Measurement of the CP Asymmetry and Branching Fraction of $B^0 \rightarrow \rho^0 K^0$
051803-3


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Decays of $B^0$ mesons to the $\rho^0K^0$ final state are expected to be dominated by $b \rightarrow s$ penguin amplitudes. Neglecting Cabibbo-Kobayashi-Maskawa (CKM) suppressed amplitudes, the mixing-induced CP violation parameter $S_{\rho^0K^0}$ should equal $\sin2\beta$, which is well measured in $B^0 \rightarrow J/\psi K^0$ decays [1]. Within the standard model (SM), only limited deviations from this prediction are expected [2]. In the standard model, a single phase in the CKM matrix governs CP violation [3], but if heavy non-SM particles appear in additional penguin diagrams, new CP-violating phases could enter and $S_{\rho^0K^0}$ would not equal $\sin2\beta$ [4].

Observation of such a discrepancy would be a clear signal of new physics. Current estimates [2] predict a greater deviation of $S_{\rho^0K^0}$ from $S_{J/\psi K^0}$ to SM processes than in related charmless $B$ decays. However, distinct from predictions in other channels, this deviation is expected to be such that $S_{\rho^0K^0}$ is less than $S_{J/\psi K^0}$.

In this Letter, we present the first observation of the decay $B^0 \rightarrow \rho^0K^0$ and a measurement of the CP-violating asymmetries $S_{\rho^0K^0}$ and $C_{\rho^0K^0}$ from a time-dependent maximum likelihood analysis. A nonzero value of $S_{\rho^0K^0}$ indicates CP violation due to the interference between decays with and without mixing. Direct CP violation leads to a nonzero value of $C_{\rho^0K^0}$. We take a quasi-two-body approach, restricting ourselves to the region of the $B^0 \rightarrow \pi^+\pi^-K^0$ Dalitz plot dominated by the $\rho^0$ and treating other $B^0 \rightarrow \pi^+\pi^-K^0$ contributions as a noninterfering background. The effects of interference with other resonances are estimated and taken as systematic uncertainties.

The data were collected with the BABAR detector at the PEP-II asymmetric-energy $e^+e^-$ storage ring at Stanford Linear Accelerator Center (SLAC). An integrated luminosity of 205 fb$^{-1}$, corresponding to $227 \times 10^6 BB$ pairs, was collected at the $\Upsilon(4S)$ resonance [center-of-mass (c.m.) energy $\sqrt{s} = 10.56$ GeV], and 16 fb$^{-1}$ was collected about 40 MeV below the resonance (off-resonance data). The BABAR detector is described in detail elsewhere [5]. Charged particles are detected and their momenta measured by the combination of a silicon vertex tracker, consisting of five layers of double sided detectors, and a 40-layer central drift chamber, both operating in the 1.5 T magnetic field of a solenoid. Charged-particle identification is provided by the average energy loss in the tracking devices and by an internally reflecting ring-imaging Cherenkov detector covering the central region.

We present a measurement of the branching fraction and time-dependent CP asymmetry of $B^0 \rightarrow \rho^0K^0$. The results are obtained from a data sample of $227 \times 10^6 \Upsilon(4S) \rightarrow BB$ decays collected with the BABAR detector at the PEP-II asymmetric-energy $B$ factory at Stanford Linear Accelerator Center. From a time-dependent maximum likelihood fit yielding $111 \pm 19$ signal events, we find $B(B^0 \rightarrow \rho^0K^0) = (4.9 \pm 0.8 \pm 0.9) \times 10^{-6}$, where the first error is statistical and the second systematic. We report the measurement of the CP parameters $S_{\rho^0K^0} = 0.20 \pm 0.52 \pm 0.24$ and $C_{\rho^0K^0} = 0.64 \pm 0.41 \pm 0.20$. 

