit. In this case the system is essentially symmetrical in that an immediate rebate may be claimed. Losses due to capital allowances may be carried back up to three years. These carry back provisions are ignored in the model, although they are incorporated in the empirical work, where the crucial issue is whether losses are carried forward.

\(^5\)In the UK, losses may be carried forward indefinitely, and this is what has been modelled here. However, this is not always the case: for example, losses may only be carried forward 15 years in the US and 5 years in France, Italy and Germany (until 1990, when indefinite loss carry forward was introduced).
termed Advanced Corporation Tax (ACT). ACT can be deducted from the main component of the corporation tax if gross dividends do not exceed taxable profits. This upper limit introduces a second type of non-linearity in the tax system which is of particular interest concerning the choice between raising funds through retentions or new equity issues. If gross dividends exceed taxable profits, the "unrelieved ACT" (defined here as $U_{t+1}$) must again be carried forward indefinitely, without an interest markup, to be set against the main component of corporation tax in later years.

The main features of the tax system can therefore be summarised by the following expressions for $X_t$, $L_{t+1}$ and $U_{t+1}$.

$$X_t = \tau \max \{ \psi_t, 0 \} + \frac{c}{1-c} D_t - \min \left\{ \frac{c}{1-c} D_t + U_t, \max \{ \psi_t, 0 \} \right\}$$  \hspace{1cm} (2.8)

$$L_{t+1} = \max \{ -\psi_t, 0 \}$$ \hspace{1cm} (2.9)

$$U_{t+1} = \max \left\{ \frac{c}{1-c} D_t + U_t - \max \{ \psi_t, 0 \}, 0 \right\}$$ \hspace{1cm} (2.10)

where $\tau$ is the statutory rate of corporation tax. In (2.8), the first term represents the principal corporation tax charge and the second is ACT. The third term shows the reduction in the corporation tax charge available under the imputation system. The first term net of the third term is known as the "mainstream corporation tax" charge (MCT). If $\psi_t < 0$, losses, $L_{t+1}$, are carried forward. Similarly, unrelieved ACT, $U_{t+1}$, is carried forward if gross dividends exceed taxable profits. Note that classical corporation tax (in force in the US, for example) is a special case of (2.8)-(2.10) in which $c = 0$. Clearly taxable losses, $L_{t+1}$, and unrelieved ACT, $U_{t+1}$, represent two state variables additional to the standard model.

As for losses, in the UK ACT not offset against current taxable profit may be set against taxable profit of earlier periods (since 1984 up to 6 years earlier). Again, these carry back provisions are ignored in the model, but are allowed for in the empirical work.
Given this characterisation of the tax system, there are three possible tax positions in which the firm might find itself.

(a) Full Tax Exhaustion:

Full tax exhaustion occurs when \( \psi_t \leq 0 \). The firm has no mainstream tax liability and therefore, since \( \text{ACT} \) cannot be set against corporation tax, it simply adds to the stock of unrelieved \( \text{ACT} \), \( U_{t+1} \). Using (2.1), (2.2), (2.5) and (2.8), the net dividend paid is:

\[
D_t = (1-c) \left\{ p_t^{V_t} I_t - p_t I_t + V_t^N + B_t^{t+1} - (1+1_t)B_t - A_t K_t - p_t R_t L_t \right\} \quad (2.11a)
\]

while, using (2.9) and (2.10), tax losses and unrelieved \( \text{ACT} \) evolve according to:

\[
L_{t+1} = - p_t^{V_t} t_t + \Gamma_t + 1_t B_t + L_t - p_t R_t L_t \quad (2.12a)
\]

\[
U_{t+1} = U_t + c \left\{ p_t^{V_t} I_t - p_t I_t + V_t^N + B_t^{t+1} - (1+1_t)B_t - A_t K_t - p_t R_t L_t \right\} \quad (2.13a)
\]

From (2.2), (2.5) and (2.11a) this case arises for values of the stochastic component of profits, \( \alpha_t \), for which \( \alpha_t \leq \alpha_t^* \), where \( \alpha_t^* \) is defined from

\[
p_t^{V_t} (\alpha_t, K_t) = \Gamma_t + L_t + 1_t B_t - A_t K_t - p_t R_t L_t - p_t G_t I_t K_t \quad (2.14)
\]

(b) \( \text{ACT} \) exhaustion:

\( \text{ACT} \) exhaustion occurs when the mainstream corporation tax liability is positive (ie. \( \psi_t > 0 \)) but is insufficient to absorb all current and accumulated unrecovered \( \text{ACT} \): ie. \( cD_t / (1-c) + U_{t+1} > \psi_t \). In this case

\[
D_t = (1-c) \left\{ \left[ 1-(\tau-c) \right] p_t^{V_t} I_t - p_t I_t + V_t^N + B_t^{t+1} - \left\{ 1-(1+1_t) \right\} t_t B_t \\
- A_t K_t - \left[ 1-(\tau-c) \right] p_t R_t L_t + (\tau-c)(\Gamma_t + L_t) \right\} \quad (2.11b)
\]

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\[ L_{t+1} = 0 \] 

(2.12b)

and

\[ U_{t+1} = U_t + c\left\{ -(\tau-c)p_t^{\tilde{P}} - p_t I_t + V_t^N + B_{t+1} - \left[ 1-(\tau-c)I_t \right] B_t \right. \]

\[-A(B_{t+1},K_t) + (\tau-c)p_t R_{t+1}^L + (1+\tau-c)\left( \Gamma_{t+1}^t + L_t \right) \right\} \] 

(2.13b)

Using (2.1), (2.2), (2.5), (2.8) and (2.10), this is the outcome if \( \alpha_t > b_t \), where \( b_t \) is defined from\(^7\)

\[ c(\tau-c)p_t^{\tilde{P}}(b_t,K_t) = U_t + c\left\{ -(\tau-c)p_t I_t + V_t^N + B_{t+1} - \left[ 1-(\tau-c)I_t \right] B_t \right. \]

\[-A(B_t,K_t) + (\tau-c)p_t R_t^L + (1+\tau-c)\left( \Gamma_t + L_t \right) + (\tau-c)p_t^G (I_t, I_t^L, K_t) \right\} \] 

(2.15)

\((c)\) Full tax-paying:

The only remaining possibility is that \( \psi_t > 0 \) and \( c_D/(1-c)^t + U_t \neq c\psi_t \), which occurs when \( \alpha_t > b_t \). The firm is then a full taxpayer. In this case

\[ D_t = (1-\tau)p_t^{\tilde{P}} - p_t I_t + V_t^N + B_{t+1} - [1+(\tau-c)]I_t B_t + U_t \]

\[-A(B_t,K_t) - p_t R_t^L + \tau(\Gamma_t + L_t) \] 

(2.11c)

and

\[ L_{t+1} = U_{t+1} = 0 \] 

(2.12c, 2.13c)

In addition to describing the corporate tax system in detail, we also wish to examine the role of other financial factors in firms' decisions. Agency costs on debt have already been included in the model. In

\(^7\)It can be shown that \( D_t \geq 0 \Rightarrow b_t \geq a_t \). Since below we impose the condition that \( D_t \geq 0 \), we can therefore ignore the possibility that \( b_t < a_t \).
addition to these agency costs, it is necessary to specify the legal constraints under which firms operate. Following much of the literature (see, for example, King, 1974 and 1977, and Edwards and Keen, 1985), the conditions of non-negative dividends and share issues are imposed. Under UK law, the former are prohibited. The latter were also prohibited until 1981; since then, payments for repurchase are taxed as capital gains rather than income only if certain stringent conditions are met (Gammie, 1982). As discussed in King (1977), the constraint that debt is non-negative is also imposed. This is more problematic, but without it, for some possible configurations of tax parameters, the firm would set \( B_t \) and \( V_t^N \) infinitely large.

More generally, there exists a large literature examining why dividends are paid given that they are tax disadvantaged (especially in the US which operates a classical system). Several reasons have been offered, such as that dividends play a role in signalling the prospects of the firms to investors, or that by paying dividends, firms are forced to subject themselves to the scrutiny of the market (Easterbrook, 1984, and Rozeff, 1982). For a critical survey of this literature see Edwards (1987). The important point here, though, is simply that firms may be forced to pay some positive level of dividends, for reasons which are not fully understood. (As shown in chapter 4, for the sample of companies analysed in this study, dividends were positive 94% of the time.) For the purposes of this model, we simply impose the constraints that:

\[
D_t \geq d_t; \quad V_t^N \geq 0; \quad B_{t+1} \geq 0 \quad (2.16)
\]

Ignoring the issues which lead to the possibility that \( d_t = 0 \), these constraints can be written simply with \( d_t = 0 \) on the grounds of the legal constraints. This has no effect on the results.

A final feature of the financial side of the company which is introduced into the model is to add costs of issuing new shares. In particular, it is assumed that there is a premium on new share issues which is a proportion \( \omega_t \) of the value of the new shares. This premium may reflect transactions costs or it may reflect the possibility that issuing new shares signals that the company is a lemon, based on the arguments of Myers and Majluf (1984), discussed in Chapter 1.
To complete the specification of the firm maximisation problem we add the usual capital market equilibrium condition:

\[(1-m)R_t V_t = \theta E_t(D_t) + (1-z) \left\{ E_t(V_{t+1}) - V_t + V_t^N(1+\omega_t) \right\} \quad (2.17)\]

where \(V_t\) is the market value of equity at the beginning of period \(t\), \(R_t\) is the (gross) return on comparable assets, \(m\) the marginal rate of personal income taxation, \(z\) the tax rate on capital gains, \(\theta\) is a parameter representing the tax on net dividend payments (so that \((1-\theta)\) is the tax rate payable by shareholders on net dividends received; note that \((1-\theta)\) is not constrained to be positive), and \(E_t[\cdot]\) denotes conditional expectations given the information set available at the beginning of period \(t\). Note that it is convenient to assume that \(\alpha_t\) is the only item occurring in period \(t\) not known at the beginning of the period. Thus all decisions regarding investment and leasing are taken at the beginning of the period. This implies that \(K_{t+1}^T, K_{t+1}^L\) and \(K_{t+1}\) are all known at the beginning of period \(t\). It is also convenient to assume that dividends do not depend on \(\alpha_t\); it is therefore assumed that new debt issues are adjusted to maintain the equality of sources and uses of funds.

If \(V_t\) is the maximum value function defined on the predetermined variables at the beginning of the period, then rearranging (2.17), the firm's problem is to maximise:

\[M = \frac{V_t}{\rho_t} = \gamma E_t(D_t) - V_t^N(1+\omega_t) + E_t \left\{ V_{t+1}(K_{t+1}^T, K_{t+1}^L, L_{t+1}, U_{t+1}, B_{t+1}) \right\} \quad (2.18)\]

subject to the definitions (2.1), (2.2), (2.5), (2.6), (2.8), the equations of motions (2.3), (2.4), (2.7), (2.9), (2.10), and the constraints (2.16). The latter are associated with multipliers \(\lambda_t^D, \lambda_t^N\).

---

8This assumption avoids complications in the section below on optimal financial policy, which would arise if \(\lambda_t^D\) were state dependent.
and $\lambda_{t+1}^B$. In (2.18), $\gamma$ is sometimes referred to as the tax discrimination variable, since it is essentially the tax paid on a unit of dividends compared to the tax paid on a unit of new equity. It, and the discount factor, $\rho_t$, are defined as

$$\gamma = \frac{(1-\alpha)/(1-c)}{(1-z)}$$

and

$$\rho_t = \left\{ \frac{(1-\alpha)R_t}{1-c} \right\}^{-1}$$

For simplicity, we assume that $R_t$ is equal to the market rate of interest, $r_t$. Weighting (2.12a)-(2.12c) by their corresponding probabilities and collecting terms, expected dividends can be written as:

$$E_t(D_t) = \left\{ (1-\gamma)\left[(1-\gamma)H(a_t)+\gamma(b_t)\right] \right\} - p^\gamma_t \left[ (I_t^L K_t^L I_t^L ) - p_t^L I_t^L K_t^L I_t^L B_t \right]$$

$$+ \left\{ 1-cH(b_t) \right\} \left[ V_t^N + B_{t+1}^N - B_t - p_t^L I_t^L - A(B_t, K_t) \right] \left\{ 1-H(b_t) \right\} U_t$$

$$+ \left\{ (1+c)H(b_t) \right\} \left[ (1-c)\gamma(1-c)H(a_t) \right] \left[ (1-j) p_t^L I_t^L + \delta^T K_t^T I_t^L \right]$$

$$+ \left\{ (1+c)H(b_t) \right\} \left[ (1-c)\gamma(1-c)H(a_t) \right] \left[ (1-j) p_t^L I_t^L + \delta^T K_t^T I_t^L \right]$$

$$+ N(\tau, c, a_t, b_t) p^\gamma_t (\alpha_t, K_t)$$

(2.19)

where $H$ denotes the cumulative density function for $\alpha_t$. The value function, $E_t(V_{t+1}(\cdot))$ in (2.18) can also be written as the sum of three components, each relating to one of the possible tax regimes, and substituting for $L_{t+1}$ and $U_{t+1}$ the expressions in (2.13a)-(2.13c) and (2.14a)-(2.14c). Thus,

$$N(\tau, c, a_t, b_t) p^\gamma_t (\alpha_t, K_t) = (1-c) \int_{-\infty}^a p^\gamma_t (\alpha_t, K_t) h(\alpha_t) d\alpha_t$$

$$+ (1-c)(1-\gamma+c) \int_{-\infty}^a p^\gamma_t (\alpha_t, K_t) h(\alpha_t) d\alpha_t + (1-\gamma) \int_{-\infty}^a p^\gamma_t (\alpha_t, K_t) h(\alpha_t) d\alpha_t.$$
2.3 The Choice of Optimal Investment

This section illustrates the marginal conditions for investment and discusses the equilibrium condition that must hold between the rental cost of leased capital and the cost of purchased capital. The concepts of the "effective" tax rate and the "effective" price of new machines are defined and illustrated.
Differentiating (2.18) with respect to $I_t$ and rearranging, the first order conditions for new machines bought can be written as:

$$E_t \left[ \frac{\partial V_{t+1}}{\partial K_{t+1}} \right] = (\gamma + \lambda_t^D)(1-\tau_t^*)G_I(I_t, I_{t+1}^L, K_t) + (\gamma + \lambda_t^D)p_t^* \quad (2.21)$$

$G_I$ denotes the derivative of the adjustment cost function with respect to investment (new fixed assets purchased). $E_t[\partial V_t/\partial K_{t+1}]$ is the shadow value of capital used in production at time $t+1$ expected in period $t$, where the timing here depends upon the assumption that there is a one period gestation lag for new investment. $\tau_t^*$ and $p_t^*$ denote respectively the "effective" corporate tax rate and the "effective" price of investment goods and are defined below. Apart from $\tau_t^*$ and $p_t^*$ the structure of condition (2.19) is quite standard and it states that the number of new machines purchased depends upon the difference between their shadow value and their effective replacement cost (including adjustment costs).

The definitions of $\tau_t^*$ and $p_t^*$ are quite forbidding but have simple intuitive explanations. Beginning with $\tau_t^*$, it is defined as

$$\tau_t^* = \tau \left\{ H(b_t) - H(a_t) \right\}$$

$$+ \left\{ \tau - c(\tau-c) + \frac{c(\tau-c)}{\gamma + \lambda_t^L} E_t \left[ \frac{\partial V_{t+1}}{\partial K_{t+1}} \right] \left\{ a_t \leq b_t \right\} \right\} \left\{ H(b_t) - H(a_t) \right\}$$

$$+ \left\{ \frac{1}{\gamma + \lambda_t^D} E_t \left[ \frac{\partial V_{t+1}}{\partial L_{t+1}} \right] \right\} \left\{ a_t \leq b_t \right\} - \left\{ \frac{c}{\gamma + \lambda_t^D} E_t \left[ \frac{\partial V_{t+1}}{\partial U_{t+1}} \right] \right\} \left\{ a_t \leq b_t \right\} \right\} \left\{ H(a_t) \right\} \quad (2.22)$$

Note that $E_t[\partial V_t/\partial L_{t+1}]$ and $E_t[\partial V_t/\partial U_{t+1}]$ appear in the definition of $\tau_t^*$. These are the shadow values of the new state variables, taxable losses and unrelieved ACT respectively. Clearly, in the empirical application below, some measure of these terms will be needed. To interpret the expression in (2.22) in period $t$, consider the case in which the profit of the firm is increased by $\mu_t^*$. It is possible to show
that evaluating the derivative of $V_t$ at $\mu_t=0$:

$$1-\tau_t^* = \left( \frac{1}{\gamma \rho_t} \right) \frac{\partial}{\partial \mu_t} \left[ \sum_{j=0}^{\infty} \rho_t^{j+1} \left( \gamma E_t(D_{t+j}) - \lambda_t^{N+1} \right) \right]$$

(2.23)

Thus $\tau_t^*$ is an "effective" rate of corporation tax in the sense that $(1-\tau_t^*)$ is the answer to the question: "How much would the present value of net distributions to shareholders increase if the firm operating profit in the current period (only) were to be one pound higher?".

The expression in (2.22) is made up of three parts, corresponding to the probabilities of being in each of the tax regimes. With probability $1-H(b_t)$ the firm will be a full taxpayer, in which case $\tau_t^* = \tau$. With probability $H(b_t)-H(a_t)$, the firm is in a position of ACT exhaustion in which case $\tau_t^*$ depends on the shadow value of an increment to the stock of unrelieved ACT. With probability $H(a_t)$ the firm is fully tax exhausted, in which case $\tau_t^*$ depends on the shadow values of an increment to taxable losses and unrelieved ACT. The effective corporate tax rate is bounded below by the statutory rate of imputation $c$. The bound is attained if the firm expects with probability of one to be tax exhausted from time $t$ onwards (so that any increment to taxable losses or to unrelieved ACT is worthless).

Note that by setting $c=0$, and $H(b_t) = H(a_t)$ and $\lambda_t^{D}=0$, it is possible to obtain the effective tax rate under the classical system of company taxation. In this case:

$$\tau_t^* = \tau \left( 1-H(b_t) \right) + \frac{1}{\gamma} E_t \left[ \frac{\partial V_t}{\partial \mu_t} \mid \alpha_t=b_t \right] H(b_t)$$

(2.24)

\[\text{\footnotesize{10}}\]

It is clear from (2.19) and (2.20) that $-G(.)$ appears in the maximand in exactly the same way as $\Pi(.)$. Suppose that $\mu_t$ is added to $\Pi_t$, and regard $\mu_t$ as a state variable at the beginning of period $t$. The value function can therefore be written as $V_t(K_t, \ldots, \mu_t)=\rho_t M_t$. Then, $\partial V_t/\partial \mu_t = p_t \delta M_t/\partial \mu_t = -p_t \delta M_t/\partial G_t = -\gamma p_t (1-\tau_t^*)$ and so $(1-\tau_t^*)=(1/\gamma \rho_t) \partial V_t/\partial \mu_t$. 

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This is identical to equation (1.16) in Mayer (1986), when dividend payments are strictly positive, and it defines the effective tax rate as a weighted average of the statutory rate and the value to the firm of one more unit of losses carried forward, where the weights are the probabilities of being a regular taxpayer or of being tax exhausted respectively.

\[ p^*_t \text{ is defined as } \]

\[
p^*_t = p_t \left[ 1 - c \left( 1 + \frac{1}{\gamma} E_t \left[ \frac{\partial V_{t+1}}{\partial \theta_{t+1}} \left| a_t \leq b_t \right. \right] \right) H(b_t) \right] - (1-j) \left[ \tau \left( 1 - H(b_t) \right) \right]
\]

\[
+ \left\{ (1-c)(1-c) + \frac{c(1+\tau-c)}{\gamma} E_t \left[ \frac{\partial V_{t+1}}{\partial \theta_{t+1}} \left| a_t \leq a_t \right. \right] \right\} \left\{ H(b_t) - H(a_t) \right\}
\]

\[
+ \frac{1}{\gamma} E_t \left[ \frac{\partial V_{t+1}}{\partial \alpha_{t+1}} \left| a_t \leq a_t \right. \right] H(a_t)
\]

\[
- \frac{j}{\gamma} E_t \left[ \frac{\partial V_{t+1}}{\partial k_{t+1}} \right]
\]

(2.25)

To consider its interpretation, consider the case in which the number of new machines bought in period \( t \) is increased by \( \mu_t \) but that the additional machines incur no adjustment costs and generate no return (so that \( G(.) \) and \( K \) are unaffected). Again, it can be shown that, for \( \mu_t = 0 \):

\[
p^*_t = - \left( \frac{1}{\gamma p_t} \right) \frac{\partial}{\partial \mu_t} \left[ \sum_{j=0}^{\infty} \rho_t^{j+1} \left\{ \gamma E_t (D_{t+j}) - V^N_{t+j} \right\} \right]
\]

(2.26)

Thus \( p^*_t \) is the "effective" price of capital in the sense of being the answer to the question: "suppose the firm were to buy one more physical unit of capital, but this was never turned into productive capacity and involved no adjustment costs; what would be the effect on the present value of net distribution to shareholders?".

More directly, (2.25) has three elements. The first element is the cost to the firm of purchasing the asset if there were no depreciation
allowances for tax purposes. If the firm is a full tax payer, the cost of an extra unit of investment goods is simply \( p_t \). However, if the firm is ACT exhausted, the cost of purchasing the asset is actually lower. This is most easily seen in the case of retention finance, since being ACT exhausted implies that cutting net dividends by \( 1-c \) generates a tax saving of \( c \) (the ACT due on the dividend) and hence frees \( 1 \) for additional investment. If the firm were a full tax payer, it would need to reduce its dividends by the full amount. Of course, this effect is only temporary: the reduction in unrelieved ACT will eventually increase tax when the firm moves out of being ACT exhausted.

It is useful for further analysis below to define here \( s = c/(1-c) \) as the rate of credit. It is then also useful to define

\[
 s_t^* = s \left\{ 1-H(b_t) + E_t \left[ \frac{\partial V_{t+1}}{\partial U_{t+1}} \bigg| a_{t+1}^* b_{t+1}^* \right] H(b_t) \right\} \tag{2.27}
\]

to be the "effective" rate of credit. It is straightforward to show that \( 0 \leq s_t^* \leq s \). Intuitively, \( s_t^* \) may be lower than \( s \) if there is a possibility that the firm may be ACT exhausted \( (H(b_t) > 0) \). If \( H(b_t) = 0 \), so that the firm will definitely not be ACT exhausted, \( s_t^* = s \). If the firm is certain to be ACT exhausted, with \( H(b_t) = 1 \) and \( a_{t+1}^* b_{t+1}^* \), \( s_t^* = s E_t [\partial V_{t+1}/\partial U_{t+1}] \). Here the benefit of any ACT tax credit is postponed until the firm moves out of the position of ACT exhaustion; in the current period, the tax credit simply serves to increase the stock of unrelieved ACT, \( U_{t+1} \). Using \( s_t^* \), we can rewrite the first term of the expression for \( p_t \) as:

\[
 1 - c \left\{ 1 + \frac{1}{r} E_t \left[ \frac{\partial V_{t+1}}{\partial U_{t+1}} \bigg| a_{t+1}^* b_{t+1}^* \right] \right\} H(b_t) = \frac{(1+s_t^*)}{(1+s)} \tag{2.28}
\]

The second part of \( p_t^* \) reflects the fact that of each pound spent on investment a fraction \( 1-j \) receives free depreciation. The value of this relief clearly depends on the tax position of the firm in a similar way to the effect of additional profit (it is not identical to the value of additional profit, because an additional relief yields no return to the shareholder until the firm can reduce its tax liability, whereas an additional unit of profit yields an immediate return). The remainder, \( j \), is not available for tax depreciation purposes until period \( t+1 \), and so simply adds to the pool of capital available for tax depreciation next.
period, $K_{t+1}^T$. It is easy to see that if there is 100% depreciation ($j=0$) and the firm is a full tax payer, then $p_t^* = p_t (1-\tau)$. The second two parts of $p_t^*$ are therefore familiar: they simply represent the present value of allowances due on the purchase of one additional unit of capital. If this present value is denoted $\eta_t^*$ (the $^*$ signifying the effect of tax exhaustion), the expression for $p_t^*$ can be rewritten as

$$p_t^* = p_t \left( \frac{(1+s)^t}{(1+s)} - \eta_t^* \right)$$

(2.29)

(b) Leasing

Turning to the first order condition for machines leased (in or out), differentiation of the objective function with respect to $I_t^L$ yields:

$$E_t \left[ \frac{\partial V_{t+1}}{\partial K_{t+1}^L} \right] = (\gamma + \lambda_t^D)(1-\tau_t^*)G_t^L(I_t^L, I_t^L, K_t^L) - E_t \left[ \frac{\partial V_{t+1}}{\partial K_{t+1}^L} \right]$$

(2.30)

This indicates that the leasing decision depends upon the difference between the expected shadow value of productive capital and the sum of the marginal adjustment costs resulting from an increase in leased assets and the (negative) shadow value of the stock of leased capital. Differentiating (2.18) with respect to $K_{t+1}^L$ and solving forward the resulting difference equation gives

$$E_t \left[ \frac{\partial V_{t+1}}{\partial K_{t+1}^L} \right] = -\sum_{j=1}^{\infty} \left( \prod_{k=t+1}^j \rho_k \right) E_t \left[ (\gamma + \lambda_{t+j}^D)(1-\tau_{t+j}^*)p_{t+j}^* R_{t+j}^L (1-\delta)^j \right]$$

(2.31)

Thus $E_t [\partial V_{t+1} / \partial K_{t+1}^L]$ represents the expected present value of the reduction in net dividends due to the stream of rental payments on a machine leased at time $t$. The rental payments start in period $t+1$ and are net of the tax savings they generate.

If the firm is a lessee, so that $G_t^L = G_t^L$ (on the assumption that there is no difference in the adjustment costs between purchasing and leasing an
additional asset), then at the optimum, for convenience setting $\lambda^D = 0$ for all $s$, equations (2.21), (2.30) and (2.31) imply:

$$P_t = \sum_{j=1}^{\infty} \left( \prod_{k=t+1}^{j} \rho_k \right) E_t \left[ (1-\tau^*) p_{t+1} R_{t+1} (1-\delta)^j \right]$$  \hspace{1cm} (2.32)

ie. the "effective" price of a machine is therefore equal to the present value of rental rate payments. In the case of 100% free depreciation and constant $\tau^*$, rates of interest, $R$ and $R^L$, and rate of increase in the price of capital goods, $g$, the equilibrium condition can be given a slightly different simple interpretation. After some manipulation, (2.32) yields

$$R^L = \frac{(1-m)}{(1-z)} R + \delta - g + \delta g$$  \hspace{1cm} (2.33)

The left-hand side is the rental rate on a machine leased during period $t$, which in equilibrium equals the effective cost of using a machine for one period, buying it at $t$ and reselling it at $t+1$ (the right-hand side). For $\delta g = 0$, this is simply the Jorgensen cost of capital: $R(1-m)/(1-z)$ is the required post-personal tax rate of return on capital, $\delta$ is the depreciation rate and $g$ is the capital gain on holding the asset for one period. Equation (2.33) is equivalent to the one that can be obtained from Edwards and Mayer (1987) by equating their equations (2.22) and (2.24) and by assuming that dividends paid are strictly positive.

More generally, however, (2.32) indicates that the equilibrium rental rate depends on the tax position of the company. It is not the case that a tax exhausted company can substitute purchases of assets by leased assets at the "full tax" cost. This suggests that the effects of tax exhaustion cannot simply be nullified by use of leasing$^{11}$.

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$^{11}$Of course, a full analysis of equilibrium in the leasing market requires consideration of the tax position of the lessor and lessee. See Edwards and Mayer (1987) for such an analysis.
(c) Q investment model

The estimation of a Q investment equation from (2.21) requires the relationship between the marginal value of capital, \( E_t[\partial V_{t+1}/\partial K_{t+1}] \) and its average value to be specified. Following the approach of Hayashi (1982), by assuming that \( \Pi, G, \) and \( A \) are all linear homogeneous, it can be shown that

\[
V_t = \frac{\partial V_t}{\partial K_t} K_t + \frac{\partial V_t}{\partial L_t} L_t + \frac{\partial V_t}{\partial B_t} B_t + \frac{\partial V_t}{\partial L_{t+1}} L_{t+1} + \frac{\partial V_t}{\partial U_t} U_t
\]

(2.34)

(2.34) simply states that the market value of the firm equals the sum of the shadow value of each state variable multiplied by its quantity. Taking expectation of period \( t+1 \) help in period \( t \) and solving for \( E_t[\partial V_{t+1}/\partial K_{t+1}] \) yields

\[
E_t[\partial V_{t+1}/\partial K_{t+1}] = \frac{1}{k_{t+1}} \left( E_t(V_{t+1}) - K_{t+1} E_t \left[ \frac{\partial V_t}{\partial K_t} K_t \right] - L_{t+1} E_t \left[ \frac{\partial V_t}{\partial L_{t+1}} L_{t+1} \right] - B_{t+1} E_t \left[ \frac{\partial V_t}{\partial B_t} B_t \right] - U_{t+1} E_t \left[ \frac{\partial V_t}{\partial U_t} U_t \right] \right)
\]

(2.35)

Thus, the possibility of tax exhaustion and leasing introduces a wedge between the marginal and average value of capital due to the presence of three additional state variables, losses carried forward and unrelieved ACT due to tax exhaustion and the stock of leased capital. In order to estimate a Q investment equation like (2.21), there remains the problem of providing empirical analogues for the unobservable terms in (2.35). This issue is discussed in the empirical section 2.5 below.

We are now in a position to define the empirical model to be estimated. Re-arranging the first-order condition on investment (2.21) yields

\[
G_I(I_t, \bar{I}_t, K_t) = \frac{E_t \left[ \frac{\partial V_{t+1}}{\partial K_{t+1}} \right]}{(1-t^*_t)} - \frac{p_t^*}{(1-t^*_t)}
\]

(2.36)
The interpretation of (2.36) should perhaps be noted. Since it is simply a rearrangement of the first order condition for investment, it can be still be interpreted as simply showing that at the optimum, marginal costs (marginal adjustment costs, $G_I(1-t^*_{12})$, plus the effective price of capital goods, $p^*_{13}$) are equal to marginal benefits (the shadow value of an additional unit of capital, $\partial V_{t+1}/\partial K_{t+1}$). Writing the expression in the form given in (2.36) implicitly suggests that investment decisions are taken with a view to optimising adjustment costs for a given "tax-adjusted Q". It is clear that no such implication is derived from the theoretical model - indeed it is just as possible to argue that investment decisions are taken with a view to optimising the shadow value of capital for a given total marginal cost. While neither view is particularly enlightening, this does serve as a reminder that stochastic shocks to the investment process are unlikely to be independent of Q.

To form an empirical investment model from (2.36) requires the specification of the adjustment cost function. Here we follow the literature in using a quadratic function (see, for example, Summers (1981)):

$$G(I_t, I^L_t, K_t)=\frac{b}{2} \left\{ \frac{I^L_{t+1}}{K_t} - c - \epsilon_t \right\}^2$$

(2.37)

This formulation yields the following investment equation:

$$\left( \frac{I^L_{t+1}}{K_t} \right) = c + \frac{1}{b} Q_t + \epsilon_t$$

(2.38)

---

12 Adjustment costs are assumed to be deductible from tax.
13 Both of these terms are valued in terms of net dividends and are therefore multiplied by $(\gamma^t + \lambda^t)$ to measure the marginal cost in terms of shareholders’ wealth.
14 Assuming that $I^L_t > 0$. 
where

\[
Q_t = \frac{E_t \left[ \frac{\partial V_{t+1}}{\partial K_{t+1}} \right]}{(1-\tau_t^*)} - \frac{p_t^*}{(1-\tau_t^*)} \tag{2.39}
\]

and \(\tau_t^*, p_t^*\) and \(\frac{\partial V}{\partial K_{t+1}}\) are defined in (2.22), (2.25) and (2.35) respectively. The constant \(c\) can be interpreted as the "normal" rate of investment at which adjustment costs are zero. The only stochastic term is the unobservable factor \(\epsilon_t\) in the adjustment cost function. This may contain firm specific effects and time effects common to all firms, in addition to an idiosyncratic time-varying shock. Moreover, the latter may be serially correlated, giving rise to a dynamic specification of the investment equation characterised by common factor restrictions. This is discussed further in the empirical section below.

The construction of empirical measures are discussed in the section 2.5 below. However, it is perhaps worth noting here that, because of the unavailability of reasonable data on leasing, all of the leasing terms in the investment equation (2.38) have been excluded from the empirical work. This may lead to an overestimate or an underestimate of total investment as a proportion of the capital stock (the left hand side of (2.38)), depending on the relative size of leasing and new purchases in period \(t\) relative to earlier periods. It will cause an underestimate of marginal \(Q\), since the market value of the company is depressed by the present value of future expected payments on the current stock of leased capital. This present value should therefore be added to the numerator of \(Q\), and its absence will reduce the estimated value of \(Q\).

2.4: Optimal Financial Policy

In this section we make further use of \(s_t^*\), the effective rate of credit, defined in (2.27). It is also useful to define the "effective" tax discrimination variable as \(\gamma_t^*\), where \(\gamma_t^*\) is

\[
\gamma_t^* = \frac{(1+s_t^*)}{(1-m)/(1-z)} \tag{2.40}
\]
Using these terms, we can now examine the marginal condition for new equity issues, which can be written as:

\[ \gamma^* t + \lambda^D_t \frac{\gamma^*}{\gamma} = 1 + \omega_t - \lambda^N_t \]  

(2.41)

where \( \lambda^N_t \) is the multiplier associated with the non-negativity condition for \( V^*_t \), \( \lambda^D_t \) is the multiplier associated with the non-negativity condition for \( D_t \) and \( \omega_t \) is the lemons premium on new share issues.

Ignoring tax exhaustion and the lemons premium, (2.41) is simply the standard marginal condition for new equity issues, (derived by, for example, Poterba and Summers (1983)), \( \gamma^* \lambda^D_t = 1 - \lambda^N_t \). This familiar result implies that firms will not simultaneously issue new shares (implying \( \lambda^N_t = 0 \)) and pay dividends (implying \( \lambda^D_t = 0 \)) unless \( \gamma = 1 \). Most studies do not allow this possibility (and Poterba and Summers rely on \( \gamma \neq 1 \) to examine the marginal source of finance for the firm). However, as discussed below, it is, in principle, possible to allow shareholders for whom \( \gamma = 1 \) to select companies which simultaneously issue new shares and pay dividends.

Here we investigate the role played by the imputation system, which has two important effects. The first is that, for certain configurations of personal tax parameters the standard dividend puzzle - why do firms both issue new shares and pay dividends despite the tax disadvantages of doing so? - is turned on its head. This is because there is a tax advantage to implementing this strategy. From (2.18) it is clear that this would be the case if \( \gamma > 1 \). Recalling that \( \gamma = (1 - m)/(1 - c)(1 - z) \) this can occur in at least two common cases. The first is a basic rate personal income tax payer, for whom \( m = c \), who additionally has a positive marginal tax rate for capital gains. The second is a shareholder exempt from income tax and capital gains tax (so that \( m = z = 0 \)) but can claim the benefit of the tax credit associated with the dividend payment. Institutions, such as pension funds are in this position.

In the absence of tax exhaustion, with \( \gamma > 1 \), firms should continually issue new shares and pay out the proceeds as dividends, since on every unit of the transaction shareholders gain \( \gamma - 1 \). Some models explicitly
prevent this by imposing a constraint that dividends cannot exceed some legal limit (Edwards and Keen (1985) explore this constraint in some detail; basically, firms cannot pay dividends which exceed current and past profits). However, it is clearly not the case in the UK that this upper constraint on dividends is frequently met. Evidence is presented in Chapter 4 that the average dividend payout ratio is around 25% - far below a level prohibited by law.

However, the second effect of the imputation system captured in (2.41) is that tax exhaustion introduces a further reason why firms may be observed simultaneously issuing new shares and paying dividends. In the absence of the lemons premium, from (2.41) $\lambda_t^D = \lambda_t^N = 0$ implies $\gamma^* = 1$. An intuitive explanation of this position is as follows. Assume that $\gamma > 1$ and that the firm is initially certain to be a regular taxpayer. From (2.12c) net dividends can be increased by the same amount of the share issue. The cost of the share issue is one, but the post-tax value of the net dividend to the shareholder is $\gamma > 1$, so that issuing equity increases shareholders' wealth. As the firm continues to sell new equities and to pay dividends however, the possibility emerges that it will become ACT exhausted, in which case (2.12b) implies that it could only finance a dividend of $1-c=1/(1+s)$, worth only $\gamma(1-c) = (1-m)/(1-z)$ to shareholders. These two extremes of the value of the dual transaction to shareholders is captured precisely by $\gamma^*$. If a full tax-paying position is certain, $s^* = s$ and so $\gamma^* = \gamma$. If ACT exhaustion is certain, $s^* = 0$ and $\gamma = (1-m)/(1-z)$. If $m > z$, there will come a point where $\gamma^* = 1$ and the marginal issue will leave wealth unchanged. Therefore, if, for the marginal shareholder, $\gamma > 1$, the firm should exploit the arbitrage possibility of issuing new shares and paying dividends up to the point at which the probability of ACT exhaustion is such that $\gamma^* = 1$.

This discussion implies that if $\gamma > 1$ the company will be driven to a position in which it both issues new shares and pays dividends. This need not hold, however, in the presence of a lemons premium on new share issues. In this case, the arbitrage opportunity only arises if $\gamma > 1+\omega_t$. If this is true, the same argument applies and the company will end up at the point at which $\gamma^* = 1+\omega_t$. If $\gamma < 1+\omega_t$, it is not worth issuing a new share in order to fund a dividend payment.
The first order condition for debt can be written:

$$\gamma_t^* + \lambda_D^* \frac{\gamma_t^*}{\gamma} = -E_t \left[ \frac{\partial V_{t+1}}{\partial B_{t+1}} \right] - \lambda_{t+1}^B \tag{2.42}$$

Using (2.32) and (2.33) together, the first order conditions for debt and new equity imply

$$E_t \left[ \frac{\partial V_{t+1}}{\partial B_{t+1}} \right] = -(1+\omega_t) + \lambda_t^N - \lambda_{t+1}^B \tag{2.43}$$

In the case in which there is no lemons premium and the firm issues debt and new equity, so that $\lambda_{t+1}^B = \lambda_t^N = 0$, this is a Miller (1977) type of result: at the optimum, the sum of the value of the firm's equity and debt is independent of the financial structure (i.e. the level of debt), despite the presence of personal taxes.

To explore (2.42) further, we need to find an expression for $E_t \left[ \frac{\partial V_{t+1}}{\partial B_{t+1}} \right]$. Differentiating (2.18) with respect to $B_t$ gives

$$\frac{\partial V_t}{\partial B_t} = \rho_t \left\{ -(\gamma + \lambda_t^D)(1-\tau_t^*) I_t - (\gamma_t^* + \lambda_t^D)(1+s^*)(1+\lambda_t^N \frac{B}{K_t}) \right\} \tag{2.44}$$

Taking the expectation in period $t$ of (2.44) for period $t+1$ and substituting into (2.42) gives

$$\gamma_t^* + \lambda_D^* \frac{\gamma_t^*}{\gamma} = E_t \left[ \rho_t \left\{ -(\gamma + \lambda_t^D)(1-\tau_{t+1}^*) I_{t+1} - (\gamma_t^* + \lambda_t^D)(1+\lambda_t^N \frac{B}{K_{t+1}}) \right\} \right] - \lambda_{t+1}^B \tag{2.45}$$

and using (2.41) and rearranging, it can be shown that:

$$E_t \left[ r_{t+1}^* (1+\frac{\lambda^D}{\gamma} t_{t+1}^*) \right] = c + \frac{\lambda_t^N - \omega_t - \lambda_{t+1}^B}{\rho_{t+1} \gamma_{t+1}^*} - \frac{E_t (\lambda_{t+1}^N)}{\gamma_{t+1}^*} + \frac{1}{\gamma_{t+1}^*} E_t (\lambda_t^D) + \frac{1}{\gamma_{t+1}^*} E_t \left[ \lambda_t^N B_{t+1} K_{t+1} (1-\lambda_t^N) (1+\lambda_t^N \frac{B_{t+1} K_{t+1}}{K_{t+1}}) \right] \tag{2.46}$$
In the absence of agency costs and theemons premium, internal optima for all new equity and retentions in period t and period t+1 imply that $E_t(\tau_{t+1}^*)=c$. As noted by Keen and Schiantarelli (1987) (their Proposition 1), this implies permanent and certain tax exhaustion. The presence of agency costs in (2.36) implies that, even with internal optima for all sources of finance, $E_t(\tau_{t+1}^*)$ is strictly greater than c. Thus, in this sense, then, the role played by agency costs in this model is to justify why an internal solution on all three margins is consistent with the observation that corporate tax revenues are not zero.

