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# Full-Duplex Amplify-and-Forward Relay Selection in Cooperative Cognitive Radio Networks

Bakhtiar Ali, Jawad Mirza, Senior Member, IEEE, Juping Zhang, Gan Zheng, Senior Member, IEEE, Saqib Saleem, Kai-Kit Wong, Fellow, IEEE

Abstract—This paper investigates relay selection policies in a full-duplex (FD) based underlay cooperative cognitive radio network. We consider a scenario consisting of multiple pairs of primary users (PUs) with access to the spectrum, and multiple pairs of secondary users (SUs) with no dedicated spectrum access. Each pair (PU or SU) comprises of a unique set of transmitter and receiver. The secondary transmitter (ST) supports the FD transmission with the simultaneous transmission of the information to its own receiver and reception of the designated PU signal for relaying. Firstly, we formulate an optimization problem to maximize the PU rate by optimizing the transmission power at the relay (i.e., ST), while keeping the SU rate fixed. Secondly, we propose two low-complexity relay selection schemes based on the sum rate maximization, namely, centralized successive and distributed prioritized matching schemes. Numerical results show that the proposed schemes perform close to exhaustive selection

Index Terms—Cooperative communication, Cognitive radio

#### I. Introduction

**F**ULL-DUPLEX (FD) communication aspires to accomplish high data rates required in the fifth generation (5G) networks. An FD transceiver can attain higher data rates, compared to its half-duplex (HD) counterpart, owing to its concurrent transmission and reception capabilities [1]. Another popular paradigm to improve spectral efficiency is the cognitive radio network, where an unlicensed secondary user (SU) shares the spectrum allocated to a licensed primary user (PU) [2]. The cooperation between PUs and SUs can provide an additional spectral efficiency gain. Such cooperative networks are also known as cooperative cognitive radio networks (CCRNs). When SUs act as relays for PUs, the spectral efficiency of CCRNs improves, as SUs get more opportunity to utilize the spectrum [3]. Moreover, it has been shown that utilizing FD assisted relaying in CCRNs further improves the spectral efficiency [1], [4], [5]. Despite these gains, an FD transceiver suffers from the impact of self-interference (SI) that originates due to the simultaneous transmission and reception.

Previous studies have investigated FD relaying in CCRNs with a single relay [1] and multiple relays [4], [5]. The

- B. Ali and J. Mirza are with the Department of Electrical and Computer Engineering, COMSATS University Islamabad, Islamabad 45550, Pakistan (Email: bakhtiar\_ali@comsats.edu.pk; jaydee.mirza@gmail.com).
- J. Zhang and G. Zheng are with the Wolfson School of Mechanical, Electrical and Manufacturing Engineering, Loughborough University, Leicestershire, LE11 3TU, UK (Email: j.zhang3@lboro.ac.uk; g.zheng@lboro.ac.uk).
- S. Saleem is with the Department of Electrical Engineering, COMSATS University Islamabad, Sahiwal Campus, Sahiwal 57000, Pakistan (Email: saqibsaleem2002@hotmail.com).
- K.-K. Wong is with the Department of Electronic and Electrical Engineering, University College London, London, WC1E 6BT, UK (Email: kai-kit.wong@ucl.ac.uk).

network model in [4], [5] comprises of a PU (single terminal) and a SU pair, where only the SU pair utilizes a relay. An opportunistic FD relay selection scheme has been presented in [4] for underlay CCRNs with a decode-and-forward (DF) relaying protocol. The performance of SUs with an amplify-and-forward (AF) protocol and a two-way opportunistic FD relay selection for underlay CCRNs is quantified in [5]. Specifically, lower bounds on the outage and symbol error probabilities are derived to design the opportunistic relay selection scheme. In multi-hop cooperative networks, the AF relaying strategy is generally favoured due to its easy implementation and high security, compared to the other relaying strategies [6].

Different from the seminal studies [4], [5], this paper considers an underlay CCRN with multiple PUs and SUs. The FD-assisted transceiver at the secondary transmitter (ST) acts as a potential relay for PUs and in exchange it shares the spectrum of the designated PU to communicate with its own secondary receiver (SR). Each PU pair selects the best relay (i.e., ST) using a specific relay selection policy. Therefore, a simultaneous transmission and reception is carried out at the FD relay, which gives rise to SI. A similar CCRN model has been investigated in [7] for the conventional HD mode with overlay settings, however, to the best of the authors knowledge, this model has not been investigated for the FD mode.

