Performance assessment of 3D-mapping-aided GNSS part 1: Algorithms, user equipment, and review

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Abstract
A full performance assessment of 3D mapping–aided (3DMA) GNSS in dense urban areas is presented. This first part of a two-part paper focuses on the effects of algorithm design and user equipment, based on data collected in London using Leica Viva GS15 and u-blox EVK-M8T GNSS receivers and a Nexus 9 Android tablet. Best performance is obtained by combining shadow matching with likelihood-based 3DMA GNSS ranging using hypothesis-domain integration. Improved versions of the algorithms, together with a comprehensive tuning process, are described. Root mean square horizontal position errors obtained using data from Leica, u-blox, and Nexus receivers are 3.5, 4.7, and 4.9 m, respectively, compared with 23.6, 26.4, and 31.0 m using conventional GNSS positioning, about a factor of six improvement. Optimal algorithm tuning depends on the environment and the impact of varying grid spacing of the candidate positions is assessed. Algorithms have also been shown to operate in real time on both a Raspberry Pi 3 and a Samsung Galaxy S8+ Android smartphone.

1 | INTRODUCTION

Conventional global navigation satellite system (GNSS) positioning is poor in dense urban areas because buildings block, reflect, and diffract the signals. Many different studies have now shown that 3D mapping of the buildings can be used to substantially improve GNSS-positioning accuracy in these environments. This two-part paper presents a comprehensive assessment of the performance that can be achieved by 3D mapping–aided (3DMA) GNSS and the different factors that affect it. These may be divided into four categories: algorithm design, user-equipment design, the environment, and mapping quality. This first part of the paper focuses on the effects of algorithm design and user equipment, together with a discussion of the practical implementation of 3DMA GNSS. The second part then investigates how performance is affected by the surrounding environment and the quality of the mapping data and makes recommendations for improving the algorithms based on the results of the study as a whole.

The work presented here builds on several previous studies at University College London (UCL). Shadow matching determines position by comparing the measured signal availability with that predicted over a grid of candidate positions using 3-D mapping. In dense urban environments, it significantly outperforms conventional GNSS positioning in the across-street direction. Using 3D mapping to aid a conventional ranging-based least-squares GNSS positioning algorithm improves its accuracy by about a factor of two in challenging environments. Better performance is obtained using a likelihood-based 3DMA ranging algorithm that scores candidate position...
hypotheses according to the correspondence between measured and predicted pseudoranges. In dense urban environments, 3DMA ranging algorithms substantially improve the positioning accuracy in the along-street direction, but shadow matching is typically more accurate in the across-street direction. Therefore, best performance is usually obtained by combining the two techniques together, which can be done with a suitable direction-dependent weighting of the two position solutions.

3DMA GNSS ranging and shadow matching may also be integrated in the hypothesis domain by combining the two sets of candidate position hypothesis scores before extracting a common position solution. Preliminary results, presented at ION GNSS+ 2016, showed that this gives better performance than position-domain integration. Here, we present final results using improved versions of the hypothesis-domain integration and likelihood-based 3DMA ranging algorithms tested using new experimental data from three different types of GNSS receiver. In addition, a comprehensive tuning process has been implemented and the effects of varying the candidate position hypothesis grid spacing and of tuning the algorithms using data from different environments are investigated for the first time. The second part of the paper then extends the performance analysis to assess the impact of sky visibility, building height and street width, building materials, passing vehicles, and mapping quality on 3DMA GNSS performance. All results presented in both parts of the paper are from algorithms using a single-epoch of GNSS pseudorange and carrier-power-to-noise density ratio (C/N0) measurements. However, multi-epoch positioning is also discussed.

An improvement in the real time–position accuracy of low-cost GNSS user equipment in dense urban areas to 5 m or better would benefit many different potential applications. These include the following: situation awareness of emergency, security, and military personnel and vehicles; emergency caller location; mobile mapping; tracking vulnerable people and valuable assets; intelligent mobility; location-based services; location-based charging; augmented reality; and enforcement of curfews, restraining orders, and other court orders. A further accuracy improvement to around 2 m would also enable navigation for the visually impaired; lane-level road positioning for intelligent transportation systems; aerial surveillance for law enforcement, emergency management, building management, and newsgathering; and advanced rail signaling.

This paper begins with a review of 3DMA GNSS, discussing the different algorithms that have been implemented by researchers around the world. This is followed by a description of the 3DMA GNSS position algorithms used for the present study, updated from another study, and a discussion of how variations in the design can potentially impact positioning performance. This is followed by a discussion of how the user equipment design impacts 3DMA GNSS. The experimental data collection is then described, followed by a description of the algorithm tuning process. The positioning results are then presented and discussed. Data was collected using a Nexus 9 tablet, representing smartphone applications, a u-blox EVK-M8T receiver, representing general applications, and a Leica Viva geodetic receiver to determine the best achievable performance. Finally, the practicalities of real-time implementation are discussed, conclusions are summarized, and a brief outline of part 2 of the paper is presented.

2 | REVIEW OF 3D MAPPING–AIDED GNSS

2.1 | The urban-positioning problem

Buildings and other obstacles degrade GNSS positioning in three ways. First, where signals are completely blocked, they are simply unavailable for positioning, degrading the signal geometry. Second, where the direct signal is blocked (or severely attenuated), but the signal is received via a (much stronger) reflected path, this is known as non-line-of-sight (NLOS) reception. NLOS signals exhibit positive ranging errors corresponding to the path delay (the difference between the reflected and direct paths). These are typically a few tens of meters in dense urban areas but can be much larger if a signal is reflected by a distant building. Third, where both direct line-of-sight (LOS) and reflected signals are received, multipath interference occurs. This can lead to both positive and negative ranging errors, the magnitude of which depends on the signal and receiver designs. NLOS reception and multipath interference are often grouped together and referred to simply as “multipath.” However, to do so is highly misleading as the two phenomena have different characteristics and can require different mitigation techniques.

There are many different approaches to multipath and NLOS mitigation. A good GNSS antenna is more sensitive to right-hand circularly polarized (RHCP) signals than to left-hand circularly polarized (LHCP) signals. As direct LOS signals are RHCP while most reflected signals are LHCP or mixed polarization, this reduces multipath errors by attenuating the reflected signal components with respect to the direct component. However, cheaper antennas offer less polarization discrimination and smartphone antennas none at all.

The measured signal to noise ratio (SNR) is usually lower for NLOS signals, enabling them to be eliminated from the position calculation. However, this is not always the case, as discussed later in the paper. NLOS detection
can be enhanced using a dual-polarization antenna\textsuperscript{10} or by using an antenna array to measure angle of arrival.\textsuperscript{11} However, additional hardware is not always practical.

Much of the literature on multipath mitigation is dominated by receiver-based signal-processing techniques.\textsuperscript{12} However, because they work by separating out the direct and reflected signals within the receiver, they can generally only be used to mitigate multipath; they have no effect on NLOS reception at all. An exception is synthetic aperture GNSS, which increases the receiver sensitivity for direct LOS signals but not generally for reflected components.\textsuperscript{13}

Consistency checking aims to select the optimum subset of received signals from which to compute a position solution. This is based on the principle that measurements from “clean” direct LOS signals produce a more consistent navigation solution (ie, with smaller residuals) than those from NLOS and severely multipath-contaminated signals. In dense urban areas, a subset comparison approach is more robust than conventional sequential testing.\textsuperscript{14,15}

When a filtered navigation solution is implemented, measurement innovation-based outlier detection can also be used. Furthermore, by comparing a series of innovations, NLOS reception, characterized by a bias, may be distinguished from multipath interference, characterized by a larger than normal variance.\textsuperscript{16}

### 2.2 | 3D mapping

From 2010 onwards, there has been a lot of interest in using 3D mapping data to improve GNSS positioning accuracy in dense urban areas. Many different approaches have been developed. The simplest of these is terrain-height aiding. For most land applications, the antenna is at a known height above the terrain, so by using a digital terrain model (DTM) or digital elevation model (DEM), the position solution may be constrained to a surface, effectively removing a dimension from the position solution. In conventional least-squares positioning, this is done by generating a virtual ranging measurement.\textsuperscript{17} In open areas, this only improves vertical positioning. However, in dense urban areas where the signal geometry is poor, terrain-height aiding can improve the horizontal accuracy by almost a factor of two.\textsuperscript{3}

3D models of the buildings can be used to predict which signals are blocked and which are directly visible at any location. This can be done by ray intersection,\textsuperscript{18} ray tracing,\textsuperscript{19} or image generation.\textsuperscript{20} Where multiple candidate positions must be considered, this can be very computationally intensive using a conventional central processing unit (CPU). A graphics processing unit (GPU) can perform the computations much more quickly using parallel processing but can increase the power consumption. Another approach is to use precomputed building boundaries.\textsuperscript{21} These describe the minimum elevation above, which satellite signals can be received at a series of azimuths and are precomputed for each candidate position. A signal can then be classified as LOS or NLOS simply by comparing the satellite elevation with that of the building boundary at the corresponding azimuth; the real-time processing load for this is minimal.