Financial Regimes

We are now in a position to investigate different financial regimes in which the firm may operate. This exercise has already been undertaken by a number of authors (for example, Auerbach (1984), Hayashi (1985), Mayer (1986), and King (1987)). Of these, Mayer has already examined the role of tax exhaustion, but principally only in the special case of a classical system.

Below we distinguish four possible regimes which a firm may face. In each case we assume, for simplicity, that dividends are paid in period t+1 and henceThe first distinction to make is whether $\gamma$ is greater or less than one. We begin with the case in which $\gamma<1$. In this case, as is well known, firms should not issue new equity and pay dividends simultaneously. Rather, they should use retentions in preference to new equity until dividends are zero and hence retentions exhausted. This situation is unaffected by the possibility of ACT exhaustion. This is because, as outlined above, ACT exhaustion can only reduce the amount which the firm can pay as dividends. It can therefore only increase the advantage of retention finance over new equity finance. In an extreme case, with ACT exhaustion being permanent and certain, $\gamma^*$ falls to $(1-m)/(1-z)$ - the value of $\gamma$ under a classical system.

Since retentions are preferred to new equity, we first consider the case in which new equity is not issued in period t or period t+1. From (2.41), for $\lambda^D_t=\lambda^D_{t+1}=\lambda^B_t=0$, $\lambda^N_t=1-\gamma_t^*$ and $E_t(\lambda^N_t)=1-E_t(\gamma_t^*)$. Substituting into (2.46) and rearranging yields
(2.47) is therefore the equilibrium condition for internal optima on debt and retentions, when new equity issues are zero in both period $t$ and period $t+1$. The left hand side of (2.47) represents the marginal cost of issuing a unit of debt in period $t$, consisting of an interest charge relieved at the expected effective tax rate (multiplied by $\gamma$ since it is measured in terms of the value of net dividends foregone by the shareholder) plus marginal agency costs of debt (multiplied by the expected value of $\gamma^*$), both discounted by the discount factor $\rho_{t+1}$ since they do not arise until period $t+1$. The right hand side represents the marginal cost of retaining an additional unit of retentions. This is an immediate cost to the shareholder of $\gamma^*$ since net dividends are reduced, matched by a discounted increase in dividends worth $E^*_t(\gamma^*_{t+1})$ in period $t+1$.

If the left hand side of (2.47) were less than the right hand side, debt would be preferred to retentions. In this case, however, the firm would continually issue debt and pay dividends. Three factors would eventually end this arbitrage possibility. The first is that marginal agency costs are assumed to increase with debt ($A_{BB}>0$) and so force up the cost of issuing debt. The second is that as expected interest payments increase in period $t+1$, then the expected value of the effective tax rate in period $t+1$, $E_t(\tau^*_{t+1})$, falls since taxable profits become lower and hence full tax exhaustion becomes more probable. This also increases the expected cost of issuing debt since the tax relief on interest payments is lower. It should be noted, though, that there is an offsetting factor here, namely that as expected taxable profits fall, so the probability of ACT exhaustion in period $t+1$ rises and hence $E_t(\gamma^*_{t+1})$ falls. This reduces the impact of agency costs (lowering the marginal cost of debt) and also reduces the benefit of higher dividends in period $t+1$ (raising the marginal cost of retentions). However, the third factor is that this effect is also present in period $t$. Thus as dividends increase, so does the probability of gross dividends exceeding taxable profit and hence the probability of ACT exhaustion in period $t$. Thus $\gamma^*_t$ falls, lowering...
the cost of reducing dividends in period $t$.

These effects will therefore limit the arbitrage possibility of continually issuing debt and paying dividends, so that an equilibrium will be reached when (2.47) holds, in which the company is indifferent between debt and retention finance. Denote this regime 1. This regime can exist with or without the presence of agency costs of debt since, as noted above, tax exhaustion also limits the arbitrage possibilities. In this regime, new investment may be financed partly by retentions and partly by debt or may be financed solely by debt. To see this, consider the two cases in which a firm uses only debt or only retentions.

If only debt is used to purchase new fixed assets, there are two groups of effects. First, as before, an increase in debt causes marginal agency costs to increase. Offsetting this, however, the increase in the capital stock, assuming $A_{BK} < 0$, is that marginal agency costs tend to fall. Under most reasonable formulations of the agency cost function, though, $A_{BB} > -A_{BK}$, so that the impact of the increase in debt dominates (at least if the new investment is wholly financed by debt)\(^\text{15}\). Intuitively, this is because an investment wholly financed by debt must increase in gearing of the company, on which agency costs are likely to depend.

Second, an increase in debt will increase interest payments which will tend to reduce taxable profit and hence $E_t(\tau_{t+1})$, increasing the marginal cost of debt. However, now there is also an offsetting effect here, namely that taxable profits will tend to rise as a result of the return earned on the new investment, increasing $E_t(\tau_{t+1})$ and reducing the cost of debt (this effect is absent if the new debt raised is used simply to pay dividends). The relative size of these effects depends on the definition of taxable profit: if the tax system is neutral, a marginal investment project financed wholly by debt would leave taxable profit unchanged. More generally, however, $E_t(\tau_{t+1})$ could either rise or

\(^{15}\)For example, if $A = \frac{B}{2} (B/K)^2 K$, then $A_{BB} = b/K$ and $A_{BK} = -bB/K^2 = -A_{BB} (B/K)$. Thus, $A_{BB} > -A_{BK}$ for $B < K$.  

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fall, depending on the tax system. Note that $E_t(\gamma_{t+1}^*)$ will move in the same direction since the probability of ACT exhaustion falls with higher taxable profit. An increase in $E_t(\tau_{t+1}^*)$ will reduce the marginal cost of debt (although the corresponding increase in $E_t(\gamma_{t+1}^*)$ will increase the impact of agency costs) and vice versa. It is possible that the reduction in the marginal cost of debt due to the increase in $E_t(\gamma_{t+1}^*)$ will outweigh the increase in the marginal cost of debt due to agency costs. An increase in $E_t(\gamma_{t+1}^*)$ will reduce the marginal cost of retentions, and vice versa.

These effects give rise to three possibilities. The first is that the marginal cost of retentions falls by more than the marginal cost of debt\textsuperscript{16}, which would imply that the company would also use retention finance in order to maintain the equilibrium condition (2.47). Alternatively, it is possible that $E_t(\tau_{t+1}^*)$ and $E_t(\gamma_{t+1}^*)$ are unchanged, in which case the only effect would be an increase in the marginal cost of debt. In this case as well, the company would also use retentions. The second is that the marginal costs of debt and retentions are reduced by the same amount, in which case the firm would not need to use retention finance (although this is extremely unlikely since marginal agency costs must exactly match the difference in marginal tax costs between retentions and debt). The third is that the marginal cost of debt falls by more than the marginal cost of retentions. In this case, the company has further arbitrage possibilities of the kind already discussed, and would issue even more debt and pay the proceeds out as dividends. This third possibility must also be counted as unlikely.

An investment wholly financed by retentions will reduce the probability of ACT exhaustion since dividends are lower. This will increase $\gamma_t^*$ and hence the marginal cost of retentions (intuitively, the lower the probability of ACT exhaustion, the more shareholders give up for each pound in the firm that might otherwise be distributed). However, if the investment yields a taxable return, then $E_t(\tau_{t+1}^*)$ will increase. As

\textsuperscript{16}Or, alternatively, the marginal cost of retentions may increase less than the marginal cost of debt or fall while the marginal cost of debt rises.

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discussed above, this will reduce the marginal cost of debt and also tend to reduce the marginal cost of retentions. In addition, the increase in the capital stock will reduce the marginal agency costs of debt. Unless the increase in \( E_t(\gamma_{t+1}) \) outweighs the increase in \( \gamma_t \) to the extent that the overall effect on the marginal cost of retentions is to lower it by as much as the reduction in the marginal cost of debt, then the company will also issue debt.

This rather detailed discussion therefore suggests that it is probable that both debt and retentions will be used in regime 1 (although there are possible configurations in which this might not be the case). It is not the case, however, (as in some other models) that the marginal cost of finance is constant in this regime. Clearly it will depend on the probability of tax exhaustion and the size of agency costs, both of which will depend on the size of investment, debt and retentions. Also, the proportions in which debt and retentions are used need not be constant.

If there were no tax exhaustion, and agency costs were a function of the ratio of the stock of debt to the replacement value of the capital stock, then in regime 1 debt would constitute a constant proportion of new finance (Hayashi (1985) makes these assumptions, although several papers investigating Q based investment equations assume this to be the case - see, for example, Summers (1981) and Poterba and Summers (1983)). This can easily be seen by considering (2.48), which is derived from (2.47) under the conditions of no tax exhaustion and agency costs depend on the debt to capital stock ratio:

\[
\rho_{t+1}\left[\gamma(1-\tau)_{t+1} \gamma A_B(B_{t+1}/K_{t+1})\right] = \gamma - \rho_{t+1} \gamma
\]

(2.48)

If the tax parameters \((\tau, \gamma, m \text{ and } z)\) are constant over time, and the interest rate in period \(t+1\) is unaffected by the behaviour of the company, then the only element in (2.48) which depends on the company are marginal agency costs. If agency costs depend only on the ratio of \(B_{t+1}\) to \(K_{t+1}\), then in order to maintain the equality in (2.48) this ratio must remain constant, so that \(A_B\) is constant.
It is worth noting that a regime similar to this can be derived for a classical corporation tax. Both De Angelo and Masulis (1980) and Mayer (1986) introduce the possibility that $E_t(\tau_{t+1})$ falls as interest payments increase. However, both papers consider only a classical system, for which $s=s_t=s_{t+1}=0$. Ignoring agency costs, (2.47) becomes

$$1-E_t(\tau_{t+1}) = \frac{(1-m)}{(1-z)}$$

which is, for example, equation (2.2) of Mayer (1986). In this case, as described above, it is probable that both retentions and debt are used, even though the marginal cost of retentions is fixed. This is because an increase in taxable profit due to investment financed by retentions will increase $E_t(\tau_{t+1})$. To maintain $E_t(\tau_{t+1})$ constant requires an increase in interest payments. However, the increase in interest payments must exactly match the increase in taxable profit due to the return from the investment. In the case of a marginal investment, and a neutral tax system, this implies that the marginal source of finance must be entirely debt. It is also possible that an investment earns a taxable return higher than the interest rate, this implies that debt must be issued in excess of that needed to finance the investment. Note that an identical result is obtained under an imputation system if there is no possibility of ACT exhaustion, and hence $s=s_t=s_{t+1}=0$. This is perhaps not surprising: the additional effect of ACT exhaustion mentioned above was that the value of additional funds to pay dividends are reduced by the possibility of ACT exhaustion. If there is no such possibility, then the role played by tax exhaustion here is simply that of additional interest payments reducing taxable profit while additional returns increase taxable profit.

King (1987) argues that a regime similar to that described can exist even if there are no agency costs on debt and no tax exhaustion. In this case, (2.47) reduces to $(1-\tau) = (1-m)/(1-z)$, which is the knife-edge condition familiar from King's earlier work (1974 and 1977). King claims that this condition can be met since the condition essentially defines the marginal shareholder, who is indifferent between debt and retention finance. Other investors invest their wealth wholly in debt or wholly in
equity, depending on their tax preference.

Note that under King's assumptions, the firm is indifferent to its debt to retention finance ratio, which can therefore be described as a Miller (1977) equilibrium. However, this is not generally true of the condition (2.47), since if the equilibrium depends on agency costs or tax exhaustion, the firm has a determinate ratio for the two sources of finance. The Modigliani-Miller (1958) theorem therefore only holds under assumptions regarding the marginal investor. One problem with King's result is simply that τ, m and z are statutory tax rates and therefore have discrete values (although z is sometimes taken to be an accruals-equivalent tax rate). In this case, there is no reason to suppose that there is any investor for whom the knife-edge condition holds. It does not hold for common groups of investors such as tax free institutions (m=z=0), basic rate taxpayers with annual capital gains less than around £5000 (m=25%, z=0 in 1990 in the UK), or higher rate taxpayers with large capital gains (m=z=40% in 1989); in all cases τ=35%. It is, however, possible that these tax rates may be held to be uncertain, which would introduce the possibility that the knife-edge condition may hold.

Returning to the general case in regime 1 above, it is also useful to describe the equilibrium value of marginal \( q \). This is the shadow value of an additional unit of the capital stock to the firm when it is in a steady-state. The steady state is usually defined to mean when adjustment costs, \( G(I_t, I_{t+1}^L, K_t) \), are zero. From (2.21), the first order condition for investment, assuming also that the derivative of \( G(I_t, I_{t+1}^L, K_t) \) with respect to \( I_t \) is also zero, the equilibrium value of marginal \( q \) is

\[
q^e_t = \frac{1}{p_t} \mathbb{E}_t \left[ \frac{\delta V_t}{\delta K_t} \right] = \frac{(\gamma + \lambda^D_t)p^*_t}{p_t} \quad (2.50)
\]

In (2.50) \( p_t \) is simply the price of the additional unit of capital and \( p^*_t \) is the "effective" price, after allowing for the various allowances in the tax system. Intuitively, \( q^e_t \) answers the question "what is the value to the shareholder of the firm in steady state being given one additional unit of capital?". Using the expression for \( p^*_t \) in (2.29), \( q^e_t \)
can also be written

\[ q_t^e = (\gamma + \lambda^D_t) \left[ \frac{\gamma_t^e}{\gamma} - \eta_t^* \right] \]  

(2.51)

In regime 1, \( \lambda^D_t = 0 \), and so \( q_t^e = \frac{\lambda^e_t}{p_t} \). The reason for this value is clear: if the firm is given a gift of one additional unit of capital it will respond by purchasing one less unit (since, by assumption, it already has the optimal stock of capital). The gain to the shareholder of purchasing one less unit of capital is the effective price of capital, \( p_t^e \), multiplied by any relevant personal tax parameters. In regime 1, net dividends will be increased by \( p_t^e \) and so the shareholder will gain by \( \frac{\lambda^e_t}{p_t} \).

As long as the firm remains paying dividends (or at least until it reaches some minimum level), it will remain in regime 1. As investment increases, dividends must eventually reach their minimum level, at which point debt will become the sole marginal source of finance, with dividends and new equity set at zero. This is regime 2. Since there is no equality condition between any two sources of finance in the regime, it is impossible to derive an expression similar to that in (2.47).

Similarly, although the value of \( q_t^e \) can be specified in terms of \( E_t(\tau_{t+1}^*) \) as

\[ q_t^e = \frac{p_t}{p_t^e} \left[ \frac{\gamma}{\gamma_t} - 1 \right] E_t \left[ \rho_{t+1}^* \left( 1 - E_t(\tau_{t+1}^*) \right) + \rho_{t+1}^* \left( 1 + A_{t+1}^B(B_{t+1}, K_{t+1}) \right) \right] \]  

(2.52)

this does not give a unique value of \( q_t^e \), because from (2.46) \( E_t(\tau_{t+1}^*) \) depends on \( \lambda_t^D, \lambda_{t+1}^D, \lambda_t^N \) and \( \lambda_{t+1}^N \), all of which may be non-zero and unknown. The equilibrium value of \( q \) in regime 2 therefore depends on the level of the capital stock, and hence investment, through its effect on \( E_t(\tau_{t+1}^*) \) and agency costs of debt. This mirrors the results of Hayashi (1985). Edwards and Keen (1985) do give an expression for equilibrium \( q \) in this regime, but their expression depends on the discounted costs of eventually winding up the firm.

Regime 3 occurs when the costs of issuing debt have risen to the level of issuing new equity. Following the arguments set out above, it should
be noted that it is possible that the marginal cost of issuing debt does not rise in regime 2 - indeed it is possible that it should fall. Regime 3 would not be reached unless it rose. Assuming that it is reached, at this point the firm begins to issue new shares so that $\lambda_t^N$ becomes zero. The characterisation of regime 3 depends on $E_t(\lambda_{t+1}^N)$. If $E_t(\lambda_{t+1}^N)=0$, but continuing to assume that $E_t(\lambda_{t+1}^D)=0$, then

$$\rho_t\left[\gamma(1-E_t(\tau_{t+1}^*)1_{t+1}^*)1_{t+1}^*+E_t(\gamma_{t+1}^*)A_B(B_{t+1},K_{t+1})\right] = 1+w_t^*-p_t^*E_t(\gamma_{t+1}^*) \quad (2.53)$$

This condition is very similar to that in (2.47) for regime 1. The only difference is that the cost of financing the marginal investment through retentions, $\gamma_t^*$, is replaced by the cost of financing the marginal investment through new equity, namely $1+w_t^*$. Following the intuition given for (2.47), then, the left hand side of (2.53) represents the marginal cost of debt finance, and the right hand side the marginal cost of new equity finance.

The second term on the right hand side of (2.53) is the value of the return to the shareholder given that the return will be paid in the form of dividends. If it is further assumed that $E_t(\gamma_{t+1}^*)=1+w_{t+1}$, so that the firm expects to be indifferent between retentions and new equity in period $t+1$, and hence that $E_t(\lambda_{t+1}^N)=0$, then this substitution can be made in (2.53). If the lemons premium on new shares issues is ignored, so that $\omega_t=\omega_{t+1}=0$, (2.53) can be simplified to

$$\gamma(1-E_t(\tau_{t+1}^*)1_{t+1}^*)1_{t+1}^* + A_B(B_{t+1},K_{t+1}) = \frac{(1-m)}{(1-z)}i_{t+1} \quad (2.54)$$

Both debt and new equity are issued in this regime. If the capital stock rose, financed solely from new equity, marginal agency costs of debt would fall on the assumption that $A_{BK}<0$, and $E_t(\tau_{t+1}^*)$ would rise, reducing the net interest cost of issuing debt. This would make debt the cheaper source of finance again. Partly substituting debt for equity would both reduce $E_t(\tau_{t+1}^*)$ (given that the taxable return is unchanged) and increase marginal agency costs, both of which would increase the marginal costs of debt. To maintain equilibrium between debt and new equity, they must therefore both be used. In particular the
amount used must be such as to maintain a constant marginal cost of debt (which, as in regime 1, cannot necessarily be achieved without new issues of debt exceeding new investment). As in regime 1, debt would only be a constant proportion of new finance used if the effective tax rate were unaffected by further changes in the level of debt and if agency costs depended only on the ratio of the stock of debt to the replacement value of the capital stock.

Substituting (2.42) into (2.50) and (2.51), with $\lambda_t^N=0$, yields the following expression for $q_t^e$ in regime 3

$$q_t^e = \frac{p_t^*}{p_t} \gamma_t (1+\omega_t) = \left[ 1 - \frac{\gamma_t}{\gamma_t^*} \eta_t^* \right] (1+\omega_t) \quad (2.55)$$

If there is no ACT exhaustion and $\omega_t=0$, then $q_t^e/p_t^*$, which is the value commonly assigned to it when new equity is the marginal source of finance. The possibility of ACT exhaustion raises the possibility that $q_t^e>p_t^*/p_t$. An intuition for this possibility is as follows. As in regime 1, the effective gain to shareholders of increasing dividends as a result of purchasing one less unit of capital is $\gamma p_t^*$. However, in this case the marginal source of finance is new equity. But from (2.32) the value in terms of a potential reduction of new share issues derived from cutting dividends by one is $\gamma_t^*$. The overall effect of these two transactions is therefore $\gamma p_t^*/\gamma_t^*$, which is identical to the expression in (2.43) (divided by $p_t$)\(^{17}\). When $q_t^e=p_t^*/p_t$, the question arises as to whether investors would be better off investing through an identical unincorporated business. However, this would only be the case if the effective price of capital for the unincorporated business were also $p_t^*$ (or at least no greater than $\gamma p_t^*/\gamma_t^*$).

This summarises the three financial regimes when $\gamma<1$. This, of course,

\(^{17}\)The reason why $q_t^e$ is not simply equal to $p_t^*$ is that $p_t^*$ measures the effective price of capital goods in terms of net dividends foregone. This includes the effect on the probability of ACT exhaustion of reducing dividends. If dividends are not reduced, this effect must be corrected for. This correction is measured by $\gamma/\gamma_t^*$.

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is the standard case examined in the literature. However, in the UK, and in many other European countries, many shareholders face a situation in which \( \gamma > 1 \).

The possibility of ACT exhaustion provides one reason why firms do not engage in more-or-less unlimited arbitrage when \( \gamma > 1 \). As explained above, if there is a possibility of ACT exhaustion, a unit of new equity cannot necessarily be transformed into a unit of net dividends, worth \( \gamma \) to the shareholder. This because part of it may be needed to pay the ACT charge arising on the payment of the dividend. In this situation, the effective rate of credit falls from \( s \) to \( s^* \) (defined in (2.27)) and the value of the transaction to the shareholder falls to \( \gamma^* \). (2.42) confirms that an equilibrium is reached when \( \gamma^* = 1 \).

If \( \gamma^* \) were less than 1, the opposite arbitrage possibility would arise, since the shareholder would gain by reducing dividends and reducing new equity issues. This will not therefore occur as long as \( \gamma > 1 \), and dividends and new equity issues are both positive. However, it is possible that \( \gamma^* < 1 \) and \( v^N = 0 \). This might arise if the firm has carried forward a large stock of unrelieved ACT into the current period. In this case the shareholder cannot benefit by reducing dividends and new equity. If \( \gamma^* < 1 \), the three regimes already described are again the relevant cases.

However, the possibility that \( \gamma > 1 \) and \( \gamma^* = 1 \) introduces a fourth regime. In regime 4, the marginal cost of retained earnings is equal to that of new equity. From (2.42) we can write

\[
\gamma^* = 1 + \omega_t \quad \text{or} \quad (1-m)(1+s^*) = (1-z)(1+\omega_t) \tag{2.56}
\]

This characterises the values of the personal tax parameters in this regime. In equilibrium, however, the marginal cost of debt will be equal to the marginal cost of the other two sources of finance. This implies that the condition specified in (2.53) (from regime 3) must also hold.

In this regime, an extra unit of capital will probably be financed by debt and new equity. To see this, consider an additional unit of investment financed by new equity. The marginal cost of new equity
finance is unaffected by the size of the investment or of the share issue\textsuperscript{18}. To maintain \( \gamma_t^* = 1 + \omega_t \), however, requires that dividends are unchanged from the equilibrium level (since taxable profits will only change in period \( t+1 \)). This does not imply that the firm does not use retention finance, but merely that the marginal source of finance does not include retentions. However, as the capital stock increases, agency costs of debt will fall and \( E_t(\tau_{t+1}^*) \) will rise, both of which reduce the marginal cost of debt. This suggests that more debt must be issued in order to raise the marginal cost back up to its equilibrium level (although as discussed above, it is strictly possible that increasing the level of debt does not increase its marginal cost).

The value of \( q_t^e \) in regime 4 is straightforward. As in regime 1, \( \lambda_t^D = 0 \), and so \( q_t^e = \gamma p_t^*/p_t \). The difference between regimes 1 and 4 is that in the former \( \gamma < 1 \) and in the latter \( \gamma > 1 \). This raises the possibility that \( q_t^e > 1 \). However, it is straightforward to show that this requires that the lemons premium be greater than the present value of tax allowances on the new investment (ie. \( \omega_t > \gamma \eta_t^* \)), which requires a very large premium. This can be seen by using the alternative expression for \( q_t^e \) in (2.51) which shows that \( q_t^e = 1 + \omega_t - \gamma \eta_t^* \). Since both \( \gamma \) and \( \eta_t^* \) are positive, \( q_t^e < 1 \) for \( \gamma \eta_t^* > \omega_t \).

Note that regime 4 is clearly not a Miller equilibrium. Although the marginal cost of all three sources of finance must be the same, the firm is not indifferent to which it uses: there is an optimal debt equity ratio, dependent on agency costs and tax exhaustion. However, it should be noted that a regime of this type may occur even in the absence of tax exhaustion and the lemons premium, in which case \( \gamma = 1 \) and (2.53) holds with \( \tau \) replacing \( E_t(\tau_{t+1}^*) \). Thus, it could be argued that \( \gamma = 1 \) defines the marginal investor. Note that \( \gamma = 1 \) implies that

\textsuperscript{18}The marginal cost of new equity issues would depend on the size of the issue if there were transactions costs: there are relatively high fixed costs to making a new equity issue, so that the overall cost of any single issue is inversely related to the size of the issue.
\[
m - c \\
\frac{z}{1-c} \tag{2.57}
\]

This possibility is, in fact, rather more likely than the knife-edge condition \((1-\tau) = (1-m)/(1-z)\) used by King (1987) and discussed above. This is because most individuals pay income tax at the basic rate, but pay no capital gains tax (ie. their gains are less than £5000 per year in the UK in 1990). Thus, for them, \(m = c\) and \(z = 0\), and so (2.57) holds. Indeed, this is far from surprising: this condition is what the imputation system was designed to achieve. Given (2.57), the firm will also use debt finance if

\[
\frac{A_B(B_{t+1}, K_{t+1})}{1-\tau} = 1-c \tag{2.58}
\]

which is simply the condition in (2.54) in the absence of tax exhaustion. (2.57) and (2.58) are therefore a version of regime 4 in the absence of tax exhaustion. Notice that in this case, dividend policy is irrelevant, since the marginal shareholder is indifferent between the three sources of finance. The firm is also unconcerned about the marginal tax rates of its investors - this is consistent with the survey results of Edwards and Mayer (1986). These points are also claimed by King (1987) for his regime 1 (equivalent to the regime 1 here in the absence of tax exhaustion). This regime differs from King's regime 1 in a number of ways, however. First, new equity may be used. Second, there is a unique debt equity ratio, determined by the scale of agency costs of debt. Because of this, debt must always constitute some proportion of new finance raised.

A final point to note is the implication for the cost of increasing the capital stock by purchasing another firm, rather than purchasing the asset directly. The regime 1 here mirrors King's regime 1, in that the cost of purchasing one unit of another firm \(\gamma p^*_t\) is less than the cost of the direct purchase of one unit of the asset \(p^*_t\) since \(\gamma < 1\). In regime 4, the cost of purchasing another firm is also \(\gamma p^*_t\), but here \(\gamma > 1\),

\[19\] If the target firm is in regime 1.
and so it would be cheaper to purchase the asset directly.

This description of possible financial regimes indicates that consideration of tax exhaustion, especially in combination with the imputation system (which has been in effect in the UK since 1973 and is also common elsewhere in Europe), adds substantially to the richness of the relevant analysis. It also suggests a fourth regime which is ignored in the literature, but which may prove to be common in the UK and in other European countries.

2.5. Issues in the empirical construction of tax parameters and Q

In this section we discuss the derivation of the terms needed to estimate the Q investment equation given in (2.38). Three items in particular need to be identified: \( \tau_t \), \( p_t \), and the version in this model of the commonly used empirical analogue to the shadow value of capital \( \frac{\partial V}{\partial K_{t+1}} \) defined in (2.35). Although the usual use will be made of the equity market value of the firm, \( V_t \), from (2.35), all three of these variables include terms which depend on tax exhaustion. In particular, all three expressions include expected values of stocks of taxable losses or unrelieved ACT, \( E_t[\frac{\partial V}{\partial L_{t+1}}] \) and \( E_t[\frac{\partial V}{\partial U_{t+1}}] \) i.e. the shadow values of the two additional state variables introduced by tax exhaustion.

Two issues are important here. The first is simply how to find empirical measures of \( \frac{\partial V}{\partial L_{t+1}} \) and \( \frac{\partial V}{\partial U_{t+1}} \) even if periods of tax exhaustion were known by the firm with certainty. The second concerns how expectations are formed. These issues are discussed in turn.

In general, the effect of either form of tax exhaustion is simply to delay any tax payment or rebate. This can be seen by differentiating the maximand in (2.18) with respect to \( U_t \) and \( L_t \). Beginning with \( U_t \) we have
\[
\frac{\partial V}{\partial U_t} = \rho \left( (\gamma + \lambda_t^D)[1-H(b_{t+1})] + E \left[ \frac{\partial V_{t+1}}{\partial U_{t+1}} \bigg| \alpha_{t+1}, b_t \right] H(b_t) \right)
\]

(2.59)

(2.59) says that with probability \(1-H(b_t)\) the firm will not be ACT exhausted or fully tax exhausted at the end of period. In this case, unrelieved ACT brought forward into period \(t\) will be worth \((\gamma + \lambda_t^D)\) (discounted back to the beginning of the period to find the value of \(\frac{\partial V}{\partial U_t}\)). This is because if the firm resumes a tax-paying position, any unrelieved ACT can be converted directly into an increase in net dividend payments, worth \((\gamma + \lambda_t^D)\) to the shareholder. With probability \(H(b_t)\), the firm will remain in a position of having unrelieved ACT at the end of period \(t\), which then has expected value \(E_t(\frac{\partial V_t}{\partial U_t})\).

Taking (2.59) one period forward and solving forward yields

\[
E_t \left[ \frac{\partial V_{t+1}}{\partial U_{t+1}} \right] = \rho \left( (\gamma + \lambda_{t+1}^D)[1-H(b_{t+1})] + \sum_{s=t+2}^{\infty} \left( \prod_{j=t+2}^{s} \rho^j H(b_{j-1}) \right) \left( (\gamma + \lambda_s^D)[1-H(b_s)] \right) \right)
\]

(2.60)

Ignoring expectations, this defines \(\frac{\partial V_{t+1}}{\partial U_{t+1}}\) as \(\rho_t (\gamma + \lambda_t^D)\) with probability \(1-H(b_{t+1})\) plus future values of \((\gamma + \lambda_t^D)\) weighted by the discount rate and the probability of resuming a full tax-paying position. In the simple case in which it is known with certainty that the firm will be ACT exhausted for \(n\) periods from period \(t\), then \(H(b_{t+1})=H(b_{t+2})=\ldots=H(b_{t+n})=1\) and \(H(b_{t+n})=0\). Treating the discount rate as constant over time, \(\rho=\rho_t=\rho_{t+1}=\ldots\), (2.60) becomes

\[
E_t \left[ \frac{\partial V_{t+1}}{\partial U_{t+1}} \right] = \rho^n (\gamma + \lambda_t^D)
\]

(2.61)

which clearly states that \(\frac{\partial V_{t+1}}{\partial U_{t+1}}\) is simply an increment to net dividends in period \(t+n\) discounted over the period of tax exhaustion.

The definition of \(E_t[\frac{\partial V_{t+1}}{\partial L_{t+1}}]\) can be analysed in the same way, although it is slightly more complex. Differentiating (2.18) with respect to \(L_t\) yields
\[
\frac{\partial V_t}{\partial L_t} = (\gamma + \lambda_t^D) \rho_t \left\{ \tau(1-H(b_t)) + \left\{ (\tau-c)(1-c) + \frac{c(1+\tau-c)}{\gamma + \lambda_t^D} E_t\left[ \frac{\partial V_{t+1}}{\partial L_{t+1}} \big| a_{t+s} \leq b_{t+s}, \alpha_{t+s} \leq b_{t+s} \right] \right\} \{H(b_t) - H(a_t)\} \right. \\
+ \left. \left\{ \frac{1}{\gamma + \lambda_t^D} E_t\left[ \frac{\partial V_{t+1}}{\partial L_{t+1}} \big| a_{t+s} \leq b_{t+s} \right] \right\} H(a_t) \right\} 
\]

(2.62)

Just as in the expression for \( \tau_t^* \), there are three parts to this expression, corresponding to the three tax regimes in which the firm can be in period \( t \) - tax-paying with probability \( 1-H(b_t) \), ACT exhausted with probability \( H(b_t)-H(a_t) \), and full tax exhaustion with probability \( H(a_t) \). Using the last term, (2.62) can again be solved forward to produce an expression for \( E_t [\partial V_{t+1}/\partial L_{t+1}] \):

\[
E_t \left[ \frac{\partial V_{t+1}}{\partial L_{t+1}} \right] = E_t \left\{ \sum_{s=t+1}^{\infty} \prod_{j=t+1}^{s} \rho_j H(a_{j-1}) \left\{ (\gamma + \lambda_s^D) \tau(1-H(b_s)) \right. \right. \\
+ \left. \left. \left\{ (\tau-c)(1-c) + \frac{c(1+\tau-c)}{\gamma + \lambda_s^D} \right\} \left\{ \frac{\partial V_{t+1}}{\partial L_{t+1}} \big| a_{t+s} \leq b_{t+s}, \alpha_{t+s} \leq b_{t+s} \right\} \right\} \{H(b_s) - H(a_s)\} \right\} \right\} 
\]

(2.63)

where, for convenience of notation, \( H(a_t) = 1 \). While the firm remains fully tax exhausted, \( H(a_t) = 1 \), so that the two parts of the expression in (2.63) are only relevant when the firm regains either a full tax paying position or an ACT exhausted position. For example, in the simple case in which it is known with certainty that the firm will be fully tax exhausted for \( m \) periods and ACT exhausted for a further \( n \) periods - so that \( H(a_s) = 1 \) for \( s < t+m \) and \( H(b_s) = 1 \) for \( s > t+m+n \) - then using the expression for \( E_t [\partial V_{t+1}/\partial L_{t+1}] \) in (2.61) and treating the discount rate as constant over time, we can write

\[
E_t [\partial V_{t+1}/\partial L_{t+1}] = \rho (\gamma + \lambda_{t+m}^D)(\tau-c)(1-c) + \rho^{m+n}(\gamma + \lambda_{t+m+n}^D) c(1+\tau-c) 
\]

(2.64)

If it is known with certainty that the firm will resume a full tax paying position in period \( t+m \), but will not then be ACT exhausted, so that \( n=0 \), (2.64) simplifies to

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Expressions (2.61), (2.64) and (2.65) give values of $E_t[\partial V_t / \partial L_{t+1}]$ and $E_t[\partial V_t / \partial L_{t+1}]$ if periods of tax exhaustion are known with certainty at period $t$, which can be used to construct empirical measures of these terms and hence of $\tau^*_t$ and $p^*_t$.

Having defined $E_t[\partial V_t / \partial L_{t+1}]$ and $E_t[\partial V_t / \partial L_{t+1}]$ in the case of certainty it is now necessary to find some empirical measures. The approach taken here follow the approach of Hansen (1982) in replacing the expected values with actual values and instrumenting. Thus, it is assumed that $E_t[\partial V_t / \partial L_{t+1}] = \partial V_t / \partial L_{t+1} + v_{t+1}$ and $E_t[\partial V_t / \partial L_{t+1}] = E_t[\partial V_t / \partial L_{t+1}] + w_{t+1}$, where $v_{t+1}$ and $w_{t+1}$ are assumed to be orthogonal to information available in period $t$. Altshuler and Auerbach (1987) provide the only other attempt to form values of such terms. They assume that the forecast can be proxied by some distributed lag. However, it is not clear why this approach should be preferred to the one used here.

We are now in a position to define the values of $\tau^*$ and $p^*$ used below. For each, three cases can be distinguished:

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20 Three issues should be noted regarding concerning the use of actual values. First, 'actual' values are based on the model of the UK corporation tax system described in Appendix B, and are therefore subject to some measurement error. Second, perfect foresight assumptions are clearly not available for periods beyond which data is available (1986). In this case, the accounting data has been predicted using simple autoregressive models, and the tax model applied to the resulting forecasts. Third, using the 'actual' tax system for future periods, given that UK tax reforms are generally unanticipated (an exception being the transition period in the 1984 reforms), may not minimise the forecast error, compared with the assumption of static expectations for the tax system and perfect foresight for accounting data. In practice, there was no significant difference in the Q model under these two approaches. The results below assume static expectations for the tax system.
(a) fully tax paying in period \( t \): \( H(b_t) = H(a_t) = 0 \)
(b) Fully tax exhausted for \( m \) periods and ACT exhausted for \( m+n \) periods:
\[ H(b_s) = 1 \text{ if } s < m \text{ and } H(b_{t+m}) = 0; \quad H(a_s) = 1 \text{ if } s < m+n \text{ and } H(a_{t+n}) = 0 \]
(c) ACT exhausted for \( n \) periods: \( H(b_t) = 1; \quad H(a_s) = 0 \text{ if } s < n \text{ and } H(a_{t+n}) = 1 \)

Beginning with \( \tau \), we can write:

\[
\tau = \begin{cases} 
\tau & \text{case (a)} \\
c + (\tau - c)(1-c)\rho^m + c(1+\tau - c)\rho^n & \text{case (b)} \\
\tau - c(\tau - c) + c(\tau - c)\rho^n & \text{case (c)}
\end{cases}
\tag{2.66}
\]

It should be noted that in these definitions, and below, it is assumed that \( \lambda^D \) is constant over time. Clearly the simplest assumption to achieve this is that dividends are always paid, so that \( \lambda^D_s = 0 \) for all \( s \).

The definition of \( p^* \) is complicated by the term \( E_t[\delta V_t / \delta K_{t+1}^T] \) in (2.25). However, \( K_t^T \) is simply the tax-written down value of the asset at the end of period \( t \) – i.e. the value of the asset after receiving depreciation allowances in period \( t \). Suppose there is a constant exponential depreciation rate for tax purposes for the asset, at rate \( \delta_t \), so that (abstracting from new investment) \( K_t^T = (1-\delta_t)K_{t-1}^T \). Further define the depreciation allowance claimable in period \( s \) in the absence of tax exhaustion as \( Z = \delta_t K_{t-s}^T \). In the presence of tax exhaustion, the value of \( Z \) falls to \( Z^* \). In this case, \( \eta^* \) is just the discounted present value of the \( Z^* \) for \( s \geq t \). \( \eta^* \) and hence \( p^* \) are calculated in exactly this way, where, for any future period \( s \), \( Z^*_s \) is

\[
Z^* = \begin{cases} 
\tau Z & \text{case (a)} \\
(\tau - c)(1-c)\rho^m Z + c(1+\tau - c)\rho^n Z & \text{case (b)} \\
(1-c)(\tau - c)Z + c(1+\tau - c)\rho^n Z & \text{case (c)}
\end{cases}
\tag{2.67}
\]

21 A fourth case can be distinguished in which the firm is fully tax exhausted, but not ACT exhausted, which can occur because of the carry-back provisions in the imputation system. This creates the same expressions as under a classical corporation tax with tax exhaustion, namely \( \tau = \rho^m \tau \) and \( z = \rho^m z \) (see below for the definition of \( z \)). This case has been allowed for in the empirical work.
It only remains to define the various terms used as a proxy for \( E_t [\partial V_{t+1} / \partial K_{t+1}] \), or \( Q \), from (2.35). \( V_{t+1} \) is simply the market value of the firm at the beginning of period \( t+1 \). The second term, \( T^T_t E_t [\partial V_{t+1} / \partial K_{t+1}] \), the present value of capital allowances for tax purposes on past investment, is calculated in exactly the same way as \( Z^* \) in (2.55). The third term is the shadow value of the stock of leased assets, for which, as noted above, insufficient data exists for this term to be included in the empirical work. It is therefore ignored. The fourth term, \(-B_t E_t [\partial V_{t+1} / \partial B_{t+1}]\), can be estimated using the expression in (2.43). If the firm issues debt and new equity, then this term is equal to minus the stock of debt held at the beginning of period \( t+1 \). This would be the case, for example, in regimes 3 and 4 discussed above. However, we further make use of the assumption that positive dividends are paid, which restricts us to regime 4. It is therefore assumed that the firm is in regime 4\(^{22}\). The last two terms are the shadow values of the stock of losses and unrelieved ACT brought forward into period \( t+1 \). Given estimates of these stocks from the tax model in Appendix B, and the discussion above, these terms are straightforward to calculate.