When STs act as FD relays, they increase their spectral efficiency, as well as the spectral efficiency of PUs, therefore, it is important to investigate this sophisticated CCRN model. Moreover, this CCRN model is best suited for ultra-dense networks in order to improve spectrum utilization and spectral efficiency, where spectrum resources become more scarce. A major challenge in this CCRN model is that of the FD relay assignment in a fair manner among multiple competing PUs, while also considering the effects of SI. Therefore, the main motivation of this study is to improve the spectral efficiency of the CCRN model with FD relays, by developing lowcomplexity relay selection strategies. It is also important to design an effective power control strategy at the FD relay that favours the PU communication. In this study, SUs benefit the most in the FD mode as they share the entire transmission time slot allocated for PUs. The main contributions of the paper are summarized below:

- For the CCRN model, we introduce an optimization approach to maximize the PU rate by optimizing the transmission power at the ST, while setting the SU rate as a fraction of the PU rate.
- We propose two low complexity relay assignment policies based on the sum rate maximization of the PUs: centralized successive matching (CSM) and distributed prioritized matching (DPM).

## II. SYSTEM MODEL

Consider a CCRN as shown in Fig. 1, with N primary transmitter (PT) and primary receiver (PR) pairs, also known as PUs. Each PU pair has its own dedicated spectrum bandwidth, which is assumed to be equal for all pairs. Each PT is equipped with  $M_p$  number of transmit antennas, whereas, each PR has  $N_p$  number of receive antennas. There are total K number of SU pairs, where each pair comprises of ST and SR. The total number of transmit antennas at ST and receive antennas at SR are denoted by  $M_s$  and  $N_s$ , respectively.

Each PT-PR pair communication takes place in two time slots. In the first time slot, the PT transmits a signal to the PR, whereas in the second time slot, the selected ST acts as a relay and re-transmits the previously received signal towards the PR using the AF protocol. On the other hand, STs can transmit information to their respective SRs in both time slots. In the first time slot, the FD-capable ST simultaneously performs transmission (to the SR) using  $M_s$  antennas and reception (from the PT) with a single receive antenna. This single receive antenna is an additional antenna apart from the  $M_s$  transmit antennas at the relay. In the second time slot, the ST serves both PR and SR by employing a multiuser beamforming technique.

The transmit power of all the PTs is assumed to be the same, denoted by  $P_{\rm t}$ . In the second time slot, the ST transmits the signals to the PR and SR with power  $P_{\rm p,s}$  and  $P_{\rm s,s}$ , respectively, whereas, in the first time slot, the ST allocates the maximum power  $P_{\rm s}$  to the associated SR. During the first time slot, the received signal vector at the  $i^{\rm th}$  PR via a direct link is

$$\mathbf{y}_{i}^{(1)} = \sqrt{P_{t}} \mathbf{H}_{i}^{(1)} \mathbf{v}_{j,i} + \sqrt{P_{s}} \mathbf{A}_{i,j}^{(1)} \mathbf{u}_{j}^{(1)} + \mathbf{n}_{i}^{(1)}, \tag{1}$$

where  $\mathbf{H}_i^{(1)}$  is the channel matrix of size  $N_p \times M_p$  between the  $i^{\text{th}}$  PU pair and the entries of  $\mathbf{H}_i^{(1)}$  are assumed to follow an independent and identically distributed (i.i.d.) complex Gaussian distribution i.e.,  $CN(0, \eta_i^{(1)})$ , where  $\eta_i^{(1)}$  is the path loss between the  $i^{\text{th}}$  PU pair. The transmitted signal is denoted by  $\mathbf{v}_{j,i} = \mathbf{w}_{j,p}x_i$ , where  $x_i \sim CN(0,1)$  denotes the primary signal sent from the  $i^{\text{th}}$  PT and  $\mathbf{w}_{j,p}$  represents the beamforming vector for the  $j^{\text{th}}$  relay, assuming that the  $i^{\text{th}}$  PU pair selects the  $j^{\text{th}}$  relay in the first time slot. The interfering channel matrix between the  $j^{\text{th}}$  SR and  $i^{\text{th}}$  PR, is given by  $\mathbf{A}_{i,j}^{(1)}$ , whose entries follow the  $CN(0,\eta_{i,j}^{(1)})$  distribution, where  $\eta_{i,j}^{(1)}$  is the associated path loss of the link. The signal transmitted from the selected  $j^{\text{th}}$  relay to its SR in the first time slot is given by  $\mathbf{u}_j^{(1)} = \mathbf{w}_{s,j}^{(1)} s_j^{(1)}$ , where  $\mathbf{w}_{s,j}^{(1)}$  denotes the beamforming vector at the  $j^{\text{th}}$  relay (for the serving SR) and  $s_j^{(1)} \sim CN(0,1)$  represents the data symbol for the  $j^{\text{th}}$  SR. The additive white Gaussian noise (AWGN) vector at the  $i^{\text{th}}$  PR is denoted by  $\mathbf{n}_i^{(1)} \sim CN(0,N_0\mathbf{I}_{N_p})$ . Similarly, the received signal at the selected  $j^{\text{th}}$  ST from the  $i^{\text{th}}$  PT, in the first time slot, can