We reconstruct $B^0 \rightarrow \rho^0K^0$ candidates ($B^0_{rec}$ in the following) from combinations of $\rho^0$ and $K^0$ candidates, both reconstructed in their $\pi^+\pi^-$ decay mode. For the $\pi^+\pi^-$ pair from the $\rho^0$ candidate, we remove tracks identified as very likely to be electrons, kaons, or protons. The mass of the $\rho^0$ candidate is restricted to the interval $0.4 < m(\pi^+\pi^-) < 0.9$ GeV/$c^2$. The $K^0$ candidate is required to have a mass within $13$ MeV/$c^2$ of the nominal $K^0$ mass [6] and a decay vertex separated from the $\rho^0$ decay vertex by at least 3 times the estimated separation measurement uncertainty. In addition, the cosine of the angle in the lab frame between the $K^0$ flight direction and the vector between the $\rho^0$ decay vertex and the $K^0$ decay vertex must be greater than 0.995. Vetoing against $B^0 \rightarrow D^+\pi^-$ and $B^0 \rightarrow K^+\pi^-(K^0 \rightarrow \pi^+\pi^-)$ are imposed by requiring that the invariant masses of both $K^0\pi$ combinations are more than 0.055 and 0.040 GeV/$c^2$ from the $K^+\pi^-$ and $D^+\pi^-$ masses [6], respectively. To exclude events with poorly reconstructed vertices, we require the estimated error on $\Delta t$ to be less than 2.5 ps and that $|\Delta t|$ must be less than 20 ps, where $\Delta t$ is the proper time difference between the decay of the reconstructed $B$ meson ($B^0_{rec}$) and its unreconstructed partner ($B^0_{tag}$), $t_{rec} - t_{tag}$. It is determined from the measured relative displacement of the two $B$-decay vertices and the known boost of the $e^+e^-$ system.

Two kinematic variables are used to discriminate between the signal and the combinatorial background. The first is $\Delta E$, the difference between the measured c.m. energy of the $B$ candidate and $\sqrt{s}/2$, where $\sqrt{s}$ is the c.m. beam energy. The second is the beam-energy-substituted mass $m_{ES} \equiv \sqrt{(s/2 + \mathbf{p}_B \cdot \mathbf{p}_B)^2/E_{ES}^2 - \mathbf{p}_B^2}$, where the $B^0_{rec}$ momentum $\mathbf{p}_B$ and the four-momentum of the initial $\Upsilon(4S)$ state ($E_{\Upsilon}, \mathbf{p}_\Upsilon$) are defined in the laboratory frame. We require $|\Delta E| < 0.15$ GeV and $5.23 < m_{ES} < 5.29$ GeV/$c^2$.

Continuum $e^+e^- \rightarrow q\bar{q}$ ($q = u, d, s, c$) events are the dominant background. To enhance discrimination between the signal and the continuum, we use a neural network (NN) to combine five variables: the cosine of the angle between the $B^0_{rec}$ direction and the beam axis in the c.m., the cosine of the angle between the thrust axis of the $B^0_{rec}$ candidate and the beam axis, the sum of momenta transverse to the direction of flight of the $B^0_{rec}$, and the zeroth and second angular moments $L_{0,2}$ of the energy flow about the $B^0_{rec}$ thrust axis. The moments are defined by $L_i = \sum_j \mathbf{p}_j \times |\cos\theta_j|$, where $\mathbf{p}_j$ is its momentum and $\theta_j$ is the
angle with respect to the $B_{\text{rec}}^0$ thrust axis of the track or neutral cluster $i$ excluding the tracks that make up the $B^0_{\text{rec}}$ candidate. The NN is trained with off-resonance data and Monte Carlo (MC) [7] simulated signal events.

The efficiency to reconstruct signal events is determined to be 0.29 from large samples of MC events. When more than one candidate per event passes all selection (less than 10% of events), we choose among them randomly. We estimate that 16% of the selected signal events are reconstructed incorrectly with low momentum tracks from the other $B$ meson being used to form the $\rho^0$ candidate. In total, 20,073 events pass all selection criteria in the on-resonance sample.