Finally it should be noted that the investment equation, (2.34), includes a term \( (\gamma + \lambda^D_t) \). Since it is assumed elsewhere that \( \lambda^D_t = 0 \), this implies that marginal \( Q \) should be divided by \( \gamma \). The distinction between the case of dividing or not dividing by \( \gamma \) was the distinction drawn by Poterba and Summers (1983) between what are termed here regimes 1 and 3 (since in their model \( \gamma + \lambda^D_t = 1 \) in regime 3). Here, since we are assuming that the firm is in regime 4, it is possible that \( \gamma \neq 1 \). This includes two common possibilities - institutional investors, for whom \( m=z=0 \) and \( \gamma=1/(1-c)>1 \), and basic rate taxpayers for whom \( m=c, z=0 \) and so \( \gamma=1 \). Empirically, there is little to choose between these two possibilities, and so the results presented below are for the case in which \( \gamma=1 \).

\(^{22}\)In principle, this has the disadvantage that this implies that new equity is always issued. However, restricting the empirical work to only those observations for which new equity is issued would greatly reduce the size of the sample and introduce sample selection problems.
2.6. Data

The data are described in Chapter 1 and Appendix A, and only a brief description of the relevant variables is given here, before some discussion of the values of Q and the tax parameters. As noted elsewhere, the main data sources are company accounting data from Datastream and their stock market values from the London Share Price Database. In addition, estimates of the tax position of these companies from the model described in Appendix B is used. An unbalanced panel of 729 UK manufacturing companies with at least four years of continuous data is used.

Inflation-adjusted estimates of the net capital stock have been constructed from the historic cost data available in company accounts using the perpetual inventory method, with depreciation rates of 8.19% for plant and machinery and 2.5% for buildings (taken from King and Fullerton (1984)). Gross investment includes both new fixed assets purchased and fixed assets acquired through takeovers. This is because data on each form of investment separately is not available; firms involved in major acquisitions have been excluded from the sample. The book value of debt reported in the accounts is taken as a proxy for its market value. The market value of equity, $V_{t-1}$, is measured as the average of three stock market valuations representing the midpoint of each month prior to the start of the firm's accounting year.

Descriptive statistics concerning the main accounting variables, such as the investment rate, are given in Appendix A. Here we discuss values of $\tau^*$, $p^*$ and Q. In particular, we wish to examine the impact on the effective tax rate, the effective price of capital goods and Q of tax exhaustion. One method of doing so is to construct the tax parameters and Q under various alternative models of the tax system, and to compare the results. We therefore consider four different methods of accounting for taxation:

1. Ignoring tax exhaustion (full tax).
2. Accounting for full tax exhaustion, but ignoring the imputation system altogether, using the model described in Appendix B for estimates
of periods of tax exhaustion.

3. Accounting for full tax exhaustion and ACT exhaustion, again using the model in Appendix B.

4. Accounting for full tax exhaustion and ACT exhaustion, but using estimates of tax exhaustion directly from tax data in company accounts.

The reason for including the fourth method is as a partial check on the corporation tax model described in Appendix B. If the results using methods 3 and 4 were very different, this may be due to one of the sources of estimates of tax exhaustion estimates being poor. Below means and standard deviations of each of $\tau^*$, $p^*$ and $Q$ are presented using each of these four methods. In addition, the same information on $Q$ ignoring tax altogether is presented. Finally, correlation coefficients are presented: if these variables vary across firms because of tax exhaustion (as the theory predicts), correlation coefficients will be low.

Beginning with $\tau^*$, Table 2.1(a) presents estimates of the unweighted mean value of $\tau^*$ across all the companies available in each year. Table 2.1(b) presents the standard deviations on the same basis. These tables reveal that tax exhaustion can have a significant effect on the average effective tax rate, and that variation in the effective tax rate across companies in the same year was considerable in the 1970s and early 1980s. Considering first the preferred measure in the third column of each table – allowing for full tax exhaustion and ACT exhaustion using the model in Appendix B – the average tax rate was around 3 and 4 percentage points lower than the full statutory rate (column 1) for much of the 1970s and early 1980s. Moreover, the standard deviation ranged between 5 and 7 percentage points (compared with zero in the full tax case, since the statutory tax rate was constant over much of this period; positive values for the standard deviation for column 1, ignoring tax exhaustion, arise only because of changes in the statutory tax rate). Clearly the effective tax rate, $\tau^*$ cannot exceed the

---

23 This is essentially the approach taken, in a somewhat ad hoc form, in Blundell et al (1989).
Table 2.1(a) Estimated average effective tax rate, $\tau^*$

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<th>Year</th>
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Notes
1. Accounting years are attributed to the calendar year in which the year end occurs.
2. The estimates presented are an unweighted average of the effective tax rates facing all of the companies available in each year.

Table 2.1(b) Estimated standard deviation of effective tax rate, $\tau^*$

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<th>Year</th>
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statutory tax rate, \( \tau \), and so the high standard deviation of \( \tau^* \) (greater than the difference between \( \tau^* \) and \( \tau \)) mostly reflects values below the average value. This gives a strong indication that there are a large number of companies facing a very low effective tax rate.

These estimates are reasonably close to those in the last column, based on tax data directly from company accounts. The last column does not exhibit such a low average value of \( \tau^* \), nor such a high standard deviation, which reflects that this method does not suggest quite such a high prevalence of tax exhaustion. However, this may simply reflect that estimates based on tax data in company accounts may underestimate tax exhaustion due to a desire on the part of companies (or their accountants) to show a reasonably high figure for tax paid.

The second column presents estimates of \( \tau^* \) ignoring ACT exhaustion. The figures here appear to be even more striking in the size of the standard deviation and the difference of the average value from \( \tau \). At first sight this may appear counterintuitive since less tax exhaustion is being allowed for in these estimates. However, the reason is that ACT to some extent has an offsetting effect to full tax exhaustion. Consider the value of \( \tau^* \) in case (b) in (2.66), for example, in which the firm is fully tax exhausted and ACT exhausted in period \( t \). Ignoring ACT, tax due in period \( t \) on an additional pound of earnings is zero; tax is deferred until the firm resumes a tax-paying position. However, allowing for ACT, an ACT charge of \( c \) is due immediately if the additional pound of earnings is paid out as a dividend (which is what \( \tau^* \) measures). At some point in the future the firm will increase this to \( \tau \) (ignoring discounting) by paying a corporation tax charge of \( \tau \) and claiming ACT relief of \( c \). The effect of this is that in the latter case \( \tau^* \) cannot fall below \( c \), whereas in the former case it can fall to zero.

Table 2.2(a) and Table 2.2(b) present similar statistics for \( \rho^*/\rho \), a measure of the effective reduction in the price of capital goods due to tax reliefs and allowances. The full tax case in column 1 is not, in this case, simply a statutory tax rate. In fact it is simply \( 1-\eta_t \) where \( \eta_t \) is the present value of depreciation allowances, averaged over assets. Assets are split into three categories for this purpose: plant and machinery, industrial buildings and commercial buildings. Weights
used are the investment in each asset in each period, and so variation across companies occurs in $p^*/p$ because of variation in the asset structure of their investments. The minimum value that $p^*/p$ could take would be 48% - this would be the case for an investment (in plant and machinery) which received immediate expensing at a tax rate of 52%. The relative generosity of the allowance for plant and machinery and industrial buildings is reflected in the low numbers in column 1 of Table 2.2(a) in the 1970s and up to the 1984 reforms to corporation tax. Commercial buildings have never received an allowance.

Tax exhaustion has two effects as can easily be seen from the definition of $p^*$ in (2.29): $p^* = \frac{(1+s^*)}{(1+s)} - \frac{\gamma^*}{\gamma - \eta_t^*}$. If ACT exhaustion is present in period $t$ then $\gamma^* < \gamma$, which reduces the effective price of the asset in terms of dividends foregone by the shareholder, since the firm does not need to reduce net dividends by the full amount of the price of the asset (since in the absence of imputation reducing net dividends by $1-c$ it saves ACT of $c$). However, tax exhaustion need not occur in period $t$ for it to affect the effective price of capital goods (unless there is immediate expensing). A subsequent period of tax exhaustion would still reduce the present value of allowances, $\eta_t^*$, and so increase $p^*$ if allowances to be claimed in that period are delayed.

It is generally the case that the second effect, through $\eta_t^*$, outweighs the first effect, through $s_t^*$, so that tax exhaustion significantly increases the effective price (although in the mid-1980s the reverse becomes true). As would be expected, ignoring ACT exhaustion (as in column 2) leads to an overestimate of the effective price since the first effect is ignored. Compared with ignoring tax exhaustion altogether, the standard deviation across companies is also much greater when tax exhaustion is accounted for. Once again, the standard deviation is higher if ACT exhaustion is ignored. Columns 3 and 4 of Table 2.2 are once more quite similar, although this partly reflects the fact that forecasts of tax exhaustion beyond the end of the period for which data are available are the same in both cases.
Table 2.2(a) Estimated average effective price of capital goods, $p/p$ (%).

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Notes
1. Figures are shown as a percentage of the pre-tax price, $p$.

Table 2.2(b) Estimated standard deviation of effective price, $p/p$.

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<td>602</td>
<td>5.9</td>
<td>10.1</td>
<td>7.4</td>
<td>7.1</td>
</tr>
<tr>
<td>1985</td>
<td>565</td>
<td>5.5</td>
<td>8.3</td>
<td>6.5</td>
<td>5.9</td>
</tr>
<tr>
<td>1986</td>
<td>492</td>
<td>5.3</td>
<td>6.4</td>
<td>6.2</td>
<td>5.8</td>
</tr>
</tbody>
</table>
Tables 2.3(a) and 2.3(b) present similar descriptive statistics for Q. In this case there are five versions of Q: the four corresponding to the four cases above together with an estimate of Q without any correction for taxation. Several points may be noted regarding the estimates in Table 2.3(a). To understand them it is helpful to repeat the definition of Q:

$$Q_t = \frac{\mathbb{E}_t \left[ \frac{\partial V_{t+1}}{\partial K_{t+1}} \right]}{(1-\tau_t)p_t^Y} - \frac{p_t^*}{(1-\tau_t)p_t^Y}$$  \hspace{1cm} (2.39)$$

In long run equilibrium, Q=0, implying zero adjustment costs. Note then, that for most of the period considered - from 1975 onwards - average Q is negative for all versions of Q, which implies that it appears to have been the stock market's view that, on average, firms should have been reducing their capital stock. According to the figures given in Appendix A, firms were increasing their capital stocks over the period. However, these figures are subject to some measurement error, and depend crucially on the assumed rates of depreciation. Sales were static or falling for much of the period.

Perhaps surprisingly, adding taxes increases the value of Q. However, the first term in Q, reflecting the shadow value of an additional unit of capital to the firm increases with \( \tau^* \) for \( \tau^* > 0 \). Note that \( \mathbb{E}_t \left[ \frac{\partial V_{t+1}}{\partial K_{t+1}} \right] \) should reflect taxation, so that this simply means that Q unadjusted for taxes is too low since taxes are implicitly deducted from the numerator but not the denominator. This effect outweighs the additional cost to the firm of taxation measured by the second term, which becomes greater than \( 1/p_t^Y \) unless \( p_t^* \) is at its minimum value of \( 1-\tau^* \). The drop in the value of Q when tax exhaustion is introduced can partly be explained by the fact that \( \tau^* \) falls as a result of tax exhaustion. The fact that the value of Q continues to fall, moving along

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24 Bond and Devereux (1989) examine the impact of measurement error in the capital stock on the empirical performance of the Q model of investment, concluding that the results were insensitive to varying the measure used.
Table 2.3(a) Estimated average tax-adjusted Q.

<table>
<thead>
<tr>
<th>year no of cos</th>
<th>no tax exhaustion</th>
<th>no tax exhaustion</th>
<th>full tax exhaustion</th>
<th>full tax and ACT exhaustion</th>
<th>full tax and ACT exhaustion (accounts) %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1971 190</td>
<td>0.28</td>
<td>0.81</td>
<td>0.81</td>
<td>0.81</td>
<td>0.81</td>
</tr>
<tr>
<td>1972 297</td>
<td>0.29</td>
<td>0.88</td>
<td>0.89</td>
<td>0.89</td>
<td>0.89</td>
</tr>
<tr>
<td>1973 374</td>
<td>0.57</td>
<td>1.67</td>
<td>1.67</td>
<td>1.56</td>
<td>1.66</td>
</tr>
<tr>
<td>1974 409</td>
<td>0.12</td>
<td>1.18</td>
<td>1.09</td>
<td>0.92</td>
<td>1.15</td>
</tr>
<tr>
<td>1975 431</td>
<td>-0.65</td>
<td>-0.35</td>
<td>-0.40</td>
<td>-0.57</td>
<td>-0.43</td>
</tr>
<tr>
<td>1976 655</td>
<td>-0.82</td>
<td>-0.63</td>
<td>-0.66</td>
<td>-0.83</td>
<td>-0.75</td>
</tr>
<tr>
<td>1977 673</td>
<td>-0.79</td>
<td>-0.54</td>
<td>-0.57</td>
<td>-0.74</td>
<td>-0.71</td>
</tr>
<tr>
<td>1978 685</td>
<td>-0.69</td>
<td>-0.39</td>
<td>-0.42</td>
<td>-0.58</td>
<td>-0.57</td>
</tr>
<tr>
<td>1979 693</td>
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<td>-0.25</td>
<td>-0.29</td>
<td>-0.44</td>
<td>-0.44</td>
</tr>
<tr>
<td>1980 690</td>
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<td>-0.25</td>
<td>-0.30</td>
<td>-0.43</td>
<td>-0.45</td>
</tr>
<tr>
<td>1981 673</td>
<td>-0.64</td>
<td>-0.33</td>
<td>-0.37</td>
<td>-0.48</td>
<td>-0.53</td>
</tr>
<tr>
<td>1982 660</td>
<td>-0.69</td>
<td>-0.40</td>
<td>-0.41</td>
<td>-0.53</td>
<td>-0.58</td>
</tr>
<tr>
<td>1983 639</td>
<td>-0.60</td>
<td>-0.22</td>
<td>-0.25</td>
<td>-0.34</td>
<td>-0.38</td>
</tr>
<tr>
<td>1984 602</td>
<td>-0.49</td>
<td>-0.11</td>
<td>-0.13</td>
<td>-0.21</td>
<td>-0.24</td>
</tr>
<tr>
<td>1985 565</td>
<td>-0.38</td>
<td>-0.02</td>
<td>-0.03</td>
<td>-0.08</td>
<td>-0.08</td>
</tr>
<tr>
<td>1986 492</td>
<td>-0.37</td>
<td>-0.13</td>
<td>-0.14</td>
<td>-0.17</td>
<td>-0.16</td>
</tr>
</tbody>
</table>

Notes
1. Figures are shown as a percentage of the pre-tax price, p.

Table 2.3(b) Estimated standard deviation of tax-adjusted Q

<table>
<thead>
<tr>
<th>year no of cos</th>
<th>no tax exhaustion</th>
<th>no tax exhaustion</th>
<th>full tax exhaustion</th>
<th>full tax and ACT exhaustion</th>
<th>full tax and ACT exhaustion (accounts) %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1971 190</td>
<td>1.23</td>
<td>2.06</td>
<td>2.06</td>
<td>2.06</td>
<td>2.06</td>
</tr>
<tr>
<td>1972 297</td>
<td>1.16</td>
<td>1.96</td>
<td>1.97</td>
<td>1.97</td>
<td>1.97</td>
</tr>
<tr>
<td>1973 374</td>
<td>1.17</td>
<td>2.20</td>
<td>2.20</td>
<td>2.18</td>
<td>2.17</td>
</tr>
<tr>
<td>1974 409</td>
<td>0.94</td>
<td>1.95</td>
<td>1.89</td>
<td>1.91</td>
<td>1.94</td>
</tr>
<tr>
<td>1975 431</td>
<td>0.64</td>
<td>1.34</td>
<td>1.35</td>
<td>1.35</td>
<td>1.33</td>
</tr>
<tr>
<td>1976 655</td>
<td>0.51</td>
<td>1.05</td>
<td>1.03</td>
<td>1.06</td>
<td>1.06</td>
</tr>
<tr>
<td>1977 673</td>
<td>0.50</td>
<td>1.04</td>
<td>1.00</td>
<td>1.04</td>
<td>1.06</td>
</tr>
<tr>
<td>1978 685</td>
<td>0.66</td>
<td>1.37</td>
<td>1.34</td>
<td>1.36</td>
<td>1.39</td>
</tr>
<tr>
<td>1979 693</td>
<td>0.59</td>
<td>1.23</td>
<td>1.21</td>
<td>1.23</td>
<td>1.26</td>
</tr>
<tr>
<td>1980 690</td>
<td>0.58</td>
<td>1.21</td>
<td>1.18</td>
<td>1.23</td>
<td>1.23</td>
</tr>
<tr>
<td>1981 673</td>
<td>0.65</td>
<td>1.39</td>
<td>1.38</td>
<td>1.39</td>
<td>1.40</td>
</tr>
<tr>
<td>1982 660</td>
<td>0.62</td>
<td>1.30</td>
<td>1.29</td>
<td>1.31</td>
<td>1.32</td>
</tr>
<tr>
<td>1983 639</td>
<td>0.73</td>
<td>1.49</td>
<td>1.51</td>
<td>1.51</td>
<td>1.50</td>
</tr>
<tr>
<td>1984 602</td>
<td>0.81</td>
<td>1.57</td>
<td>1.56</td>
<td>1.58</td>
<td>1.56</td>
</tr>
<tr>
<td>1985 565</td>
<td>0.91</td>
<td>1.59</td>
<td>1.59</td>
<td>1.61</td>
<td>1.60</td>
</tr>
<tr>
<td>1986 492</td>
<td>0.90</td>
<td>1.45</td>
<td>1.45</td>
<td>1.46</td>
<td>1.46</td>
</tr>
</tbody>
</table>
the columns in Table 2.3(a), also reflects the fact that the effective price of investment goods rises, as shown in Table 2.2(a).

More surprising results come from Table 2.3(b), indicating the standard deviation of Q across companies for each year. The standard deviation shows a large increase moving from the "no-tax" case to the "full-tax" case, which is mainly due to variation in the asset structure across firms leading to a different average p* (since there is virtually no variation in t across firms, except during a period of tax reform). However, the standard deviation of Q is virtually constant across all four versions which allow for taxation in some way. The variation, across the different definitions, of the standard deviation of t* and p* noted above disappears when both tax effects are included in the definition of Q.

Further evidence on this issue is presented in Table 2.4, which gives correlation coefficients between the different definitions of t*, p* and Q across all firms and years. Table 2.4(a) gives the correlation t*. The correlation between the full tax case and the other cases is very weak, as would be expected, since the statutory tax rate, t, changes little. There is a relatively high coefficient on the correlation between the main tax exhaustion case and the case ignoring the imputation system, since the same estimates of periods of tax exhaustion were used and it is likely that both forms of tax exhaustion might occur together. The correlation with the method of estimating tax exhaustion directly from accounts is much lower, since different estimates are used. A similar pattern emerges for p*/p.

However, Table 2.4(c) shows that all of the definitions of Q are highly correlated, with no coefficient less than 0.98. There are two possible reasons for this. The first is that the effects of tax exhaustion tend to cancel each other out. This can certainly occur (to some extent) in the second term of Q, but not to a large extent in the first term (the empirical proxy for $E_t[\frac{\partial V_{t+1}}{\partial K_{t+1}}]$, defined in (2.35), contains the shadow value of future depreciation allowances on past investment, which varies with the assumption on tax, but this term is very small compared to the market value of debt and equity). In any case, the average values of Q continue to differ, as shown in Table 2.3(a).
The second reason is then that variation in Q is dominated by variation in factors other than tax, notably the market value of equity, which is common to all of the definitions discussed. If this is true, it suggests that the average Q model as set up in this chapter may not be a very fruitful model in which to investigate the effects of taxation. More evidence on this is presented in the next section, which gives estimates the investment model.

**Table 2.4(a) Correlation of different definitions of \( \tau \)**

<table>
<thead>
<tr>
<th></th>
<th>no tax exhn</th>
<th>full tax exhn</th>
<th>full and ACT exhn</th>
<th>full and ACT exhn</th>
</tr>
</thead>
<tbody>
<tr>
<td>no tax exhn</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>full tax exhn</td>
<td>0.30</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>full and ACT exhn</td>
<td>0.52</td>
<td>0.85</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>full and ACT exhn</td>
<td>0.64</td>
<td>0.41</td>
<td>0.53</td>
<td>1.00</td>
</tr>
</tbody>
</table>

(accounts)

**Table 2.4(b) Correlation of different definitions of \( p /p \)**

<table>
<thead>
<tr>
<th></th>
<th>no tax exhn</th>
<th>full tax exhn</th>
<th>full and ACT exhn</th>
<th>full and ACT exhn</th>
</tr>
</thead>
<tbody>
<tr>
<td>no tax exhn</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>full tax exhn</td>
<td>0.66</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>full and ACT exhn</td>
<td>0.83</td>
<td>0.86</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>full and ACT exhn</td>
<td>0.86</td>
<td>0.68</td>
<td>0.77</td>
<td>1.00</td>
</tr>
</tbody>
</table>

(accounts)

**Table 2.4(c) Correlation of different definitions of Q**

<table>
<thead>
<tr>
<th></th>
<th>no tax</th>
<th>no tax exhn</th>
<th>full exhn</th>
<th>full and ACT exhn</th>
<th>full and ACT exhn</th>
</tr>
</thead>
<tbody>
<tr>
<td>no tax</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>no tax exhn</td>
<td>0.98</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>full tax exhn</td>
<td>0.98</td>
<td>0.99</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>full and ACT exhn</td>
<td>0.98</td>
<td>0.99</td>
<td>0.99</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>full and ACT exhn</td>
<td>0.98</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
<td>1.00</td>
</tr>
</tbody>
</table>

(accounts)
2.7. Estimation and Results

This section has two aims. The first is to examine the econometric issues which arise in estimating the investment model specified above. The second is to examine the role played by taxation, and in particular to compare models which ignore taxation altogether, which allow for taxation but ignore tax exhaustion and, finally, which also allow for tax exhaustion.

The investment model to be estimated for firm 1 can be written

\[
\left( \frac{I}{K} \right)_{it} = c + \beta Q_{it} + \epsilon_{it}
\]

for \( i=1,2,\ldots,N, \ t = 1,2,\ldots,T \), and where \( Q \) is defined from (2.39). As mentioned above, the error term, \( \epsilon_{it} \), may contain firm specific effects, \( \alpha_i \) and time specific effects, \( \alpha_t \) as well as an idiosyncratic shock, \( v_{it} \). Thus the following structure is adopted for \( \epsilon_{it} \)

\[
\epsilon_{it} = \alpha_i + \alpha_t + v_{it}
\]

The last term is not restricted by the theory to be an innovation, although more general dynamic relationships between the investment rate and \( Q \) will not necessarily be consistent with the theory.

The estimation of (2.68) requires assessment of the stochastic properties of \( \epsilon_{it} \) and \( Q_{it} \). If \( Q_{it} \) is strictly exogenous (ie. it is uncorrelated with \( \alpha_i \) and \( v_{is} \) for all \( s \) and \( t \)) and \( v_{it} \) is uncorrelated over time, then the standard variance-components GLS estimator can be used. If, however, \( Q_{it} \) is correlated with the fixed effect, \( \alpha_i \), but not with \( v_{is} \), then the GLS estimator is inconsistent, but the within groups estimator may be used.

However, it is likely both that \( Q_{it} \) is correlated with \( v_{is} \), and that \( v_{it} \) is correlated over time. It has already been noted above in the context of the theoretical model that contemporaneous \( Q \) is simultaneously determined with investment, which suggests that it should be treated as endogenous, and as has also been noted, there is nothing in the theory
to prevent adjustment cost shocks being correlated over time. In addition, we may wish to test the possibility of more general dynamic structures involving the lagged dependent variable and the possibility that $Q_{it}$ is correlated with the fixed effect $\alpha_i$. In any of these cases, both the GLS and within groups estimators are inconsistent.

The approach taken here is therefore to estimate (2.68) in first-differences, using a Generalised Methods of Moments (GMM) estimator (see Hansen (1982)). This is an instrumental variables estimator in which the instruments are weighted optimally. In first differences, the error term becomes $(\alpha_t - \alpha_{t-1}) + (\nu_{it} - \nu_{it-1})$ since the firm specific effect drops out. If $Q_{it}$ is endogenous but $\nu_{it}$ is serially uncorrelated, $Q_{it-2}$ and further lags are valid instruments. If $Q_{it}$ is, in fact, predetermined (as it may be given the construction of $Q$ discussed above), $Q_{it-1}$ is also a valid instrument. Clearly, as $t$ increases over the sample period, more data points are available as instruments.

Omitting time effects for notational simplicity, the GMM estimator is

$$\hat{\beta} = (x'Z_N Z'x)^{-1}x'Z_N Z'y$$

(2.70)

where $x$ is the stacked vector of observations on $\Delta Q_{it}$ and $y$ is the stacked vector of observations on $\Delta(I/K)_{it}$. The instrument matrix, $Z$, allows the number of instruments in each cross section to increase as we proceed through the panel. In the presence of general heteroskedasticity across both firms and time, the optimal choice for $A_N$ is

$$A_N = \left(1 - \sum_{i=1}^{N} Z_i^\prime \hat{\Delta} \hat{\nu}_i \hat{\nu}_i^\prime Z_i \right)^{-1}$$

(2.71)

where the vectors $\hat{\Delta} \hat{\nu}_i$ are consistent estimates of the first-differenced residuals for each firm (White (1982)). They are obtained from a preliminary consistent estimator of $\beta$, setting $A_N = (H_{i=1}^{N} Z_i^\prime H Z_i)^{-1}$ where $H$ is a matrix with twos on the leading diagonal, minus ones on the first off-diagonal and zeros elsewhere. With large $N$, this first stage estimation gives similar, though less well determined, coefficient estimates.

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The validity of the instruments used depends on whether \( v_{1t} \) is serially correlated. Only if \( v_{1t} \) is serially uncorrelated (implying a MA(1) error term in the differenced model) will \( Q_{it-2} \) be a valid instrument (given the endogeneity of \( Q \)). Second order serial correlation is tested using the one degree of freedom test (\( m2 \)) proposed by Arellano and Bond (1989). Robust Sargan tests of the over-identifying restrictions exploited by the estimator are also reported.

One other important factor influencing the validity of the instrument set is the possibility of measurement error. For example, if \( Q_{it} \) is measured with error, inducing a negative correlation between it and the current shock, \( v_{it} \), then \( Q_{it-1} \) is no longer a valid instrument even if \( Q \) is otherwise exogenous. Note that measurement error derived from including \( Q_{it-1} \) as an instrument would be expected to induce downwards bias in the estimated value of \( \beta \), while the endogeneity of \( Q_{it-1} \) would be expected to result in upwards bias. Although differencing may exacerbate measurement error problems, the first-difference estimates are robust to permanent measurement error. Although omitting \( Q_{it-1} \) might allow us to make some inference of the importance of white noise measurement error (see Griliches and Hausman (1986)), this procedure may also remove endogeneity in the instrument set. Below, various experiments are tried to try to separate these two effects, including the use of future instruments as suggested by Hayashi and Inoue (1989).

We now turn to the results themselves. Following the strategy outlined above, we first examine the effects of using different instrument sets to analyse the stochastic elements in the model. To do this we use the value of \( Q \) given in the model above - that is the value which allows for full tax exhaustion and ACT exhaustion. Below we test whether this version of \( Q \) in fact fits the data better than alternative versions which do not model the tax system in as much detail (if at all).

Table 2.5 presents estimates of the \( Q \) investment model in first-differences. Heteroskedastic-consistent standard errors are reported. The sample period is 1975-86, although as noted in Appendix A, the sample consists of an unbalanced panel. In most columns \( Q_{1t-2} \) is used which implies an additional two periods of data. In addition, in
Table 2.5 Testing the econometric specification of the Q model

<table>
<thead>
<tr>
<th>Dependent variable Δ(I/K)_{1t}</th>
<th>(a) UNRESTRICTED (1)</th>
<th>(ii)</th>
<th>(iii)</th>
<th>(iv)</th>
<th>(v)</th>
<th>(vi)</th>
</tr>
</thead>
<tbody>
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<td>ΔQ</td>
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<td>0.0099</td>
<td>0.0021</td>
<td>0.0088</td>
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<td>(0.0014)</td>
<td>(0.0035)</td>
<td>(0.0026)</td>
<td>(0.0009)</td>
</tr>
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<td>(0.0007)</td>
<td>(0.0028)</td>
<td>(0.0012)</td>
<td>(0.0007)</td>
<td></td>
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<tr>
<td>Δ(I/K)</td>
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<td>-0.38</td>
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<td>(0.0007)</td>
<td>(0.0028)</td>
<td>(0.0012)</td>
<td>(0.0007)</td>
<td></td>
</tr>
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<td>418.2</td>
<td>380.0</td>
<td>347.1</td>
<td>484.4</td>
</tr>
<tr>
<td>Sargan</td>
<td>71.5(75)</td>
<td>68.7(78)</td>
<td>77.9(72)</td>
<td>69.8(73)</td>
<td>98.6(81)</td>
<td>128.6(75)</td>
</tr>
<tr>
<td>Stability</td>
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<td>3.13(3)</td>
<td>5.07(3)</td>
<td>1.10(3)</td>
<td>19.7(3)</td>
<td>17.4(3)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(b) RESTRICTED (1)</th>
<th>(ii)</th>
<th>(iii)</th>
<th>(iv)</th>
<th>(v)</th>
<th>(vi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ΔQ</td>
<td>-0.0105</td>
<td>0.0037</td>
<td>0.0080</td>
<td>0.0175</td>
<td>0.0030</td>
</tr>
<tr>
<td></td>
<td>(0.0034)</td>
<td>(0.0010)</td>
<td>(0.0035)</td>
<td>(0.0025)</td>
<td>(0.0009)</td>
</tr>
<tr>
<td>ρ</td>
<td>-0.2549</td>
<td>0.2433</td>
<td>0.2367</td>
<td>0.2440</td>
<td>0.2467</td>
</tr>
<tr>
<td></td>
<td>(0.0186)</td>
<td>(0.0198)</td>
<td>(0.0203)</td>
<td>(0.0190)</td>
<td>(0.0208)</td>
</tr>
<tr>
<td>Comfac</td>
<td>4.47</td>
<td>3.02</td>
<td>4.66</td>
<td>0.02</td>
<td>3.40</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Instruments</th>
<th>Q_{-2}Q_{-8}</th>
<th>Q_{-2}Q_{-7}</th>
<th>Q_{-1}Q_{-6}</th>
<th>Q_{-3}Q_{-8}</th>
<th>Q_{-2}Q_{-4}</th>
<th>Q_{-3}Q_{-2}</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(I/K)_{-2}</td>
<td>(I/K)_{-2}</td>
<td>(I/K)_{-2}</td>
<td>(I/K)_{-2}</td>
<td>(I/K)_{-2}</td>
<td>(I/K)_{-2}</td>
</tr>
</tbody>
</table>

Notes:
1. Time dummies are included as regressors and instruments in all equations.
2. Asymptotic standard errors are reported in parentheses. Standard errors and test statistics are robust to general time-series and cross-section heteroskedasticity.
3. m2 is a test for second order serial correlation in the residuals, asymptotically distributed as N(0,1) under the null of no serial correlation.
4. z1 is a Wald test of the joint significance of the reported coefficients, asymptotically distributed as χ² under the null of no relationship. The number of degrees of freedom is equal to the number of reported coefficients (excluding time dummies).
5. z2 is a Wald test of the joint significance of the time dummies. There are 12 degrees of freedom for each model in the table.
6. The Sargan statistic is a test of the over-identifying restrictions, asymptotically distributed as χ²(k) under the null. The number of degrees of freedom is given in parentheses.

... continued
continued....

7. The stability test is a Wald test of the hypothesis that the reported coefficients are common across the sub-periods 1975-80 and 1981-86. The number of degrees of freedom is equal to the number of reported coefficients.

8. The comfac statistic is a test of the common factor restrictions that the dynamics are generated by an AR(1) disturbance, asymptotically distributed as $\chi^2(1)$.

In order to use lagged instruments data back to 1971 is used. Of the 729 firms used here, only 125 have data for every year from 1971 to 1986. The number of observations used here is 6541.

Column (i) presents the results for a model that allows Q to be endogenous (so that the latest value of Q used as an instrument is dated t-2) and correlated with the fixed effect, but assumes that $v_{it}$ is not serially correlated. Although the estimate of $\beta$ is positive, it is very small and not significant. Further, the m2 statistic signals that there is potentially dynamic misspecification. Residual autocorrelation remained when lags of Q were added to the model and when the lagged investment rate was added to the instrument set.

In column (ii), a more general specification is therefore presented which includes lags of both Q and I/K. The estimate of $\beta$ is considerably higher, while its standard error falls, so that it becomes significantly different from zero. The lagged dependent variable is strongly significant. The m2 statistic falls to close to zero under this specification, indicating absence of second order serial correlation. However, as noted above, a dynamic model such as that presented in column (ii) is only consistent with the theoretical model if the dynamics can be represented by persistence in the stochastic factor in the adjustment cost function. In the case of the model in column (ii), this would require that $v_{it}$ follows an AR(1) process and that the parameter estimates satisfy the common factor restrictions for such a process — that is, if $v_{it} = \rho v_{i,t-1} + w_{it}$, where $w_{it}$ is white noise, then the coefficient on $Q_{i,t-1}$ must be equal to $-\beta \rho$. The sign and size suggest that this restriction might be accepted by the data. A formal test is presented in the lower half of the table. Here the comfac restriction is imposed on the unrestricted parameter estimates by the
minimum distance approach\textsuperscript{25}. The test statistic for these restrictions is the minimised value of this criteria function, which is asymptotically distributed as $\chi^2(k)$ under the null, where $k$ is the number of restrictions; in this case $k=1$. The data slightly reject the restriction at 5% confidence levels.

It could therefore be argued that column (ii) represents a reasonable specification of the model. As a further test, the table also shows that the estimates in column (ii) do not reject a Wald test for the stability of the slope coefficients over the two halves of the sample period, split in 1981. One remaining problem, however, is the size of the coefficient on $Q_{1t}$. The estimate in column (ii) (and indeed throughout the results presented here) is roughly in line with results found on aggregate data (see for example, Summers (1981), Poterba and Summers (1983), von Furstenburg (1977) and Poret and Torres (1989)), which, as shown by Summers (1981), implies an extremely long adjustment process for new investment. These possible shortcomings of the Q model are discussed further below.

Column (iii) of Table 2.5 begins to test some of the assumptions underlying the specification in column (ii), in particular the possible endogeneity of $Q_{1t}$ and possible measurement error in $Q_{1t}$. Column (iii) explores these issues by including $Q_{1,t-1}$ in the instrument set. This would be valid in the absence of measurement error and if $Q_{1t}$ is predetermined with respect to $v_{1t}$. Contrary to what would be expected if the endogeneity issue were the most important, the coefficient on $Q_{1t}$ falls when $Q_{1,t-1}$ is added to the instrument set. Indeed, it is no longer significantly different from zero. The other parameter estimates and the test statistics are largely unchanged: although the fact that the Sargan rises by a relatively small amount is further evidence that the endogeneity issue is not dominant. Overall, these results suggest, then, that measurement error in Q is leading to downwards bias which more than offsets any upward bias due to the simultaneous determination of $Q_{1,t-1}$ and $v_{1,t-1}$.

\textsuperscript{25}This approach is described, for example, in Blundell et al (1989).
However, in the presence of measurement error, in principle, $Q_{1,t-2}$ also becomes an invalid instrument although its significance in contributing to any bias may well be small. This is because measurement error in $Q_{1,t-2}$ can only constitute a small part of the whole error term – if $w_{it}$ is in the innovation part of the AR(1) process, and $m_{it}$ is the measurement error in $Q_{it}$, then the error term in column (ii) is $w_{it}-w_{1,t-1}+\beta m_{it}-\beta(1+\rho)m_{1,t-1}+\beta \rho m_{1,t-2}$. Given the sizes of $\beta$ and $\rho$, the last term must be relatively small. Column (iv) therefore explores the effect of removing $Q_{1,t-2}$ from the instrument set. The result is a relatively small drop in the estimate of $\beta$ compared with column (ii) and a large rise compared with column (iii). Comparison of columns (ii) and (iii) do not therefore support the hypothesis of large biases due to measurement error resulting from the inclusion of $Q_{1,t-2}$ in the instrument set.

Such a comparison cannot reject the possibility entirely, though, since it may simply the case that including only lags of $t-3$ and earlier in the instrument set reduces the correlation of the instrument set with the regressors: in other words the instruments are not very powerful. Some light on this possibility is, however, shed by considering the standard error of $\beta$ in column (iv). Since it is only very slightly higher than in column (ii) this suggests that the power of the two instrument sets are not very different. Note also that, as in column (ii) the comfac restriction slightly rejects in column (iv) and that the stability test also holds.

In column (v) we examine the possibility that conditions hold under which future instruments may be valid by including $Q_{1,t+1}, Q_{1,t+2}$ and $Q_{1,t+3}$ in the instrument set, again in their GMM form ($Q_{1t}$ and $Q_{1,t-1}$ continue to be excluded). This follows the arguments of Hayashi and Inoue (1989), that if $v_{it}$ is serially independent, then under efficient markets, $Q_{1,t+1}$ can be shown to be independent of current and past innovations to the investment process. If this argument is valid the efficiency of the GMM estimator will be increased by including future instruments. It is apparent from column (v) of Table 2.5 that the standard error on the coefficient of $\beta$ does indeed fall when these additional future instruments are included. However, it should also be
noted that the coefficient estimate almost doubles compared with the estimate in column (ii). Thus, as predicted by the theory, the presence of serial correlation in \( v_{it} \) induces upwards bias when future instruments are included.

Finally, in column (vi) the effect of assuming strict exogeneity of \( Q \) (as would be required for the consistency of the GLS or within groups estimators) is tested by including values of \( Q \) dated between \( t-2 \) and \( t+3 \) inclusive in the instrument set. The estimate of \( \beta \) under these conditions falls once more to below that found in columns (ii) and (iv), which suggests that downwards bias due to measurement error is once again dominating any upwards bias due to simultaneity. The large differences in the coefficient estimates in this table suggest that using any estimator which requires strict conditions on the stochastic properties of the model, such as strict exogeneity of \( Q \) or lack of serial correlation in \( v_{it} \) is likely to be biased and inconsistent.

Clearly, this rules out the use of OLS as well as GLS and within groups. Of the estimates shown in Table 2.5, the results in column (ii) seem most reasonable, showing a reasonably precise and relatively large estimate of \( \beta \) in a theoretically consistent and stable model.

Having found such a preferred specification, we can now turn to the issue of taxation, and, in particular, to test the effect of using different measures of \( Q \) in the model. The results of this procedure are shown in Table 2.6.