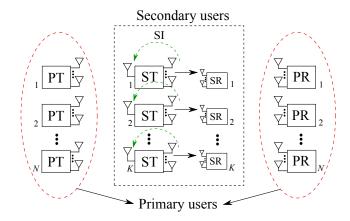


Fig. 1. The system model of the CCRN with FD AF relays.

be written as

$$r_{j,i}^{(1)} = \sqrt{P_{t}} \mathbf{h}_{j,i}^{(1)H} \mathbf{v}_{j,i} + \sqrt{P_{s}} \mathbf{h}_{SI,j}^{H} \mathbf{u}_{j}^{(1)} + z_{j}^{(1)},$$
(2)

where  $\mathbf{h}_{j,i}^{(1)} \sim \mathcal{CN}(0, \eta_{j,i}^{(1)} \mathbf{I}_{M_p})$  denotes the channel between the  $i^{\text{th}}$  PT and the selected  $j^{\text{th}}$  relay (ST). Due to the simultaneous transmission and reception at the  $j^{\text{th}}$  ST in the first time slot, SI is also present. In this work, we assume that SI is suppressed such that there is some residual SI (RSI) present at the  $j^{\text{th}}$  relay, denoted by  $\mathbf{h}_{\text{SI},j}$ . Here we assume that  $\mathbf{h}_{\text{SI},j} \sim \mathcal{CN}(0,\sigma_{\text{SI}}^2 \mathbf{I}_{M_s})$ . The noise term at the  $j^{\text{th}}$  ST is denoted by  $z_i^{(1)} \sim \mathcal{CN}(0,N_0)$ .

In the second time slot, the  $j^{th}$  ST forwards the previously received signal from the  $i^{th}$  PT towards the  $i^{th}$  PR, while also serving its own SR, by utilizing a multiuser MIMO technique. The received signal at the  $i^{th}$  PR can be expressed as

$$\mathbf{y}_{i,i}^{(2)} = \sqrt{P_{p,s}} \mathbf{A}_{i,i}^{(2)} \mathbf{u}_{i}^{(2)} + \sqrt{P_{s,s}} \mathbf{A}_{i,i}^{(2)} \mathbf{u}_{i}^{(2)} + \mathbf{n}_{i}^{(2)},$$
(3)

where the matrix  $\mathbf{A}_{i,j}^{(2)}$  of size  $N_p \times M_s$  denotes the channel between the  $j^{\text{th}}$  ST (relay) and the  $i^{\text{th}}$  PR, with entries following the i.i.d. complex Gaussian distribution  $\mathcal{CN}(0,\eta_{i,j}^{(2)})$ . The transmitted signal from the relay to the  $i^{\text{th}}$  PR is given by  $\mathbf{u}_i^{(2)} = \mathbf{w}_{\mathrm{p},j}\Delta_j\left(r_{j,i}^{(1)}\right)$ . The amplification factor at the  $j^{\text{th}}$  ST is given by  $\Delta_j = \sqrt{1/(P_{\mathrm{t}}|\mathbf{h}_{j,i}^{(1)H}\mathbf{w}_{j,\mathrm{p}}|^2 + P_{\mathrm{s}}|\mathbf{h}_{\mathrm{SI},j}^H\mathbf{w}_{\mathrm{s},j}^{(1)}|^2 + N_0}$ . The beamforming vector at the  $j^{\text{th}}$  ST for the  $i^{\text{th}}$  PR is denoted by  $\mathbf{w}_{\mathrm{p},j}$ . The transmitted signal from the  $j^{\text{th}}$  ST intended for the  $j^{\text{th}}$  SR is given by  $\mathbf{u}_j^{(2)} = \mathbf{w}_{\mathrm{s},j}^{(2)} s_j^{(2)}$ , where  $\mathbf{w}_{\mathrm{s},j}^{(2)}$  denotes the beamforming vector for the associated SR and  $s_j^{(2)}$  represents the data symbol for the  $j^{\text{th}}$  SR in the second time slot. The noise vector at the  $i^{\text{th}}$  PR in the second time slot is denoted by  $\mathbf{n}_i^{(2)} \sim \mathcal{CN}(0,N_0\mathbf{I}_{N_p})$ . The received signals at the  $j^{\text{th}}$  SR from its ST in the first and second time slots are given by

$$\mathbf{d}_{j}^{(1)} = \sqrt{P_{s}} \mathbf{C}_{j}^{(1)} \mathbf{u}_{j}^{(1)} + \sqrt{P_{t}} \mathbf{B}_{j,i}^{(1)} \mathbf{v}_{j,i} + \mathbf{e}_{j}^{(1)}, \tag{4}$$

and

$$\mathbf{d}_{i}^{(2)} = \sqrt{P_{s,s}} \mathbf{C}_{i}^{(2)} \mathbf{u}_{i}^{(2)} + \sqrt{P_{p,s}} \mathbf{C}_{i}^{(2)} \mathbf{u}_{i}^{(2)} + \mathbf{e}_{i}^{(2)}, \tag{5}$$

respectively. Here,  $C_j^{(t)}$  represents the channel matrix between the  $j^{th}$  SU pair in the time slot t whose entries follow