Positioning algorithms that make use of satellite visibility predictions from 3D mapping fall into two main categories. The first are 3DMA GNSS ranging techniques that generally use pseudorange measurements, like conventional GNSS positioning algorithms. The second are shadow-matching techniques, which make use of GNSS signal to noise ratio (SNR) or carrier-power-to-noise-density ratio, $C/N_0$, measurements. Both approaches can be implemented either using data from a single-epoch or using a Bayesian filter to combine data from multiple epochs. For applications such as navigation that require continuous positioning, filtering improves accuracy by reducing the impact of noise-like errors on the position solution, just as in conventional GNSS positioning.

3DMA GNSS ranging and shadow matching are reviewed in turn, followed by a discussion of integrated techniques that combine GNSS ranging with shadow matching and/or other navigation technologies. It is difficult to compare the performance of the different techniques based on the published literature as each author uses different test sites, different user equipment, and different sources of 3D mapping, all of which impact positioning performance. Similarly, single-epoch algorithms cannot be directly compared with filtered methods. However, all of the different techniques achieve significantly better results than conventional GNSS positioning in dense urban areas.

### 2.3 | 3D mapping–aided ranging

The simplest 3D mapping–aided GNSS ranging techniques assume that the user position is already approximately known and simply use a 3D city model to predict the NLOS signals and eliminate them from the position solution.\textsuperscript{22–24} However, for most urban positioning applications, the position uncertainty is too large to be able to make confident satellite visibility predictions. A relatively simple solution to this problem is to consider a search area centered on the conventional GNSS position solution and compute the proportion of candidate positions within this area at which each signal is predicted to be receivable via direct LOS. This can then be used to
reweight a least-squares position solution and also aid consistency checking.  

More sophisticated approaches score a series of position hypotheses using the GNSS pseudo-range measurements and the satellite visibility predictions at each of the candidate positions. In another study, candidate positions are scored according to the correspondence between the measured and predicted pseudoranges, using different distributions for satellites predicted to be direct LOS and NLOS. In a previous study, a least-squares position solution is computed using only those signals predicted to be direct LOS, and the candidate position is then scored according to its Mahalanobis distance from the least-squares position solution.

Several groups have extended 3D mapping–aided GNSS ranging by using the 3D city model to predict the path delay of the NLOS signals across an array of candidate positions. This then enables more accurate scoring of candidate positions by comparing measured and predicted pseudoranges. However, this approach is more computationally intensive as the path delays cannot easily be precomputed. The processing load can be reduced by using predicted pseudoranges based on the satellite elevation angle and distance to the reflected surface. The urban trench approach presented in an existing study enables the path delays of NLOS signals to be computed very efficiently but only if the building layout is highly symmetric, so it can only be used in suitable environments.

For multipath-contaminated signals, 3D mapping can also be used to predict the path delay(s) of the reflected component(s). Predicting the pseudorange errors due to multipath is impractical because this also requires estimates of the relative amplitude of the reflected signal and the phase delay. The former requires building reflectivity data at GNSS wavelengths, which 3D mapping does not provide, while the latter requires centimeter accuracy for both the 3D mapping and the receiver antenna position. However, by using signal processing techniques to separate out the different components of a received multipath signal, the reflected components can be treated as additional NLOS signals that can be used to enhance the position solution, while the direct LOS components have much smaller ranging errors. A bespoke receiver design, which is easiest to implement using a software receiver, is needed for the component separation. Channel estimation techniques can be used to extract multiple signal components from 10 or more correlation channels separated in the range domain. If correlation is also performed at different Doppler shifts, the signal components can also be separated in the Doppler domain, provided the receiver antenna is moving with respect to the reflecting surfaces. Synthetic aperture GNSS could also be used for dynamic applications.

3DMA GNSS ranging has also been combined with “direct positioning,” which uses the receiver correlator outputs to score an array of position hypothesis. A similar approach determines position by matching the predicted path delay(s) of the reflected component(s) at each candidate position with the outputs of a signal correlator bank.

2.4 | Shadow matching

GNSS shadow matching determines position by comparing the measured signal availability and strength with signal visibility predictions made using a 3D city model over a range of candidate positions. The concept was independently conceived by four different research groups, each publishing before becoming aware of the work of the others. Several groups then demonstrated this experimentally in 2011 to 2012 for static positioning, using both single and multiple epochs of GNSS data. In another study, reflection and diffraction of GNSS signals is considered in addition to the direct signal visibility. A real-time demonstration of shadow matching on an Android smartphone then followed in 2013. NLOS signals can be strong when the reflecting surface is metallic or metallized glass, while direct LOS signals can sometimes be attenuated by people or foliage. Moreover, a smartphone uses a linear antenna, which does not attenuate LHCP reflected signals with respect to RHCP direct signals. Thus, better performance is obtained by inferring a direct LOS probability from the SNR or C/N0 measurements instead of a hard threshold.

For moving pedestrians, several research groups have demonstrated multi-epoch shadow matching using a particle filter. For vehicle applications, it has been shown that shadow matching can be used for road lane identification in relatively open areas by making use of the shadows cast by mobile-phone masts. However, detecting these at speed requires receiver firmware modifications in order to obtain C/N0 measurement with a high spatial resolution.

2.5 | Integrated solutions

Shadow matching provides across street–accuracy of a few meters in dense urban areas, but along-street performance can be poor, particularly where there is a high degree of translational symmetry along the street. Conversely, ranging-based GNSS positioning, whether conventional or aided by 3D mapping, is generally more...
accurate in the along-street direction in dense urban areas due to the geometry of the direct LOS signals. Best performance is thus obtained by combining shadow matching with ranging-based GNSS positioning, a concept sometimes known as “intelligent urban positioning.” This was first demonstrated in a previous research\textsuperscript{49} simply by combining the across-street component of the shadow-matching solution with the along-street component of a conventional GNSS solution. In another study,\textsuperscript{50} a particle filter is used to integrate shadow matching with a conventional GNSS position solution, modeled with a non-Gaussian error distribution. In a previous study,\textsuperscript{5} shadow matching is integrated with 3DMA least-squares GNSS ranging in the position domain using two different weighting approaches. Error covariance-based weighting was found to perform slightly better than weighting using the street azimuth. In another existing research,\textsuperscript{6} shadow matching is integrated with likelihood-based 3DMA GNSS ranging in both the position and hypothesis domains. A hypothesis-domain integration of shadow matching with Mahalanobis distance-based 3DMA GNSS ranging is presented in a previous study.\textsuperscript{25}

3DMA GNSS has also been combined with other navigation technologies. In another research,\textsuperscript{51} 3DMA GNSS ranging is integrated with an IMU and odometer for road vehicle applications. In an existing study,\textsuperscript{52} 3DMA GNSS ranging is integrated with pedestrian dead reckoning (PDR) using step detection. In both cases, the integrated navigation solution using 3DMA GNSS is shown to be better than that using conventional GNSS positioning.