The five columns in Table 2.6 represent five measures of \( Q \). They are as follows:

1. Ignoring taxation entirely.
2. Allowing taxes, but assuming no tax exhaustion.
3. Allowing taxable losses, but not unrelieved ACT.
4. Allowing both forms of tax exhaustion.
5. As (iv), but using estimates of tax exhaustion based on an algorithm for using data on taxation in company accounts, rather than using the model described in Appendix B.
Table 2.6 Alternative measures of Q

<table>
<thead>
<tr>
<th>Dependent variable $\Delta(I/K)_{1t}$</th>
<th>(a) UNRESTRICTED</th>
<th>(i)</th>
<th>(ii)</th>
<th>(iii)</th>
<th>(iv)</th>
<th>(v)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta Q_{1t}$</td>
<td>0.0219</td>
<td>0.0103</td>
<td>0.0100</td>
<td>0.0099</td>
<td>0.0102</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0073)</td>
<td>(0.0034)</td>
<td>(0.0032)</td>
<td>(0.0034)</td>
<td>(0.0033)</td>
<td></td>
</tr>
<tr>
<td>$\Delta Q_{1,t-1}$</td>
<td>-0.0083</td>
<td>-0.0040</td>
<td>-0.0039</td>
<td>-0.0047</td>
<td>-0.0038</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0024)</td>
<td>(0.0012)</td>
<td>(0.0012)</td>
<td>(0.0012)</td>
<td>(0.0012)</td>
<td></td>
</tr>
<tr>
<td>$\Delta(I/K)_{1t}$</td>
<td>0.2316</td>
<td>0.2206</td>
<td>0.2282</td>
<td>0.2365</td>
<td>0.2281</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0206)</td>
<td>(0.0205)</td>
<td>(0.0206)</td>
<td>(0.0205)</td>
<td>(0.0203)</td>
<td></td>
</tr>
<tr>
<td>$m^2$</td>
<td>-0.52</td>
<td>-0.49</td>
<td>-0.33</td>
<td>-0.29</td>
<td>-0.43</td>
<td></td>
</tr>
<tr>
<td>$z^2$</td>
<td>388.7</td>
<td>388.8</td>
<td>389.9</td>
<td>400.8</td>
<td>376.3</td>
<td></td>
</tr>
<tr>
<td>Sargan</td>
<td>61.9(78)</td>
<td>67.9(78)</td>
<td>68.0(78)</td>
<td>68.7(78)</td>
<td>71.8(78)</td>
<td></td>
</tr>
<tr>
<td>Stability</td>
<td>2.91(3)</td>
<td>2.45(3)</td>
<td>3.60(3)</td>
<td>3.13(3)</td>
<td>4.15(3)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(b) RESTRICTED</th>
<th>(i)</th>
<th>(ii)</th>
<th>(iii)</th>
<th>(iv)</th>
<th>(v)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta Q$</td>
<td>0.0233</td>
<td>0.0105</td>
<td>0.0103</td>
<td>0.0105</td>
<td>0.0104</td>
</tr>
<tr>
<td></td>
<td>(0.0072)</td>
<td>(0.0033)</td>
<td>(0.0032)</td>
<td>(0.0034)</td>
<td>(0.0033)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.2443</td>
<td>0.2338</td>
<td>0.2395</td>
<td>0.2549</td>
<td>0.2385</td>
</tr>
<tr>
<td></td>
<td>(0.0187)</td>
<td>(0.0187)</td>
<td>(0.0192)</td>
<td>(0.0186)</td>
<td>(0.0189)</td>
</tr>
<tr>
<td>Comfac</td>
<td>2.21</td>
<td>2.46</td>
<td>2.34</td>
<td>4.47</td>
<td>2.03</td>
</tr>
<tr>
<td>Instruments</td>
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<td>$(I/K)^{-2}_2$</td>
<td>$Q_2^{-Q_7}$</td>
<td>$(I/K)^{-2}_2$</td>
<td>$Q_2^{-Q_7}$</td>
</tr>
</tbody>
</table>

Notes. See notes to Table 2.5

As might have been expected, given the very high correlations between the different measures of Q presented in Table 2.4, there is very little variation across the columns in Table 2.6. In particular, the four columns in which Q is adjusted for taxation in some fashion produce virtually identical results. In the light of such similarity there is clearly no point in performing formal specification tests to determine whether any measure of Q outperforms the other measures. The result would obviously be that no measure can be preferred by consideration of the data.
It is somewhat surprising that the measure of $Q$ unadjusted for taxation yields a coefficient estimate roughly double that of the measures of $Q$ adjusted for taxation, contrary to results presented elsewhere (see, for example, Salinger and Summers (1984)). It should also be noted, however, that the standard errors on $Q_{1t}$ and $Q_{1,t-1}$ in column (1) are also double those in the other columns, indicating that the 'no tax' model is less well determined.

2.8 Conclusions

This chapter has developed a model of firm behaviour which explicitly allows for the two forms of tax exhaustion found in the UK and elsewhere in Europe, full tax exhaustion and ACT exhaustion. The model has been used to analyse the financial behaviour of the firm and the investment behaviour through the Q model of investment. In the course of this analysis, two important tax variables are developed, the effective tax rate and the effective price of capital goods, both of which depend on the tax position of the firm, and, in particular, on whether it is currently tax exhausted or whether it expects to be in the future.

Four possible financial regimes in which the firm may find itself are discussed. Which regime the firm is in depends partly on the value of $\gamma$, the 'tax discrimination' variable. If $\gamma < 1$ (the case which is commonly analysed in the literature) there are three possible regimes. In regime 1, the firm will use debt and retention finance, in regime 2 only debt finance, and in regime 3 debt and new equity finance. The proportions of each type of finance used is, however, difficult to determine since any activity by the firm tends to alter the probability of tax exhaustion and therefore affect effective tax rates and the relative cost of each form of finance. If $\gamma > 1$, a fourth regime is possible, in which the firm uses all three sources of finance. This regime does not constitute a 'Miller' equilibrium, however, since once again, the proportions of each source of finance used is determined by the probability of tax exhaustion and agency costs on debt.
The definition of average Q is slightly different from the normal formulation when tax exhaustion is allowed for. In particular, the shadow values of the two additional state variables, losses carried forward and unrelieved ACT must be deducted from the market value of the firm in order to find an expression for marginal Q. Despite considerable variation in the effective tax rate and effective price of capital goods across different measures of taxation (essentially allowing for tax exhaustion and not allowing for tax exhaustion), the equivalent variation in Q was minimal.

In estimating the Q model, considerable attention was paid to the possibility of endogeneity of Q and measurement error in Q, with results suggesting that measurement error was significant and dominated problems of endogeneity. In comparing the preferred specification across alternative measures of taxation, however, there was very little difference in the empirical results. While this may be accounted for by the fact that tax exhaustion is simply irrelevant to investment decisions, such a view is weakened by the fact that the effective tax rate and effective price of capital goods varied considerably across the alternative measures of taxation. An alternative view is therefore that the Q model is a poor model for the study of the effects of taxation on investment. This view is supported by the very small variation in Q across the different measures which suggests that the variation in taxes is dominated by variation in the market value of the firm. The low coefficient found on Q (in line with other empirical work) together with the apparently important role played by measurement error, may suggest that the firm's market value may be too volatile to be used as a reasonable proxy for marginal Q, and therefore casts doubt on the value of investment equations based on average Q.

These points mirror the debates between Jorgenson, Eisner and others beginning in the late 1960s\(^{26}\): essentially the question then was whether

\(^{26}\)See, for example, Eisner and Nadiri (1968) and (1970) and Hall and Jorgensen (1969).
the investment equation should include a term combining output and the cost of capital or whether the two should be kept separate. The more restricted model, in which the composite term is used tends to overstate the impact of the cost of capital on investment (Chirinko and Eisner (1983)). In the case of the Q model, the reverse may be true. For some reason, possibly volatility of market values, the coefficient is very low. This may result in understating the impact of taxation on investment, since, once again, a composite term is used.
3.1 Introduction

In this chapter the model developed in chapter 2 is used to develop measures of the cost of capital under different financial regimes. The impact of these measures on the level on investment is then tested in an Euler equation approach, based directly on the first order conditions for investment and the capital stock. This model differs from the standard cost of capital procedure of Jorgensen (1963), Jorgensen and Hall (1967) and many others in a number of ways, primarily due to the treatment of adjustment costs and expectations in the maximisation process (for a critical survey of the vast literature on investment and the cost of capital, see Chirinko (1987 and 1988)).

The traditional empirical approach is to consider a standard neoclassical model in the absence of adjustment costs, optimisation of which yields the result that the cost of capital should be equal to the marginal product of capital. Specifying a functional form for the production function yields an equation determining the optimal, or desired, capital stock. The presence of adjustment costs is taken to mean that the firm does not reach this optimal point in every period, but rather that it makes some partial adjustment towards this desired level from the current level. Introducing the possibility of adaptive expectations and delivery and other lags yields an equation for the level of investment in a given period, which generally depends on output, lagged output, lagged investment and the cost of capital.

The precise form of the traditional equation varies with the assumptions made, and various features of the equation can vary considerably.

1Although theoretical work has incorporated adjustment costs into explicit optimising models since Elsner and Strotz (1963), Lucas (1967), Gould (1968) and Treadway (1969).
However, this type of approach has two common elements. First, following this procedure clearly results in an ad hoc investment equation. The reasons for the partial adjustment to the desired level of the capital stock are generally not explicitly incorporated into the optimisation which yields the equality between the cost of capital and the marginal product of capital.

Second, the eventual model is backward looking, lags representing adaptive expectations, partial adjustment due to some form of adjustment costs and lags between the placing of an order for a capital asset and the beginning of its productive use. As a result of each of these features this approach has several disadvantages. It suffers from the Lucas (1976) critique, in that the estimated parameters are some (unknown) combination of effects derived from expectations, tastes and technology. However, expectations may well change as a result of government policy, for example a change in taxation. Consider, for example, the 1984 reforms to corporation tax in the UK, in which changes to the tax system were announced for up to two years ahead. Inasmuch as the actual level of investment responded to the announcement of future changes to the tax system, the parameters of such backward-looking models cannot incorporate such effects.

The model described below has neither of these features. It is derived explicitly from what is essentially the same model as discussed in Chapter 2. The presence of adjustment costs in that model enable a forward-looking investment equation to be derived, in which the rate of investment in the current period depends positively on the rate of investment, the rate of investment squared and the marginal product of capital, all in the next period and, negatively on the cost of capital. Assuming rational expectations permits the expected values of the future variables to be replaced with their actual outcomes, with instrumental variables being used for estimation.

In the next section, this basic investment model is derived, and the definition of the cost of capital in the model is analysed. In particular, alternative financial regimes are taken into account, and the effects of the UK imputation system and tax exhaustion are considered. The cost of capital depends on the financial regime in which
the firm operates in both in the current period and in the next period. Allowing for each source of finance to be the marginal source in each period yields a 3x3 matrix of possible values of the cost of capital, which (as argued by Edwards and Keen (1984)) shows that the tax system creates most distortion when financial regimes change over time.

Section 3.3 then considers the investment model in more detail, paying particular attention to the role of expectations and the response of investment to various types of tax reform. In particular, the response of investment to announcements of tax reforms, temporary tax reforms and short and long run effects of permanent reforms are all considered. Section 3.4 presents some evidence on the value of the cost of capital in the UK over the period considered, and indicates the wide disparity between the cost of capital for different investment projects. Section 3.5 presents estimates of the investment model and section 3.6 concludes.

3.2 The cost of capital in alternative financial regimes

One additional feature is added and two minor simplifications are made to the model in Chapter 2, although none of these are necessary to derive measures of the cost of capital or an investment equation. First, the possibility of imperfect competition in the product market is allowed for. This permits a unique steady-state optimal capital stock in a model with a constant returns to scale. In combination with this, the model now explicitly deals with labour as a variable factor. To incorporate these additions the definitions of the variables are slightly changed. We now define \( \Pi(\alpha_t, K_t, L_t, I_t, p_t^V) \) as a net revenue function, where \( \Pi(\alpha_t, K_t, L_t, I_t, p_t^V) = F(\alpha_t, K_t, L_t, p_t^V) - G(K_t, I_t, p_t^V) \). Thus \( F(\cdot) \) is the gross revenue function, which depends on the stochastic shock \( \alpha_t \), as well as the stock of capital at the beginning of the period, \( K_t \), labour used during the period, \( L_t \) and the price of output, \( p_t^V \). \( G(\cdot) \) is the adjustment cost function, assumed separable from \( F(\cdot) \), the arguments of which are \( K_t, p_t^V \) and new investment in period \( t, I_t \). Thus,

\[2\] The stochastic shock is again assumed to be separable from the other arguments of the revenue function.
as in Chapter 2, adjustment costs are assumed to be internal and result in lost output.

Second, in order to simplify the analysis and allow the focus to be on the effect of taxation in different financial regimes, both leasing and the lemons premium on new share issues is ignored. Otherwise the objective function in (2.18) is unchanged.

The first order conditions used to derive the basic investment equation are those for investment and the beginning of the period capital stock. The condition for investment is:

\[
E_t \left[ \frac{\partial V_t^{t+1}}{\partial K_t} \right] = E_t \left\{ \left( \gamma + \lambda t \right) \left[ (1-\tau_t^*) G_I(t, K_t, p_t^Y) + p_t^* \right] \right\} \quad (3.1)
\]

which is familiar as the basic Q investment equation from chapter 2. The condition for K_t is

\[
\frac{\partial V_t}{\partial K_t} = E_t \int \left\{ \left( \gamma + \lambda t^D \right) \left[ (1-\tau_t^*) \Pi K(\alpha_t, K_t, L_t, I_t, p_t^Y) - A_K(B_t, K_t) \right] + (1-\delta) \left[ \frac{\partial V_t^{t+1}}{\partial K_t} \right] \right\} \quad (3.2)
\]

Using the values of (3.1) and (3.2) for t+1, expected in t, writing \( \Pi K(.) \) as \( F(.) - G(.) \), and for notational simplicity replacing the full arguments of F and G by their time period^3, they can be rearranged to give:

\[
(\gamma + \lambda t^D) [1-E_t (\tau_t^*)] G_I(t)
\]

\[
= E_t \left\{ \rho_{t+1} (\gamma + \lambda t^D) \left[ (1-\tau_{t+1}^*) \left[ F_K(t+1) - G_K(t+1) + (1-\delta)G_I(t+1) \right] - A_K(t+1) \right] \right\}
\]

\[
\left\{ (\gamma + \lambda t^D) p_t^* - (1-\delta) E_t \left[ \rho_{t+1} (\gamma + \lambda t^D^P p_{t+1}^* \right] \right\} \quad (3.3)
\]

^3So that, for example, \( F(\alpha_t, K_t, L_t, p_t^Y) = F(t) \).
Parameterising the adjustment cost function, $G$, allows the left hand side of (3.3) to be written in terms of an observable level (or rate) of investment in period $t$. This depends positively on marginal revenue product of capital expected in period $t+1$, negatively on the derivative of the adjustment cost function in period $t+1$ with respect to $K_{t+1}$, positively on the derivative of the same function with respect to $I_{t+1}$, negatively on the derivative of the agency cost function with respect to $K_{t+1}$ and negatively on a term involving the effective price of investment goods, personal tax parameters and the rate of depreciation$^4$.

In most formulations of cost of capital models, adjustment costs are ignored (some arbitrary movement of the capital stock towards its desired level being in their place). In this case, (3.3) can be rewritten to define the cost of capital as equal to the marginal revenue product of capital, $F_{K}$, in the absence of adjustment costs (possibly in a steady-state equilibrium). In order to follow this definition, we can take the last term in (3.3) into the previous term, and arrive at a definition of the cost of capital as

\begin{equation}
E_t(c_t) = \frac{(\gamma + \lambda^D) p^*_t - (1-\delta) \rho_{t+1} (\gamma + \lambda^D) p^*_t}{\rho_{t+1} p^*_t \{ (\gamma + \lambda^D) (1-\tau^*_t) \}}
\end{equation}

(3.4)

Thus, in the absence of adjustment costs, $E_t [F_{K}(t+1)] = E_t [p^*_t c_t]$.

In estimation below, the same procedure as in Chapter 2 is followed, replacing expected values with actual outturns, and adding a stochastic error term $\epsilon_t$ which is orthogonal to information in period $t$. Making this substitution the investment equation can therefore be written as

$^4$Of course no causation is implied by the Euler conditions, and (3.1) and (3.2) could equally well be solved for $G_t(t+1)$, for example, and other interpretations are discussed further below. However, given the forward-looking nature of investment decisions and the assumed one period lag in capital becoming productive, the formulation in (3.3) has a natural interpretation.
Using the definition of $E_t(c_t)$ from (3.4) and dealing with the expectational errors through $c_t$, we are now in a position to identify the value of the cost of capital under various financial regimes.

Both $\lambda_t^D$ and $\lambda_{t+1}^D$ enter (3.4). This implies that the source of finance used by the firm in both periods $t$ and $t+1$ are relevant to the period $t$ cost of capital. This is not surprising: the cost of capital is defined to be forward-looking inasmuch as it includes a term representing the capital gain on the asset between period $t$ and period $t+1$ and depends on the tax rate in the period in which the return on the investment is earned. To see this, consider the simplest case in which $\lambda_t^D=\lambda_{t+1}^D=0$. Then (3.4) can be rearranged to yield:

$$c_t = \frac{1}{(1-\tau^*_t)\delta p^*_t + (1-\tau^*_t+1)\delta p^*_{t+1}}$$

This is clearly closely related to the usual Jorgensen user cost of capital. Inside the brackets are three terms: the required financial rate of return (here the rate of interest) adjusted for personal taxes, the cost of depreciation of the asset (valued at period $t+1$ prices) and a capital gains term. All of these are grossed up by a factor depending on the tax rate in period $t+1$. It is worth noting that all of the price terms reflect the "effective" price of the asset. Intuitively, this is because the cost of capital is a one period cost, measuring the cost to the firm of accelerating a unit of expenditure from period $t+1$ to period $t$.

---

\[ (\gamma+\lambda_t^D)(1-\tau^*_t)G_1(I_t, K_t) \]

\[ = \rho_{t+1}(\gamma+\lambda_{t+1}^D) \left( (1-\tau^*_t+1) \left[ G_1(t+1) - G_1(t+1)(1-\delta)G_1(t+1) - p_{t+1}^T \right] \right) \]

\[ \quad - \lambda_{t+1}(t+1) + e_t \] (3.5)

---

5 In the absence of taxes, this is identical to the definition given in Nickell (1978, p.258)
t. Thus, for example, the capital gain reflects the change in the
effective price of the asset between the two dates. However, the
effective price in period t+1 depends in the source of finance used
then, and hence on \( \lambda_{t+1}^D \).

Table 3.1 examines the value of the cost of capital for alternative
financial regimes. It extends the results of Edwards and Keen (1984) and
(1985) in two principal ways. First, it allows for tax exhaustion.
Second it considers the possibility that the source of finance in either
period may be debt (as well as new equity and retentions). This results
in there being nine possible values of the cost of capital (compared
with the four considered by Edwards and Keen (1984)). These nine
possibilities are defined by the values of \( \lambda_t^D, \lambda_{t+1}^D, \lambda_t^N, \lambda_{t+1}^N, \lambda_t^B \), and
\( \lambda_{t+1}^B \). Thus \( \lambda_t^D = 0 \) implies that retentions are the marginal source of
finance in period t, \( \lambda_t^N = 0 \) implies that the marginal source of finance is
new equity, and \( \lambda_t^B = 0 \) implies that the marginal source of finance is
debt. Below we consider cases in which the firm is indifferent between
two sources of finance and relate the results here to the financial
regimes discussed in Chapter 2.

Recall from Chapter 2 that the first order condition for new equity
(ignoring the lemons premium) is:

\[
\gamma_t^* + \lambda_t^D \frac{\gamma_t^*}{\gamma} = 1 - \lambda_t^N
\]  

(3.7)

A similar expression holds for period t+1. Similarly, the first order
condition for debt can be rearranged to yield

\[
\gamma_t^* + \lambda_t^D \frac{\gamma_t^*}{\gamma} = -E_t \left[ \rho_{t+1} (\gamma + \lambda_{t+1}^D) \left( (1 - \tau_{t+1}^*)_t \frac{\gamma_{t+1}^*}{\gamma} + \frac{\gamma_{t+1}^*}{\gamma} (1 + A_B (t+1)) \right) \right] \lambda_{t+1}^B
\]  

(3.8)

Combining (3.4), (3.7) and (3.8) enable values for the cost of capital
to be derived for the nine cases shown in Table 3.1. The only elements
of (3.6) which differ in the different financial regimes are the
coefficients of \( p_t^* \). The table therefore shows only these coefficients.
All the other terms are as shown in (3.6).
Table 3.1 Definitions of the cost of capital

<table>
<thead>
<tr>
<th>$\lambda_{t+1}^D = 0$</th>
<th>$\lambda_{t+1}^N = 0$</th>
<th>$\lambda_{t+2}^B = 0; \lambda_{t+2}^D = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_{t+1}^* \left[ 1 + \frac{(1-m)}{(1-z)} i_{t+1} \right]$</td>
<td>$\gamma_{t+1}^* \left[ 1 + \frac{(1-m)}{(1-z)} i_{t+1} \right]$</td>
<td>$\gamma_{t+1}^* \left[ 1 + \frac{(1-m)}{(1-z)} i_{t+1} \right]$</td>
</tr>
<tr>
<td>$\gamma_{t+1}^* \left( 1 + A_B(t+1) \right) \gamma_t^* \frac{(1-\tau^<em>_{t+1})}{\gamma_t^</em>} i_{t+1}$</td>
<td>$\gamma_{t+1}^* \left( 1 + A_B(t+1) \right) \gamma_t^* \frac{(1-\tau^<em>_{t+1})}{\gamma_t^</em>} i_{t+1}$</td>
<td>$\gamma_{t+1}^* \left( 1 + A_B(t+1) \right) \gamma_t^* \frac{(1-\tau^<em>_{t+1})}{\gamma_t^</em>} i_{t+1}$</td>
</tr>
</tbody>
</table>

Notes:

1. The table gives the values of the coefficient of $p_t^*$ in the cost of capital, i.e. $\lambda_{t+1}^D = 0$, under different financial conditions. $p_t^* \equiv \left( 1-\tau^*_{t+1} \right) c_t^* + \left( 1-\delta^* \right) p_{t+1}^*$. $p_t = \gamma r_s + \gamma^* (1+A_B(s))$.

2. $\xi_s = \gamma(1-\tau^*_{t+1}) r_s + \gamma^* (1+A_B(s))$.

3. The cases in which $\lambda_{t+2}^B = 0$ and $\lambda_{t+1}^D = 0$ require an additional assumption regarding the marginal source of finance in period $t+2$ (and thereafter is the marginal source in period $t+2$ is debt). Here it is assumed that $\lambda_{t+2}^D = 0$.

Expressions commonly found in the literature generally assume (implicitly or explicitly) that $\lambda_{t+1}^D = 0$, and so correspond to the values in the first column of Table 3.1. Thus, for example, the values given in the first column correspond exactly to those of Edwards and Keen (1985) when, following their assumptions, $\delta = 0$, $p_t = p_t^*$, $A_B(t+1) = 0$ and $E(p_{t+1}^*) = E(p_{t+1})$, $s_t = s$ and $E(s_{t+1}) = s$. In addition, ignoring tax...
parameters between period t and period t+1 (in which case γ=γ=γ and p=p(1-η)), the values in column 1 match those given in the widely-used methodology of King and Fullerton (1984). There are two differences from the King and Fullerton formulae, however, which are discussed below.

Finally, it can also be shown that the formulae in Table 3.1 correspond to those derived from retention and debt finance by Devereux (1989). These last results (based on an approach by Edwards (1984)), which also assume that p=p, were based on a similar model of the investment process which again assumed a one period lag between the acquisition of the capital good and its contribution to output. Although some of the definitions of the terms used in Devereux (1989) differ from those used here, the underlying results are the same.

While it is comforting to realise that the formulae in Table 3.1 are in line with the literature, several issues can now be examined. First, what role does tax exhaustion play in the way in which dividend taxation (summarised by γ, γ and γ) affects the cost of capital? Second, what is the cost of capital if the marginal source of finance in period t or period t+1 is debt? Third, how do changes in the position of the firm with regard to both full tax exhaustion and ACT exhaustion affect the cost of capital? Fourth, how do the results in the table compare to the financial regimes outlined in Chapter 2? Fifth, what is the appropriate measure to use in estimating an investment equation? We examine these issues in turn.

The table extends the results of Edwards and Keen (1984) regarding the relevance of the trapped equity model. Thus, if D=D=0, so that retentions are the marginal source of finance in both period t and period t+1, then the coefficient of p shown in the table is simply the shareholder’s discount factor, p, and so dividend taxation does not affect the cost of capital (this was first shown by Auerbach (1979)). This result therefore holds whether or not the firm is tax exhausted in either period. As Edwards and Keen show, in the absence of tax exhaustion and assuming no change in tax parameters (i.e. so that γ=γ=γ), the same would be true if N=N=0. However, if γ=γ, then the result does not hold for the case in which N=N=0; that
is, the way in which dividends are taxed will affect the cost of capital. Thus the conclusion of Edwards and Keen (1984) that "the dividend tax affects the cost of capital ... only when the marginal source of finance is about to change" does not extend to the case in which tax exhaustion is allowed for, because the effective rate of dividend taxation may in any case change between period $t$ and period $t+1$.

If the marginal source of finance changes between period $t$ and period $t+1$, then, once again, the taxation of dividends may affect the cost of capital. The "traditional" view of the effect of dividend taxation is given by the assumptions that $\lambda^N_t=0$, $\lambda^D_t \neq 0$, $\lambda^N_{t+1} = 0$, $\lambda^D_{t+1} = 0$. This is shown in the second row of the first column of Table 3.1. In the absence of taxation, with $\gamma < 1$, this shows that the cost of capital is increased by dividend taxation. This would also be true in the presence of tax exhaustion if $\gamma < 1$. However, it is interesting to consider the case in which $\gamma > 1$. As discussed in Chapter 2, $\gamma > 1$ would imply that the firm would benefit from the arbitrage of issuing new equity to fund dividend payments. It was also suggested that this would increase the probability of ACT exhaustion, thus reducing the value of $\gamma^*$. An equilibrium would be reached when $\gamma^* = 1$. In this case, the discrimination against new equity finance would disappear: since $\lambda^D_t = 0$, the values given in the first and second rows of the table would be equal. However, it is possible that if there were some other constraint which prevented this arbitrage (some reason for which dividends could not be paid, for example) then the firm might reach this regime, with $\gamma^* > 1$. In this case, the cost of capital would be reduced by dividend taxation.

It might also be noted that the result here is different from that of King and Fullerton (1984)$^6$. They only consider the role of dividend taxation on the required rate of return. Thus, ignoring tax exhaustion,

$^6$The second difference from the King Fullerton methodology, which I brought out more clearly in Keen (1990) concerns the definitions of $p_t^*$ and $p_{t+1}^*$. Here the definitions of these effective prices are independent of the financial regime. In the King Fullerton methodology, however, the discount factor applied to future tax allowances varies with the regime which the firm is in during period $t$.  

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they correctly find that the required rate of return in the absence of taxation, \( i_{t+1}' \), should be multiplied by \( (1-m)/\gamma(1-z)=1+s \). However, they ignore the fact that this multiplier \((1/\gamma)\) should also be applied to the capital invested. Essentially the marginal value of the firm is multiplied by this factor as a result of changing financial regimes between period \( t \) and period \( t+1 \). For \( \gamma<1 \), this implies that there is a much larger cost of new equity finance (in period \( t \) only) than simply the effect on the company's discount rate. Similarly, if \( \gamma>1 \) (or \( \gamma_t>1 \) if tax exhaustion is considered) then there is a very large benefit to being in this regime.

It is clear from the result in the first row of the second column that similar arguments can be used when the marginal source of funds in period \( t \) is retentions and in period \( t+1 \) is new equity. Here, though, the multiplier on \( 1+i_{t+1}'(1-m)/(1-z) \) is \( \gamma_{t+1}' \) rather than \( 1/\gamma_t' \), so that all of the results are reversed.

It is interesting to note that all of the regimes in the third row of the table (in all of which debt is the marginal source of finance in period \( t \)) yield the same cost of capital. In the absence of tax exhaustion (so that \( \gamma=\gamma_t'=\gamma_{t+1}' \)) the cost of capital is independent of dividend taxation, not only if the marginal source of finance in period \( t \) is debt. Once again, tax exhaustion introduces the possibility that dividend taxation does affect the cost of capital in these regimes. For example, if \( \gamma_t'\gamma_{t+1}'<\gamma \) (ACT exhaustion in period \( t \) and period \( t+1 \) to an equal extent) then the cost of capital would be raised. (Tax exhaustion also raises the cost of capital through \( \tau_{t+1}' \), reducing the value of interest deductibility). Finally, it should also be noted that marginal agency costs appear in the definition of the cost of capital, as well as separately in the investment equation.

The final two elements of the table are where the marginal source of finance in period \( t+1 \) is debt, but in period \( t \) is either retained earnings or new equity. It is impossible to define the cost of capital independently of some multiplier in these cases unless further assumptions are made. The most convenient is to assume that the marginal source of finance in period \( t+2 \) is retained earnings. The resulting cost of capital is a combination of those in column 1. For example, defining
the top left hand corner of the table as $c_{1t}$ and the bottom left hand corner as $c_{13}$, the cost of capital in the top right hand corner, $c_{31}$, can be expressed as: $c_{31} = c_{1t} \cdot \frac{c_{13}}{c_{11}}$. This makes some intuitive sense: in periods $t$ and $t+2$ the marginal source of finance is retentions and period $t+1$ it is debt. Thus the cost of capital in period $t+1$ is $c_{13}$. If retained earnings had been the marginal source of finance in period $t+1$, the costs of capital in periods $t$ and $t+1$ would have been $c_{11}$ and $c_{13}$ respectively. The actual cost of capital in period $t$ is therefore a combination of these terms.

Tax exhaustion is probably most important when companies move into and out of tax exhaustion, since it is in these cases that the relevant tax parameters can vary widely between consecutive years. (Thus studies which do try to take account of tax exhaustion, but assume that a company is either a permanent tax payer or is permanently tax exhausted miss these effects). The two principal effects here are as follows.

To abstract from the effects of tax exhaustion via dividend taxation, consider the impact of full tax exhaustion on the cost of capital in the "trapped equity" regime in which $\lambda_t^D = \lambda_{t+1}^D = 0$, given in (3.6). Consider first a company which becomes fully tax exhausted in period $t+1$ and remains in that state for a number of years but which was not fully tax exhausted in period $t$. Differentiating the expression for $c_t$ with respect to $\tau_{t+1}$ gives

$$
\frac{\partial c_t}{\partial \tau_{t+1}} = \frac{c_t}{(1-\tau_{t+1}^*)} \left\{ \frac{1}{(1-z)\tau_{t+1}^*} \left[ (1-m) \frac{\partial p_t^*}{\partial \tau_{t+1}^*} - (1-\delta) \frac{\partial p_{t+1}^*}{\partial \tau_{t+1}^*} \right] \right\} \quad (3.9)
$$

A reduction in $\tau_{t+1}^*$ will tend to increase $p_{t+1}^*$, since the value of investment allowances will be reduced. Although this may be offset, to some extent, by the fact that $\frac{\partial p_t^*}{\partial \tau_{t+1}^*} < 0$, it is unlikely that the effect through $p_t^*$ can outweigh the effect through $p_{t+1}^*$, since at least part of the allowance due on investment in period $t$ will already have been claimed before period $t+1$. In the extreme case in which the whole purchase price can be written off against taxable profit in the year in which the expense was incurred (as was the case in the UK for plant and machinery between 1972 and 1984) and the firm was a full tax payer in
period $t$, then $\partial p_t^*/\partial r_{t+1}^*=0$. These arguments imply that, if $c_t^*>0$, then
$\partial c_t^*/\partial r_{t+1}^*>0$, so that moving into a period of tax exhaustion in period
$t+1$ (and hence reducing $r_{t+1}^*$) will reduce the cost of capital in period
$t$. Intuitively, this is because the return on investment (earned in
period $t+1$) will be taxed at a lower rate, while the present value of
allowances on the investment made in period $t$ will not be reduced so
substantially. In addition, the last term of (3.9) notes that, since
$p_{t+1}^*$ is reduced by full tax exhaustion in period $t+1$, the capital gain
on the effective price of the asset is also reduced$^7$.

Conversely, a company which moves out of a period of tax exhaustion
faces a higher cost of capital. To see this differentiate $c_t$ with
respect to $r_t^*$ in the same financial regime:

$$\frac{\partial c_t}{\partial r_t^*} = \frac{1}{(1-r_{t+1}^*)p_{t+1}^*} \left\{ \frac{(1-m)}{(1-z)^{t+1}} \right\} \frac{\partial p_t^*}{\partial r_t^*} \tag{3.10}$$

As before, $\partial p_t^*/\partial r_t^*<0$ and so $\partial c_t/\partial r_t^*<0$. The reason for this is clear.
Full tax exhaustion in period $t$ reduces the present value of tax
allowances on investments made in period $t$. However, if the firm is a
full tax payer in period $t+1$, the return to each investment is taxed at
the full rate, and, in addition, the effective price of capital in
period $t+1$ is lower. All of these effects tend to raise the cost of
capital if the firm is fully tax exhausted in period $t$ (only). It should
be noted that this effect has considerably less impact than the former
case. This is because it only arises if a fully tax exhausted firm is
about to become tax paying. In this case, the reduction of $r_t^*$ below $r$
will be small, since tax effects are essentially only discounted one
period. However, a firm becoming fully tax exhausted in period $t+1$ may
remain so for an unlimited period, so that additional tax liabilities
arising in period $t+1$ may have a very small present value, with the
reduction in $r_{t+1}^*$ below $r$ being consequently large.

$^7$The "capital gain" here is best interpreted as the gain from bringing
forward the investment by one period. In this case it is clear that the
firm's effective price in periods $t$ and $t+1$ is relevant; the effective
price in period $t+1$ should not be interpreted as a market value.
These considerations suggest that full tax exhaustion may have a counter-cyclical effect. To the extent that widespread tax exhaustion occurs towards the bottom of the business cycle, firms will tend to move into positions of full tax exhaustion during the downswing of a recession. This reduces their cost of capital (possibly by a considerable amount). The opposite effect occurs during the upswing (although this is not likely to be as important).

Some comments regarding the effects of ACT exhaustion have already been made regarding the effect through dividend taxation. Thus, apart from the two cases in which $\lambda_t^D=0$ and either $\lambda_{t+1}^D=0$ or $\lambda_t^N=0$, ACT exhaustion in period $t$ reduces the value of $y_t$ and, ceteris paribus, raises the cost of capital. Similarly, apart from the two cases in which $\lambda_{t+1}^D=0$ and either $\lambda_t^D=0$ or $\lambda_t^N=0$, ACT exhaustion in period $t+1$ reduces the value of $y_{t+1}$ and, ceteris paribus, raises the cost of capital.

More generally, however, the impact of ACT exhaustion on the cost of capital is ambiguous. Consider again the "trapped equity" regime in which $\lambda_t^D=\lambda_{t+1}^D=0$. Using a reduction in $y_t$ and $y_{t+1}$ to reflect the existence of ACT exhaustion in periods $t$ and $t+1$, respectively, then

$$\frac{\partial c_t}{\partial y_{t+1}} = \frac{c_t}{(1-\tau_{t+1})} \left[ 1+ \frac{(1-m)}{(1-z)_{t+1}} \right] \frac{\partial p_t}{\partial y_t} - \frac{(1-\delta)}{(1-\gamma_{t+1})} \frac{\partial p_{t+1}}{\partial y_{t+1}}$$

(3.11)

and

$$\frac{\partial c_t}{\partial y_t} = \frac{1}{(1-\tau_{t+1})} \left[ 1+ \frac{(1-m)}{(1-z)_{t+1}} \right] \frac{\partial p_t}{\partial y_t}$$

(3.12)

The problem in finding the sign of these effects is that the signs of $\partial p_t/\partial y_t$, $\partial p_{t+1}/\partial y_{t+1}$ and $\partial p_{t+1}/\partial y_{t+1}$ are ambiguous. To see this, recall from (2.29) that $p_t$ can be written

$$p_t = \left\{ y_t - \eta_t \right\}$$

(3.13)
Increasing $\gamma^*_t$ (by reducing ACT exhaustion) increases the first term in (3.13) which is the cost to the shareholder of forgoing one unit of dividends. But it also increases $\eta^*_t$, the present value of allowances. The net effect of these two terms is therefore ambiguous$^8$.

One other important point to note from the formulae in Table 3.1 does not depend on tax exhaustion, but concerns the nature of the capital gain term. As already noted, the capital gain in each of the formulae reflects the change in the "effective" price of capital goods, not the change in the nominal price. This means that it depends on, among other things, the depreciation provisions in the tax system. To consider two extreme cases, assume that there is no tax exhaustion, and consider the case of purchases of plant and machinery and commercial buildings under the pre-1984 UK tax system, for which the relevant depreciation rates were 100% and zero, respectively. The "effective" price of a unit of plant and machinery was therefore $p_t(1-\tau)$ and that of a unit of commercial buildings was $p_t$. Assuming no change in the statutory tax system between periods $t$ and $t+1$, then it is clear that for retention finance, for example, the statutory tax rate both multiplies and divides

$^8$ It is worth noting that there are at least two other features of the UK tax system which, because of their asymmetric treatment, give rise to large effects on the cost of capital. The first is double taxation relief, which allows part of the foreign tax paid on foreign source income to be deducted from the UK liability. However, if foreign tax paid is greater than the UK liability on the repatriated profit, no UK tax is paid, but, more importantly, the difference is not eligible to be carried forward to be set against future liabilities. In this case it is possible for small changes in taxable profit before deduction of foreign taxes to make no difference to the final tax liability in this, or any other, period. This may have very large effects on the cost of capital in a dynamic context. For example, if there is excess foreign tax in period $t$, and depreciation allowance due on a marginal investment in period $t$ is lost, with the result that $\eta^*_t$ is very low or zero, $p^*_t$ and $c^*_t$ are very large and positive. If it occurs in period $t+1$, $\tau^*_{t+1}=0$, $\eta^*_t$ is very low and $p^*_t$ is very large. As in the discussion above, for $\delta<1$, the effects on $\tau^*_{t+1}$ and $p^*_t$ have opposite effects on $c^*_t$. One further asymmetry is in the treatment of net dividend receipts. Where dividends received by the company are greater than dividends paid, the company faces no ACT liability. The excess of receipts over payments can either be carried forward to offset against future ACT liabilities, or used as an offset against taxable losses, allowing an immediate tax credit. These situations act in some ways as ACT exhaustion and, again, the dynamic effects on the cost of capital can be large.

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each term and therefore drops out of the expression for $c_t^9$. For commercial buildings, the financial cost of capital (abstracting from the economic depreciation charge, $\delta$) is larger by a factor of $1/(1-\tau)$. However, if $c_t$ for plant and machinery is negative because the capital gain term outweighed the other two terms (as, according to the results presented below, was sometimes the case), then the cost of capital for commercial buildings will be negative and larger; in this case, the depreciation allowance reduces the incentive to invest.

So far we have considered the cost of capital in conventional financial regimes (that is with retentions, debt or new equity as the marginal source of finance and no agency costs on debt). However, it is easy to relate the expressions derived to the discussion of financial regimes in chapter 2.

In regime 1, the marginal source of finance was retentions and debt. Since the marginal cost of these two sources must be equal in regime 1, the expressions given in the first and third rows of Table 3.1 must be equal. Equating them yields the same condition for equality as given in (2.47), defining the regime. Note that it was assumed in the discussion of the financial regimes that $\lambda^D_{t+1}=0$, corresponding to the first column of Table 3.1. In regime 2, the marginal source of finance was just debt in which case there is no similar condition. In regime 3, new equity and debt were the marginal sources of finance; along the same lines of argument as for regime 1, the expressions in rows 2 and 3 of Table 3.1 must be equal. Here the conditions (2.53) and (2.54) can be derived\(^{10}\), corresponding to whether or not $\lambda^N_{t+1}=0$.

In regime 4, the marginal cost of finance is the same from each of the three sources. In this case, the cost of capital in each row of Table 3.1 must be equal (again assuming $\lambda^D_{t+1}=0$). Equating the first and second

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\(^9\)This is of course the neutrality result of a flow of funds corporation tax, advocated by the Meade Committee (1977) and Edwards (1982) among others.