 $<sup>^1</sup>$ We use  $(\cdot)^H$  and  $(\cdot)^{-1}$  to denote the conjugate transpose and the inverse operations, respectively.  $\|\cdot\|$  and  $|\cdot|$  stand for vector and scalar norms respectively.  $\mathbf{A}(:,n)$  represents the  $n^{\text{th}}$  column of a matrix  $\mathbf{A}$ . The complex normal distribution with mean  $\mu$  and variance  $\sigma^2$  is denoted by  $\mathcal{CN}(\mu,\sigma^2)$ . We use a superscript  $(\cdot)$  to represent the time slot.

the  $C\mathcal{N}(0,\eta_j^{(t)})$  distribution. As we have assumed the block Rayleigh fading channel model, therefore we have,  $\mathbf{C}_j^{(1)} = \mathbf{C}_j^{(2)}$ . The matrix  $\mathbf{B}_{j,i}^{(1)}$  represents the interfering channel matrix from the  $i^{\text{th}}$  PT to the  $j^{\text{th}}$  SR in the first time slot, and its entries are also assumed to have  $C\mathcal{N}(0,\hat{\eta}_{j,i})$  distribution.  $\mathbf{e}_j^{(1)}$  and  $\mathbf{e}_j^{(2)}$  are the noise vectors at the  $j^{\text{th}}$  SR in the first and second time slots, respectively, following  $C\mathcal{N}(0,N_0\mathbf{I}_{N_s})$  distribution. The achievable rate of the  $i^{\text{th}}$  PR is given by

$$C_{p,i} = \frac{1}{2} \log_2 \left( 1 + \gamma_i^{(1)} + \frac{\gamma_{j,i}^{(1)} \gamma_{i,j}^{(2)}}{\gamma_{j,i}^{(1)} + \gamma_{i,j}^{(2)} + 1} \right), \tag{6}$$

where  $\gamma_i^{(1)}$ ,  $\gamma_{j,i}^{(1)}$  and  $\gamma_{i,j}^{(2)}$  represent signal-to-interference-plusnoise ratios (SINRs) at  $i^{\text{th}}$  PR (in the first time slot),  $j^{\text{th}}$  ST (in the first time slot) and  $i^{\text{th}}$  PR (in the second time slot), respectively, which are explicitly expressed as

$$\gamma_i^{(1)} = \frac{P_{\rm t} \left\| \mathbf{H}_i^{(1)} \mathbf{w}_{j, \rm p} \right\|^2}{P_{\rm s} \left\| \mathbf{A}_{i, i}^{(1)} \mathbf{w}_{s, i}^{(1)} \right\|^2 + N_0},\tag{7}$$

$$\gamma_{j,i}^{(1)} = \frac{P_{t} \left\| \mathbf{h}_{j,i}^{(1)H} \mathbf{w}_{j,p} \right\|^{2}}{P_{s} \left\| \mathbf{h}_{SI}^{H} \mathbf{w}_{s,j}^{(1)} \right\|^{2} + N_{0}},$$
(8)

and

$$\gamma_{i,j}^{(2)} = \frac{P_{p,s} \left\| \mathbf{A}_{i,j}^{(2)} \mathbf{w}_{p,j} \right\|^2}{P_{s,s} \left\| \mathbf{A}_{i,j}^{(2)} \mathbf{w}_{s,j}^{(2)} \right\|^2 + N_0},$$
(9)

respectively. For the  $j^{th}$  SR, the achievable rates for the first and second time slots can be written as

$$C_{s,j}^{(1)} = \frac{1}{2} \log_2 \left( 1 + \frac{P_s \left\| \mathbf{C}_j^{(1)} \mathbf{w}_{s,j}^{(1)} \right\|^2}{P_t \left\| \mathbf{B}_{j,i}^{(1)} \mathbf{w}_{j,p} \right\|^2 + N_0} \right)$$
(10)

and

$$C_{s,j}^{(2)} = \frac{1}{2} \log_2 \left( 1 + \frac{P_{s,s} \left\| \mathbf{C}_j^{(2)} \mathbf{w}_{s,j}^{(2)} \right\|^2}{P_{p,s} \left\| \mathbf{C}_j^{(2)} \mathbf{w}_{p,j} \right\|^2 + N_0} \right), \tag{11}$$

respectively.

