Supplemental Material for

”Topological phase transitions driven by strains in monolayer tellurium”

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The SOC energy bands for structures with different strains in BS phase I are shown below.

Fig. S1 Energy bands for structures with different strains in BS phase I, when considering SOC interaction. (a) When $\Delta = -5\%$, the band gap is 0.016 eV. (b) When $\Delta = -6\%$, the band gap is 0.048 eV. (c) When $\Delta = -7\%$, the band gap is 0.08 eV.
In equation (1), \( H(k) = A k_x + B k_y + (a k_x + c k_y) \sigma_y + (b k_x + d k_y) \sigma_x + m \sigma_z \).

\( m \) is the mass term which opens the band gap. Other parameters near the crossing points are listed below.

Table S1: Parameters used in equation (1) for BS structures with different strains and BS Te/SrTiO\(_3\), respectively.

<table>
<thead>
<tr>
<th>Strain</th>
<th>A (10^5 m/s)</th>
<th>B (10^5 m/s)</th>
<th>( \sqrt{a^2 + b^2} ) (10^5 m/s)</th>
<th>( \sqrt{c^2 + d^2} ) (10^5 m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4.5%</td>
<td>8.30</td>
<td>8.27</td>
<td>1.06</td>
<td>1.16</td>
</tr>
<tr>
<td>-5%</td>
<td>8.36</td>
<td>8.39</td>
<td>0.96</td>
<td>1.05</td>
</tr>
<tr>
<td>-6%</td>
<td>8.42</td>
<td>8.41</td>
<td>0.85</td>
<td>0.93</td>
</tr>
<tr>
<td>-7%</td>
<td>8.50</td>
<td>8.50</td>
<td>0.73</td>
<td>0.77</td>
</tr>
<tr>
<td>BS Te/SrTiO(_3)</td>
<td>7.98</td>
<td>7.96</td>
<td>0.99</td>
<td>1.03</td>
</tr>
</tbody>
</table>

To estimate the accuracy of the parameters, the parameters are fit to the first-principles energy bands of BS structures with various strains and BS Te/SrTiO\(_3\). The red dots are parameter fitting bands, while black lines are energy bands from first-principles calculations.

Fig. S2 (a) \( \Delta = -4.5\% \), (b) \( \Delta = -5\% \), (c) \( \Delta = -6\% \), (d) \( \Delta = -7\% \), (e) BS Te/SrTiO\(_3\)