Here, the aim is to assess 3DMA GNSS, so other navigation technologies are not used. In order to achieve real-time operation while also handling large initialization errors, only algorithms that predict satellite visibility using precomputed building boundaries are considered. For best performance, both pseudorange and C/N\textsubscript{0} measurements should be used. Therefore, the shadow-matching algorithm from the previous research\textsuperscript{2} is combined with an improved version of the likelihood-based 3DMA-ranging algorithm from the previous study\textsuperscript{4} using the position-domain integration algorithm from another study\textsuperscript{5} and an improved version of the hypothesis-domain integration algorithm from.\textsuperscript{6} The 3DMA least-squares GNSS ranging algorithm from an existing research\textsuperscript{3} is used for initialization in order to reduce the number of candidate positions that subsequent algorithms must handle.

The present study is limited to single-epoch positioning. Several authors have demonstrated that filtering over multiple epochs can improve 3DMA GNSS performance.\textsuperscript{25,46,47} A filtered version of the algorithm set presented here is currently under development.

### 3 | POSITIONING ALGORITHMS

The integrated 3DMA GNSS positioning system assessed here comprises four main algorithms as shown in Figure 1. The least-squares 3DMA GNSS ranging algorithm is used to initialize the likelihood-based 3DMA GNSS ranging algorithm and the shadow-matching algorithm, enabling them to use a much smaller search area than if the conventional GNSS position was used for initialization. The integration algorithms then compute a joint position solution from likelihood-based 3DMA GNSS ranging and shadow matching. Both a position-domain integration algorithm and a hypothesis-domain integration algorithm are presented. The following subsections summarize each algorithm and comprise updated versions of the descriptions in the other study.\textsuperscript{6} This is followed by a discussion on the impact of variations in the algorithm design on positioning performance and processing efficiency.

Both 3DMA GNSS ranging algorithms and the shadow matching algorithm use precomputed building boundaries, derived from 3D mapping as described in the existing research.\textsuperscript{2} Each building boundary comprises the elevation above which GNSS signals are directly visible across a set of discrete azimuths; an azimuth interval of $1^\circ$ is currently in use, and the elevation resolution is 1/128th of a right angle (about 0.7°). This is sufficient resolution for the test sites, as discussed further in part 2 of this paper.\textsuperscript{1} A building boundary is computed for every outdoor location within a one-meter-interval grid of candidate positions. For the results presented here, building heights were obtained from the 2.5D building model, which is generated by combining two Ordnance Survey (OS) data sets. The first is the OS MasterMap Building Heights (Beta) Layer. The Height attributes provided for each building are the following: ground level, the base of the roof, and the highest part of the roof\textsuperscript{53}; the mean of the two roof heights

![FIGURE 1 Intelligent urban positioning→algorithm configuration](https://wileyonlinelibrary.com)
is used to calculate the building boundaries. The second data set is OS Terrain 5. This is a digital terrain model (DTM), which consists of a grid of height values at 5 m horizontal intervals. This is taken as the base height on which the OS MasterMap Building Heights layer floats.

Most of the algorithms incorporate empirically determined tuning parameters. The values of those parameters are given here, while the tuning process is described in a separate section between the data collection and results sections. For most of the results presented here, calibration data from all test sites was used.

3.1 | Least-squares 3DMA GNSS ranging

Figure 2 shows the least-squares 3DMA GNSS ranging algorithm, comprising the following six steps:

1. A search area is determined using the conventional GNSS position solution on the first iteration and the previous solution on subsequent iterations, together with an appropriate confidence interval.
2. Using the precomputed building boundaries, the proportion of the search area within which each satellite is directly visible is computed, giving the probability that the signal is direct LOS.
3. A consistency-checking process is applied to the ranging measurements, using the direct LOS probabilities from the 3D mapping.
4. The set of signals resulting from the consistency checking process is subjected to a weighting strategy based on the previously determined LOS probabilities and \( C/N_0 \) measurements.
5. Terrain height is extracted from the 3D mapping and a virtual range measurement is generated using the position at the center of the search area.
6. Finally, a position solution is derived from the pseudoranges and virtual range measurement using weighted least-squares estimation.

The algorithm is then iterated several times to improve the position solution. Full details are presented in an existing study.

Projected coordinates (eastings and northings) are used for the 3D mapping while Cartesian ECEF (Earth Centered, Earth Fixed) coordinates are used for the least-squares position solution. Conversion between Cartesian ECEF and projected coordinates can be simplified using a nearby reference point.

3.2 | Likelihood-based 3DMA GNSS ranging

In likelihood-based 3DMA GNSS ranging, an array of candidate position hypotheses are scored according to the correspondence between the predicted and measured pseudoranges. This enables different error distributions to be assumed for a given GNSS signal at different candidate positions. Thus, at positions where a signal is predicted from the 3D mapping (via precomputed building boundaries), to be NLOS, a skew normal (Gaussian) distribution is assumed, biased towards positive ranging errors. Elsewhere, a conventional symmetric normal distribution is assumed.

Terrain-height aiding is inherent in generating the position hypotheses, enabling a single height to be associated with each horizontal position and thus avoiding the computational load of a 3D search area. For airborne applications, a barometric altimeter can be used to provide the height. All pseudorange measurements are differenced across satellites in order to cancel out the receiver clock bias so that this does not have to be estimated as part of the position solution.

Figure 3 shows the likelihood-based 3D model–aided ranging algorithm, comprising the following six steps:

1. A circular search area of radius 40 m is defined with its center at the least-squares 3DMA GNSS ranging position solution. This was the smallest radius that gave reliable performance. Within this search area, a grid of candidate positions is set up with a spacing of one meter except where stated otherwise.
2. For each candidate position, the satellite visibility is predicted using the building boundaries precomputed...
from the 3D city model. At each candidate position, the highest elevation satellite predicted to be direct LOS is selected as the reference satellite.

3. At each candidate position, the direct LOS range to each satellite is computed. Measurement innovations are then computed by subtracting the computed ranges from the measured pseudoranges and then differencing with respect to the reference satellite. The error standard deviation is computed as a function of $\frac{C}{N_0}$ and satellite elevation angle as described below.

4. At each candidate position, the measurement innovation for each satellite predicted to be NLOS is remapped. The skew normal distribution is used to determine the cumulative probability. The corresponding direct LOS innovation with the same cumulative probability is then substituted.

5. A likelihood score for each candidate position, $p$, is computed using

$$\Lambda_{Rp} = \exp\left(-\delta z_p^T C_{\delta z_p}^{-1} \delta z_p\right),$$

where $\delta z_p$ is the vector of measurement innovations and $C_{\delta z_p}$ is the measurement error covariance matrix, computed using the direct-LOS-hypothesis measurement error standard deviations, which are the same for all candidate positions.

6. A position solution is derived from the scores of the candidate positions using

$$\hat{E}_R = \frac{\sum_p \Lambda_{Rp} E_p}{\sum_p \Lambda_{Rp}}, \quad \hat{N}_R = \frac{\sum_p \Lambda_{Rp} N_p}{\sum_p \Lambda_{Rp}},$$

where $E_p$ and $N_p$ are the easting and northing coordinates of the $p$th candidate position.

Full details of the algorithm are described in the previous study with the exception of the pseudorange error standard deviation model, which has been modified as follows. The LOS error standard deviations are computed using Brunner et al:

$$\sigma_{j,LOS} = \sqrt{b_L + a_L 10^{-\left(\frac{(C/N_0)_j}{10}\right) - m_L \delta_j}},$$

where $j$ denotes the satellite ($j = r$ for the reference satellite); $(C/N_0)_j = 10\log_{10}(c/n_0)_j$ is the measured carrier-power-to-noise density in dB-Hz; $d_j$ is the discrepancy between the measured $C/N_0$ and the expected value for the satellite elevation; and $a_L$, $b_L$, and $m_L$ are empirically determined coefficients. The values used are given in Table 1. The discrepancy term helps to account for the effects of multipath interference as constructive interference results in an increase in $C/N_0$ while destructive interference results in a decrease. The larger the $C/N_0$ perturbation, the larger the potential multipath error. The discrepancy is calculated using

$$d_j = |(C/N_0)_j - T(\theta_j)|,$$

where $\theta_j$ is the elevation angle and $T$ is an empirically determined $C/N_0$ template function. Figure 4 shows an example.