The maximum ratio transmission beamforming scheme is utilized at the PT to transmit the information to the  $j^{th}$  ST in the first time slot, given by  $\mathbf{w}_{j,p} = \mathbf{h}_{j,i}^{(1)}/\|\mathbf{h}_{j,i}^{(1)}\|$ . Similarly, the relay employs a dominant eigenmode transmission based beamforming technique in the first time slot to serve its own SR, given by  $\mathbf{w}_{s,j}^{(1)}$ , which is the right singular vector of  $\mathbf{C}_{j}^{(1)}$  (obtained via a singular value decomposition), corresponding to the largest singular value. The dominant eigenmode transmission maximizes the signal-to-noise ratio (SNR) of the user and it also achieves the full diversity order.

In the second time slot, ST serves two users i.e., both PR and SR. Therefore, to suppress the inter-user interference, we employ the signal-to-leakage-and-noise ratio (SLNR) maximizing beamforming scheme [8]. Unlike zero-forcing beamforming, the SLNR beamforming scheme does not impose any

restrictions on the number of transmit and receive antennas and it also takes into account the impact of noise when designing the beamforming vectors [8]. At the  $j^{\text{th}}$  ST, the SLNR beamforming vector for the PR,  $\mathbf{w}_{p,j}$ , is the eigenvector corresponding to the largest eigenvalue of the matrix  $\mathbf{W}_{p,j}$ , where  $\mathbf{W}_{p,j} = \left(\frac{N_s N_0}{P_{p,s}}\mathbf{I}_{M_s} + \mathbf{C}_j^{(2)^H}\mathbf{C}_j^{(2)}\right)^{-1} \left(\mathbf{A}_{i,j}^{(2)^H}\mathbf{A}_{i,j}^{(2)}\right)$ . Similarly, the beamforming vector at the relay for the SR,  $\mathbf{w}_{s,j}^{(2)}$ , in the second time slot is the eigenvector corresponding to the largest eigenvalue of the matrix  $\mathbf{W}_{s,j}^{(2)}$ , where  $\mathbf{W}_{s,j}^{(2)} = \left(\frac{N_s N_0}{P_{s,s}}\mathbf{I}_{M_s} + \mathbf{A}_{i,j}^{(2)^H}\mathbf{A}_{i,j}^{(2)}\right)^{-1} \left(\mathbf{C}_j^{(2)^H}\mathbf{C}_j^{(2)}\right)$ . The transmission power at the ST (relay) can be optimized to maximize the rate of the PR in the second time slot. For this purpose, we formulate an optimization problem with the objective to maximize the PR rate by optimizing the transmit powers for PR and SR at the ST, while fixing the SINR of the SR to be equal or less than a fraction of the PR SINR, such that

$$\max_{\left\{P_{p,s}, P_{s,s}, \gamma_{i,j}^{(2)}, \gamma_{s,j}^{(2)}\right\}} \log_{2}\left(1 + \gamma_{i,j}^{(2)}\right) \tag{12}$$
s.t.
$$\gamma_{i,j}^{(2)} \leq \frac{P_{p,s} \left\|\mathbf{A}_{i,j}^{(2)} \mathbf{w}_{p,j}\right\|^{2}}{P_{s,s} \left\|\mathbf{A}_{i,j}^{(2)} \mathbf{w}_{s,j}^{(2)}\right\|^{2} + N_{0}},$$

$$\gamma_{s,j}^{(2)} \leq \frac{P_{s,s} \left\|\mathbf{C}_{j}^{(2)} \mathbf{w}_{s,j}^{(2)}\right\|^{2}}{P_{p,s} \left\|\mathbf{C}_{j}^{(2)} \mathbf{w}_{p,j}\right\|^{2} + N_{0}},$$

$$\gamma_{s,j}^{(2)} \leq \zeta \left(\gamma_{i,j}^{(2)}\right),$$

$$P_{p,s} + P_{s,s} \leq P_{s},$$

where  $\gamma_{s,j}^{(2)}$  is the SINR of the  $j^{\text{th}}$  SR in the second time slot, which is explicitly visible in (11) and  $\zeta$  is a constant value, such that  $0 < \zeta < 1$ . The SINR of the SR is fixed to a relatively small fraction of the PR SINR. By removing the third constraint in the problem (12), the power allocated for the SR will be negligible, hence, resulting in a low SU rate. Therefore, the quantity  $\zeta$  is used to achieve a balance between the PU performance and the SU performance. To efficiently solve the optimization problem (12), it is transformed into an equivalent complementary geometric program (CGP) [9] using standard reformulation techniques. This CGP problem is then converted to an approximated geometric program (GP) via monomial approximations<sup>2</sup>, which yields a sub-optimal solution for the problem (12). The steps for obtaining the sub-optimal solution  $\{P_{p,s}^*, P_{s,s}^*\}$  for the problem (12) via the GP approximation are given in Algorithm 1. The initial SINR guess for q=1,  $\hat{\gamma}_{i,j,q}^{(2)}=\hat{\gamma}_{i,j,1}^{(2)}$ , is obtained from (9) with  $P_{p,s}=P_{s,s}=P_s/2$ , and  $\epsilon$  represents the stopping criteria.

