Measurement of the Quark Mixing Parameter $\cos 2\phi_1$ Using Time-Dependent Dalitz Analysis of $B^0 \rightarrow D[K_S^0 \pi^+ \pi^-]h^0$


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We present a measurement of the angle $\phi_1$ of the Cabibbo-Kobayashi-Maskawa unitarity triangle using a time-dependent Dalitz analysis of $D \rightarrow K_S^0\pi^+\pi^-$ decays produced in neutral $B$ meson decay to a neutral $D$ meson and a light meson ($\bar{B}^0 \rightarrow D^*(0)h^0$). The method allows a direct extraction of $2\phi_1$ and, therefore, helps to resolve the ambiguity between $2\phi_1$ and $\pi - 2\phi_1$ in the measurement of $\sin 2\phi_1$. We obtain $\sin 2\phi_1 = 0.78 \pm 0.44 \pm 0.22$ and $\cos 2\phi_1 = 1.87^{+0.40+0.22}_{-0.53-0.32}$. The sign of $\cos 2\phi_1$ is determined to be positive at 98.3% C.L.

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Precise determination of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements [1] is important to check the consistency of the standard model (SM) and search for new physics. The value of $\sin 2\phi_1$, where $\phi_1$ is one of the angles of the unitarity triangle, is now measured with high precision: $\sin 2\phi_1 = 0.725 \pm 0.037$ [2,3]. This leads to four solutions in $\phi_1$: $23^\circ$, $67^\circ$, $(23 + 180)^\circ$, and $(67 + 180)^\circ$. Resolution of this ambiguity has been attempted using time-dependent angular analysis in the $B^0 \rightarrow J/\psi K^{*0}(K^0_S\pi^+)\phi(2060)$ decay. This technique provides a measurement of $\cos 2\phi_1$ and therefore helps to distinguish between the solutions at $23^\circ$ and $67^\circ$ [4,5].

A new technique based on the analysis of $\bar{B}^0 \rightarrow D[K^0_S\pi^+\pi^-]h^0$ has been recently suggested [6]. Here we use $h^0$ to denote light neutral mesons, $\pi^0$, $\eta$, and $\omega$. The neutral $D$ meson is reconstructed in the $K^0_S\pi^+\pi^-$ decay mode; its resonant substructure has been measured [7,8].

Consider a neutral $B$ meson that is known to be a $\bar{B}^0$ at time $t_{\text{tag}}$. At another time, $t_{\text{sig}}$, its state is given by

$$|\bar{B}^0(\Delta t)\rangle = e^{-|\Delta t|/\tau_{\bar{B}^0}} \times \left[|\bar{B}^0\rangle \cos(\Delta m \Delta t/2) - \frac{i}{q} \frac{p}{q} \langle |\bar{B}^0\rangle \sin(\Delta m \Delta t/2)\right]$$

where $\Delta t = t_{\text{sig}} - t_{\text{tag}}$, $\tau_{\bar{B}^0}$ is the average lifetime of the $\bar{B}^0$, $\Delta m$, $p$, and $q$ are parameters of $B^0-\bar{B}^0$ mixing. Here we have assumed $CPT$ invariance and neglected terms related to the lifetime difference of neutral $B$ mesons. In the SM, $|q/p| = 1$ to a good approximation, and, in the usual phase convention, $\arg(p/q) = 2\phi_1$.

The $B \rightarrow D^0$ decay amplitude is dominated by the CKM favored $b \rightarrow c\bar{u}d$ diagram as shown in Fig. 1, with roughly a 2% contribution from the CKM suppressed $b \rightarrow u\bar{c}d$ diagram. Ignoring the latter, a neutral $D$ meson produced in a $\bar{B}^0$ decay is a $D^0$, while that produced in a $B^0$ decay is a $\bar{D}^0$. The $D$ meson state produced at time $\Delta t$ is then given by $|D^0\rangle \cos(\Delta m \Delta t/2) - i e^{2\phi_1} \xi_{\bar{B}^0}(-1)^l |\bar{D}^0\rangle \sin(\Delta m \Delta t/2)$, where we use $\xi_{\bar{B}^0}$ to denote the CP eigenvalue of $h^0$, and $l$ gives the orbital angular momentum in the $Dh^0$ system. In the case of $\bar{B}^0 \rightarrow D^*h^0$, an additional factor arises due to the CP properties of the particle emitted in the $D^*$ decay (either $D^* \rightarrow D^0\pi^0$ or $D^* \rightarrow D\gamma$) [9].