For the signals predicted to be NLOS, the $C/N_0$ template function is not applicable, so the “LOS error standard

<table>
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<th>Parameter</th>
<th>Leica Viva GS15, All Sites</th>
<th>u-blox EVK MST, All Sites</th>
<th>u-blox EVK MST, Canary Wharf Only</th>
<th>Nexus 9 Tablet, All Aites</th>
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<tr>
<td>LOS error variance coefficient, $a_L$, m$^2$Hz</td>
<td>19 500</td>
<td>205 700</td>
<td>200 600</td>
<td>238 200</td>
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<td>LOS error variance coefficient, $b_L$, m$^2$</td>
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<td>$C/N_0$ discrepancy coefficient, $m_L$</td>
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<td>2.2</td>
<td>2.4</td>
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</tr>
<tr>
<td>NLOS error mean, $\mu_N$, m</td>
<td>42</td>
<td>42</td>
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<td>68</td>
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<tr>
<td>NLOS error standard deviation, $\sigma_N$, m</td>
<td>33</td>
<td>36</td>
<td>20</td>
<td>58</td>
</tr>
</tbody>
</table>
deviation,” which accounts for all pseudorange errors except for the NLOS reception error is computed using

$$\sigma^{\text{NLOS}}_j = \sqrt{b_0 + \frac{a_1}{(C/N_0)_j}}.$$  (5)

with the coefficients taking the same values as in the direct LOS case. The NLOS reception error is modeled separately with mean $\mu^\text{N}$ and standard deviation $\sigma^\text{N}$. As NLOS reception and multipath interference can occur together (ie, when there are multiple reflected paths), multipath must then be accounted for within the NLOS reception error standard deviation, $\sigma^\text{N}$. This is reasonable as, in the NLOS case, the multipath error can be considered to be a perturbation of the NLOS reception error.

Where there is a single-reflected signal path, the mean, $\mu^\text{N}$, and standard deviation, $\sigma^\text{N}$, of the NLOS reception error should be receiver independent. However, the standard deviation of multipath-induced perturbations to the NLOS reception error will potentially vary between receivers. Empirically determined values for each receiver are included in Table 1.

### 3.3 | Shadow matching

The shadow-matching algorithm is a modified version of that presented in previous study.³ Again, positioning is horizontal only with a terrain-height database used to associate a height with each horizontal position. Figure 5 shows the algorithm, comprising the following five steps:

1. A circular search area of radius 40 m is defined with its center at the least-squares 3DMA GNSS ranging position solution. This was the smallest radius that gave reliable performance. Within this search area, a grid of candidate positions is set up with a spacing of one meter except where stated otherwise.

2. For each candidate position, $p$, the satellite visibility is predicted using the building boundaries pre-computed from the 3D city model. If the satellite elevation is above the building boundary at the relevant azimuth, the LOS probability predicted from the building boundary, $p(LOS|BB)_{BB}$, is set to 0.8. Otherwise, it is set to 0.2. These values were determined empirically and allow for diffraction and 3D model errors.

3. The observed satellite visibility is determined from the GNSS receiver’s $C/N_0$ measurements. From these, a probability that each received signal is direct LOS, $p(LOS|C/N_0)$, is estimated using

$$p(LOS|C/N_0) = \begin{cases} P_{0-\min} & (C/N_0) < s_{\min} \\ a_2 \left( (C/N_0) \right)^2 + a_1 (C/N_0) + a_0 & s_{\min} < (C/N_0) < s_{\max} \\ P_{0-\max} & s_{\max} < (C/N_0) \end{cases}$$  (6)
where the empirically determined coefficients are listed in Table 2. If no signal is received at all, \( p(\text{LOS} | C/N_0) \) is set to \( p_{0-\text{min}} \).

4. Each candidate position is scored according to the match between the predicted and measured satellite visibility. For a given satellite, the probability that the predicted and measured satellite visibility match is

\[
P_{\text{Mp}} = p(\text{LOS} | C/N_0) p(\text{LOS} | BB)_p + \left[ 1 - p(\text{LOS} | C/N_0) \right] \left[ 1 - p(\text{LOS} | BB)_p \right] + 2p(\text{LOS} | C/N_0) p(\text{LOS} | BB)_p.
\]

The overall likelihood score, \( \Lambda_{Sp} \), for each position, \( p \), is then the product of the individual satellite probabilities:

\[
\Lambda_{Sp} = \prod_p P_{Mp}.
\]

5. A position solution is derived from the scores of the candidate positions using

\[
\hat{E}_S = \frac{\sum_p \Lambda_{Sp} E_p}{\sum_p \Lambda_{Sp}}, \quad \hat{N}_S = \frac{\sum_p \Lambda_{Sp} N_p}{\sum_p \Lambda_{Sp}},
\]

where \( E_p \) and \( N_p \) are the easting and northing coordinates of the \( p \)th candidate position.

### 3.4 Position-domain integration

The position domain–integration algorithm uses the error covariance matrices of the 3DMA GNSS ranging and shadow-matching position solutions to compute a weighted average of the two positions using

\[
\mathbf{x}_u^{\text{EN}} = \left[ (\mathbf{C}_S^{\text{EN}})^{-1} + (\mathbf{C}_R^{\text{EN}})^{-1} \right]^{-1} \left[ (\mathbf{C}_S^{\text{EN}})^{-1} \mathbf{x}_S^{\text{EN}} + (\mathbf{C}_R^{\text{EN}})^{-1} \mathbf{x}_R^{\text{EN}} \right].
\]

where \( \mathbf{x}_u^{\text{EN}} = \left( \hat{E}_a \hat{N}_u \right)^T \) is the integrated position solution of the user antenna, \( a \); \( \mathbf{x}_S^{\text{EN}} = \left( \hat{E}_S \hat{N}_S \right)^T \) is the shadow-matching solution; \( \mathbf{x}_R^{\text{EN}} = \left( \hat{E}_R \hat{N}_R \right)^T \) is the 3DMA GNSS ranging solution; \( \mathbf{C}_S^{\text{EN}} \) is the shadow-matching error covariance; and \( \mathbf{C}_R^{\text{EN}} \) is the 3DMA GNSS ranging error covariance.

For shadow matching and likelihood-based 3DMA GNSS ranging, an error covariance must be extracted from a likelihood surface that is non-Gaussian and potentially multimodal. The error covariance therefore needs to be larger for multimodal distributions than it is for unimodal. The error covariance is therefore calculated using the following steps:

1. Compute an initial error covariance from the second statistical moments of the likelihood surface.
2. Determine the directions of the maximum and minimum of the error ellipse corresponding to the initial error covariance.
3. Transform to coordinates aligned with the maximum-covariance (\( x \)) and minimum-covariance (\( y \)) directions.
4. Compute the kurtoses of the likelihood surface in each direction, \( \kappa_x \) and \( \kappa_y \).
5. Rescale the error ellipse using the two kurtoses using

\[
S_x = \begin{cases}
S_{0x}, & 1.8 \geq \kappa_x \\
(S_{0x} - 1) \frac{3 - \kappa_x}{1.2} + 1, & 3 \geq \kappa_x \geq 1.8 \\
1, & \kappa_x \geq 3
\end{cases},
\]

\[
S_y = \begin{cases}
S_{0y}, & 1.8 \geq \kappa_y \\
(S_{0y} - 1) \frac{3 - \kappa_y}{1.2} + 1, & 3 \geq \kappa_y \geq 1.8 \\
1, & \kappa_y \geq 3
\end{cases},
\]

where \( S_x \) and \( S_y \) are the variance scaling factors in each direction and the coefficients \( S_{0x} \) and \( S_{0y} \) are determined empirically from the calibration data. Table 3 lists the values used.