\(^{10}\)Except for the lemons premium which is now ignored.
rows yields the condition $\gamma_t^* = 1$. Equating the first and third rows, given $\gamma_t^* = 1$ yields

$$(1-\gamma_{t+1}^*) = (1-c)1_{t+1} - \frac{[1-\gamma_{t+1}^* (1+A_{t+1})]}{\gamma} \quad (3.14)$$

where $c$ (without a subscript) is the imputation rate. If it is further assumed that the regime also holds in period $t+1$, so that $\gamma_{t+1}^* = 1$, and marginal agency costs are zero then this implies $r_t^* = c$ which, as noted by Keen and Schiantarelli (1989), implies that the firm must be permanently tax exhausted with certainty. As noted in Chapter 2, agency costs of debt were included in the model partly to avoid this implication of the model.

The equality $\gamma_t^* = 1$, or $(1-m)/(1-z) = 1/(1+s_t^*)$ can be used to substitute into the expression for retention finance in Table 3.1. This yields the following expression for the cost of capital in regime 4:

$$c_t = \frac{1}{(1-\gamma_{t+1}^*)p_{t+1}} \left\{ \frac{1}{(1+s_t^*)} 1_{t+1} p_t^* + \delta p_{t+1}^* - (p_{t+1}^* - p_t^*) \right\} \quad (3.15)$$

It is this expression for the cost of capital that is used in the estimation below, for two reasons. First, under the imputation system, with $\gamma > 1$ for a number of investors (including large financial institutions such as pension funds), it is likely that a firm would aim to be in this regime. Second, for empirical purposes, it removes the need to stipulate values of $m$ and $z$, the income tax and capital gains tax rates of the marginal investor. This is a significant advantage given that the identity of the marginal investor is unknown. By contrast, $s$ is simply the cost of credit under the imputation system, which applies to all investors. It is true that $s_t^*$ depends on future profitability, but as in Chapter 2, this can be replaced by the actual outturn within the estimation strategy used. The main alternatives to this procedure are to arbitrarily choose a marginal investor (eg a pension fund for which $m = z = 0$), or to approximate marginal personal tax

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111 Equivalent to (2.56) in the absence of the lemons premium.
rates by estimates of average tax rates (Robson (1988) provides estimates of average personal tax rates among those who invest in the UK). While none of these methods avoid problems, heroic assumptions are kept to a minimum by taking the route of the expression in (3.15) from regime 4.

Of course using the expression in (3.15) implies that the empirical section below ignores most of the preceding discussion of financial policy. However, this discussion should provide a basis for some empirical examination of the role of taxation and tax exhaustion in financial decisions. The main difficulty is to assess which financial regime a firm is likely to be in at any point in time. This is beyond the scope of this thesis.

3.3 Taxes in a forward-looking investment model

Returning now to the investment equation (3.5), it is necessary to specify a functional form for the adjustment cost function, and to make some restrictions on the production function in order to derive an estimable model. Partly to compare this approach with the Q approach discussed in Chapter 2, we assume that the adjustment cost function is of the same form\(^{12}\). Hence

\[
G(I_t, K_t, p_t^y) = p_t^y \frac{b}{2} \left( \frac{I_t}{K_t} - a \right)^2 K_t
\]

(3.16)

where \(a\) is the rate of investment at which adjustment costs are zero. It is sometimes useful to assume that \(a=\delta\), so that adjustment costs are only incurred on net, rather than gross, investment. A similar form is assumed for the agency cost function:

\(^{12}\)Apart from the stochastic element, which is now neglected.
Using the definition in (3.16) it is now necessary to specify the form of the revenue function. Begin by defining the gross revenue function (i.e. gross of adjustment costs) to be 

\[ F(\alpha_t, K_t, L_t, p_t^Y) = p_t^{YF^1(\alpha_t, K_t, L_t)} - w_t L_t, \]

where \( L_t \) is the amount of labour used in period \( t \) and \( w_t \) is the wage rate in period \( t \). Further, assume (as in Chapter 2) that labour is paid its real marginal product, so that \( F^1_L = \frac{w_t}{p_t^Y} \), and that the production function, \( F^1(\alpha_t, K_t, L_t) \), is constant returns to scale. Further, define \( p_t^Z(\cdot) = p_t^{YF^1(\cdot)} - G(\cdot) \) to be the value of gross output. We do not observe \( p_t^{YF^1} \), but we do observe \( p_t^Z(\cdot) \). Using the fact that both \( F(\cdot) \) and \( G(\cdot)/p_t^Y \) are linear homogeneous, the marginal revenue product of capital, \( F_K(\cdot)G_K(\cdot) \), can be substituted out of the equation (3.5) using \( Z/K, WL/K \) and \( I/K \). In particular,

\[
F_K(t+1) = G_K(t+1) = p_t^Y \left[ 1 - \frac{1}{\varepsilon} \right] \left\{ \frac{Z_{t+1}}{K_{t+1}} - \frac{w_{t+1}L_{t+1}}{p_t^{YF^1(\cdot)}K_{t+1}(1-1/\varepsilon)} \right\} + b \left[ \frac{I_{t+1}}{K_{t+1}} \right]^2 - ab \left[ \frac{I_{t+1}}{K_{t+1}} \right] \]

\[ (3.18) \]

where \( \varepsilon \) is the elasticity of demand. Similarly, the derivative of the adjustment cost function with respect to investment is

\[
G_I(t) = p_t^Y \left[ 1 - \frac{1}{\varepsilon} \right] \left\{ \frac{bI_t}{K_t} - ab \right\} \]

\[ (3.19) \]

Finally, it is useful to combine the variables in the equation with the tax rates by which they are multiplied. Defining, for example,

\[
\begin{bmatrix}
\frac{I_s}{K_s} \\
\rho_s
\end{bmatrix} = \left( 1 - \tau_s \right) \frac{I_s}{K_s}
\]

\[ (3.20) \]

we can then write the investment equation to be estimated as:
\[
\begin{align*}
\left[ \frac{I_t}{K_t} \right] &= \alpha + \beta_1 \left[ \frac{Z_{t+1}}{K_{t+1}} \right] + \beta_2 \left[ \frac{w_{t+1} L_{t+1}}{P_{t+1} K_{t+1}} \right] + \beta_3 \left[ \frac{I_{t+1}}{K_{t+1}} \right] \\
&\quad + \beta_4 \left[ \frac{I_{t+1}}{K_{t+1}} \right]^2 + \beta_5 \frac{C_t}{P_{t+1} K_{t+1}} + \beta_6 \left[ \frac{B_{t+1}}{P_{t+1} K_{t+1}} \right]^2 \omega_t 
\end{align*}
\]

(3.21)

where

\[
\beta_1 = \frac{1}{b} - \frac{1}{b(1-1/c)}; \quad \beta_2 = \frac{1}{b} - \frac{1}{b(1-1/c)}; \quad \beta_3 = 1 - \delta - a; \quad \beta_4 = 1; \quad \beta_5 = \frac{-1}{b(1-1/c)}; \quad \beta_6 = \frac{\Sigma}{b(1-1/c)}; \quad \alpha = a \left[ (1-\tau^*_t) p_t^Y - (1-\delta) \rho_{t+1} (1-\tau^*_t) p_{t+1}^Y \right] 
\]

(3.22)

The equation derived here is clearly not without problems for estimation. First, the intercept, \( \alpha \), in principle varies across both firms and years because it depends on both \( \tau^*_t \) and \( \tau^*_t \) (and \( \rho_{t+1} \)). In estimation, approximate notice of this issue will be taken by allowing for firm specific effects and time effects. Second, the inclusion of the two squared terms, \((I/K)^2\) and \((B/pK)^2\), may be problematic if (as is likely), the replacement cost of the capital stock is measured with error, since any error will enter the error term squared.

Third, the equation has been written as if the elasticity of demand, \( \epsilon \), is constant across both firms and time. This is rather unlikely, although an approximation may be to allow it to vary between firms in different industries. Of course, the model would revert to perfect competition by setting \( \epsilon \) to be infinite. Clearly, allowing for a constant, but not infinite, \( \epsilon \) is a less restrictive version of the model (and therefore less restrictive than the \( Q \) model estimated in Chapter 2). However, since the size of the sample available falls if remuneration, \( wL \), is added to the database, most of the estimation makes use of the larger database by imposing perfect competition. In the case of perfect competition, \( p_t^Y Z - wL \) is equal to cash flow, and the first two explanatory variables can be merged with a single coefficient equal to \( 1/b \).

In principle, there are five parameters of interest in the model: \( b \), \( a \), \( \delta \), \( \epsilon \) and \( \Sigma \). However, two of these, \( a \) and \( \delta \) only appear in \( \beta_2 \), the
coefficient on \((I/K)_{t+1}\). They cannot therefore be separately identified. However, under the reasonable assumption that adjustment costs on net investment are zero, then \(\delta = a\), and thus an estimate can be found. The third and fourth parameters, \(b\) and \(c\) are overidentified, since they can be found from \(\beta_1\), \(\beta_2\) and \(\beta_5\). In the case of perfect competition, the coefficients of cash flow and the cost of capital should be equal and of opposite sign. More generally, \(\beta_2\) and \(\beta_5\) should be equal and opposite. This, and the fact that \(\beta_3 = 1\) yields two testable restrictions in the model. Thus, for example, for \(\epsilon = \omega\), as well as being estimated in the form given in (3.21), the restrictions could be imposed to yield

\[
\begin{pmatrix}
[I_t]^* - [I_{t+1}]^* \\
[K_t] - [K_{t+1}]
\end{pmatrix}
\begin{pmatrix}
[I_{t+1}]^* \\
[K_{t+1}]
\end{pmatrix}^2
= \alpha + \beta_1\begin{pmatrix}
[Z_{t+1}]^* \\
[K_{t+1}]
\end{pmatrix}^* + \beta_3\begin{pmatrix}
[I_{t+1}]^* \\
[K_{t+1}]
\end{pmatrix}^* + \beta_6\begin{pmatrix}
[B_{t+1}]^* \\
[K_{t+1}]
\end{pmatrix}^* + \omega_t
\] (3.23)

One feature of the way that the model is presented here is that it is forward-looking: the expected profitability of new investment is captured by the next period's marginal product of capital, proxied as shown in (3.18). The role played by the future investment rate and the future investment rate squared is due to the nature of the adjustment cost function. With quadratic adjustment costs, the firm will aim to even out its investment pattern. For example, if next period's investment rate is expected to be high, then the firm should increase this year's investment up to the point at which total adjustment costs over the two periods are minimised. The convexity of the adjustment cost function is reflected in the presence of the squared term. Finally, increasing the capital stock reduces marginal agency costs of debt.

Two issues which can be investigated within the model are the role played by shocks to the investment process and the effects of different kinds of tax reform. As is clear from the discussion above, the cost of capital model as presented predicts that investment will depend on expectations held in period \(t\) of various period \(t+1\) variables. This is clearly similar to the Q model discussed in Chapter 2. However, the difference from the Q model is that the latter explicitly uses the expectation itself. Thus, \(V_t\), the market value of the firm, is held to reflect all expectations of relevant future events conditional on the
information available at the time the investment decision is taken. It is clear from this that any subsequent shock cannot affect either $Q_t$, via $V_t$, or $(I/K)_t$. Any such new information is simply irrelevant.

By contrast, the estimation strategy used here replaces the currently held expectations of future variables with the actual outturn. The difference between the expectation and the outturn is the stochastic shock, $w_t$. An innovation which takes place after the investment decision in period $t$ has been taken (and therefore not included in the relevant information set) is therefore a form of measurement error in the model. Predicting the effect of a tax reform known in period $t$, but using outturn values from period $t+1$ is therefore subject to a form of error not present in the Q model.

It should be noted that this formulation is somewhat different from other uses of the Euler equation framework. The investment equation (3.21) could equally well be written with $(I/K)_{t+1}$ as the dependent variable (see Bond and Meghir (1990), for example). In this interpretation the model would be predicting the investment rate in period $t+1$ given information in period $t$ and expectations held in period $t$ of other variables in period $t+1$. In this context the innovation, labelled $w_t$ in (3.21), but taking place in period $t+1$, would represent a shock to investment in period $t+1$ unanticipated at period $t$. In the context of analysing the impact of tax reform on investment, this implies that an unanticipated reform taking place in period $t+1$ would influence investment in period $t+1$ in a manner not captured by the model.

The formulation presented above can, however, be used to analyse the impact of a number of different forms of tax reform. It can, for example, be used to compare the effect on the investment rate in the short and long run of temporary and permanent reforms, and anticipated and unanticipated reforms.

The long-run properties of a model with a linear homogeneous production

13 And therefore uses instrumental variables in estimation.
and adjustment cost function and perfect competition in the product market have been analysed by Lucas (1967), who pointed out that, in the long run, the capital stock will grow or decline at a constant rate. Investment and output grow or decline at the same rate. In this case the model therefore does not yield a constant long-run level of the capital stock, $K^*$, although it does yield a constant capital to output ratio, $(K/Z)^{14}$. However, there are several ways of avoiding this result. For example, Nickell (1978) does not impose linear homogeneity of the adjustment cost function, so that adjustment costs define an optimal level of the capital stock. Alternatively, if the elasticity of demand varied with output in a product market with imperfect competition, the optimal level of output and hence $K^*$ would be fixed\textsuperscript{15}.

The short-run properties of the model have been analysed by Nickell (1978), Abel (1978) and (1982) and Summers (1981). While the papers by Abel and Summers are in the context of a Q model, as has been emphasised, the two models are essentially the same and so the results of these papers apply to the formulation presented here. The main difference in approach is therefore in estimation, using the estimated parameters from (3.21) rather than from the Q model. However, it should be noted that the model here assumes a one period lag between investment and the asset having productive capacity (possibly because of delivery lags). As noted above, this introduces a number of complications regarding the impact of the reform on the cost of capital in successive periods. The impact of a change in $\tau$ on the cost of capital, $c_\tau$, has already been discussed in the context of tax exhaustion since the impact of an increase in $\tau^*$ will be in the same direction as an increase in $\tau^*$, ceteris paribus. Thus (3.9) and (3.10) show that $\delta c_\tau / \delta \tau < 0$, and $\delta c_\tau / \delta \tau^{t+1} > 0$, for reasons given above.

In order to discuss the impact of tax reform on investment in this

\textsuperscript{14} Summers (1981) assumes that adjustment costs apply only to net investment (so that $a = \delta$ in (3.16)). In the long-run steady state, adjustment costs are minimised at zero by $I/K = \delta$. However, this still does not imply a unique solution for $K$ dependent only on prices, since it also depends on the initial value of $K$.

\textsuperscript{15} Although this raises insuperable problems for estimation of the investment equation.
model, we therefore briefly consider the impact of temporary and permanent changes to the statutory tax rate. To simplify the analysis, assume that output is constant and that prior to the reform the firm had been in its steady state position.

(a) unanticipated temporary increase in $\tau_t$:

The most obvious effect of a temporary increase in $\tau_t$ is clearly to reduce $c_t$ (since the present value of investment allowances rises, reducing the net cost of investment). Since the change in $\tau$ is reversed in period $t+1$, there is no offsetting effect from period $t+1$, nor is the cost of capital in any other period affected. The reduction in $c_t$ has a direct impact on $(I/K)_t$ with a coefficient of $-1/b$.

However, this is not the only impact on $(I/K)_t$. If investment in period $t$ rises as a result of the reduction in the cost of capital, then the capital stock used in period $t+1$ will be higher. Assuming that $F_{KK}<0$, and noting from (3.16) that $G_{KK}>0$, then $Z_{KK}=F_{KK}-G_{KK}<0$. This suggests that there will be a downward effect on $(I/K)_t$ via the expected impact on production and adjustment costs. To see this within the context of the equation (3.21), holding output constant and abstracting from the impact of adjustment costs via $(I/K)_{t+1}$, it would be expected that $(Z/K)_{t+1}$ would fall since production in period $t+1$ depends on the capital stock at the beginning of the period, which depends directly on $I_t$. It can easily be demonstrated that, with the one period lag in capital becoming productive, $0<\delta(Z/K)_{t+1}/\delta c_t<1$, implying that $I_t$ and $K_{t+1}$ both rise$^{16}$.

To consider the effects of adjustment costs, consider the return to the steady state position. Eventually, $I/K$ must return to $\delta$, with $K$ returning to the steady-state capital stock, $K^*$. Since the initial

$^{16}$To confirm this, consider the bounds on $\partial(Z/K)_{t+1}/\partial c_t$. Is it possible that $\partial(Z/K)_{t+1}/\partial c_t=1$? For example, if $\partial(Z/K)_{t+1}/\partial c_t=1$, then the two effects via $c_t$ and $(Z/K)_{t+1}$ must cancel out, leaving $I_t$ unchanged. But if $I_t$ is unchanged then $(Z/K)_{t+1}$ cannot change, and so $\partial(Z/K)_{t+1}/\partial c_t$ cannot equal 1. Similar considerations rule out other values outside the range 0 to 1.
effect is to increase $K_{t+1}$, the future path of $K$ must be downwards, with $(I/K)_{t+n} < \delta$, for $n=1,2,...$, until the firm has returned to the steady state. Since $(I/K)_{t+1} < \delta$, this will provide a further moderating influence on $I_t$. Intuitively, the presence of adjustment costs mean that it is not worthwhile increasing the capital stock to fully take advantage of the lower cost of capital in period $t$, because quadratic adjustment costs will be incurred both when $K$ is raised and when it is reduced\(^{17}\).

(b) temporary increase in $\tau_{t+1}$, announced in period $t$:

As discussed above, a increase in $\tau_{t+1}$ both reduces $c_{t+1}$ and increases $c_t$. To consider the path of the capital stock in this case, it is useful to examine these effects separately. Begin with the reduction in $c_{t+1}$, which is virtually identical to the case just considered. Neglecting, for the moment, the impact via $c_t$, the reduction in $c_{t+1}$ would lead to changes in the path of $K$ similar to that described above. However, there is an additional effect here, namely that since $(I/K)_{t+1}$ will be higher as a result of the reduction in $c_{t+1}$, there will also be an upward pressure on $(I/K)_t$ independent of the effect via $c_t$.

The impact via $c_t$ can be analysed in much the same way, and is simply the opposite of the effect discussed above of a reduction in $c_t$. Since $c_t$ is increased in this case, however, the initial impact on $I_t$ will be downwards. This will offset the announcement effect derived through $c_{t+1}$.

The relative effects on $c_t$ and $c_{t+1}$ depend on the tax system. From (3.9) and (3.10), ignoring tax exhaustion and changes in prices, it can be shown that:

\(^{17}\)This path of investment and the capital stock is similar to that which can be derived from, for example, Abel (1982) when the firm is initially at its steady state level. Abel also analyses the case in which there is a temporary tax rate change when the firm is on the stable path to the steady state with similar results.
\[
\frac{\partial c_t}{\partial \tau_{t+1}} \frac{\partial c_{t+1}}{\partial \tau_{t+1}} = \frac{1}{(1-\tau)p_t^*} \left\{ \left[ p^{*-1} - (1-\delta) \right] \left[ \frac{\partial p_{t+1}^*}{\partial \tau_{t+1}} + \rho^{-1} \frac{\partial p_t^*}{\partial \tau_{t+1}} \right] + \rho^{-1} \frac{\partial p_t^*}{\partial \tau_{t+1}} \right\}
\]

(3.24)

In the case of 100% first year allowances, \( \frac{\partial p_{t+1}^*}{\partial \tau_{t+1}} = 0 \) and \( \frac{\partial p_t^*}{\partial \tau_{t+1}} = -p \). In this case, the right hand side of (3.24) becomes zero, indicating that the change in \( c_t \) will be equal and opposite to the change in \( c_{t+1} \). In this case, \( (I/K)_t \) must fall, as the impact of \( c_t \) must outweigh the announcement effect through \( c_{t+1} \). However, \( (I/K)_{t+1} \) must rise (above \( \delta \)) because of the upward impact of the reduction in \( c_{t+1} \). Since the announcement effect moderates the reduction in \( I_t \), an effect which is absent in period \( t+1 \), it is likely that \( K_{t+2} \) will be above the steady state level, \( K^* \), so that subsequent investment rates must gradually decline back to the steady state rate \( \delta \).

However, with less than 100% first year allowances, the sign of the right hand side of (3.24) is ambiguous. In this case, it is possible that the announcement effects via \( c_{t+1}^* \) may outweigh the direct effects through \( c_t^* \), so that the initial movement in \( I_t \) and hence \( K_{t+1} \) is upwards. However, this would require (3.24) to be strongly negative, which is unlikely.

(c) permanent increase in \( \tau \), with effect from period \( t \):

A permanent increase in \( \tau \) has a different effect on the cost of capital. Since this is a permanent change there is no difference between the effect on the cost of capital in different periods, so the issue above, concerning the relative size of the change in \( c_t \) and \( c_{t+1} \), does not arise here. As noted above, under a neutral corporation tax the long run cost of capital is independent of the tax rate. In this case, the long run steady state will not change and there would not even be a temporary effect.

However, if the tax system were not neutral, then there would be both a short term and a long term effect of the increase in the tax rate. Specifically, considering the trapped equity regime shown in (3.6), if capital allowances were less than 100% in the first year, then corporation tax would not be neutral and the increase in the tax rate
would increase the cost of capital.

The effect of an increase in the cost of capital in period $t$ and all subsequent periods on the level of the capital stock and the rate of investment has been analysed by Abel (1982). There would be an initial drop in $I_t$ and hence $K_{t+1}$, with the capital stock continuing to fall until the new steady-state was reached.

(d) permanent or temporary change in $\tau$, with effect from period $t+n$, but announced in period $t$:

Many of the comments made in the previous section remain valid in the case of the announcement in period $t$ of a permanent or temporary increase in $\tau$ to take effect from period $t+n$. The additional factor here, of course, is that the rate of investment will begin to change immediately in order to minimise adjustment costs over the course of the total adjustment to the new steady-state level of the capital stock which has to take place.

It should perhaps be noted that, although the forward-looking nature of this model in principle permits the analysis of various forms of tax reform, whether permanent, temporary or simply announced, tax simulation exercises would need to be able to predict the effect of the tax reform also on the marginal product of capital. This would require separate estimation of parameters of the production function. (This is, of course, similar to exercises that have been done in the context of the Q model - see, for example, Summers (1981)).

We turn now to the empirical implementation of the model, considering in the next section the value of the cost of capital over time and between uses, paying particular attention to the role of tax exhaustion. In section 3.5, the model described above is estimated on the sample of data described in Appendix A.
3.4. Empirical measurements of the cost of capital

This section presents empirical estimates of the cost of capital as defined in the previous section, based on the panel of company accounting data described in Appendix A. We begin by summarising the means and standard deviations of the 'average' cost of capital for each year of data. As noted above, these estimates are based on the assumption that companies are in financial regime 4. The overall average cost of capital is constructed by first computing the cost of capital for each firm in each period for nine different forms of investment: three assets purchased (plant and machinery, industrial buildings and commercial buildings) each financed from three sources (retained earnings, debt and new equity). Second a weighted average cost of capital for each company in each period is constructed, weighted across the different types of investment using actual investment in each asset and finance from each source in that period as the weight. Third, an unweighted average across all of the companies in the sample in each year is computed. The standard deviations are constructed in the same way. After examining this overall average, some estimates of the variation between the cost of capital for investment in different assets and financed from different sources are presented.

As in Chapter 2, the cost of capital is constructed under five different possible ways of taking account of taxation: (a) ignoring tax altogether; (b) allowing for tax, but ignoring all forms of tax exhaustion; (c) allowing for tax including full tax exhaustion (given by the presence of positive losses carried forward) but ignoring ACT exhaustion; (d) allowing for tax, including both full tax exhaustion and ACT exhaustion; and (e) as (d), but using estimates of periods of tax exhaustion using the data on tax payments from accounts, rather than from the model described in Appendix B. As in the discussion of Q in Chapter 2, the correlation between these different definitions (over the whole sample) are also presented.

Table 3.2 presents descriptive statistics on the average cost of capital. As noted in Appendix A, the measure chosen for the required financial rate of return, \( r_{t+1} \), is simply a long run rate of interest.
### Table 3.2(a) Estimated average cost of capital (%)

<table>
<thead>
<tr>
<th>Year</th>
<th>No of Cos</th>
<th>No Tax</th>
<th>No Tax Exhaustion</th>
<th>Full Tax</th>
<th>Full Tax Exhaustion</th>
<th>Full Tax and ACT Exhaustion</th>
</tr>
</thead>
<tbody>
<tr>
<td>1971</td>
<td>190</td>
<td>6.46</td>
<td>3.56</td>
<td>3.33</td>
<td>3.33</td>
<td>3.60</td>
</tr>
<tr>
<td>1972</td>
<td>297</td>
<td>2.89</td>
<td>0.15</td>
<td>-0.16</td>
<td>-0.16</td>
<td>-0.12</td>
</tr>
<tr>
<td>1973</td>
<td>374</td>
<td>-1.32</td>
<td>-5.00</td>
<td>-7.21</td>
<td>-4.20</td>
<td>-5.04</td>
</tr>
<tr>
<td>1974</td>
<td>409</td>
<td>1.12</td>
<td>-6.13</td>
<td>-5.35</td>
<td>-5.06</td>
<td>-3.85</td>
</tr>
<tr>
<td>1975</td>
<td>431</td>
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<td>-2.75</td>
<td>-2.36</td>
<td>-1.73</td>
<td>0.15</td>
</tr>
<tr>
<td>1976</td>
<td>655</td>
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<td>-1.71</td>
<td>-1.99</td>
<td>-2.02</td>
<td>-0.10</td>
</tr>
<tr>
<td>1977</td>
<td>673</td>
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<td>2.43</td>
<td>2.71</td>
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<tr>
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</tr>
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<td>1979</td>
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<td>-0.32</td>
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<td>0.83</td>
</tr>
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<td>-0.12</td>
<td>3.49</td>
<td>3.13</td>
</tr>
</tbody>
</table>

**Notes:** The average is constructed as described in the text.

### Table 3.2(b) Estimated standard deviation of cost of capital

<table>
<thead>
<tr>
<th>Year</th>
<th>No of Cos</th>
<th>No Tax</th>
<th>No Tax Exhaustion</th>
<th>Full Tax</th>
<th>Full Tax Exhaustion</th>
<th>Full Tax and ACT Exhaustion</th>
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<tbody>
<tr>
<td>1971</td>
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<td>1.69</td>
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<td>2.84</td>
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<td>5.34</td>
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<td>3.15</td>
<td>6.48</td>
<td>4.89</td>
<td>6.42</td>
</tr>
<tr>
<td>1978</td>
<td>685</td>
<td>2.46</td>
<td>2.43</td>
<td>8.04</td>
<td>5.14</td>
<td>7.19</td>
</tr>
<tr>
<td>1979</td>
<td>693</td>
<td>1.57</td>
<td>1.94</td>
<td>8.90</td>
<td>4.90</td>
<td>6.61</td>
</tr>
<tr>
<td>1980</td>
<td>690</td>
<td>3.55</td>
<td>3.42</td>
<td>11.96</td>
<td>7.37</td>
<td>7.98</td>
</tr>
<tr>
<td>1981</td>
<td>673</td>
<td>2.80</td>
<td>1.54</td>
<td>9.60</td>
<td>5.54</td>
<td>9.20</td>
</tr>
<tr>
<td>1982</td>
<td>660</td>
<td>2.50</td>
<td>1.85</td>
<td>12.26</td>
<td>7.97</td>
<td>9.44</td>
</tr>
<tr>
<td>1983</td>
<td>639</td>
<td>2.06</td>
<td>1.34</td>
<td>13.92</td>
<td>6.60</td>
<td>7.84</td>
</tr>
<tr>
<td>1984</td>
<td>602</td>
<td>2.36</td>
<td>15.62</td>
<td>13.44</td>
<td>10.03</td>
<td>8.99</td>
</tr>
<tr>
<td>1985</td>
<td>565</td>
<td>1.83</td>
<td>10.33</td>
<td>6.16</td>
<td>5.77</td>
<td>5.12</td>
</tr>
<tr>
<td>1986</td>
<td>492</td>
<td>1.80</td>
<td>5.03</td>
<td>4.14</td>
<td>5.72</td>
<td>4.55</td>
</tr>
</tbody>
</table>
No attempt has been made to distinguish required rates of return on equity as opposed to debt. The economic depreciation rates are taken from King and Fullerton (1984) and are 8.19% for plant and machinery and 2.5% for buildings, both on an exponential basis. The investment goods price series and the general price series are constructed from deflators available in the Blue Book (various years). The latter is industry specific.

Table 3.2(a) presents the mean of the average cost of capital ie. the cost of capital averaged across different types of investment and across all firms in the sample. As noted above, the figures given correspond to the cost of capital in the financial regime 4 defined above. Even in the 'no tax' case, it is clear that there has been substantial variation over the period considered (1971 to 1986). Throughout much of the 1970s the real interest rate was negative, and the cost of capital is only positive because of the cost of depreciation. By the recession in the early 1980s, however, real interest rates were very high, and this is reflected in very high figures for the cost of capital in this period.

Introducing taxation in all cases reduces the cost of capital, indicating that throughout this period the tax system acted, on average, as an incentive to invest. This, of course, reflects the generosity of the tax system during this period - the existence of 100% allowances for plant and machinery, and allowances climbing to 75% in the first year for industrial buildings, in combination with an imputation system for taxation of dividends and full interest deductibility.

However, the most striking example of this is during the transition period after the 1984 reforms. This occurred because for t=1984 and t=1985, \( \tau_{t+1} < \tau_t \) and \( \eta_{t+1} < \eta_t \), implying that \( p_{t+1}^* > p_t^* \). As already noted, \( \frac{\partial c_t}{\partial \tau_t} > 0 \) and it is also the case (as can be seen from (3.6)) that \( \frac{\partial c_t}{\partial p_{t+1}^*} < 0 \). Both the reduction in the tax rate and the increase in the effective price of capital goods in period \( t+1 \) therefore reduce the cost of capital in period \( t \). This is exactly what happened during the transition period between 1984 and 1986, with the result that the cost of capital, ignoring tax exhaustion, was of the order of 15 percentage
points below the 'no tax' cost of capital in 1984 and 1985. Intuitively, an investment made in 1984, say, received a capital allowance in 1984 at a relatively high rate; in 1985 the return on that investment was taxed at a relatively low rate, and the rise in the effective price of capital goods also created a high capital gain. The change in the effective price effectively captures the advantage of bringing forward investment from 1985 to 1984, and similarly from 1986 to 1985. Since the transitional period did not end until April 1986, some of this transitional effect is also present in 1986.

In fact, the second column in Table 3.2(a), ignoring tax exhaustion, exaggerates the effect of the transitional rules. As can be seen in the other columns, the effect of tax exhaustion was to reduce the impact of the provisions on $\tau^*_t$ and $p^*_t$, compared with $\tau_{t+1}$ and $p_{t+1}$. Otherwise, however, the average values of the cost of capital when tax exhaustion is allowed for are not very different from the case in which tax exhaustion is not allowed for.

It is clear from Table 3.2(b), however, that this measure hides large variation in the cost of capital between companies when tax exhaustion is allowed for. The standard deviation shown for the 'full tax' case is around 1 to 2 percentage points (up to 1984). Since all firms face the same statutory tax regime, this variation is due almost exclusively to differences in the weights used between firms to create an average cost of capital for each firm. Further, since these figures are for regime 4, the variation is due solely to differences in the composition of the capital stock rather than due to differences in the use of different forms of finance. By contrast, the standard deviation in the third column (allowing for full tax exhaustion but not ACT exhaustion), ranges from around 6 to around 12 percentage points, and that for the fourth column (including ACT exhaustion as well) ranges from around 4 to around 8 percentage points. As would be expected from the discussion above and in Chapter 2, these are clearly an order of magnitude greater than those in column 2. The standard deviations in column 5, using the alternative estimates of tax exhaustion are generally between those in columns 3 and 4.

Table 3.2(c) presents simple correlation coefficients between the
various measures of the average cost of capital across both firms and time periods. Recall that a similar table showing correlations between the various definitions of Q, in Chapter 2, revealed that any variation due to tax exhaustion was dwarfed by variation in the market value of firms. One possible advantage of using an investment model based on the cost of capital is that such an effect is clearly not present in Table 3.2(c). The correlation coefficients between the 'full tax' cost of capital and the two 'tax exhaustion' measures based on the model in Appendix B are 0.61 and 0.62. The correlation with the other estimates based on tax exhaustion are even lower. This suggests that the effects of tax exhaustion on investment might be more easily investigated using such cost of capital measures than in the Q model used in Chapter 2.

Table 3.2(c) Correlation of different definitions of cost of capital

<table>
<thead>
<tr>
<th></th>
<th>no tax</th>
<th>no tax exhn</th>
<th>full exhn</th>
<th>full and ACT exhn (accounts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>no tax</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>no tax exhn</td>
<td>0.57</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>full tax exhn</td>
<td>0.50</td>
<td>0.61</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>full and ACT exhn</td>
<td>0.64</td>
<td>0.62</td>
<td>0.84</td>
<td>1.00</td>
</tr>
<tr>
<td>full and ACT exhn (accounts)</td>
<td>0.56</td>
<td>0.50</td>
<td>0.48</td>
<td>0.58</td>
</tr>
</tbody>
</table>

Before turning to the estimation of the investment model, we briefly discuss differences in the cost of capital due to alternative forms of finance being used, and alternative types of asset purchased. Table 3.3 presents estimates based on the definitions in the first column of Table 3.1 ie. it is assumed that $\lambda_{t+1}^D = 0$, so that the firm pays dividends in period $t+1$. The first three columns of Table 3.3 present the cases in which $\lambda_t^D = 0$, $\lambda_t^N = 0$ and $\lambda_t^{B_{t+1}} = 0$, which correspond to the marginal source of finance being retentions, new equity and debt respectively. Column 4 presents the cost of capital under the regime in which the firms uses all three forms of finance, described above as regime 4 (and which is used elsewhere in the empirical work in this chapter). The figures presented correspond to those in column 4 of Table 3.2(a) ie. they allow for both full tax exhaustion and ACT exhaustion. For the purposes of
presentation it is assumed that the marginal rate of personal income tax is equal to the basic rate and that the marginal rate of capital gains tax is zero.

Table 3.3 Average cost of capital by source of finance (%)

<table>
<thead>
<tr>
<th>year</th>
<th>no of earnings</th>
<th>retained</th>
<th>new equity</th>
<th>debt</th>
<th>regime</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>cos</td>
<td>earnings</td>
<td></td>
<td></td>
<td>4</td>
</tr>
<tr>
<td>1971</td>
<td>190</td>
<td>3.36</td>
<td>80.04</td>
<td>3.41</td>
<td>3.33</td>
</tr>
<tr>
<td>1972</td>
<td>297</td>
<td>-0.67</td>
<td>69.10</td>
<td>-0.63</td>
<td>-0.16</td>
</tr>
<tr>
<td>1973</td>
<td>374</td>
<td>-3.07</td>
<td>25.18</td>
<td>-6.17</td>
<td>-4.19</td>
</tr>
<tr>
<td>1974</td>
<td>409</td>
<td>-5.79</td>
<td>10.19</td>
<td>-14.31</td>
<td>-5.06</td>
</tr>
<tr>
<td>1975</td>
<td>431</td>
<td>-2.58</td>
<td>12.71</td>
<td>-11.91</td>
<td>-1.73</td>
</tr>
<tr>
<td>1976</td>
<td>655</td>
<td>-3.54</td>
<td>10.83</td>
<td>-8.22</td>
<td>-2.02</td>
</tr>
<tr>
<td>1977</td>
<td>673</td>
<td>3.76</td>
<td>19.91</td>
<td>-1.19</td>
<td>2.97</td>
</tr>
<tr>
<td>1978</td>
<td>685</td>
<td>0.34</td>
<td>17.02</td>
<td>-5.81</td>
<td>0.80</td>
</tr>
<tr>
<td>1979</td>
<td>693</td>
<td>0.03</td>
<td>12.86</td>
<td>-5.90</td>
<td>0.65</td>
</tr>
<tr>
<td>1980</td>
<td>690</td>
<td>6.26</td>
<td>18.61</td>
<td>-0.66</td>
<td>5.94</td>
</tr>
<tr>
<td>1981</td>
<td>673</td>
<td>9.66</td>
<td>19.73</td>
<td>1.90</td>
<td>9.61</td>
</tr>
<tr>
<td>1982</td>
<td>660</td>
<td>11.39</td>
<td>19.30</td>
<td>4.08</td>
<td>11.62</td>
</tr>
<tr>
<td>1984</td>
<td>602</td>
<td>0.12</td>
<td>3.38</td>
<td>-5.60</td>
<td>3.39</td>
</tr>
<tr>
<td>1985</td>
<td>565</td>
<td>-6.84</td>
<td>-3.44</td>
<td>-14.62</td>
<td>-2.75</td>
</tr>
<tr>
<td>1986</td>
<td>492</td>
<td>3.20</td>
<td>4.77</td>
<td>0.86</td>
<td>3.13</td>
</tr>
</tbody>
</table>

There is a clear ordering of the relative sizes of the cost of capital. Debt is clearly the cheapest form of finance - because it is fully deductible from corporation tax. By contrast, the cost of capital for new equity is substantially higher than either of the other two sources. The size of this effect depends crucially on the assumption made regarding the marginal rate of personal income tax. Under the assumptions used, of a shareholder facing the basic rate of income tax and a zero rate of capital gains tax, then, in the absence of tax exhaustion, $\gamma=1$. From Table 3.1, this implies that the firm should be indifferent between retention and new equity finance. In the presence of ACT exhaustion, however, it is possible that $s_t<s$, so that $\gamma_t<1$. In this case, the cost of capital for new equity finance is raised. In an extreme case in which $s_t^*$ falls to zero (as under a classical corporation tax) the cost of capital for new equity would be much higher than shown in the table. Note that if, instead, it were assumed that the marginal rate of income tax was zero (as for a pension fund, for example), then $\gamma>1$, and it is probable that $\gamma^*=1$, which would make the cost of capital...
for new equity lower than that for retained earnings. The difficulty of empirically choosing between these two cases is one of the principle reasons for choosing the 'regime 4' estimates (defined in (3.15) in empirical work. Using regime 4 does not require marginal rates of income tax and capital gains tax rates to be specified. It can be seen that, in practice, the average cost of capital under regime 4 is very close to that for retained earnings. Within the context of the model used here, the difference between the regime 4 cost of capital and the cost of capital under debt finance may be attributed to marginal agency cost of debt.

Table 3.4 turns to the cost of capital on each of the three types of asset considered, under the regime 4 form of finance. The three assets are plant and machinery, industrial buildings and commercial buildings (buildings are split in this way because they are treated differently by the corporation tax system; in particular, commercial buildings receive no allowance). The relative treatment of the different assets depends mainly on the rate of capital allowance, but also on the rate of

<table>
<thead>
<tr>
<th>year</th>
<th>no of cos</th>
<th>plant and machinery</th>
<th>industrial buildings</th>
<th>commercial buildings</th>
</tr>
</thead>
<tbody>
<tr>
<td>1971</td>
<td>190</td>
<td>-2.89</td>
<td>-3.73</td>
<td>-3.43</td>
</tr>
<tr>
<td>1972</td>
<td>297</td>
<td>-2.24</td>
<td>-2.49</td>
<td>-4.62</td>
</tr>
<tr>
<td>1973</td>
<td>374</td>
<td>-0.36</td>
<td>-1.37</td>
<td>-5.27</td>
</tr>
<tr>
<td>1975</td>
<td>431</td>
<td>-3.16</td>
<td>-4.74</td>
<td>-8.94</td>
</tr>
<tr>
<td>1976</td>
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<td>-6.11</td>
<td>-3.31</td>
<td>-2.55</td>
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<td>-4.32</td>
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<tr>
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<td>-5.86</td>
</tr>
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<td>-2.53</td>
<td>-2.42</td>
<td>-5.47</td>
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<tr>
<td>1980</td>
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<td>-2.55</td>
<td>-4.63</td>
</tr>
<tr>
<td>1981</td>
<td>673</td>
<td>-2.79</td>
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</tr>
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<td>1982</td>
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<tr>
<td>1983</td>
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<td>-1.92</td>
<td>-2.34</td>
<td>-1.10</td>
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<td>1984</td>
<td>602</td>
<td>-9.94</td>
<td>-12.36</td>
<td>-2.51</td>
</tr>
<tr>
<td>1986</td>
<td>492</td>
<td>-4.35</td>
<td>-8.35</td>
<td>0.56</td>
</tr>
</tbody>
</table>
economic depreciation assumed, which as noted above, is 8.19% for plant and machinery and 2.5% for buildings. Because of variation in the cost of capital due differences in the economic depreciation rate between plant and machinery and buildings, Table 3.4 presents estimates of the marginal tax wedge. This is simply the difference between the cost of capital in the absence of taxation and the cost of capital in the presence of taxation for each asset. This is therefore a comparable measure of the direct effect of taxation on the cost of capital for each asset.