An unbinned extended maximum likelihood fit is used to extract the $\rho^0 K^0_s$ CP asymmetry and branching fraction. There are ten components in the fit: signal, continuum background, and eight separate background components from $B$ decays. Large samples of MC-simulated events are used to identify these specific $B$ backgrounds. Where an individual decay mode makes a significant contribution to the data set (one or more events expected in the data), we include it as a separate contribution to the fit. Probability density functions (PDFs) are taken from simulation with the expected number of $B$ background events fixed to values estimated from known branching fractions [6] and MC efficiencies (Table I). Where only upper limits are available, decay modes are not included in the default fit but are used in alternate fits to evaluate systematics.

Events from $B$ decays that do not come from individually significant channels are collected together into two “bulk” $B$ contributions to the fit ($B^0$ and $B^+ \rightarrow K^+ \pi^+ \pi^-$ decays). The assumption is made that $B^0 \rightarrow f_0(600)K^0$ can be neglected, with support from Refs. [8,9], which do not require this mode to describe $B^+ \rightarrow K^+ \pi^+ \pi^-$. The $f_0(600)$, a broad scalar resonance decaying to $\pi\pi$, could potentially interfere with the $\rho$, and $S_{f_0(600)K^0}$ would be $-1 \times S_{f_0(600)\rho}$. A reinterpretation of the result would be required if it were seen to be a significant component in $B^0 \rightarrow K^0_s \pi^+ \pi^-$ decays.

The events in the data sample have their unreconstructed $B$ meson flavor tagged as $B^0$ or $\bar{B}^0$ with the method described in Ref. [10]. Events are separated into four flavor-tagging categories and an “untagged” category, depending upon the method used to determine the flavor. Each category has a different expected purity and accuracy of tagging. The likelihood function for the $N_k$ candidates in flavor-tagging category $k$ is

$$L_k = e^{-N_k} \prod_{i=1}^{N_k} \left[ N_j \epsilon_k \left( (1 - f_{\text{MR}}) \mathcal{P}_{j,k}^{\text{SR}} + f_{\text{MR}} \mathcal{P}_{j,k}^{\text{MR}} \right) + N_{C,k} \mathcal{P}_{j,k}^{\text{C}} + \sum_{j=1}^{n_k} N_{B,j} \epsilon_{j,k} \mathcal{P}_{j,k}^{\text{B}} \right],$$

(1)

where $N_k$ is the sum of the signal and background yields for events tagged in category $k$, $N_j$ is the number of $\rho^0 K^0$ signal events in the sample, $\epsilon_k$ is the fraction of signal events tagged in category $k$, $f_{\text{MR}}$ is the fraction of mis-reconstructed (MR) signal events in tagging category $k$, and the superscript CR implies a correctly reconstructed signal. $N_{C,k}$ is the number of continuum background events that are tagged in category $k$, and $N_{B,j} \epsilon_{j,k}$ is the number of $B$-background events of class $j$ that are tagged in category $k$. The $B$-background event yields are fixed in the default fit to values shown in Table I. The values $\epsilon_k$ and $f_k$ are determined from MC calculations for $B$ backgrounds and from a sample of $B$ decays of known flavor for signal. The total likelihood $L$ is the product of the likelihoods for each tagging category.

Each signal and background PDF is defined as:

$$\mathcal{P}_k = \mathcal{P}(m_{\text{ES}}) \mathcal{P}(\Delta E) \mathcal{P}(NN) \mathcal{P}^{(\cos \theta_\pi)} \mathcal{P}(\Delta t) \mathcal{P}(m_{\pi^+ \pi^-}),$$

where $m_{\text{ES}}$, $\Delta E$, $NN$, and $m_{\pi^+ \pi^-}$ are the variables described previously and $\cos \theta_\pi$ is the angle between the $K^0_s$ and the $\pi^+$ from the $\rho^0$ in the $\rho^0$ meson’s center-of-mass frame. The distributions of these variables were studied in depth using large control samples.