We follow Ref. [8] and describe the amplitude for a $\bar{B}^0 \rightarrow K^0_S\pi^+\pi^-$ decay as $f(m^2_{\pi^+}, m^2_{\pi^-})$, where $m^2_{\pi^+}$ and $m^2_{\pi^-}$ are the squares of the two-body invariant masses of the $K^0_S\pi^+$ and $K^0_S\pi^-$ combinations. Assuming no CP violation in the neutral $D$ meson system, the amplitude for a $D^0$ decay is then given by $f(m^2_{\pi^+}, m^2_{\pi^-})$. The time-dependent Dalitz plot density is defined by

$$P(m^2_{\pi^+}, m^2_{\pi^-}, \Delta t, q_B) = \frac{e^{-|\Delta t|/\tau_{\bar{B}^0}} F(m^2_{\pi^+}, m^2_{\pi^-})}{8\tau_{\bar{B}^0} N} \left(1 + q_B \times \left\{ |\mathcal{A}(m^2_{\pi^+}, m^2_{\pi^-})\cos(\Delta m \Delta t) + S(m^2_{\pi^+}, m^2_{\pi^-})\sin(\Delta m \Delta t)|, \right\} \right),$$

$$\mathcal{A} = (|f(m^2_{\pi^+}, m^2_{\pi^-})|^2 - |f(m^2_{\pi^+}, m^2_{\pi^-})|^2) F(m^2_{\pi^+}, m^2_{\pi^-}),$$

$$F = |f(m^2_{\pi^+}, m^2_{\pi^-})|^2 + |f(m^2_{\pi^+}, m^2_{\pi^-})|^2,$$

$$S = -\frac{2\xi_{\bar{B}^0}(-1)^l}{\text{Im} \{ f(m^2_{\pi^+}, m^2_{\pi^-})f^*(m^2_{\pi^+}, m^2_{\pi^-}) e^{2i\phi_1} \} / F(m^2_{\pi^+}, m^2_{\pi^-})},$$

$$N = \int |f(m^2_{\pi^+}, m^2_{\pi^-})|^2 dm^2_{\pi^+} dm^2_{\pi^-},$$

where the $b$-flavor charge is $q_B = +1$ (−1) when the tagging $B$ meson is a $\bar{B}^0$ ($\bar{B}^0$). Thus the phase $2\phi_1$ can be extracted from a time-dependent Dalitz plot fit to $\bar{B}^0$ and $\bar{B}^0$ data if $f(m^2_{\pi^+}, m^2_{\pi^-})$ is known. Note that this formulation assumes that there is no direct CP violation in the $B$ decay amplitudes.

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This analysis is based on $386 \times 10^6$ $B\bar{B}$ events collected with the Belle detector at the asymmetric energy $e^+e^-$ collider [10]. The Belle detector has been described elsewhere [11]. We reconstruct the decays $B^0 \to Dh^0$ for $h^0 = \pi^0, \eta$ and $\omega$ and $\bar{B}^0 \to D^*h^0$ for $h^0 = \pi^0$ and $\eta$.

Charged tracks are selected based on the number of hits and impact parameter relative to the interaction point (IP). To reduce combinatorial background, a transverse momentum of at least 0.1 GeV/$c$ is required of each track. All charged tracks that are not positively identified as electrons are treated as pions.

Neutral kaons are reconstructed via the decay $K^0_S \to \pi^+\pi^-$. The $\pi^\pm$ invariant mass is required to be within 9 MeV/$c^2$ ($\sim 3\sigma$) of the $K^0_S$ mass, and the displacement of the $\pi^+\pi^-$ vertex from the IP in the transverse ($r$-$\varphi$) plane is required to have a magnitude between 0.2 cm and 20 cm and a direction that agrees within 0.2 radians with the combined momentum of the two particles.

Photon candidates are selected from calorimeter showers not associated with charged tracks. An energy deposition of at least 50 MeV and a photonlike shape are required for each candidate. A pair of photons with an invariant mass within 12 MeV/$c^2$ ($2.5\sigma$) of the $\pi^0$ mass is considered as a $\pi^0$ candidate.