# III. RELAY ASSIGNMENT SCHEMES

In this section, we present two low-complexity relay selection schemes for the CCRN model described in Section II. We use a network wide performance metric for the relay

<sup>&</sup>lt;sup>2</sup>Due to space constraints, we refer the reader to [9, Section 3.2.2] for the details of the GP approximation of the problem (12).

# Algorithm 1 Successive Approximation to Maximize PU Rate

- 1: **Initialization**: q = 1,  $\epsilon > 0$ , initial SINR guess,  $\hat{\gamma}_{i,i,q}^{(2)}$
- 2: Solve the GP

Solve the GP 
$$\begin{aligned} \min_{\left\{P_{p,s}, P_{s,s}, \gamma_{i,j}^{(2)}, \gamma_{s,j}^{(2)}\right\}} & \left\{\gamma_{i,j}^{(2)}\right\}^{-\frac{\gamma_{i,j,q}^{(2)}}{1+\gamma_{i,j,q}^{(2)}}} \\ \text{s.t.} & \gamma_{i,j}^{(2)} \leq \frac{P_{p,s} \left\|\mathbf{A}_{i,j}^{(2)} \mathbf{w}_{p,j}\right\|^{2}}{P_{s,s} \left\|\mathbf{A}_{i,j}^{(2)} \mathbf{w}_{s,j}^{(2)}\right\|^{2} + N_{0}}, \\ & \gamma_{s,j}^{(2)} \leq \frac{P_{s,s} \left\|\mathbf{C}_{j}^{(2)} \mathbf{w}_{s,j}^{(2)}\right\|^{2}}{P_{p,s} \left\|\mathbf{C}_{j}^{(2)} \mathbf{w}_{p,j}\right\|^{2} + N_{0}}, \\ & \gamma_{s,j}^{(2)} \leq \zeta \left(\gamma_{i,j}^{(2)}\right), \\ & \gamma_{p,s}^{(2)} \leq \zeta \left(\gamma_{i,j}^{(2)}\right), \\ & P_{p,s} + P_{s,s} \leq P_{s}. \end{aligned}$$
Obtain the sub-optimal solution  $P_{p,s}^{\star}, P_{s,s}^{\star}, \gamma_{i,j}^{(2)^{\star}}, \gamma_{s,j}^{(2)^{\star}}. \end{aligned}$ 

- 3: If  $\left| \gamma_{i,j}^{(2)^*} \hat{\gamma}_{i,j,q}^{(2)} \right| \le \epsilon$ , then stop.
- 4: Else q = q + 1;  $\hat{\gamma}_{i,j,q}^{(2)} = \gamma_{i,j}^{(2)*}$  and **go to** step 2.

assignment which is defined as the sum of achievable rates of all the PU pairs, given by  $R = \sum_{i} C_{p,i}$ . The main objective of the relay selection is to assign relays to the PUs, such that R is maximized. In the beginning, all PTs use orthogonal channels to transmit their information towards respective PRs. Those STs who decode this information correctly from all PTs declare themselves as possible candidates for relaying. Here, we assume that there are a total of K such STs. In this study, only PUs communicate via relays, however, using FDcapable relays (ST) also help SUs, as they can communicate throughout the transmission frame, along side the associated PU. The PU uses a relay only when the rate of a direct (noncooperative) link is less than the PU rate with cooperation, i.e.,  $C_{\mathrm{p},i}^{\mathrm{no \, relay}} < C_{\mathrm{p},i}$ , where  $C_{\mathrm{p},i}^{\mathrm{no \, relay}} = \log_2(1 + (P_{\mathrm{t}}/N_0) \|\mathbf{H}_i^{(1)}\mathbf{w}_i\|^2)$ and  $\mathbf{w}_i$  is the right singular vector of  $\mathbf{H}_i^{(1)}$  that corresponds to the largest singular value. The proposed relay assignment policies require the knowledge of global channel state information at a central controller (centralized scheme) and the PUs (distributed scheme). The CSM scheme does not guarantee a fairness due to the selfish approach of PUs, however, the fairness among the PUs is assured in the DPM scheme.