### TABLE 2 Coefficients for determining direct LOS probability from measured SNR

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Leica Viva GS15, All Sites</th>
<th>u-blox EVK M8T, All Sites</th>
<th>u-blox EVK M8T, City Only</th>
<th>u-blox EVK M8T, Canary Wharf Only</th>
<th>Nexus 9 Tablet, All Sites</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_{0-\text{min}} )</td>
<td>0.33</td>
<td>0.23</td>
<td>0.24</td>
<td>0.21</td>
<td>0.17</td>
</tr>
<tr>
<td>( s_{\text{min}}, \text{dB-Hz} )</td>
<td>26</td>
<td>23</td>
<td>24</td>
<td>22</td>
<td>20</td>
</tr>
<tr>
<td>( a_0 )</td>
<td>−4.834</td>
<td>−3.482</td>
<td>−3.025</td>
<td>−1.963</td>
<td>−0.9455</td>
</tr>
<tr>
<td>( a_{10}, (\text{dB-Hz})^{-1} )</td>
<td>0.2976</td>
<td>0.2259</td>
<td>0.2035</td>
<td>0.1182</td>
<td>0.0617</td>
</tr>
<tr>
<td>( a_{2}, (\text{dB-Hz})^{-2} )</td>
<td>−0.004821</td>
<td>−0.003036</td>
<td>−0.001922</td>
<td>−0.000892</td>
<td>0.00053</td>
</tr>
<tr>
<td>( p_{0-\text{max}} )</td>
<td>0.84</td>
<td>0.9</td>
<td>0.88</td>
<td>0.91</td>
<td>0.9</td>
</tr>
<tr>
<td>( s_{\text{max}}, \text{dB-Hz} )</td>
<td>29</td>
<td>31</td>
<td>30</td>
<td>33</td>
<td>33</td>
</tr>
</tbody>
</table>
TABLE 3  Covariance scaling coefficients for position-domain integration

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Leica Viva GS15, All Sites</th>
<th>u-blox EVK M8T, All Sites</th>
<th>Nexus 9 Tablet, All Sites</th>
</tr>
</thead>
<tbody>
<tr>
<td>(S_{0x}) for likelihood-based 3DMA ranging</td>
<td>1.6</td>
<td>1.9</td>
<td>2.5</td>
</tr>
<tr>
<td>(S_{0y}) for likelihood-based 3DMA ranging</td>
<td>1.6</td>
<td>1.8</td>
<td>2.3</td>
</tr>
<tr>
<td>(S_{0x}) for shadow matching</td>
<td>1.9</td>
<td>2.2</td>
<td>2.1</td>
</tr>
<tr>
<td>(S_{0y}) for shadow matching</td>
<td>2.0</td>
<td>2.3</td>
<td>2.4</td>
</tr>
</tbody>
</table>

6. Transform back to east- and north-aligned coordinates

Full details of this process are presented in the previous study.\(^{5}\)

3.5  Hypothesis-domain integration

Both shadow matching and likelihood-based 3DMA GNSS ranging can produce multimodal position distributions where there is a good match between predictions and measurements in more than one part of the search area. These will typically comprise the true position hypothesis and one or more false hypotheses. In general, the true position hypothesis will be consistent across the two positioning methods whereas the false hypotheses will not be. Hypothesis-domain integration therefore helps to eliminate false position hypotheses by computing a joint ranging and shadow matching–likelihood surface prior to determining a position solution. Here, it is only applied to likelihood-based 3DMA GNSS ranging.

The joint likelihoods are computed using

\[
\Lambda_p = \Lambda_r p \Lambda_s p^W_p \forall p, \tag{12}
\]

where \(\Lambda_r p\) is the ranging-based likelihood of point \(p\), given by (1); \(\Lambda_s p\) is the shadow-matching-based likelihood, given by (8); and \(W_p\) is the shadow-matching weighting factor. The weighting approach here is heuristic. If \(W_p\) is greater than one, the combined likelihood will be biased in favor of shadow matching. If it is set to less than one, it will be biased in favor of ranging. Best performance was obtained by selecting a weighting factor proportional to the number of satellites that are NLOS. This is logical as shadow matching doesn’t work in open environments where all of the satellites are direct LOS. The weighting factor is given by

\[
W_p = \frac{\alpha \times m_p^{NLOS}}{m_p^{LOS} + m_p^{NLOS}}, \tag{13}
\]

where \(m_p^{LOS}\) and \(m_p^{NLOS}\) are, respectively, the number of satellites at grid point \(p\) predicted to be direct LOS and NLOS, and \(\alpha\) is an empirically determined positive constant. Table 4 shows the values of \(\alpha\) used for each receiver. Shadow matching is given the highest weighting for the geodetic receiver and the lowest weighting for the tablet. This reflects the fact that the lower the quality of the antenna, the more the LOS and NLOS \(C/N_0\) distributions overlap and thus the more difficult it is to infer satellite visibility from the \(C/N_0\) measurements.

Finally, the position solution is obtained using

\[
\hat{E}_a = \frac{\sum_p \Lambda_r p E_p}{\sum_p \Lambda_r p}, \quad \hat{N}_a = \frac{\sum_p \Lambda_r p N_p}{\sum_p \Lambda_r p}, \tag{14}
\]

where \(E_p\) and \(N_p\) are the easting and northing coordinates of the \(p\)th candidate position.

Figure 6 shows example likelihood surfaces from 3DMA GNSS ranging, shadow matching, and the hypothesis-domain-integrated solution using a u-blox GNSS receiver at test site 2A (see the experimental data collection section below). In each case, the highest likelihood area is shown in red. Here, 3DMA GNSS ranging gives a clear position solution with the true position within the highest scoring region, but it is more accurate in the along-street direction than the across-street direction. Conversely, the shadow matching–likelihood surface shows a sharp maximum in the across-street direction incorporating the true position. However, along-street precision is poor with the maximum likelihood area extending about 20 m along the street. The integrated likelihood surface has a much sharper maximum than either 3DMA GNSS ranging or shadow matching with the center of the highest scoring region very close to the true position.

3.6  Algorithm design variations

The detailed design of the 3DMA GNSS algorithms is largely a trade-off between performance and processing load. Thus, a smaller grid spacing between candidate positions could improve the accuracy, while a larger grid spacing will reduce the processing load, which varies approximately as the inverse square of the grid spacing.

TABLE 4  Values of the empirically determined constant, \(\alpha\), used in the hypothesis-domain integration algorithm

<table>
<thead>
<tr>
<th>Data set</th>
<th>Leica Viva GS15, All sites</th>
<th>u-blox EVK M8T, All sites</th>
<th>u-blox EVK M8T, Canary Wharf only</th>
<th>u-blox EVK M8T, City only</th>
<th>Nexus 9 tablet, All sites</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha)</td>
<td>2.4</td>
<td>1.1</td>
<td>1.3</td>
<td>0.8</td>
<td>0.2</td>
</tr>
</tbody>
</table>
This is investigated at the end of the experimental results section.

Another design factor is the size of the search area. Shrinking the search area reduces the processing load, but also increases the chance of the true position being outside the search area, in which case the 3DMA GNSS algorithm cannot generate a correct positioning solution. Furthermore, if the confidence interval of the initialization position is not circular, an elliptical search area may be more efficient.56

The final design factor to consider is the statistical distributions of the direct LOS and NLOS pseudorange measurement errors used in the likelihood-based 3DMA GNSS ranging algorithm and of the direct LOS and NLOS C/N0 measurements in the shadow-matching algorithm. The coefficients of these models are empirically determined from calibration data. However, different tuning parameters are required for different types of user equipment, and their optimum values depend on the environment, as shown in the experimental results section. More sophisticated models are proposed in part 2 of this paper.1

4 USER EQUIPMENT DESIGN

3DMA GNSS performance is impacted by the design of both the receiver and the antenna. Receivers vary in their multipath-mitigation performance with survey receivers typically implementing more sophisticated signal-processing techniques and benefiting from a higher bandwidth.8,9,12 New consumer-grade receivers and smartphone GNSS chips that process the new L5/E5 signals are also less sensitive to multipath interference due to the higher chipping rate of those signals. Smaller multipath-induced pseudorange errors directly improve the performance of 3DMA (and conventional) GNSS ranging, while less multipath-induced variation in the measured C/N0 helps shadow matching.

A survey antenna offers better polarization discrimination than that of a consumer-grade antenna, further reducing multipath interference and making it easier for shadow matching to distinguish NLOS and direct LOS signals using C/N0. For both types of antenna, the polarization discrimination and the gain both drop as the angle of incidence at the antenna increases. As these antennas are normally held flat, a high angle of incidence corresponds to a low satellite elevation angle. Thus, the 3DMA GNSS algorithms could be potentially modified to account for this variation in antenna performance with elevation.