3.5 Estimation of the investment model

The basic model to be estimated for firm $i$ is

$$
[I_{1t}^*]_{K_{1t}}^* = \alpha + \beta_1 \frac{H_{1,t+1}}{K_{1,t+1}} + \beta_2 \frac{I_{1,t+1}}{K_{1,t+1}} + \beta_3 \frac{I_{1,t+1}}{K_{1,t+1}} + \beta_4 C_{it}^* + w_{it} \tag{3.25}
$$

for $i=1,2,\ldots,N$, $t=1,2,\ldots,T$, where $H_t = z_t L_t W_t / p_t^y$, i.e. cash flow, and

where:

$$
[I_{1s}^*]_{K_{1s}}^* = \frac{\rho_s}{\rho_t} (1 - \tau_{1s}) \left[ I_{1s} \right]_{K_{1s}} \tag{3.26}
$$

and

$$
\beta_1 = \frac{1}{b}; \quad \beta_2 = 1 - \delta - a; \quad \beta_3 = 1; \quad \beta_4 = \frac{1}{b}; \quad a = \alpha \left[ (1 - \tau_{1t}^*) p_t^y (1 - \delta) \rho_{t+1} (1 - \tau_{1t+1}^*) p_t^y \right] \tag{3.27}
$$

Data on remuneration is not available for all the observations of the sample, so the estimation strategy effectively imposes perfect competition\(^\text{18}\). This implies replacing the output and wages terms by cash

\(^{18}\) The imperfect competition model was estimated using a smaller data set which included data on wages. However, this did not lend support to the more general model. The wages term was positive and significant when the
flow. In addition, the significance of the debt term from the original equation is investigated below. The presence of time and firm variation in the constant, $\alpha$, can be approximated by the inclusion of firm fixed effects and time dummies. Thus the estimation procedure needs to allow for fixed effects even if they are not present in the error term, $w_{it}$.

We therefore assume that

$$\alpha + w_{it} = \varphi'_1 + \varphi_t + \varepsilon_{it}$$

(3.28)

The same estimation procedure as in Chapter 2 is followed. The presence of endogenous terms in (3.25) makes the standard variance-components GLS estimator inconsistent, and the inclusion of $(I/K)_{i,t+1}$ in the equation would introduce bias if within groups estimation were used. As in Chapter 2, the estimation is therefore in first-differences, using a GMM estimator. Estimating the equation in first differences implies that the fixed effect, $\varphi'_1$, drops out of the equation. The GMM procedure weights the instruments optimally. However, first-differencing the equation introduces $w_{i,t-1}$ into the equation. Since $w_{i,t-1}$ must be correlated with $(I/K)_{i,t-1}$, then the latter cannot be used as an instrument. While in principle $(I/K)_{i,t-2}$ is uncorrelated with $w_{i,t-1}$, it is necessary to test for more general dynamic structures particularly due to the expanded error structure defined in (3.28). As in Chapter 2, the appropriateness of the instrument set is tested using the Sargan test of over-identifying restrictions and a test for second order serial correlation (Arellano and Bond (1989)).

Table 3.5 examines the econometric specification of the investment model (3.25). Column (1) begins by estimating the unrestricted form of the model, using each of the right hand side variables as instruments, dated period $t-2$ or earlier. Each of these instruments is in its GMM form$^{19}$. In addition to these instruments, the 'full tax' cost of capital is used as an instrument, on the grounds that it is unlikely to be correlated with

---

model predicted that it should be negative. Probably as a result, the output term (replacing cash flow) was small and insignificant. The other variables were broadly similar to those presented in Table 3.5.

$^{19}$The number of periods used for each instrument is restricted mainly by the need to restrict the overall number of instruments to 90 in order to compute the estimates using the econometric software GAUSS 1.49.
the error term since it does not depend on the behaviour of the company. This table uses the measure of the cost of capital and developed in the model, and therefore takes account of full tax exhaustion and ACT exhaustion.

The Sargan statistic in column (i) indicates that this set of instruments is not valid, and is therefore probably introducing bias into the estimated coefficients. Column (ii) therefore takes all the GMM instruments back to period t-3 (and earlier). The Sargan statistic is now (marginally) acceptable at normal confidence levels. The results in column (ii) are mixed. Three of the four coefficients have the correct sign and reasonable values. Thus the coefficient on $(I/K)_{t,t+1}$ is 0.4385, which assuming the $\alpha=\delta$, gives a value of $\delta$, the average economic depreciation rate of around 28%. While this may be considered rather high, it is not unreasonably so.

The coefficients on $(Z/K)_{t,t+1}$ and $c_{it}$ are predicted to be equal and opposite, and this is confirmed in that the difference between the two coefficients is within one standard error. Furthermore, this coefficient measures precisely the same parameter as the Q coefficient in the Q model, that is, it is $1/b$, where $b$ is given in the adjustment cost function (3.16). However, the estimate of $1/b$ is between ten and fifteen times larger than that found in the Q model (see Table 2.5, and many other estimates in the literature). Given that one of the weaknesses of the Q model is that empirical estimates of $1/b$ are always very small, this is an encouraging feature of the results in column (ii). It suggests that the value of $b$ is between 6 and 10, rather than 100.

The only feature of the results in column (ii) not to be in line with the predictions of the model in (3.25) is therefore the coefficient on $(I/K)_{t,t+1}^2$. The intuition underlying the model is that this coefficient should be positive. This reflects the fact that adjustment costs are quadratic; if the company expects investment in period $t+1$ to be high, it should increase investment in period $t$ by an amount which minimises adjustment costs (ceteris paribus). One point to note about this result

---

20 The DPD programme developed by Arellano and Bond (1988) was used for estimation.
Table 3.5 Testing the econometric specification of the model

<table>
<thead>
<tr>
<th>Dep variable</th>
<th>(i)</th>
<th>(ii)</th>
<th>(iii)</th>
<th>(iv)</th>
<th>(v)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δ(I/K)_{1,t+1}</td>
<td>0.6054</td>
<td>0.4385</td>
<td>0.4043</td>
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<td>0.4233</td>
</tr>
<tr>
<td></td>
<td>(0.0627)</td>
<td>(0.0743)</td>
<td>(0.0682)</td>
<td>(0.0597)</td>
<td>(0.0517)</td>
</tr>
<tr>
<td>Δ(I/K)_{1,t+1}^2</td>
<td>-0.1968</td>
<td>-0.1814</td>
<td>-0.1694</td>
<td>-2.2668</td>
<td>-0.1712</td>
</tr>
<tr>
<td></td>
<td>(0.0248)</td>
<td>(0.0260)</td>
<td>(0.0239)</td>
<td>(0.0597)</td>
<td>(0.0201)</td>
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<tr>
<td>Δ(H/K)_{1,t+1}</td>
<td>0.0090</td>
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<td>0.1216</td>
<td>0.7048</td>
<td>0.0487</td>
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<td></td>
<td>(0.0308)</td>
<td>(0.0651)</td>
<td>(0.0594)</td>
<td>(0.1129)</td>
<td>(0.0548)</td>
</tr>
<tr>
<td>Δc_{1,t}</td>
<td>-0.0332</td>
<td>-0.1539</td>
<td>-0.1279</td>
<td>-0.1702</td>
<td>-0.0960</td>
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<tr>
<td></td>
<td>(0.0449)</td>
<td>(0.0469)</td>
<td>(0.0462)</td>
<td>(0.0520)</td>
<td>(0.0412)</td>
</tr>
<tr>
<td>Δ(B/pK)_{1,t+1}^2</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.0223</td>
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<tr>
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<td>(0.0217)</td>
<td>(0.0217)</td>
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</table>

\[
m2 = -0.39 -1.21 -1.25 0.45 -1.78
\]
\[
z = 227.0 (11) 232.4 (11) 228.4 (11) 235.5 (11) 209.2 (11)
\]
\[
Sargan = 120.5 (74) 95.3 (71) 95.9 (71) 101.8 (72) 92.2 (70)
\]

<table>
<thead>
<tr>
<th>Instruments</th>
</tr>
</thead>
<tbody>
<tr>
<td>G(I/K)_{-2}</td>
</tr>
<tr>
<td>G(I/K)_{-2}</td>
</tr>
<tr>
<td>G(H/K)_{-3}</td>
</tr>
<tr>
<td>G(H/K)_{-4}</td>
</tr>
<tr>
<td>Gc_{t} \Delta c_t</td>
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<tr>
<td>Gc_{t} \Delta c_t</td>
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<td>Gc_{t} \Delta c_t</td>
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<td>Gc_{t} \Delta c_t</td>
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<tr>
<td>Gc_{t} \Delta c_t</td>
</tr>
<tr>
<td>G(B/pK)_{-3}</td>
</tr>
<tr>
<td>G(B/pK)_{-4}</td>
</tr>
</tbody>
</table>

Notes:
1. Time dummies are included as regressors and instruments in all equations.
2. Asymptotic standard errors are reported in parentheses. Standard errors and test statistics are robust to general time-series and cross-section heteroskedasticity.
3. m2 is a test for second order serial correlation in the residuals, asymptotically distributed as N(0,1) under the null of no serial correlation.
4. z2 is a Wald test of the joint significance of the time dummies. There are 12 degrees of freedom for each model in the table.

Continued...
5. The Sargan statistic is a test of the over-identifying restrictions, asymptotically distributed as $\chi^2(k)$ under the null. The number of degrees of freedom is given in parentheses.

6. Instruments preceded by G are in their GMM form. c refers to the cost of capital allowing for taxation but ignoring tax exhaustion (and therefore unaffected by actions of individual companies).

is, however, the possibility of measurement error, especially in the estimates of the capital stock. Any measurement error (which could well be important given the lack of data on the replacement cost values of the capital stock) is exacerbated by the squared term, and this may be reflected in a biased coefficient.

Column (iii) is a slight variation on column (ii). Here the definitions of each variable are slightly different from those given in (3.26), since the discount factor is removed. The purpose here is simply to allow the discount factor to be estimated by forcing it to become part of the coefficient, and hence fixed across companies and over time. As can be seen, the coefficients are slightly lower than in column (ii), as would be expected, since the right hand side variables are now slightly larger. Imposing constant discount factors therefore does little harm to the model.

Column (iv) returns to the problem of the coefficient on $(I/K)_{1,t-1}$ by imposing the restriction that it is equal to one, and forming a new dependent variable. It is clear from the results that this does not improve the fit of the model. Indeed, the performance of all the other terms is now worse than in the unrestricted case. It should perhaps be noted here that, if at least part of the problem is due to measurement error, this would not be solved by imposing the restriction.

Finally, column (v) adds the square of the ratio of the stock of debt to the replacement cost of the capital stock to the specification. As indicated in the model, if agency costs are present and are a function of this ratio, this term should enter with a positive sign. As can be seen, the coefficient on this term is positive but it is insignificantly different from zero. This suggests that agency costs dependent on the level of debt do not play a major role in investment decisions by the
firm. The role played by cash flow in period \( t+1 \) is weakened somewhat by the inclusion of the debt term, due possibly to collinearity between these two terms.

Table 3.6 turns to the issue of whether the results are affected by using different measures of the cost of capital. The five columns of the table make different assumptions regarding the role of the tax system. The assumptions are the same as those investigated earlier in considering values of the cost of capital. They are

1. ignoring tax altogether
2. allowing for tax, but ignoring all forms of tax exhaustion
3. allowing for tax including full tax exhaustion (given by the presence of positive losses carried forward) but ignoring ACT exhaustion
4. allowing for tax, including both full tax exhaustion and ACT exhaustion
5. as (iv), but using estimates of periods of tax exhaustion using the data on tax payments from accounts, rather than from the model described in Appendix B

Table 3.6 reproduces the model in column (ii) of Table 3.5 for each of these measures of the cost of capital (note also that \( \tau^*_{t+1} \), and hence the regressors, are also affected by the assumption). It can be seen that only column (iv), reproduced from Table 3.5 since it allows for both full tax exhaustion and ACT exhaustion, fits the predictions of the model at all well. Although the coefficients on the cost of capital do not vary greatly between the different columns (and for columns (ii), (iii) and (iv) they are very close), the overall results, and hence predictions of the model, do vary. In particular, the coefficient on \( (I/K)_{1,t+1} \) is lower in all of the other columns (compared with (iv)), which, interpreted in the light of the underlying model, suggests very high depreciation rates. The fact that these coefficients are low contributes to the presence of serial correlation, as given by the M2 statistic.

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21 This conclusion is supported by the evidence in the next chapter.
Table 3.6 Testing alternative measures of the cost of capital

<table>
<thead>
<tr>
<th>Dependent variable $\Delta(I/K)_{i,t+1}$</th>
<th>(a)</th>
<th>(1)</th>
<th>(11)</th>
<th>(111)</th>
<th>(iv)</th>
<th>(v)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta(I/K)_{i,t+1}$</td>
<td>0.1233</td>
<td>0.1288</td>
<td>0.0104</td>
<td>0.4385</td>
<td>0.2416</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0705)</td>
<td>(0.0722)</td>
<td>(0.0725)</td>
<td>(0.0743)</td>
<td>(0.0039)</td>
<td></td>
</tr>
<tr>
<td>$\Delta(I/K)_{i,t+1}^2$</td>
<td>-0.1692</td>
<td>-0.1363</td>
<td>-0.1198</td>
<td>-0.1814</td>
<td>-0.0643</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0265)</td>
<td>(0.0270)</td>
<td>(0.0262)</td>
<td>(0.0260)</td>
<td>(0.0292)</td>
<td></td>
</tr>
<tr>
<td>$\Delta(H/K)_{i,t+1}$</td>
<td>0.3461</td>
<td>0.2799</td>
<td>0.2115</td>
<td>0.1068</td>
<td>0.3913</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0653)</td>
<td>(0.0651)</td>
<td>(0.0666)</td>
<td>(0.0651)</td>
<td>(0.0844)</td>
<td></td>
</tr>
<tr>
<td>$\Delta c_{i,t}$</td>
<td>-0.3466</td>
<td>-0.1552</td>
<td>-0.1557</td>
<td>-0.1539</td>
<td>-0.2553</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0753)</td>
<td>(0.0342)</td>
<td>(0.0415)</td>
<td>(0.0469)</td>
<td>(0.0436)</td>
<td></td>
</tr>
</tbody>
</table>

| m2                                      | -2.47     | -2.42  | -2.41  | -1.21  | -3.20   |
| z                                       | 153.8 (11)| 234.0 (11)| 236.0 (11)| 232.4 (11)| 151.1 (11)|
| Sargan                                  | 96.9 (71) | 95.7 (71)| 97.5 (71)| 95.3 (71)| 97.1 (71)|

| Instruments                              | $G(I/K)_{-3}$ | $G(I/K)_{-3}$ | $G(I/K)_{-3}$ | $G(I/K)_{-3}$ | $G(I/K)_{-3}$ |
|                                         | $G(I/K)_{-3}$ | $G(I/K)_{-3}$ | $G(I/K)_{-3}$ | $G(I/K)_{-3}$ | $G(I/K)_{-3}$ |
|                                         | $G(H/K)_{-3}$ | $G(H/K)_{-3}$ | $G(H/K)_{-3}$ | $G(H/K)_{-3}$ | $G(H/K)_{-3}$ |
|                                         | $G(H/K)_{-4}$ | $G(H/K)_{-4}$ | $G(H/K)_{-4}$ | $G(H/K)_{-4}$ | $G(H/K)_{-4}$ |

A further experiment was to include more than one of the definitions of the cost of capital in the equation simultaneously. This was tried using the definitions from columns (11) and (iv). However, the collinearity between the two definitions meant that sensible estimates could not be derived (the coefficients were almost exactly equal and opposite, and very large). An alternative experiment was to include the cost of capital from column (11) (the 'full tax' case) and the difference between the two definitions in columns (11) and (iv) (a measure of the additional information due to tax exhaustion). The results of this were as follows:
\[
(I/K)^*_{t+1} = 0.1237 (I/K)^*_{t} - 0.1080 (I/K)^*_{t+1} + 0.2466 (Z/K)^*_{t+1} \\
\quad - 0.0712 c^*_t + 0.4556 (c^*_t - c^*_t) \\
\quad (0.0786) \quad (0.0303) \quad (0.0764) \quad (0.0474) \quad (0.1001)
\]

where \( c^*_t \) is the cost of capital under the 'full tax' case, and \( c^*_x \) is the cost of capital allowing for tax exhaustion. This has the somewhat surprising result that an increase in the cost of capital due to tax exhaustion tends to increase the rate of investment.

There are two possible interpretations of these results. The first is that all models which do not take account of tax exhaustion are misspecified, since it is only in the case in which tax exhaustion is properly allowed for that the results can be interpreted to be in line with the predictions of the model. However, the fact that small changes to the specification can lead to dramatic changes in the estimated coefficients and to the fit of the model also suggest that the formulation of the model in (3.25) is not robust.

3.6. Conclusions

This chapter has constructed a model of investment based directly on an explicit maximisation process in which investment in the short term depends on adjustment costs. The model is essentially the same as that in Chapter 2 and thus allows for both forms of tax exhaustion. This chapter analysed the effects of tax exhaustion on the cost of capital in various financial regimes, and considered the dynamic effects of moving into and out of tax exhaustion.

The investment model generated by the optimisation is well suited to the simulation of tax changes to investigate the likely response of investment. In particular, the prediction of the model for various forms of tax reform, whether temporary or permanent, or pre-announced or unannounced, was discussed. The model fitted the data reasonably well. In particular, the coefficient on the cost of capital, which according
to the model has the same interpretation of that on Q in Q investment models i.e. the parameter of the adjustment cost function, was relatively high, at around (minus) 0.15, compared to only around 0.01 in the Q model shown in Chapter 2. This suggests that this model may be a more fruitful tool for the examination of the impact of taxation on investment than the Q model.

However, the model does not appear to be very robust to small changes in its specification. This suggests further exploration of some of the assumptions used in the model might identify its weaknesses. There are several possibilities for such further examination. First, the model could be allowed to have rather more general timing: for example, the assumption of a one year lag in capital becoming productive may not be reasonable. More general timings should also be considered. Second, the non-quadratic forms for the adjustment cost function might be incorporated into the model. The quadratic form used here was to provide a direct comparison with the Q model, and it is a simple form to use. However, other forms may prove to be more robust. Third, more work needs to be done to compare the perfect competition and imperfect competition models. Finally, as much of the earlier part of the chapter indicated, the cost of capital should depend on personal tax parameters. Problems involved in defining those parameters were largely defined away in the empirical work, but other assumptions could be incorporated into further empirical work.

Finally, some comment should be made on the performance of tax variables adjusted for tax exhaustion. Tax exhaustion has been tested into two separate (although related) models of investment, with a view to discovering whether it has an independent effect on investment decisions as theory would predict. However, it has proved difficult to test this hypothesis in a theoretically consistent model of investment because there are, in any case, underlying problems with both models. Having said this, there is one other possible factor which would reduce the impact of tax exhaustion, namely the possibility available to firms to lease, rather than to purchase, assets, and thereby probably capture at least part of the tax allowance. Unfortunately, owing to the lack of data on leasing, it has proved impossible to test this possibility.
CHAPTER 4

INVESTMENT, FINANCIAL FACTORS AND CASH FLOW

4.1. Introduction

Most empirical models of company investment rely on the assumption of perfect capital markets. In a world without taxes, one implication of this assumption is that firms are indifferent to funding their investment programmes from internal or external funds. However, there is a rapidly growing body of literature examining the possible existence of imperfections in capital markets and their effects on firms' financial and real decisions. This chapter provides some econometric evidence on the impact of financial factors like cash flow, debt and stock measures of liquidity on the investment decisions of firms in the sample used in this thesis. It also investigates whether the effect of financial factors varies across different types of firm.

One of the basic models of company financial policy, described briefly in Chapter 1 is an attempt to distinguish firms according to three specific regimes. In regime 1, the firm uses only retention finance (the cheapest source of finance). In regime 2, retention finance is exhausted - or, equivalently, dividend payments have reached zero - and the firm uses debt finance. The cost of debt finance increases with the amount borrowed, so that eventually, debt becomes as expensive as new equity finance. At this point, the latter becomes the marginal source of finance for the firm. As Hayashi (1985) argued, changes in the firm's cash flow will increase the availability of retention finance, and will therefore reduce the marginal cost of finance if the firm is in regime 2 (regimes 1 and 3 will be unaffected). It follows from this that investment should be positively related to cash flow if the firm is in regime 2.

This model therefore predicts that there will be cross sectional variation in the way that companies respond to a change in their cash
flow, and that it is not the case that all companies would increase their investment in response to an increase in cash flow. Before discussing papers which have attempted to test the predictions of this model of financial policy, the outline of the approach here will be given.

As will be noted from the detailed discussion of financial regimes in Chapter 2, the simple three regime model is unconvincing. Perhaps its most unfavourable feature is that it predicts that companies would never simultaneously issue new shares and pay dividends - and indeed that they would never simultaneously issue new debt and pay dividends. As indicated below, this is completely at variance with the data, at least in the UK. In this chapter a simple extension of the model of the firm from Chapter 2 is therefore presented. The earlier model is simplified by ignoring tax exhaustion, but allows for agency costs of debt, for reasons given at length in Chapter 1, relating to problems of asymmetric information and principal-agent issues. In this model, agency costs are assumed to depend on the company's cash flow, as well as the stock of debt, the size of the capital stock and the level of internal funds. The prediction of such a model is that it is possible that all companies may face a positive relationship between cash flow and investment, and not just those in a specific financial regime.

Part of the aim of the chapter is therefore to examine whether it is indeed the case that the data is consistent with the view that all companies may face a positive relationship between cash flow and investment. Such a positive relationship will be referred to, somewhat loosely, as a credit constraint. A second aim is to consider whether it is likely that some companies are more likely to face credit constraints than others, and again to test any such predictions against the data.

This chapter investigates in particular the possibility that small companies (and young companies) face a more severe disadvantage in raising external finance, and therefore have a more severe 'credit constraint' problem. There are several reasons why this may be true. First, smaller companies are likely to face greater problems of asymmetric information. A lot of information is provided by financial analysts on the state and prospects of quoted companies, but this is
undoubtedly greater for larger companies. As a result, lemons arguments might apply more significantly to small companies - external investors cannot distinguish between small companies with good and bad prospects, and will therefore charge a premium to cover the risk. This raises the cost of external finance for such companies.

Second, smaller companies often tend to be less diversified, to display greater earnings volatility, and to be more prone to bankruptcy (Titman and Wessels, 1988). The higher degree of risk creates a greater distinction between the shareholder and debtholders of the company, in that the returns to the latter in a successful outcome will be limited, while in an unsuccessful outcome may be zero. As a result, debtholders are likely to impose a higher charge. This issue may be worsened for small companies by asymmetric information: the lemons premium is likely to be higher because small firms as a group ten to be more risky.

However, there are also reasons why it might be the case that incentive problems are more severe for firms in which insiders own a smaller proportion of the firm and outside ownership is more dispersed. This is because of principal agent problems, where shareholders need to control the actions of the managers. Since size may proxy for ownership structure, there is some ambiguity in assessing the effect of size on the cost of external finance.

Most of the arguments regarding the impact of the size of the firm on the cost of external finance apply equally well to the distinction between older firms which are more established, and younger firms. It is therefore also interesting to compare the impact of cash flow on investment decisions for these two categories of companies.

The cross-sectional variation of the impact on investment of flow and stock measures of liquidity has been analysed also by Fazzari et al (1988) and by Gertler and Hubbard (1988) for US firms and by Hoshi et al (1988) for Japanese firms. The former studies distinguish between firms according to their dividend payment behaviour, while the latter classify firms according to the strength of their institutional relationships with banks. However, neither of these approaches is suitable for the UK. The latter is not suitable because the close relationship between some
firms and their bankers is not observed in the UK to the same extent as it is in Japan. In any case, no data exist for the UK to analyse this kind of issue.

It might be considered that the former approach is suitable for use on the UK data used in this thesis. Fazzari et al are essentially trying to test the simple model described above. To distinguish firms in regime 2, they use the dividend payout ratio: they split their sample of firms into three groups, corresponding to a high, medium and low payout. The idea is presumably that firms with low dividend payout ratios are closer to having exhausted their retention finance, and therefore it is more likely that a relationship between cash flow and investment will be observed for such firms. However, this argument is fallacious. The empirical approach implicitly requires that there must be some reason why firms continue to pay any dividends - indeed there is a large literature on why firms pay dividends despite the apparent tax disadvantages of doing so - for example, firms may need to send a signal to investors (see for instance John and Williams (1985), Ambarish et al. (1987) and Edwards (1987) for a critical discussion) or investors may want firms to be subject to scrutiny from the market (see Easterbrook (1984) and Rozeff (1982)). However, without explicitly modelling why dividends are paid, it is not clear which firms are constrained in raising finance by their earnings and which are not. It may well be the case, for example, that firms which have a higher dividend payout ratio face a constraint that demands a higher payment, and so are forced to use external finance more quickly.

This ambiguity is reflected in the empirical results of Fazzari et al: cash flow proved to a significant determinant of investment for all three groups of firms; although the coefficient on cash flow proved to be highest for firms with low dividend payout ratios and lowest for those firms with high payout ratios, it is not clear from this simple model what the implications of this result are. Certainly one possible interpretation consistent with the simple model is simply that the elasticity of the cost of debt finance with respect to the level of debt is lowest for firms with high dividend payout ratios and highest for firms with low payout ratios (although this in itself is one reason for supposing the latter group to be more constrained).
The existing literature thus leaves open a number of questions. First, does cash flow have an empirical significance in Q investment equations on UK panel data? Second, if so, does this empirical significance really represent some form of financial constraint on firms' activities— or at least, an increasing cost of external funds— as opposed to merely proxying better than Q, investment opportunities open to the firm. Third, if financial factors are really important, it is not clear why they should necessarily be best proxied by cash flow: for example, internal funds may be better proxied by the stock of cash or liquid assets held within the firm; are these variables important empirically? Fourth, if financial factors do play a role, do they appear to do so for all companies? Fifth, do financial factors appear to play a more important role for small, or young, companies? Sixth, the empirical work of Fazzari et al. is open to objection on the grounds that their sample selection is based on an endogenous variable (dividend payout) and so their results may be biased and inconsistent; can significant differences in the role played by financial factors across groups of firms selected exogenously be identified?

This chapter addresses itself to these questions. Financial factors are introduced into the model of the firm from the previous two chapters (although tax exhaustion is ignored in the exposition of the model in order to focus on the issues raised by financial factors). In particular, financial factors are introduced via an agency costs of debt finance; agency costs depend on the level of liquid assets in the firm and cash flow in the current period. While this is obviously an ad hoc means of introducing financial factors, potential lenders, such as bank managers, appear to take such items into account in deciding whether to lend to a firm, and if so, at what rate. In addition to agency costs on debt, a lemons premium is added to the cost of new equity, following the arguments of Myers and Majluf (1984). As noted above, they argued that if managers acted in the interests of the existing shareholders, and had information regarding a profitable investment opportunity that outsiders did not have, then it would be impossible for the managers to issue shares to finance that investment at a price which reflected the true value of the project.
The position of the agency cost function is allowed to vary across firms of different size, age and sector reflecting the arguments discussed above. Observations are therefore grouped here according to firm size and age (and controlling for the type of industry). The empirical importance of this breakdown is a natural subject of investigation and the problems of endogenous selection are minimised.

In Section 4.2, the simple extension of the investment model is outlined, illustrating how cash flow can be introduced in Q models. The determinants of the size of the cash flow effect are discussed, and it is explained why caution must be exercised in attributing inter-firm differences only to differences in the importance of agency or financial distress costs. Section 4.3 describes the behaviour and performance of the sample of firms, split by size and age, and section 4.4 presents some econometric results which indicate that financial factors, principally in the form of lagged cash flow, do have an independent effect on investment. Section 4.5 briefly concludes.

4.2. Financial factors in an investment model

This section presents an extension of the model used in the previous two chapters to investigate the role of financial factors in the investment decision. To concentrate on financial factors other than taxation, tax is here modelled only by using the corporate tax rate, $\tau$. For clarity of exposition, depreciation rates are ignored, although they are included in the empirical work below. The model includes agency or financial distress costs $A(.)$, which are a function of the stock of debt at the end of the period, $B_{t+1}$, the replacement value of the capital stock at the end of the period, $P_{t+1}$, the stock of liquid assets at the end of the period $L_{t+1}$, and cash flow during the period, $X_t$. Debt and liquid assets are chosen endogenously, together with investment and new share issues. On the basis of the arguments of the previous section and in chapter 1, agency costs are assumed to be an increasing function of debt

1Note that stocks held at the beginning of the period are dated $t$ and at the end of the period are dated $t+1$; flows during the period are dated $t$. 143
and a decreasing function of cash flow, the capital stock and of liquid assets. As explained above, the agency cost function is allowed to vary for firms in different age and size classes and in different industries. In addition, the lemons premium introduced by Mayers and Majluf (1984) is again included.

This way of summarising informational asymmetries and the risk of bankruptcy is obviously ad hoc. It is adopted here to provide some unifying principle to the discussion and to the empirical testing and to make clear the implicit assumptions underlying the type of equations that have been used so far to test for the importance of financial factors in equations containing average Q. In particular, the aim is to specify a model that is consistent with the fact that cash flow may matter (albeit differently) for all firms, and not only for those that have used up all retentions and are not issuing any new shares.

As in chapter 2, we begin in (4.1) by specifying the sources and uses of funds:

\[
(1-\tau)p_t^v \Pi(K_t, I_t) - A_t(\pi_t, B_{t+1}, L_{t+1}, p_t K_t, \rho_{t+1}) + \nu_{t+1} B_t + L_t (1+(1-\tau)L_t) + \nu_{t+1} B_t + L_t (1+(1-\tau)L_t)
\]

\[= D_t + p_t I_t + (1+(1-\tau)L_t)B_t + L_{t+1} \]  \hspace{1cm} (4.1)

where \(p_t^v\) is the price of output and \(p_t\) the price of investment goods, both during period \(t\), \(\Pi\) real revenues net of variable costs, \(K_t\) the capital stock at the beginning of the period (there is a one period lag in capital becoming productive), \(I_t\) investment during the period, \(B_t\) debt at the beginning of the period, \(\nu_{t+1}\) new equity issued during period \(t\), \(L_t\) liquid assets at the beginning of the period, \(D_t\) dividends paid during the period, \(L\) the rate of interest on debt and \(i^L_t\) the rate of interest on liquid assets, both, for simplicity, assumed to be constant over time. \(X_t\) is cash flow generated by operations, after interest payments but before dividend payments and new investment:

\[
X_t = (1-\tau)\Pi_t - (1+(1-\tau)L_t) + (1+(1-\tau)L_t)L_t \tag{4.2}
\]

The capital accumulation equation is
\[ K_{t+1} = (1-\tau)K_t + I_t \quad (4.3) \]

The firm is assumed to maximise the market value of the firm, which is given by
\[ V_t = \mathbb{E}_t \sum_{j=0}^{\infty} \rho^{j+1} \{ \gamma D_t - V^N_t (1+\omega_t) \} \quad (4.4) \]

where \( \omega_t \) is the lemons premium associated with issuing new shares and \( \rho \) and \( \gamma \) are, respectively, the discount factor (assumed constant over time for expositional purposes) and the tax discrimination parameter between dividends and retentions, defined as in chapters 2 and 3 as
\[ \rho = \frac{(1-m)}{(1-z)} \quad \text{and} \quad \gamma = \frac{(1-m)}{(1-z)(1-c)} \]

where \( m \) is the marginal rate of personal income tax, \( z \) the marginal rate of capital gains tax and \( c \) the imputation rate. Finally, it is assumed that \( \Pi(K_t, I_t) \) is linear homogeneous and separable, and can therefore be written as \( F(K_t) - G(I_t, K_t) \), where \( F \) is the production function and \( G \) is the adjustment cost function. Further, \( G \) is assumed to have the following functional form, familiar from the Q literature (see Summers (1981)):
\[ G(I_t, K_t) = \frac{b}{2} \left( \frac{I_t}{K_t} - c - \varepsilon_t \right)^2 \quad (4.5) \]

where \( c \) is the rate of investment at which adjustment costs are zero and \( \varepsilon_t \) is a stochastic error term. Introducing non-negativity equations on new equity issues and dividend payments, \( V^N_t \geq 0 \) and \( D_t \geq 0 \), with associated multipliers \( \mu^D_t \) and \( \mu^N_t \) and denoting the multiplier associated with (4.3) as \( \lambda^K_t \), the first order condition for investment can be written as:
\[ \frac{I_t}{K_t} = \left( \frac{\lambda^K_t/(\gamma + \lambda^D_t) - \mu^D_t}{b(1-A^\lambda) \left( \frac{\lambda^K_t/(\gamma + \lambda^D_t) - \mu^D_t}{(1-\tau)p_t^Y} \right)} \right) \quad (4.6) \]

where \( A^\lambda \) denotes the partial derivative of the agency cost function with
respect to cash flow\(^2\). If it is further assumed that the agency cost function is linear homogeneous, it is possible to show that the following relationship holds (this is equivalent to the usual equality between average and marginal Q first derived by Hayashi (1982))

\[ \lambda_{K_t}^K (1-\delta) + \lambda_{B_t}^B + \lambda_{L_t}^L = V_t \tag{4.7} \]

where \( \lambda_{t}^B \) and \( \lambda_{t}^L \) are the shadow values of debt and liquid assets respectively. If the firm is on its optimal path, it is possible to show that \( \lambda_{t}^B = -(\gamma + \mu_{t}^D) \) and \( \lambda_{t}^L = (\gamma - \mu_{t}^D) \). If positive dividends are paid, as is almost always the case in our sample, then \( \mu_{t}^D = 0 \). Using this result in (4.7) and taking a first order approximation of (4.6) around sample averages or steady state values we can write:

\[ \frac{I_t}{K_t} = \beta_0 + \beta_1 Q + \beta_2 \left( \frac{X_t}{p_t K_t} \right) + \beta_3 \left( \frac{B_t}{p_t K_t} \right) + \beta_4 \left( \frac{L_t}{p_t K_t} \right) \tag{4.8} \]

where

\[ Q_t = \frac{(V_t/\gamma) + B_t - L_t}{(1-\delta)K_t (1-\tau)P_t} - \frac{p_t}{(1-\tau)P_t} \tag{4.9} \]

The coefficients, denoting sample averages or steady state values by bars, are:

\[ \beta_1 = \frac{1}{b(1-A_x)}; \beta_2 = \frac{1}{1-A_x}; \beta_3 = \frac{1}{1-A_x}; \beta_4 = \frac{1}{1-A_x} \tag{4.10} \]

where subscripts again denote partial derivatives.

This suggests that the coefficient of average Q, i.e. \( \beta_1 \), reflects both the adjustment cost parameter \( b \) and the derivative of the agency cost

---

\(^2\)It is important here that cash flow is defined net of adjustment costs. If cash flow is defined gross of adjustment costs, then agency costs of debt do not depend on \( I \), and the investment equation is identical to the basic Q model, with \( A_x = 0 \).
function with respect to cash flow. The coefficient of cash flow, ie. \( \beta_2 \), is positive if \( A_{x/k} > 0 \), as is reasonable to assume (ie. increasing cash flow reduces agency costs at a decreasing rate). The coefficient increases with the average investment rate. It also depends upon average ratios of cash flow, debt and liquid assets to the capital stock. Similar comments apply to the coefficients of \( B/K \) and \( L/K \) ie. \( \beta_3 \) and \( \beta_4 \) respectively, the signs of which depend on the cross partial derivatives of \( A \). If the agency cost function is additively separable in the pairs \((X, K)\), \((B, K)\) and \((L, K)\), the last two regressors can be omitted and the coefficient of \( X/K \) depends only upon the average cash flow to beginning of period capital stock ratio (in addition to the investment rate). Unless more specific assumptions are made about the functional form of \( A \) little can be said a priori regarding its effect on the size of the coefficient and this is a source of ambiguity in forecasting the expected strength of the effect of cash flow, debt and liquid assets on investment for different types of firms.

Aside from this ambiguity, the agency cost function may be displaced upward or downward by a multiplicative constant which is specific for each group of firms and therefore varies according to size, age and sector. An increase in the constant unambiguously increases the coefficient of cash flow, debt and liquid assets.

It has so far been assumed that positive dividends are being paid, because the data show that this occurs most of the time. In this case the first order condition on new shares issues implies that \( \gamma - I- \omega_t \mu_t^N = 0 \). If \( \omega_t \) is independent of \( \nu_t^N \) as in Fazzari et al. (1988), then, within the context of this model it is necessary to assume that \( \gamma = 1 + \omega_t \), otherwise it would pay to finance continuous new dividend distributions by issuing new shares. Of course, the possibility that this condition might not hold was discussed extensively in Chapter 2 and will not be repeated here, except to note that the possibility of ACT exhaustion rules out such an arbitrage.
4.3 Intra-Firm Differences in Financing, Investment and Profitability in the UK

The discussion above implied that there are several reasons why one might expect the location of the agency cost function to differ across firms. Given its location, the expectation of the relative effect of financial factors on investment would also depend on their relative investment rates, and their cash flow, debt and other liquid assets relative to their capital stock. This section presents some evidence on the relative sizes of these ratios and more generally on firms' characteristics according to size, age and whether they operate in a growing or declining sector.

Table 4.1 presents some summary statistics when each observation on each firm is classified into one of three size categories according to the real value of the capital stock (1980 prices) at the beginning of the preceding period (\( p'K_{t-2} \), where \( p' \) is the 1980 price). The observation is classified as small if \( p'K_{t-2} \) is less than £6 million, medium if \( p'K_{t-2} \) is between £6 million and £50 million, and large if \( p'K_{t-2} \) is above £50 million. Note that as a firm grows, it may move from one group to another. As explained in the next section, the sample is split according to the size of \( p'K_{t-2} \) in order to minimise problems of endogenous selection in estimation.3 The table indicates that investment and cash flow, each as a percentage of the end of period capital stock, decrease with size. This is particularly true of cash flow, with small firms generating a return of 18% compared to only 11% for large firms. Ceteris paribus, the existence of higher cash flows for small firms makes it less likely that they will face financial constraints. The dividend payout ratio is higher for larger firms, although this appears to be mainly due to the fact that depreciation (the difference between cash flow and profit) represents a higher proportion of cash flow for large firms; the average dividend to cash flow ratio is remarkably constant across the three size categories. The frequency with which dividends are paid increases with size, but even for small firms

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3 In order to allow for any distortion to these results arising from measurement error in \( K \), a similar split was performed using the real value of sales two periods earlier as a measure of size. The results were very similar.
however, the average dividend payout ratio is approximately 34% and dividends are paid 89% of the time.

