
fig 1 Question LA1:
Joe and Fred are thinking about the pair of numbers 3 and \(11\).

They notice that the **SUM** \((3 + 11)\) is **EVEN**.

They notice that the **PRODUCT** \((3 \times 11)\) is **ODD**.

Joe says: If the **SUM** of two whole numbers is **EVEN**, their **PRODUCT** is **ODD**.

Fred says: If the **PRODUCT** of two whole numbers is **ODD**, their **SUM** is **EVEN**.

a) Are Joe’ s and Fred’ s statements saying the same thing?  

b) The **PRODUCT** of two whole numbers is 1271.

Suppose Fred is right.

Which one of these must also be right? Tick (\(\square\)) one box.

- You can be sure that the **SUM** of the two numbers is **EVEN**.
- You can be sure that the **SUM** of the two numbers is **ODD**.
- You can't be sure whether the **SUM** is **ODD** or **EVEN** until you know what the two numbers are.

c) Is Joe’ s statement true?  

Explain your answer .

d) Is Fred’ s statement true?  

Explain your answer .

fig 2 schematic diagram:

```
Data → Because ← So → Conclusion
         ↓
Since
         ↓
Warrant
         ↓
On account of
         ↓
Backing
```

fig 3 type A:

```
The propositions $p$ and $q$ are just the other way round

Data → Because ← So → Conclusion
         ↓
Since
Reversing the order of propositions makes no difference to an implication
         ↓
On account of
Both statements are true
ON
Both statements are false

```

fig 4 type B:

fig 5 type C:

**Warrant**

(If two numbers are even, their sum is even but their product is also even)

**Data**

For 10 and 8, say, the sum is 18 (even, as required) and the product is 80 (even, not as required)

**Conclusion/Data**

Because

So

**Warrant**

If there are numbers which make \( p \) true and \( q \) false then \( p \Rightarrow q \) is false

**Data**

There are numbers which make \( p \) true and \( q \) false

**Conclusion/Data**

Since

So

**Warrant**

Joe's statement is false (see above) and Fred's statement is true (see below)

**Conclusion**

Because

So

**Data**

For 3 and 3, say, the product is 9 (odd, as required) and the sum is 6 (even, as required)

**Conclusion/Data**

Because

So

**Warrant**

There are numbers which make \( q \) true and \( p \) true

**Data**

If the product of two whole numbers is odd, then the numbers are odd (so their sum is even)

**Conclusion/Data**

Since

So

**Warrant**

If the numbers which make \( q \) true make \( p \) true then \( q \Rightarrow p \) is true

fig 6 type D:

**The propositions \( p \) and \( q \) are the other way round**

**Data**

Because

So

**Conclusion**

If \( p \Rightarrow q \) is not the same as \( q \Rightarrow p \)

On account of

Joe's statement is false and Fred's statement is true

**Conclusion**