A smartphone antenna is linearly polarized with a roughly omni-directional gain pattern.57 This offers no discrimination at all between RHCP and LHCP signals,

FIGURE 6 Normalised log-likelihoods of candidate positions at City of London test location 2A from likelihood-based ranging (top), shadow matching (middle), and hypothesis-domain integration (bottom). The cross shows the true position. White areas are indoors [Color figure can be viewed at wileyonlinelibrary.com and www.ion.org]
increasing the susceptibility to multipath interference. There is therefore a larger overlap between the \( C/N_0 \) measurement distributions of the direct LOS and NLOS signals, making it difficult to distinguish them.\(^2\) Thus, effectively, only the weakest and strongest signals are useful for shadow matching.

Here, results from a tablet (effectively a smartphone), a consumer-grade receiver, and a survey-grade receiver are compared to see how the user equipment design impacts 3DMA GNSS performance.

5 | EXPERIMENTAL DATA COLLECTION

GNSS measurements, comprising GPS and GLONASS, were collected over three days in October 2016 using an HTC Nexus 9 tablet, a u-blox EVK-M8T GNSS receiver, and a Leica Viva GS15 survey-grade GNSS receiver, all shown in Figure 7. On each day, one of the three sets of GNSS equipment was used and all test sites were visited. The Nexus 9 tablet was running Android version 7.0 (Nougat), enabling capture of GNSS “raw data,” including GNSS satellite pseudoranges, alongside conventional NMEA sentences.\(^5\) Data collection was performed using a purposely written application. The tablet’s GNSS receiver and antenna are similar to those found on smartphones, so the results should be a good prediction of the performance of smartphones that provide access to raw GNSS measurements. The u-blox receiver was interfaced to a Raspberry Pi via USB for data logging and also to provide power from a battery pack. A smartphone, connected via Wi-Fi, served as a user interface using the mobile SSH App. Leica Viva data was collected using the receiver’s standard software and stored on an SD card. At the beginning of each data collection session, time was allowed for the receivers to download the satellite ephemeris data and synchronize their clocks. Time-synchronization requirements are the same as for conventional GNSS positioning. The true positions were established to decimeter-level accuracy using a 3D city model to identify landmarks and a tape measure to measure the relative position of the user from those identified landmarks. Interaction with the 3D mapping does not impose additional constraints.

Data was collected using all three devices in two different areas of London: five pairs of points (10 in total) in the City of London and four pairs and a triplet of points (11 in total) in Canary Wharf. Figures 8 and 9 illustrate these sites. The paired locations corresponded to data collected on opposite sides of the street on the edge of the footpath next to the road. The Canary Wharf triplet of locations included a collection point located on an island in the middle of the road. The City of London area is typical of a traditional European city with narrow streets and buildings packed close together. The Canary Wharf area is representative of a modern city environment, found more commonly in North American and East Asian cities. The streets are wider and the buildings taller with more space between them. There is also a greater ratio of glass and steel to brick and stone than in the City of London district.

At each test site, two 4-minute rounds of data were collected using each receiver. These were separated by approximately 2 hours, ensuring that the satellite positions in the two datasets were independent. Each test location is independent of the others because 3DMA GNSS performance depends on the interaction of the satellite signals with the buildings, which are different for each location. The first dataset was used for calibration, as described in
the positioning algorithms section above. The second dataset was then used for testing the positioning algorithms as reported in the experimental results section below. This experimental data was also used to generate the results presented in the previous study. However, only the conventional GNSS positioning results are previously published; the results from all of the 3D mapping-aided positioning algorithms are new, noting that the likelihood-based 3DMA GNSS algorithms have been modified following the work presented in the study.

6 | ALGORITHM TUNING PROCESS

Most of the 3DMA GNSS algorithms described in this paper incorporate empirically determined tuning parameters. The values used are listed in Tables 1–4. This section describes how those values were obtained from the calibration dataset. The optimum values will depend on both the user equipment and the environment. However, algorithms that adapt to the environment are a subject for future research, so calibration data from all test sites was generally used. For the u-blox receiver only, separate calibrations using the City of London and Canary Wharf data were also performed.

6.1 | Likelihood-based 3DMA GNSS ranging

FIGURE 8  Data collection sites in the City of London (GoogleTM earth) [Color figure can be viewed at wileyonlinelibrary.com and www.ion.org]

FIGURE 9  Data collection sites in the Canary Wharf area–London–3-D view (GoogleTM earth) [Color figure can be viewed at wileyonlinelibrary.com and www.ion.org]
both the multipath error and tracking noise depend on the receiver design. Independent values were therefore determined for each of the three receivers using the direct LOS measurements from the calibration data at all test sites, with separate City and Canary Wharf calibrations also performed for the u-blox data. As described in the previous study, a pseudorange error variance, $\sigma^2$, was computed for each value of $C/N_0$, and values of $a_L$ and $b_L$ were determined by fitting a linear function, $\sigma^2 = b_L + a_L/(c/n_0)$, approximating (3), to the data.

The $C/N_0$ template function is obtained empirically from the calibration data by fitting a polynomial function of elevation to the $C/N_0$ measurements of all of the direct LOS signals (as determined using the building boundaries at the true positions). Separate values were determined for each receiver-antenna combination with separate City and Canary Wharf calibrations also performed for the u-blox data.

Values of $m_L$ coefficient of the LOS error variance model (3) were then determined for each receiver. The likelihood-based 3DMA GNSS ranging position solution was calculated from the calibration data using values of $m_L$ that were varied between 1 and 3 with a step of 0.1. A value of $m$ for each receiver was then selected that minimized the root mean square (RMS) position error across all of the test sites, with separate City and Canary Wharf calibrations also performed for the u-blox data.

Next, the mean and standard deviation of the NLOS reception error were then determined. The likelihood-based 3DMA GNSS ranging position solution was calculated from the calibration data using values of $\mu_N$ and $\sigma_N$ that were each independently varied from 20 to 60 m with a step of 1 m. Values of these parameters were then selected that minimized the root mean square (RMS) position error across all of the test sites. The mean, $\mu_N$, was shared across the three receivers while the standard deviation, $\sigma_N$, was determined independently.

### 6.2 Shadow matching

For shadow matching, the coefficients for determining the direct LOS probability from the measured $C/N_0$ in Equation 6 were determined empirically for each receiver from the calibration data collected at all sites using the process described in the other study. Signals were classified as direct LOS or NLOS using the building boundaries at the true positions. Separate $C/N_0$ distributions for the LOS and NLOS signals were then generated from the data and used to infer a LOS probability for each $C/N_0$ value. The coefficients $p_{o_{\text{max}}}$, $p_{o_{\text{min}}}$, $s_{\text{max}}$, and $s_{\text{min}}$ were then determined by manual inspection, and suitable values of $a_0$, $a_1$, and $a_2$ were determined by fitting a quadratic function to the LOS probabilities in the range $s_{\text{min}}$ to $s_{\text{max}}$. Additional calibrations for the u-blox receiver were performed using City data only and Canary Wharf data only. Suitable values for the LOS probability predicted from the building boundary were selected that minimized the root mean square (RMS) position error across all of the test sites. The LOS probability for elevations above the building boundary was varied from 0.7 to 0.9 with a step of the 0.05 and the LOS probability for elevations below the building boundary were set to one minus this value.

### 6.3 Position-domain integration

The covariance scaling coefficients for position-domain integration (see Equation 11) were determined separately for each receiver using the calibration data collected at all sites. The position-domain integrated 3DMA GNSS position solution was calculated using maximum- and minimum-covariance direction-scaling coefficients for ranging and for shadow matching that were each independently varied from 1.5 to 3 with a step of 0.1. Values of these parameters were then selected that minimized the root mean square (RMS) position error across all of the test sites.

### 6.4 Hypothesis-domain integration

The constant $\alpha$ used for determining the weighting of the shadow-matching hypothesis scores using (14) was determined empirically by calculating the integrated 3DMA GNSS position solution from the calibration data using values of $\alpha$ that were varied from 0 to 3 with a step of 0.1. Values of $\alpha$ for each receiver were then selected that minimized the root mean square (RMS) position error across all of the test sites.