Table 4.1 Description of the sample split by size

| Case | Description               | No of observations | Investment/Capital Stock | Sales/Capital stock | Cash flow/Capital stock | Profit/Capital stock | Dividends/Cash flow | Dividends/Profit | Investment/Total funds | Retentions/Total funds | New Equity/Total funds | Change in long term debt/Total funds | Change in short term debt/Total funds | Change in bank debt/Total funds | Long term debt/Market value | Interest paid/(Interest+cash flow) | Current assets/Capital stock | Average Q | Std deviation of real sales growth | Frequency of dividend payments | Frequency of new equity issues |
|------|-----------------------------|--------------------|--------------------------|---------------------|--------------------------|-----------------------|--------------------|-------------------|------------------------|------------------------|------------------------|---------------------------------|---------------------------------|-------------------------------|-------------------------|------------------|---------------------------------|-----------------------------|-----------------------------|
| 1    | $p_{K_{t-2}} < £6m$       | 2681               | 13.4                     | 318.8               | 17.8                     | 12.4                 | 33.5               | 66.4              | 67.9                   | 13.2                   | 12.1                   | 5.7                             | 13.2                             | 12.1                          | 7.6                      | 16.6                          | 24.5                          | -0.13                      |
| 2    | £6m < $p_{K_{t-2}} < £50m$ | 3966               | 11.1                     | 232.9               | 13.6                     | 8.8                  | 23.8               | 70.0              | 65.5                   | 14.8                   | 11.9                   | 7.8                             | 11.9                             | 10.8                          | 12.5                      | 18.1                          | 20.6                          | -0.19                      |
| 3    | $p_{K_{t-2}} > £50m$      | 2059               | 10.2                     | 170.8               | 11.4                     | 6.6                  | 22.4               | 78.3              | 68.0                   | 12.3                   | 6.5                    | 13.3                            | 6.5                             | 5.2                           | 23.3                      | 20.3                          | 23.2                          | 0.11                       |

Notes:
1. Total funds are the sum of retentions, new equity and the change in long term and short term debt.
2. "Market value" is taken as the market value of equity plus the book value of debt.
3. Current assets comprise inventories and work in progress, financial investments, the stock of cash and trade debtors less trade creditors and other short term liabilities (excluding short term debt).
4. Q is defined in equation (4). $V_t$ is measured at the beginning of the period.
Prima facie evidence that internal sources of finance are preferred to external sources is represented by the fact that investment is financed mainly through retentions, which constitutes about 67% of the total sources of funds. Perhaps surprisingly, the proportion of funds raised from retentions by large firms is almost identical to that raised by small firms. New equity varies between 12 and 15% of total new funds. The frequency of new share issues increases with size. The lower frequency of new equity issues for small firms is consistent with the observation that flotation and underwriting costs are an decreasing function of the value of the issue.

Long term debt represents a small percentage of investment finance especially for smaller firms. This suggests that it is expensive for small firms to rely on market debt. Note, however, that the percentage of new finance derived from short term debt (with maturity of less than one year) is greater for smaller firms. The vast majority of their short term debt is provided by banks. This indicates that the difficulty of borrowing in the open market may be partly relieved by the ability to borrow from institutions that can more easily monitor the borrower through a continuing relationship. It is not clear, however, that the duration of bank debt matches the requirements imposed by investment projects that will provide a return over a long period of time.

A final piece of interesting evidence from Table 4.1 is that the standard deviation of real sales growth falls with size although this effect is not very large. The slightly higher figure for small firms may be reflected in the relatively high ratio of current assets to the capital stock, in that such firms may find it useful to maintain a sizable reserve of liquid assets in order to buffer the volatility of sales revenues and to avoid being forced to borrow on unfavourable terms. Moreover, this ratio is one of the indices commonly used by lenders to judge the credit worthiness of potential borrowers. Another indicator of the ability to meet financial obligations is the ratio of interest payments to cash flow, which is smaller for smaller firms. By

---

4 Mayer (1988) claims that the proportion of funds raised from new share issues is somewhat lower, although our figures appear to be reasonably in line with official statistics (DTI Business Monitor, MA3).
presenting a healthy liquid asset position firms may be able to reduce the cost of borrowing.

Table 4.2 presents some independent evidence on the degree to which financial factors are perceived to influence the investment decision of different sizes of firms. The figures are taken from the quarterly survey of UK manufacturing industry conducted by the Confederation of British Industry. It indicates that over the period 1981 to 1986 virtually a third of the respondents cited some financial factor as constraining their investment (although it is hard to distinguish the

Table 4.2  Evidence from CBI Industrial Trends Survey of UK Manufacturing companies

Average response to the question: "What factors are likely to limit (wholly or partly) your capital expenditure authorisations over the next 12 months?" over the period 1981 to 1986 (24 quarterly surveys)

<table>
<thead>
<tr>
<th>Percent</th>
<th>Size by number of employees</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Whole Sample</td>
</tr>
<tr>
<td>Inadequate net return on proposed investment</td>
<td>39.5</td>
</tr>
<tr>
<td>Shortage of internal finance</td>
<td>21.2</td>
</tr>
<tr>
<td>Inability to raise external finance</td>
<td>2.6</td>
</tr>
<tr>
<td>Cost of finance</td>
<td>8.5</td>
</tr>
<tr>
<td>Uncertainty about demand</td>
<td>46.3</td>
</tr>
<tr>
<td>Shortage of labour(^1)</td>
<td>3.1</td>
</tr>
<tr>
<td>Other</td>
<td>2.3</td>
</tr>
<tr>
<td>N/A</td>
<td>12.2</td>
</tr>
</tbody>
</table>

Notes:

1. Including manual and technical staff
three questions related to financial factors). The most striking feature of the table is, however, the proportion of the largest firms which cited "shortage of internal finance" as a significant constraint on their investment. While the sample of firms in this category is low, this does suggest that very large firms may face financial constraints. The table suggests, however, that slightly less large firms (in the third category) face somewhat lower financial constraints.

Table 4.3 Description of the sample split by size and age

<table>
<thead>
<tr>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>pK&lt;sub&gt;t-2&lt;/sub&gt; ≤ £50m; less than 12 years since first quotation</td>
<td>pK&lt;sub&gt;t-2&lt;/sub&gt; ≤ £50m; more than 12 years since first quotation</td>
</tr>
<tr>
<td>No of observations</td>
<td>773</td>
</tr>
<tr>
<td>Investment/Capital Stock</td>
<td>14.4</td>
</tr>
<tr>
<td>Sales/Capital stock</td>
<td>282.5</td>
</tr>
<tr>
<td>Cash flow/Capital stock</td>
<td>18.0</td>
</tr>
<tr>
<td>Profit/Capital stock</td>
<td>12.3</td>
</tr>
<tr>
<td>Dividends/Cash flow</td>
<td>23.6</td>
</tr>
<tr>
<td>Dividends/Profit</td>
<td>34.5</td>
</tr>
<tr>
<td>Investment/Total funds&lt;sup&gt;1&lt;/sup&gt;</td>
<td>72.3</td>
</tr>
<tr>
<td>Retentions/Total funds</td>
<td>69.0</td>
</tr>
<tr>
<td>New Equity/Total funds</td>
<td>15.3</td>
</tr>
<tr>
<td>Change in long term debt/Total funds</td>
<td>5.9</td>
</tr>
<tr>
<td>Change in short term debt/Total fund</td>
<td>9.8</td>
</tr>
<tr>
<td>Change in bank debt/Total funds</td>
<td>9.3</td>
</tr>
<tr>
<td>Long term debt/Market value&lt;sup&gt;2&lt;/sup&gt;</td>
<td>10.1</td>
</tr>
<tr>
<td>Interest paid/(Interest+cash flow)</td>
<td>17.4</td>
</tr>
<tr>
<td>Current assets/Capital stock</td>
<td>13.2</td>
</tr>
<tr>
<td>Average Q&lt;sup&gt;4&lt;/sup&gt;</td>
<td>0.82</td>
</tr>
<tr>
<td>Std deviation of real sales growth</td>
<td>17.1</td>
</tr>
<tr>
<td>Frequency of dividend payments</td>
<td>95.5</td>
</tr>
<tr>
<td>Frequency of new equity issues</td>
<td>24.1</td>
</tr>
</tbody>
</table>

Notes: See notes to Table 4.1

<sup>1</sup>Between 25 and 60 out of a sample of around 1250.
Within this age category, new companies have a higher investment rate and cash flow. The payout ratio is fairly stable across the two categories. Younger firms make greater use of retentions, and also derive a slightly larger fraction of new funds from new share issues. The higher profitability and investment of new companies is reflected in a higher value of Q. There is little variation in the standard deviation of sales growth, thus suggesting that sales volatility does not depend to any great extent on firm age.

Another dimension that has a potential bearing on investment and financing decisions, especially in the presence of asymmetric information, is the firm's age. Although accounting data do not include information on each firm's age, it is known when firms went public. Table 4.3 distinguishes between observations on companies that have been quoted for at least 12 years and observations on companies that have been quoted for less than 12 years. This table examines only small and medium-sized firms (i.e. \( p_{t-2} < £50 \) million). Since larger firms are almost exclusively more than 12 years since their first quotation, they would all fall into the "old" category. By concentrating on the remainder, we consider firms which, apart from age, are more nearly alike.

4.4 Empirical Results

We turn now to estimating equation (4.8) for the entire sample. Many of the econometric issues involved in estimating this equation have been discussed at length in chapter 2 and only a cursory discussion is given here. We wish to allow for the possibility of time specific and firm specific effects. Introducing the subscript \( i \) to distinguish companies, we therefore wish to estimate

\[
\begin{align*}
\left( \frac{1}{K} \right)_{it} &= \beta_0 + \beta_1 Q_{it} + \beta_2 \left( \frac{X}{pK} \right)_{it} + \beta_3 \left( \frac{B}{pK} \right)_{it} + \beta_4 \left( \frac{L}{pK} \right)_{it} + \alpha_i + \alpha_t + v_{it} \\
\end{align*}
\]

The stochastic term, \( v_{it} \), arises from disturbances to the adjustment cost function, as in the standard Q model. There is nothing in the
theory which restricts this term to be an innovation error, and indeed, as discussed in chapter 2, in the standard Q model \( \Delta t \) appears to follow an AR(1) process. To allow for the possibility that this is true also of the extended model shown in (4.11), lagged values of the dependent variable and of each regressor are included in the equation (the model is again estimated without imposing the common factor restriction). The lagged values may, of course, also reflect the ambiguities involved in choosing the timing of the various variables.

The model is again estimated in first differences to allow for firm specific, time invariant effects and an instrumental variable procedure is used to allow for the endogeneity of the regressors. This endogeneity arises because current cash flow, debt, current assets, Q and investment may all be simultaneously determined (although Q, unlike the other variables, is constructed by dating it at the beginning of the period). In addition, care must be taken to allow for the possibility of measurement error, particularly in Q. For reasons discussed in chapter 2, a GMM estimator is used, and the instrument set used is denoted in the form eg. \( Q(n,m) \), where \( n \) indicates that the latest lag used is dated \( t-n \), and \( m \) indicates the number of lags used.

Column 1 of Table 4.4 presents the estimated coefficients for the equation containing, in addition to Q and lagged investment, both flow and stock measures of liquidity and the stock of debt. Time dummies are included as regressors and instruments in all equations. The results suggest that contemporaneous Q remains a significant determinant of investment despite the inclusion of the other terms, although the size of its coefficient remains small. Cash flow, especially dated \( t-1 \), plays an important role with a large coefficient. The coefficient on contemporaneous debt is negative and significant, as one would expect if an increase in cash flow decreases the marginal agency cost of debt, so that \( A_X, (B/X) < 0 \) (see (4.10)). The stock of liquid assets does not play a significant role in this equation. Dropping liquid assets from the model in column 1 has very little effect on the other terms in the equation.

---

Experiments have also been undertaken to assess the effect of alternative empirical measures of \( \gamma \). The results do not appear to be sensitive to alternative measures. The tables report the results for \( \gamma = 1 \).

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These results are generally robust to variations in the instrument set. The equation does not exhibit second order serial correlation (see the M2 statistic) which would invalidate the instrument set. Moreover, the Sargan test of overidentifying restrictions suggests that the instruments are not correlated with the error term. As in chapter 2, if \( Q_{t-1} \) is included in the instrument set (the most recent value of \( Q \) used in the instrument set in column 1 is dated \( t-2 \)), the coefficient on contemporaneous \( Q \) falls, which is consistent with the possibility that downwards bias due to measurement error in \( Q \) outweighs any upward bias due to the possible endogeneity of \( Q \). This result is also found when the same comparison is made for the other equations presented below, and so \( Q_{t-1} \) is generally excluded from the instrument set.

The positive effect of the lagged investment rate and the negative coefficient on the lagged \( Q \) term are consistent with an AR(1) error term in the underlying equation. However, the positive coefficients on both the cash flow terms is inconsistent with this explanation of the dynamic structure. (Replacing \( (X/pK)_t \) with \( (X/pK)_{t-2} \) provides a result consistent with the AR(1) process although this would imply that lagged cash flow, not current cash flow, should be in the specification in equation (4.8)). This suggests that the timing of the impact of cash flow in investment is more complex than suggested by the model in section 4.2. Intuitively, the significance of lagged cash flow may be explained if external investors may observe only cash flow in the previous period, or, more generally, may judge the firm's credit worthiness using a weighted average of past cash flows.

Column 2 of Table 4.4 explores what happens when debt is excluded from the model (debt is rarely significant in the subsamples of the data examined below, which may be mainly due to the fact that less data is available). The positive effect of contemporaneous cash flow disappears in the absence of the negative effect of contemporaneous debt, while

---

4 As suggested in chapter 2, including \( Q_{t-2} \) in the instrument set may also introduce bias since it also appears as a regressor in the differenced equation, although in later tables the first differenced \( Q_{t-1} \) is omitted since it is not significant for subsamples of the data.
### Table 4.4 The Full Sample

<table>
<thead>
<tr>
<th>Dependent Variable $\Delta(I/K)_{tt}$</th>
<th>Period 1972-1986</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>$\Delta(I/K)_{t}$</td>
<td>0.1896</td>
</tr>
<tr>
<td></td>
<td>(0.0306)</td>
</tr>
<tr>
<td>$\Delta Q_{t}$</td>
<td>0.0180</td>
</tr>
<tr>
<td></td>
<td>(0.0051)</td>
</tr>
<tr>
<td>$\Delta Q_{t-1}$</td>
<td>-0.0044</td>
</tr>
<tr>
<td></td>
<td>(0.0019)</td>
</tr>
<tr>
<td>$\Delta(X/K)_{t}$</td>
<td>0.1168</td>
</tr>
<tr>
<td></td>
<td>(0.0788)</td>
</tr>
<tr>
<td>$\Delta(X/K)_{t-1}$</td>
<td>0.1584</td>
</tr>
<tr>
<td></td>
<td>(0.0582)</td>
</tr>
<tr>
<td>$\Delta(B/pK)_{t}$</td>
<td>-0.0772</td>
</tr>
<tr>
<td></td>
<td>(0.0300)</td>
</tr>
<tr>
<td>$\Delta(B/pK)_{t-1}$</td>
<td>0.0581</td>
</tr>
<tr>
<td></td>
<td>(0.0418)</td>
</tr>
<tr>
<td>$\Delta(L/pK)_{t}$</td>
<td>-0.0149</td>
</tr>
<tr>
<td></td>
<td>(0.0130)</td>
</tr>
<tr>
<td>$\Delta(L/pK)_{t-1}$</td>
<td>0.0153</td>
</tr>
<tr>
<td></td>
<td>(0.0138)</td>
</tr>
<tr>
<td>$\Delta(Y/pK)_{t-1}$</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta(Y/pK)_{t}$</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>$m^2$</td>
<td>-1.26</td>
</tr>
<tr>
<td>WT</td>
<td>52.1 (15)</td>
</tr>
<tr>
<td>Sargan</td>
<td>59.0 (72)</td>
</tr>
<tr>
<td>Instruments</td>
<td>Q(2,2), X/pK(2,1)</td>
</tr>
<tr>
<td></td>
<td>B/pk(2,1),</td>
</tr>
<tr>
<td></td>
<td>I/K_{t-2}' Y/pK_{t-3}</td>
</tr>
<tr>
<td></td>
<td>L/K_{t-2}' L/K_{t-3}</td>
</tr>
</tbody>
</table>

**Notes.**
1. Time dummies are included in all equations.
2. Asymptotic standard errors are reported in parentheses. Standard errors and test statistics are asymptotically robust to heteroscedasticity across companies.
3. $m^2$ is a test for second-order serial correlation in the residuals, asymptotically distributed as $N(0,1)$ under the null of no serial correlation. See Arellano and Bond (1988).
4. The Sargan statistic is a test of the over-identifying restrictions, asymptotically distributed as $\chi^2(k)$.
5. W is a Wald test of the joint significance of the time dummies, asymptotically distributed as $\chi^2(k)$, under the null of no significance.
6. The instrument sets are explained in the text.
lagged cash flow becomes more important. The coefficient on current Q falls slightly.

In Column 3 lagged and twice lagged output as a proportion of the replacement value of the capital stock is added to this specification (contemporaneous output is not significant). Their coefficients are neither individually nor jointly significant. However, note that the negative coefficient on current output is consistent with the presence of imperfect competition, which introduces an additional wedge between marginal and average Q, which depends on the present value of current and future output. The wedge captures the loss of monopoly profits due to the decrease in price associated with the additional output produced by new investment. Adding output to the equation to some extent proxies for the wedge, and therefore we would expect a negative coefficient. This issue is explored further below for different subsamples of the data.

The presence of output in the equation has little effect on the coefficient of lagged cash flow. Its remaining significance suggests that even if cash flow is to some extent proxying for demand, this is not the main reason for its importance. The principal model investigated below is a parsimonious version of column 2 of Table 4, dropping lagged Q and current cash flow (which are individually and jointly insignificant). The size and significance of the other variables is virtually unchanged when these two terms are omitted.

One reason for the significance of cash flow is that it may be a better

---

8 More precisely, omitting debt, liquid assets and taxes, it can be shown that

\[ V_t - \sum_{i=1}^{\infty} \frac{1}{(1+R)^i} Y_{t+i} \frac{1}{(1-\delta)K_t} \]

where \( \varepsilon_{t+1} \) is the elasticity of demand.

9 However, if the equation were estimated in a quasi-differenced form, as suggested by Schiantarelli and Georgoutsos (1988), the contemporaneous investment rate, given 'scaled' past investment should be positively related to \( \frac{V}{K} \) \( t-1 \). When this variable is added to the specification in column 2 on its own, it is rarely significant (depending on the instrument set used).
proxy for market fundamentals than the market value of the firm and entrepreneurs may respond only to fundamentals (Blanchard et al., 1988). In this case one would expect that during periods of potential speculative bubbles or fads in the stock market, the coefficient for Q and cash flow should be different, compared with other periods. In particular one may expect that Q matters less relative to cash flow in such periods. It is obviously difficult to identify unambiguously when bubbles or fads caused share prices to be a poor reflection of fundamentals. During the period available in the data, the years between 1981 and 1986 are possible candidates; average price-earnings ratios have been consistently higher from 1981 onwards then over the previous 10 years. While this may, of course, simply reflect more optimistic expectations, this may also reflect the existence of a bubble.

The specification used below, for example in Table 5, has therefore been re-estimated allowing all of the slope coefficients to differ between the two subperiods. However, there is not strong evidence of a structural break. The Wald test statistic for the joint significance of the three additional terms (each variable interacted with a dummy equal to 1 for the period 1981 to 1986 and 0 otherwise) is 6.83 (compared with a critical value of 7.81 at the 5% significance level). In addition, the coefficient on lagged cash flow for the whole period was 0.2951 (with standard error 0.0462), while that for the additional variable lagged cash flow from 1981 to 1986 only was -0.0982 (with standard error of 0.0607). If $Q_{t-1}$ is included in the instrument set, the three additional terms become jointly significant (with a Wald statistic of 15.3). The same pattern arises for the cash flow terms, and additionally in this case, the coefficient on Q from 1981 to 1986 only is positive and significant. Any support for a structural break which might be found in these results would therefore be in the opposite direction to what would be expected if cash flow were merely proxying for market fundamentals. Rather, it seems that in the relative boom years of the 1980s firms were simply less financially constrained and hence cash flow was less important. The asymmetric effect of cash flow on investment during booms and recessions is emphasised by Gertler and Hubbard (1988). Of course, it may be that cash flow proxies both for market fundamentals and financial constraints, but that the change in the latter dominate in the 1980s. This is an issue that deserves further investigation. However
these initial results suggest that fads and bubbles are not the key explanation as to why cash flow is significantly related to investment.

The arguments in the previous section suggest that cash flow and other financial variables may have a differential impact across different types of firms. In Table 4.5 we present the results on the effect on cash flow for firms of three different sizes ("small", "medium" and "large"). We also consider "very large" firms (which are a subset of the

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δ(I/K)_{1,t-1}</td>
<td>0.1723</td>
<td>0.1550</td>
<td>0.1056</td>
<td>0.1032</td>
</tr>
<tr>
<td></td>
<td>(0.0485)</td>
<td>(0.0355)</td>
<td>(0.0493)</td>
<td>(0.0480)</td>
</tr>
<tr>
<td>ΔQ_{1,t}</td>
<td>0.0011</td>
<td>0.0144</td>
<td>0.0188</td>
<td>0.0085</td>
</tr>
<tr>
<td></td>
<td>(0.0052)</td>
<td>(0.0082)</td>
<td>(0.0101)</td>
<td>(0.0058)</td>
</tr>
<tr>
<td>Δ(X/K)_{1,t-1}</td>
<td>0.2275</td>
<td>0.2263</td>
<td>0.3163</td>
<td>0.4050</td>
</tr>
<tr>
<td></td>
<td>(0.0413)</td>
<td>(0.0385)</td>
<td>(0.0667)</td>
<td>(0.1113)</td>
</tr>
<tr>
<td>m2</td>
<td>-2.14</td>
<td>-0.52</td>
<td>-0.18</td>
<td>0.03</td>
</tr>
<tr>
<td>WT</td>
<td>67.3 (15)</td>
<td>67.1 (15)</td>
<td>38.0 (15)</td>
<td>59.7 (15)</td>
</tr>
<tr>
<td>Sargan</td>
<td>82.1 (72)</td>
<td>89.4 (72)</td>
<td>85.0 (72)</td>
<td>73.8 (72)</td>
</tr>
<tr>
<td>Instruments</td>
<td>I/K(2,1), Q(2,2), X/pK(2,1), Y/pK(2,1)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes. See notes to Table 4.4.
group of "large" firms). Note that observations are classified according to the real value (1980 prices) of the capital stock at the end of time \( t-2 \), \( p'K_{t-1} \) (where \( p' \) is the price of capital goods in 1980). Under the assumption that the error term in the levels equation is not serially correlated, \( p'K_{t-1} \) is predetermined with respect to the error term in the differenced equation. Current assets were not significant when added to the various equations. In addition, current cash flow and further lags of cash flow and \( Q \) were generally insignificant when added to the equations presented.

Consider, first, cases 1, 2 and 3. The coefficient on cash flow is significant for all classes of firms. Perhaps surprisingly, it is greater for large firms, although there is not a statistically significant difference between the coefficients for large and small firms at normal significance levels (the \( t \)-statistic for the significance of the difference between the two coefficients is 1.13). The coefficient and the significance of current \( Q \) increases across the size categories; for small firms \( Q \) appears to have no impact on investment, while for large firms, the coefficient on \( Q \) is much greater. Given the increasing coefficient on cash flow as size increases, it is also worth considering whether the impact of cash flow for large firms is dominated by very large firms. The results shown in case 4 show that this may be the case; although the coefficient on cash flow for very large firms is less precisely determined (as would be expected since there are fewer observations), the point estimate is considerably higher even than that for 'large' firms and the significance of the difference between it and that for small firms is slightly higher (with a \( t \)-statistic of 1.50).

These qualitative results are invariant to alternative instrument sets.

---

\(^{10}\) On the assumption that the error terms are independent across the two categories, the appropriate standard error is simply the square root of the sum of the squares of the two standard errors on the two coefficients. This allows a simple \( t \)-test to be performed on the difference between the coefficients.

\(^{11}\) This also suggests that cash flow is not significant merely because it is a better proxy for expectations of future prospects than \( Q \): \( Q \) has a more significant impact for large firms, yet the coefficient on cash flow is also higher for these firms.
However, the significance of both the Q and cash flow does vary with the instrument set. In particular, if $Q_{t-1}$ is included in the instrument set, current Q is statistically significant for medium, large and very large firms although the estimated coefficients are slightly lower. In addition, the differences between the cash flow coefficients are more significant (with t-statistics of 1.68 for the difference between small and large firms and 1.88 for the difference between small and very large firms).

With one main exception, adding other regressors has little impact on the coefficients and standard errors presented in Table 5. The exception occurs when current output is added to the model for large firms. The coefficient on current output for large firms is -0.0106 with standard error 0.0026. Its negative sign is again consistent with the possibility that output is reflecting the existence of imperfect competition since large firms are more likely to be in a position to exploit the benefits of monopolistic competition. The coefficient on current cash flow for large firms increases substantially when current output is included, although it is less precisely estimated. Current debt also has a negative sign but is not significant when added to the models in Table 4.5. Adding debt tends to increase the difference in the coefficients on cash flow between case 1 and case 3 firms, although their standard errors also increase.

In the context of the model described in the section 4.2, the size of the coefficient on cash flow for large firms cannot be accounted for by a higher investment rate of large firms (see (4.10)), because it is, in fact, lower. It could be explained by the lower cash flow/capital ratio that characterises larger firms, if the coefficient of cash flow decreases with this ratio. It is easy to find parameterisations of the agency cost function that yield this result. This factor may be dominant since differences in the investment rate are not very large and

\[ A = \begin{cases} 
-a(X/K)^\alpha + b(B/K)^\beta K, & \text{where } 0<\alpha<1, \text{ or if} \\
(X/K)^\alpha(B/K)^\beta, & \text{where } \alpha<0.
\]
neither is the difference in the riskiness as measured by the variance of sales. It is also possible that the differential according to size may capture industry effects. Finally, it is possible that, ceteris paribus, agency costs may be more severe when insiders effectively controlling the firm hold a lower fraction of the equity and/or outside equity holdings are more dispersed. Size may proxy for the effect of these factors on the severity of the incentive problems.

Two criticisms might be made with regard to splitting firms according to the replacement cost value of the capital stock two periods ago. One is that there may remain some endogeneity introduced by serial correlation in the error term (although the results do not suggest such a correlation). The second is that whatever effects size is proxying for, an alternative would be to split by the size of a firm relative to the size of other firms in the industry in which that firm operates. Thus a "small" firm overall may seem larger relative to other firms in its own industry. To meet these possible criticisms, two other sample splits have been considered. Firstly, firms are split simply according to their initial size (that is, their size, in 1980 prices, when they first entered the database). Of course, this takes no account of the rate of growth of a firm since it entered the database, and possibly as a result, there is much less variation in the value of the cash flow coefficient between different size classes of firms measured by initial size.

Secondly, however, in Table 4.6, we present the results of splitting firms according to their initial size relative to that of other firms in their industry which are also in the database; again, the comparison is made in 1980 prices. Thus, case 1 firms are among the smallest 75% of firms in their industry measured by initial size and case 2 firms are among the largest 25%. It is clear from the table that the results concerning cash flow are similar to those in Table 5 (indeed the size and significance of the difference across the two categories is greater in Table 4.6; the t-statistic on the difference between the two cash flow coefficients is 1.84). By contrast, however, Q appears more important for the smaller firms. This latter result may be partly due to grouping together all "non-large" firms in the first column.
Table 4.6  Split by Initial Size relative to Distribution of Industry Initial Size

<table>
<thead>
<tr>
<th>Dependent Variable $\Delta(I/K)_{it}$</th>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1 $p'K_0$ within smallest 75% of firms in the same industry</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 2 $p'K_0$ within largest 25% of firms in the same industry</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of firms</td>
<td>4530</td>
<td>2016</td>
</tr>
<tr>
<td>Number of observations</td>
<td>541</td>
<td>179</td>
</tr>
<tr>
<td>$\Delta(I/K)_{1,t-1}$</td>
<td>0.1741</td>
<td>0.1782</td>
</tr>
<tr>
<td>(0.0325)</td>
<td>(0.0546)</td>
<td></td>
</tr>
<tr>
<td>$\Delta Q_{1,t}$</td>
<td>0.0130</td>
<td>0.0060</td>
</tr>
<tr>
<td>(0.0082)</td>
<td>(0.0032)</td>
<td></td>
</tr>
<tr>
<td>$\Delta(X/K)_{1,t-1}$</td>
<td>0.2303</td>
<td>0.3613</td>
</tr>
<tr>
<td>(0.0293)</td>
<td>(0.0648)</td>
<td></td>
</tr>
<tr>
<td>$m2$</td>
<td>-1.67</td>
<td>-0.33</td>
</tr>
<tr>
<td>WT</td>
<td>96.9 (15)</td>
<td>38.5 (15)</td>
</tr>
<tr>
<td>Sargan</td>
<td>102.0 (72)</td>
<td>85.1 (72)</td>
</tr>
<tr>
<td>Instruments</td>
<td>$I/K(2,1), Q(2,2), X/pK(2,1), Y/pK(2,1)$</td>
<td></td>
</tr>
</tbody>
</table>

Notes. See notes to Table 4.4

Table 4.7 takes the issue of examining the effects of size while controlling for industry one stage further by exploring the distinction between growing and declining sectors. The reason for this is that it is intuitively more probable that companies in declining sectors may face financial distress. The second hand market for capital goods is likely to be less active, the liquidation value of assets to be smaller and, therefore, the cost of financial problems greater. Here the two size categories are used, $p'K_{t-1}<10$ and $p'K_{t-1}>10$. Within each category, companies are divided according to whether they are in a growing or declining sector. Due to the small number of observations in some of the categories, parameters are estimated with less precision than in other tables. As can be seen from Table 4.7, it remains the case that larger

163
companies have higher coefficients on the cash flow term. This is true both for companies in growing sectors and companies in declining sectors. However, the impact of size appears to be reduced when the sector is taken into account.

Table 4.7 Split by Size and Sector

<table>
<thead>
<tr>
<th>Case</th>
<th>p'K_{t-1} &lt; £10m; growing sectors</th>
<th>p'K_{t-1} &lt; £10m; declining sectors</th>
<th>p'K_{t-1} &gt; £10m; growing sectors</th>
<th>p'K_{t-1} &gt; £10m; declining sectors</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number of firms</td>
<td>Number of observations</td>
<td>Δ(I/K)_{1,t-1}</td>
<td>ΔQ_{1t}</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 1</td>
<td>157</td>
<td>859</td>
<td>0.2222 (0.0674)</td>
<td>0.0086 (0.0080)</td>
</tr>
<tr>
<td>Case 2</td>
<td>298</td>
<td>1775</td>
<td>0.1246 (0.0454)</td>
<td>0.0142 (0.0056)</td>
</tr>
<tr>
<td>Case 3</td>
<td>132</td>
<td>1356</td>
<td>0.0614 (0.0145)</td>
<td>0.0299 (0.0145)</td>
</tr>
<tr>
<td>Case 4</td>
<td>279</td>
<td>2556</td>
<td>0.1149 (0.0413)</td>
<td>0.0061 (0.0030)</td>
</tr>
<tr>
<td>Δ(X/K)_{1,t-1}</td>
<td>0.2719 (0.0648)</td>
<td>0.1786 (0.0400)</td>
<td>0.3234 (0.0683)</td>
<td>0.2055 (0.0433)</td>
</tr>
<tr>
<td>m2</td>
<td>-3.05</td>
<td>-1.24</td>
<td>-0.66</td>
<td>0.02</td>
</tr>
<tr>
<td>WT</td>
<td>39.8 (15)</td>
<td>55.8 (15)</td>
<td>30.9 (15)</td>
<td>48.5 (15)</td>
</tr>
<tr>
<td>Sargan</td>
<td>67.1 (72)</td>
<td>85.8 (72)</td>
<td>82.2 (72)</td>
<td>89.3 (72)</td>
</tr>
</tbody>
</table>

Notes.

1. See notes to Table 4.4
2. Growing sectors are: chemicals and man-made fibres, electrical and instrument engineering and food, drink and tobacco. Declining sectors are: metals and metal goods, other minerals and mineral products, mechanical engineering, motor vehicles and parts and other transport equipment, textiles, clothing, leather and footwear and other industries.
The perhaps surprising result from Table 4.7 is that the coefficient on cash flow is greater for firms operating in growing sectors. This is true even if the long run impact of cash flow is considered. This result is not sensitive to the instrument set used. One explanation for this effect may be that the lower investment rate of firms in declining sectors dominates empirically their lower cash flow and their higher agency costs which, ceteris paribus, would be expected to arise. The table indicates that the impact of Q is mixed: among small firms it is more important for firms in declining sectors but among large firms it is more important for firms in growing sectors.

Table 4.8 Split by Size and Age

<table>
<thead>
<tr>
<th>Dependent Variable Δ(I/K)_it</th>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of firms</td>
<td>99</td>
<td>574</td>
</tr>
<tr>
<td>Number of observations</td>
<td>450</td>
<td>4370</td>
</tr>
<tr>
<td>Δ(I/K)_it, t-1</td>
<td>0.0935</td>
<td>0.1939</td>
</tr>
<tr>
<td>(0.0610)</td>
<td></td>
<td>(0.0342)</td>
</tr>
<tr>
<td>ΔQ1,t</td>
<td>0.0122</td>
<td>0.0095</td>
</tr>
<tr>
<td>(0.0099)</td>
<td></td>
<td>(0.0066)</td>
</tr>
<tr>
<td>Δ(X/K)_it, t-1</td>
<td>0.2720</td>
<td>0.2242</td>
</tr>
<tr>
<td>(0.0662)</td>
<td></td>
<td>(0.0302)</td>
</tr>
<tr>
<td>m2</td>
<td>-1.57</td>
<td>-0.97</td>
</tr>
<tr>
<td>WT</td>
<td>36.7 (15)</td>
<td>88.4 (15)</td>
</tr>
<tr>
<td>Sargan</td>
<td>48.3 (72)</td>
<td>100.7 (72)</td>
</tr>
<tr>
<td>Instruments</td>
<td>I/K(2,1), Q(2,2), X/pK(2,1), Y/pK(2,1)</td>
<td></td>
</tr>
</tbody>
</table>

Notes. See notes to Table 4.4
The final issue to be explored is the effect of age on the relevance of cash flow. Table 4.8 reports the results obtained when, excluding large firms, we distinguish between firms that have been quoted for more or less than twelve years. Twelve years may seem rather long, but it is imposed by the necessity of having enough observations in the "new" firms category for the purposes of estimation. The results suggest that cash flow is somewhat more important for new firms, although the differences between the two categories are not large. Once again, it should be noted that the category of new firms is very small, and that the variables consequently tend to be less significant.

4.7 Conclusions

The results presented in this chapter suggest that in all cases cash flow is significantly associated with investment. Stock measures of liquidity do not play an important empirical role. The stock of debt does appear to have a negative impact on investment, although the significance of this term depends on the size of the sample. The performance of Q is mixed. While it plays a significant role in the full sample, there are subsamples, typically of small firms, in which it does not appear to have an independent effect on investment. The results for the full sample over different time periods suggest that the significance of cash flow is not due solely to the fact that, in proxying for demand, it is a better measure of fundamentals than Q, nor simply that it contains new information not captured by beginning of period Q, although more research is needed on this issue.

Cash flow does appear to differ across firms in the magnitude of its impact on investment. In particular, it appears to play a more important role for large firms than for small firms. While this may be surprising at first sight, there are several reasons why this effect might be observed. For example, it may reflect the fact that large firms tend to have a lower relative cash flow. In addition, it may reflect the possibility that large firms have a more diverse ownership structure, which tends to increase agency costs. Given size, the effect of cash flow tends to be larger for firms in growing sectors, contrary to what
one would expect since collateralizable net worth is likely to be larger in this case and the risk of bankruptcy lower. However, firms in growing sectors need to finance a higher rate of investment. Finally, when firms are classified according to age, it appears that cash flow matters somewhat more for newer firms, as would be expected since information asymmetries are likely to be larger for such firms, and they need to finance a higher investment rate.


Anderson G. (1987) 'Is the cost of capital really independent of the level of investment and the financial structure of the firm?', mimeo, University of Toronto.


Inland Revenue (1982) 'The computer simulation model for forecasting and costing corporation tax in the United Kingdom'.


OECD (1985) Two reports by the Committee on Fiscal Affairs on Quantitative Aspects of Corporate Taxation, Paris: OECD.


APPENDIX A

DESCRIPTION OF DATA

This appendix describes the calculation of the main variables used in this thesis, and provides some summary statistics. There are two sources of data for company information: company accounting records from Datastream and share price and related data from the London Share Price Database. A sample of 729 companies, whose principal activity was in manufacturing was taken from those companies which could be identified as being available from both data sources. Companies were required to have at least 4 continuous years of data. The companies were allocated to 9 industry groups as shown in Table A.1.

<table>
<thead>
<tr>
<th>Industry</th>
<th>SIC classes</th>
<th>Number of companies</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Metals and metal goods</td>
<td>21, 22, 31</td>
<td>43</td>
</tr>
<tr>
<td>2 Other minerals and mineral products</td>
<td>23, 24</td>
<td>34</td>
</tr>
<tr>
<td>3 Chemicals and man-made fibres</td>
<td>25, 26</td>
<td>54</td>
</tr>
<tr>
<td>4 Mechanical engineering</td>
<td>32, 33</td>
<td>162</td>
</tr>
<tr>
<td>5 Electrical and instrument engineering</td>
<td>34, 37</td>
<td>87</td>
</tr>
<tr>
<td>6 Motor vehicles, parts, other transport</td>
<td>35, 36</td>
<td>27</td>
</tr>
<tr>
<td>7 Food, drink and tobacco</td>
<td>41, 42</td>
<td>76</td>
</tr>
<tr>
<td>8 Textiles, clothing, leather and footwear</td>
<td>43, 44, 45</td>
<td>128</td>
</tr>
<tr>
<td>9 Other</td>
<td>46, 47, 48, 49</td>
<td>118</td>
</tr>
</tbody>
</table>

Notes
1. 'Other' includes 'timber and wooden furniture', 'paper and publishing', 'rubber and plastics' and 'other manufacturing'.

The number of years of data available for each of the companies is shown in Table A.2.
### Table A.2 Sample of companies by period of data

<table>
<thead>
<tr>
<th>Number of records per company</th>
<th>Number of companies</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>19</td>
</tr>
<tr>
<td>5</td>
<td>13</td>
</tr>
<tr>
<td>6</td>
<td>17</td>
</tr>
<tr>
<td>7</td>
<td>22</td>
</tr>
<tr>
<td>8</td>
<td>20</td>
</tr>
<tr>
<td>9</td>
<td>35</td>
</tr>
<tr>
<td>10</td>
<td>64</td>
</tr>
<tr>
<td>11</td>
<td>174</td>
</tr>
<tr>
<td>12</td>
<td>30</td>
</tr>
<tr>
<td>13</td>
<td>44</td>
</tr>
<tr>
<td>14</td>
<td>72</td>
</tr>
<tr>
<td>15</td>
<td>88</td>
</tr>
<tr>
<td>16</td>
<td>125</td>
</tr>
</tbody>
</table>

The variables used in the main part of this thesis were constructed as follows; Datastream numbers are referred to as, for example, D435.

**Investment**: Total new fixed assets (D435). This item includes both purchase of new assets and fixed assets of new subsidiaries: the split between these two is rarely available.

**Market value**: An average price for the the three months preceding the accounting year is constructed from monthly observations (on the last day of trading in each month) on share prices from the London Share Price Database. This average price is then multiplied by the total number of issued shares at the beginning of the accounting year. Adjustments sometimes need to be made for new share issues.

**Output**: A proxy is taken by using total sales from Datastream (D101), deflated by the output price.

**Cash Flow**: This is a measure of the total funds available for investment and distributions and is therefore defined as the sum of accounting profit before dividends (D182) and the accounting depreciation charge.
Debt: Long term debt is taken to include any loan with repayment due in excess of one year (D321).

Replacement value of the capital stock: For most of the period considered, accounts gave only historic cost information regarding the value of the capital stock. Historic cost information on the historic cost value of the capital stock split into plant and machinery (D328) and buildings (D327) is first used to estimate the proportion of investment in each category, as follows:

\[
I_{t}^{P} = I_{t}^{I} \left( \frac{HK_{t}^{P} - HK_{t-1}^{P}}{HK_{t}^{P} - HK_{t-1}^{P} + HK_{t}^{B} - HK_{t-1}^{B}} \right) \tag{A.1}
\]

\[
I_{t}^{B} = I_{t}^{I} - I_{t}^{I}^{P}
\tag{A.2}
\]

where \(I_{t}^{I}\) is the price of capital goods, \(I_{t}^{I}\) is total investment, \(I_{t}^{P}\) is investment in plant and machinery and \(I_{t}^{B}\) is investment in buildings, all in period t, and \(HK_{t}^{P}\) is the historic cost value of plant and machinery and \(HK_{t}^{B}\) the historic cost value of buildings both at the end of period t.