The $\Delta t$ PDF for signal events is defined as

$$\mathcal{P}(\Delta t) = \frac{e^{-|\Delta t|/\tau_\rho}}{4\tau_\rho} \left[ 1 + \frac{\Delta D}{2} + q(D)(S_{\rho K^0_s} \sin(\Delta m\Delta t)) - C_{\rho K^0_s} \cos(\Delta m\Delta t) \right] \Theta R_{\text{sig}}(\Delta t, \sigma_{\Delta t}),$$

(2)

where $\tau_\rho$ and $\Delta m$ are the average lifetime and eigenstate mass difference of the neutral $B$ meson, $q = +1(-1)$ when $B^0_\text{rec} = B^0$ ($\bar{B}^0$), $\langle D \rangle$ describes the dilution effect from imperfect flavor tagging, and $\Delta D$ is the difference in this dilution between $B^0$ and $\bar{B}^0$ tags. This formalism is found to effectively describe both correctly and incorrectly reconstructed signals, $\langle D \rangle$, $\Delta D$, and the $\Delta t$ resolution function $R_{\text{sig}}(\Delta t, \sigma_{\Delta t})$ have parameters fixed to values taken from a sample where $B$ mesons of known flavor can be reconstructed [10]. Untagged events have a $\langle D \rangle$ of 0, reflecting the lack of tag information.

The $m_{\text{ES}}$, $\Delta E$, $NN$, $\cos \theta_\pi$, and $m_{\pi^+ \pi^-}$ PDFs for signal and $B$ background are taken from MC simulation. In general, they are nonparametric, with the exception of $m_{\text{ES}}$.
and $\Delta E$ for signal. Signal PDFs appear as solid curves in Fig. 1. The CP parameters for $\eta'K^0_S$ and $f_0K^0_S$ backgrounds are fixed to $C = 0$ and $S = \sin 2\beta$ (for $\eta'K^0_S$) and $S = -\sin 2\beta$ (for $f_0K^0_S$), in accordance with SM expectations. For the remaining $B$ backgrounds, the parameters $C$ and $S$ are fixed to 0. The PDF parameters describing the continuum background are either allowed to vary freely in the fit or else are determined separately from off-resonance data.

There are 16 free parameters in the fit: the yield of signal events, $S$, $C$, and $S$. The continuum parameters are the yields (5), those associated with the second order polynomial describing the $\Delta E$ distribution (2), the ARGUS [11] function describing the $m_{ES}$ distribution (1), and the double Gaussian used to model the $\Delta t$ distribution (5).

The fit yields $111 \pm 19$ signal events. We calculate the branching fraction from the measured signal yield, efficiency (including the $\rho^0 \to \pi^+\pi^-$, $K^0 \to K^0_S$, and $K^0_S \to \pi^0\pi^0$ branching fractions), and the number of $BB$ events. The result is $B(B^0 \to \rho^0 K^0) = (4.9 \pm 0.8 \pm 0.9) \times 10^{-6}$, where the first error is statistical and the second systematic. The likelihood ratio between the fit result of 111 signal events and the null hypothesis of zero signal shows that this is excluded at the $8.7\sigma$ level. When additive systematic effects are included, we exclude the null hypothesis at the $5.0\sigma$ level. The fit for CP parameters gives $S = 0.20 \pm 0.52 \pm 0.24$ and $C = 0.64 \pm 0.41 \pm 0.20$.

Figure 1 shows the plots of the discriminating variables in the fit. Knowledge of the level of background and our ability to distinguish it from signal can be gained from the errors in these plots. In addition, Fig. 1(f) shows the ratio $L_S/(L_S + L_B)$ for all events, where $L_S$ and $L_B$ are the likelihoods for each event to be signal or background, respectively.

**FIG. 1 (color online).** Plots of maximum likelihood fit discriminating variables: (a) $m_{ES}$, (b) $\Delta E$, (c) neural network output, (d) $\cos \theta_{\pi^+\pi^-}$, and (e) invariant mass of the $\pi^+\pi^-$ combination. Lines are projections of signal PDFs for each variable. (f) is a plot of the likelihood of an event being signal calculated for all events in our data set and compared to the predictions of our PDF (predicted continuum in light gray, $B$ background in dark gray, and signal unshaded).