### 7 EXPERIMENTAL RESULTS

Table 5 presents the root mean square (RMS) along-street, across-street, and horizontal (2D) position errors for each receiver and positioning method across the City of London sites, Canary Wharf sites, and all sites. The all-site results are also shown in Figure 10. In each case, the 3DMA GNSS algorithms have been tuned using data from all sites. Comparing the different positioning methods, it can be seen that the likelihood-based 3DMA GNSS ranging solution is significantly more accurate than conventional GNSS positioning in all cases. Shadow matching is slightly more accurate than 3DMA GNSS ranging in the across-street direction. However, in the along-street direction, it is less accurate than
TABLE 5  RMS-position errors across all test sites (tuned for all sites)

<table>
<thead>
<tr>
<th>Receiver</th>
<th>Method</th>
<th>Along-Street RMS Error, m</th>
<th>Across-Street RMS Error, m</th>
<th>Horizontal RMS Error, m</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>City</td>
<td>Canary Wharf</td>
<td>All</td>
</tr>
<tr>
<td>Leica</td>
<td>Conventional</td>
<td>4.8</td>
<td>4.8</td>
<td>4.8</td>
</tr>
<tr>
<td>Viva</td>
<td>Shadow Matching</td>
<td>5.3</td>
<td>6.2</td>
<td>5.8</td>
</tr>
<tr>
<td></td>
<td>Likelihood-based 3DMA ranging</td>
<td>1.3</td>
<td>1.6</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>Position-domain integration</td>
<td>2.4</td>
<td>2.5</td>
<td>2.4</td>
</tr>
<tr>
<td></td>
<td>Hypothesis-domain integration</td>
<td>1.6</td>
<td>1.7</td>
<td>1.7</td>
</tr>
<tr>
<td>u‐blox</td>
<td>Conventional</td>
<td>7.7</td>
<td>11.9</td>
<td>9.9</td>
</tr>
<tr>
<td>EVK</td>
<td>Shadow Matching</td>
<td>9.3</td>
<td>15.3</td>
<td>12.6</td>
</tr>
<tr>
<td>M8T</td>
<td>Likelihood-based 3DMA ranging</td>
<td>1.2</td>
<td>3.0</td>
<td>2.3</td>
</tr>
<tr>
<td></td>
<td>Position-domain integration</td>
<td>2.1</td>
<td>3.9</td>
<td>3.1</td>
</tr>
<tr>
<td></td>
<td>Hypothesis-domain integration</td>
<td>1.4</td>
<td>3.2</td>
<td>2.5</td>
</tr>
<tr>
<td>Nexus 9</td>
<td>Conventional</td>
<td>11.3</td>
<td>20.3</td>
<td>16.6</td>
</tr>
<tr>
<td>tablet</td>
<td>Shadow matching</td>
<td>12.4</td>
<td>25.8</td>
<td>20.7</td>
</tr>
<tr>
<td></td>
<td>Likelihood-based 3DMA ranging</td>
<td>2.1</td>
<td>3.0</td>
<td>2.6</td>
</tr>
<tr>
<td></td>
<td>Position-domain integration</td>
<td>3.0</td>
<td>3.8</td>
<td>3.4</td>
</tr>
<tr>
<td></td>
<td>Hypothesis-domain integration</td>
<td>2.3</td>
<td>3.1</td>
<td>2.8</td>
</tr>
</tbody>
</table>

FIGURE 10  RMS position errors using data from all receivers across all test sites (tuned for all sites). Conv is the conventional GNSS positioning solution, SM is the shadow matching solution, LBR is the likelihood-based 3DMA GNSS ranging solution, PDI is the position-domain integrated SM and LBR solution, and HDI is the hypothesis-domain integrated SM and LBR solution [Color figure can be viewed at wileyonlinelibrary.com and www.ion.org]
conventional GNSS positioning. Comparing the two integrated solutions, hypothesis-domain integration gives a position solution that is 10% to 25% more accurate than position-domain integration. Thus, hypothesis-domain is the preferred integration approach. In the across-street direction and overall, the hypothesis-domain integrated solution is the most accurate. However the 3DMA solution is most accurate in the along-street direction.

Comparing the three receivers, it can be seen that for all of the different positioning methods, the Nexus 9 tablet provides the least accurate solution, and the Leica Viva provides the most accurate position solution. The tablet results are due to the inferior characteristics of its antenna, affecting shadow matching the most. Conventional positioning is least affected because it is dominated by the NLOS ranging errors that 3DMA positioning helps to minimize; these are not affected by the antenna design. The Leica Viva results are better than the u-blox results due to the higher quality antenna and higher bandwidth receiver. Both enable greater multipath mitigation, improving both the conventional and likelihood-based 3DMA GNSS ranging solutions.

Figure 11 shows separate City of London and Canary Wharf results using the u-blox receiver with the 3DMA GNSS algorithms tuned using data from all sites. All positioning algorithms give much more accurate results in the City of London than at Canary Wharf. Overall, likelihood-based 3DMA GNSS ranging is 2.2 times more accurate in the City than at Canary Wharf, while conventional GNSS positioning is 2.1 times more accurate in the City. This is likely to be for two main reasons. First, the Canary Wharf buildings are taller and further apart, so the path delay of NLOS signals is likely to be larger, resulting in commensurately larger ranging errors. Second, many of the buildings have metallized glass surfaces that are strong reflectors of GNSS signals. Consequently, multipath interference will be stronger.

Shadow matching in the City is three times more accurate in the across-street direction compared to Canary Wharf. Again, there are two likely causes. First, the stronger reflected signals make it more difficult to distinguish NLOS from direct LOS signals using $C/N_0$ measurements. Second, buildings that are taller and further apart produce larger shadows, effectively reducing the spatial resolution of shadow matching. The hypothesis-domain integrated 3DMA GNSS solution is 2.5 times more accurate overall in the City than at Canary Wharf. The effect of the environment on 3DMA GNSS performance is explored in more detail in the second part of the paper.1

Different tuning of the 3DMA GNSS algorithms to suit different environments may produce better results. To test this hypothesis, the City of London and Canary Wharf calibration data was separated. Figure 12 shows the RMS position errors of shadow matching, likelihood-based 3DMA GNSS ranging, and hypothesis-domain integrated 3DMA GNSS positioning for the City of London and Canary Wharf using only City of London calibration data. Figure 13 shows the corresponding results using only Canary Wharf calibration data. Calibrating using City data only improves the hypothesis domain-integrated 3DMA GNSS solution accuracy by 24% compared to using all-site data for calibration. Similarly, calibrating using Canary Wharf data only improves the Canary Wharf positioning accuracy by 17%. Using the calibration data from the other test area degrades positioning

![FIGURE 11](Color figure can be viewed at wileyonlinelibrary.com and www.ion.org)
performance, particularly in the City area. These results suggest that development of 3DMA GNSS algorithms that adapt to the surrounding environment is likely to be beneficial. This is also explored further in the second part of the paper.1

For all of the preceding results, candidate positions were generated and scored with a grid spacing of one meter. However, a smaller grid spacing could improve the accuracy, while a larger grid spacing will reduce the processing load. Table 6 presents the root mean square (RMS) along-street, across-street, and horizontal (2D) position errors for each receiver and positioning method across all sites with grid spacings of 0.5, 1, 2, 3, 5, and 10 m. Figure 14 shows the RMS horizontal position error for the hypothesis–domain integrated SM and LBR solution for grid spacings up to 5 m. Performance with a 10 m grid spacing was found to be poor with the integrated solution often less accurate than the conventional GNSS solution (Table 5). Thus, the maximum viable grid spacing is about 5 m. Considering all positioning methods and all directions, the position errors are about 30% larger with a 5 m grid spacing than with a 1 m spacing using the Leica Viva data, about 40% larger using the u-blox data, and about 35% larger using the Nexus data. Thus, the performance is less sensitive to the grid spacing than to the algorithm tuning. By contrast, the processing load with the 5 m grid spacing is about 25 times smaller than that with the 1 m grid spacing. Reducing the grid spacing from

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**FIGURE 12** RMS-position errors using u-blox receiver data across all City of London and all Canary Wharf sites (tuned for City of London only). SM is the shadow matching solution, LBR is the likelihood-based 3DMA GNSS ranging solution, and HDI is the hypothesis-domain integrated SM and LBR solution [Color figure can be viewed at wileyonlinelibrary.com and www.ion.org]

**FIGURE 13** RMS-position errors using u-blox receiver data across all City of London and all Canary Wharf sites (tuned for Canary Wharf only). SM is the shadow-matching solution, LBR is the likelihood-based 3DMA GNSS ranging solution, and HDI is the hypothesis domain-integrated SM and LBR solution [Color figure can be viewed at wileyonlinelibrary.com and www.ion.org]
1 to 0.5 m brings only a 4% accuracy improvement in exchange for a factor of four increase in processing load.