The replacement cost value of the capital stock is calculated on a perpetual inventory method, ie:

\[
RK_{t}^{P} = (1 - \delta^{P})RK_{t-1}^{P} \frac{I_{t}^{I}}{p_{t}} + \frac{I_{t}^{I} P}{p_{t}}
\tag{A.3}
\]

\[
RK_{t}^{B} = (1 - \delta^{B})RK_{t-1}^{B} \frac{I_{t}^{I}}{p_{t}} + \frac{I_{t}^{I} B}{p_{t}}
\]

where \(RK_{t}\) is the replacement cost value of the capital stock at the end of period t, \(\delta^{P}\) is the economic depreciation rate for plant and machinery and \(\delta^{B}\) is the economic depreciation rate for buildings. Economic depreciation rates are taken from King and Fullerton (1984), namely 8.19% for plant and machinery and 2.5% for buildings. This method requires an initial valuation of the capital stock. For this an
adjustment is made to the historic cost value in the first year of data, multiplying by a factor reflecting price changes over the average life of the capital stock (taken to be three years). This is similar to the method used by King and Mairesse (1983), and also suggested by Wadhwani and Wall (1986). To reduce the impact of this rather arbitrary assumption on the estimates, the first three years of data (1968 to 1970) are excluded from the sample used in estimation.

Other assets: Other assets, including stocks and work in progress and financial assets, must be included in the definition of Q; they must either be deducted from the numerator of Q (so that the numerator is a measure of the market value of the capital stock) or deducted from the denominator (so that the denominator is a measure of the replacement cost value of the entire company). Since we are concerned here with the capital stock, these variables are deducted from the numerator of Q.

Price indices: The output prices \( p^Y_t \) are implicit value-added price deflators for each of the 9 sectors of manufacturing industry. They were constructed from current price GDP and constant price GDP figures published by industry in various Blue Books. The price of investment goods \( p^I_t \) was derived as an implicit price deflator for gross fixed investment by manufacturing industry, taken from data in Economic Trends Annual Supplement (1988).

Table A.3 presents some descriptive statistics of the relevant variables. It shows (in two parts) the average value of each variable across the whole of the sample in each year. The table closely follows accounting conventions, dividing the variables according to where they are found in company accounts. The second row of the table indicates how many companies are available in each year. Note that there is a significant jump in the size of the sample after 1975. This is because this was the first year for which the London Share Price Database provided the entire population of UK quoted companies. It should also be noted that accounting records are included in the average of the year in which the accounting year end falls. All figures are in 1980 prices.
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>No of companies</td>
<td>374</td>
<td>409</td>
<td>431</td>
<td>655</td>
<td>673</td>
<td>685</td>
<td>693</td>
</tr>
</tbody>
</table>

**Balance sheet assets**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital stock</td>
<td>113.7</td>
<td>115.1</td>
<td>114.9</td>
<td>79.8</td>
<td>81.3</td>
<td>82.8</td>
<td>88.4</td>
</tr>
<tr>
<td>Stock and WIP</td>
<td>47.6</td>
<td>56.4</td>
<td>51.0</td>
<td>34.3</td>
<td>33.4</td>
<td>31.9</td>
<td>32.2</td>
</tr>
<tr>
<td>Intangible assets</td>
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**Balance sheet liabilities**

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**Profit and loss variables**

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**Sources and uses of funds variables**

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**Other variable**

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**Notes**

1. All variables are taken from Datastream accounting data apart from market value and Q. The capital stock estimates (of the replacement cost) are derived from the accounting data as described in the text.
2. All figures are in 1980 prices.
Table A.3(b) Variable means by year; Whole sample, 1980-1986  £m1980

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**Balance sheet assets**

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**Balance sheet liabilities**

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**Profit and loss variables**

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**Sources and uses of funds variables**

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**Other variable**

| Q       | -0.4 | -0.5 | -0.5 | -0.3 | -0.2 | -0.1 | -0.2 |

**Notes**

1. See notes to Table A.3(a).
The variation in the size of the sample over time in Table A.3 makes it difficult to assess the average movement of each variable over time. Table A.4 therefore gives similar statistics for a balanced panel of 274 companies which have data for every year between 1973 and 1986.

**Table A.4(a) Variable means by year; panel of 274 companies 1973-1979**

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**Notes**
1. See notes to Table A.3(a).
### Table A.4(b) Variable means by year; panel of 274 companies 1980-1986

£m1980

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<td>28.5</td>
<td>27.4</td>
<td>27.0</td>
<td>26.4</td>
<td>26.6</td>
<td>27.6</td>
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<tr>
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<td>9.2</td>
<td>8.8</td>
<td>13.1</td>
<td>14.9</td>
<td>14.6</td>
<td>13.9</td>
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<td>280.4</td>
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<td>293.7</td>
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<td>5.5</td>
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<td>1.6</td>
<td>1.4</td>
<td>4.3</td>
<td>4.4</td>
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<td>3.2</td>
<td>1.4</td>
<td>1.2</td>
<td>0.5</td>
<td>2.4</td>
<td>3.3</td>
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<td>2.1</td>
<td>0.6</td>
<td>0.8</td>
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<tr>
<td>New bank debt</td>
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<td>1.7</td>
<td>0.7</td>
<td>0.9</td>
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<td>18.6</td>
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<td>17.4</td>
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<td>23.3</td>
<td>25.5</td>
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<tr>
<td>Q</td>
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<td>-0.2</td>
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**Notes**

1. See notes to Table A.3(a).
This section describes in more detail the most important features of the UK corporation tax system, and describes how they are treated by the model briefly described in Chapter 1. The aim of the model is to provide estimates of corporation tax and ACT liabilities for the sample of companies considered, together with estimates of the relative size of the various reliefs and allowances and estimates of the degree to which companies in the data have faced either full tax exhaustion or ACT exhaustion. None of this data is readily available from company accounts. The main reason for the absence of tax liabilities is that profit and loss statements include a provision for taxation which may include "deferred" tax, rather than simply the tax due in the current period. This deferred tax arose in particular because the generous capital allowances of the 1970s and early 1980s exceeded the depreciation rates normally charged in accounts. Accountants attempted to estimate the tax that would have been due had capital allowance rates equalled accounting depreciation rates, and this was the basis of the charge against profit.

The approach taken is to apply, as far as possible, the actual rules of the tax system to company accounting data on such items as profit, investment and dividend payments. This approach has several benefits: estimates of the importance of different provisions within the tax system can be provided, and, in particular, estimates of the size of losses carried forward and unrelieved ACT are made. In addition, the model can be used to simulate the effects of tax reform, although in its basic form, the model cannot make any allowance for behavioural change as a result of reform. It is obvious that the model cannot fully represent the whole of the corporation tax system - for example, only two types of fixed asset are dealt with: plant and machinery and industrial buildings. Thus, other assets with special regimes, such as hotels, oil wells or motor vehicles are not treated separately. However, modelling of such detailed parts of the tax system would be wholly
unrealistic due to lack of data.

The model, and this description are divided into six parts, corresponding to the main areas of the system. They are: capital allowances, stock relief, taxable profit, ACT together with the imputation system and income tax, double taxation relief and overseas earnings, and the payments mechanism. In most of these cases, the model mirrors the actual rules of the tax system as closely as possible. In the last two areas, however, data problems or deviations in practice from the precise rules of the system provide too great an obstacle for such an approach. In these cases various estimates are made to overcome the problems. In each part we outline the rules of the tax system and provide an algebraic listing of the relevant parts of the model. A complete variable listing is provided following the equations.

This section draws on Mayer and Morris (1982), which described an earlier version of the corporation tax model. Various amendments have been undertaken since the earlier paper. These have arisen partly because of changes to the tax system and partly because of other improvements which have been made in attempts to model the system more precisely. The main changes from the earlier working paper are: the new system of stock relief from 1981; various provisions arising out at the 1984 reforms; the modelling of payments and taxation of overseas earnings; and, the extension of the losses carried back provisions. In addition, parts of the model relating to investment grants and capital gains have been dropped.

It should be mentioned that an implicit assumption needs to be made regarding group relief. This is that any group of companies so arranges its affairs in order to qualify for maximum relief in any year. This means that losses in some companies can immediately be offset against profits in other companies within the same group. This seems reasonable, and is also the only assumption that could realistically be made given that the data describes only company groups.

1. Capital Allowances

Since 1945 there have been a wide variety of schemes within the tax
system for encouraging investment in manufacturing industry. While some incentives have taken the form of grants both national and restricted to particular regions of the country, the tax system has had various tax allowances, the main ones being in respect of investment in plant and machinery and industrial buildings.

Considering first expenditure on plant and machinery, incentives have included initial allowances, investment allowances, first-year allowances and writing down allowances. The first three are a tax deduction on a certain percentage of the initial capital expenditure, which is treated as a deductible expense in computing taxable profits. The only difference between them comes in the determination of writing down allowances - the tax provisions for depreciation. In the case of initial allowances and first-year allowances the expenditure on which the depreciation provisions is based is computed net of the allowance, whereas investment allowances are not subtracted from the purchase price. Investment allowances operated in place of initial allowances between 1954 and 1956 and in addition to investment allowances between 1959 and 1966 and during those periods the effective depreciation provision was thus enhanced. In 1972, first-year allowances replaced both investment and initial allowances, but they were phased out between 1984 and 1986. Below, IVAP_t denotes the rate of initial of first year allowance on purchases of plant and machinery in period t and ITAP_t the rate of investment allowance on purchases of plant and machinery in period t.

Writing down allowances are determined by the tax depreciated value of the investment in plant and machinery. An investment is viewed as contributing to a pool of plant and machinery expenditure (PP_t) which is depreciated at the rate assumed for tax purposes (d^P_t) and carried forward to the next period. By applying the stipulated depreciation rate to the pool of expenditure the writing down allowance is derived. If an asset is sold then a balancing charge is applied to the difference between the tax written down value of the asset and the amount for which it is sold. This will reduce the firm's initial and investment allowances by the value of the sale over the amount carried forward in the pool. The balancing charge thus presents no difficulty provided that investment in plant and machinery is net of disposals. This is precisely...
the way in which it is recorded in company accounts.

Buildings have also enjoyed initial and investment allowances in the post World War II period (at rates denoted ITAB_t and IVAB_t, respectively). Depreciation allowances for buildings are calculated on a straight-line basis over the life of the asset assumed for tax purposes, denoted T. This life is computed net of initial allowances but gross of investment allowances. Equations (B.1) to (B.5) below describe the model calculations regarding capital allowances. A full variable listing is given at the end of the Appendix.

a) Plant and Machinery:

\[ CA_t^P = (IVAP_t + ITAP_t)I_t^P + d_t^P PP_t \]  
\[ PP_t = PP_{t-1} + I_t^P (1 - ITAP_t) - d_t^P PP_{t-1} \]  

(b.2)

b) Buildings:

\[ CA_t^B = (IVAB_t + ITAB_t)I_t^B + \sum_{i=1}^{T_t} d_t^B I_t^B \]  
for \( T_t \geq t \),

where \( T_t = \frac{1 - ITAB_t}{d_t^B} \)

c) Total capital allowances:

\[ CA_t = CA_t^P + CA_t^B \]  

(B.4)

2. Stock Relief

There were a variety of provisions for stock relief (SR_t) from its inception in 1973 to its abandonment in 1984. The general structure remained unchanged until 1981. During this period companies could treat as an allowable expense the increase in the value of their stockholdings less
a certain proportion ($\beta$) of their income ($\Pi_t$) for that period. For the years 1973 and 1974 this proportion was set at 10% but thereafter was increased to 15%. In the first two years income was defined as gross of capital allowances but subsequently net of allowances.

Stock relief was not actually introduced until the Finance Act 1975, where it was made retrospective for accounting years ending in 1973/4 and 1974/5. This raises problems for modelling when the relief actually affected a firm's cash flow. There is evidence that some relief was received for mainstream tax payments in early 1975. However, the system was not made permanent until 1976, and it is probable that many firms claimed stock relief for earlier years only at this stage. These problems relate to the payment, as opposed to the liability, of mainstream CT, and are discussed in more detail below.

In the event of a fall in the value of stocks ($S_t$) a company was liable to a clawback on its previous relief. This, prior to 1979, was limited to the lower of the previous relief unrecovered ($\text{URSR}_t$ denotes relief in period $a$ unrecovered by period $t$) and the fall in the value of stocks ($S_t - S_{t-1}$). From 1979, however, relief given in 1973 and 1974 was deemed unrecoverable and recoverable relief was restricted to a six year period preceding the accounting year. $T$ in equations (B.8) to (B.14) is thus equal to 4 in 1979 and 5 in 1980. Earlier years' relief are recovered before later years so that the series of equations (B.8) to (B.14) determine the extent of any unrecovered stock relief and the amount of clawback that can be recovered. In equation (B.6) clawback is then diminished by the amount that remains unrecovered ($\text{USRCBT}_t$). In addition post 1979 companies could defer the clawback less 5% of the opening stock. This deferral is shown as $\text{DEFCB}_t$ in (B.6).

The system was changed completely in November 1981. Stock relief was then set at the opening value of stock less £2000, multiplied by the increase in the all stocks index (ASI). Clawback and deferred clawback were abolished, although any clawback already deferred remained payable. Finally, stock relief was completely abolished in 1984, removing that protection against inflation.
a) 1973 to 1981:

\[ SR_t = \lambda \max\{S_t - S_{t-1} - \beta (T_t - CA_t), 0\} \]

\[ + (1 - \lambda)(S_t - S_{t-1} + USRCBT_t + \Gamma \cdot DEFCB_t) - DEFCB_{t-1} \]  \hspace{1cm} (B.6)

where \( DEFCB_t = \max\{- (S_t - S_{t-1} + USRCBT_t + 0.05S_{t-1}), 0\} \) \hspace{1cm} (B.7)

and \( \lambda = \begin{cases} 
1 & \text{if } S_t - S_{t-1} \geq 0 \\
0 & \text{if } S_t - S_{t-1} < 0
\end{cases} \)

\( \beta = \begin{cases} 
0.1 & \text{if } 1973 \leq t \leq 1974 \\
0.15 & \text{if } 1975 \leq t \leq 1980
\end{cases} \)

\( \gamma = \begin{cases} 
0 & \text{if } t=1973 \text{ or } t=1974 \\
1 & \text{if } 1975 \leq t \leq 1980
\end{cases} \)

\( \Gamma = \begin{cases} 
0 & \text{if } 1973 \leq t \leq 1978 \\
1 & \text{if } 1979 \leq t \leq 1980
\end{cases} \)

and USRCBT is computed from the following set of equations

\[ URSR_{t-T} = \max\{URSR_{t-T-1} + (1-\lambda)(S_t - S_{t-1})\} \]  \hspace{1cm} (B.8)

\[ USRCB1 = \max\{- (1-\lambda)(S_t - S_{t-1}) - URSR_{t-T-1}, 0\} \]  \hspace{1cm} (B.9)

\[ URSR_{t-T+1} = \max\{URSR_{t-T+1} - USRCB1, 0\} \]  \hspace{1cm} (B.10)

\[ USRCB2 = \max\{USRCB1 - URSR_{t-T+1}, 0\} \]  \hspace{1cm} (B.11)

\[ \vdots \]

\[ URSR_{t-1} = \max\{URSR_{t-1} - USRCB(T-1), 0\} \]  \hspace{1cm} (B.12)

\[ USRCBT = \max\{USRCB(T-1) - URSR_{t-1}, 0\} \]  \hspace{1cm} (B.13)

\[ URSR_t = \lambda ASR_t \]  \hspace{1cm} (B.14)
where $\Gamma = \begin{cases} 
1 & \text{if } t \leq 1982 \\
0 & \text{if } t > 1983 
\end{cases}$

b) 1981 to March 1984:

$$SR_t = (S_{t-1} - 2,000) \left( \frac{ASI_t}{ASI_{t-1}} - 1 \right) - \Gamma.DEFB_{t-1}$$  \hspace{1cm} (B.15)

where $\Gamma = \begin{cases} 
1 & \text{if } t \leq 1982 \\
0 & \text{if } t > 1983 
\end{cases}$

c) From April 1984:

$$SR_t = 0$$  \hspace{1cm} (B.16)

3. Taxable Profit and Corporation Tax Liabilities

Taxable profits (denoted $\Pi_t$) are defined as gross income and chargeable gains less allowable expenses. The expenses permitted for tax purposes do not correspond exactly to those used in establishing accounting profits. For example the entertainment of UK customers, political donations and gifts do not qualify as expenses. In addition, capital gains are included although effectively taxed after 1973 at a lower rate. But of considerably more importance than these are the additional items which can be treated as deductions against tax; these include capital allowances, stock relief and interest payments ($r^P_t$). The basic definition of taxable profits in equation (B.17) is therefore operating profit plus other income (denoted $\Pi_t$) plus net interest received ($r^N_t-r^P_t$) less stock relief and capital allowances. Other items are omitted through lack of data.

The corporation tax rate is applied to taxable profits only if they are positive ($\Pi_t$ denotes the amount due to be taxed in period $t$). If there is a net taxable loss ($\Pi_t < 0$), it can then be either carried back to
the previous year and set against last year's taxable profits, or failing that carried forward to the next year. Losses carried forward in period \( t \) are denoted \( LCF_t \) and losses carried back are denoted \( LCB_t \). Thus, (B.19) shows that the value of losses offset against the profits of the previous period \( (LCBA_{t-1}) \) may be no greater than the previous year's taxable profit. After subtracting the loss carried back, taxable profits from the last year are recomputed in the light of period \( t \) information (denoted \( TII_t \)) as shown in (B.20). If taxable profits of an earlier year are reduced by losses carried back, then that year's tax is recomputed, including any ACT set off and carry back (see below).

In addition, from 1972, taxable losses due to capital allowances for plant and machinery can be carried back up to three years. This is shown in equations (B.21), (B.22) and (B.23), where \( LCBCA_{t-1} \) is the amount of losses due to capital allowances which can be carried back to periods \( t-2 \) and \( t-3 \). This additional carry-back is then compared with any remaining taxable profit from those periods in order to determine the level of any additional offset. Thus, for \( n=2 \) and \( 3 \), equations (B.24), (B.25) and (B.26) summarise the amount that can be offset against taxable profit of earlier periods \( (LCBA_{t-n}) \) and the value of any remaining taxable profit from those periods not used in this way \( (TII_{t-n}) \). Total losses carried back from period \( t \) \( (LCB_t) \) are defined in (B.27) as the sum of \( LCBA_{t-n} \), for \( n=1,2 \) and \( 3 \). Hence:

**a) Taxable profits and losses carried back:**

\[
\Pi_t^t = \Pi_t + P_{t-1}^{t-1} - d_{t-1} - LFC_{t-1} - \min(\min(\max(d_{t}, 0), EICF_{t-1} - LCF_{t-1}, 0)) \] (B.17)

\[
\Pi_t = \max(\Pi_t^t, 0) \] (B.19)

if \( \Pi_t < 0 \) then

\[
LCBA_{t-1} = \min(\Pi_t, TII_{t-1}) \] (B.19)

\[
TII_{t-1} = \max(\Pi_t, LCB_{t-1}, 0) \] (B.20)

(1) if \( \Pi_t + CAP_t > 0 \) then \( LCBCA_{t-1} = \Pi_t^t - LCB_t \) (B.21)
(11) if $\Pi_t^t + \text{CAP}_t < 0$ and $\Pi_t^t + \text{CAP}_t < -\text{LCBA}_t$ then $\text{LCBCA}_{t-1} = \text{CA}_t$  \hspace{1cm} (B.22)

(111) if $\Pi_t^t + \text{CAP}_t < 0$ and $\Pi_t^t + \text{CAP}_t > -\text{LCBA}_t$ then $\text{LCBCA}_{t-1} = -\text{LCBA}_t - \Pi_t^t$ \hspace{1cm} (B.23)

for $n = 2, 3$:

$$\text{LCBA}_{t-n} = \min[\text{LCBA}_{t-n+1}, \text{TH}_{t-n}^{t-1}]$$  \hspace{1cm} (B.24)

$$\text{LCB}_{t-n} = \max[\text{LCB}_{t-n+1} - \text{TH}_{t-n}^{t-1}, 0]$$  \hspace{1cm} (B.25)

$$\text{TH}_{t-n}^t = \max[\text{TH}_{t-n}^{t-1} - \text{LCBA}_{t-n-1}^- - 0]$$  \hspace{1cm} (B.26)

$$\text{LCB}_t = \sum_{n} \text{LCBA}_{t-n}$$  \hspace{1cm} (B.27)

if $\text{LCB}_t > 0$, equations (B.29) to (B.56) are recomputed for each year to which losses are carried back, with $\text{TH}_{t-n}^t$ substituted by $\text{TH}_{t-n}^{t-1}$.

There is one further complication which means that losses carried forward are not simply the difference between the taxable loss and losses carried back. This is that if a company is a net recipient of dividends then the excess of receipts over payments of franked investment income (ie. net dividends received: $d^R - d^P$) can be treated as profits for the purpose of obtaining relief against trading losses. Therefore if tax losses cannot be fully relieved against past profits ($\Pi_t^t + \text{LCB}_t < 0$) then the firm will be able to realise at least a part of the remaining tax loss if dividend receipts in the current period are greater than dividend payments. The amount that the firm can realize as the tax credit is equal to the ACT rate (the basic rate of income tax) multiplied by the difference between the gross dividend receipt and the gross dividend payment (ie. in both cases gross of ACT). The repayment on the residual loss will therefore be the lower of the loss and the net dividend receipts (EFI) multiplied by the basic income tax rate ($\tau_p$), as shown in equation (B.29) and the last part of equation (B.31). The remaining loss which the firm can neither carry back nor set against net dividend receipts is carried forward to next year as shown in the first part of (B.28).
The dividend provision is a means of bringing forward the tax credit due on net dividend receipts to a period in which a company is incurring a trading loss. It is not therefore in practice a means of realizing the taxable loss at all despite the fact that losses carried forward are reduced accordingly. As a consequence when the firm becomes a net payer of dividends once again, the trading loss is reestablished at a rate equal to the excess of dividend payments until the original loss offset has been exhausted (see the last bracket in equation (B.28). The cumulative amount of net dividend receipts at any point in time which a company has used to claim a tax credit in the past is shown by the variable \( \text{EFICF}_t \) in equation (B.30) and the rate of increase of losses carried forward in (B.28) is restricted to the lesser of current net dividend payments and \( \text{EFICF}_t \).

If a company is earning a taxable profit \((\Pi_t > 0)\) then its corporation tax liability is obtained by applying the corporation tax rate, \( \tau_c \), to its profits provided that its taxable profits (including dividend receipts) are in excess of an upper band, \( Z^U \). For companies with taxable profits plus dividend receipts below a lower band, \( Z^L \), the small companies tax rate, \( \tau_c^L \), applies. This lower rate is applied to the taxable income of the firm which includes its profits and dividend receipts. Between the lower and upper profits levels the average corporation tax rate moves from its reduced to its full rate with the result that in that band the marginal rate of Corporation Tax exceeds \( \tau_c^L \) (and is approximately equal to \( \tau_c^L + \theta' \), where \( \theta' \) is the small companies marginal relief fraction, or SCMR). Hence:

b) Losses carried forward:

\[
\text{LCF}_t = (1-\varphi_t)(-\Pi_t - \text{LCB}_t - \text{EFI}_t) + \min(\max[d_t^L - d_t^R, 0], \text{EFICF}_{t-1})
\]

(B.28)

where \( \varphi_t = \begin{cases} 
1 & \text{if } \Pi_t > 0 \\
0 & \text{if } \Pi_t < 0 
\end{cases} \)

c) Net dividends used as loss offset:

\[
\text{EFI}_t = \min(-\Pi_t + \text{LCB}_t, \max[d_t^R - d_t^P, 0])
\]

(B.29)
\[ \text{EFICF}_t = \text{EFICF}_{t-1} + \text{EFI}_t - \min \left( \max \left( \frac{d_t^P - d_t^R}{0}, \text{EFICF}_{t-1} \right), \text{EFICF}_{t-1} \right) \quad (B.30) \]

d) Corporation tax liability before ACT and DTR offsets:

\[ \text{CTRL}_t = \varphi_t \left\{ \tau_c \frac{T_t^U - T_t^R + d_t^R}{T_t + d_t^R} \right\} + \left( 1 - \varphi_t \right) \text{EFI}_t 
\]

where \( \varphi' = \left\{ \begin{array}{ll}
\text{SMCR} & \text{if } Z^U_t < T_t^R + d_t^R < Z^U_t \\
0 & \text{otherwise}
\end{array} \right\} \]

and \( \varphi'' = \left\{ \begin{array}{ll}
1 & \text{if } T_t^R + d_t^R < Z^U_t \\
0 & \text{if } T_t^R + d_t^R > Z^U_t 
\end{array} \right\} \)

4. Advance Corporation Tax, the Imputation System and Income Tax

Before the introduction of the imputation system in the UK in 1973 companies were required to deduct income tax at the standard rate on gross dividends paid. On receiving dividends from other UK companies a firm could set the income tax associated with the dividend receipt against the amount due on its payments. If receipts exceeded payments then no income tax was repayable and the excess could be carried forward to future periods.

The above system continued to prevail after the introduction of the imputation provisions though the terminology employed and the precise definition of some of the relevant variables was revised. The payment of income tax on dividend distributions received the misleading title of advance corporation tax, the 'standard rate' of income tax had by then become the 'basic rate' and the rates of ACT payment were redefined in terms of net as opposed to the previously gross distributions. Equations (B.32) for the payment of ACT and (B.33) for unrelieved net receipts of dividends carried forward (\( \text{Sd}_t^N \)) are, however, applicable to the entire corporation tax period; the equations are defined in gross terms throughout. The net franked investment income that can be set against trading losses, \( \text{EFI}_t' \), is subtracted from the excess in determining the carry-forward to the next year.
Hence:

a) Advance Corporation Tax

\[
\text{ACT}_t = \max \{\tau_p(d_t^P - d_t^R - S_d^N), 0\} \quad \text{(B.32)}
\]

\[
S_d^N = \max (d_t^R - d_t^P - S_d^N - \text{EFI}_t, 0) \quad \text{(B.33)}
\]

The major reform in 1973 was, of course, the provision of setting payments of ACT against mainstream corporation tax liabilities thereby in theory avoiding the previous double taxation of distributions. In practice this provision is complicated by the requirement that the gross distribution that forms the basis of the ACT deduction should not exceed the taxable profits against which they are being set. Until 1984, the definition of taxable profits for this purpose included dividends received from foreign subsidiaries, denoted \(\text{FTT}_t\). Then double tax relief was allowed as a deduction before ACT, and hence ACT can now only be set off against taxable profits net of double tax relief. In addition, even before 1984, as discussed below, double tax relief was often effectively offset before ACT.

Until 1984, under the UK system a company could set its ACT payments against its mainstream liabilities of either of the two preceding years as well as the current year, although always subject to the conditions of the previous paragraph. From 1984, this was increased to include the previous six years. Relief is paid against the nearest period’s profits first before carrying back to earlier years. These provisions are depicted in the set of equations (B.36) to (B.40) which show the total offsets against each year’s liabilities (for example, \(\text{OFST}_{t-n}^t\) is the offset of the ACT liability in period \(t\) claimed against taxable profits in period \(t-n\)) and the residual amount which remains unrelieved and carried forward to the next year (\(\text{SACT}_t\)). \(\text{NOFST}_{t-n}^t\) is that part of the ACT payment in year \(t\) not offset against corporation tax in the current year or the previous \(n\) years and \(\text{FTT}_{t-n}^{t-1}\) is the taxable profit in period \(t-n\) against which ACT in period \(t\) can be set (i.e. after ACT from period \(t-1\) has been potentially set against it) which after 1984 includes gross dividends received from overseas subsidiaries.
Finally, if a company has brought forward the tax credit on a net dividend receipt from a period of a trading loss its ACT liability when it once more becomes a net distributor of dividends will be correspondingly increased. Under the imputation system if it can then offset the ACT payment against its mainstream corporation tax it will have gained from what is intended to be merely a timing facility. To avoid this possibility, the ACT that it can offset against mainstream is reduced (as shown equation (B.36)) by the amount of the tax credits that the company has claimed in the past (EFICFP in equation (B.41)).

Hence:

b) Corporation tax liability net of ACT

Pre 1973:

\[ CTL_t = CTRL_t \]  \hspace{1cm} (B.34)

Post 1973:

\[ CTL_t = CTRL_t - (SACT_{t-1} + Act_t - SACT_t) + \min(Act_t, EFICFP_{t-1}) \]  \hspace{1cm} (B.35)

where SACT_t is computed as follows:

\[ OFST_t = \min\{SACT_{t-1} + Act_t - \min[Act_t, EFICFP_{t-1}], r_p \Pi_t\} \]  \hspace{1cm} (B.36)

\[ NOFST_t = \max\{SACT_{t-1} + Act_t - \min[Act_t, EFICFP_{t-1}] - r_p \Pi_t, 0\} \]  \hspace{1cm} (B.37)

\[ FTI_t = \frac{1}{r_p} \max\{r_p \Pi_{t-1} - SACT_{t-1} - Act_t + \min[Act_t, EFICFP_{t-1}], 0\} \]  \hspace{1cm} (B.38)

and, more generally, for \( n \leq m \)

\[ OFST_{t-n} = \min\{OFST_{t-n-1} + NOFST_{t-n-1}, r_p \Pi_{t-n-1}\} \]  \hspace{1cm} (B.39)

\[ NOFST_{t-n} = NOFST_{t-n+1} - [OFST_{t-n} - OFST_{t-n+1}] \]  \hspace{1cm} (B.40)

\[ FTI_{t-n} = \frac{1}{r_p} (r_p \Pi_{t-n-1} - OFST_{t-n}) \]  \hspace{1cm} (B.41)
\[ SACT_t = \text{NOFSt}_{t-m} \]  

(B.42)

where \( m = \begin{cases} 2 & \text{for } t \leq 1984 \text{ Q1} \\ 6 & \text{for } t \geq 1984 \text{ Q2} \end{cases} \)

\[ EFICFP_t = EFICFP_{t-1} + EFI_t - \min (ACT_t, EFICFP_{t-1}) \]  

(B.43)

Turning to income tax, in much the same way that companies are required to deduct the personal liabilities of shareholders on dividends at source, income tax is due on certain classes of interest payments, in particular annual interest charges but not bank interest. Income tax is assessed on the net interest paid; if a company is a net recipient of the relevant interest it can set the tax credit associated with the receipt against its corporation tax liability. If the corporation tax liability is zero, then the tax credit is reclaimable (see (B.55)). Treating ACT as a payment on behalf of shareholders, the firm's total liability to income tax is shown as the sum of its interest and dividend tax obligations (see (B.56)). Since income tax is deducted after overseas earnings, the algebra is postponed until after the next section.

5. Double Taxation Relief and Taxation of Overseas Earnings

There are essentially three classes of double taxation relief (DTR). At the most general level, any company resident in the UK that receives a dividend from a non-resident company can deduct from its UK liability the overseas tax liability that has been a direct consequence of the dividend payment. But unilateral relief against the overseas taxes on the profits underlying the distribution is only available for those domestic firms that control at least 10 per cent of the voting power of the overseas company. Otherwise DTR is restricted to the increment of the overseas tax burden arising from the payment of the dividend itself. Finally there are a number of bilateral agreements with individual countries that have established specific arrangements.
In all cases, the relief is limited to the lesser of the overseas tax paid and the amount of UK tax which would otherwise have been chargeable on the foreign income. Therefore the rate of DTR is restricted to the minimum of the UK Corporation Tax rate (applicable to the firm's liabilities) and the overseas tax rate incurred on foreign activities. There is no provision for carrying forward or back DTR that cannot be claimed in the current period.

There is an important difference regarding the taxation of earning by overseas branches and overseas subsidiaries. Any branch of a UK company which is based overseas is liable to UK corporation tax on its profits in exactly the same way as if it had been in the UK. DTR is then available to reduce this liability if the branch has also been liable to foreign tax. However, a UK company is only liable to UK tax regarding an overseas subsidiary on that part of the subsidiary's earnings that is paid to the parent in the form of dividend payments. Again any UK tax liability may be reduced by DTR.

Data deficiencies cause several problems in attempting to model such a system. Some issues are as follows. First, does a company have either branches or subsidiaries based overseas? Second, if it is a subsidiary, what proportion of the group's accounting data (which is what is available) is directly due to the subsidiary, and what dividend payments has the subsidiary made to the parent? Third, how much tax has the branch or subsidiary faced abroad? Fifth, does the parent launder its DTR through a UK subsidiary so that it can claim a full amount even though it has unrelieved ACT? (This last question only arises before 1984, when ACT had to be deducted from tax liabilities before DTR. Any DTR that could not be deducted - because the mainstream tax liability was zero - was lost. A common method of overcoming this was to offset the whole DTR against a UK subsidiary's profits, thus reducing group taxable profit before offsetting the parent company's ACT payments.)

There are no clear answers to these problems. The method that the model currently uses is to assume that all overseas earnings are through a branch. Consequently the UK tax system is applied to the full group accounts. Any DTR (which is shown in the group accounts) is deductible. As noted in Devereux (1986), estimates of corporation tax liabilities
making several alternative assumptions regarding foreign activity all yielded similar results. In the absence of better accounting data it therefore seems reasonable to use the simplest approach.

Making this assumption, taxable profit does not need to be adjusted for gross dividends received by subsidiaries, and so

$$F\Pi_t = T\Pi_t$$  \hspace{1cm} (B.44)

Similarly, no adjustments need be made to the definition of the tax liability already derived other than to deduct double tax relief. Hence

$$CTA_t = \text{Max} (CTL_t - DTR_t, 0)$$  \hspace{1cm} (B.45)

Returning to income tax and the final mainstream corporation tax liabilities, equation (B.46) summarises the taxes on income (dividends and interest) withheld by the company, and (B.47) notes the reduction in the mainstream corporation tax liability due to income tax already paid on net interest receipts. The DCTI terms in (B.48) reflect changes in the liability of previous years which result from losses carried back from the current period.

**Income tax:**

$$IT_t = ACT_t + \max(\tau_p (r^P - r^R), 0)$$  \hspace{1cm} (B.46)

$$CTI_t = CTA_t - \max(\tau_p (r^R - r^P), 0)$$  \hspace{1cm} (B.47)

**Final mainstream corporation tax liability:**

$$\text{if } LCB < 0: \quad CT_t = CTI_t + DCTI_{t-1} + DCTI_{t-2} + DCTI_{t-3}$$  \hspace{1cm} (B.48)

$$\text{if } LCB \geq 0: \quad CT_t = CTI_t$$  \hspace{1cm} (B.49)
<table>
<thead>
<tr>
<th>VARIABLE LISTING</th>
<th>Description</th>
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<tr>
<td>ACT</td>
<td>Advance corporation tax</td>
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<td>ASI</td>
<td>All stocks index</td>
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<tr>
<td>CA</td>
<td>Capital allowances on plant, machinery and buildings</td>
</tr>
<tr>
<td>CAB</td>
<td>Capital allowances on buildings</td>
</tr>
<tr>
<td>CAP</td>
<td>Capital allowances on plant and machinery</td>
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<tr>
<td>CT</td>
<td>Corporation tax liability with ACT, investment income, DTR and LCB Offsets</td>
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<tr>
<td>CTA</td>
<td>Corporation tax liability with ACT and DTR offsets</td>
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<tr>
<td>CTI</td>
<td>Corporation tax liability with ACT, investment income and DTR offsets</td>
</tr>
<tr>
<td>CTL</td>
<td>Corporation tax liability net of ACT offset but gross of DTR offset</td>
</tr>
<tr>
<td>CTLR</td>
<td>Corporation tax liability gross of ACT and DTR offsets</td>
</tr>
<tr>
<td>CTP</td>
<td>Corporation tax payment due</td>
</tr>
<tr>
<td>DCTI(_{t-n})</td>
<td>Change in CTI for year (t-n) as a result of recomputation due to losses carried back</td>
</tr>
<tr>
<td>DEFCB</td>
<td>Deferral on clawback of stock relief</td>
</tr>
<tr>
<td>D(_B^)</td>
<td>Depreciation rate allowed for tax purposes on industrial buildings</td>
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<td>D(_P^)</td>
<td>Depreciation rate allowed for tax purposes on pool of plant and machinery expenditures</td>
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<tr>
<td>DTR</td>
<td>Double taxation relief</td>
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<tr>
<td>EFI</td>
<td>Net dividend receipts which are set against taxable losses to effect tax credit</td>
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<td>EFICF</td>
<td>Cumulation of past net dividend receipts on which tax credits have been paid</td>
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<td>EFICFP</td>
<td>Cumulation of past payment of tax credits</td>
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<td>d(_P^)</td>
<td>Gross payments of dividends</td>
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<td>d(_R^)</td>
<td>Gross receipts of dividends</td>
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<tr>
<td>FTII</td>
<td>Total of taxable profits on home activities and gross dividends received from overseas subsidiary</td>
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<tr>
<td>IB</td>
<td>Investment expenditure in industrial buildings</td>
</tr>
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<td>Investment expenditure in plant and machinery</td>
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<tr>
<td>IT</td>
<td>Income tax due (including ACT)</td>
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<td>ITAB</td>
<td>Initial allowance rate on industrial buildings</td>
</tr>
<tr>
<td>ITAP</td>
<td>Initial or first year allowance rate on plant and machinery</td>
</tr>
<tr>
<td>IVAB</td>
<td>Investment allowance rate on industrial buildings</td>
</tr>
</tbody>
</table>
IVAP  Investment allowance rate on plant and machinery
LCB  Total losses carried back
LCBA\textsubscript{t-n}  Losses carried back and set against profits of year t-n
LCBC\textsubscript{t-n}  Losses carried back up to three years because they are due to capital allowances
LCF  Losses carried forward to next year
NOFST\textsubscript{t-n}  ACT payments which cannot be offset against taxable income of year t-n
OFST\textsubscript{t-n}  ACT payments which can be offset against taxable income of year t-n
\Pi  Gross trading profits including 'other income'
\Pi T  Taxable profits or losses
PP  Pool of written down plant and machinery expenditure
r\textsuperscript{P}  Gross payments of interest
r\textsuperscript{R}  Gross receipts of interest
S  Closing value of stocks held
SACT  Cumulated value of ACT payments which remain unrelieved against taxable income
SCMR  Small companies rate of marginal relief
Sd\textsuperscript{N}  Net excess of receipts of dividends over payments to be carried forward to subsequent years
SR  Stock relief
\Pi T  Taxable profits - constrained to be non-negative
URSR\textsubscript{t-n}  Recoverable, as yet unrecovered stock relief periods t-n
USRCBT  Amount of destocking that remains unrecovered
Z\textsuperscript{L}  Small companies marginal relief: lower profit limit
Z\textsuperscript{U}  Small companies marginal relief: upper profit limit

PARAMETERS

\beta  Proportion of trading profits that are set against stockbuilding in determining stock relief
\gamma  Determines whether profits in stock relief computation are gross or net of capital allowances
\Gamma  Determines whether clawback deferral is operative
\phi'  Small companies rate of marginal relief
\phi''  Determines whether company is liable to tax at the small companies
rate

\( \lambda \) Dependent on whether company is stockbuilding or destocking

\( \varphi \) Dependent on whether corporation tax liability gross of DTR offset is positive or negative

\( \tau_c \) Full rate of corporation tax

\( \tau_{cL} \) Small companies corporation tax rate

\( \tau_p \) Basic rate of personal income tax (and thus advance corporation tax rate)

\( \psi \) Dependent on whether company is making taxable profit or loss