**FIG. 2.** Plots of $\Delta t$, overlaid with projected signal PDFs, split into (a) $B^0_{uag}$ tags, (b) $B^0_{ag}$ tags, and (c) the asymmetry $(N_{B^0_{uag}} - N_{B^0_{ag}})/(N_{B^0_{uag}} + N_{B^0_{ag}})$ as a function of $\Delta t$. 

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Table II. Summary of contributions to the systematic error.

<table>
<thead>
<tr>
<th>Contribution</th>
<th>$\sigma_{s_{p_{K_{S}}}}$</th>
<th>$\sigma_{c_{p_{K_{S}}}}$</th>
<th>$\sigma_{BF}(%)$</th>
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</thead>
<tbody>
<tr>
<td>Misreconstructed events and fit bias</td>
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<td>0.09</td>
<td>10</td>
</tr>
<tr>
<td>PDF uncertainties</td>
<td>0.13</td>
<td>0.18</td>
<td>2</td>
</tr>
<tr>
<td>Neglect of interference</td>
<td>0.14</td>
<td>0.09</td>
<td>7</td>
</tr>
<tr>
<td>$\rho^0$ mass shape</td>
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<td>0.05</td>
<td>3</td>
</tr>
<tr>
<td>$B$ background BF</td>
<td>0.02</td>
<td>0.10</td>
<td>13</td>
</tr>
<tr>
<td>$CP$ of background</td>
<td>0.04</td>
<td>0.00</td>
<td>· · ·</td>
</tr>
<tr>
<td>Tracking efficiency and $B$ counting</td>
<td>· · ·</td>
<td>· · ·</td>
<td>6</td>
</tr>
<tr>
<td>Total</td>
<td>0.24</td>
<td>0.20</td>
<td>19</td>
</tr>
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</table>

Figure 2 shows plots of $\Delta t$. Untagged events are removed, and events are split into $B_{0\text{tag}}^0$ and $\bar{B}_{0\text{tag}}^0$ tags. An asymmetry of $\langle N_{B_{0\text{tag}}}^0 - N_{\bar{B}_{0\text{tag}}}^0 \rangle / \langle N_{B_{0\text{tag}}}^0 + N_{\bar{B}_{0\text{tag}}}^0 \rangle$ as a function of $\Delta t$ is also shown.

Systematic errors are listed in Table II and are discussed here in the same order. We estimate biases due to the fit procedure from fits to a large number of simulated experiments and vary the fraction of misreconstructed events within estimated limits to determine the resulting systematic error. The effect of alternate models was also studied. We vary aspects of the model fixed in the nominal fit (for example, signal $\Delta t$ resolution) by their estimated uncertainties (including where appropriate the estimated effects of discrepancies between data and MC calculations) and take the change in result as the systematic error. We estimate the systematic uncertainty due to neglecting the interference between $B^0 \to K^0_S \pi^+ \pi^-$ from both parametrized and full simulations that take interference into account. We include contributions from $\rho^0(770)K^0_S, f_0(980)K^0_S, K^*_0(1430)^+ \pi^-$, $K^*_0(892)^+ \pi^-$, and $f_2(1270)K^0_S$, as well as two $K^0_S \pi \pi$ nonresonant contributions. Uncertainties from the amplitude and phase of each mode are added in quadrature. Additionally, we calculate the systematic effect uncertainties from alternative models for resonances, uncertainties in $BF$ and $CP$ of $B$ backgrounds, the effects of finite knowledge of tracking efficiency, and the number of $B$ mesons in the sample.

In summary, we have established the existence of the decay $B^0 \to \rho^0 K^0_S$ and measured its branching fraction with the significance of 5 standard deviations. Our measurement agrees within errors with $B(B^0 \to \omega K^0)$ as measured in Ref. [13], as expected if a single penguin amplitude dominates these decays. We have extracted $CP$ violating parameters $S$ and $C$ for $B^0 \to \rho^0 K^0_S$ which are consistent with those measured in charmonium channels [1].

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