We reconstruct neutral $D$ mesons in the $K^0_S\pi^+\pi^-$ decay channel and require the invariant mass to be within 15 MeV/$c^2$ ($2.5\sigma$) of the nominal $D^0$ mass. $D^0$ candidates are reconstructed in the $D^0\pi^0$ decay channel. The mass difference between $D^{*0}$ and $D^0$ candidates is required to be within 3 MeV/$c^2$ of the expected value ($3\sigma$). $\omega$ candidates are reconstructed in the $\pi^+\pi^-\pi^0$ decay channel. Their invariant mass is required to be within 20 MeV/$c^2$ ($2.5\sigma$) of the $\omega$ mass. We define the angle $\theta_\omega$ between the normal to the $\omega$ decay plane and opposite of the $B$ direction in the rest frame of $\omega$ and require $|\cos \theta_\omega| > 0.3$. We reconstruct $\eta$ candidates in the $\gamma\gamma$ and $\pi^+\pi^-\pi^0$ final states and require the invariant mass to be within 10 and 30 MeV/$c^2$ ($2.5\sigma$) of the $\eta$ mass, respectively. The photon energy threshold for the prompt $\pi^0$ and $\eta$ candidates coming from $B$ decays is increased to 200 MeV in order to reduce combinatorial background. We remove $\eta$ candidates if either of the daughter photons can be combined with any other photon with $E_\gamma > 100$ MeV to form a $\pi^0$ candidate.

We combine either $D$ and $h^0 = \{\pi^0, \omega, \eta\}$ or $D^*$ and $h^0 = \{\pi^0, \eta\}$ to form $B$ mesons. Signal candidates are identified by their energy difference in the center-of-mass (c.m.) system of the $Y(4S)$, $\Delta E = (\sum \not{p}_f) - E_{\text{beam}}$, and the beam-energy constrained mass, $M_{bc} = \sqrt{(E_{\text{beam}} - \sum \frac{\not{p}_i}{2})^2}$, where $E_{\text{beam}}$ is the beam energy and $\not{p}_i$ and $E_f$ are the momenta and energies of the decay products of the $B$ meson in the c.m. frame. The masses of $\pi^0$, $\eta$, and $D^{(*)}$ candidates are constrained to their nominal values to improve $\Delta E$ resolution. We select events with $M_{bc} > 5.2$ GeV/$c^2$ and $|\Delta E| < 0.3$ GeV, and define the signal region to be $5.272 \text{ GeV}/c^2 < M_{bc} < 5.287 \text{ GeV}/c^2$, $-0.1 \text{ GeV} < \Delta E < 0.06 \text{ GeV} (\pi^0)$, $\omega \to \gamma\gamma$), or $|\Delta E| < 0.03$ GeV ($\omega, \eta \to \pi^+\pi^-\pi^0$). In cases with more than one candidate in an event, the one with $D$ and $h^0$ masses closest to the nominal values is chosen.

To suppress the large combinatorial background dominated by the two-jetlike $e^+e^- \to q\bar{q}$ continuum process, variables that characterize the event topology are used. We require $|\cos \theta_{\text{thr}}| < 0.80$, where $\theta_{\text{thr}}$ is the angle between the thrust axis of the $B$ candidate and that of the rest of the event. This requirement eliminates 77% of the continuum background and retains 78% of the signal. We also construct a Fisher discriminant, $F$, which is based on the production angle of the $B$ candidate, the angle of the $B$ candidate thrust axis with respect to the beam axis, and nine parameters that characterize the momentum flow in the event relative to the $B$ candidate thrust axis in the c.m. frame [12]. We impose a requirement on $F$ that rejects 67% of the remaining continuum background and retains 83% of the signal.

Signal yields and background levels are determined by fitting distributions in $\Delta E$ for candidates in the $M_{bc}$ signal region. For each mode, the $\Delta E$ distribution is fitted with an asymmetric Gaussian for signal and a linear function for background. The signal shape is fixed, based on MC simulation. The region $\Delta E < -0.1$ GeV is excluded from the fit to avoid contributions from other $B$ decays. The results from our fits to the data are shown in Fig. 2 and Table I. We study the systematic error of the fit by varying the shapes for signal and background and changing the fit range. The difference in the signal yields does not exceed 5%. We also confirm that there is no feed across between channels and other peaking background by using generic $B\bar{B}$ MC calculations.

The signal $B$ decay vertex is reconstructed using the $D$ trajectory and the IP constraint. The tagging $B$ vertex is obtained with well-reconstructed tracks not assigned to the signal $B$ candidate and the IP constraint [13]. The time difference between signal and tagging $B$ candidates is calculated using $\Delta t = \Delta z / \gamma \beta c$ and $\Delta z = z_{CP} - z_{\text{tag}}$. The proper-time interval resolution function $R_{\text{tag}}(\Delta t)$ is formed by convoluting four components: the event-by-event detector resolutions for $z_{CP}$ and $z_{\text{tag}}$, the shift in the $z_{\text{tag}}$ vertex position due to secondary tracks originating from $B\bar{B}$ decays.
from charmed particle decays, and the kinematic approximation that B mesons are at rest in the c.m. frame [13]. A small component of broad outliers in the \( \Delta z \) distribution, caused by misreconstruction, is represented by a Gaussian function. Charged leptons, pions, kaons, and \( \Lambda \) baryons that are not associated with a reconstructed \( \bar{B}^0 \to D[K^0 \pi^+ \pi^-]B^0 \) decay are used to identify the \( b \) flavor of the accompanying \( B \) meson. The tagging algorithm is described in detail elsewhere [14].