8 | PRACTICAL IMPLEMENTATION

There are three ways in which 3D mapping-aided GNSS could be implemented in a practical system. The first is by postprocessing recorded data. This is suited to detail surveys and generating map information such as pollution levels and wheelchair accessibility. It is also useful for monitoring the movement of people, animals, or vehicles for research purposes. Finally, post-processing is ideally suited to the development, tuning, and testing of the 3DMA GNSS algorithms themselves as it enables different configurations to be compared using exactly the same experimental data. It was therefore used for the results presented here.

The second option is real-time implementation on a remote server that can operate using existing assisted GNSS protocols for sending GNSS measurement data from a mobile device to the server. Thus it has the advantage of being compatible with any mobile device that incorporates a GNSS receiver. However, the number of position fixes that can be provided within a given time

### TABLE 6 RMS position errors (m) across all test sites (tuned for all sites) with different grid spacings

<table>
<thead>
<tr>
<th>Receiver</th>
<th>Method</th>
<th>Grid spacing, m</th>
<th>Shadow Matching</th>
<th>Likelihood-Based 3DMA Ranging</th>
<th>Hypothesis-Domain Integration</th>
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<tbody>
<tr>
<td></td>
<td>m</td>
<td></td>
<td>Along-street</td>
<td>Across-street</td>
<td>Horizontal</td>
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<tr>
<td>Leica</td>
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<td>3</td>
<td>6.4</td>
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<tr>
<td>Viva</td>
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<td>5.8</td>
<td>3</td>
<td>6.6</td>
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<tr>
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</table>

**FIGURE 14** RMS position errors for the hypothesis-domain integrated SM and LBR solution with different grid spacings across all test sites (tuned for all sites) using data from all receivers [Color figure can be viewed at wileyonlinelibrary.com and www.ion.org]
interval is limited by the processing capacity of the server. Therefore, this approach is most suited to location-based services that only require a one-time position fix, emergency caller location, and tracking applications with long update intervals.

For consumer and professional navigation and continuous tracking applications, implementation of the 3DMA GNSS algorithms on the mobile device itself is the most efficient approach. This requires algorithms that are efficient enough to run in real-time on a mobile device with real-time access to GNSS pseudorange and C/N₀ (or other SNR) measurements from the device’s application processor. Access to 3D mapping data is also required. This can be preloaded onto the device, particularly if it operates within a limited area. However, unconstrained operation requires the mapping data to be streamed to the device when it is needed. This should be feasible using third generation (3G) and higher mobile communications as discussed in another study.⁵⁹

In terms of GNSS measurement data access, survey receivers have always provided pseudorange and SNR measurements but are not practical for most 3DMA GNSS applications. Historically, consumer receivers have not output all of these measurements. However, today, receivers such as the u-blox EVK-M8T provide pseudorange and SNR measurements from all GNSS constellations, and this information is now available through the application programming interface (API) on Android smartphones that have a compatible GNSS chipset and run the Nougat or later version of the Android operating system. Compatible devices are listed at another study.⁶⁰

Predicting GNSS signal propagation using 3D mapping directly is computationally intensive. The 3DMA GNSS algorithms presented here consider several thousand candidate receiver positions. Performing ray tracing of a signal’s path for 20 satellite positions and all receiver positions can take several minutes of central processing unit (CPU) time or several seconds of graphics processing unit (GPU) time. Here, this is circumvented by using precomputed building boundaries.

For postprocessed implementation, a position solution using the algorithms presented here can be computed from a single epoch of GNSS measurement data in 233 ms on a Dell Precision M2800 laptop computer (running the Microsoft Windows 7 Professional 64-bit operating system equipped with 16 GB RAM and a quad-core processor with a 2.5 GHz base frequency). A real-time version of 3DMA GNSS, using the same algorithms and preloaded building boundary data, has been implemented on a Raspberry Pi 3 (running the Raspbian operating system based on Debian Jessie equipped with 1 GB RAM and a quad-core processor with 1.2 GHz 64 bit CPU), using measurements from a u-blox EVK-M8T GNSS receiver. This requires 410 ms to process a single epoch of GNSS measurement data. A real-time Android demonstration application has also been developed that interacts with a compatible mobile device’s GNSS sensor through the Android 7 (Nougat) framework-application programming interface.⁶⁰ This also uses preloaded building boundary data and requires 387 ms to process a single epoch of GNSS measurement data on a Samsung Galaxy S8+. The Android Native Development Kit (NDK) toolset was used to embed the existing C++ code within the application. Thus, all three implementations use the same code base to run the 3DMA GNSS algorithms, enabling modifications to be implemented on all three platforms simultaneously.

9 | CONCLUSIONS

A full assessment of integrated 3D mapping–aided GNSS algorithms, combining likelihood-based 3DMA GNSS ranging with shadow matching, has been presented. Hypothesis-domain and position-domain integration algorithms have been compared, and a 3DMA least-squares ranging algorithm is used for initialization. Following publication of preliminary results at ION GNSS+ 2016, improvements have been made to the hypothesis-domain integration and likelihood-based 3DMA GNSS ranging algorithms, a comprehensive tuning process implemented, and new experimental data collected using three different types of GNSS receiver. The algorithms have also been implemented in real time on both a Raspberry Pi 3 and a Galaxy S8+ Android smartphone, taking about 400 ms to process an epoch of data on both devices with a 1 m candidate position grid spacing.

Best performance in the along-street direction is obtained using the likelihood-based 3DMA GNSS ranging algorithm, while shadow-matching performance is no better than conventional GNSS positioning. In the across-street direction, shadow matching gives slightly better performance than likelihood-based 3DMA GNSS ranging, with best results obtained by combining them using hypothesis-domain integration. The hypothesis-domain integrated solution also gives the best overall horizontal position accuracy. Position-domain integration gives slightly poorer results than likelihood-based 3DMA GNSS ranging on its own. These trends are consistent across all of the receivers. For all positioning methods, the Leica Viva gives better results than the u-blox receiver, which, in turn, gives better results than the Nexus 9 Android tablet. This is largely due to the difference in antenna design. The RMS horizontal position errors using the Leica, u-blox, and Nexus receivers with a 1 m grid spacing are 3.5, 4.7, and 4.9 m, respectively,
compared with 23.6, 26.4, and 31.0 m using conventional GNSS positioning, about a factor of six improvement.

All positioning methods are approximately twice as accurate in the City of London, a traditional European city environment, than in the Canary Wharf district, a modern urban environment. This is because the Canary Wharf buildings are taller, further apart, and reflect GNSS signals more strongly than most City of London buildings. Better performance is obtained in both districts by calibrating the 3DMA GNSS algorithms using data from only that environment. Therefore, further development of the algorithms to account for environmental variation is likely to be beneficial.

Finally, the impact of varying the grid spacing of the candidate positions has been assessed. The maximum viable grid spacing is about 5 m. Compared with a 1 m grid spacing, this reduces the position accuracy by 30% to 40% and reduces the processing load by about a factor of 25.

10 | PART TWO

The second part of this paper begins with an explanation of how the environment affects 3DMA GNSS performance, considering conventional GNSS error sources, the building geometry, building materials, and environmental complexity. Experimental results are then presented showing how 3DMA GNSS performance is affected by the surrounding environment, including sky visibility, building height and street width, building materials, and passing vehicles. This is followed by a discussion of the impact of the mapping data quality on 3DMA GNSS, including further experimental results. Recommendations are then made for improvements to the design of the 3DMA GNSS algorithms to improve accuracy and resilience.

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