## A. Centralized Successive Matching (CSM)

In the CSM scheme, the PU pair having the maximum rate in the system, given by (6), is given the first priority to select the highest rate yielding relay and this relay cannot be selected by remaining N-1 PUs. In the next step, the PU pair with the second best rate in the system is given the priority to select the highest rate yielding relay from the remaining K-1 relays. Subsequently, the process continues until all the relays in the system are assigned. Due to the successive assignment, the computational complexity of the CSM scheme is significantly lower than that of the centralized exhaustive matching (CEM) scheme, where the central controller selects the best (based on the sum rate) permutation from all the possible permutations.

# B. Distributed Prioritized Matching (DPM)

The DPM scheme presented in this study relies on the formation of grand coalition between cooperating PUs to

# Algorithm 2 Distributed Prioritized Matching Algorithm

- 1: **Initialization**: i = 1 $\mathcal{P}_1 = \{ PT_1, PT_2, \dots, PT_N \}, \{ PU_{LIST,1,1}, \dots, PU_{LIST,1,N} \}$
- 2: **Update** the priority list  $\mathcal{P}_i$  and  $PU_{LIST,i,n}$ ,  $\forall n$
- 3: Set j = 1
- 4: **Find** the  $j^{\text{th}}$  PU in  $\mathcal{P}_i$  i.e.,  $\mathcal{P}_i(j)$
- 5:  $\mathcal{P}_i(j)$  selects the best available relay from  $PU_{LIST,i,j}$ , which has not been selected by previous prioritized PUs i.e.,  $\mathcal{P}_i(1), \cdots, \mathcal{P}_i(j-1)$
- 6: If  $PU_{LIST,i,j}$  is empty then  $\mathcal{P}_i(j)$  is left unmatched
- 7: Set j = j + 1
- 8: Until j = N, go to step 4
- 9: Set i = i + 1
- 10: Until i = N, go to step 2

increase their sum rate as discussed in [7]. The PUs form a coalition to prioritize their relay selection, where the priority list of the PUs in the  $i^{th}$  round is given by  $\mathcal{P}_i$ , and for the case when i = 1, we have  $\mathcal{P}_1 = \{PT_1, PT_2, \dots, PT_N\}$ . The first PU in the ith round has the first choice to select its best (highest rate yielding relay) ST. The nth PU in the ith round has its own preference list given by PU<sub>LIST,i,n</sub> containing indices of all relays which are sorted in descending order according to the offered rate. In the next round, the priority list is updated in a round robin rotation manner, giving a new priority list  $\mathcal{P}_2 = \{PT_N, PT_1, PT_2, \dots, PT_{N-1}\}$ . The DPM scheme lasts for the total of N number of rounds such that each PU leads the priority list at least once. The implementation steps involved in DPM are summarized in Algorithm 2. Unlike the CSM scheme, the DPM scheme assures the fairness in the relay assignment process among the competing PUs.

## IV. NUMERICAL RESULTS

In this section, we evaluate the performance of relay selection schemes outlined in Section III for the underlay FD CCRN shown in Fig. 1. The path loss of a link is given by  $\eta = (d/d_0)^{-\alpha}$ , where  $d_0 = 100$ m and  $\alpha = 4$ . We assume that PUs and SUs are randomly placed on the y-axis ranging from 0-200m. The relays (STs) are placed towards the right side of the PTs on the x-axis with random distance of 0-100m. Whereas, SRs are located on the further right on the x-axis with random distance of 0-100m from STs. Similarly, PRs are randomly placed on the right side of the x-axis from PTs at a fixed distance of 600m. We set  $M_p = N_p = M_s = N_s = 4$  and assume that the RSI power level,  $\sigma_{\rm SI}^2$ , to be -10 dB, unless stated otherwise. The values of  $\zeta$  and  $\epsilon$  are set to 0.25 and 0.1, respectively. For comparison, we have also plotted the results of CEM, centralized max-min matching (CMM) [10] and random allocation (RA) relay selection schemes.

Figure 2 shows the average sum rate of PUs, where the transmit SNRs of the PU link and the SU link are given by  $\rho_{\rm PU} = P_t/N_0 = 10$  dB and  $\rho_{\rm SU} = P_s/N_0 = 10$  dB, respectively. Here, we set N = 5 and K = 2, ..., 8. For the HD case [7], the time slot allocation factor  $\beta$  is set to 0.6, meaning that the PU will use a fraction  $\beta$  of the time slot while  $(1 - \beta)$ of the time slot is used by the SU. It can be seen that the FD based relay selection schemes perform better than the HD

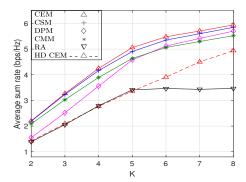


Fig. 2. The average sum rate of PUs for the varying number of SUs.

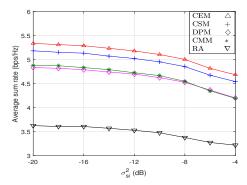


Fig. 3. The average sum rate of PUs for the range of RSI power levels.