We perform an unbinned time-dependent Dalitz plot fit. The negative logarithm of the unbinned likelihood function is minimized:

\[
-2 \log L = -2 \sum_{i=1}^{n} \log \left[ (1 - f_{bg}) P_{\text{sig}} + f_{bg} P_{bg} \right]. \tag{3}
\]

where \( n \) is the number of events. The function \( P_{\text{sig}}(m_B^2, m^2, \Delta t) \) is the time-dependent Dalitz plot density for the signal events, which is calculated according to Eq. (2) and incorporates reconstruction efficiency, flavor-tagging efficiency, wrong tagging probability, and \( \Delta t \) resolution. The function \( P_{bg} \) is the probability density function (PDF) for the background. Both \( P_{\text{sig}} \) and \( P_{bg} \) are normalized by \( \int P_{\text{sig/bg}}(m_B^2, m^2, \Delta t) dm_B^2 dm^2 d\Delta t = 1 \). The event-by-event background fraction \( f_{bg}(\Delta E, M_{bc}) \) is based on signal and background levels found by fitting \( \Delta E \) as described above, the \( \Delta E \) shape used in the fit, and an \( M_{bc} \) shape that is the sum of a Gaussian signal and an empirical background function with kinematic threshold and shape parameters determined from off-resonance data.

We describe the background by the sum of four components: \( B \) decays containing (a) real \( D \) mesons and (b) combinatorial \( D \) mesons, and \( q\bar{q} \) events containing (c) real \( D \) mesons and (d) combinatorial \( D \) mesons. The Dalitz plot is described by the function \( f\left(m_B^2, m^2\right) \) for (a) and (c). For (b) and (d) we use an empirical background function which includes enhancements near the edges of the Dalitz plot as well as an incoherently added \( K^*(892) \) contribution [8]. The shape of this function is obtained from an analysis of events in the \( D \) mass sideband. The \( \Delta t \) distribution for the \( B \) decay backgrounds is described by an exponential convolved with the detector resolution. For the \( q\bar{q} \) background, a triple Gaussian form is used, which is obtained from events with \( |\cos\theta_{\text{det}}| > 0.8 \). The use of this sideband region has been validated using MC calculations. We use the experimental data and generic MC calculations to fix the fractions of background components.

Figure 3 shows the Dalitz plot distributions for candidates in signal and \( M_{bc} \) sideband region, integrated over the entire \( \Delta t \) range and \( B^0 \) and \( \bar{B}^0 \) combined. We can see clear differences in these distributions.

The procedure for the \( \Delta t \) fit is tested by extracting \( \tau_{B^+} \) using \( B^+ \to D^{(*)}\bar{B}^0 \) decay. We obtain \( \tau_{B^+} = 1.678 \pm 0.043 \) ps (statistical error only), consistent with the PDG [2] value 1.638 \( \pm 0.011 \) ps.

We perform a fit by fixing \( \tau_{B^0} \) and \( \Delta m \) at the PDG values with a fixed background shape and using \( \sin2\phi_1, \cos2\phi_1 \) as fitting parameters. The results are given in Table II for each of the three final states separately and for the simultaneous fit over all modes.

### Table I. Number of events in the signal region \( (N_{\text{sig}}) \), detection efficiency, number of signal events from the \( \Delta E \) fit \( (N_{\text{sig}}) \), and signal purity for the \( \to D^{(*)}\bar{B}^0 \) final states.

<table>
<thead>
<tr>
<th>Process</th>
<th>( N_{\text{tot}} )</th>
<th>Efficiency (%)</th>
<th>( N_{\text{sig}} )</th>
<th>Purity</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D\pi^0 )</td>
<td>265</td>
<td>8.7</td>
<td>157 ( \pm 24 )</td>
<td>59%</td>
</tr>
<tr>
<td>( D\omega )</td>
<td>88</td>
<td>4.1</td>
<td>67 ( \pm 10 )</td>
<td>76%</td>
</tr>
<tr>
<td>( D\eta )</td>
<td>101</td>
<td>3.9</td>
<td>58 ( \pm 13 )</td>
<td>57%</td>
</tr>
<tr>
<td>( D^<em>\pi^0, D^</em>\eta )</td>
<td>67</td>
<td>3.9</td>
<td>43 ( \pm 12 )</td>
<td>64%</td>
</tr>
<tr>
<td>Sum</td>
<td>521</td>
<td></td>
<td>325 ( \pm 31 )</td>
<td>62%</td>
</tr>
</tbody>
</table>