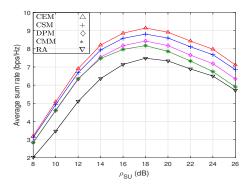


Fig. 4. The average sum rate of PUs for different values of  $ho_{SU}$ .

relaying scheme. This is because in the FD based schemes, the SUs utilize the full time slot along side the PUs. We also notice that the performance of the proposed CSM scheme is marginally lower than that of the exhaustive CEM scheme.

The impact of the RSI power,  $\sigma_{SI}^2$ , on the sum rate performance is shown in Fig. 3 with N=K=5. As expected, when the RSI power increases the performance of PUs decreases. The SI should be suppressed effectively otherwise the performance degrades significantly, specifically a greater than 200 % performance drop is observed in the achievable rate when SI increases from -20 dB to -2 dB. Fig. 4 shows the performance of PUs when  $\rho_{SU}$  increases, while  $\rho_{PU}$  is set to 10 dB. Initially, the average sum rate increases, however, it degrades at high SNR values. This is due to the fact that at high SU SNRs, interference at PRs dominate, hence, yielding a lower sum rate. Fig. 5 shows the average rate of a single SU for various K values. As each SU communicates in a different frequency resource, the SU rate does not vary with K. We also

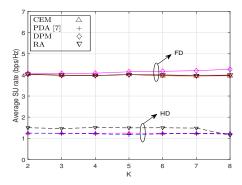


Fig. 5. The average rate of a single SU against various values of K.

plot the results for the pragmatic distributed algorithm based relay selection [7] for the HD relays. We noticed that the FD mode provides a higher rate for the SU than the HD mode.

Besides these results, it has also been observed that the PU rate with the optimized transmission power at the FD relay are significantly higher than the rate with no power optimization. With no power optimization in (12), i.e., an equal power allocation at the relay, the SU rate improves and the PU rate decreases.

## V. CONCLUSION

We proposed two relay selection strategies, namely CSM and DPM, which are computationally efficient than the exhaustive CEM and CMM schemes. Numerical results also revealed that the sum rate performance of the proposed schemes matches to that of the high complexity relay selection schemes.

#### REFERENCES

- G. Zheng, I. Krikidis, and B. Ottersten, "Full-duplex cooperative cognitive radio with transmit imperfections," *IEEE Trans. Wireless Commun.*, vol. 12, no. 5, pp. 2498–2511, May 2013.
- [2] L. Liu, Y. Zhou, L. Tian, and J. Shi, "CPC-based backward-compatible network access for LTE cognitive radio cellular networks," *IEEE Commun. Mag.*, vol. 53, no. 7, pp. 93–99, July 2015.
- [3] G. Zheng, S. Song, K. Wong, and B. Ottersten, "Cooperative cognitive networks: Optimal, distributed and low-complexity algorithms," *IEEE Trans. Signal Process.*, vol. 61, no. 11, pp. 2778–2790, June 2013.
- [4] B. Zhong, Z. Zhang, X. Chai, Z. Pan, K. Long, and H. Cao, "Performance analysis for opportunistic full-duplex relay selection in underlay cognitive networks," *IEEE Trans. Veh. Technol.*, vol. 64, no. 10, pp. 4905–4910, Oct 2015.
- [5] B. Zhong and Z. Zhang, "Opportunistic two-way full-duplex relay selection in underlay cognitive networks," *IEEE Syst. J.*, vol. 12, no. 1, pp. 725–734, March 2018.
- [6] C. Xing, Y. Ma, Y. Zhou, and F. Gao, "Transceiver optimization for multi-hop communications with per-antenna power constraints," *IEEE Trans. Signal Process.*, vol. 64, no. 6, pp. 1519–1534, March 2016.
- [7] W. Liang, S. X. Ng, J. Feng, and L. Hanzo, "Pragmatic distributed algorithm for spectral access in cooperative cognitive radio networks," *IEEE Trans. Commun.*, vol. 62, no. 4, pp. 1188–1200, April 2014.
- [8] M. Sadek, A. Tarighat, and A. H. Sayed, "A leakage-based precoding scheme for downlink multi-user MIMO channels," *IEEE Trans. Wireless Commun.*, vol. 6, no. 5, pp. 1711–1721, May 2007.
- [9] P. C. Weeraddana, M. Codreanu, M. Latva-aho, A. Ephremides, and C. Fischione, "Weighted sum-rate maximization in wireless networks: A review," *Found. Trends Netw.*, vol. 6, no. 1-2, pp. 1–163, 2012.
- [10] X. Zhang, M. Hasna, and A. Ghrayeb, "Performance analysis of relay assignment schemes for cooperative networks with multiple sourcedestination pairs," *IEEE Trans. Wireless Commun.*, vol. 11, no. 1, pp. 166–177, Jan. 2012.