### FIG. 3. Dalitz plot distribution for the \( Dh^0 \) candidates from \( B \) signal region (left) and \( M_{bc} \) sideband.
We check goodness-of-fit using one-dimensional projections to $K_S^0 \pi^\pm$ and $\pi^+ \pi^-$ invariant masses and $\delta t$ and find no pathological behavior. To illustrate, the raw $CP$ asymmetry distribution for $D^{(*)}h^0$ candidates with an additional constraint $|M_{\pi^+ \pi^-}| < 0.15 \text{ GeV}/c^2$, to select events consistent with $D \to K_S^0 \rho$, is displayed in Fig. 4. For $D^* h^0$ candidates we take into account the opposite $CP$ asymmetry. In this case the system behaves approximately as a $CP$ eigenstate, with an asymmetry proportional to $-\sin 2\phi_1$.

Uncertainty of the $D \to K_S^0 \pi^+ \pi^-$ decay model is one of the main sources of systematic error for our analysis. We repeat the fit using two additional decay models from CLEO [7] and similar Belle analysis [15]. The difference between these models and our primary model [8] is in describing of wide resonances in $\pi^+ \pi^-$ and $K_S^0 \pi$, non-resonant part and doubly Cabibbo suppressed channels. The CLEO [7] does not include wide resonances $\sigma(600)$ and $f_0(1370)$, and the doubly Cabibbo suppressed channel $D^0 \to K^{++}(1430)\pi^-$. Another Belle model [15] has an additional contributions from $K^*(1410)^\pm \pi^\mp$ and $K_S^0 p^0(1450).$ The difference in fitted values for $\sin 2\phi_1$ and $\cos 2\phi_1$ between the nominal model [8] and others is found not to exceed 0.1, and we assign this value as a model uncertainty.

We vary the background descriptions to estimate the systematic uncertainty due to the background parametrization. We use only a combinatorial and only a signal $D$ PDF for the Dalitz plot distribution. For the time dependence, we consider cases with only a $q\bar{q}$ component or only a $B\bar{B}$ component. The differences do not exceed 0.2 and we take this value as a systematic error.

Other contributions to the systematic error are found to be small: vertexing and flavor tagging (0.02), neglecting suppressed amplitudes (0.01), and signal yield determination (0.02).

The measurement of $\sin 2\phi_1 = 0.78 \pm 0.44 \pm 0.22$ is consistent with the high statistics measurement in the $J/\psi K^0$ channel [2]. The result of $\cos 2\phi_1 = 1.87 \pm 0.33 \pm 0.32$ allows one to distinguish between two solutions in $\phi_1$: 23$^\circ$ and 67$^\circ$. We define the confidence level at which the 67$^\circ$ solution (negative value of $\cos 2\phi_1$) can be excluded as C.L.$(x) = f_+(x)/[f_+(x) + f_-(x)]$, where $f_+(x)$ and $f_-(x)$ is the likelihood to obtain the fit result $\cos 2\phi_1 = x$ when true $\cos 2\phi_1$ value of 0.689 (−0.689). To evaluate $f_+$ and $f_-$ we use sample of 2500 pseudoexperiments with the same size as data for both hypotheses. We fit these distributions with a sum of two Gaussians. We calculate C.L. for $x = 1.87, 1.55$, and 2.09 to take into account systematic uncertainties of $+0.22$ in our $\cos 2\phi_1$ measurement. As a final result we use the smallest value C.L.(1.55) = 98.5 ± 0.2%, excluding the 67$^\circ$ solution at 98.3% C.L.

In summary, we have presented a new method to measure the unitarity triangle angle $\phi_1$ using a time-dependent amplitude analysis of the $D \to K_S^0 \pi^+ \pi^-$ decay produced in the processes $B^0 \to D^{(*)}h^0$. We find $\sin 2\phi_1 = 0.78 \pm 0.44 \pm 0.22$ and $\cos 2\phi_1 = 1.87 \pm 0.33 \pm 0.32$. The sign of $\cos 2\phi_1$ is determined to be positive at 98.3% C.L., favoring the $\phi_1 = 23^\circ$ solution.

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![FIG. 4. Raw asymmetry distribution for the $D^{(*)}[K_S^0 \rho^0]h^0$ candidates. The smooth curve is the result of the fit to the full Dalitz plot.](image-url)


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